Gravitation

1. Introduction

The motion of celestial bodies such as the moon the earth, the planets, etc, has been a subjected of great interest for a long time. Famous Indian astronomer and mathematician, Aryabhatta, studied these motions in great detail, most likely in the 5th century A.D., and wrote his conclusions in his book Aryabhatta. He established that the earth revolves about its own axis. He also gave description of motion of other celestial bodies as seen from the earth.

After Aryabhatta, further developments in the study of gravitational forces are as under :

(i) The hypothesis about planetary motion given by Nicolaus Copernicus (1473-1543).

(ii) The careful experimental measurements of the positions of the planets and the sun by Tycho Brahe (1546 - 1601)

(iii) Analysis of the data and the formulation of empirical laws by Johannes Kepler (1571 - 1630).

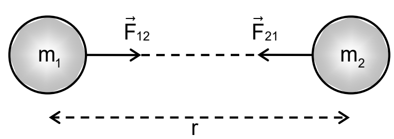
(iv) The development of a general theory by Isaac Newton (1642 - 1727)

2. Newton's Law of Gravitation

It states that every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

If $|\vec{F}_{12}| = |\vec{F}_{21}| = F$, then

$F \propto m_1 m_2$ and $F \propto \dfrac{1}{r^2}$



so $F \propto \dfrac{m_1 m_2}{r^2}$

$$\therefore \qquad F = \frac{Gm_1 m_2}{r^2}$$

Newton further generalised the law by saying that not only the earth but all material bodies in the universe attract each other according to equation with same value of G. The constant G is called universal constant of gravitation and its value is found to be 6.67×10^{-11} N–m^2/kg^2. Equation is known as the universal law of gravitation.

Note : This formula is applicable only for spherically symmetric masses or point masses.

2.1 Vector form of Law of Gravitation

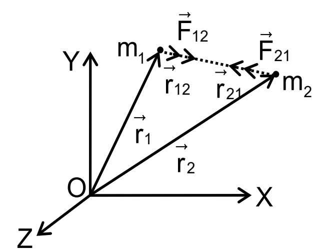
Let $\vec{r}_{12}$ = Displacement vector from m_1 to m_2

$\vec{r}_{21}$ = Displacement vector from m_2 to m_1

$\vec{F}_{2,1}$ = Gravitational force exerted on m_2 by m_1

$\vec{F}_{1,2}$ = Gravitational force exerted on m_1 by m_2

$$\vec{F}_{1,2} = \frac{Gm_1 m_2}{r_{21}^2}\,\hat{r}_{21} = -\frac{Gm_1 m_2}{r_{21}^3}\,\vec{r}_{21}$$



Negative sign shows that

(i) The direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$

(ii) The gravitational force is attractive in nature

Similarly $\vec{F}_{12} = -\frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21}$

or $\vec{F}_{21} = -\frac{Gm_1m_2}{r_{12}^3}\vec{r}_{12} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$

The gravitational force between two bodies are equal in magnitude and opposite in directions.

- **Universal Gravitational Constant 'G'**
- Universal Gravitational Constant is a scalar quantity.
- **Value of G : SI :** $G = 6.67 \times 10^{-11}$ N-m^2/kg^2;
  CGS : $G = 6.67 \times 10^{-8}$ dyne-cm^2/g^2 **Dimensions :** $[M^{-1}L^3T^{-2}]$
- Its value is same throughout the universe; G does not depend on the nature and size of the bodies; it does not depend even upon the nature of the medium between the bodies.
- Its value was first found out by the scientist **"Henry Cavendish"** with the help of "Torsion Balance" experiment.

2.2 Properties of Gravitational Force

- Gravitational force is always attractive.
- Gravitational forces are developed in the from of action and reaction pair. Hence they obey Newton's third law of motion.
- It is independent of the nature of medium in between two masses.
- Gravitational forces are central forces as they act along the line joining the centres of the two bodies.
- Gravitational forces are conservative forces so work done by gravitational force does not depends on path.
- If any particle moves along a closed path under the action of gravitational force then the work done by this force is always zero.
- Gravitational force is weaker than the electrostatic and nuclear forces.
- Force developed between any two masses is called gravitational force and force between Earth and any body is called force of gravity.
- Gravitational force holds good over a wide range of distances. It is found true from interplanetary distances to interatomic distances.
- It is a two body interaction i.e., gravitational force between the two particles is independent of the presence or absence of other bodies or particles.

Example 1:

Two spherical balls of mass 10 kg each are placed 100 m apart. Find the gravitational force of attraction between them.

Solution:

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 10 \times 10}{(100)^2} = 6.67 \times 10^{-13}\ N$$

Example 2:

Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial acceleration of the heavier particle.

Solution:

Force exerted by one particle on another is

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times2}{(0.5)^2} \qquad = 5.34 \times 10^{-10}\ \text{N}$$

Acceleration of heavier particle $= \dfrac{F}{m_2} = \dfrac{5.3\times10^{-10}}{2} = 2.6 \times 10^{-10}\ \text{ms}^{-2}$

Note : This example shows that gravitational force is quite weak but this is the only force keep binds our solar system and also the universe comprising of all galaxies and other interstellar system.

3. Law of Superposition of Forces

Gravitational force follows law of superposition which states

The total gravitational force on a particle due to a number of particles is the resultant of the forces of attraction exerted on the given particle due to the individual particles i.e. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 ....$

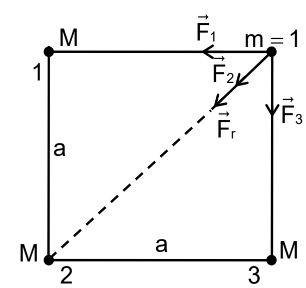
Key Points

- It may be worth noting that a uniform spherical shell of matter or a uniform solid sphere attracts a particle outside their periphery as if their mass is concentrated at their centre.

Example 3:

Three masses, each equal to M are placed at the three corners of a square of side a. Calculate the force of attraction on unit mass placed at the fourth corner.

Solution:



Force on m due to masses at corners 1 and 3 are $\vec{F}_1$ and $\vec{F}_3$

with $F_1 = F_3 = \dfrac{GMm}{a^2}$

resultant of $\vec{F}_1$ and $\vec{F}_3$ is $F_r = \sqrt{2}\,\dfrac{GMm}{a^2}$, and its direction is along the diagonal

i.e., towards corner 2

Force on m due to mass M at 2 is

$$\vec{F}_r = \frac{GMm}{\left(\sqrt{2}a\right)^2} = \frac{GMm}{2a};$$

F_r and F_2 act in the same direction. Resultant of these two is the net force :

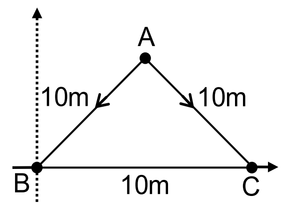
$$F_{net} = F_r + F_2 = \frac{GMm}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

along the diagonal

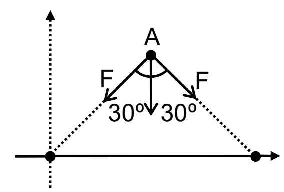
Since m = 1 kg So. $F_{net} = \dfrac{GM}{a^2}\left(\sqrt{2}+\dfrac{1}{2}\right)$

Example 4:

Three particles of mass 2kg each are placed as shown. Find out net gravitational force on 'A'.



Solution:



Net force on 'A' = $2\,F\cos\dfrac{60^\circ}{2} = F\sqrt{3}$

Where F is the force exerted by B & C on A

$$F_{net} = \frac{G(2)(3)}{(10)^2}\sqrt{3}$$

$$F_{net} = 0.46 \times 10^{-11}\ \text{N}$$

Example 5:

Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side 'a'. The only forces acting on the particles are their mutual gravitational forces. It is intended that each particle moves along a circle while maintaining their original separation 'a'. Determine the initial velocity that should be given to each particle and the time period of the circular motion.

Solution:

The resultant force on particle at A due to other two particles is

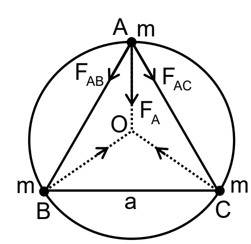
$$F_A = \sqrt{F_{AB}^2 + F_{AC}^2 + 2\,F_{AB}F_{AC}\cos 60^\circ}$$

$$= \sqrt{3}\,\frac{Gm^2}{a^2} \qquad \text{......(i)}$$

$$\left[\because F_{AB} = F_{AC} = \frac{Gm^2}{a^2}\right]$$

Radius of the circle $r = \dfrac{a}{\sqrt{3}}$

If each particle is given a tangential velocity v, so that the resultant force acts as the centripetal force,



then $\dfrac{mv^2}{r} = \sqrt{3}\,\dfrac{mv^2}{a}$ ....(ii)

From (i) and (ii)

$$\sqrt{3}\,\frac{mv^2}{a} = \frac{Gm^2\sqrt{3}}{a^2} \Rightarrow v = \sqrt{\frac{Gm}{a}}$$

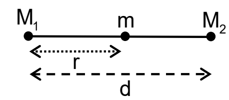
$$\text{Time period } T = \frac{2\pi r}{v} = \frac{2\pi a}{\sqrt{3}}\sqrt{\frac{a}{Gm}} = 2\pi\sqrt{\frac{a^2}{3Gm}}$$

Example 6:

Two stationary particles of masses M_1 and M_2 are 'd' distance apart. A third particle lying on the line joining the particles, experiences no resultant gravitational force. What is the distance of this particle from M_1?

Solution:

Let m be the mass of the third particle



Force on m towards M_1 is $F_1 = \dfrac{GM_1m}{r^2}$

Force on m towards M_2 is $F_2 = \dfrac{GM_2m}{(d-r)^2}$

Since net force on m is zero $\therefore$ $F_1 = F_2$

$$\Rightarrow \frac{GM_1m}{r^2} = \frac{GM_2m}{(d-r)^2} \Rightarrow \left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1}$$

$$\Rightarrow \frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}} \Rightarrow r = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right]$$

Example 7:

A mass (M) is split into two parts (m) and (M-m) which are then separated by a certain distance. What ratio $\dfrac{m}{M}$ will maximise the gravitational force between them?

Solution:

If r is the distance between m and (M - m), the gravitational force will be

$$F = G\frac{m(M-m)}{r^2} = \frac{G}{r^2}(mM - m^2)$$

for F to be maximum $\dfrac{dF}{dm} = 0$ and $\dfrac{d^2F}{dm^2} < 0$ as

M and r are constants,

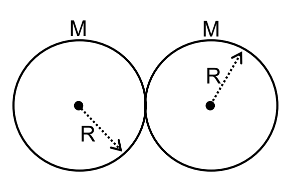
i.e., $\dfrac{d}{dm}\left[\dfrac{G}{r^2}(mM - m^2)\right] = 0$

i.e., $M - 2m = 0$ $\quad\left[\because \dfrac{G}{r^2} \neq 0\right]$

or $\dfrac{m}{M} = \dfrac{1}{2}$, i.e., the force will be maximum when the two parts are identical

Example 8:

Two solid spheres of same size of a certain metal are placed in contact with each other. Prove that the gravitational force acting between them is directly proportional to the fourth power of their radius.



Solution:

The weights of the spheres may be assumed to be concentrated at their centres.

$$\text{So } F = \frac{G\left[\frac{4}{3}\pi R^3\rho\right]\times\left[\frac{4}{3}\pi R^3\rho\right]}{(2R)^2} = \frac{4}{9}(G\pi^2\rho^2)\ R^4$$

$\therefore \qquad F \propto R^4$

Concept Builder-1

Q.1 Two bodies of mass 3 kg and 4 kg each placed 50 m apart. Find the gravitational force of attraction between them in C.G.S. unit.

Q.2 Gravitational force on 2 kg and 5 kg bodies is 10 newton then find separation between bodies ?

Q.3 Four masses, each equal to M are placed at the four corners of a square of side a. Calculate the force of attraction on another mass m_1 kept at the centre of the square.

Q.4 Three identical particles each of mass (m) are placed at the three corners of an equilateral triangle of side "a". Find the gravitational force exerted on one body due to the other two.

Q.5 Three identical point masses, each of mass 1 kg lie in the x-y plane at points (0, 0) (0, 0.2m) and (0.2m, 0) respectively. The gravitational force on the mass at the origin is :

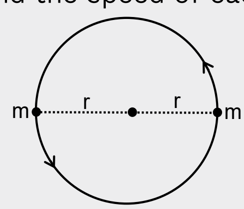
(1) $1.67 \times 10^{-11}\ (\hat{i}+\hat{j})$ N

(2) $3.34 \times 10^{-10}\ (\hat{i}+\hat{j})$ N

(3) $1.67 \times 10^{-9}\ (\hat{i}+\hat{j})$ N

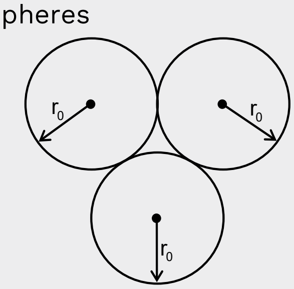
(4) $3.34 \times 10^{-10}\ (\hat{i}-\hat{j})$ N

Q.6 Two particles each of equal mass (m) move along a circle of radius (r) under the action of their mutual gravitational attraction. Find the speed of each particle.



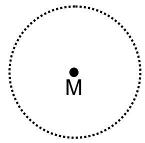
Q.7 Where a mass of 3 kg has to be placed near a combination of two masses 9 kg and 16 kg placed 14 m apart, such that it experiences no force.

Q.8 There identical sphere (density ρ_0 & radius r_0) are arranged as shown what is the net force experienced by any one of the spheres



4. Gravitational Field and it's Intensity Gravitational Field

The gravitational field is the space around a mass or an assembly of masses within which it can exert gravitational forces on other masses.



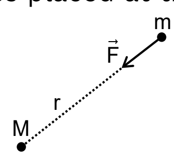
Gravitational field

Theoretically speaking, the gravitational field extends up to infinity. However, in actual practice the gravitational field may become too weak to be measured beyond a particular distance.

- **Gravitational Field Intensity**

  The gravitational field intensity at a point within a gravitational field is defined as the gravitational force exerted on unit mass placed at that point.

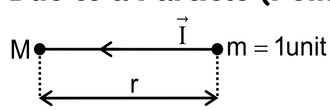
$$\boxed{\vec{I} = \frac{\vec{F}}{m}}$$



Gravitational field intensity is a vector quantity whose direction is same as that of the gravitational force. Its SI unit is 'N/kg'.

Dimensions of intensity $\frac{[F]}{[m]} = \frac{[M^1L^1T^{-2}]}{[M^1]} = [M^0L^1T^{-2}]$

5. Gravitational Field Intensity Due to a Particle (Point-Mass)



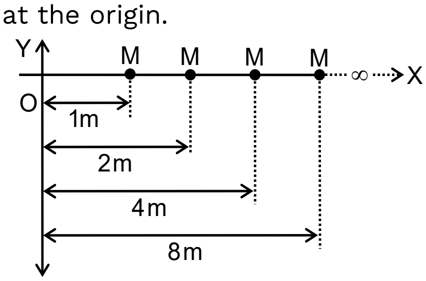
Gravitational field intensity = gravitational force exerted on unit mass

$$\Rightarrow \vec{I} = \frac{GM}{r^2}(-\hat{r}) = \frac{GM}{r^3}(-\hat{r})$$

where 'M' is the mass of that particle due to which intensity is to be found.

Example 9:

Infinite particles each of mass 'M' are placed at positions x = 1 m, x = 2m, x = 4m ..... ∝. Find the gravitational field intensity at the origin.



Solution:

$$\vec{I}_{net} = \vec{I}_1 + \vec{I}_2 + \vec{I}_3 + \vec{I}_4 + ...\infty \text{ terms}$$

$$\frac{GM}{(1)^2}\hat{i} + \frac{GM}{(2)^2}\hat{i} + \frac{GM\hat{i}}{(4)^2} ....\infty$$

terms = $GM\hat{i}\left(1+\frac{1}{4}+\frac{1}{16}+.......\infty\right)$

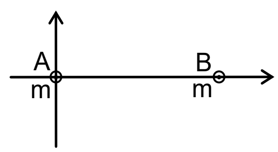
$\left[\text{Here in the GP } a = 1 \text{ and } r = \frac{1}{4}\right]$

So, $\vec{I}_{net} = GM\hat{i}\left[\frac{1}{1-\frac{1}{4}}\right] = GM\hat{i}\left[\frac{1}{\left(\frac{3}{4}\right)}\right]$

$\Rightarrow \vec{I}_{net} = \frac{4}{3}GM\hat{i}$

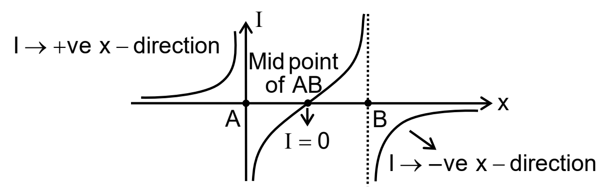
Example 10:

Draw the variation of Intensity along x-axis If two masses are arranged as shown in figure.



