Level - I

Chapter 3

Motion in a Straight Line

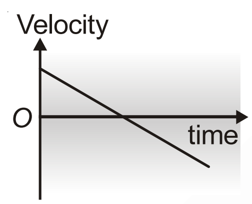
Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. A ball is thrown upwards, draw its velocity-time graph.

Sol. 

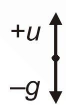
When a ball is thrown upwards, its velocity first decreases then, becomes zero at highest point and, again start increasing in the opposite direction as it comes back to the ground.

2. An object is thrown vertically upwards from the surface of earth. If the upward direction is taken as positive. What is the direction of the velocity and acceleration of the object during its upward and downward motion?

Sol. <u>Upward motion</u>

Velocity = +ve

Acceleration = –ve



<u>Downward motion</u>

Velocity = – ve

Acceleration = – ve

3. A ball is thrown upwards. What is the velocity and acceleration at the highest point?

Sol. At the top, the velocity of the ball is zero and acceleration of the ball is 9.8 m/s^2 downward.

4. What is the acceleration of a body when its velocity-time graph is (i) perpendicular to time axis (ii) parallel to time axis?

Sol. (i) infinity, (ii) zero.

5. Is it possible that an object moving with decreasing speed have constant acceleration?

Sol. Yes, during upward motion of an object thrown up under gravity.

6. The position s of a particle moving along a straight line at time t is given by $s = at^3 + bt^5$. Find its velocity at $t = 1$ s.

Sol. $v = \dfrac{ds}{dt} = 3at^2 + 5bt^4$.

at $t = 1$ s

$v = 3a + 5\,b$

7. What is the distance travelled by a body thrown upward with a speed of 20 m/s, under the effect of gravity in the first second of its motion? (use g = 10 m/s^2)

Sol. u = + 20 m/s, g = 10 m/s^2, t = 1 s.

Using,

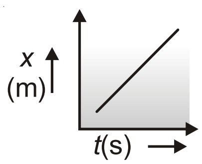
$$s = ut - \frac{1}{2}gt^2$$

$$s = 20 - \frac{1}{2}\times 10 = 15 \text{ m}$$

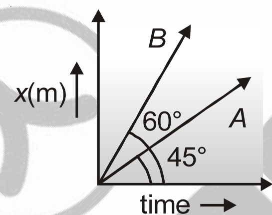
s = 15 m

8. Draw the position-time graph of an object moving with zero acceleration.

Sol.



9. The position-time graph of two objects A and B is shown below. Which one is moving with greater velocity?



Sol. Slope of position-time graph gives velocity and slope of graph = tanθ,

Slope of A = tan45° = 1,

Slope of B = tan60° = $\sqrt{3}$

B is moving with greater velocity as its slope is greater than A.

10. Mention the condition, when an object in motion can be considered as a point object.

Sol. When its size is negligible as compared to the distance travelled by object.

Short Answer Type Questions :

11. A car starts moving from rest with a constant acceleration of 5 m/s^2 along a straight line. Find

(i) The distance travelled by it in the first 2 seconds.

(ii) The distance travelled by it in the 2nd second

Sol. u = 0, a = 5 m/s^2, t = 2

using $s = ut + \frac{1}{2}at^2$

$$s = \frac{1}{2}\times 5\times 2^2 = 10 \text{ m}$$

10 m is the distance travelled by the car in 2 seconds. The distance travelled by the car in 1 second can be calculated as

$$s = \frac{1}{2}\times 5\times 1 = 2.5 \text{ m}$$

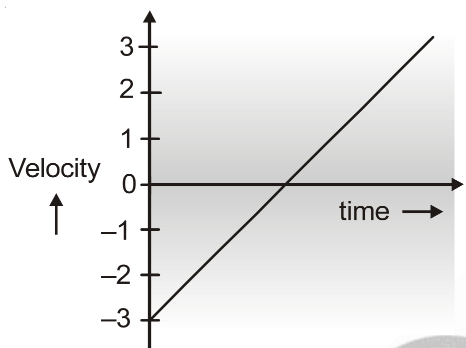
2.5 m is the distance travelled by the car in the 1st second. Hence the distance travelled by it in 2nd second = 10 – 2.5 = 7.5 m.

