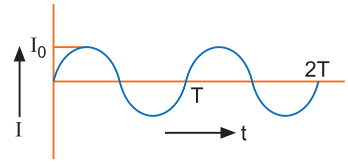
Chapter –7

Alternating Current

basic concepts

1. Alternating Current



Alternating current is the one which changes in magnitude continuously and in direction periodically. The maximum value of current is called **current-amplitude or peak value of current**.

It is expressed as

$$I = I_0 \sin \omega t$$

Similarly alternating voltage (or emf) is

$$V = V_0 \sin \omega t$$

2. Mean and RMS Value of Alternating Currents

The mean value of alternating current over complete cycle is **zero**

$$(I_{mean})_{full\ cycle} = 0$$

While for half cycle it is

$$(I_{mean})_{half\ cycle} = \frac{2I_0}{\pi} = 0.636 I_0$$

$$V_{av} = \frac{2V_0}{\pi} = 0.636\, V_0$$

An electrical device reads root mean square value as

$$I_{rms} = \sqrt{(I^2)_{mean}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0\, ; V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

3. Phase Difference between Voltage and Current

In a circuit having a reactive component, there is always a phase difference between applied voltage and the alternating current.

If $E = E_0 \sin \omega t$

Current is $I = I_0 \sin (\omega t + \phi)$

where ϕ is the phase difference between voltage and current.

4. Impedance and Reactance

Impedance: The opposition offered by an electric circuit to an alternating current is called impedance. It is denoted as Z. Its unit is ohm.

$$Z = \frac{V}{I} = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$$

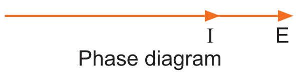
Reactance: The opposition offered by inductance and capacitance or both in *ac* circuit is called reactance. It is denoted by X_C or X_L.

The opposition due to inductor alone is called the inductive reactance while that due to capacitance alone is called the capacitive reactance.

Inductive reactance, $X_L = \omega L$

Capacitive reactance, $X_C = \dfrac{1}{\omega C}$

5. Purely Resistive Circuit



If a circuit contains pure resistance, then phase difference $\phi=0$ *i.e.*, current and voltage are in the same phase.

Impedance, $Z = R$

6. Purely Inductive Circuit

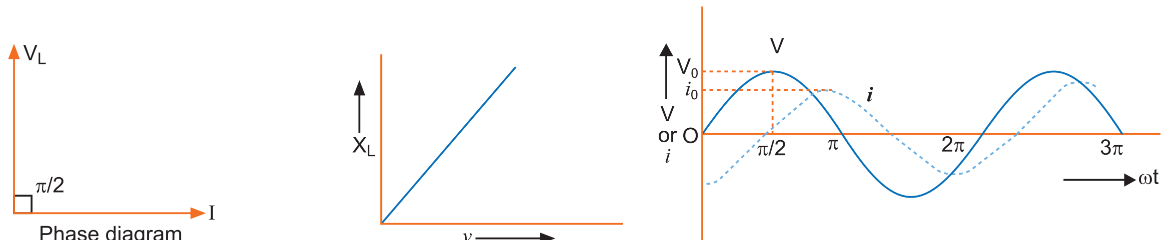
If a circuit contains pure inductance, then $\phi = -\dfrac{\pi}{2}$, *i.e.*, current lags behind the applied voltage by an angle $\dfrac{\pi}{2}$.

i.e., If $V = V_0 \sin \omega t$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

In this case inductive reactance, $X_L = \omega L$

The inductive reactance increases with the increase of frequency of AC linearly (fig. b).



(a) (b) (c)

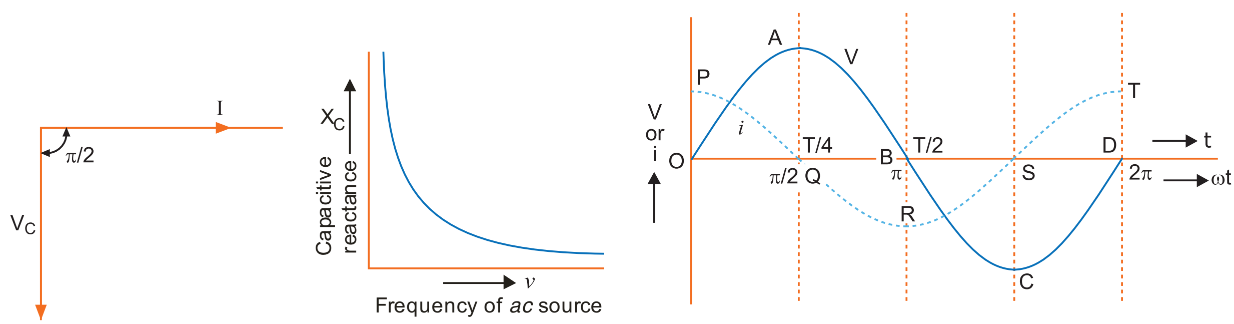
7. Purely Capacitive Circuit

If circuit contains pure capacitance, then $\phi = \dfrac{\pi}{2}$, *i.e.*, current leads the applied voltage by angle $\dfrac{\pi}{2}$ *i.e.*,

$$V = V_0 \sin\omega t, I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Capacitance reactance, $X_C = \dfrac{1}{\omega C}$

Clearly capacitance reactance (X_C) is inversely proportional to the frequency ν (fig. b).



(a) (b) (c)

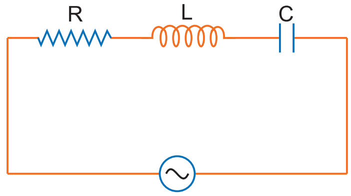
8. LC Oscillations

A circuit containing inductance L and capacitance C is called an LC circuit. If capacitor is charged initially and *ac* source is removed, then electrostatic energy of capacitor ($q_0^2/2C$) is converted into magnetic energy of inductor $\left(\frac{1}{2}LI^2\right)$ and vice versa periodically; such oscillations of energy are called LC oscillations. The frequency is given by

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow 2\pi\nu = \frac{1}{\sqrt{LC}}$$

9. Series LCR Circuit

If a circuit contains inductance L, capacitance C and resistance R, connected in series to an alternating voltage $V = V_0 \sin \omega t$

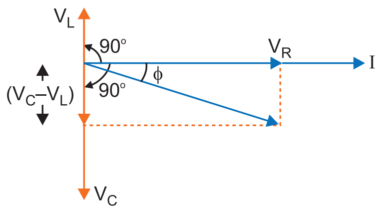


(a)

then impedance $Z = \sqrt{R^2 + (X_C - X_L)^2}$

and phase $\phi = \tan^{-1}\frac{X_C - X_L}{R}$

Net voltage $V = \sqrt{V_R^2 + (V_C - V_L)^2}$



(b)

10. Resonant Circuits

Series LCR circuit: In series LCR circuit, when phase (ϕ) between current and voltage is zero, the circuit is said to be resonant circuit. In resonant circuit $X_C = X_L$ or $\frac{1}{\omega C} = \omega L$

$$\Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}$$

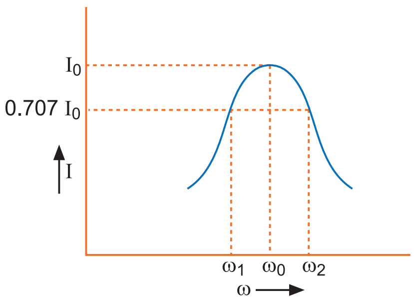
Resonant angular frequency $\omega_r = \frac{1}{\sqrt{LC}}$

(linear) frequency, $\nu_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

At resonant frequency $\phi = 0$, $V = V_R$

Quality factor (Q)

The quality factor (Q) of series LCR circuit is defined as the ratio of the resonant frequency to frequency band width of the resonant curve.



$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$$

Clearly, smaller the value of R, larger is the quality factor and sharper the resonance. Thus quality factor determines the nature of sharpness of resonance. It has no unit.

11. Power Dissipation in AC Circuit is

$$P = V_{rms} I_{rms} \cos\phi = \frac{1}{2} V_0 I_0 \cos\phi$$

where $\cos\phi = \frac{R}{Z}$ is the power factor.

For maximum power

$\cos\phi = 1$ or $Z = R$

i.e., circuit is purely resistive.

For minimum power

$\cos\phi = 0$ or $R = 0$

i.e., circuit should be free from ohmic resistance.

Power loss, $P = I^2 R$

12. Wattless Current

In purely inductive or purely capacitive circuit, power loss is zero. In such a circuit, current flowing is called wattless current.

$$I_{wattless} = I\sin\phi = I\left(\frac{X_C}{Z}\right) = I\left(\frac{X_L}{Z}\right)$$

13. AC Generator

It is a device used to convert mechanical energy into electrical energy and is based on the phenomenon of electromagnetic induction. If a coil of N turns, area A is rotated at frequency ν in uniform magnetic field of induction B, then motional emf in coil (if initially it is perpendicular to field) is

$$\varepsilon = NBA\,\omega\,\sin\omega t \text{ with } \omega = 2\pi\nu$$

Peak emf, $\varepsilon_0 = NBA\,\omega$

14. Transformer

A transformer is a device which converts low *ac* voltage into high *ac* voltage and vice versa. It works on the principle of mutual induction. If N_p and N_S are the number of turns in primary and secondary coils, V_P and I_P are voltage and current in primary coil, then voltage (V_S) and current (I_S) in secondary coil will be

$$V_S = \left(\frac{N_S}{N_P}\right)V_P \quad \text{and} \quad I_S = \left(\frac{N_P}{N_S}\right)I_P$$

Step up transformer increases the voltage while step down transformer decreases the voltage.

In step up transformer $V_S > V_P$ so $N_S > N_P$

In step down transformer $V_S < V_P$ so $N_S < N_P$

Energy Losses and Efficiency of a Transformer

(*i*) **Copper Losses:** When current flows in primary and secondary coils, heat is produced. The power loss due to Joule heating in coils will be i^2R where R is resistance and i is the current.

(*ii*) **Iron Losses (Eddy currents):** The varying magnetic flux produces eddy currents in iron-core, which leads to dissipation of energy in the core of transformer. This is minimised by using a laminated iron core or by cutting slots in the plate.

(*iii*) **Flux Leakage:** In actual transformer, the coupling of primary and secondary coils is never perfect, *i.e.*, the whole of magnetic flux generated in primary coil is never linked up with the secondary coil. This causes loss of energy.

(*iv*) **Hysteresis Loss:** The alternating current flowing through the coils magnetises and demagnetises the iron core repeatedly. The complete cycle of magnetisation and demagnetisation is termed as hysteresis. During each cycle some energy is dissipated. However, this loss of energy is minimised by choosing silicon-iron core having a thin hysteresis loop.

(*v*) **Humming Losses:** Due to the passage of alternating current, the core of transformer starts vibrating and produces humming sound. Due to this a feeble part of electrical energy is lost in the form of humming sound.

On account of these losses the output power obtained across secondary coil is less than input power given to primary. Therefore, the efficiency of a practical transformer is always less than 100%.

$$\text{Percentage efficiency of transformer} = \frac{\text{output power obtained from secondary}}{\text{input power given to primary}} \times 100\%$$

$$= \frac{V_S i_S}{V_P i_P} \times 100\%$$

Selected NCERT Textbook Questions

AC Circuit

Q. 1. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply:

(*a*) What is the rms value of current in the circuit?

(*b*) What is the net power consumed over a full cycle?

Ans. The given voltage of 220 V is the rms or effective voltage.

Given $V_{rms} = 220$ V, $\nu = 50$ Hz, $R = 100\ \Omega$

(*a*) RMS value of current $I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = \mathbf{2.2\ A}$

(*b*) Net power consumed $P = I^2_{\text{rms}} R = (2.20)^2 \times 100 = \mathbf{484\ W}$

Q. 2. (*a*) The peak voltage of an ac supply is 300 V. What is the rms voltage?

(*b*) The rms value of current in an ac circuit is 10 A. What is the peak current?

Ans. (*a*) Given $V_0 = 300$ V

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \approx \mathbf{212\ V}$$

(*b*) Given $I_{rms} = 10$ A

$$I_0 = I_{rms}\sqrt{2} = 10 \times 1.41 = \mathbf{14.1\ A}$$

Q. 3. (*a*) A 44 mH inductor is connected to 220 V, 50 Hz *ac* supply. Determine the rms value of current in the circuit. ***[CBSE (AI) 2013, 2012]***

(*b*) What is the net power absorbed by the circuit in a complete cycle?

Ans. (*a*) Given $L = 44$ mH $= 44 \times 10^{-3}$ H, $V_{rms} = 220$ V, $\nu = 50$ Hz

Inductive reactance of current $X_C = \omega L$

$\therefore$ RMS value of current, $I_{rms} = \frac{V_{rms}}{\omega L} = \frac{V_{rms}}{2\pi\nu L}$

$$= \frac{220}{2 \times \left(\frac{22}{7}\right) \times 50 \times 44 \times 10^{-3}} = \frac{220 \times 7 \times 10^3}{2 \times 22 \times 50 \times 44} = \frac{700}{44} = \mathbf{15.9\ A}$$

(*b*) $P = V_{rms} . I_{rms} . \cos\phi$

In pure inductor circuit $\phi = \frac{\pi}{2}$ radians $\Rightarrow$ $\cos\frac{\pi}{2} = 0$

As such net power consumed $= V_{rms} I_{rms} \cos\frac{\pi}{2} = \mathbf{0}$

Q. 4. (*a*) A 60 μF capacitor is connected to a 110 V, 60 Hz *ac* supply. Determine the rms value of current in the circuit.

(*b*) What is the net power absorbed by the circuit in a complete cycle?

Ans. (*a*) Given $C = 60\ \mu F = 60 \times 10^{-6}$ F, $V_{rms} = 110$ V, $\nu = 60$ Hz

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

RMS value of current, $I_{rms} = \frac{V_{rms}}{X_C} = 2\pi\nu C V_{rms}$

$$= 2 \times 3.14 \times 60 \times (60 \times 10^{-6}) \times 110\ \text{A} = \mathbf{2.49\ A}$$

(*b*) In a purely capacitive circuit, the current leads the applied p.d. by an angle $\frac{\pi}{2}$, therefore,

$$\cos\phi = \cos\frac{\pi}{2} = 0$$

$$\therefore \quad P_{av} = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \cos\frac{\pi}{2} = \mathbf{0}$$

i.e., in purely capacitive circuit the power absorbed by the circuit is zero.