Solution:

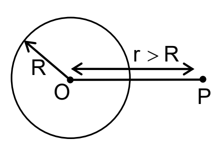
Since two masses are identical. Intensity will be zero at the mid point of AB. Closer to A intensity will be directed towards A & Closer to B intensity will be directed towards A & closer to B intensity will be directed towards B. As we know $I \propto \frac{I}{r^2}$ so variation will be as follows



6. Gravitational Field Intensity Due to a Spherical Shell

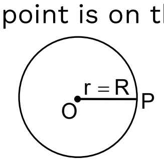
Case I : If r > R, the point is outside the shell then

$\vec{I}_{out} = \frac{GM}{r^2}(-\hat{r})$



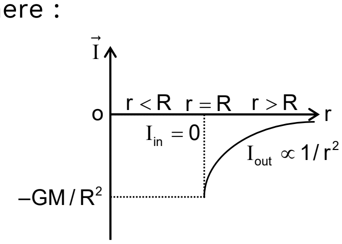
Case II : If r = R, the point is on the surface then

$\vec{I}_{surface} = \frac{GM}{R^2}(-\hat{r})$



Case III : If r < R, the point is inside the shell then I = 0

$\Rightarrow \vec{I}$ v/s r graph for hollow sphere :

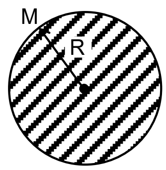


7. Gravitational Field Intensity Due to Spherical Mass Distribution

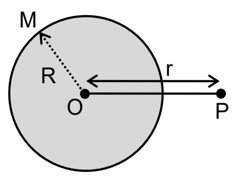
If the observation point is located on the surface or outside the surface then the spherical mass can be taken as a particle which is situated at the centre of the sphere. ie. point mass.

For solid sphere

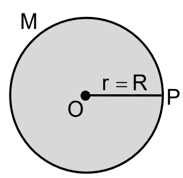
Let 'M' be the mass of sphere, 'R' the radius of sphere and 'r' the distance of the point under consideration from the centre of sphere.



$\Rightarrow$ **Case I :** When r > R, i.e., outside the sphere then $\vec{I}_{out} = \frac{GM}{r^2}(-\hat{r})$

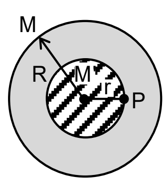


$\Rightarrow$ **Case II :** When r = R, i.e., at the surface then $\vec{I}_{surface} = \frac{GM}{R^2}(-\hat{r})$



$\Rightarrow$ **Case III :** When r < R, i.e., inside the sphere then $\vec{I}_{in} = \frac{GM'}{r^2}(-\hat{r})$ ....(i)

We know, $\frac{M'}{M} = \frac{V' \times \rho'}{V \times \rho} = \frac{\frac{4}{3}\pi r^3 \times \rho}{\frac{4}{3}\pi R^3 \times \rho} = \frac{r^3}{R^3}$ $\Rightarrow$ $M' = \frac{Mr^3}{R^3}$



Putting the expression for M' in above equation (i) we get

$I_{in} = \frac{GMr^3}{r^2R^3}(-\hat{r})$ $\Rightarrow$ $I_{in} = \frac{GMr}{R^3}(-\hat{r})$

Important conclusions

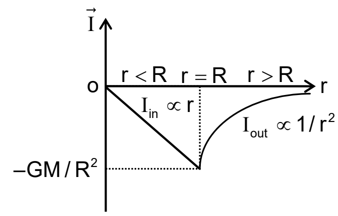
(1) $I_{out} = \frac{GM}{r^2}$ $\therefore I_{out} \propto \frac{1}{r^2}$

(2) $I_{sur} = \frac{GM}{R^2}$

(3) $I_{in} = \frac{GMr}{R^3}$ $\therefore I_{in} \propto r$

(4) So, $I_{max} = I_{sur} = \frac{GM}{R^2}$

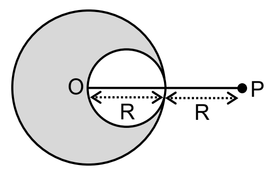
(5) $I_{centre} = \frac{GM(0)}{R^3}$ $\therefore I_{min} = I_{centre} = 0$



Graph between $\vec{I}$ and 'r' for a solid sphere

Example 11:

A solid sphere of uniform density and radius R exerts a gravitational force of attraction F_1 on a particle P, distant 2R from the centre of the sphere. A spherical cavity of radius R/2 is now formed in the sphere as shown in figure. The sphere with cavity now applies a gravitational force F_2 on the same particle P. Find the ratio F_2/F_1.



Solution:

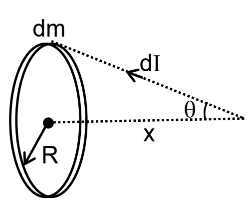
$F_1 = \frac{GMm}{4R^2}$, F_2 = force due to whole sphere – force due to the sphere forming the cavity

$= \frac{GMm}{4R^2} - \frac{GMm}{18R^2} \Rightarrow \frac{7GMm}{36R^2} \quad \therefore \frac{F_2}{F_1} = \frac{7}{9}$

8. Gravitational Field Intensity Due to a Ring

8.1 Intensity Due to a Ring at Axial Point

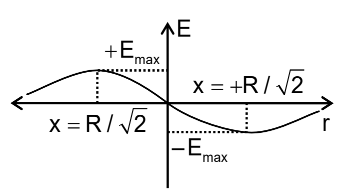
$dI = \frac{Gdm}{\left(\sqrt{R^2 + x^2}\right)}$



Now only axial component will be added other components will be concluded.

So $I = \int dI \cos\theta = \int \frac{Gdm}{R^2 + x^2} \cdot \frac{x}{(R^2 + x^2)^{1/2}}$

$I = \frac{GMx}{(R^2 + x^2)^{3/2}}$



- When x <<< R

  $I = \frac{GMx}{R^3}$

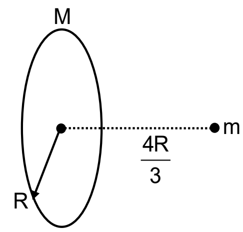
  At the centre of ring x = 0 so $E_0 = 0$

  I will be maximum when $\frac{dI}{dx}$ at $x = \pm \frac{R}{\sqrt{2}}$,

  $I_{max} = \frac{2GM}{3\sqrt{3}R^2}$

Example 12:

Find the force exerted by the particle on the ring as shown in figure.



Solution:

Intensity due to the ring at the location of point mass

$$I = \frac{GM\left(\frac{4R}{3}\right)}{\left[R^2 + \left(\frac{4R}{3}\right)^2\right]^{3/2}}$$

$$I = \frac{36}{125}\frac{GMm}{R^2}$$

Force on point mass

F =

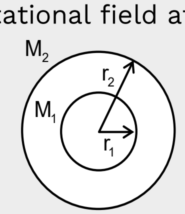
Now according to newton's third law,

|Force exerted by ring on particle| = |Force exerted by particle on ring|

So $F_{Ring} = \frac{36}{125}\frac{GMm}{R^2}$

Concept Builder-2

Q.1 Two concentric shells of masses M_1 and M_2 are having radii r_1 and r_2. Which of the following is the correct expression for the gravitational field at a distance r ?



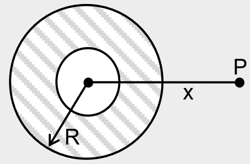
(1) $\frac{G(M_1+M_2)}{r^2}$, for $r < r_1$

(2) $\frac{G(M_1+M_2)}{r^2}$, for $r < r_2$

(3) $\frac{GM_2}{r^2}$, for $r_1 < r < r_2$

(4) $\frac{GM_1}{r^2}$, for $r_1 < r < r_2$

Q.2 From a solid sphere of mass M & radius r, as sphere of diameter r is cut off as shown. What is the intensity at a point x distance away from the centre.



9. Acceleration Due to Gravity

9.1 Gravity

In Newton's law of gravitation, the force of attraction between any two bodies is gravitation force. If one of the bodies is Earth then the gravitation is called 'gravity'. Hence, gravity is the force by which Earth attracts a body towards its centre. It is a special case of gravitation.

9.2 Acceleration Due to Gravity Near Earth's Surface

Let us assume that Earth is a uniform sphere of mass M and radius R. The magnitude of the gravitational force of Earth on a particle of mass m, located outside the Earth at a distance r from its centre, is $F = \frac{GMm}{r^2}$. Now according to Newton's second law $F = ma_g$

Therefore $a_g = \frac{GM}{r^2}$ ......(i)

At the surface of Earth, acceleration due to gravity $g = \frac{GM}{R^2} = 9.8 \text{ m/s}^2$

However any a_g value measured at a given location will differ from the a_g value calculated according to equation due to any three reasons :-

(i) Earth's mass is not distributed uniformly.

(ii) Earth is not a perfect sphere and

(iii) Earth rotates.

Percentage Change in 'g'

In terms of density $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \times \rho$

$\therefore\ g = \frac{4}{3}\pi G R \rho$ If ρ is constant then $g \propto R$

- If M is constant then $g \propto \frac{1}{R^2}$; For % variation in 'g' upto 5% $\boxed{\frac{\Delta g}{g} = -2\frac{\Delta R}{R}}$
- If mass (M) and radius (R) correspond to a planet and if small changes ΔM and ΔR occur in (M) and (R) respectively then

  by $g = \frac{GM}{R^2}$ $\therefore\ \frac{\Delta g}{g} = \frac{\Delta M}{M} - 2\frac{\Delta R}{R}$

  If R is constant then $\frac{\Delta g}{g} = \frac{\Delta M}{M}$;

  If M is constant then $\frac{\Delta g}{g} = -2\frac{\Delta R}{R}$

Example 13:

What will be acceleration due to gravity at a planet having diameter twice the diameter of earth & mass 3 times that of earth.

Solution:

As we know $g_e = \dfrac{GM_e}{R_e^2}$ & $g_P = \dfrac{GM_P}{R_e^2}$

So $\dfrac{g_p}{g_e} = \dfrac{M_p}{M_e}\left(\dfrac{R_e}{R_p}\right)^2 = \dfrac{3}{4}$

$\Rightarrow g_p = \dfrac{3}{4}g_e = 7.362\ \text{m/s}^2$

Example 14:

What will be the acceleration due to gravity on a planet having density 3 times that of earth & radius half of earth.

Solution:

As we know

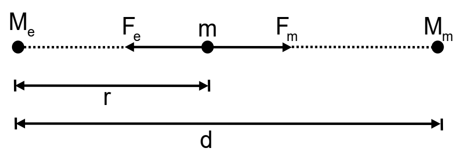
$g_e = \dfrac{4}{3}\pi G\rho_e R_e,$ $g_p = \dfrac{4}{3}\pi G\rho_p R_p,$

$\Rightarrow \dfrac{g_p}{g_e} = \dfrac{\rho_p}{\rho_e}\left(\dfrac{R_p}{R_e}\right) = \dfrac{3}{2} \Rightarrow g_p = \dfrac{3}{2}g_e$

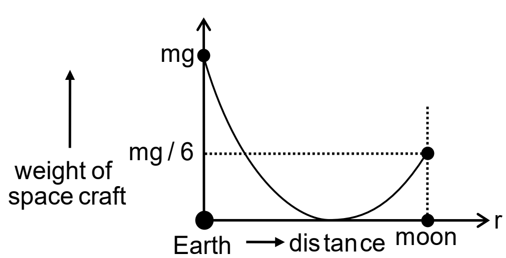
Example 15:

Draw a rough sketch of the variation in weight of a spacecraft which moves from earth to moon.

Solution:



Net weight = $F_e \sim F_m = \left|\dfrac{GM_e}{r^2} - \dfrac{GM_m}{(d-r)^2}\right| m$

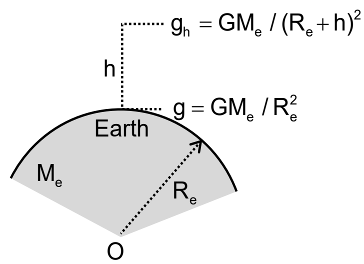


10. Variation in Acceleration Due to Gravity

10.1 Above the Surface of Earth Due to Altitude (Height)

From diagram

$$\frac{g_h}{g} = \frac{R_e^2}{(R_e+h)^2} = \frac{R_e^2}{R_e^2\left[1+\frac{h}{R_e}\right]^2} = \left(1+\frac{h}{R_e}\right)^{-2}$$



By binomial expansion $\left(1+\frac{h}{R_e}\right)^{-2} = \left(1-\frac{2h}{R_e}\right)$

[If h << R_e, then higher power terms become negligible]

$\therefore\ g_h = g\left[1-\frac{2h}{R_e}\right]$

Note :

(i) This formula is valid if h is upto 5% of earth's radius. (320 km from earth's surface)

(ii) If h is greater than 5% of the earth's radius we use $g_n = \frac{GM_e}{(R_e+h)^2}$

Example 16:

At what height above the earth surface value of gravity is 98% of gravity at earth surface :

Solution:

As we know $\frac{\Delta g}{g} = -\frac{2h}{R_e}$

So $-2\% = -\frac{2h}{R_e} \Rightarrow h = 64\text{Km}$

Note : It can be understood (& memorised) that value of g decreases by 1% after moving every 32 Km.

Example 17:

At what height above the Earth's surface will be the acceleration due to gravity $1/9^{th}$ of its value at the Earth's surface ? (Radius of Earth is 6400 km)

Solution:

Acceleration due to gravity at height h is

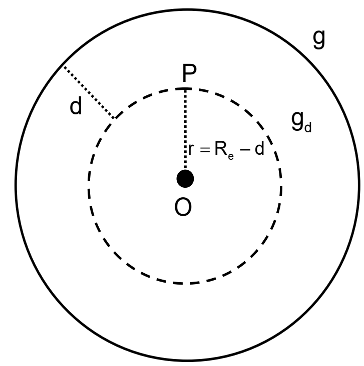
$$g' = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = \frac{g}{9} \quad \Rightarrow \quad \left(1+\frac{h}{R_e}\right) = 3$$

$\Rightarrow \quad h = 2R_e = 12800$ km.

10.2 Variation in 'g' inside Earth Surface Due to Depth

Assuming that the density of Earth remains same throughout the volume

At Earth's surface : $g = \frac{4}{3}\pi G R_e \rho$ ......(i)



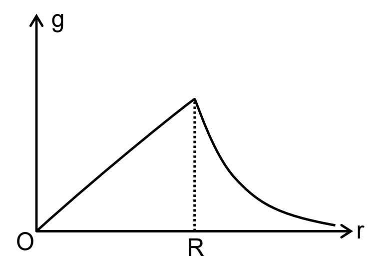
At a depth d inside the Earth :

For point P only mass of the inner sphere is effective $g_d = \frac{GM'}{r^2}$

Mass of sphere of radius r = M'

$$M' = \frac{4}{3} pr^3\ r = \frac{4}{3} pr^3 \times \frac{M_e}{4/3\pi R^3} = M' = \frac{M_e}{R^3}\ r^3$$

$$g_d = \frac{G}{r^2} \times \frac{M_e r^3}{R_e^3} = \frac{GM_e}{R_e^2} \times \frac{r}{R_e} = \frac{GM_e}{R_e^2} \times \frac{R_e - d}{R_e}$$



(Taking direction towards centre of earth as positive)

$g_d = g\left[1 - \frac{d}{R_e}\right]$ valid for any depth

$\Delta g_d = g - g_d$ = decrement in g with depth

$$= g - g\left[1 - \frac{d}{R_e}\right] \quad \Rightarrow \quad \frac{\Delta g_e}{g} = \frac{d}{R_e}$$

Example 18:

At what depth below the Earth's surface the acceleration due to gravity is decreased by 1%?

Solution:

$$\frac{\Delta g_d}{g} = \frac{d}{R_e} \Rightarrow \frac{1}{100} = \frac{d}{6400} \qquad \therefore d = 64 \text{ km}$$

Example 19:

At which height above earth's surface is the value of 'g' same as in a 100 km deep mine?

Solution:

At height 'h' above the earth surface (h < 5% R_e)

$$\frac{\Delta g}{g} = \frac{-2h}{R_e}$$

At depth x below the earth surface

$$\frac{\Delta g}{g} = \frac{-x}{R_e}$$

For g to be same, Δg will also be same

So $\frac{2h}{R_e} = \frac{x}{R_e} \Rightarrow h = \frac{x}{2} = \frac{100}{2}$ km, So h = 50 km

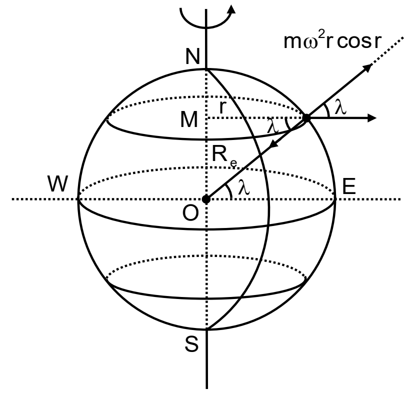
10.3 Variation in 'g' Due to Rotation

Net force on particle at P

$mg' = mg - mr\omega^2\cos\lambda$

$g' = g - r\omega^2\cos\lambda$ from ΔOMP $r = R_e\cos\lambda$ where λ = Latitude

Substituting for r we have $g' = g - R_e\,\omega^2\cos^2\lambda$



- At the equator ($\lambda = 0°$)

  $g_{eq} = g - \omega^2 R\cos^2(0°)$ g_{min} or
- At the poles ($\lambda = 90°$)

  $g_{pole} = g - \omega^2 R\cos^2(90°) = g - \omega^2 R(0)$: g_{max} or $g_{pole} = g$

It means that acceleration due to gravity at the poles does not depend upon the angular velocity or rotation of earth.