12. The position s of a particle moving along x-axis at time t is given by $s = 5 - 3t + 2t^2$. Draw its velocity-time graph.

Sol. $v = \frac{ds}{dt} = -3 + 4t$

$v = 4t - 3$

as $v \propto t \Rightarrow$ graph will be a straight line and at $t = 0$, $v = -3$ so graph will be a straight line cutting y-axis at $v = -3$



13. Distinguish between speed and velocity.

Sol.

Speed	Velocity
(i) Speed is define as the rate of coverage of distance w.r.t. time.	(i) Velocity is the speed of the object in a specific direction
(ii) Speed is a scalar quantity	(ii) Velocity is a vector quantity
(iii) The speed of an object can be zero or positive but never negative	(iii) The velocity of an object can be zero, positive and negative

14. A person travels along a straight road for the first half distance with a velocity v_1 and the second half distance with velocity v_2. What is the average speed of the person?

Sol. Time taken by person to travel first half length $t_1 = \frac{\left(\frac{d}{2}\right)}{v_1} = \frac{d}{2v_1}$

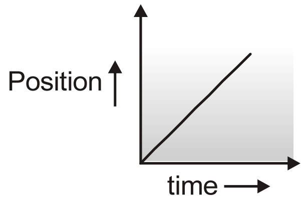
Time taken by person to travel second half length $t_2 = \frac{\left(\frac{d}{2}\right)}{v_2} = \frac{d}{2v_2}$

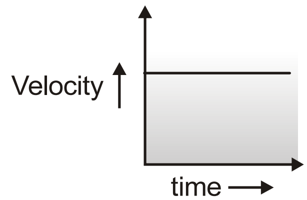
Total time = $t_1 + t_2 = \frac{d}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right) = \frac{d(v_1 + v_2)}{2v_1v_2}$

Average speed = $\frac{\text{Distance}}{\text{Total time}} = \frac{d}{t_1 + t_2} = \frac{2v_1v_2}{(v_1 + v_2)}$

15. Define uniform motion of an object moving along a straight line. Draw position-time and velocity-time graphs of such a motion.

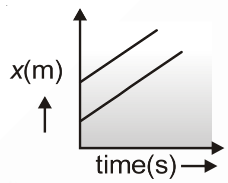
Sol. An object is said to be moving in uniform motion if it covers equal distances in equal intervals of time.





16. Define relative velocity of an object w.r.t. another. Draw the position-time graphs of two objects moving along a straight line in the same direction, when their relative velocity is zero.

Sol. Relative velocity is the velocity with which one object moves w.r.t. another object. The $x - t$ graphs of two objects moving with zero relative velocity along a straight line in the same direction is shown below.



17. What do you understand by non-uniform motion? Explain, average velocity and instantaneous velocity of an object moving along a straight line.

Sol. An object is said to be in non-uniform motion, if its speed, direction or both speed and direction changes with time. Average velocity gives the constant velocity with which the object is moving over an interval of time whereas instantaneous velocity gives the velocity of the object at a particular instant of time during its motion.

18. Distinguish between path length and displacement of an object.

Sol.

Path length	Displacement
(i) It is the actual length of the path traversed by the body	(i) It is the shortest distance traversed by the body between initial and final position
(ii) It is a scalar quantity	(ii) It is a vector quantity

19. When two bodies move uniformly towards each other, the distance between them decreases by 4 m/s. If both the bodies move in the same direction, with the same speeds, the distance between them increases by 2 m/s. What are the speeds of two bodies?

Sol. Let the speeds of the bodies be u and v according to the question.

$u + v = 4$ and $u - v = 2$

On solving these equation $u = 3$ m/s and $v = 1$ m/s

20. A train of 150 m length is going towards north direction at a speed of 10 m/s. A bird flies at a speed of 5 m/s towards the south direction parallel to the railway track. Find the time taken by the bird to cross the train.

Sol. Relative velocity of bird w.r.t. train = 5 + 10 = 15 m/s.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time} = \frac{150}{15} = 10 \text{ s}$$

Time taken by the bird to cross the train is 10 s.

21. A boy travelling along a straight line, traversed one third of the total distance with a velocity of 4 m/s. The remaining part of the distance was covered with a velocity of 2 m/s and 6 m/s for the equal time interval. Calculate the average velocity of the boy during his journey.