Q. 5. A light bulb is rated 100 W for 220 V *ac* supply of 50 Hz. Calculate

(*a*) the resistance of the bulb;

(*b*) the rms current through the bulb.

Ans. (*a*) $R = \frac{V_{rms}^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$

(*b*) $I_{rms} = \frac{P}{V_{rms}} = \frac{100}{220} = \mathbf{0.45\ A}$

LR Circuit

Q. 6. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz *ac* supply.

(*a*) What is the maximum current in the coil?

(*b*) What is the time lag between the voltage maximum and the current maximum?

Ans. Given $L = 0.50$ H, $R = 100\ \Omega$, $V = 240$ V, $\nu = 50$ Hz

(*a*) Maximum (or peak) voltage $V_0 = V\sqrt{2}$

Maximum current, $I_0 = \frac{V_0}{Z}$

Inductive reactance, $X_L = \omega L = 2\pi\nu L$

$$= 2 \times 3.14 \times 50 \times 0.50 = 157\ \Omega$$

Impedance of circuit, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (157)^2} = 186.14\ \Omega$

$\therefore$ Maximum current $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{240 \times 1.41}{186.14}$ $\mathbf{= 1.82\ A}$

(*b*) Phase (lag) angle ϕ is given by

$$\tan\phi = \frac{X_L}{R} = \frac{157}{100} = 1.57$$

$$\therefore \quad \phi = \tan^{-1}(1.57) = 57.5°$$

Time lag $\Delta T = \frac{\phi}{2\pi} \times T = \frac{\phi}{2\pi} \times \frac{1}{\nu} = \frac{57.5}{360} \times \frac{1}{50}$ s

$$= 3.2 \times 10^{-3} \text{ s} = \mathbf{3.2\ ms}$$

Q. 7. In above prob., if the circuit is connected to a high frequency supply (240 V, 10 kHz); find :

(*a*) The maximum current in the coil.

(*b*) The time lag between the voltage maximum and the current maximum.

(*c*) Hence explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Ans. Here $R = 100\ \Omega$, $L = 0.50$ H, V=240V, $\nu = 10 \times 10^3$ Hz

(*a*) Inductive reactance $X_L = \omega L$

$$= 2\pi\nu L = 2 \times 3.14 \times (10 \times 10^3) \times 0.50 \text{ ohm} = 3.14 \times 10^4\ \Omega$$

Impedance of circuit $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{(100)^2 + (3.14 \times 10^4)^2} \approx 3.14 \times 10^4\ \Omega$$

Maximum current, $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{240 \times 1.41}{3.14 \times 10^4}$ A

$= 107 \times 10^{-4}$ A = **10.7 mA**

(*b*) Phase lag $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{3.14 \times 10^4}{100}\right) = \tan^{-1} 314 = 89.8^\circ \approx \boldsymbol{\frac{\pi}{2}}$

(*c*) Maximum current in high frequency circuit is much smaller than that in low frequency circuit; this implies that at high frequencies an inductor behaves like an open circuit.

In a *dc* circuit after steady state $\omega = 0$, so, $X_L = \omega_L = 0$, *i.e.*, inductor offers no hindrance and hence it acts as a pure conductor.

LC Circuit

Q. 8. (*a*) A charged 30 μF capacitor having initial charge 6 mC is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

(*b*) What is the total energy stored in the circuit initially? What is the total energy at later time?

Ans. Given $C = 30\ \mu\text{F} = 30 \times 10^{-6}$ F, $L = 27$ mH $= 27 \times 10^{-3}$ H

Initial Charge $q_0 = 6$ mC $= 6 \times 10^{-3}$ C

(*a*) Angular frequency of free oscillations

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(27 \times 10^{-3} \times 30 \times 10^{-6})}}$$

$$= \frac{10^4}{9} = \mathbf{1.1 \times 10^3\ rad/s}$$

(*b*) Initial energy stored in circuit = Initial energy stored in capacitor $= \frac{q_0^2}{2C} = \frac{(6 \times 10^{-3})^2}{2 \times 30 \times 10^{-6}} = \mathbf{0.6\ J}$

Energy is lost only in resistance.

As circuit is free from ohmic resistance; so the total energy at later time remains **0.6 J**.

Q. 9. A radio can tune over the frequency range of a portion of medium wave (MW) broadcast band (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of variable capacitor?

Ans. Given $\nu_1 = 800$ kHz $= 800 \times 10^3$ Hz, $\nu_2 = 1200$ kHz $= 1200 \times 10^3$ Hz

$L = 200\ \mu\text{H} = 200 \times 10^{-6}$ H

$C_1 = ?$, $C_2 = ?$

The natural frequency of LC circuit is

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

i.e.,
$$C = \frac{1}{4\pi^2\nu^2 L}$$

For $\nu = \nu_1 = 800 \times 10^3$ Hz,

$$C_1 = \frac{1}{4 \times (3.14)^2 \times (800 \times 10^3)^2 \times 200 \times 10^{-6}}\ \text{F} = 198.09 \times 10^{-12}\ \text{F} \approx \mathbf{198\ pF}$$

For $\nu = \nu_2 = 1200 \times 10^3$ Hz

$$C_2 = \frac{1}{4 \times (3.14)^2 \times (1200 \times 10^3)^2 \times 200 \times 10^{-6}} \approx \mathbf{88\ pF}$$

The variable capacitor should have a range of about **88 pF to 198 pF**.

Q. 10. An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant when the circuit is closed be *t* = 0.

(*a*) What is the total energy stored initially? Is it conserved during LC oscillations?

(*b*) What is the natural frequency of the circuit?

(c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

(d) At what time the total energy stored equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Ans. Given $L = 20$ mH $= 20 \times 10^{-3}$ H, C = 50 μF = 50×10^{-6} F, $q_0 = 10$ mC $= 10 \times 10^{-3}$ C

(a) Total energy stored initially $= \dfrac{q_0^2}{2C} = \dfrac{(10\times10^{-3})^2}{2\times50\times10^{-6}}$J = **1.0 J**

Yes, the total energy is conserved during *LC* oscillations (because circuit is free from ohmic resistance).

(b) Angular frequency of circuit, $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{20\times10^{-3}\times50\times10^{-6}}} = 10^3$ rad/s

Natural linear frequency, $\nu = \dfrac{\omega}{2\pi} = \dfrac{10^3}{2\times3.14}$ = **159 Hz**

(c) When circuit is closed at $t = 0$ then equation of charge on capacitor is $q = q_0 \cos \omega t$

(i) Energy is completely electrical when $q = q_0$ *i.e.*, when $\cos \omega t = \pm 1$ or $\omega t = r\pi$ where $r = 0, 1, 2, 3, ...$

$$t = \frac{r\pi}{\omega}, T = \frac{2\pi}{\omega} \text{ or } \omega = \frac{2\pi}{T},$$

$$t = \frac{r\pi}{2\pi/T} = r.\frac{T}{2}, (r = 0, 1, 2, 3, ...)$$

$$i.e.,\ t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...,$$

(ii) Energy is completely magnetic when electrical energy is zero,

i.e., when $\cos \omega t = 0$ or $\omega t = (2r+1)\dfrac{\pi}{2}, r = 0, 1, 2, ...$

$$t = (2r+1)\frac{\pi}{2\omega} = (2r+1)\frac{\pi}{2(2\pi/T)} = (2r+1)\frac{T}{4} \quad (r = 0, 1, 2, ...)$$

or $$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, ...$$

(d) Energy is equally divided between inductor and capacitor, when half the energy is electrical. Let charge, in this state, be q, then

$$\frac{q^2}{2C} = \frac{1}{2}\frac{q_0^2}{2C} \quad \Rightarrow \quad q = \pm\frac{q_0}{\sqrt{2}}$$

$$\Rightarrow \quad q_0 \cos \omega t = \pm\frac{q_0}{\sqrt{2}} \quad \text{or} \quad \cos \omega t = \pm\frac{1}{\sqrt{2}}$$

or $$\omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, ...$$

or $$\frac{2\pi}{T}t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, ... \quad \text{or} \quad t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$$

(e) When R is inserted in the circuit, the oscillations become damped and in each oscillation some energy is dissipated as heat. As time passes, the whole of the initial energy (1.0 J) is eventually dissipated as heat.

LCR Circuit

Q. 11. A series *LCR* circuit with $R = 20\ \Omega$, $L = 1.5$ H and $C = 35$ μF is connected to a variable frequency 200 V *ac* supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Ans. When frequency of supply is equal to natural frequency of circuit, then resonance is obtained. At resonance $X_C = X_L$

Impedance $Z = \sqrt{R^2 + (X_C - X_L)^2} = R = 20\ \Omega$

Current in circuit, $I_{rms} = \frac{V_{rms}}{R} = \frac{200}{20} = 10$ A

Power factor $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

Average power $\overline{P} = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} = 200 \times 10 = 2000$ W=**2 kW**

Q. 12. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? (Average implies average over one cycle).

Ans. Given V=230 V, ν =50 Hz, L =80 mH = 80 × 10^{-3} H, C = 60 μF = 60 ×10^{-6} F

(a) Inductive reactance $X_L = \omega L = 2\pi\nu L$

$$=2 \times 3.14 \times 50 \times 80 \times 10^{-3} = 25.1\ \Omega$$

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.1\,\Omega$

Impedance, Z= Net reactance $\left|\frac{1}{\omega C} - \omega L\right| = 53.1 - 25.1 = 28.0\ \Omega$

Current amplitude $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{230 \times 1.41}{28.0} = \mathbf{11.6\ A}$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{11.6}{1.41} = \mathbf{8.23\ A}$$

(b) *RMS* value of potential drops across L and C are $V_L = X_L I_{rms} = 25.1 \times 8.23 = \mathbf{207\ V}$

$V_C = X_C I_{rms} = 53.1 \times 8.23 = \mathbf{437\ V}$

Net voltage = $V_C - V_L$ = 230 V

(c) The voltage across L leads the current by angle $\frac{\pi}{2}$ therefore, average power

$$P_{av} = V_{rms} I_{rms} \cos\frac{\pi}{2} = \mathbf{0\ (zero)}$$

(d) The voltage across C lags behind the current by angle $\frac{\pi}{2}$.

$$\therefore \quad P_{av} = V_{rms} I_{rms} \cos\frac{\pi}{2} = \mathbf{0}$$

(e) As circuit contains pure L and pure C, average power consumed by LC circuit is **zero**.

Q. 13. A circuit containing a 80 mH inductor, a 60 μF capacitor and a 15 Ω resistor are connected to a 230 V, 50 Hz supply. Obtain the average power transferred to each element of the circuit and total power absorbed.

Ans. Given L= 80 mH = 80 × 10^{-3} H, C = 60 μF =60 ×10^{-6} F, R = 15 Ω, V_{rms} = 230 V, ν=50 Hz

Inductive reactance $X_L = \omega L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 80 \times 10^{-3} = 25.1\ \Omega$

Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\times3.14\times50\times60\times10^{-6}} = 53.1\,\Omega$

Impedance of circuit $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$= \sqrt{(15)^2 + (53.1-25.1)^2} = \sqrt{(15)^2 + (28)^2} = 31.8\,\Omega$$

RMS current, $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{31.8} = 7.23$ A

Average power transferred to resistance $= I^2_{\text{rms}}R = (7.23)^2 \times 15 =$ **784 W**

Average power transferred to inductor = Average power transferred to capacitor

$$= V_{rms}\, I_{rms} \cos\frac{\pi}{2} = \textbf{zero}$$

Total power absorbed ≅ **784 W**

Q. 14. A series LCR circuit with $L = 0.12$ H, $C = 480$ nF, $R = 23\ \Omega$ is connected to a 230 V variable frequency supply.

(*a*) What is the source frequency for which current amplitude is maximum? Obtain the maximum value.

(*b*) What is the source frequency for which average power observed by the circuit is maximum? Obtain the value of this maximum power.

(*c*) For which frequencies of the source is the power transferred to circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(*d*) What is the Q-factor of the given circuit?

Ans. Given : $L = 0.12$ H, $C = 480$ nF $= 480 \times 10^{-9}$ F, $R = 23\ \Omega$, $V_{rms} = 230$ V

(*a*) Current amplitude $= \dfrac{V_0}{Z} = \dfrac{V_{rms}\sqrt{2}}{\sqrt{R^2 + (X_C - X_L)^2}}$

Clearly current amplitude is maximum when $X_C - X_L = 0$

$$\Rightarrow \quad X_C = X_L$$

$$\Rightarrow \quad \frac{1}{\omega C} = \omega L \quad \text{or} \quad \omega = \frac{1}{\sqrt{LC}}.\ \text{This is resonant frequency.}$$

$$\text{Resonant frequency} \quad \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.12 \times 480 \times 10^{-9})}} = \frac{10^5}{24} = 4.167 \times 10^3 \ \text{rad / s}$$

$$\text{Resonant linear frequency,}\ \nu_r = \frac{\omega_r}{2\pi} = \frac{4.167 \times 10^3}{2 \times 3.14} = \textbf{663 Hz}$$

At resonant frequency $Z = R$

(*b*) Average power, $\overline{P} = V_{rms}\ I_{rms} \cos\phi$

$$\text{For maximum power,}\ \cos\phi = 1; I_{rms} = \frac{V_{rms}}{R} = \frac{230}{23} = 10 \text{ A}$$

$$\therefore \quad \overline{P}_{\max} = V_{rms}\ I_{rms} = 230 \times 10 = \textbf{2300 watt}$$

(*c*) Power absorbed, $P = \dfrac{1}{2} \times$ maximum power

$$I^2 R = \frac{1}{2} I^2_{rms} R \Rightarrow I = \frac{I_{rms}}{\sqrt{2}}$$

$$\frac{V_{rms}}{\sqrt{R^2 + \left(\dfrac{1}{\omega C} - \omega L\right)^2}} = \frac{1}{\sqrt{2}} \frac{V_{rms}}{R}$$

$$\Rightarrow \quad R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2 = 2R^2 \Rightarrow \frac{1}{\omega C} - \omega L = \pm R$$

If $\omega_1 < \omega_r$, then $\dfrac{1}{\omega_1 C} - \omega_1 L = + R$ ...(*i*)

If $\omega_2 < \omega_r$, then $\dfrac{1}{\omega_2 C} - \omega_2 L = - R$ ...(*ii*)

Adding (*i*) and (*ii*),

$$\frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)-(\omega_1+\omega_2)L=0$$

$$\frac{\omega_1+\omega_2}{C\,\omega_1\omega_2}-(\omega_1+\omega_2)L=0 \quad\Rightarrow\quad \omega_1\omega_2=\frac{1}{LC} \qquad \ldots(iii)$$

As $\omega_r^2=\frac{1}{LC}\Rightarrow\omega_r=\sqrt{\omega_1\omega_2}=\frac{1}{\sqrt{LC}}$ resonant frequency.