Condition of weightlessness on Earth's surface:

If apparent weight of body is zero then angular speed of Earth can be calculated as $mg' = mg - mR_e\omega^2\cos^2\lambda$

$$0 = mg - mR_e\omega^2\cos^2\lambda \Rightarrow w = \frac{1}{\cos\lambda}\sqrt{\frac{g}{R_e}}$$

But at equator $\lambda = 0°$

$$\therefore w = \sqrt{\frac{g}{R_e}} = \frac{1}{800} \text{ rad/s} = 0.00125 \text{ rad/s} = 1.25 \times 10^{-3} \text{ rad/s}$$

Note:

- If Earth were to rotate with 17 times of its present angular speed then bodies lying on equator would fly off into the space. Time period of Earth's rotation in this case would be 1.4 h.
- If Earth stops rotating about its own axis, then the apparent weight of bodies or effective acceleration due to gravity will increase at all the places except poles.

Example 20:

Determine the speed with which Earth would have to rotate about its axis so that a person on the equator weighs $\frac{3}{5}$th of its present value. Write your answer in terms of g and R.

Solution:

Weight on the equator

$$W' = \frac{3}{5}W \Rightarrow \frac{3}{5}mg = mg - m\omega^2 R \quad \Rightarrow \omega = \sqrt{\frac{2g}{5R}}$$

Example 21:

Which of the following statements are true about acceleration due to gravity?

(1) 'g' decreases in moving away from the centre of earth if r > R

(2) 'g' decreases in moving away from the centre of earth if r < R

(3) 'g' is zero at the centre of earth

(4) 'g' decreases if earth stops rotating on its axis

Solution:

Variation of g with distance : if r > R then $g \propto \frac{1}{r^2}$

$\therefore$ (1) is correct

If r < R then $g \propto r$

$\therefore$ (2) is incorrect & (3) is correct

variation of g with ω : $g' = g - \omega^2 R\cos^2\lambda$

If $\omega = 0$ then g will not change at poles where $\cos\lambda = 0$. While at other points g increases

$\therefore$ (4) is incorrect

10.4 Variation due to Shape of Earth:

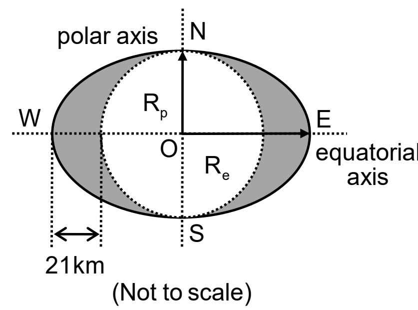
From the diagram

$R_p = R_e$ ($R_e = R_p + 21$ km) $g_p = \frac{GM_e}{R_p^2}$

and $g_e = \frac{GM_e}{(R_p + 21)^2}$

$\therefore$ $g_e < g_p$

By putting the values



Example 22:

At sea level the value of g is minimum at :

(1) the equator (2) 45° north latitude

(3) 45° south latitude (4) the pole

Solution:

At the equator, radius of earth is largest so g at equator will be minimum.

Concept Builder-3

Q.1 The radius of Earth is about 6400 km and that of Mars is 3200 km. The mass of Earth is 10 times that of Mars. An object weighs 200 N on the surface of Earth. Its weight on the surface of Mars will be :-

(1) 80 N (2) 40 N (3) 20 N (4) 8 N

Q.2 What will be the weight of a person on a planet having mass M (=4 Me) & diameter D(= 2Re). If he weighs 600 N on earth.

Q.3 What will be the acceleration due to gravity on a planet having density 2 times that of earth & diameter half of radius of earth ?

Q.4 A stone dropped from a height 'h' reaches the Earth's surface in 1 s. If the same stone is taken to Moon and dropped freely from the same height then it will reach the surface of the Moon in a time (The 'g' of Moon is 1/6 times that of Earth) :-

(1) $\sqrt{6}$ seconds (2) 9 seconds (3) $\sqrt{3}$ seconds (4) 6 seconds

Q.5 The maximum vertical distance through which an astronaut can jump on the earth is 0.5 m. Estimate the corresponding distance on the moon.

Q.6 Find the percentage decrement in the weight of a body when taken to a height of 16 km above the surface of earth. (radius of earth is 6400 km)

Q.7 At which height from the earth's surface does the acceleration due to gravity decrease by 1%

Q.8 If earth is assumed to be a sphere of uniform density then plot a graph between acceleration due to gravity (g) and distance from the centre of earth.

Q.9 What is the value of acceleration due to gravity at a height equal to half the radius of earth, from surface of earth? [take $g = 10\ m/s^2$ on earth's surface]

Q.10 At which height from the earth's surface does the acceleration due to gravity decrease by 75% of its value at earth's surface?

Q.11 At what depth below the surface does the acceleration due to gravity becomes 70% of its value on the surface of earth ?

Q.12 At what depth from earth's surface does the acceleration due to gravity becomes $\frac{1}{4}$ times that of its value at surface ?

Q.13 Weight of a body decreases by 1% when it is raised to a height h above the Earth's surface. If the body is taken to a depth h in a mine, then its weight will :
(1) decrease by 0.5 % (2) decrease by 2% (3) increase by 0.5% (4) increase by 1%

Q.14 Calculate the angular velocity of earth so that an object at equator may feel weightlessness.

Q.15 Calculate the angular speed of earth such that an object at latitude 45° feels half of its weight.

Q.16 As we go from the equator to the poles, the value of g :
(1) remains the same
(2) decreases
(3) increases
(4) decreases up to latitude of 45° and then increases

11. Gravitational Potential Energy

11.1 Potential Energy Corresponding to a Conservative Field

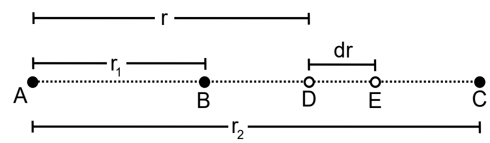
The potential energy of a system corresponding to a conservative force was defined as

$U_f - U_i = \int_i^f \vec{F}.d\vec{r}$.

The change in potential energy is equal to the negative of the work done by the internal forces.

11.2 Gravitational Potential Energy of a Two Mass System

Change in potential energy of the system of the two particles as the distance changes from r_1 to r_2.



Consider a small displacement when the distance between the particles changes from r to r + dr. In the figure, this corresponds to the second particle going from D to E.

The force on the second particle is

$$F = \frac{Gm_1m_2}{r^2} \text{ along } \overrightarrow{DA}.$$

The work done by the gravitational force in the displacement is

$$dW = -\frac{Gm_1m_2}{r^2}\,dr$$

The increase in potential energy of the two-particle system during this displacement is

$$dU = -dW = \frac{Gm_1m_2}{r^2}\,dr$$

The increase in potential energy as the distance between the particles change from r_1 to r_2 is

$$U(r_2) - U(r_1) = \int dU$$

$$= \int_{r_1}^{r_2} \frac{Gm_1m_2}{r^2}\,dr = Gm_1m_2 \int_{r_1}^{r_2} \frac{1}{r^2}\,dr$$

$$= Gm_1m_2 \left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$= Gm_1m_2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right). \qquad \text{...(11.2)}$$

We choose the potential energy of the two particle system to be zero when the distance between them is infinity. This means that we choose $U(\infty) = 0$. By (11.2) the potential energy U(r), when the separation between the particles is r, is

$$U(r) = U(r) - U(\infty)$$

$$= Gm_1m_2\left[\frac{1}{\infty} - \frac{1}{r}\right] = -\frac{Gm_1m_2}{r}$$

The gravitational potential energy of a two particle system is

$$U(r) = -\frac{Gm_1m_2}{r} \qquad \text{...(11.3)}$$

where m_1 and m_2 are the masses of the particles r is the separation between the particles and the potential energy is chosen to be zero when the separation is infinite.

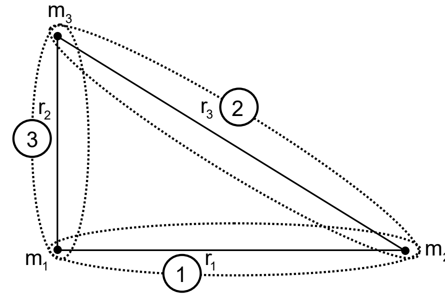
11.3 Gravitational Potential Energy of a Three Particle System

If there are more than two particles in a system then the net gravitational potential energy of the whole system is the sum of gravitational potential energies of all the possible pairs in that system.

For eg :

$$U_{system} = \left(-\frac{Gm_1m_2}{r_1}\right) + \left(-\frac{Gm_2m_3}{r_2}\right) + \left(-\frac{Gm_1m_3}{r_3}\right)$$

$$U_{system} = -\frac{Gm_1m_1}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$$



Example 23:

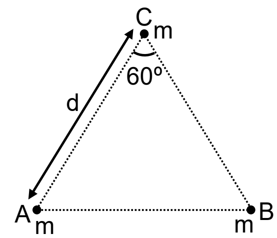
Three particles each of mass m are placed at the corners of an equilateral triangle of side d. Calculate (a) the potential energy of the system, (b) work done on this system if the side of the triangle is changed from d to 2d.

Solution:

(a) As in case of two-particle system potential energy is given by $-(Gm_1m_2/r)$, so

$U_A = U_{12} + U_{23} + U_{31}$

So, $U_A = -3\dfrac{Gmm}{d} = -\dfrac{3Gm^2}{d}$



(b) When d is changed to 2d, $U_B = -\dfrac{3Gm^2}{2d}$

So, work done = $U_B - U_A = \dfrac{3Gm^2}{2d}$

11.4 Change in Gravitational Potential Energy of Earth - Mass System

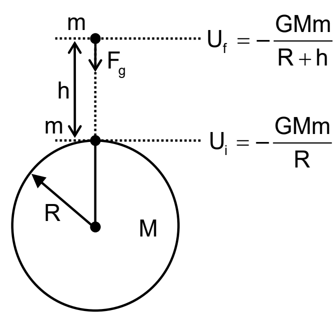
(a) We change in gravitational potential energy of the earth-particle system when the particle was raised through a small height over earth's surface. In this case the force Mg may be treated as constant and the change in potential energy is

$U_f - U_i = mgh$

where the symbols have their usual meanings.

(b) $W = \Delta U = U_f - U_i$

$$\Rightarrow W = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$



$$\Rightarrow W = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow W = GMm\left(\frac{R+h-R}{R(R+h)}\right)$$

$$\Rightarrow W = gR^2m\left(\frac{h}{R^2\left(1+\frac{h}{R}\right)}\right) \qquad [\because GM = gR^2]$$

$$W = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

Special cases

(i) If h << R, then $\frac{h}{R} \in 0$ $\therefore$ $W \in \frac{mgh}{1+0} = mgh$

(ii) If h = R, then $W = \frac{mgh}{\left(1+\frac{R}{R}\right)} = \frac{mgR}{2}$

11.5 Application based on Earth-Mass System

(a) The velocity required to project a particle to a height 'h' from the surface of earth.

Applying 'COME' on the surface and at a height 'h'.

$(K.E. + U)_{surface} = (K.E. + U)_{final}$

$$\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R+h}$$

$$\Rightarrow = \frac{1}{2}mv^2 - \frac{GMm}{R} = -\left[\frac{GMm}{R+h}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$$

$$\Rightarrow v^2 = \frac{2gh}{1+\frac{h}{R}} \Rightarrow v = \sqrt{\frac{2gh}{1+\frac{h}{R}}}$$

Note : If a body is released from a height 'h' above the surface of earth, then its velocity on reaching the earth's surface is also given by :

$$v = \sqrt{\frac{2gh}{1+\frac{h}{R}}}$$

(b) To find the maximum height attained by a body when it is projected with velocity 'v' from the surface of earth.

Form $v^2 = \frac{2gh}{1+\frac{h}{R}}$

$$\Rightarrow v^2 + \frac{v^2h}{R} = 2gh$$

$$\Rightarrow v^2 = 2gh - \frac{v^2h}{R}$$

$$\Rightarrow v^2 = h\left(2g - \frac{v^2}{R}\right)$$

$$\Rightarrow h = \frac{v^2}{2g - \frac{v^2}{R}} = \frac{v^2R}{2gR - v^2} \Rightarrow h = \frac{v^2R}{2gR - v^2}$$

Example 24:

With what velocity must a body be thrown from the earth's surface so that it may reach a height $4R_e$ above the Earth's surface? (Radius of the Earth R_e = 6400 km, g = 9.8 ms^{-2}).

Solution:

By using conservation of mechanical energy

$$\frac{1}{2}m_0v^2 - \frac{GMm_0}{R_e} = 0 - \frac{GMm_0}{(R_e + 4R_e)}$$

$$\frac{1}{2}m_0v^2 = -\frac{GMm_0}{5R_e} + \frac{GMm_0}{R_e}$$

$$\Rightarrow \frac{1}{2}m_0v^2 = \frac{4}{5}\frac{GMm_0}{R_e} \Rightarrow v^2 = \frac{8}{5}\frac{GM}{R_e} = \frac{8}{5}\frac{gR_e^2}{R_e}$$

$$v^2 = \frac{8}{5} \times 9.8 \times 6400 \times 10^3 = 10^8 \qquad \Rightarrow v = 10^4 \text{ km/s.}$$

Example 25:

A body of mass m kg starts falling from a distance 2R above the earth's surface. What is its kinetic energy when it has fallen to a distance 'R' above the earth's surface ? (Where R is the radius of Earth)

Solution:

By conservation of mechanical energy,

$$-\frac{GMm}{3R} + 0 = -\frac{GMm}{2R} + \text{K.E.}$$

$$\Rightarrow \text{K.E.} = \frac{GMm}{R}\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{6}\frac{GMm}{R}$$

$$= \frac{1}{6}\frac{(gR^2)m}{R} = \frac{1}{6}mgR$$

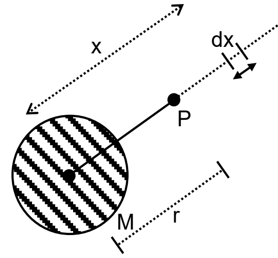
12.1 Definition of Gravitational Potential:

Gravitational field around a material body can be described not only by gravitational intensity $\vec{I}_g$, but also by a scalar function, the gravitational potential V. Gravitational potential is the amount of work done by external agent in bringing a body of unit mass from infinity to that point without changing its kinetic energy. $V = \frac{W_{ext}}{m}$

Gravitational force on unit mass at (P) will be = $\frac{GM}{x^2}$

Work done by this force when the unit mass is displaced through the distance dx is

$dW_{ext} = Fdx = \frac{GM}{x^2} . dx$



Total work done in bringing the body of unit mass from infinity to point (P) is

$$W_{ext} = \int_{\infty}^{r} \frac{GM}{x^2} dx = -\left(\frac{GM}{x}\right)_{\infty}^{r} = -\frac{GM}{r}$$

This work done is the measure of gravitational potential at point (P) $\quad \therefore V_P = -\frac{GM}{r}$

- If $r = \infty$ then $V_\infty = 0$. Hence gravitational potential is maximum at infinity (as it is a negative quantity at point P)
- If $r = R_e$ (on the surface of Earth) $V_S = -\frac{GM_e}{R_e}$

12.2 Relation Between Intensity and Potential Gradient:

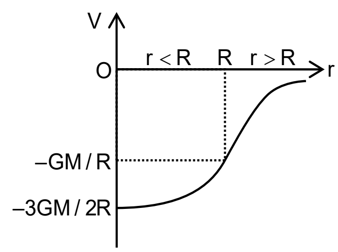
$$V = -\int \vec{I} . \vec{d}r \Rightarrow dV = \vec{I} . \vec{d}r$$

$\therefore I = -\frac{dV}{dr}$ = – ve potential gradient.