Sol. Let total distance = d.

time taken by the boy to travel $\frac{d}{3}$ distance $t_1 = \frac{d}{3\times4} - = \frac{d}{12}$

Let t be the time for the remaining journey

d_1 distance moved by the boy in $\frac{t}{2} = 2t$

d_2 distance moved by the boy in $\frac{t}{2} = 6t$

$$d_1 + d_2 = \frac{2d}{3}$$

$$2t + 6t = \frac{2d}{3}$$

$$8t = \frac{2d}{3}$$

$$t = \frac{2d}{24}$$

Total time $t' = t_1 + t = \frac{d}{12} + \frac{2d}{24} = \frac{48d}{12\times24}$

Average velocity = $\frac{\text{Displacement}}{\text{Total time}}$

$$= \frac{d}{48d}\times12\times24 = 6 \text{ m/s}$$

22. A car travels along a straight line for first half time with a velocity 20 km/h and the second half time with a velocity of 30 km/h. Calculate the average speed of the car.

Sol. Average speed of the car will be the mean of the given velocities.

Average speed = $\frac{30+20}{2} = \frac{50}{2} = 25\,\text{km/hr}$.

23. A car travelling at 50 km/h overtakes another car travelling at 32 km/h. Assuming each car to be 5 m long, find the time taken during the overtake.

Sol. Velocity of car 1 = 50 km/h

Velocity of car 2 = 32 km/h

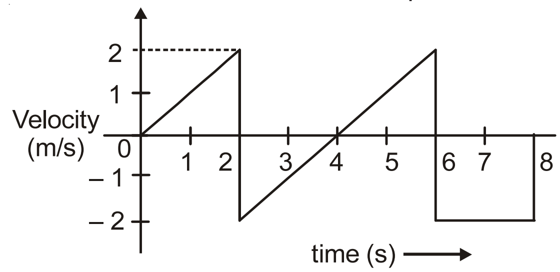
v_{12} = velocity of car 1 w.r.t. car 2 = $v_1 - v_2$

= 50 – 32 = 18 km/h = 5 m/s

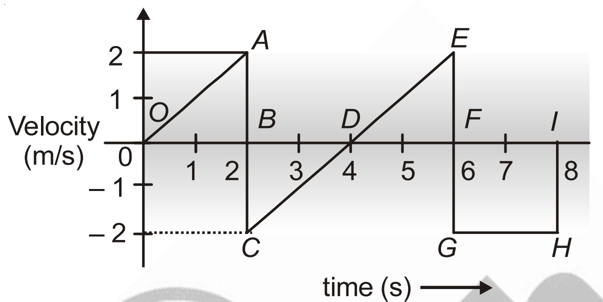
Relative velocity = $\frac{\text{Relative distance of separation}}{\text{Time}}$

Time = $\frac{5+5}{5} = \frac{10}{5} = 2$ s

24. The velocity-time graph of a particle moving along a straight line is as shown below. Calculate the distance covered between t = 0 to t = 8 seconds. Also calculate the displacement between the same interval.



Sol. Distance covered by the particle = area under the velocity-time curve without considering the signs.



= $ar(\Delta OAB) + ar(\Delta BCD) + ar(\Delta DEF) + ar(\text{square } FGHI)$

$= \frac{1}{2}\times 2\times 2+\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 2\times 2+4$

Distance = 10 m.

Displacement = area under the velocity-time curve (considering the signs).

= $ar(\Delta OAB) - ar(\Delta BCD) + ar(\Delta DEF) - ar(\text{square } FGHI)$

$= \frac{1}{2}\times 2\times 2-\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 2\times 2-4$

Displacement = – 2 m

Negative sign shows that the displacement is in the negative direction.

25. The acceleration 'a' of a particle in m/s^2 is given by $a = 2t^2 + 3t + 5$, where t is the time. If the velocity of the particle at t = 0 was 2 m/s, then calculate its velocity at t = 3 s.