Subtracting (*ii*) from (*i*), $\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right)\frac{1}{C}+(\omega_2-\omega_1)L=2R$

$$\frac{\omega_2-\omega_1}{\omega_1\omega_2}\cdot\frac{1}{C}+(\omega_2-\omega_1)L=2R$$

Using (*iii*), we get

$$(\omega_2-\omega_1)L+(\omega_2-\omega_1)L=2R$$

$$\Rightarrow \qquad \omega_2-\omega_1=\frac{R}{L}$$

If $\Delta\omega$ is the difference of ω_1 and ω_2 from ω_r, then $\omega_r+\Delta\omega-(\omega_r-\Delta\omega)=\frac{R}{L}$

$$\Rightarrow \qquad 2\Delta\omega=\frac{R}{L}$$

or $$\Delta\omega=\frac{R}{2L}=\frac{23}{2\times0.12}=95.8\text{ rad / s}$$

$$\Delta\nu=\frac{\Delta\omega}{2\pi}=\frac{95.8}{2\times3.14}=\mathbf{15.2\ Hz}$$

$$\therefore \qquad \nu_1=\nu_r-\Delta\nu=663-15.2=647.8\text{ Hz}$$

$$\nu_2=\nu_r+\Delta\nu=663+15.2=678.2\text{ Hz}$$

Thus, power absorbed is half the power at resonant frequency at frequencies 647.8 Hz and 678.2 Hz.

(*d*) *Q*–value of given circuit,

$$Q=\frac{\omega_r L}{R}$$

$$=\frac{4.167\times10^3\times0.12}{23}=\mathbf{21.7}$$

Q. 15. Obtain the resonant frequency ω_r of a series *LCR* circuit with L = 2.0 H, C = 32 μF and R = 10 Ω What is the quality factor (*Q*) of this circuit?

Ans. Resonant frequency, $\omega_r=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2.0\times32\times10^{-6}}}=\frac{1}{8}\times10^3=\mathbf{125\ rad\ s^{-1}}$

$$Q\text{-value of circuit}=\frac{\omega_r L}{R}=\frac{125\times2.0}{10}=\mathbf{25}$$

Transformer

Q. 16. A power transmission line needs input power at 2300 V to a step down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary windings in order to get output power at 230 V?

Ans. Given V_P= 2300 V, N_P =4000 turns, V_S =230 V, N_S =?

We have $$\frac{V_S}{V_P}=\frac{N_S}{N_P}$$

$$\Rightarrow \qquad N_S = \frac{V_S}{V_P} \times N_P = \frac{230}{2300} \times 4000 = \textbf{400 turns}$$

Q. 17. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000 V – 220 V step down transformer at a sub-station in the town. Calculate (*i*) the line power loss in the form of heat (*ii*) how much power must the plant supply, assuming there is negligible power loss due to leakage (*iii*) characterise the step up transformer at the plant.

Ans. Length of wire line $= 15 \times 2 = 30$ km

Resistance of wire line, $R = 30 \times 0.5 = 15\ \Omega$

(*i*) Power to be supplied P = 800 kW = 800×10^3 W

Voltage at which power is transmitted = 4000 V

$$P = VI \Rightarrow I = \frac{P}{V} = \frac{800 \times 10^3}{4000} = 200 \text{ A}$$

$\therefore$ Line power loss $= I^2 \times R = (200)^2 \times 15 = 6 \times 10^5$ watt = **600 kW**

(*ii*) Power that must be supplied = 800 kW + 600 kW = **1400 kW**

(*iii*) Voltage drop across to wire line $= I^2R = 200 \times 15 = 3000$ V

The plant generates power at 440 V and it has to be stepped up so that after a voltage drop of 3000 V, across the line, the power at 4000 V is received at the sub-station in the town. Therefore the output voltage is

$$3000 + 4000 = 7000 \text{ V}$$

Here step up transformer at the plant is

$$440 \text{ V} \rightarrow 7000 \text{ V}$$

Multiple Choice Questions [1 mark]

Choose and write the correct option(s) in the following questions.

1. If the *rms* current in a 50 Hz *ac* circuit is 5 A, the value of the current 1/300 seconds after its value becomes zero is [*NCERT Examplar*]

(*a*) $5\sqrt{2}$ A (*b*) $5\sqrt{\frac{3}{2}}$ A (*c*) 5/6 A (*d*) $5/\sqrt{2}$ A

2. An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to [*NCERT Examplar*]

(*a*) zero (*b*) X_g (*c*) $-X_g$ (*d*) R_g

3. In an ac circuit, the maximum value of voltage is 423 volts. Its effective voltage is

(*a*) 400 volt (*b*) 300 volt (*c*) 323 volt (*d*) 340 volt

4. The peak voltage of 220 V *ac* mains is

(*a*) 155.6 V (*b*) 220.0 V (*c*) 311 V (*d*) 440 V

5. An inductive circuit have zero resistance. When *ac* voltage is applied across this circuit, then the current lags behind the applied voltage by an angle

(*a*) 30° (*b*) 45° (*c*) 90° (*d*) 0°

6. If an *LCR* circuit contains $L = 8$ henry; $C = 0.5\ \mu$F, $R = 100\ \Omega$ in series. Then the resonant angular frequency will be:

(*a*) 600 rad/s (*b*) 500 rad/s (*c*) 600 Hz (*d*) 500 Hz

7. When a voltage measuring device is connected to *ac* mains, the meter shows the steady input voltage of 220 V. This means [*NCERT Examplar*]

(*a*) input voltage cannot be *ac* voltage, but a *dc* voltage.

(*b*) maximum input voltage is 220 V.

(*c*) the meter reads not V but $<V^2>$ and is calibrated to read $\sqrt{<V^2>}$.

(*d*) the pointer of the meter is stuck by some mechanical defect.

8. To reduce the resonant frequency in an *LCR* series circuit with a generator [*NCERT Examplar*]

(*a*) the generator frequency should be reduced.

(*b*) another capacitor should be added in parallel to the first.

(*c*) the iron core of the inductor should be removed.

(*d*) dielectric in the capacitor should be removed.

9. In a pure capacitive circuit, the current

(*a*) lags behind the applied emf by angle $\pi/2$ (*b*) leads the applied emf by an angle π

(*c*) leads the applied emf by angle $\pi/2$ (*d*) and applied emf are in same phase

10. In an *ac* circuit, the emf (ε) and the current (*i*) at any instant are given by

$$\varepsilon = E_0 \sin \omega t, \; i = I_0 \sin(\omega t - \phi)$$

Then average power transferred to the circuit in one complete cycle of *ac* is

(*a*) $E_0 I_0$ (*b*) $\frac{1}{2}E_0 I_0$ (*c*) $\frac{1}{2}E_0 I_0 \sin\phi$ (*d*) $\frac{1}{2}E_0 I_0 \cos\phi$

11. The average power dissipation in pure inductance is

(*a*) $\frac{1}{2}LI^2$ (*b*) $\frac{1}{4}LI^2$ (*c*) $2LI^2$ (*d*) zero

12. Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct? [*NCERT Examplar*]

(*a*) For a given power level, there is a lower current.

(*b*) Lower current implies less power loss.

(*c*) Transmission lines can be made thinner.

(*d*) It is easy to reduce the voltage at the receiving end using step-down transformers.

13. The reactance of a capacitance at 50 Hz is 5 Ω. If the frequency is increased to 100 Hz, the new reactance is

(*a*) 5 Ω (*b*) 10 Ω (*c*) 2.5 Ω (*d*) 125 Ω

14. In a pure inductive circuit, the current

(*a*) lags behind the applied emf by an angle π

(*b*) lags behind the applied emf by an angle $\pi / 2$

(*c*) leads the applied emf by an angle $\pi / 2$

(*d*) and applied emf are in same phase

15. When an *ac* voltage of 220 V is applied to the capacitor C [*NCERT Examplar*]

(*a*) the maximum voltage between plates is 220 V.

(*b*) the current is in phase with the applied voltage.

(*c*) the charge on the plates is in phase with the applied voltage.

(*d*) power delivered to the capacitor is zero.

16. In an *ac* circuit, voltage *V* and current *i* are given by

$$V = 100 \sin 100\,t \text{ volt}$$

$$i = 100 \sin(100t + \pi/3) \text{ mA}$$

The power dissipated in the circuit is

(*a*) 10^4W (*b*) 10 W (*c*) 2.5 W (*d*) 5 W.

17. Which of the following combinations should be selected for better tuning of an *LCR* circuit used for communication? *[NCERT Examplar]*

(*a*) $R = 20\ \Omega, L = 1.5$ H, $C = 35\ \mu$F (*b*) $R = 25\ \Omega, L = 2.5$ H, $C = 45\ \mu$F

(*c*) $R = 15\ \Omega, L = 3.5$ H, $C = 30\ \mu$F (*d*) $R = 25\ \Omega, L = 1.5$ H, $C = 45\ \mu$F

18. An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6 V (rms) *ac* source. The power dissipated in the circuit is *[NCERT Examplar]*

(*a*) 8 W (*b*) 12 W (*c*) 14.4 W (*d*) 18 W

19. The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an L-C-R circuit, the power factor for this circuit is

(*a*) 0.4 (*b*) 0.5 (*c*) 0.8 (*d*) 1.0

20. The output of a step-down transformer is measured to be 24 V when connected to a 12 watt light bulb. The value of the peak current is *[NCERT Examplar]*

(*a*) $1/\sqrt{2}$ A (*b*) $\sqrt{2}$ A (*c*) 2 A (*d*) $2\sqrt{2}$ A

Answers

1. (*b*) **2.** (*c*) **3.** (*b*) **4.** (*c*) **5.** (*c*) **6.** (*b*)
7. (*c*) **8.** (*b*) **9.** (*c*) **10.** (*d*) **11.** (*d*) **12.** (*a*), (*b*), (*d*)
13. (*c*) **14.** (*b*) **15.** (*c*), (*d*) **16.** (*c*) **17.** (*c*) **18.** (*c*)
19. (*c*) **20.** (*a*)

Fill in the Blanks [1 mark]

1. The average power supplied to an inductor over one complete cycle is ______________.
2. The inductive reactance is directly proportional to the inductance and to the ______________ of the circuit.
3. The capacitive reactance limits the ______________ in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit.
4. The phenomenon of resistance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's ______________.
5. The quantity $\frac{\omega_0}{2\Delta\omega}$ is regarded as a measure of the ______________.
6. The average power dissipated depends not only on the voltage and current but also on the ______________ of the phase angle ϕ between them.
7. For many purposes, it is necessary to change an alternating voltage from one to another of greater or smaller value. This is done with a device called ______________ using the principle of mutual induction.
8. In an ac circuit, containing pure resistance, the voltage and current are in ______________ phase.
9. In a pure inductive circuit current ______________ the voltage by a phase angle of $\frac{\pi}{2}$.
10. In a pure capacitive circuit, the current ______________ the voltage by a phase angle of $\frac{\pi}{2}$.

Answers

1. zero **2.** frequency **3.** amplitude of the current **4.** natural frequency
5. sharpness of resonance **6.** cosine **7.** transformer **8.** same **9.** lags
10. leads

Very Short Answer Questions [1 mark]

Q. 1. Define capacitor reactance. Write its SI units? *[CBSE Delhi 2015]*

Ans. The imaginary/virtual resistance offered by a capacitor to the flow of an alternating current is called capacitor reactance, $X_C = \frac{1}{\omega C}$. Its SI unit is ohm.

Q. 2. Explain why current flows through an ideal capacitor when it is connected to an *ac* source but not when it is connected to a *dc* source in a steady state. *[CBSE (East) 2016]*

Ans. For *ac* source, circuit is complete due to the presence of displacement current in the capacitor. For steady *dc*, there is no displacement current, therefore, circuit is not complete.

Mathematically, Capacitive reactance $X_C = \frac{1}{2\pi\nu C} = \frac{1}{\omega C}$

So, capacitor allows easy path for *ac* source.

For *dc*, $\nu = 0$, so X_c = infinity,

So capacitor blocks *dc*.

Q. 3. Define 'quality factor' of resonance in series *LCR* circuit. What is its SI unit? *[CBSE Delhi 2016]*

Ans. The quality factor (Q) of series *LCR* circuit is defined as the ratio of the resonant frequency to frequency band width of the resonant curve.

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$$

Clearly, smaller the value of R, larger is the quality factor and sharper the resonance. Thus quality factor determines the nature of sharpness of resonance.

It has no units.

Q. 4. In a series *LCR* circuit, $V_L = V_C \neq V_R$.

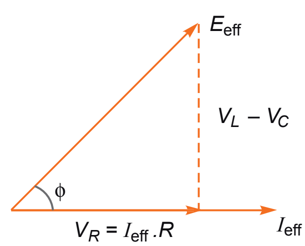
What is the value of power factor for this circuit? *[CBSE Panchkula 2015]*

Ans. Power factor,

$$\cos\phi = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}}$$

Since $V_L = V_C$; $\cos\phi = \frac{V_R}{V_R} = 1$

The value of power factor is 1.



Q. 5. The power factor of an *ac* circuit is 0.5. What is the phase difference between voltage and current in this circuit? *[CBSE (F) 2015, (South) 2016]*

Ans. Power factor between voltage and current is given by $\cos\phi$, where ϕ is phase difference

$$\cos\phi = 0.5 = \frac{1}{2} \Rightarrow \quad \phi = \cos^{-1}\left(\frac{1}{2}\right) = \boldsymbol{\frac{\pi}{3}}$$

Q. 6. What is wattless current? *[CBSE Delhi 2011, Chennai 2015]*

Ans. When pure inductor and/or pure capacitor is connected to ac source, the current flows in the circuit, but with no power loss; the phase difference between voltage and current is $\frac{\pi}{2}$. Such a current is called the wattless current.