12.3 Gravitational Potential Due to Regular Bodies:

Solid Sphere:

Case I : r > R (outside the sphere); $V_{out} = -\frac{GM}{r}$



Case II : r = R (on the surface); $V_{surface} = -\frac{GM}{R}$

Case III : r < R (inside the sphere);

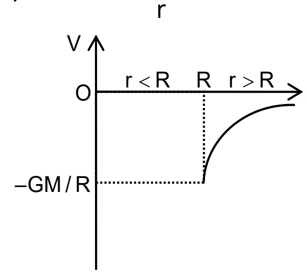
$$V_{in} = -\frac{GM}{2R^3}[3R^2 - r^2]$$

It is clear that the potential |V| will be maximum at the centre (r = 0)

$$|V_{centre}| = \frac{3}{2}\frac{GM}{R}, \ V_{centre} = \frac{3}{2} V_{surface}$$

Spherical Shell:

Case I: r > R (outside the sphere) $V_{out} = -\frac{GM}{r}$



Case II: r = R (on the surface); $V_{surface} = -\frac{GM}{R}$

Case III: r < R (inside the sphere);

Potential is same every where and is equal to its value at the surface $V_{in} = -\frac{GM}{R}$

Example 26:

In a certain region of space gravitational field is given by I = – (K/r) (Where r is the distance from a fixed point and K is constant). Taking the reference point to be at $r = r_0$ with $V = V_0$. Find the potential at a distance r.

Solution:

$$\int dV = -\int E.dr, \int dV = \int \frac{K}{r} dr$$

$V = K \log r + c$ at $r = r_0$; $V = V_0$

$\Rightarrow V_0 = K \log r_0 + c \Rightarrow c = V_0 - K \log r_0$

By substituting the value c in equation

$$V = K \log \left(\frac{r}{r_0}\right) + V_0$$

Example 27:

Two bodies of respective masses m and M are placed d distance apart. The gravitational potential (V) at the position where the gravitational field due to them is zero is :-

(1) $V = -\frac{G}{d}(m + M)$ (2) $V = -\frac{G}{d}$ (3) $V = -\frac{GM}{d}$ (4) $V = -\frac{G}{d}(\sqrt{m} + \sqrt{M})^2$

Solution:

Equilibrium position of the neutral point from mass 'm' is :

$$= \left(\frac{\sqrt{m}}{\sqrt{m} + \sqrt{M}}\right) d$$

$$V_1 = \frac{-Gm_1}{r_1}; V_2 = \frac{-Gm_2}{r_1}$$

$$V_1 = \frac{-Gm}{\sqrt{m}\,d}(\sqrt{M} + \sqrt{m});$$

$$V_2 = \frac{-Gm}{\sqrt{m}\,d}(\sqrt{M} + \sqrt{m})$$

$$V_1 = \frac{-G}{d}\sqrt{M}(\sqrt{M} + \sqrt{m});$$

$$V_2 = \frac{-G}{d}\sqrt{M}(\sqrt{M} + \sqrt{m})$$

$$V = \frac{-G}{d}(\sqrt{M} + \sqrt{m})$$

Concept Builder-4

Q.1 Three particles each of mass m are placed at the corners of an equilateral triangle of side 'a'. Energy is given to their system so that they are now at the corners of an equilateral triangle of side 20a. Energy given to the system in this process is :

(1) $\frac{Gm^2}{20a}$ (2) $\frac{19Gm^2}{20a}$ (3) $\frac{3Gm^2}{20a}$ (4) $\frac{57Gm^2}{20a}$

Q.2 What is the potential energy of a body of mass m relative to the surface of earth at a (a) height h = R above its surface ? (b) depth d = R below its surface ?

Q.3 Gravitational potential difference between a point on the surface of a planet and point 10 m above is 4 J/kg. Considering the gravitational field to be uniform, how much work is done in moving a mass of 2 kg from the surface to a point 5 m above the surface ?

(1) 4 J (2) 5 J (3) 6 J (4) 7 J

Q.4 The gravitational acceleration on the surface of earth is g. Find the increase in potential energy in lifting an object of mass m to a height equal to the radius of earth.

Q.5 A particle is fired vertically upwards from the surface of earth and reaches a height 6400 km. Find the initial velocity of the particle if R = 6400km and g at the surface of earth is 10 m/s^2.

Q.6 The intensity of gravitational field at a point situated at a distance 8000 km from the centre of Earth is 6.0 N/kg. The gravitational potential at that point in N-m/kg will be :-

(1) 6 (2) 4.8×10^7 (3) 8×10^5 (4) 4.8×10^2

Q.7 The gravitational field due to a certain mass distribution is $E = \frac{K}{x^3}$ in the x-direction (K is a constant). Taking the gravitational potential to be zero at infinity, its value corresponding to distance x is :-

(1) $\frac{K}{x}$ (2) $\frac{K}{2x}$ (3) $\frac{K}{x^2}$ (4) $\frac{K}{2x^2}$

Q.8 Two masses of 10^2 kg and 10^3 kg are separated by 1 m distance. Find the gravitational potential at the mid point of the line joining them.

13. Escape Energy & Escape Velocity

Escape Velocity (V_e)

It is the minimum velocity required for an object located at the planet's surface so that it just escapes the planet's gravitational field.

Consider a projectile of mass m, leaving the surface of a planet (or some other astronomical body or system), of radius R and mass M with escape speed v_e.

when the projectile just escapes to infinity it has neither kinetic energy nor potential energy.

From conservation of mechanical energy

$$\frac{1}{2}mv_e^2 + \left(-\frac{GMm}{R}\right) = 0 + 0 \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

The escape velocity of a body from a location which is at height 'h' above the surface of planet, we can use:

$$v_{es} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}} \qquad (\because r = R + h)$$

Where, r = distance from the centre of the planet.
h = Height above the surface of the planet.

Escape speed depends on

(i) Mass (M) and radius (R) of the planet
(ii) Position from where the particle is projected.

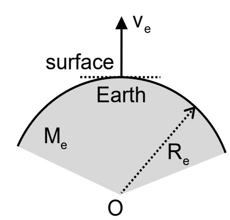
Escape speed does not depend on

(i) Mass (m) of the body which is projected
(ii) Angle of projection.

If a body is thrown from the Earth's surface with escape speed, it goes out of earth's gravitational field and never returns back to the earth's surface.

- **Escape energy**

Minimum energy given to a particle in the form of kinetic energy so that it can just escape the Earth's gravitational field.



Magnitude of escape energy = $\frac{GMm}{R}$ (– ve of PE on the Earth's surface)

Escape energy = Kinetic Energy corresponding to the escape velocity

$$\Rightarrow \frac{GMm}{R} = \frac{1}{2}mv_e^2$$

Note : In the above discussion it can be any planet for that matter

Key Points

- $v_e = \sqrt{\frac{2GM}{R}}$ If M = constant then $v_e \propto \frac{1}{\sqrt{R}}$
- $v_e = \sqrt{2gR}$ If g = constant $v_e \propto \sqrt{R}$
- $v_e = R\sqrt{\frac{8\pi G\rho}{3}}$ If ρ = constant then $v_e \propto R$
- Escape velocity does not depend on the mass of the body being projected, angle of projection or direction of projection.
  $v_e \propto m^0$ and $v_e \propto \theta^0$
- Escape velocity at : Earth's surface v_e = 11.2 km/s, Moon surface v_e = 2.31 km/s.
- Atmosphere on Moon is missing because root mean square velocity of gas particles is greater than escape velocity. i.e., $v_{rms} > v_e$

Example 28:

A mass of 6×10^{24} kg (= mass of earth) is to be compressed in a sphere in such a way that the escape velocity from its surface is 3×10^{8} m/s (equal to the velocity of light). What should be the radius of the sphere?

(1) 9 mm (2) 8 mm (3) 7 mm (4) 6 mm

Solution:

As, $v_e = \sqrt{\left(\frac{2GM}{R}\right)}$, $R = \left(\frac{2GM}{v_e^2}\right)$,

$$\therefore\ R = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^{8})^2} = 9 \times 10^{-3}\ \text{m} = 9\text{mm}$$

Example 29:

The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at distance d apart. What is the minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity ?

Solution:

Potential energy of m when it is midway between M_1 and M_2,

$$U = m(V_1 + V_2) = m\left[-\frac{GM_1}{d/2} + \frac{-GM_2}{d/2}\right]$$

$$= \frac{-2Gm}{d}\ [M_1 + M_2]$$

And as potential energy at infinity is zero, so work required to shift m from the given position to infinity,

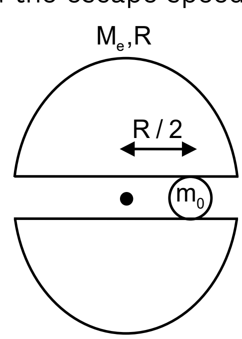
$W = 0 - U = 2Gm\ (M_1 + M_2)/d$

As this work is provided by initial kinetic energy,

$$\frac{1}{2}m\upsilon^2 = \frac{2Gm(M_1 + M_2)}{d} \text{ or } u = 2\sqrt{\frac{G(M_1 + M_2)}{d}}$$

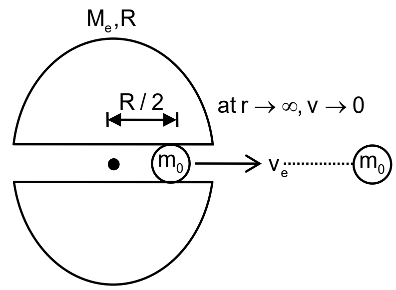
Example 30:

A narrow tunnel is dug along the diameter of the earth, and a particle of mass m_0 is placed at R/2 distance from the centre. Find the escape speed of the particle from that place.



Solution:

Suppose we project the particle with speed v_e, so that it just reaches infinity ($r \rightarrow \infty$).



Applying energy conservation principle

$K_i + U_i = K_f + U_f$

$$\frac{1}{2} m_0 v_e^2 + m_0 \left[-\frac{GM_e}{2R^3} \left\{ 3R^2 - \left(\frac{R}{2} \right)^2 \right\} \right] = 0$$

$$\Rightarrow v_e = \sqrt{\frac{11GM_e}{4R}}$$

Example 31:

If velocity given to an object from the surface of the Earth is n times the escape velocity then what will be its residual velocity at infinity?

Solution:

Let the residual velocity be v, then from energy conservation $\frac{1}{2} m(nv_e)^2 - \frac{GMm}{R} = \frac{1}{2} mv^2 + 0$

$$\Rightarrow v^2 = n^2 v_e^2 - \frac{2GM}{R}$$

$$= n^2 v_e^2 - v_e^2 = (n^2 - 1) v_e^2$$

$$\Rightarrow v = \left(\sqrt{n^2 - 1} \right) v_e$$

Concept Builder-5

Q.1 A planet has mass 100 times that of earth, what has to be its Diameter so that no physical body can escape from it.

Q.2 A body of mass m is situated at a distance $4R_e$ above the Earth's surface, where R_e is the radius of Earth. What minimum energy should be given to the body so that it may escape ?

(1) mgR_e (2) $2mgR_e$ (3) $\frac{mgR_e}{5}$ (4) $\frac{mgR_e}{16}$

Q.3 The escape velocity for a planet is v_e. A particle starts from rest at a large distance from the planet, reaches the planet only under gravitational attraction and passes through a smooth tunnel through its centre. Its speed at the centre of the planet will be :-

(1) $\sqrt{1.5} v_e$ (2) $\frac{v_e}{\sqrt{2}}$ (3) v_e (4) zero

Q.4 A rocket is fired with a speed $v = 2\sqrt{gR}$ near the earth's surface and directed upwards. (a) Show that it will escape from the earth. (b) Show that in interstellar space its speed is $v = \sqrt{2gR}$.

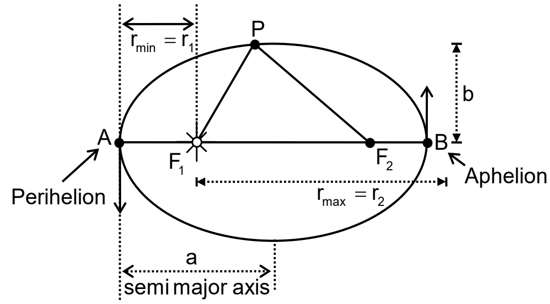
Q.5 A projectile is fired vertically upward from the surface of earth with a velocity Kv_e where v_e is the escape velocity and K < 1. Neglecting air resistance, show that the maximum height to which it will rise measured from the centre of earth is $R/(1 - K^2)$ where R is the radius of the earth.

14. Kepler's Laws of Planetary Motion

Kepler found important regularities in the motion of the planets. These regularities are known as 'Kepler's three laws of planetary motion.

(a) First Law (Law of Orbits)

All planets move around the Sun in elliptical orbits having the Sun at one focus of the orbit.



When a particle moves with respect to two fixed points in such a way that the sum of the distances from these two points is always constant then the path of the particle is an ellipse. And the two fixed points are called focal points.

According to Figure :-

$PF_1 + PF_2 = AF_1 + AF_2 = BF_1 + BF_2 = \text{constant}$

But in ellipse $AF_1 = BF_2$ (minimum distance from both focal is same)

$PF_1 + PF_2 = BF_2 + AF_2 = BF_1 + AF_1 = 2a = \text{length of major axis}$

$r_1 + r_2 = r_{min} + r_{max} = 2a$

$$\therefore \qquad a = \frac{r_1 + r_2}{2} = \frac{r_{min} + r_{max}}{2} \qquad \text{(Mean distance)}$$

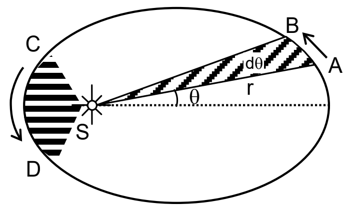
(b) Second Law (Law of Areas)

A line joining any planet to the Sun sweeps out equal areas in equal intervals of time, i.e., the areal speed of the planet remains constant.

According to the second law, if a planet moves from A to B in a given time interval, and from C to D in the same time interval then the areas ASB and CSD will be equal.

dA = area of the curved triangle

$$SAB = \frac{1}{2}(AB \times SA) = \frac{1}{2}(rd\theta \times r) = \frac{1}{2} r^2 2\theta$$



Thus, the instantaneous areal speed of the planet is

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega = \frac{1}{2} rv \qquad .....(i)$$

where ω is the angular speed of the planet

Let L be the angular momentum of the planet about the Sun S and m is the mass of the planet.

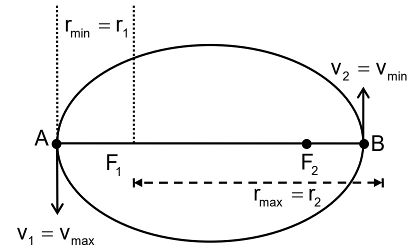
Then, $L = I\omega = mr^2 \omega = mvr$ ....(ii)

where I (= mr^2) is the instantaneous moment of inertia of the planet about the Sun S.

From equation (i) and (ii), $\dfrac{dA}{dt} = \dfrac{L}{2m}$ ....(iii)

Now, the areal speed dA/dt of the planet is constant according to Kepler's second law. Therefore according to eq. (iii), the angular momentum L of the planet is also constant, that is, the angular momentum of the planet is conserved. Thus, Kepler's second law is equivalent to conservation of angular momentum.

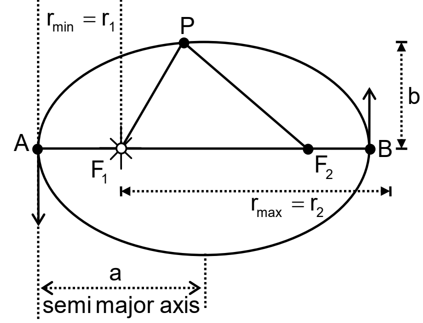
Applying conservation of angular momentum between points A and B



$L_A = L_B \Rightarrow mv_{max}r_{min} = mv_{min}r_{max}$

$\Rightarrow v_{max}r_{min} = v_{min}r_{max}$

A planet moves around the sun in an elliptical orbit of semi major axis a and eccentricity e



For an ellipse : its general equation is given as

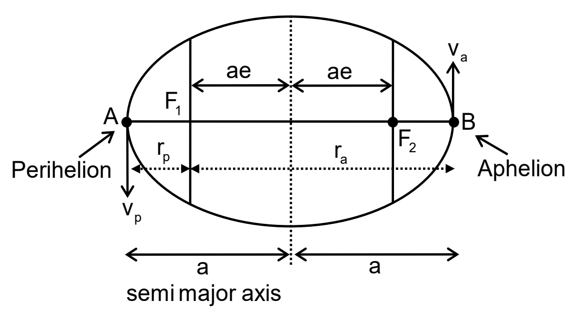
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If a > b then a is semi major axis b is semi minor axis and e is eccentricity where

$$b^2 = a^2(1 - e^2) \qquad \Rightarrow e = \sqrt{1-\left(\frac{b}{a}\right)^2}$$

Applying the conservation of angular momentum (COAM) at the perihelion and aphelion

$mv_pr_p = mv_ar_a$



$r_{max} = a(1 + e)$; $r_{min} = a(1 - e)$

$$\Rightarrow \boxed{\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1+e}{1-e}} \qquad .....(i)$$

By conservation of mechanical energy

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = -\frac{GMm}{2a} \quad .....(ii)$$

By solving equation (i) and (ii)

$$\Rightarrow v_a = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)};\ v_p = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

(c) Third Law (Law of Periods)

The square of the period of revolution of any planet around the Sun is directly proportional to the cube of the semi-major axis of its elliptical orbit.