Sol. $a = 2t^2 + 3t + 5$

$a = \frac{dv}{dt}$

$dv = adt = (2t^2 + 3t + 5)dt$

On integrating the above equation we get

$v = \frac{2t^3}{3}+\frac{3t^2}{2}+5t+c$

at t = 0, v = 2 m/s therefore, c = 2 m/s

so $v = \frac{2t^3}{3}+\frac{3t^2}{2}+5t+2$ at t = 3

$= \frac{2\times 27}{3}+\frac{3\times 9}{2}+15+2$

= 18 + 13.5 + 15 + 2

= 35 + 13.5 = 48.5 m/s

26. A car was moving at a rate of 18 km/h when the brakes were applied. If it comes to rest after travelling a distance of 5 m, calculate the retardation produced in the car.

Sol. Using

$v^2 - u^2 = 2as$

$v = 0$, $u = 18$ km/h, 5 m/s, $a = ?$, $s = 5$ m

$-25 = 2 \times a \times 5$

$$a = -\frac{25}{10} = -2.5 \text{ m/s}^2$$

Retardation produced in the car is 2.5 m/s^2.

27. A ball is dropped from the top of a tower. The distance travelled by the ball in the last second is 40 m. Find the height of the tower.

Sol. Let the height of the tower be h. Time taken by the ball to reach the ground is t.

$$s = -\frac{1}{2}gt^2$$

$$s = -\frac{1}{2}gt^2 \qquad \ldots\text{(i)}$$

The distance travelled by the ball in $(t - 1)$ second.

$$s' = -\frac{1}{2}g(t-1)^2 \qquad \ldots\text{(ii)}$$

Distance travelled by the ball in the last second, subtraction of equation (ii) from (i).

$$s - s' = \frac{1}{2}g(t^2 - (t-1)^2)$$

$\Rightarrow$ $40 = 5(2t - 1)$

$\Rightarrow$ $2t - 1 = 8$

$\Rightarrow$ $t = 9/2$

Substituting this value of t in (i)

$$s = -\frac{1}{2}\times 10\left(\frac{9}{2}\right)^2 = -\frac{81\times 5}{4} = -101.25 \text{ m}$$

Height of the tower is 101.25 m

28. A stone is dropped from the roof of a tower of height h. The total distance covered by the stone in the last 2 seconds of its motion is equal to the distance covered by it in the first four seconds. Find the height of the tower.

Sol. Let t be the time taken by the stone to reach the ground and h be the height of the tower.

Then, using $s = ut - \frac{1}{2}gt^2$

$$h = -\frac{1}{2}gt^2$$

Distance covered by the stone in $(t - 2)$ seconds

$$h' = -\frac{1}{2}g(t-2)^2$$

Distance covered in last 2 seconds = $h - h'$

$$= -\frac{1}{2}g(t^2 - t^2 - 4 + 4t)$$

$$= -\frac{1}{2}g(4t - 4) \qquad \dots\text{(i)}$$

Distance covered by the stone in the first 4 seconds.

$$s = -\frac{1}{2}g \times (4)^2 = -\frac{16}{2}g \qquad \dots\text{(ii)}$$

Equating (i) and (ii),

$$-\frac{1}{2}g(4t - 4) = -\frac{16}{2}g$$

$4t - 4 = 16$

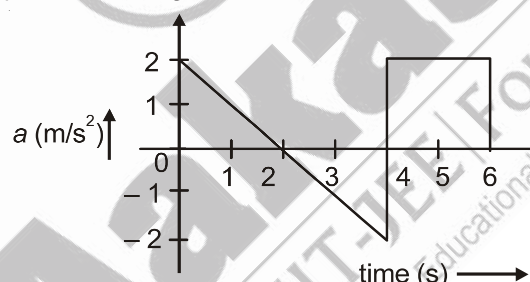
$4t = 20$, $t = 5$ seconds

Substituting the value of t, $h = -\frac{1}{2} \times 9.8 \times 25$

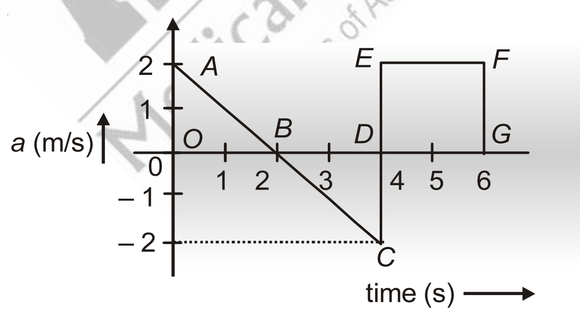
$h = -122.5$ m

So the height of tower is 122.5 m

29. The acceleration-time graph of a particle moving along a straight line is shown in the figure given below, calculate the maximum velocity of the particle starting from rest in 6 seconds.