Q. 7. Mention the two characteristic properties of the material suitable for making core of a transformer. *[CBSE (AI) 2012]*

Ans. Two characteristic properties:

(*i*) Low hysteresis loss (*ii*) Low coercivity

Q. 8. A light bulb and a solenoid are connected in series across an ac source of voltage. Explain, how the glow of the light bulb will be affected when an iron rod is inserted in the solenoid. *[CBSE (F) 2017]*

Ans. When iron rod is inserted in the coil, the inductance of coil increases; so impedance of circuit increases and hence, current in circuit $I = \dfrac{V}{\sqrt{R^2 + (\omega L)^2}}$ decreases. Consequently, the glow of bulb decreases.

Q. 9. Why is the use of *ac* voltage preferred over *dc* voltage? Give two reasons. [*CBSE (AI) 2014*]

Ans. (*i*) The generation of *ac* is more economical than *dc*.

(*ii*) Alternating voltage can be stepped up or stepped down as per requirement during transmission from power generating station to the consumer.

(*iii*) Alternating current in a circuit can be controlled by using wattless devices like the choke coil.

(*iv*) Alternating voltages can be transmitted from one place to another, with much lower energy loss in the transmission line.

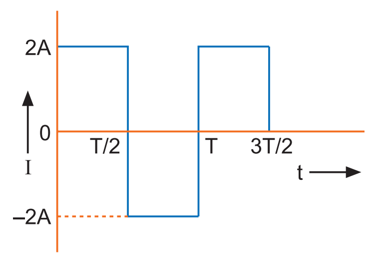
Q. 10. What is the average value of *ac* voltage

$$V = V_0 \sin \omega t$$

over the time interval $t = 0$ to $t = \dfrac{\pi}{\omega}$. [*HOTS*]

Ans. $$V_{av} = \frac{\int_0^{\pi/\omega} V dt}{\int_0^{\pi/\omega} dt} = \frac{\int_0^{\pi/\omega} V_0 \sin \omega t \, dt}{[t]_0^{\pi/\omega}} = \frac{V_0 \left\{-\dfrac{\cos\omega t}{\omega}\right\}_0^{\pi/\omega}}{\pi/\omega} = -\frac{V_0}{\pi}[\cos\pi - \cos 0] = \frac{2V_0}{\pi}$$

Q. 11. What is the rms value of alternating current shown in figure? [*HOTS*]



Ans. In given *ac*, there are identical positive and negative half cycles, so the mean value of current is zero; but the rms value is not zero.

$$(I^2)_{mean} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{\int_0^{T/2} (2)^2 dt + \int_{T/2}^{T} (-2)^2 dt}{T} = \frac{\int_0^T 4dt}{T} = 4$$

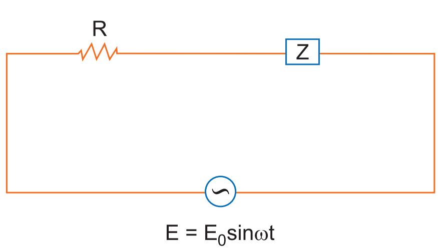
$$I_{rms} = \sqrt{4} = \mathbf{2\ A}$$

Short Answer Questions–I [2 marks]

Q. 1. An alternating voltage $E = E_0 \sin \omega t$ is applied to a circuit containing a resistor *R* connected in series with a black box. The current in the circuit is found to be $I = I_o \sin(\omega t + \pi/4)$.

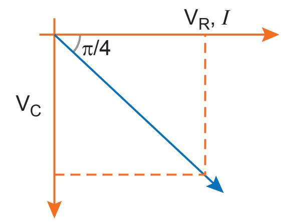
(*i*) State whether the element in the black box is a capacitor or inductor.

(*ii*) Draw the corresponding phasor diagram and find the impedance in terms of *R*.



Ans. (*i*) As the current leads the voltage by $\dfrac{\pi}{4}$, the element used in black box is a 'capacitor'.

(*ii*) Here, $\tan\frac{\pi}{4} = V_C/V_R \Rightarrow 1 = \frac{V_C}{V_R}$



$\Rightarrow \quad V_C = V_R \quad \Rightarrow \quad X_C = R$

$\therefore$ Impedance $Z = \sqrt{(X_C)^2 + R^2} = \sqrt{R^2 + R^2} = \sqrt{2R^2}$

$\therefore \; Z = \sqrt{2}R$

Q. 2. Define power factor. State the conditions under which it is (*i*) maximum and (*ii*) minimum. [*CBSE Delhi 2010*]

Ans. The power factor ($\cos\phi$) is the ratio of resistance and impedance of an *ac* circuit *i.e.*,

Power factor, $\cos\phi = \frac{R}{Z}$

Maximum power factor is 1 when $Z = R$ *i.e.*, when circuit is purely resistive. Minimum power factor is 0 when $R = 0$ *i.e.*, when circuit is purely inductive or capacitive.

Q. 3. When an *ac* source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero. [*CBSE (Central) 2016*]

Ans. For an ideal inductor phase difference between current and applied voltage = $\pi/2$

$\therefore$ Power, $P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \cos\frac{\pi}{2} = 0.$

Thus the power consumed in a pure inductor is **zero**.

Q. 4. When an *ac* source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero. [*CBSE (North) 2016*]

Ans. Power dissipated in *ac* circuit, $P = V_{rms} I_{rms} \cos\phi$ where $\cos\phi = \frac{R}{Z}$

For an ideal capacitor $R = 0 \;\therefore \cos\phi = \frac{R}{Z} = 0$

$\therefore \quad P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \times 0 = 0$ (zero).

i.e., power dissipated in an ideal capacitor is **zero**.

Q. 5. The current flowing through a pure inductor of inductance 2 mH is $i = 15 \cos 300\, t$ ampere. What is the (*i*) rms and (*ii*) average value of current for a complete cycle? [*CBSE (F) 2011*]

Ans. Peak value of current (i_0) = 15 A

(*i*) $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} = \frac{15}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \mathbf{7.5\sqrt{2}\,A}$

(*ii*) $i_{av} = \mathbf{0}$

Q. 6. In a series *LCR* circuit with an ac source of effective voltage 50 V, frequency $\nu = 50/\pi$ Hz, R = 300 Ω, C = 20 μF and *L* = 1.0 H. Find the rms current in the circuit. [*CBSE (F) 2014*]

Ans. Given, $L = 1.0$ H; $C = 20\,\mu F = 20 \times 10^{-6}$ F

$R = 300\,\Omega;\; V_{rms} = 50\text{ V};\; \nu = \frac{50}{\pi}$ Hz

Inductive reactance $X_L = \omega L = 2\pi\nu L = 2 \times \pi \times \frac{50}{\pi} \times 1 = 100\,\Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times \pi \times \frac{50}{\pi} \times 20 \times 10^{-6}} = 500\,\Omega$

Impedance of circuit

$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$$= \sqrt{(300)^2 + (500 - 100)^2} = \sqrt{90000 + 160000} = \sqrt{250000} = 500\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{50}{500} = \mathbf{0.1\ A}$$

Q. 7. Calculate the quality factor of a series *LCR* circuit with *L* = 2.0 H, *C* = 2 μF and *R* = 10 Ω. Mention the significance of quality factor in *LCR* circuit. **[*CBSE (F) 2012*]**

Ans. We have, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{10}\sqrt{\frac{2}{2 \times 10^{-6}}} = \mathbf{100}$$

It signifies the sharpness of resonance.

Q. 8. The instantaneous current in an *ac* circuit is *I* = 0.5 sin 314 *t*, what is (*i*) rms value and (*ii*) frequency of the current.

Ans. Given, $I = 0.5 \sin 314\, t$ ... (*i*)

Standard equation of current is $I = I_0 \sin \omega t$ ... (*ii*)

Comparing (*i*) and (*ii*), we get $I_0 = 0.5$ A, $\omega = 314$

∴ (*i*) rms value $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{0.5}{\sqrt{2}}\text{A} = \mathbf{0.35\ A}$

(*ii*) Frequency $\nu = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = \mathbf{50\,Hz}$

Q. 9. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current? **[*NCERT Exemplar*]**

Ans. An *ac* current changes direction with the source frequency and the attractive force would average to zero. Thus, the *ac* ampere must be defined in terms of some property that is independent of the direction of current. Joule's heating effect is such property and hence it is used to define rms value of *ac*.

Q. 10. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used. **[*NCERT Exemplar*]**

Ans. Here $P_L = 60$ W, $I_L = 0.54$ A

$$V_L = \frac{60}{0.54} = 111.1\ \text{V}$$

The transformer is step-down and have $\frac{1}{2}$ input voltage. Hence

$$I_P = \frac{1}{2} \times I_L = \frac{1}{2} \times 0.54 = \mathbf{0.27\ A.}$$

Q. 11. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency. **[*NCERT Exemplar*]**

Ans. A capacitor does not allow flow of direct current through it as the resistance across the gap is infinite. When an alternating voltage is applied across the capacitor plates, the plates are alternately charged and discharged. The current through the capacitor is a result of this changing voltage (or charge). Thus, a capacitor will pass more current through it if the voltage is changing at a faster rate, *i.e.*, if the frequency of supply is higher. This implies that the reactance offered by a capacitor is less with increasing frequency; it is given by $1/\omega C$.

Q. 12. Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage. **[*NCERT Exemplar*]**

Ans. An inductor opposes flow of current through it by developing an induced emf according to Lenz's law. The induced voltage has a polarity so as to maintain the current at its present value. If the current is decreasing, the polarity of the induced emf will be so as to increase the current and vice versa. Since the induced emf is proportional to the rate of change of current, it will provide greater reactance to the flow of current if the rate of change is faster, *i.e.*, if the frequency is higher. The reactance of an inductor, therefore, is proportional to the frequency, being given by ωL.

Short Answer Questions–II [3 marks]

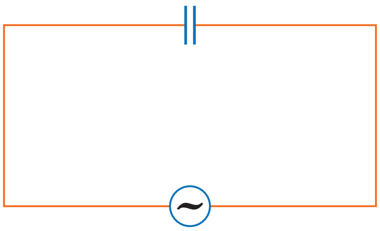
Q. 1. Show that the current leads the voltage in phase by π/2 in an *ac* circuit containing an ideal capacitor. [*CBSE (F) 2014*]

Ans. The instantaneous voltage,

$$V = V_0 \sin \omega t \quad \ldots (i)$$

Let q be the charge on capacitor and I, the current in the circuit at any instant, then instantaneous potential difference,

$$V = \frac{q}{C} \quad \ldots (ii)$$



From (*i*) and (*ii*)

$$\frac{q}{C} = V_0 \sin\omega t \Rightarrow q = CV_0 \sin\omega t$$

The instantaneous current,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CV_0 \sin \omega t) = CV_0 \frac{d}{dt}(\sin \omega t) = CV_0 \omega \cos\omega t$$

$$I = \frac{V_0}{1/\omega C} \cos\omega t$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Hence, the current leads the applied voltage in phase by $\pi/2$.

Q. 2. In a series *LCR* circuit, obtain the conditions under which (*i*) the impedance of the circuit is minimum, and (*ii*) wattless current flows in the circuit. [*CBSE (F) 2014*]

Ans. (*i*) Impedance of series *LCR* circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For the impedance, Z to be minimum

$$X_L = X_C$$

(*ii*) Power $P = V_{rms} I_{rms} \cos \phi$

When $\phi = \frac{\pi}{2}$

$$\text{Power} = V_{rms} I_{rms} \cos\frac{\pi}{2} = 0$$

Therefore, wattless current flows when the impedance of the circuit is purely inductive or purely capacitive.

In another way we can say, for wattless current to flow, circuit should not have any ohmic resistance ($R = 0$).

Q. 3. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers? [*CBSE (AI) 2012*]

Ans. The principle of transformer is based upon the principle of mutual induction which states that due to continuous change in the current in the primary coil an emf gets induced across the secondary coil. At the power generating station, the step up transformers step up the output voltage which reduces the current through the cables and hence reduce resistive power loss. Then, at the consumer end, a step down transformer steps down the voltage.

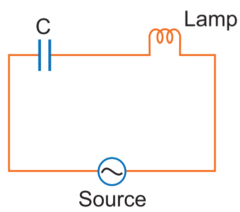
Hence, the large scale transmission of electric energy over long distances is done by stepping up the voltage at the generating station to minimise the power loss in the transmission cables.

Q. 4. An electric lamp connected in series with a capacitor and an *ac* source is glowing with of certain brightness. How does the brightness of the lamp change on reducing the (*i*) capacitance and (*ii*) frequency? **[CBSE Delhi 2010, (North) 2016]**

Ans. (*i*) When capacitance is reduced, capacitive reactance $X_C = \frac{1}{\omega C}$ increases, hence impedance of circuit

$$Z = \sqrt{R^2 + X_C^2}$$

increases and so current $I = \frac{V}{Z}$ decreases. As a result the brightness of the bulb is reduced.



(*ii*) When frequency decreases; capacitive reactance $X_C = \frac{1}{2\pi\nu C}$ increases and hence impedance of circuit increases, so current decreases. As a result brightness of bulb is reduced.

Q. 5. State the principle of working of a transformer. Can a transformer be used to step up or step down a *dc* voltage? Justify your answer. **[CBSE (AI) 2011]**

Ans. Working of a transformer is based on the principle of mutual induction. Transformer cannot step up or step down a *dc* voltage.

Reason: No change in magnetic flux.

Explanation: When *dc* voltage source is applied across a primary coil of a transformer, the current in primary coil remains same, so there is no change in magnetic flux associated with it and hence no voltage is induced across the secondary coil.

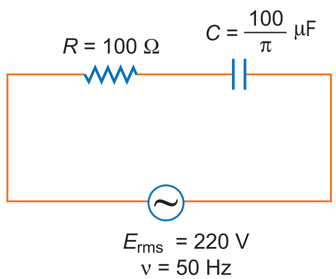
Q. 6. A resistor of 100 Ω and a capacitor of 100/π μF are connected in series to a 220 V, 50 Hz *ac* supply.

(*a*) Calculate the current in the circuit.