$\boxed{T^2 \propto a^3}$

Note : For a circular orbit semi major axis = Radius of the orbit

$\boxed{T^2 \propto R^3}$

Example 32:

Calculate the mass of the sun if the mean radius of the earth's orbit is 1.5×10^8 km and G = 6.67×10^{-11} N × m^2/kg^2.

Solution:

In case of orbital motion as $\upsilon = \sqrt{(GM/r)}$ so

$$T = \frac{2\pi r}{\upsilon} = 2\pi r\sqrt{\frac{r}{GM}}, \quad \text{i.e., } M = \frac{4\pi^2 r^3}{GT^2}$$

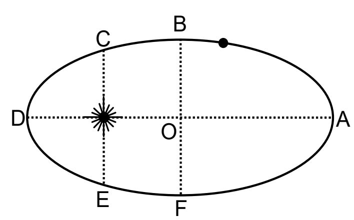
$$\therefore \quad M = \frac{4 \times \pi^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11} \times (3.15 \times 10^7)^2}$$

[as T = 1 year = 3.15×10^7 s]

i.e., M 2×10^{30} kg

Example 33:

A planet is revolving round the sun in an elliptical orbit as shown in figure. Select correct alternative(s)



(1) Its total energy is negative at D

(2) Its angular momentum is constant

(3) Net torque on the planet about sun is zero

(4) Linear momentum of the planet is conserved

Solution:

(1, 2, 3)

For (1) : For a bound system, the total energy is always negative.

For (2) : For central force field, angular momentum is always conserved.

For (3) : For central force field, torque = 0

For (4) : In presence of external force, linear momentum is not conserved.

Concept Builder-6

Q.1 The mean radius of the earth's orbit around the sun is 1.5×10^{11} metres. The mean radius of the orbit of mercury around the sun is 6×10^{10} metres. Calculate the year of the mercury.

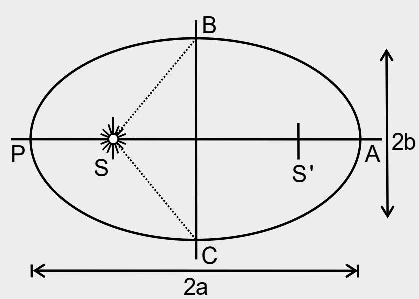
Q.2 If earth describes an orbit round the sun of double its present radius, what will be the year on earth?

Q.3 A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy (f) total energy throughout its orbit ? Neglect any mass loss of the comet when it comes very close to the Sun.

Q.4 A planet is revolving around the Sun in an elliptical orbit. Its closest distance from the Sun is r_{min}. The farthest distance from the Sun is r_{max}. If the orbital angular velocity of the planet when, it is nearest to the Sun is ω, then the orbital angular velocity at the point when it is at the farthest distance from the Sun is :-

(1) $\left(\sqrt{\frac{r_{min}}{r_{max}}}\right)\omega$ (2) $\left(\sqrt{\frac{r_{max}}{r_{min}}}\right)\omega$ (3) $\left(\frac{r_{max}}{r_{min}}\right)^2\omega$ (4) $\left(\frac{r_{min}}{r_{max}}\right)^2\omega$

Q.5 Let the speed of the planet at the perihelion P in Fig. be v_p and the Sun-planet distance SP be r_p. Relate (r_p, v_p) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB ?

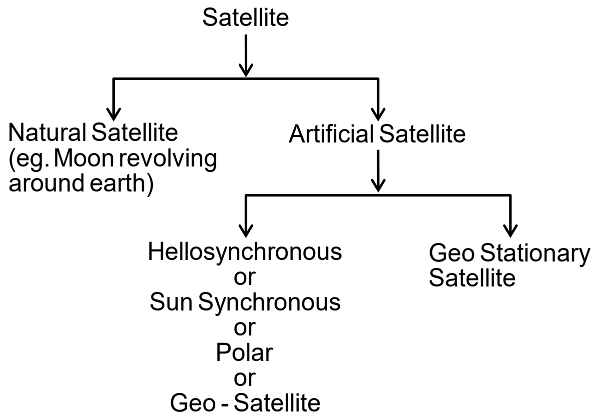


Q.6 If the gravitational force were to vary inversely as m^{th} power of the distance, then the time period of a planet in circular orbit of radius r around the Sun will be proportional to :-

(1) $r^{-3m/2}$ (2) $r^{3m/2}$ (3) $r^{m+1/2}$ (4) $r^{(m+1)/2}$

15. Satellite Motion

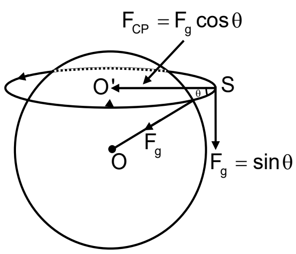
A light body revolving round a heavier planet due to gravitational attraction, is called a satellite. Moon is a natural satellite of Earth.



15.1 Essential Conditions for Satellite Motion

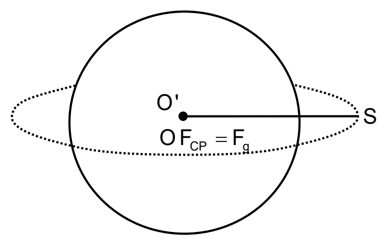
- The centre of satellite's orbit should coincide with the centre of Earth.
- Plane of the orbit of satellite should pass through the centre of Earth.

(i)



Unstable orbit
(Due to $F_g \sin\theta$, orbit will shift)

(ii)



Stable orbit

- It follows that a satellite can revolve round the earth only in those circular orbits whose centres coincide with the centre of earth. Circles drawn on globe with centres coincident with earth are known as great circles. Therefore, a satellite revolves around the earth along circles concentric with great circles.

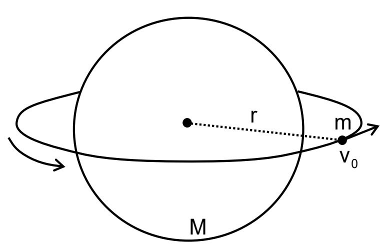
15.2 Orbital Velocity & Time Period of a Satellite

- **Orbital Velocity (v_0)**

  A satellite of mass m moving in an orbit of radius r with speed v_0. The required centripetal force is provided by gravitation.

$$F_{cp} = F_g \quad \frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R_e + h)}} \quad (r = R_e + h)$$



For a satellite very close to the Earth's surface h << R_e $\therefore$ r $\in$ R_e

$$\therefore \quad v_0 = \sqrt{\frac{GM}{R_e}} = \sqrt{gR_e} = 8 \text{ km/s}$$

- If a body is taken to some height (small) from Earth and given a horizontal velocity of magnitude 8 km/s then it becomes a satellite of Earth.
- v_0 depends upon mass of planet, Radius of the circular orbit of satellite and density of planet
- If orbital velocity of satellite becomes (or increased by 41.4%) or K.E. is doubled then it escapes from the gravitational field of Earth.

- **Time Period of a Satellite**

$$T = \frac{2\pi r}{v_0} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi r^{3/2}}{R\sqrt{g}} \qquad \Rightarrow T^2 = \frac{4\pi}{GM} r^3 \qquad \Rightarrow T^2 \propto r^3 \ (r = R + h)$$

- **For a Satellite Close to Earth's Surface**

$$v_0 = \sqrt{\frac{GM_e}{R_e}} = 8 \text{ km/s}$$

$$T_0 = 2\pi\sqrt{\frac{R_e}{g}} = 84 \text{ minutes} = 1 \text{ hour } 24 \text{ minute} = 1.4 \text{ h} = 5063 \text{ s}$$

In terms of density

$$T_0 = \frac{2\pi(R_e)^{1/2}}{(G\times 4/3\pi R_e \times \rho)^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

Time period of a near by satellite only depends on the density of the planet.

For Moon h_m = 380,000 km and T_m = 27 days

$$v_0 = \frac{2p(R_e + h_m)}{T_m} = \frac{2\pi(386400\times 10^3)}{27\times 24\times 60\times 60} = 1.04 \text{ km}$$

Example 34:

A small satellite revolves round a planet in an orbit just above planet's surface. Taking the mean density of planet as ρ, calculate the time period of the satellite.

Solution:

Let the radius of the planet be R then time period

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{GM}}$$

mass of planet $M = \frac{4}{3}\pi R^3\rho$

$$\therefore T = \frac{2\pi R^{3/2}}{\sqrt{G.\frac{4}{3}\pi R^3.\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

Example 35:

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. (a) Determine the height of the satellite above the earth's surface. (b) If the satellite is stopped suddenly in the orbit and allowed to fall onto the earth, find the speed with which it hits the surface of the earth. ($g = 9.8\ ms^{-2}$ and $R_E = 6400$ km)

Solution:

(a) We know that for satellite motion

$$\upsilon_0 = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{(R+h)}}$$

$$\left[\text{as } g = \frac{GM}{R^2} \text{ and } r = R + h\right]$$

In this problem $\upsilon_0 = \frac{1}{2}\upsilon_e = \frac{1}{2}\sqrt{2gR}$

[as $\upsilon_e = \sqrt{2gR}$]

So, $\frac{R^2 g}{R+h} = \frac{1}{2} gR$

i.e., $2R = h + R$ or $h = R = 6400$ km

(b) By conservation of ME

$$0 + \left(-\frac{GMm}{r}\right) = \frac{1}{2}m\upsilon^2 + \left(-\frac{GMm}{R}\right)$$

or $\upsilon^2 = 2GM\left[\frac{1}{R} - \frac{1}{2R}\right]$

[as $r = R + h = R + R = 2R$]

or $\upsilon = \sqrt{\frac{GM}{R}}$

$= \sqrt{gR} = \sqrt{10 \times 6.4 \times 10^6} = 8$ km/s

Example 36:

A satellite moves in a circular orbit around the earth. the radius of this orbit is one half that of the moon's orbit. Find the time in which the satellite completes one revolution.

Solution:

$T \propto r^{3/2}$

$$\frac{T_s}{T_m} = \left(\frac{r_s}{r_m}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2}$$

$\Rightarrow T_s = 9.7$ days

15.3 Energy of Satellite

Kinetic energy K.E. $= \frac{1}{2}mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$

Potential energy P.E. $= -\frac{GMm}{r} = -mv_0^2 = -\frac{L^2}{mr^2}$

Total mechanical energy T.E. = P.E. + K.E.

$$= -\frac{mv_0^2}{2} = -\frac{GMm}{2r} = -\frac{L^2}{2mr^2}$$

- **Binding Energy**

  Total mechanical energy (potential + kinetic) of a closed system is negative. The modulus of this total mechanical energy is known as the binding energy of the system. This is the energy due to which system is bound or the different parts of the system are bonded to each other.

Binding energy of a satellite (system)

B.E. = – T.E.

$$B.E. = \frac{1}{2} mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$$

Hence $B.E. = K.E. = -T.E. = \frac{-P.E.}{2}$

Escape energy and ionisation energy are the practical examples of binding energy.

Here, at point A :

$\because |(P.E.)| > K.E.$ $\because KE + PE = -ve$

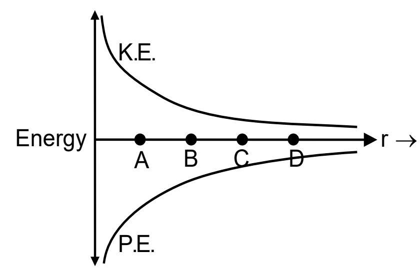
At A, B and C system is bounded.

At point D :

$\because |PE| = K.E.$

TE = KE + PE = 0

So, the system is unbounded.



15.4 Work Done in Changing the Orbit

W = Change in mechanical energy of the system

but $E = \frac{-GMm}{2r}$

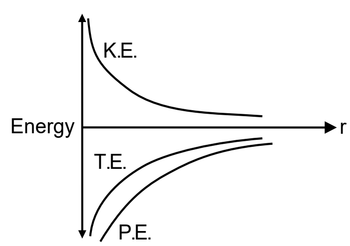
so $W = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Graphs

$$KE = -\frac{GMm}{2r}$$

$$TE = -\frac{GMm}{2r}$$

$$PE = -\frac{GMm}{r}$$



Example 37:

A space-ship is launched into a circular orbit close to the Earth's surface. What additional speed should now be imparted to the spaceship so that it overcomes the gravitational pull of the Earth.

Solution:

Let ΔK be the additional kinetic energy imparted to the spaceship to overcome the gravitation pull then

$\Delta K = -$ (total energy of spaceship) $= \frac{GMm}{2R}$

Total kinetic energy $= \frac{GMm}{2R} + \Delta K = \frac{GMm}{2R} + \frac{GMm}{2R} = \frac{GMm}{R}$

then $\frac{1}{2}mv_2^2 = \frac{GMm}{R} \Rightarrow v_2 = \sqrt{\frac{2GM}{R}}$

But $v_1 = \sqrt{\frac{GM}{R}}$. So additional velocity required

$= v_2 - v_1 = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = (\sqrt{2} - 1)\sqrt{\frac{GM}{R}}$

Alternate solution

Additional velocity = Escape velocity – orbital velocity

$= v_{es} - v_0$

$= \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} \Rightarrow (\sqrt{2} - 1)\sqrt{\frac{GM}{R}}$

Example 38:

If a satellite orbits as close to the earth's surface as possible

(1) its speed is maximum

(2) time period of its revolution is minimum

(3) the total energy of the 'earth plus satellite' system is minimum

(4) the total energy of the 'earth plus satellite' system is maximum

Solution:

(1, 2, 3)

For (1) : orbital speed $v_0 = \sqrt{\frac{GM}{r}}$, $r_{min} = R$ so v_0 = maximum

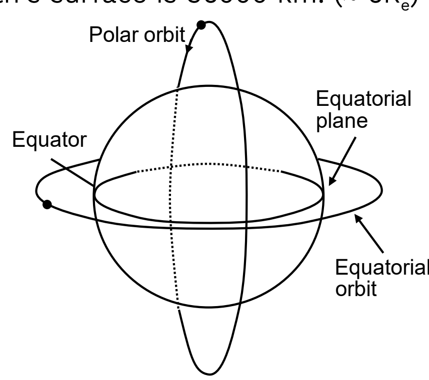
For (2) : Time period of revolution $T^2 \propto r^3$

For (3/4) : Total energy $= -\frac{GMm}{2r}$

15.5 GEO-Stationary & Polar Satellite

GEO-Stationary Satellite

- It rotates in an equatorial plane.
- Its height from the Earth's surface is 36000 km. ($\approx 6R_e$)



- Its angular velocity and time period should be same as that of Earth.
- Its rotating sense should be same as that of Earth (West to East)
- Geo Stationary/ Telecommunication/ Parking / Synchronous/ Satellite are always projected from equator (for example Singapore).
- Its orbit is called parking orbit and its orbital velocity is 3.1 km/s.

Polar Satellite (Sun - Synchronous Satellite)

It is the satellite which revolves in the polar orbit around Earth. A polar orbit is one whose angle of inclination with the equatorial plane of Earth is 90° and a satellite in polar orbit will pass over both the north and south geographical poles once per revolution. Polar satellites are Sun-synchronous satellites.

Polar satellites are employed to obtain the cloud images, atmospheric data, information regarding ozone layer in the atmosphere and to detect the ozone hole over Antarctica etc.

15.6 Weightlessness

When the weight of a body becomes zero, the body is said to be in a state of weightlessness. In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the centre of the earth which is exactly the value of earth's acceleration due to gravity at that position. Thus in the satellite everything inside it is in a state of free fall. If a body is in a satellite (which does not produce its own gravity) orbiting the Earth at a height h above its surface then

True weight = $mg_h = \boxed{\dfrac{mGM}{(R+h)^2} = \dfrac{mg}{\left(1+\dfrac{h}{R}\right)^2}}$

and Apparent weight = $m(g_h - a)$

But $a = \dfrac{v_0^2}{r} = \dfrac{GM}{r^2} = \dfrac{GM}{(R+h)^2} = g_h$

$\Rightarrow$ Apparent weight = $m(g_h - g_h) = 0$.

Note : Condition of weightlessness can be overcome by creating artificial gravity by rotating the satellite in addition to its revolution.

Key Points

- The time period of the longest pendulum on the surface of earth is given by

  $T = 2\pi\sqrt{\dfrac{R}{g}}$ = 84.6 minutes

  Note : The time period of a satellite orbiting close to the earth's surface is also 84.6 minutes.
- The angular velocity and time period of revolution of a G.S.S. is same as that of earth. It means that a G.S.S. completes its revolution around the earth once in 24 hours.
- Height of a G.S.S. (Geo-stationary satellite) from the surface of earth is about 36000 km. Therefore, its distance from the centre of earth is about R + H = 36000 + 6400 = 42400 km.
- It is used as a communication satellite. It is also known as parking satellite, telecommunication satellite or synchronous satellite.
- One G.S.S. can cover nearly one third surface area of earth. Therefore a minimum of three G.S.S. are required to cover the whole earth.
- Orbital velocity depends upon the mass of the central body and orbital radius (distance of satellite from the centre of the central body). If the distance of satellite increases, then the orbital velocity (v_0) decreases.
- Orbital velocity does not depend on the mass of satellite.
- If a body is taken to a small height and given a horizontal velocity of 8 km/s, it will start revolving around the earth in a circular orbit which means that it will also become a satellite close to the earth's surface.