Sol. Change in velocity = area under the acceleration-time graph.



= $ar(\Delta AOB) - ar(\Delta BCD) + ar(\text{rectangle } DEFG)$

$$= \frac{1}{2} \times 2 \times 2 - \frac{1}{2} \times 2 \times 2 + 2 \times 2 = 4 \text{ m/s}$$

Change in velocity $(v - u)$ = 4 m/s.

The initial velocity of the object was zero.

So the maximum velocity of the object is 4 m/s.

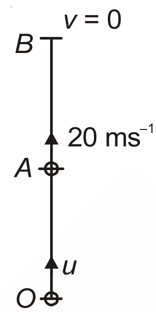
30. A ball is projected vertically upwards. Its speed at half of the maximum height is 20 m/s. Calculate the maximum height attained by it. (g = 10 m/s^2).

Sol. Let the maximum height attained by it is h.

u be the initial velocity of project ion.

Consider motion from O to A

at $\frac{h}{2}$, v = 20 m/s using $v^2 - u^2 = 2gh$



$\Rightarrow \quad 400 - u^2 = -2 \times 10 \times \frac{h}{2}$

$\Rightarrow \quad 400 + 10h = u^2$

Consider motion from O to B

At highest point v = 0

$\Rightarrow \quad -u^2 = -2gh$

$\Rightarrow \quad 400 + 10h = 2 \times 10 \times h$

$\Rightarrow \quad 10\,h = 400,\ h = 40$ m

So, the maximum height attained by the ball is 40 m

Long Answer Type Questions :

31. The position (x) of a body moving along a straight line at time t is given by $x = (3t^2 - 5t + 2)$ m. Find

(i) Velocity at t = 2 s

(ii) Acceleration at t = 2 s and draw the corresponding velocity-time (v-t) and acceleration-time (a-t) graphs

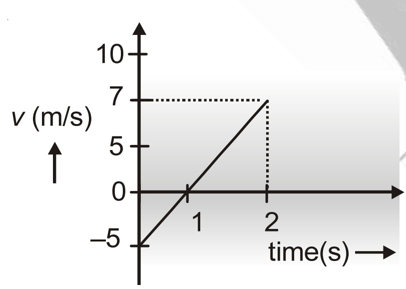
Sol. $x = 3t^2 - 5t + 2$.

$v = \frac{dx}{dt} = 6t - 5$ at t = 2 s.

v = 12 – 5 = 7 m/s, at t = 0.

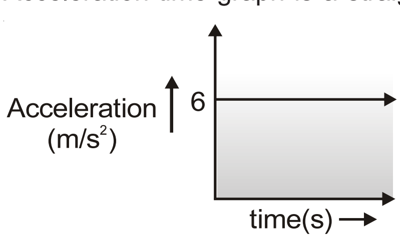
v = – 5 m/s

$v \propto t$



Acceleration $a = \frac{dv}{dt} = 6$ m/s^2 acceleration is constant.

$\therefore$ Acceleration-time graph is a straight line parallel to time axis intersecting acceleration axis at 6 m/s^2.



32. (i) Define reaction time with the help of an example.

(ii) Deduce an expression to calculate the reaction time to catch a ball dropped from the top of a tower, if you caught it after it travelled d in the downward direction.

Sol. (i) Reaction-time is that time which a person takes to observe, think and act for *e.g.*, If a person is driving a car and suddenly a boy appears on the road, then the time elapsed before he applies the car is the reaction time.

(ii) Using $s = ut - \frac{1}{2}gt^2$

$u = 0$, $s = d$, $t = t_r$ = reaction time

$$-d = -\frac{1}{2}gt_r^2$$

$$t_r = \sqrt{\frac{2d}{g}}$$

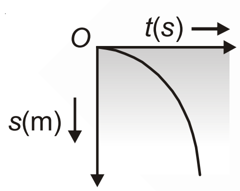
33. (i) What do you mean by free fall?

(ii) Taking upwards direction positive, point of projection as origin and neglecting air resistance, draw the position-time, velocity-time and acceleration-time graphs of an object under free fall.