(*b*) Calculate the (rms) voltage across the resistor and the capacitor. Do you find the algebraic sum of these voltages more than the source voltage? If yes, how do you resolve the paradox? **[CBSE Chennai 2015]**

Ans. (*a*) Capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

$$= \frac{1}{2\pi \times 50 \times \frac{100}{\pi} \times 10^{-6}} = 100\ \Omega$$



Impedance of the circuit, $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}$$

Current in the circuit $I_{rms} = \frac{E_{rms}}{Z} = \frac{220}{100\sqrt{2}}$ **= 1.56 A**

(*b*) Voltage across resistor, $V_R = I_{rms}\,R$

$= 1.56 \times 100 =$ **156 V**

Voltage across capacitor, $V_C = I_{rms} \times C = 1.56 \times 100\ \text{V} =$ **156 V**

The algebraic sum of voltages across the combination is

$$V_{rms} = V_R + V_C = 156\ \text{V} + 156\ \text{V} = 312\ \text{V}$$

While V_{rms} of the source is 220 V. Yes, the voltages across the combination is more than the voltage of the source. The voltage across the resistor and capacitor are not in phase.

This paradox can be resolved as when the current passes through the capacitor, it leads the voltage V_C by phase $\frac{\pi}{2}$. So, voltage of the source can be given as

$$V_{rms} = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(156)^2 + (156)^2} = 156\sqrt{2} = \mathbf{220\ V}$$

Q. 7. A capacitor of unknown capacitance, a resistor of 100 Ω and an inductor of self inductance $L = \left(\frac{4}{\pi^2}\right)$ henry are connected in series to an *ac* source of 200 V and 50 Hz. Calculate the value of the capacitance and impedance of the circuit when the current is in phase with the voltage. Calculate the power dissipated in the circuit. [*CBSE South 2016*]

Ans. Capacitance, $C = \frac{1}{L\omega^2}$

$$= \frac{1}{\frac{4}{\pi^2}(2\pi \times 50)^2} \text{F} = \frac{1}{40000} \text{F} = \mathbf{2.5 \times 10^{-5}\ F}$$

Since V and I are in same phase

Impedance = Resistance = **100 Ω**

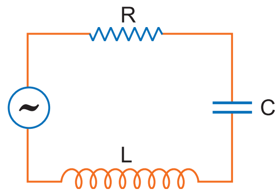
Power dissipated $= \frac{E_{rms}^2}{2} = \frac{(200)^2}{100} \text{W} = \mathbf{400\ W}$

Q. 8. The figure shows a series *LCR* circuit with L = 5.0 H, C = 80 μF, R = 40 Ω connected to a variable frequency 240 V source. Calculate.

(*i*) The angular frequency of the source which drives the circuit at resonance.

(*ii*) The current at the resonating frequency.

(*iii*) The rms potential drop across the capacitor at resonance.



[*CBSE Delhi 2012*]

Ans. (*i*) We know

$$\omega_r = \text{Angular frequency at resonance} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \mathbf{50\ rad/s}$$

(*ii*) Current at resonance, $I_{rms} = \frac{V_{rms}}{R} = \frac{240}{40} = \mathbf{6\ A}$

(*iii*) V_{rms} across capacitor

$$V_{rms} = I_{rms}X_C = 6 \times \frac{1}{50 \times 80 \times 10^{-6}} = \frac{6 \times 10^6}{4 \times 10^3} = \mathbf{1500\ V}$$

Q. 9. A series *LCR* circuit is connected to an *ac* source (200 V, 50 Hz). The voltages across the resistor, capacitor and inductor are respectively 200 V, 250 V and 250 V.

(*i*) The algebraic sum of the voltages across the three elements is greater than the voltage of the source. How is this paradox resolved?

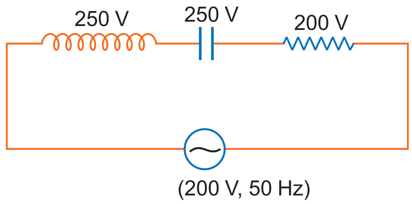
(*ii*) Given the value of the resistance of R is 40 Ω, calculate the current in the circuit.

[*CBSE (F) 2013*]

Ans. (*i*) From given parameters V_R = 200 V, V_L = 250 V and V_C = 250 V

V_{eff} should be given as

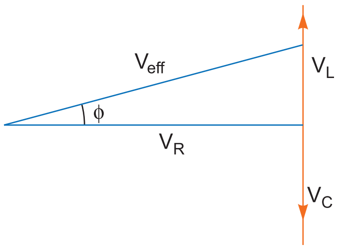
$V_{eff} = V_R + V_L + V_C$ = 200 V + 250 V + 250 V = 700 V



However, V_{eff} > 200 V of the *ac* source.

This paradox can be solved only by using phaser diagram, as given below:

$$(V_{eff}) = \sqrt{V_R^2 + (V_L - V_C)^2}$$



Since $V_L = V_C$ so $V_{eff} = V_R$ = 200 V

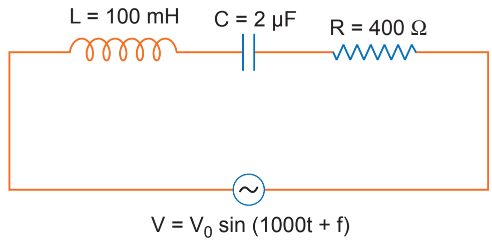
(*ii*) Given R = 40 Ω, so current in the *LCR* circuit.

$$I_{eff} = \frac{V_{eff}}{R} \quad [X_L = X_C \quad \text{or} \quad Z = R]$$

$$= \frac{200}{40} = \mathbf{5\ A}$$

Q. 10. **(*i*) Find the value of the phase difference between the current and the voltage in the series *LCR* circuit shown below. Which one leads in phase: current or voltage?**

(*ii*) Without making any other change, find the value of the additional capacitor, C_1, to be connected in parallel with the capacitor *C*, in order to make the power factor of the circuit unity. [*CBSE Delhi 2017, Allahabad 2015*]



Ans. (*i*) Inductive reactance,

$$X_L = \omega L = (1000 \times 100 \times 10^{-3})\ \Omega = 100\ \Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \left(\frac{1}{1000 \times 2 \times 10^{-6}}\right)\Omega = 500\ \Omega$$

Phase angle,

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{100-500}{400} = -1$$

$$\phi = -\frac{\pi}{4}$$

As $X_C > X_L$, (phase angle is negative), hence current leads voltage.

(*ii*) To make power factor unity

$$X_{C'} = X_L \qquad \text{(where } C' = \text{net capacitance of parallel combination)}$$

$$\frac{1}{\omega C'} = 100$$

$$C' = 10 \times 10^{-6}\ \text{F}$$

$\therefore$ $C' = 10\ \mu\text{F}$

$\because$ $C' = C + C_1$

$\Rightarrow$ $10 = 2 + C_1$ $\Rightarrow$ $C_1 = \mathbf{8\ \mu F}$

Q. 11. **(*a*) For a given *ac*, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor *R* over a complete cycle is $\frac{1}{2} i_m^2 R$.**

(*b*) A light bulb is rated at 100 W for a 220 V *ac* supply. Calculate the resistance of the bulb. [*CBSE (AI) 2013*]

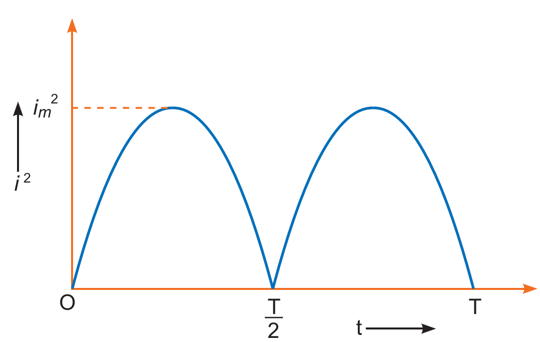
Ans. (*a*) Average power consumed in resistor *R* over a complete cycle

$$P_{av} = \frac{1}{\int_0^T dt} . \int_0^T i^2 R\, dt$$

$$= \frac{i_m^2 R}{T} \int_0^T \sin^2 \omega t\, dt \qquad ...(i)$$

$$= \frac{i_m^2 R}{2T} \int_0^T (1 - \cos 2\,\omega t)\, dt$$

$$= \frac{i_m^2 R}{2T}\left[\int_0^T dt - \int_0^T \cos 2\,\omega t\, dt\right] \qquad ...(ii)$$



$$= \frac{i_m^2 R}{2T}[T - 0] = \frac{i_m^2 R}{2}$$

(*b*) In case of ***ac***

$$P_{av} = \frac{V_{rms}^2}{R} = \frac{V_{eff}^2}{R}$$

$$R = \frac{V_{rms}^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ \Omega}$$

Q. 12. Determine the current and quality factor at resonance for a series *LCR* circuit with L = 1.00 mH, C = 1.00 nF and R = 100 Ω connected to an *ac* source having peak voltage of 100 V. **[*CBSE (F) 2011*]**

Ans. I_v = ?, Q = ?

L=1.00 mH = 1 ×10^{-3} H, C = 1.00 nF = 1×10^{-9} F, R=100 Ω, E_0= 100 V

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{E_0}{Z} \qquad \left\{\begin{array}{l}\text{at resonance } \omega L = \frac{1}{\omega C} \\ \text{Hence } Z = R\end{array}\right\}$$

$$\therefore \qquad I = \frac{V}{R} = \frac{100}{100} = 1\text{ A}$$

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1.44}{2} = \mathbf{0.707\ A} \qquad [\therefore I_0 = 1\text{ A}]$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{1.0 \times 10^{-3}}{1.0 \times 10^{-9}}} = \frac{1}{100} \times 10^3 = \mathbf{10}$$

Q. 13. A circuit is set up by connecting inductance L = 100 mH, resistor R = 100 Ω and a capacitor of reactance 200 Ω in series. An alternating emf of $150\sqrt{2}$ V, 500/π Hz is applied across this series combination. Calculate the power dissipated in the resistor. **[*CBSE (F) 2014*]**

Ans. Here, L = 100 ×10^{-3} H, R =100 Ω, X_C = 200 Ω, V_{rms} =$150\sqrt{2}$ V

$$\nu = \frac{500}{\pi}\text{Hz.}$$

Inductive reactance X_L = ωL = 2πνL

$$= 2\pi\frac{500}{\pi} \times 100 \times 10^{-3} = 100\ \Omega$$

Impedance of circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(100)^2 + (200-100)^2} = \sqrt{20000} = 100\sqrt{2}\ \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{150\sqrt{2}}{100\sqrt{2}} = \frac{3}{2}$$

Power dissipated $(I_{rms})^2 R = \frac{9}{4} \times 100 = \mathbf{225\ W}$

Q. 14. The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are 220 V and 1100 W respectively. Calculate

(*a*) the number of turns in the secondary coil.

(*b*) the current in the primary coil.

(*c*) the voltage across the secondary coil.

(*d*) the current in the secondary coil.

(*e*) the power in the secondary coil. **[*CBSE Delhi 2016*]**

Ans. (*a*) Transformation ratio $r = \dfrac{\text{Number of turns in secondary coil}(N_S)}{\text{Number of turns in primary coil}(N_P)}$

Given $N_P = 100$, $r = 100$

$\therefore$ Number of turns in secondary coil, $N_S = rN_P = 100 \times 100 = \mathbf{10{,}000}$

(*b*) Input voltage $V_P = 220$ V, Input power $P_{in} = 1100$ W

Current in primary coil $I_p = \dfrac{P_{in}}{V_P} = \dfrac{1100}{220} = \mathbf{5\ A}$

(*c*) Voltage across secondary coil (V_S) is given by

$$r = \frac{V_S}{V_P}$$

$\Rightarrow$ $V_S = rV_P = 100 \times 220 = 22{,}000\ \text{V} = \mathbf{22\ kV}$

(*d*) Current in secondary coil is given by

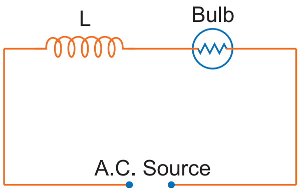
$$r = \frac{I_P}{I_S} \Rightarrow I_S = \frac{I_P}{r} = \frac{5}{100} = \mathbf{0.05\ A}$$

(*e*) Power in secondary coil, $P_{out} = V_S I_S = 22 \times 10^3 \times 0.05 = \mathbf{1100\ W}$

Obviously power in secondary coil is same as power in primary. This means that the transformer is ideal, *i.e.*, there are no energy losses.

Q. 15. An inductor *L* of reactance X_L is connected in series with a bulb *B* to an *ac* source as shown in figure. Explain briefly how does the brightness of the bulb change when (*i*) number of turns of the inductor is reduced (*ii*) an iron rod is inserted in the inductor and (*iii*) a capacitor of reactance $X_C = X_L$ is included in the circuit.

[*CBSE Delhi 2014, 2015*]



Ans. Brightness of the bulb depends on square of the I_{rms} (*i.e.*, I^2_{rms})

Impedance of the circuit, $Z = \sqrt{R^2 + (\omega L)^2}$ and

Current in the circuit, $I = \dfrac{V}{Z}$

(*i*) When the number of turns in the inductor is reduced, the self inductance of the coil decreases; so impedance of circuit reduces and so current in the circuit $\left(I = \dfrac{E}{Z}\right)$ increases. Thus, the brightness of the bulb increases.

(*ii*) When iron (being a ferromagnetic substance) rod is inserted in the coil, its inductance increases and in turn, impedance of the circuit increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Hence, brightness of the bulb decreases.

(*iii*) When capacitor of reactance $X_C = X_L$ is introduced, the net reactance of circuit becomes zero, so impedance of circuit decreases; it becomes $Z = R$; so current in circuit increases; hence brightness of bulb increases. Thus brightness of bulb in both cases increases.

Q. 16. A capacitor (*C*) and resistor (*R*) are connected in series with an *ac* source of voltage of frequency 50 Hz. The potential difference across *C* and *R* are respectively 120 V, 90 V, and the current in the circuit is 3 A. Calculate (*i*) the impedance of the circuit (*ii*) the value of the inductance, which when connected in series with *C* and *R* will make the power factor of the circuit unity.

[*CBSE 2019 (55/2/1)*]

Ans. $\because$ $R = \dfrac{V_R}{I_R} = \dfrac{90}{3} = 30\ \Omega$

$$X_C = \frac{V_C}{I_C} = \frac{120}{3} = 40\ \Omega$$

(*i*) Impedance, $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{30^2 + 40^2} = 50\,\Omega$$

(*ii*) As power factor = 1

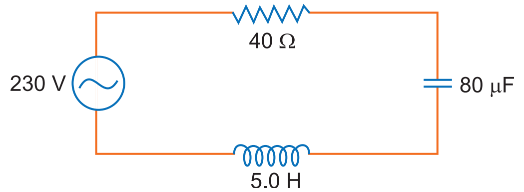
Now, $X_L = X_C$

$$2\pi\nu L = 40$$

$$100\pi L = 40$$

$$L = \mathbf{\frac{2}{5\pi}\ H.}$$

Q. 17. The figure shows a series *LCR* circuit connected to a variable frequency 230 *V* source.



(*a*) Determine the source frequency which drives the circuit in resonance.