- If a body is released from a revolving satellite, then it will continue the move in the same orbit with the same orbital velocity which means that it will also become a satellite close to the earth.
- When the total energy of a satellite is negative, it will be moving in either a circular or an elliptical orbit.
- When the total energy of a satellite is zero, it will escape away from its orbit and its path becomes parabolic.
- If the gravitational force is inversely proportional to the n^{th} power of distance r, then the orbital velocity of a satellite $v_0 \propto r^{\frac{1-n}{2}}$ and time period $T \propto r^{\frac{n+1}{2}}$.
- The total energy of any planet revolving around the sun is negative (it is bounded)
- First Satellite of Earth is Sputnik I. First Geo-satellite of India is Aryabhatt I.
  First Geo-stationary satellite of India is Apple I, Example of other Satellites of India are : Bhaskar-I, Rohini-I, Bhaskar-II (Geo Satellite); Insat-I(A) Insat-I (B) (Geo Stationary Satellite)

Example 39:

An astronaut, inside an earth's satellite experiences weightlessness because :

(1) he is falling freely

(2) no external force is acting on him

(3) no reaction is exerted by the floor of the satellite

(4) he is far away from the earth's surface

Solution:

Ans. (1, 3)

As astronaut's acceleration = g; so he is falling freely. Also no reaction is exerted by the floor of the satellite.

Example 40:

Is it possible to place an artificial satellite in an orbit such that it is always visible over Kota? Write down the reason.

Solution:

No, Kota is not in the equatorial plane.

Concept Builder-7

Q.1 The time period of revolution of moon around the earth is 28 days and radius of its orbit is 4×10^5 km. If $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 then find the mass of the earth.

Q.2 Two satellites are orbiting around the earth in circular orbits of the same radius. One of them is 100 times greater in mass than the other. Compare their periods of revolution :

(1) 100 : 1 (2) 1 : 100 (3) 1 : 10 (4) 1 : 1

Q.3 Two satellites A and B, having ratio of masses 3 : 1 are in circular orbits of radius r and 4r. Calculate the ratio of total mechanical energies of A to B.

Q.4 An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to small but continuous dissipation against atmospheric resistance. Then explain why its speed increases progressively as it comes closer and closer to the earth.

Q.4 Write the answer of the following questions in one word :-

(a) What is the orbital speed of Geo-stationary satellite ?

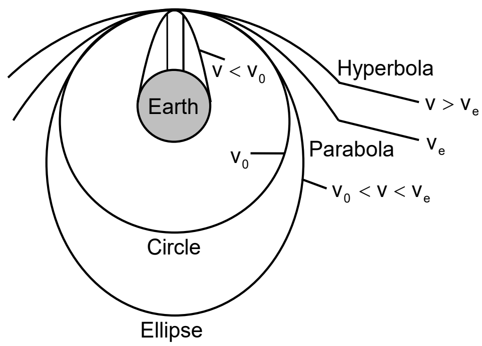
(b) For a satellite moving in an orbit around the earth what is the ratio of kinetic energy to potential energy?

Q.6 An object weighs 10 N at the north pole of the Earth. In a geostationary satellite distant 7R from the centre of the Earth (of radius R), the true weight and the apparent weight are respectively:

(1) 0, 0 (2) 0.2 N, 0 (3) 0.2 N, 9.8 N (4) 0.2 N, 0.2 N

- **Bound and Unbound Trajectories**

  Imagine a very tall building on the surface of the earth from where a projectile is fired with a velocity v parallel to the surface of the earth. The trajectory of the projectile depends on its velocity.



Velocity	Trajectory
$v < v_0$	Projectile may not orbit the arth in an elliptical path, or it may falls back on the earth's surface.
$v = v_0$	Projectile orbits the earth in a circular path.
$v_0 < v < v_e$	Projectile orbits in an elliptical path.
$v = v_e$	Projectile does not orbit. It escapes the gravitational field of earth in a parabolic path.
$v > v_e$	Projectile does not orbit. It escapes the gravitational field of earth in a hyperbolic path.

- **Geostationary Satellites**

  The plane of the orbit lies in equatorial plane of earth.
  Height from the earth surface is 36000 km. This orbit is called parking orbit.
  Orbital speed is nearly 3 km/s.
  Time period is equal to that of earth rotation i.e., 24 hours.
  Rotating direction should be same as that of earth (west to east)

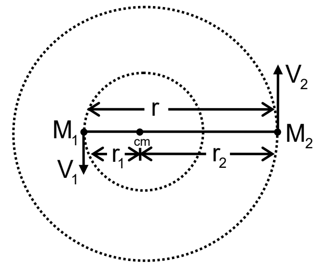
- **Polar Satellite**

  Used for remote sensing, their orbit contains axis of rotation of earth. They can cover entire earth surface for viewing.

- **Binary Star System**

  Two stars of mass M_1 and M_2 form a stable system when they move in circular orbit about their centre of mass under their mutual gravitational attraction.

- $F = \dfrac{GM_1M_2}{r^2}$,



- $M_1r_1 = M_2r_2$
- $\dfrac{GM_1M_2}{r^2} = \dfrac{M_1V_1^2}{r_1} = \dfrac{M_2V_2^2}{r_2}$
- $r_1 = \dfrac{M_2r}{M_1+M_2}$, $r_2 = \dfrac{M_1r}{M_1+M_2}$
- $V_1 = M_2\sqrt{\dfrac{G}{(M_1+M_2)r}}$
- $V_2 = M_1\sqrt{\dfrac{G}{(M_1+M_2)r}}$

  when $M_1 = M_2$
- $V_1 = V_2 = \sqrt{\dfrac{GM}{2r}}$
- $r_1 = r_2 = \dfrac{r}{2}$

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. $F = 3.2 \times 10^{-8}$ dyne.

2. $r = 0.816 \times 10^{-5}$ m

3. Zero

4. $F_{net} = \sqrt{3}F = \sqrt{3}\dfrac{Gm^2}{a^2}$

5. (3)

6. $v = \dfrac{1}{2}\sqrt{\dfrac{Gm}{r}}$

7. 6 m from 9 kg mass.

8. $\dfrac{4\pi^2}{3\sqrt{3}} G\rho_0^2 r_0^4$

CONCEPT BUILDER-2

1. (4)

2. $\dfrac{7}{8}\dfrac{GM}{x^2}$

CONCEPT BUILDER-3

1. (1)

2. 2400 N

3. $\dfrac{1}{2} g_e$

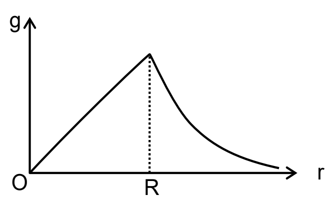
4. (1)

5. 3 m

6. 0.5%

7. 32 km

8.



9. 4.44 m/s^2

10. 6400 km

11. 1920 km

12. $\dfrac{3}{4}R$

13. (1)

14. 1.25×10^{-3} Rad/s

15. 1.25×10^{-3} Rad/s

16. (3)

CONCEPT BUILDER-4

1. (4)

2. (a) $\dfrac{1}{2}$ mgR, (b) $-\dfrac{1}{2}$ mgR

3. (1)

4. $\dfrac{1}{2}$ mgR

5. 8 km/s

6. (2)

7. (4)

8. 1.47×10^{-7} J/kg

CONCEPT BUILDER-5

1. 1.78 m

2. (3)

3. (1)

4. $\sqrt{2gR}$

5. $R/(1 - K^2)$

CONCEPT BUILDER-6

1. $T_m = \left(\dfrac{2}{5}\right)^{3/2}$ year

2. $2\sqrt{2}$ years

3. All quantities vary over an orbit except angular momentum and total energy.

4. (4)

5. The planet will take a longer time to traverse BAC than CPB.

6. (4)

CONCEPT BUILDER-7

1. 6.47×10^{24} kg

2. (4)

3. $\dfrac{12}{1}$

4. Kinetic energy increases, but potential energy decreases, and the sum decreases due to dissipation against friction.

5. (a) $\simeq$ 3.1 km/s, (b) $-\dfrac{1}{2}$

6. (2)

Exercise - I

Newton's Law of Gravitation

1. Newton's law of gravitation :
(1) is not applicable out side the solar system
(2) is used to govern the motion of satellites only
(3) control the rotational motion of satellites and planets
(4) control the rotational motion of electrons in atoms

2. Mass particles of 1 kg each are placed along x-axis at x = 1, 2, 4, 8, ...∞. Then gravitational force on a mass of 3 kg placed at origin is (G = universal gravitational constant) :
(1) 4G (2) $\frac{4G}{3}$
(3) 2G (4) ∞

3. Gravitational force between two masses at a distance 'd' apart is 6N. If these masses are taken to moon and kept at same separation, then the force between them will become :-
(1) 1N (2) $\frac{1}{6}$ N
(3) 36 N (4) 6 N

4. The value of universal gravitational constant G depends upon :
(1) Nature of material of two bodies
(2) Heat constant of two bodies
(3) Acceleration of two bodies
(4) None of these

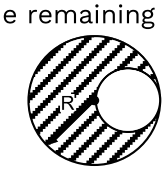
5. The tidal waves in the seas are primarily due to :
(1) The gravitational effect of the sun on the earth
(2) The gravitational effect of the moon on the earth
(3) The rotation of the earth
(4) The atmospheric effect of the earth it self

Law of Superposition of Forces

6. Four particles of masses m, 2m, 3m and 4m are kept in sequence at the corners of a square of side a. The magnitude of gravitational force acting on a particle of mass m placed at the centre of the square will be :
(1) $\frac{24m^2G}{a^2}$ (2) $\frac{6m^2G}{a^2}$
(3) $\frac{4\sqrt{2}Gm^2}{a^2}$ (4) zero

7. Three equal masses of 1 kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2 kg is placed at the centroid O of the triangle. The force in newton acting on the mass of 2 kg is :-
(1) 2 (2) $\sqrt{2}$
(3) 1 (4) zero

8. A spherical hole of radius R/2 is made in a solid sphere of radius R. The mass of the sphere before hollowing was M. The gravitational field at the centre of the hole due to the remaining mass is:



(1) zero (2) $\frac{GM}{8R^2}$
(3) $\frac{GM}{2R^2}$ (4) $\frac{GM}{R^2}$

9. Three uniform spheres of mass M and radius R each are kept in such a way that each touches the other two. The magnitude of the gravitational force on any of the sphere due to the other two is:
(1) $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$ (2) $\frac{3}{2}\frac{GM^2}{R^2}$
(3) $\frac{\sqrt{3}GM^2}{R^2}$ (4) $\frac{\sqrt{3}}{2}\frac{GM^2}{R^2}$

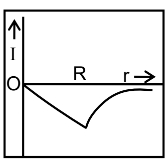
Gravitational Field Intensity Due to Particle, Spherical Shell, Spherical Mass Distribution, Ring

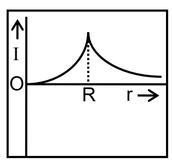
10. If the distance between the centres of earth and moon is D and mass of earth is 81 times that of moon. At what distance from the centre of earth gravitational field will be zero :

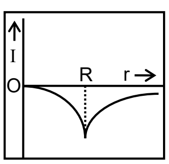
(1) $\frac{D}{2}$ (2) $\frac{2D}{3}$

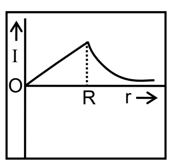
(3) $\frac{4D}{5}$ (4) $\frac{9D}{10}$

11. Following curve shows the variation of intensity of gravitational field (I) with distance from the centre of solid sphere (r) :-

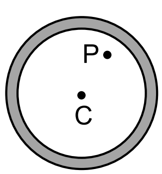
(1) 

(2) 

(3) 

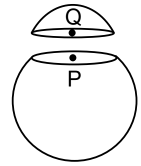
(4) 

12. The force between a hollow sphere and a point mass at P inside it as shown in figure:



(1) is attractive and constant
(2) is attractive and depends on the position of the point with respect to centre C
(3) is zero
(4) is repulsive and constant

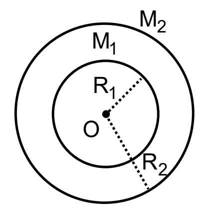
13. A spherical shell is cut into two pieces along a chord as shown in figure. For points P and Q :



(1) $I_P > I_Q$ (2) $I_P < I_Q$

(3) $I_P = I_Q = 0$ (4) $I_P = I_Q \neq 0$

14. Figure shows two concentric spherical shells of mass M_1 and M_2 and of radii R_1 and R_2 respectively. Gravitational potential at a point at a distance x from centre such that $R_1 < x < R_2$ will be :



(1) $\frac{-GM_2}{R_2}$ (2) $-G\left(\frac{M_1}{R_1}+\frac{M_2}{x}\right)$

(3) $-G\left(\frac{M_1}{x}+\frac{M_2}{R_1}\right)$ (4) $-G\left(\frac{M_1}{x}+\frac{M_2}{R_2}\right)$

15. In Q.14, gravitational intensity at a Distance x from the centre such that $x > R_2$ is :

(1) $G\left(\frac{M_1+M_2}{x^2}\right)$ (2) $G\left(\frac{M_1}{R_1^2}+\frac{M_2}{R_2^2}\right)$

(3) $G\left(\frac{M_1}{R_1^2}+\frac{M_2}{x^2}\right)$ (4) $G\left(\frac{M_1}{x}+\frac{M_2}{R_1^2}\right)$

Acceleration Due to Gravity

16. The value of 'g' on earth surface depends :
(1) only on earth's structure
(2) only on earth's rotational motion
(3) on above both
(4) on none of these and is same

17. Diameter and mass of a planet is double that of earth. Then time period of a pendulum at surface of planet is how much times of time period at earth surface :

(1) $\frac{1}{\sqrt{2}}$ times (2) $\sqrt{2}$ times
(3) Equal (4) None of these

18. Gravitation on moon is $1/6^{th}$ of that on earth. When a balloon filled with hydrogen is released on moon then this :

(1) Will rise with an acceleration less than $\left(\frac{g}{6}\right)$
(2) Will rise with acceleration $\left(\frac{g}{6}\right)$
(3) Will fall down with an acceleration less than $\left(\frac{5g}{6}\right)$
(4) Will fall down with acceleration $\left(\frac{g}{6}\right)$

19. The acceleration due to gravity g and mean density of earth ρ are related by which of the following relations? [G = gravitational constant and R = Radius of earth]:

(1) $\rho = \frac{4\pi gR^2}{3G}$ (2) $\rho = \frac{4\pi gR^2}{3G}$
(3) $\rho = \frac{3g}{4\pi gR}$ (4) $\rho = \frac{3g}{4\pi GR^3}$

20. The mass of the moon is 1% of mass of the earth. The ratio of gravitational pull of earth on moon to that of moon on earth will be :

(1) 1 : 1 (2) 1 : 10
(3) 1 : 100 (4) 2 : 1

21. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g' then :

(1) g' = 3g (2) g' = g/9
(3) g' = 9g (4) g' = 27 g

22. One can easily "weight the earth" by calculating the mass of earth using the formula
(in usual notation)

(1) $\frac{G}{g}R_E^2$ (2) $\frac{g}{G}R_E^2$
(3) $\frac{g}{G}R_E$ (4) $\frac{G}{g}R_E^3$

23. Mars has a diameter of approximately 0.5 of that of earth, and mass 0.1 of that of earth. The surface gravitational field strength on mars as compared to that on earth is greater by a factor of :

(1) 0.1 (2) 0.2
(3) 2.0 (4) 0.4

Variation in Acceleration Due to Gravity

24. Acceleration due to gravity at the centre of the earth is :

(1) g (2) $\frac{g}{2}$
(3) zero (4) infinite

25. The value of 'g' reduces to half of its value at surface of earth at a height 'h', then :-

(1) h = R
(2) h = 2R
(3) $h = (\sqrt{2} + 1)R$
(4) $h = (\sqrt{2} - 1)R$

26. At any planet 'g' is 1.96 m sec^{-2}. If it is safe to jump from a height of 2m on earth, then what should be corresponding safe height for jumping on that planet :

(1) 5 m (2) 2 m
(3) 10 m (4) 20 m

27. If the earth stops rotating suddenly the value of g at a place other than poles would :-

(1) Decrease
(2) Remain constant
(3) Increase
(4) Increase or decrease depending on the position of earth in the orbit round the sun

28. When you move from equator to pole, the value of acceleration due to gravity (g) :-
(1) increases
(2) decreases
(3) remains the same
(4) first increases then decreases

29. When the radius of earth is reduced by 1% without changing the mass, then the acceleration due to gravity will
(1) increase by 2% (2) decrease by 1.5%
(3) increase by 1% (4) decrease by 1%

30. Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, then its weight will
(1) decrease by 0.5% (2) decrease by 2%
(3) increase by 0.5% (4) increase by 1%

31. Acceleration due to gravity at earth's surface is 'g' ms^{-2}. Find the effective value of acceleration due to gravity at a height of 32 km from sea level: (R_e = 6400 Km)
(1) 0.5 g ms^{-2} (2) 0.99 g ms^{-2}
(3) 1.01 g ms^{-2} (4) 0.90 g ms^{-2}

32. The change in the value of 'g' at a height 'h' above the surface of the earth is same as at a depth 'd'. If 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct?
(1) d = h (2) d = 2h
(3) $d = \frac{3h}{2}$ (4) $d = \frac{h}{2}$

33. If the rotational speed of earth is increased then weight of a body at the equator
(1) increases
(2) decreases
(3) becomes double
(4) does not changes

34. A body weights W newton at the surface of the earth. Its weight at a height equal to half the radius of the earth will be :
(1) $\frac{W}{2}$ (2) $\frac{W}{2}$
(3) $\frac{4W}{9}$ (4) $\frac{W}{4}$

35. The imaginary angular velocity of the earth for which the effective acceleration due to gravity at the equator shall be zero is equal to
(1) 1.25×10^{-3} rad/s (2) 2.50×10^{-3} rad/s
(3) 3.75×10^{-3} rad/s (4) 5.0×10^{-3} rad/s
[Take g = 10 m/s^2 for the acceleration due to gravity if the earth were at rest and radius of earth equal to 6400 km]

36. More amount of sugar is obtained in 1 kg weight:
(1) At North pole
(2) At equator
(3) Between pole and equator
(4) At South pole

Gravitational Potential Energy

37. A particle falls on earth :
(i) from infinity, (ii) from a height 10 times the radius of earth. The ratio of the velocities gained on reaching at the earth's surface is :
(1) $\sqrt{11} : \sqrt{10}$ (2) $\sqrt{10} : \sqrt{11}$
(3) 10 : 11 (4) 11 : 10

38. Two different masses are dropped from same height. When these just strike the ground the following is same :
(1) kinetic energy
(2) potential energy
(3) linear momentum
(4) acceleration

39. If M_e is the mass of earth and M_m is the mass of moon ($M_e = 81\ M_m$). The potential energy of an object of mass m situated at a distance R from the centre of earth and r from the centre of moon will be :-
(1) $-GmM_m\left(\frac{R}{81}+r\right)\frac{1}{R^2}$
(2) $-GmM_e\left(\frac{81}{r}+\frac{1}{R}\right)$
(3) $-GmMm\left(\frac{81}{R}+\frac{1}{r}\right)$
(4) $GmMm\left(\frac{81}{R}-\frac{1}{r}\right)$

40. The gravitational potential energy is maximum at:
(1) infinity
(2) the earth's surface
(3) the centre of the earth
(4) Twice the radius of the earth

41. A body attains a height equal to the radius of the earth when projected from earth's surface. The velocity of the body with which it was projected is :

(1) $\sqrt{\frac{GM}{R}}$

(2) $\sqrt{\frac{2GM}{R}}$

(3) $\sqrt{\frac{5}{4}\frac{GM}{R}}$

(4) $\sqrt{\frac{3GM}{R}}$

42. The gravitational potential energy of a body at a distance r from the center of the earth is U. The force at that point is :

(1) $\frac{U}{r^2}$

(2) $\frac{U}{r}$

(3) Ur

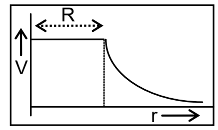
(4) Ur^2

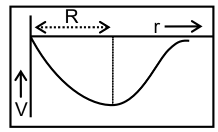
43. A projectile of mass m is thrown vertically up with an initial velocity v from the surface of earth (mass of earth = M). If it comes to rest at a height h, the change in its potential energy is
(1) GMmh/R(R + h)
(2) $GMmh^2/R(R + h)^2$
(3) GMmhR /(R + h)
(4) GMm/hR(R + h)

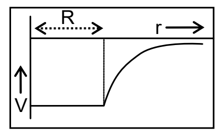
44. A particle falls from infinity to the earth. Its velocity on reaching the earth surface is :
(1) 2Rg
(2) Rg
(3) $\sqrt{Rg}$
(4) $\sqrt{2Rg}$

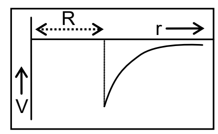
Gravitational Potential/Work Done

45. Which of the following curve expresses the variation of gravitational potential with distance for a hollow sphere of radius R :

(1) 

(2) 

(3) 

(4) 

46. Gravitational potential difference between surface of a planet and a point situated at a height of 20 m above its surface is 2 joule/kg. If gravitational field is uniform, then the work done in taking a 5 kg body upto height 4 meter above surface will be:
(1) 2 J (2) 20 J
(3) 40 J (4) 10 J

47. Two small and heavy spheres, each of mass M, are placed a distance r apart on a horizontal surface. The gravitational potential at the mid-point on the line joining the centre of the spheres is :-
(1) zero

(2) $-\frac{GM}{r}$

(3) $-\frac{2GM}{r}$

(4) $-\frac{4GM}{r}$

Escape Velocity & Escape Energy

48. Body is projected vertically upward from the surface of the earth with a velocity equal to half the escape velocity. If R is radius of the earth, the maximum height attained by the body is :-

(1) $\frac{R}{6}$ (2) $\frac{R}{3}$

(3) $\frac{2}{3}R$ (4) R

49. Escape velocity for a projectile at earth's surface is V_e. A body is projected from earth's surface with velocity $2V_e$. The velocity of the body when it is at infinite distance from the centre of the earth is :-

(1) V_e (2) $2V_e$

(3) $\sqrt{2}\,V_e$ (4) $\sqrt{3}\,V_e$

50. Potential energy of a 3 kg body at the surface of planet is –54 J, then escape velocity will be :

(1) 18 m/s (2) 162 m/s

(3) 36 m/s (4) 6 m/s

51. Escape velocity of a 1 kg body on a planet is 100 m/s. Potential energy of body at that planet is :

(1) –5000 J (2) – 1000J

(3) –2400J (4) –10000J

52. A particle on earth's surface is given a velocity equal to its escape velocity. Its total mechanical energy will be:

(1) negative (2) positive

(3) zero (4) infinite

53. A hole is drilled from the surface of earth to its centre. A particle is dropped from rest at the surface of earth. The speed of the particle when it reaches the centre of the earth in terms of its escape velocity on the surface of earth v_e is:

(1) $\frac{v_e}{2}$ (2) v_e

(3) v_e (4) $\frac{v_e}{\sqrt{2}}$

54. An electron moves in a circular orbit at a distance from a proton with kinetic energy E. To escape to infinity, the energy which must be supplied to the electron is:

(1) E (2) 2 E

(3) 0.5 E (4) $\sqrt{2}$ E

55. A particle is fired upward with a speed of 20 km/h. The speed with which it will move in interstellar space is:

(1) 8.8 km/h (2) 16.5 km/h

(3) 11.2 km/h (4) 10 km/h

Kepler's Law

56. Kepler's second law is a consequence of :

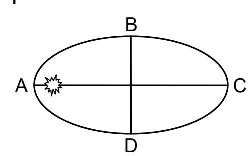
(1) conservation of kinetic energy

(2) conservation of linear momentum

(3) conservation of angular momentum

(4) conservation of speed

57. In adjoining figure earth goes around the sun in elliptical orbit on which point the orbital speed is maximum :



(1) On A (2) On B

(3) On C (4) On D

58. If a graph is plotted between T^2 and r^3 for a planet then its slope will be :-

(1) $\frac{4\pi^2}{GM}$ (2) $\frac{GM}{4\pi^2}$

(3) 4π GM (4) Zero

59. A planet of mass m is moving in an elliptical orbit about the sun (mass of sun = M). The maximum and minimum distances of the planet from the sun are r_1 and r_2 respectively. The period of revolution of the planet will be proportional to :

(1) $r_1^{3/2}$ (2) $r_2^{3/2}$

(3) $(r_1 - r_2)^{3/2}$ (4) $(r_1 + r_2)^{3/2}$

60. The earth revolves around the sun in one year. If distance between them becomes double, the new time period of revolution will be :-

(1) years (2) $2\sqrt{2}$ years

(3) 4 years (4) 8 years

61. The mean distance of mars from sun is 1.5 times that of earth from sun. What is approximately the number of years required by mars to make one revolution about sun?

(1) 2.35 years (2) 1.85 years

(3) 3.65 years (4) 2.75 years

62. The maximum and minimum distances of a comet from the sun are 8×10^{12} m and 1.6×10^{12} m respectively. If its velocity when it is nearest to the sun is 60 m/sec then what will be its velocity in m/s when it is farthest?

(1) 12 (2) 60

(3) 112 (4) 6

Satellite Motion

63. Binding energy of moon and earth is :-

(1) $\frac{GM_eM_m}{r_{em}}$ (2) $\frac{GM_eM_m}{2r_{em}}$

(3) $-\frac{GM_eM_m}{r_{em}}$ (4) $-\frac{GM_eM_m}{2r_{em}}$

64. Two artificial satellites A and B are at a distance r_A and r_B above the earth's surface. If the radius of earth is R, then the ratio of their speed will be:

(1) $\left(\frac{r_B+R}{r_A+R}\right)^{\frac{1}{2}}$ (2) $\left(\frac{r_B+R}{r_A+R}\right)^{2}$

(3) $\left(\frac{r_B}{r_A}\right)^{2}$ (4) $\left(\frac{r_B}{r_A}\right)^{\frac{1}{2}}$

65. The average radii of orbits of mercury and earth around the sun are 6×10^7 km and 1.5×10^8 km respectively. The ratio of their orbital speeds will be :-

(1) $\sqrt{5} : \sqrt{2}$ (2) $\sqrt{2} : \sqrt{5}$

(3) 2.5 : 1 (4) 1 : 25

66. A body is dropped by a satellite in its geo-stationary orbit :

(1) it will burn on entering in to the atmosphere

(2) it will remain in the same place with respect to the earth

(3) it will reach the earth in 24 hours

(4) it will perform uncertain motion

67. Two ordinary satellites are revolving round the earth in same elliptical orbit, then which of the following quantities is conserved :-

(1) velocity

(2) angular velocity

(3) Angular momentum

(4) none of above

68. One projectile after deviating from its path starts moving round the earth in a circular path of radius equal to nine times the radius of earth R. Its time period will be :-

(1) $2\pi\sqrt{\frac{R}{g}}$ (2) $27 \times 2\pi\sqrt{\frac{R}{g}}$

(3) $\pi\sqrt{\frac{R}{g}}$ (4) $0.8 \times 3\pi\sqrt{\frac{R}{g}}$

69. A satellite launching station should be :

(1) near the equatorial region

(2) near the polar region

(3) on the polar axis

(4) all locations are equally good

70. If two bodies of mass M and m are revolving around the centre of mass of the system in circular orbit of radii R and r respectively due to mutual interaction. Which of the following formula is applicable :-

(1) $\frac{GMm}{(R+r)^2} = m\omega^2 r$ (2) $\frac{GMm}{R^2} = m\omega^2 r$

(3) $-\frac{GMm}{r^2} = m\omega^2 R$ (4) $\frac{GMm}{R^2+r^2} = m\omega^2 r$

71. The relay satellite transmits the television programme continuously from one part of the world to another because its :

(1) Period is greater than the period of rotation of the earth about its axis

(2) Period is less than the period of rotation of the earth about its axis

(3) Period is equal to the period of rotation of the earth about its axis

(4) Mass is less than the mass of earth

72. If the satellite is stopped suddenly in its orbit which is at a distance equal to radius of earth from earth's surface and allowed to fall freely into the earth. The speed with which it hits the surface of earth will be :

(1) 7.919 m/s (2) 7.919 km/s

(3) 11.2 m/s (4) 11.2 km/s

73. The gravitational force between two bodies is directly proportional to $\frac{1}{R}$ (not $\frac{1}{R^2}$) where 'R' is the distance between the bodies. Then the orbital speed for this force in circular orbit is proportional to :

(1) $1/R^2$ (2) R^o

(3) R (4) 1/R

74. What will be velocity of a satellite revolving around the earth at a height h above surface of earth if radius of earth is R :

(1) $R^2\sqrt{\frac{g}{R+H}}$ (2) $R\frac{g}{(R+h)^2}$

(3) $R\sqrt{\frac{g}{R+h}}$ (4) $R\sqrt{\frac{R+h}{g}}$

75. Two artificial satellites of masses m_1 and m_2 are moving with speeds v_1 and v_2 in orbits of radii r_1 and r_2 respectively. If $r_1 > r_2$ then which of the following statements is true :

(1) $v_1 = v_2$ (2) $v_1 > v_2$

(3) $v_1 < v_2$ (4) $v_1/r_1 = v_2/r_2$

76. Orbital radius of a satellite S of earth is four times that of a communication satellite C. Period of revolution of S is :

(1) 4 days (2) 8 days

(3) 16 days (4) 32 days

77. If a satellite is revolving very close to the surface of earth, then its orbital velocity does not depend upon :

(1) Mass of satellite (2) Mass of earth

(3) Radius of earth (4) Orbital radius

78. The minimum projection velocity of a body from the earth's surface so that it becomes the satellite of the earth ($R_e = 6.4 \times 10^6$ m).

(1) $11 \times 10^3\ ms^{-1}$ (2) $8 \times 10^3\ ms^{-1}$

(3) $6.4 \times 10^3\ ms^{-1}$ (4) $4 \times 10^3\ ms^{-1}$

79. Geostationary satellite :-

(1) is situated at a great height above the surface of earth

(2) moves in equatorial plane

(3) have time period of 24 hours

(4) have time period of 24 hours and moves in equatorial plane

80. A satellite of mass m goes round the earth along a circular path of radius r. Let m_E be the mass of the earth and R_E its radius then the linear speed of the satellite depends on.

(1) m, m_E and r (2) m, R_E and r

(3) m_E only (4) m_E and r

81. Near the earth's surface time period of a satellite is 1.4 hrs. Find its time period if it is at the distance '4R' from the centre of earth :-

(1) 32 hrs. (2) $\left(\frac{1}{8\sqrt{2}}\right)$ hrs.

(3) $8\sqrt{2}$ hrs. (4) 16 hrs.

82. A communication satellite of earth which takes 24 hrs. to complete one circular orbit eventually has to be replaced by another satellite of double mass. If the new satellite also has an orbital time period of 24 hrs, then what is the ratio of the radius of the new orbit to the original orbit ?

(1) 1 : 1 (2) 2 : 1

(3) $\sqrt{2} : 1$ (4) 1 : 2

83. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v_0. The orbital velocity of satellite orbiting at an altitude of half of the radius is :-

(1) $\frac{3}{2}v_0$ (2) $\frac{2}{3}v_0$

(3) $\sqrt{\frac{2}{3}}v_0$ (4) $\sqrt{\frac{3}{2}}\ v_0$

84. An earth's satellite is moving in a circular orbit with a uniform speed v. If the gravitational force of the earth suddenly disappears, the satellite will :

(1) vanish into outer space

(2) continue to move with velocity v in original orbit

(3) fall down with increasing velocity

(4) fly off tangentially from the orbit with same velocity

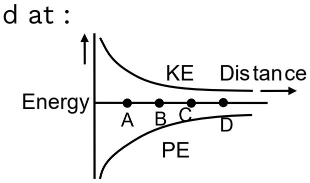
85. An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0. Its potential energy is :

(1) $-E_0$ (2) E_0

(3) $-2E_0$ (4) $2E_0$

Energy of Satellite

86. Potential energy and kinetic energy of a two particle system under imaginary force field are shown by curves KE and PE, respectively in figure. This system is bound at :



(1) only point A

(2) only point D

(3) only point A,B and C

(4) All points A, B, C and D

87. A satellite of earth of mass 'm' is taken from orbital radius 2R to 3R, then minimum work done is :-

(1) $\frac{GMm}{6R}$ (2) $\frac{GMm}{12R}$

(3) $\frac{GMm}{24R}$ (4) $\frac{GMm}{3R}$

88. A satellite is moving in a circular orbit around earth with a speed v. If its mass is m, then its total energy will be :

(1) $\frac{3}{4}\ mv^2$ (2) mv^2

(3) $\frac{1}{2}\ mv^2$ (4) $-\frac{1}{2}mv^2$

89. Two satellites of same mass m are revolving round of earth (mass M) in the same orbit of radius r. Rotational directions of the two are opposite therefore, they can collide. Total mechanical energy of the system (both satellites and earth's) is (m << M) :-

(1) $-\frac{GMm}{r}$ (2) $-\frac{2GMm}{r}$

(3) $-\frac{GMm}{2r}$ (4) Zero

90. A planet is moving in an elliptical orbit. If T, U, E and L are its kinetic energy, potential energy, total energy and magnitude of angular momentum respectively, then which of the following statement is true :-

(1) T is conserved

(2) U is always positive

(3) E is always negative

(4) L is conserved but the direction of vector will continuously change

91. Two identical satellites are at the heights R and 7R from the earth's surface. Then which of the following statement is incorrect: (R = Radius of the earth)

(1) Ratio of total energy of both is 5

(2) Ratio of kinetic energy of both is 4

(3) Ratio of potential energy of both is 4

(4) Ratio of total energy of both is 4

92. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

(1) 2 (2) 1/2

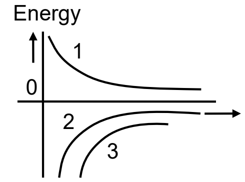
(3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

93. A satellite of mass m revolves in a circular orbit of radius R round a planet of mass M. Its total energy E is :-

(1) $-\frac{GMm}{2R}$ (2) $+\frac{GMm}{3R}$

(3) $-\frac{GMm}{R}$ (4) $+\frac{GMm}{R}$

94. A satellite is orbiting earth at a distance r. Variations of its kinetic energy, potential energy and total energy, is shown in the figure. Of the three curves shown in figure, identify the type of mechanical energy they represent.