(*b*) Calculate the impedance of the circuit and amplitude of current at resonance.

(*c*) Show that potential drop across LC combination is zero at resonating frequency.

[*CBSE 2019 (55/2/1)*]

Ans. (*a*) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{5\times80\times10^{-6}}} = \dfrac{1}{\sqrt{400\times10^{-6}}}$

$$\omega = \frac{1000}{20} = 50\ \text{rad/s} \Rightarrow f = \frac{\omega}{2\pi} = \frac{50}{2\pi} = \mathbf{\frac{25}{\pi}\ Hz}$$

(*b*) At resonance, $Z = R = \mathbf{40\ \Omega}$

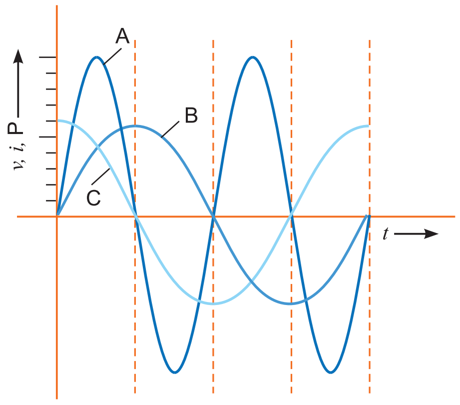
$$I_{max} = \frac{230\sqrt{2}}{R} = \frac{230\sqrt{2}}{40} = \mathbf{8.1\ A}$$

(*c*) $V_C = I_{max}X_C = \dfrac{230\sqrt{2}}{40} \times \dfrac{1}{50\times80\times10^{-6}} = 2025\ \text{V}$ $\quad [\because X_C = \dfrac{1}{\omega C}]$

$$V_L = I_{max}X_L = \frac{230\sqrt{2}}{40} \times 50 \times 5 = 2025\ \text{V} \quad [\because X_L = \omega L]$$

$$V_C - V_L = 0$$

Q. 18. A device '*X*' is connected to an *ac* source. The variation of voltage, current and power in one complete cycle is shown in the figure.



(*a*) Which curve shows power consumption over a full cycle?

(*b*) What is the average power consumption over a cycle?

(*c*) Identify the device '*X*'. **[*NCERT Exemplar*]**

Ans. (*a*) *A*

(*b*) Zero

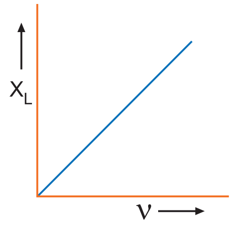
(*c*) *L* or *C* or *LC* Series combination of *L* and *C*

Q. 19. (*i*) Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied *ac* source.

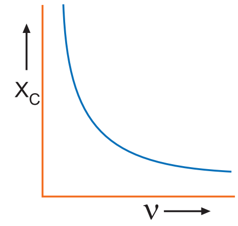
(*ii*) Can the voltage drop across the inductor or the capacitor in a series *LCR* circuit be greater than the applied voltage of the *ac* source? Justify your answer. **[*HOTS*]**

Ans. (*i*) (*a*) $X_L = \omega L = 2\pi\nu L$; graph X_L of ν and ν is a straight line

(*b*) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$, graph of X_C and ν is a rectangular hyperbola as shown in fig.



(a)

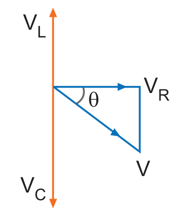


(b)

(*ii*) Yes; because $V = \sqrt{V_R^2 + (V_C - V_L)^2}$;

As V_C and V_L have opposite faces, V_C or V_L may be greater than V.

The situation may be as shown in figure where $V_C > V$.



Long Answer Questions [5 marks]

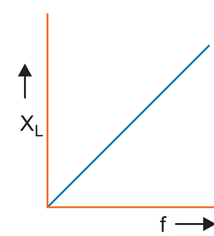
Q. 1. Explain the term inductive reactance. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage.

An *ac* voltage $V = V_0 \sin \omega t$ is applied across a pure inductor of inductance *L*. Find an expression for the current *i*, flowing in the circuit and show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\frac{\pi}{2}$. Also draw (*i*) phasor diagram (*ii*) graphs of *V* and *i* versus ω*t* for the circuit. ***[CBSE East 2016]***

Ans. Inductive Reactance: The opposition offered by an inductor to the flow of alternating current through it is called the inductive reactance. It is denoted by X_L. Its value is $X_L. = \omega L = 2\pi f L$ where L is inductance and f is the frequency of the applied voltage.

Obviously $X_L \propto f$

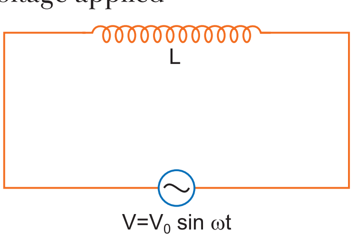
Thus, the graph between X_L and frequency f is linear (as shown in fig.).



Phase Difference between Current and Applied Voltage in Purely Inductive circuit :

AC circuit containing pure inductance: Consider a coil of self-inductance L and negligible ohmic resistance. An alternating potential difference is applied across its ends. The magnitude and direction of *ac* changes periodically, due to which there is a continual change in magnetic flux linked with the coil. Therefore according to Faraday's law, an induced emf is produced in the coil, which opposes the applied voltage. As a result the current in the circuit is reduced. That is ***inductance acts like a resistance in ac circuit***. The instantaneous value of alternating voltage applied

$$V = V_0 \sin \omega t \quad ...(i)$$



(a)

If i is the instantaneous current in the circuit and $\frac{di}{dt}$ the rate of change of current in the circuit at that instant, then instantaneous induced emf

$$\varepsilon = -L\frac{di}{dt}$$

According to Kirchhoff's loop rule

$$V + \varepsilon = 0 \Rightarrow V - L\frac{di}{dt} = 0$$

or $\quad V = L\frac{di}{dt}$ or $\frac{di}{dt} = \frac{V}{L}$

or $\quad \frac{di}{dt} = \frac{V_0 \sin\omega t}{L}$ or $di = \frac{V_0 \sin\omega t}{L}dt$

Integrating with respect to time 't',

$$i = \frac{V_0}{L}\int \sin\omega t\, dt = \frac{V_0}{L}\left\{-\frac{\cos\omega t}{\omega}\right\} = -\frac{V_0}{\omega L}\cos\omega t = -\frac{V_0}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right)$$

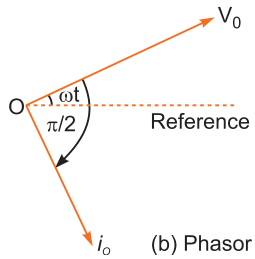
or $$i = \frac{V_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \quad \ldots(ii)$$

This is required expression for current

or $$i = i_0 \sin\left(\omega t-\frac{\pi}{2}\right) \quad \ldots(iii)$$

where $$i_0 = \frac{V_0}{\omega L} \quad \ldots(iv)$$

is the peak value of alternating current

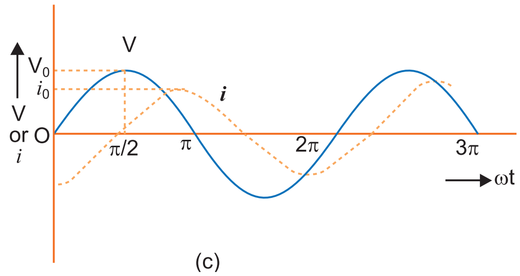


(b) Phasor diagram

Also comparing (i) and (iii), we note that current lags behind the applied voltage by an angle $\frac{\pi}{2}$ (Fig. b).

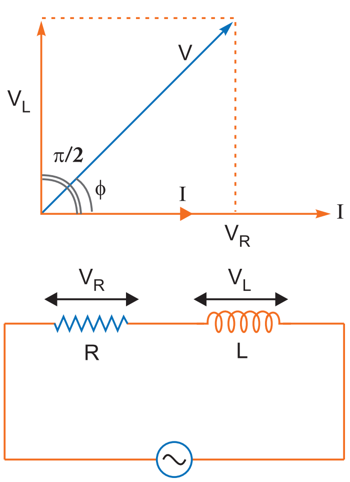
Phasor diagram: The phasor diagram of circuit containing pure inductance is shown in Fig. (b).

Graphs of V and I versus ωt for this circuit is shown in fig. (c).



(c)

Q. 2. Derive an expression for impedance of an *ac* circuit consisting of an inductor and a resistor. ***[CBSE Delhi 2008]***



Ans. Let a circuit contain a resistor of resistance R and an inductor of inductance L connected in series. The applied voltage is $V = V_0 \sin \omega t$. Suppose the voltage across resistor V_R and that across inductor is V_L. The voltage V_R and current I are in the same phase, while the voltage V_L leads the current by an angle $\frac{\pi}{2}$. Thus, V_R and V_L are mutually perpendicular. The resultant of V_R and V_L is the applied voltage *i.e.*,

$$V = \sqrt{V_R^2 + V_L^2}$$

But $\quad V_R = RI, \qquad V_L = X_L I = \omega L I$

$\therefore$ where $\quad X_L = \omega L$ is inductive reactance

$\therefore \quad V = \sqrt{(RI)^2 + (X_L I)^2}$

$\therefore \quad$ Impedance, $Z = \frac{V}{I} = \sqrt{R^2 + X_L^2} \Rightarrow Z = \sqrt{R^2 + (\omega L)^2}$

Q. 3. **(a) What is impedance?**

(b) A series *LCR* circuit is connected to an *ac* source having voltage $V = V_0 \sin \omega t$. Derive expression for the impedance, instantaneous current and its phase relationship to the applied voltage. Find the expression for resonant frequency. [*CBSE Delhi 2010*]

OR

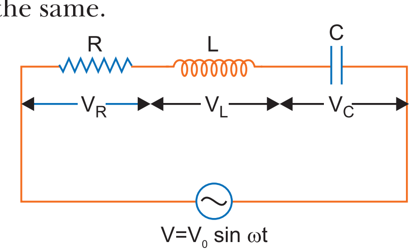
(a) An *ac* source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of *L*, *C* and *R*. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called?

(b) In a series *LR* circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance *C* such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate $\frac{P_1}{P_2}$. [*CBSE Delhi 2016*]

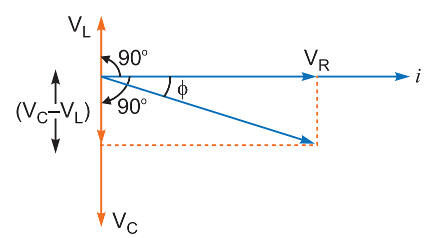
Ans. **Impedance:** The opposition offered by the combination of a resistor and reactive component to the flow of ac is called impedance. Mathematically it is the ratio of rms voltage applied and rms current produced in circuit *i.e.*, $Z = \frac{V}{I}$.

Its unit is ohm (Ω).

Expression for Impedance in *LCR* series circuit: Suppose resistance *R*, inductance *L* and capacitance *C* are connected in series and an alternating source of voltage $V = V_0 \sin \omega t$ is applied across it (fig. a). On account of being in series, the current (*i*) flowing through all of them is the same.



(a)



(b)

Suppose the voltage across resistance *R* is V_R voltage across inductance *L* is V_L and voltage across capacitance *C* is V_C. The voltage V_R and current *i* are in the same phase, the voltage V_L will lead the current by angle 90° while the voltage V_C will lag behind the current by angle 90° (fig. *b*). Clearly V_C and V_L are in opposite directions, therefore their resultant potential difference = $V_C - V_L$ (if $V_C > V_L$).

Thus V_R and $(V_C - V_L)$ are mutually perpendicular and the phase difference between them is 90°.

As applied voltage across the circuit is *V*, the resultant of V_R and $(V_C - V_L)$ will also be *V*. From fig.

$$V^2 = V_R^2 + (V_C - V_L)^2 \quad \Rightarrow \quad V = \sqrt{V_R^2 + (V_C - V_L)^2} \qquad \ldots(i)$$

But $\quad V_R = R\,i\,,\ V_C = X_C\,i$ and $V_L = X_L\,i$ $\qquad \ldots(ii)$

where $X_C = \frac{1}{\omega C}$ = capacitance reactance and $X_L = \omega L$ = inductive reactance

$$V = \sqrt{(Ri)^2 + (X_C i - X_L i)^2}$$

Impedance of circuit, $Z = \frac{V}{i} = \sqrt{R^2 + (X_C - X_L)^2}$

i.e.,

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

Instantaneous current $I = \dfrac{V_0 \sin(\omega t + \phi)}{\sqrt{R^2 + \left(\frac{1}{\omega C} - wL\right)^2}}$

The phase difference (ϕ) between current and voltage is given by, $\tan\phi = \dfrac{X_C - X_L}{R}$

Resonant Frequency: For resonance $\phi = 0$, so $X_C - X_L = 0$

$$\frac{1}{\omega C} = \omega L \Rightarrow \omega^2 = \frac{1}{LC}$$

$\therefore$ Resonant frequency $\omega_r = \dfrac{1}{\sqrt{LC}}$

Phase difference (ϕ) in series LCR circuit is given by

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{i_m(X_C - X_L)}{i_m R} = \frac{(X_C - X_L)}{R}$$

When current and voltage are in phase

$$\phi = 0 \quad \Rightarrow \quad X_C - X_L = 0 \Rightarrow \quad X_C = X_L$$

This condition is called resonance and the circuit is called resonant circuit.