(1) 1 Potential, 2 Kinetic, 3 Total

(2) 1 Total, 2 Kinetic, 3 Potential

(3) 1 Kinetic, 2 Total, 3 Potential

(4) 1 Potential, 2 Total, 3 Kinetic0

95. A space shuttle is launched in a circular orbit near the earth's surface. The additional velocity be given to the space-shuttle to get free from the influence of gravitational force, will be :

(1) 1.52 km/s

(2) 2.75 km/s

(3) 3.28 km/s

(4) 5.18 km/s

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	3	1	4	4	2	3	4	3	1	4	1	3	4	4	1	3	2	4	3	1	1	2	4	3	4
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	3	3	1	1	1	2	2	2	3	1	2	1	4	3	1	1	2	1	4	3	1	4	2	4	4
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	1	3	4	1	2	3	1	1	4	2	2	1	2	1	1	2	3	2	1	1	3	2	2	3	3
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95					
Ans.	2	1	2	4	4	3	1	3	4	4	3	2	4	1	3	1	2	1	3	3					

Exercise - II

1. Masses and radii of earth and moon are M_1, M_2 and R_1, R_2 respectively. The distance between their centre is 'd'. The minimum velocity given to mass 'M' from the mid point of the line joining their centre so that it will escape :

(1) $\sqrt{\frac{4G(M_1+M_2)}{d}}$ (2) $\sqrt{\frac{4G}{d}\frac{M_1M_2}{(M_1+M_2)}}$

(3) $\sqrt{\frac{2G}{d}\left(\frac{M_1+M_2}{M_1M_2}\right)}$ (4) $\sqrt{\frac{2G}{d}(M_1+M_2)}$

2. A projectile is fired vertically upward from the surface of earth with a velocity KV_e, where V_e is the escape velocity and K < 1. Neglecting air resistance, the maximum height to which it will rise measured from the centre of the earth is : (Where R = radius of earth)

(1) $\frac{R}{1-K^2}$ (2) $\frac{R}{K^2}$

(3) $\frac{1-K^2}{R}$ (4) $\frac{K^2}{R}$

3. If R is the average radius of earth, ω is its angular velocity about its axis and g is the gravitational acceleration on the surface of earth then the cube of the radius of orbit of a geostationary satellite will be equal to :

(1) $\frac{R^2g}{\omega}$ (2) $\frac{R^2\omega^2}{g}$

(3) $\frac{R^2\omega^2}{g}$ (4) $\frac{R^2g}{\omega^2}$

4. A geostationary satellite is orbiting the earth at a height of 6R from the earth's surface (R is the earth's radius). What is the period of rotation of another satellite at a height of 2.5 R from the earth surface?

(1) $6\sqrt{2}$ hours

(2) 10 hours

(3) $\frac{5\sqrt{5}}{\sqrt{3}}$ hours

(4) None of the above

5. A planet revolves around the sun in an elliptical orbit. If v_p and v_a are the velocities of the planet at the perigee and apogee respectively, then the eccentricity of the elliptical orbit is given by :

(1) $\frac{v_p}{v_a}$

(2) $\frac{v_a - v_p}{v_a + v_p}$

(3) $\frac{v_p + v_a}{v_p - v_a}$

(4) $\frac{v_p - v_a}{v_p + v_a}$

6. Two solid spherical planets of equal radii R having masses 4M and 9M their centre are separated by a distance 6R. A projectile of mass m is sent from the planet of mass 4M towards the heavier planet. What is the distance r of the point from the lighter planet where the gravitational force on the projectile is zero?

(1) 1.4 R (2) 1.8 R

(3) 1.5 R (4) 2.4 R

7. Read the following statements :

S_1 : An object shall weigh more at pole than at equator when weighed by using a physical balance.

S_2 : It shall weigh the same at pole and equator when weighed by using a physical balance.

S_3 : It shall weigh the same at pole and equator when weighed by using a spring balance.

S_4 : It shall weigh more at the pole than at equator when weighed using a spring balance.

Which of the above statements is/are correct?

(1) S_1 and S_2 (2) S_1 and S_4

(3) S_2 and S_3 (4) S_2 and S_4

8. Assume that a tunnel is dug through earth from North pole to south pole and that the earth is a non-rotating, uniform sphere of density ρ. The gravitational force on a particle of mass m dropped into the tunnel when it reaches a distance r from the centre of earth is :

(1) $\left(\frac{3}{4\pi}mG\rho\right)r$ (2) $\left(\frac{4\pi}{3}mG\rho\right)r$

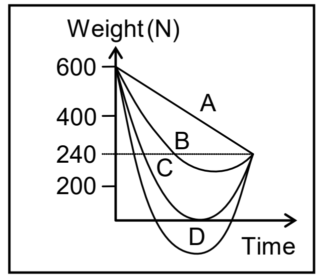
(3) $\left(\frac{4\pi}{3}mG\rho\right)r^2$ (4) $\left(\frac{4\pi}{3}m^2G\rho\right)r$

9. Three identical bodies (each mass M) are placed at vertices of an equilateral triangle of arm L, keeping the triangle as such by which angular speed the bodies should be rotated in their gravitational fields so that the triangle moves along circumference of circular orbit :

(1) $\sqrt{\frac{3GM}{L^3}}$ (2) $\sqrt{\frac{GM}{L^3}}$

(3) $\sqrt{\frac{GM}{3L^3}}$ (4) $3\sqrt{\frac{GM}{L^3}}$

10. Suppose the acceleration due to gravity at the earth's surface is 10 m/s^2 and at the surface of mars it is 4.0 m/s^2. A 60 kg passenger goes from the earth to the mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure best represent the weight (Net gravitational force) of the passenger as a function of time :



(1) A (2) B
(3) C (4) D

11. If the length of the day is T, the height of that TV satellite above the earth's surface which always appears stationary from earth, will be :

(1) $h = \left[\frac{4\pi^2 GM}{T^2}\right]^{1/3}$

(2) $h = \left[\frac{4\pi^2 GM}{T^2}\right]^{1/2}$

(3) $h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} - R$

(4) $h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} + R$

12. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then T^2 is proportional to :

(1) R^3 (2) $R^{7/2}$
(3) $R^{3/2}$ (4) $R^{9/2}$

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12
Ans.	1	1	4	1	4	4	4	2	1	3	3	2

Exercise – III (Previous Year Question)

1. Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

[AIPMT -2007]

(1) The kinetic energies of the two satellites are equal

(2) The time period of S_1 is four times that of S_2

(3) The potential energies of earth and satellite in the two cases are equal

(4) S_1 and S_2 are moving with the same speed

2. The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M, to transfer it from a circular orbit of radius R_1 to another of radius $R_2 (R_2 > R_1)$ is:

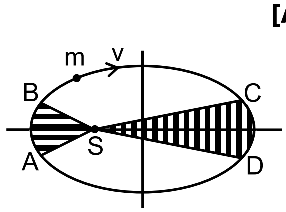
[AIPMT -2007]

(1) $GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (2) $2GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(3) $\frac{1}{2}GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (4) $GmM\left(\frac{1}{R_1^2}-\frac{1}{R_2^2}\right)$

3. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then :

[AIPMT -2009]



(1) $t_1 = t_2$ (2) $t_1 < t_2$

(3) $t_1 = 4t_2$ (4) $t_1 = 2t_2$

4. The radii of circular orbits of two satellites A and B of the earth, are 4R and R, respectively. If the speed of satellite A is 3V, then the speed of satellite B will be :

[AIPMT -2010]

(1) 3V/2 (2) 3V/4

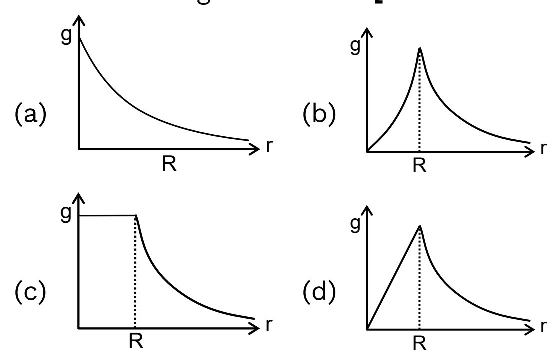
(3) 6V (4) 12V

5. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at $\frac{a}{2}$ distance from the centre, will be: **[AIPMT-2010]**

(1) $-\frac{4GM}{a}$ (2) $-\frac{3GM}{a}$

(3) $-\frac{2GM}{a}$ (4) $-\frac{GM}{a}$

6. The dependence of acceleration due to gravity 'g' on the distance 'r' from the centre of the earth, assumed to be a sphere of radius R of uniform density, is as shown in figure below: **[AIPMT -2010]**



The correct figure is :-

(1) (a) (2) (b)

(3) (c) (4) (d)

7. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio is

[AIPMT -2011]

(1) $(r_1/r_2)^2$ (2) r_2/r_1

(3) $(r_2/r_1)^2$ (4) r_1/r_2

8. A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity equal to : **[AIPMT -2012]**

(1) GM_p / D_p^2 (2) $4GM_p m / D_p^2$

(3) $4GM_p / D_p^2$ (4) $GM_p m / D_p^2$

9. A geostationary satellite is orbiting the earth at a height of 5R above that surface of the earth, R being the radius of the earth. the time period of another satellite in hours at a height of 2R from the surface of the earth is : **[AIPMT -2012]**

(1) $6\sqrt{2}$ (2) $6/\sqrt{2}$

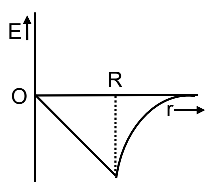
(3) 5 (4) 10

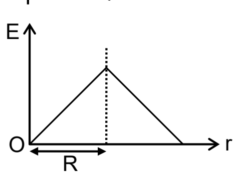
10. The height at which the weight of a body becomes $1/16^{th}$ of its weight on the surface of earth (radius R), is : **[AIPMT -2012]**

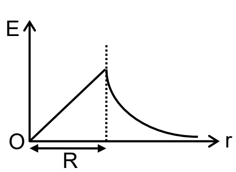
(1) 3R (2) 4R

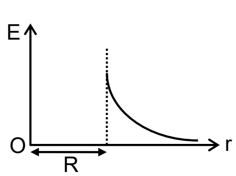
(3) 5R (4) 15R

11. Which one of the following plots represents the variation of gravitational field on a particle with distance r due to a thin spherical shell of radius R? (r is measured from the centre of the spherical shell): **[AIPMT (Main) 2012]**

(1) 

(2) 

(3) 

(4) 

12. If v_e is escape velocity and v_0 is orbital velocity of a satellite for orbit close to the earth's surface, then these are related by : **[AIPMT (Main) 2012]**

(1) $v_e = \sqrt{2v_0}$

(2) $v_e = \sqrt{2}\, v_0$

(3) $v_0 = \sqrt{2}\, v_0$

(4) $v_0 = v_e$

13. A black hole is an object whose gravitational field is so strong that even light cannot escape from it.To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole ? **[AIPMT 2014]**

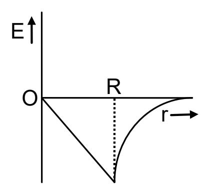
(1) 10^{-9} m

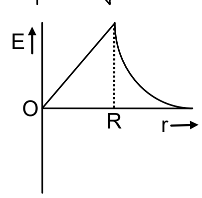
(2) 10^{-6} m

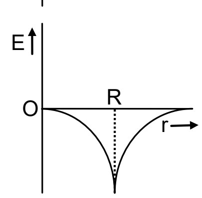
(3) 10^{-2} m

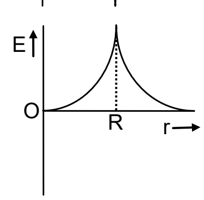
(4) 100 m

14. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by : **[AIPMT 2014]**

(1) 

(2) 

(3) 

(4) 

15. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet.
i.e. $T^2 = Kr^3$ here K is constant.
If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation between G and K is described as : **[AIPMT 2015]**

(1) $GMK = 4\pi^2$ (2) $K = G$
(3) $K = \frac{1}{G}$ (4) $GK = 4\pi^2$

16. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then, **[Re-AIPMT 2015]**

(1) the acceleration of S is always directed towards the centre of the earth.
(2) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(3) the total mechanical energy of S varies periodically with time.
(4) the linear momentum of S remains constant in magnitude.

17. A remote – sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6m and g = 9.8 ms^{-2}, then the orbital speed of the satellite is : **[Re-AIPMT 2015]**

(1) 6.67 km s^{-1} (2) 7.76 km s^{-1}
(3) 8.56 km s^{-1} (4) 9.13 km s^{-1}

18. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is : **[NEET 2016]**

(1) 1 : 2 (2) 1 : $2\sqrt{2}$
(3) 1 : 4 (4) 1 : $\sqrt{2}$

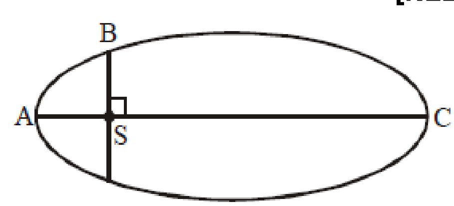
19. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then : **[NEET 2017]**

(1) $d = \frac{1}{2}$ km
(2) d = 1 km
(3) $d = \frac{3}{2}$
(4) d = 2 km

20. If the mass of the Sun were ten times smaller and the universal gravitational constant were ten time larger in magnitude, which of the following is not correct ? **[NEET 2018]**

(1) Raindrops will fall faster
(2) Walking on the ground would become more difficult
(3) Time period of a simple pendulum on the Earth would decrease
(4) 'g' on the Earth will not change

21. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then **[NEET 2018]**



(1) $K_A < K_B < K_C$ (2) $K_A > K_B > K_C$
(3) $K_B < K_A < K_C$ (4) $K_B > K_A > K_C$

22. A body weighs 200 N on the surface of the earth. How much will it weigh half way down to the centre of the earth ? **[NEET 2019]**

(1) 150 N (2) 200 N
(3) 250 N (4) 100 N

23. The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is :

[NEET 2019]

(1) mgR (2) 2 mgR

(3) $\frac{1}{2}$mgR (4) $\frac{3}{2}$mgR

24. The time period of a geostationary satellite is 24 h, at a height $6R_E$ (R_E is radius of earth) from surface of earth. The time period of another satellite whose height is 2.5 R_E from surface will be,

[NEET 2019 (Odisha)]

(1) $6\sqrt{2}$h (2) $12\sqrt{2}$h

(3) $\frac{24}{2.5}$h (4) $\frac{12}{2.5}$h

25. Assuming that the gravitational potential energy of an object at infinity is zero, the change in potential energy (final – initial) of an object of mass m, when taken to a height h from the surface of earth (of radius R), is given by,

[NEET 2019 (Odisha)]

(1) $-\frac{GMm}{R+h}$ (2) $\frac{GMmh}{R(R+h)}$

(3) mgh (4) $\frac{GMm}{R+h}$

26. A body weighs 72 N on the surface of the earth. What is the gravitational force on it, at a height equal to half the radius of the earth ? **[NEET - 2020]**

(1) 30 N (2) 24 N

(3) 48 N (4) 32 N

27. What is the depth at which the value of acceleration due to gravity becomes 1/n times the value that at the surface of earth? (radius of earth = R)

[NEET_Covid_2020]

(1) R/n^2 (2) R(n –1)/n

(3) Rn/(n –1) (4) R/n

28. A particle is released from height S from the surface of the Earth. At a certain height its kinetic energy is three times its potential energy. The height from the surface of earth and the speed of the particle at that instant are respectively :

[NEET - 2021]

(1) $\frac{S}{4}, \frac{3gS}{2}$ (2) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$

(3) $\frac{S}{2}, \frac{\sqrt{3gS}}{2}$ (4) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

29. The escape velocity from the Earth's surface is v. The escape velocity from the surface of another planet having a radius, four times that of Earth and same mass density is: **[NEET - 2021]**

(1) v (2) 2v

(3) 3v (4) 4v

30. A body of mass 60g experiences a gravitational force of 3.0 N, when placed at a particular point. The magnitude of the gravitational field intensity of that point is:

[NEET - 2021]

(1) 0.05 N/kg (2) 50 N/kg

(3) 20 N/kg (4) 180 N/kg

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	4	3	4	3	2	4	2	3	1	1	4	2	3	1	1	1	2	2	4	4	2	4	3	1	2
Que.	26	27	28	29	30																				
Ans.	4	2	4	4	2																				