Case I: $X_L = R$

$$\therefore \quad Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = \sqrt{2}R$$

Power factor, $P_1 = \cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{2}R} = \dfrac{1}{\sqrt{2}}$

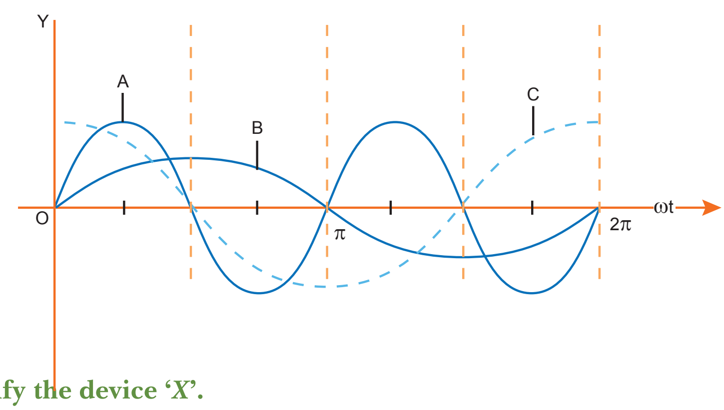
Case II: $X_L = X_C$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2} = R$$

Power factor, $P_2 = \dfrac{R}{Z} = \dfrac{R}{R} = 1$

$$\therefore \quad \frac{P_1}{P_2} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$$

Q. 4. A device 'X' is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is show in the following graph:



(*a*) Identify the device 'X'.

(*b*) Which of the curves, *A*, *B* and *C* represent the voltage, current and the power consumed in the circuit? Justify your answer.

(*c*) How does its impedance vary with frequency of the *ac* source? Show graphically.

(*d*) Obtain an expression for the current in the circuit and its phase relation with *ac* voltage.

Ans. (*a*) The device '*X*' is a capacitor.

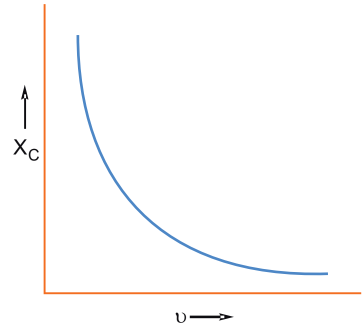
(*b*) Curve *B* : Voltage

Curve *C* : Current

Curve *A* : Power consumed in the circuit

Reason : This is because current leads the voltage in phase by $\dfrac{\pi}{2}$ for a capacitor.

(*c*) Impedance:



$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

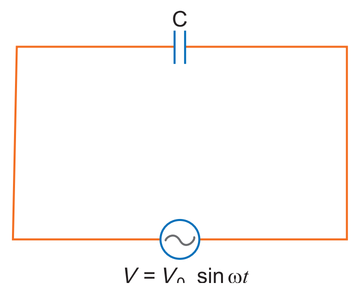
$$\Rightarrow \qquad X_C \propto \frac{1}{\nu}$$

(*d*) Voltage applied to the circuit is

$$V = V_0 \sin \omega t$$

Due to this voltage, a charge will be produced which will charge the plates of the capacitor with positive and negative charges.

$$V = \frac{Q}{C} \qquad \Rightarrow \qquad Q = CV$$



Therefore, the instantaneous value of the current in the circuit is

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$$

$$\therefore \qquad I = \omega C V_0 \cos \omega t = \frac{V_0}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where, $\quad I_0 = \dfrac{V_0}{\frac{1}{\omega C}}$ = Peak value of current

Hence, current leads the voltage in phase by $\frac{\pi}{2}$.

Q. 5. (*a*) State the condition for resonance to occur in series *LCR ac* circuit and derive an expression for resonant frequency. **[*CBSE Delhi 2010*]**

(*b*) Draw a plot showing the variation of the peak current (i_m) with frequency of the *ac* source used. Define the quality factor *Q* of the circuit.

Ans. (*a*) **Condition for resonance to occur in series LCR ac circuit:**

For resonance the current produced in the circuit and emf applied must always be in the same phase.

Phase difference (ϕ) in series *LCR* circuit is given by

$$\tan \phi = \frac{X_C - X_L}{R}$$

For resonance $\phi = 0 \quad \Rightarrow \quad X_C - X_L = 0$

or $\quad X_C = X_L$

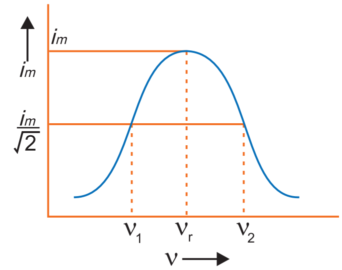
If ω_r is resonant frequency, then $X_C = \dfrac{1}{\omega_r C}$

and $\quad X_C = \omega_r L$

$$\frac{1}{\omega_r C} = \omega_r L \quad \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$

Linear resonant frequency, $\nu_r = \dfrac{\omega_r}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$

(*b*) The graph of variation of peak current i_m with frequency is shown in fig. Half power frequencies are the frequencies on either side of resonant frequency for which current reduces to half of its maximum value. In fig., ν_1 and ν_2 are half power frequencies.



Quality Factor (*Q*): The quality factor is defined as the ratio of resonant frequency to the width of half power frequencies.

i.e., $$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\nu_r}{\nu_2 - \nu_1} = \frac{\omega_r L}{R}$$

Q. 6. (*a*) An alternating voltage $V = V_m \sin \omega$ applied to a series *LCR* circuit drives a current given by $i = i_m \sin(\omega t + \phi)$. Deduce an expression for the average power dissipated over a cycle.

(*b*) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain. **[*CBSE (F) 2011*]**

OR

A voltage $V = V_0 \sin \omega t$ is applied to a series *LCR* circuit. Derive the expression for the average power dissipated over a cycle.

Under what condition is (*i*) no power dissipated even though the current flows through the circuit, (*ii*) maximum power dissipated in the circuit? **[*CBSE (AI) 2014*]**

Ans. (*a*) $V = V_m \sin \omega t$ and $i = i_m \sin(\omega t + \phi)$

and instantaneous power, $P = Vi$

$$= V_m \sin \omega t \,.\, i_m \sin(\omega t + \phi)$$

$$= V_m i_m \sin \omega t \sin(\omega t + \phi)$$

$$= \frac{1}{2} V_m i_m 2 \sin \omega t . \sin(\omega t + \phi)$$

From trigonometric formula

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$\therefore$ Instantaneous power, $P = \frac{1}{2} V_m i_m [\cos(\omega t - \omega t - \phi) - \cos(\omega t + \phi + \omega t)]$

$$= \frac{1}{2} V_m i_m [\cos \phi - \cos(2\omega t + \phi)] \quad \dots (i)$$

Average power for complete cycle

$$\overline{P} = \frac{1}{2} V_m i_m [\cos \phi - \overline{\cos(2\omega t + \phi)}]$$

where $\overline{\cos(\omega t + \phi)}$ is the mean value of $\cos(2\omega t + \phi)$ over complete cycle. But for a complete cycle, $\cos(2\omega t + \phi) = 0$

$$\therefore \text{Average power, } \overline{P} = \frac{1}{2} V_m i_m \cos \phi = \frac{V_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos \phi$$

$$\overline{P} = V_{rms} i_{rms} \cos \phi$$

(*i*) If phase angle $\phi = 90°$ (resistance R is not used in the circuit) then no power dissipated.

(*ii*) If phase angle $\phi = 0°$ or circuit is pure resistive (or $X_L = X_C$) at resonance then

Max power $P = V_{rms} \times I_{rms} = \frac{V_0 I_0}{2}$

(*b*) The power is $P = V_{rms} I_{rms} \cos \phi$. If $\cos \phi$ is small, then current considerably increases when voltage is constant. Power loss, we know is $I^2 R$. Hence, power loss increases.

Q. 7. Explain with the help of a labelled diagram, the principle and working of an *ac* generator. Write the expression for the emf generated in the coil in terms of speed of rotation. Can the current produced by an *ac* generator be measured with a moving coil galvanometer?

OR

Describe briefly, with the help of a labelled diagram, the basic elements of an *ac* generator. State its underlying principle. Show diagrammatically how an alternating emf is generated by a loop of wire rotating in a magnetic field. Write the expression for the instantaneous value of the emf induced in the rotating loop. [*CBSE Delhi 2010*]

OR

State the working of ac generator with the help of a labelled diagram.

The coil of an *ac* generator having *N* turns, each of area *A*, is rotated with a constant angular velocity ω. Deduce the expression for the alternating emf generated in the coil.

What is the source of energy generation in this device? [*CBSE (AI) 2011*]

Ans. AC generator: A dynamo or generator is a device which converts mechanical energy into electrical energy.

Principle: It works on the principle of electromagnetic induction. When a coil rotates continuously in a magnetic field, the effective area of the coil linked normally with the magnetic field lines, changes continuously with time. This variation of magnetic flux with time results in the production of an alternating emf in the coil.

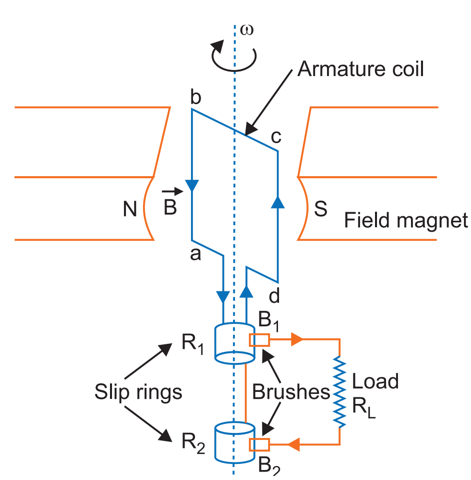
Construction: It consists of the four main parts:

(*i*) **Field Magnet:** It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.

(*ii*) **Armature:** It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve round an axle between the two poles of the field magnet. The drum or ring serves the two purposes: (*a*) It serves as a support to coils and (*b*) It increases the magnetic field due to air core being replaced by an iron core.

(*iii*) **Slip Rings:** The slip rings R_1 and R_2 are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.

(*iv*) **Brushes:** These are two flexible metal plates or carbon rods (B_1 and B_2) which are fixed and constantly touch the revolving rings. The output current in external load R_L is taken through these brushes.



Working: When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's right hand rule. Considering the armature to be in vertical position and as it rotates in clockwise direction, the wire *ab* moves downward and *cd* upward, so that the direction of induced current is shown in fig. In the external circuit, the current flows along $B_1\ R_L B_2$. The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire *ab* moves upward and *cd* downward, so the direction of current is reversed and in external circuit it flows along $B_2\ R_L B_1$. Thus the direction of induced emf and current changes in the external circuit after each half revolution.

Expression for Induced emf: When the coil is rotated with a constant angular speed ω , the angle θ between the magnetic field vector *B* and the area vector *A* of the coil at any instant

t is $\theta = \omega t$ (assuming $\theta = 0°$ at $t = 0$). As a result, the effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time t is

$$\phi_B = BA \cos \theta = BA \cos \omega t$$

From Faraday's law, the induced emf for the rotating coil of N turns is then,

$$\varepsilon = -N\frac{d\phi_B}{dt} = -NBA\frac{d}{dt}(\cos \omega t)$$

Thus, the instantaneous value of the emf is

$$\varepsilon = NBA\ \omega \sin \omega t$$

where $NBA\omega = 2\pi\upsilon NBA$ is the maximum value of the emf, which occurs when $\sin \omega t = \pm 1$. If we denote $NBA\omega$ as ε_0, then

$$\varepsilon = \varepsilon_0 \sin \omega t \qquad \Rightarrow \qquad \varepsilon = \varepsilon_0 \sin 2\pi \nu t$$

where ν is the frequency of revolution of the generator's coil.

Obviously, the emf produced is alternating and hence the current is also alternating.

Current produced by an *ac* generator cannot be measured by moving coil ammeter; because the average value of *ac* over full cycle is zero.

The source of energy generation is the mechanical energy of rotation of armature coil.

Q. 8. (*a*) Describe briefly, with the help of a labelled diagram, the working of a step up transformer.

(*b*) Write any two sources of energy loss in a transformer. ***[CBSE (F) 2012]***

(*c*) A step up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy? Explain. ***[CBSE Delhi 2011, 2009]***

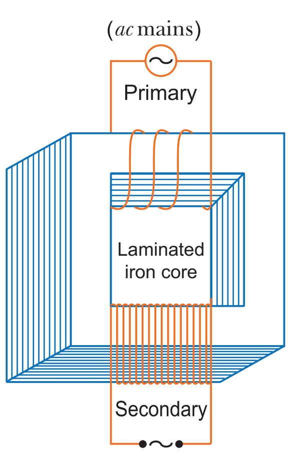
OR

Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils?

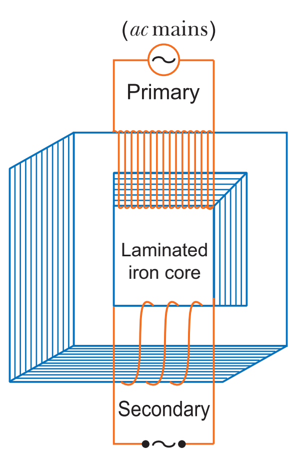
How is the transformer used in large scale transmission and distribution of electrical energy over long distances? ***[CBSE (AI) 2010, (East) 2016]***

Ans. (*a*) **Transformer:** A transformer converts low voltage into high voltage *ac* and vice-versa.

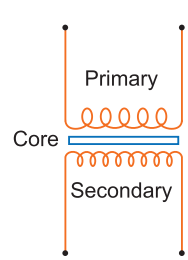
Construction: It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is called *primary coil* and other is called the *secondary coil*. The terminals of primary coils are connected to *ac* mains and the terminals of the secondary coil are connected to external circuit in which alternating current of desired voltage is required. Transformers are of two types:



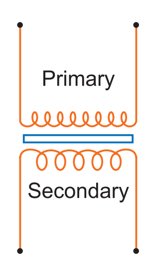
(a) Step up Transformer



(b) Step down Transformer



(a) Step up



(b) Step down

1. **Step up Transformer:** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil (*i.e.*, $N_S > N_P$).
2. **Step down Transformer:** It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (*i.e.*, $N_S < N_P$).

Working: When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let N_P be the number of turns in primary coil, N_S the number of turns in secondary coil and ϕ the magnetic flux linked with each turn. ***We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same.*** According to Faraday's laws the emf induced in the primary coil

$$\varepsilon_P = -N_P \frac{\Delta\phi}{\Delta t} \qquad ...(i)$$

and emf induced in the secondary coil

$$\varepsilon_S = -N_S \frac{\Delta\phi}{\Delta t} \qquad ...(ii)$$

From (*i*) and (*ii*)

$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} \qquad ...(iii)$$

If the resistance of primary coil is negligible, the emf (ε_P) induced in the primary coil, will be equal to the applied potential difference (V_P) across its ends. Similarly if the secondary circuit is open, then the potential difference V_S across its ends will be equal to the emf (ε_S) induced in it; therefore

$$\frac{V_S}{V_P} = \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} = r\text{(say)} \qquad ...(iv)$$

where $r = \frac{N_S}{N_P}$ is called the transformation ratio. If i_P and i_S are the instantaneous currents in primary and secondary coils and there is no loss of energy; then

For about 100% efficiency, Power in primary = Power in secondary

$$V_P\, i_P = V_S\, i_S$$

$$\frac{i_S}{i_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{1}{r} \qquad ...(v)$$

In step up transformer, $N_S > N_P \quad \rightarrow \quad r > 1$;

So $\qquad V_S > V_P$ and $i_S < i_P$

i.e., step up transformer increases the voltage, but decreases the current.

In step down transformer, $N_S < N_P \rightarrow \quad r < 1$

so $\qquad V_S < V_P$ and $i_S > i_P$

i.e., step down transformer decreases the voltage, but increases the current.

Laminated core: The core of a transformer is laminated to reduce the energy losses due to eddy currents, so that its efficiency may remain nearly 100%.

In a transformer with 100% efficiency (say),

Input power = output power $V_P I_P = V_S I_S$

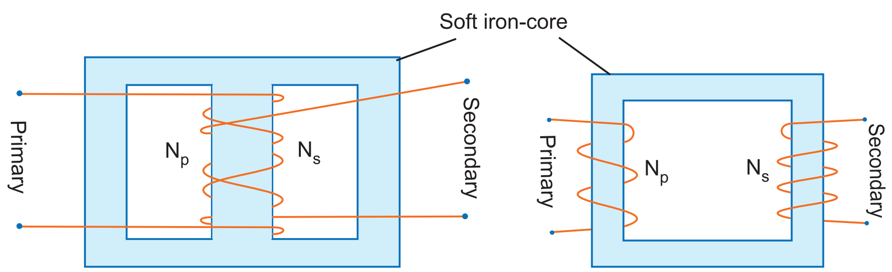
(*b*) The sources of energy loss in a transformer are (*i*) eddy current losses due to iron core (*ii*) flux leakage losses. (*iii*) copper losses due to heating up of copper wires (*iv*) hysteresis losses due to magnetisation and demagnetisation of core.

(*c*) When output voltage increases, the output current automatically decreases to keep the power same. Thus, there is no violation of conservation of energy in a step up transformer.

Q. 9. With the help of a diagram, explain the principle of a device which changes a low voltage into a high voltage but does not violate the law of conservation of energy. Give any one reason why the device may not be 100% efficient. **[*CBSE Sample Paper 2018*]**

Ans. Transformer changes a low voltage into a high voltage without voilating the law of conservation of energy.

Principle: When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.



(a) Two coils on top of each other (b)Two coils on separate limbs of the core

The device may not be 100% efficient due to following energy losses in a transformer:

(*i*) **Joule Heating:** Energy is lost due to heating of primary and secondary windings as heat (I^2Rt).

(*ii*) **Flux Leakage:** Energy is lost due to coupling of primary and secondary coils not being perfect, *i.e.*, whole of magnetic flux generated in primary coil is not linked with the secondary coil.

Q. 10. (*a*) Draw the diagram of a device which is used to decrease high *ac* voltage into a low *ac* voltage and state its working principle. Write four sources of energy loss in this device.

(*b*) A small town with a demand of 1200 kW of electric power at 220 V is situated 20 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets the power from the line through a 4000-220 V step-down transformer at a sub-station in the town. Estimate the line power loss in the form of heat. **[*CBSE 2019 (55/1/1)*]**

Ans. (*a*) Refer to Q. 8, Page no. 305.

(*b*) Demand of electric power = 1200 kW

Distance of town from power station = 20 km

Two wire = 20 × 2 = 40 km

Total resistance of line = 40 × 0.5 = 20 Ω

The town gets a power of 4000 volts

Power = voltage × current

$$I = \frac{1200 \times 10^3}{4000} = \frac{1200}{4} = 300 \text{ A}$$

The line power loss in the form of heat $= I^2 \times R$

$= (300)^2 \times 20$

$= 9000 \times 20 =$ **1800 kW**

Q. 11. A 2 μF capacitor, 100 W resistor and 8 H inductor are connected in series with an ac source.

(*i*) What should be the frequency of the source such that current drawn in the circuit is maximum? What is this frequency called?

(*ii*) If the peak value of emf of the source is 200 V, find the maximum current.

(*iii*) Draw a graph showing variation of amplitude of circuit current with changing frequency of applied voltage in a series *LRC* circuit for two different values of resistance R_1 and R_2 ($R_1 > R_2$).

(*iv*) Define the term 'Sharpness of Resonance'. Under what condition, does a circuit become more selective? **[*CBSE (F) 2016*]**

Ans. (*i*) For maximum frequency

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad 2\pi\nu \times 8 = \frac{1}{2\pi\nu \times 2 \times 10^{-6}} \quad \Rightarrow \quad (2\pi\nu)^2 = \frac{1}{16 \times 10^{-6}}$$

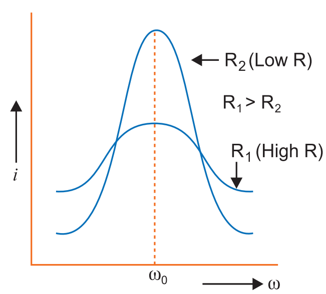
$$\Rightarrow \quad 2\pi\nu = \frac{1}{4 \times 10^{-3}} \quad \Rightarrow 2\pi\nu = \frac{10^3}{4}$$

$$\Rightarrow \quad \nu = \frac{250}{2\pi} = \mathbf{39.80\ s^{-1}}$$

This frequency is called resonance frequency.

(*ii*) Maximum current, $I_0 = \frac{E_0}{R} = \frac{200}{100} = \mathbf{2\ A}$ [E_0 maximum emf]

(*iii*)



(*iv*) $\frac{\omega_0}{2\Delta\omega}$ is measure of sharpness of resonance, where ω_0 is the resonant frequency and $2\Delta\omega$ is the bandwidth.

Circuit is more selective if it has greater value of sharpness. The circuit should have smaller bandwidth $\Delta\omega$.

Q. 12. (*i*) Draw a labelled diagram of *ac* generator. Derive the expression for the instantaneous value of the emf induced in the coil.

(*ii*) A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude 3.0 × 10^{-2} T. Calculate the maximum value of the current in the coil. **[*CBSE Delhi 2017*]**

Ans. (*i*) Refer to Q. 7, page 304.

(*ii*) Given, $N = 20$

$A = 200\ \text{cm}^2$

$= 200 \times 10^{-4}\ \text{m}^2$

$B = 3.0 \times 10^{-2}\ \text{T}$

$\omega = 50\ \text{rad s}^{-1}$

EMF induced in the coil

$$\varepsilon = NBA\omega \sin \omega t$$

Maximum emf induced

$$\varepsilon_{max} = NBA\omega$$
$$= 20 \times 3.0 \times 10^{-2} \times 200 \times 10^{-4} \times 50$$
$$= 600 \text{ mV}$$

Maximum value of current induced

$$I_{max} = \frac{\varepsilon_{max}}{R} = \frac{\mathbf{600}}{\boldsymbol{R}} \textbf{ mA}$$

Q. 13. (*i*) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(*ii*) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. [*CBSE Delhi 2017*]

Ans. (*i*) Refer to Q. 8, Page 305.

(*ii*) Given,

$$V_P = 2200 \text{ V}$$
$$N_P = 3000 \text{ turns}$$
$$V_S = 220 \text{ V}$$

We have,

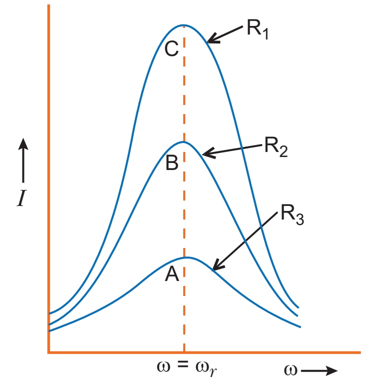
$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$N_S = \frac{V_S}{V_P} \times N_P$$

$$= \frac{220}{2200} \times 3000$$

$$N_S = \mathbf{300 \text{ turns}}$$

Q. 14. (*a*) What do you understand by 'sharpness of resonance' for a series *LCR* resonant circuit? How is it related with the quality factor '*Q*' of the circuit? Using the graphs given in the diagram, explain the factors which affect it. For which graph is the resistance (*R*) minimum? [*CBSE 2019 (55/4/1)*]



(*b*) A 2 μF capacitor , 100 Ω resistor and 8 H inductor are connected in series with an *ac* source. Find the frequency of the *ac* source for which the current drawn in the circuit is maximum.

If the peak value of emf of the source is 200 V, calculate the (*i*) maximum current, and (*ii*) inductive and capacitive reactance of the circuit at resonance.

Ans. (*a*) The circuit would be set to have a high sharpness of resonance, if the current in the circuit drops rapidly as the frequency of the applied *ac* source shifts from its resonant value.

Sharpness of resonance is measured by the quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Sharpness of resonance for given value of L and C or value of ω_r depends on R.

R is minimum for C.

(*b*)
$$\nu = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2 \times 3.14\sqrt{8 \times 2 \times 10^{-6}}}$$
$$= \frac{1000}{8 \times 3.14} = 39.81 \text{ or } \mathbf{40\ Hz} \text{ (approximately)}$$
$$V_0 = 200 \text{ V}$$

(*i*)
$$i_0 = \frac{V_0}{Z} = \frac{V_0}{R} \quad (\because Z = R \text{ at resonance})$$
$$= \frac{200}{100} = \mathbf{2\ A}$$

(*ii*) At resonance
$$X_L = X_C$$
$$X_L = \omega L = 2\pi\nu L$$
$$= 2\pi \times 39.81 \times 8 = \mathbf{2000\ \Omega}$$

Self-Assessment Test

Time allowed: 1 hour **Max. marks: 30**

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) The average power dissipation in a pure capacitance is:

(*a*) $\frac{1}{2}CV^2$ (*b*) CV^2

(*c*) $\frac{1}{4}CV^2$ (*d*) zero

(*ii*) In an *ac* series circuit, the instantaneous current is maximum when the instantaneous voltage is maximum. The circuit element connected to the source will be

(*a*) pure inductor (*b*) pure capacitor

(*c*) pure resistor (*d*) combination of a capacitor and an inductor

(*iii*) R, L and C represent the physical quantities resistance, inductance and capacitance respectively. Which one of the following combinations has dimension of frequency?

(*a*) $\frac{1}{\sqrt{RC}}$ (*b*) $\frac{R}{L}$

(*c*) $\frac{1}{LC}$ (*d*) $\frac{C}{L}$

2. Fill in the blanks. **(2 × 1 = 2)**

(*i*) One complete set of positive and negative values of alternating current or emf is called ______________.

(*ii*) The core of transformer, if laminated, ______________ eddy currents.

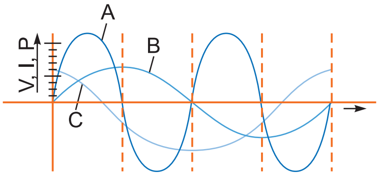
3. The power factor of an *ac* circuit is 0.5. What is the phase difference between voltage and current in this circuit? **1**

4. Draw a graph to show variation of capacitive-reactance with frequency in an *ac* circuit. **1**

5. A device '*X*' is connected to an *ac* source $V = V_0$. The variation of voltage, current and power in one complete cycle is shown in the following figure.

(*i*) Which curve shows power consumption over a full cycle?

(*ii*) Identify the device '*X*'. **1**



6. Prove that an ideal capacitor, in an *ac* circuit does not dissipate power. **2**

7. Derive an expression for the impedance of an *ac* circuit consisting of an inductor and a resistor. **2**

8. A 15.0 μF capacitor is connected to 220 V, 50 Hz source. Find the capacitive reactance and the rms current. **2**

9. How much current is drawn by the primary coil of a transformer which steps down 220 V to 22 *V* to operate a device with an impedance of 220 Ω? **2**

10. You are given three circuit elements *X*, *Y* and *Z*. When the element *X* is connected across an *ac* source of a given voltage, the current and the voltage are in the same phase. When the element *Y* is connected in series with *X* across the source, voltage is ahead of the current in phase by π/4. But the current is ahead of the voltage in phase by π/4 when *Z* is connected in series with *X* across the source. Identify the circuit elements *X*, *Y* and *Z*.

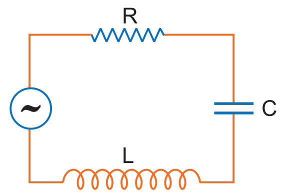
When all the three elements are connected in series across the same source, determine the impedance of the circuit.

Draw a plot of the current versus the frequency of applied source and mention the significance of this plot. **3**

11. A voltage $v = v_m \sin \omega t$ applied to a series *LCR* circuit, drives a current in the circuit given $i = i_m \sin(\omega t + \phi)$. Deduce the expression for the instantaneous power supplied by the source. Hence, obtain the expression for the average power.

Define the terms 'power factor' and 'wattless current', giving the examples where power factor is maximum and the circuit where there is wattless current. **3**

12. A series *LCR* circuit with $L = 4.0$ H, $C = 100$ μF and $R = 60$ Ω is connected to a variable frequency 240 V source as shown in figure. **3**



Calculate:

(*i*) The angular frequency of the source which drives the circuit at resonance;

(*ii*) The current at the resonating frequency;

(*iii*) The rms potential drop across the inductor at resonance.

13. (*a*) Using phasor diagram for a series *LCR* circuit connected to an ac source of voltage $v = v_0$ sin ωt, derive the relation for the current flowing in the circuit and the phase angle between the voltage across the resistor and the net voltage in the circuit.

(*b*) Draw a plot showing the variation of the current *I* as a function of angular frequency 'ω' of the applied *ac* source for the two cases of a series combination of (*i*) inductance L_1, capacitance C_1 and resistance R_1 and (*ii*) inductance L_2, capacitance C_2 and resistance R_2 where $R_2 > R_1$. Write the relation between L_1, C_1 and L_2, C_2 at resonance. Which one, of the two, would be better suited for fine tuning in a receiver set? Give reason. **5**

Answers

1. (*i*) (*d*) (*ii*) (*c*) (*iii*) (*b*)

2. (*i*) cycle (*ii*) decreases

9. 0.1 A, 0.01 A

12. (*i*) ω = 50 rad/s; (*ii*) $I = 4$ A; (*iii*) $V_L = 800$ V