Sol. If an object is released from a height near the surface of the earth. It is accelerated downwards under the influence of gravity pull, with acceleration due to gravity g. If the air resistance is neglected the object is said to be in free fall.

Position-time graph

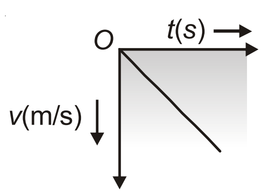
Using $s = -\frac{1}{2}gt^2 \Rightarrow s \propto t^2$



Velocity-time graph

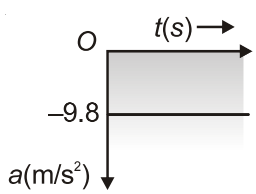
Using $v = -gt$

$v \propto t$



Acceleration-time graph

As acceleration is constant *i.e.* 9.8 m/s^2 and acting on the downward direction v negative.

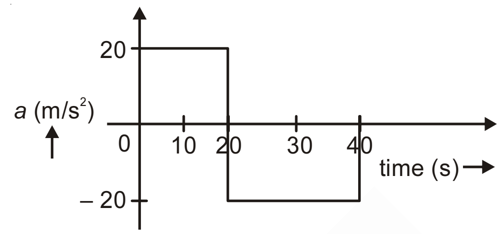


34. The acceleration-time graph of a body starting from rest is shown below. Calculate the

(i) Average acceleration

(ii) Average velocity

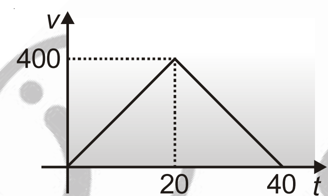
(iii) Average speed in the time interval $t = 0$ to $t = 40$ s



Sol. (i) Between $t = 0$ to $t = 20$ body was moving with constant acceleration of 20 m/s^2 and between $t = 20$ to $t = 40$ body was moving with a constant negative acceleration of 20 m/s^2. So average acceleration of the body is zero.

(ii) Corresponding v - t graph for the motion is

Displacement = Area of the Δ



$= \frac{1}{2} \times 40 \times 400$

= 8000 m

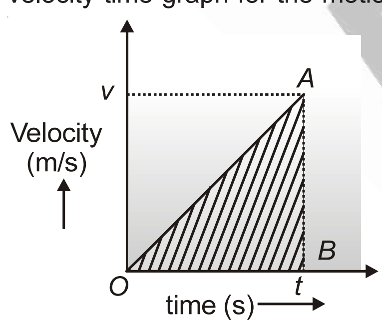
Average velocity $= \frac{8000}{40} = 200$ ms^{-1}

(iii) Distance = displacement = 8000 m

Average speed $= \frac{8000}{40} = 200$ ms^{-1}

35. A particle starts from rest and moves along a straight line with uniform acceleration *a*. Draw the velocity-time graph for the motion of the particle and deduce the kinematic equations of motion.

Sol.



Let after time t the velocity of the particle is v. Slope of the graph is equal to acceleration.

Slope = v/t

So $a = \frac{v}{t}$ $\boxed{v = at}$ …(i)

Distance travelled by the particle is given by the area under the velocity-time curve.

$s = ar\,(\Delta OAB) = \frac{1}{2} \times t \times v = \frac{vt}{2}$

$v = at$

So, $s = \frac{1}{2}at^2$ ...(ii)

From equation (i), we get $v = at$

$$t = \frac{v}{a}$$

Substituting in equation (ii), we get

$$s = \frac{1}{2}a\left(\frac{v^2}{a^2}\right) = \frac{1}{2}\frac{v^2}{a}$$

$$\boxed{v^2 = 2\,as} \quad \text{...(iii)}$$

So equation (i), (ii) and (iii) are the equation of motions for this particle.

36. Derive the equation of kinematics by the calculus method.

Sol. By the definition of acceleration, it is the rate of change of velocity,

i.e., $a = \frac{dv}{dt}$

$\Rightarrow$ $dv = adt$

Integrating both sides we get

$$\int dv = a\int dt$$

As acceleration is constant therefore we take it out side the integral. On the velocity we take limit u to v and time from 0 to t

$$\int_u^v dv = a\int_0^t dt$$

$\Rightarrow$ $v\big|_u^v = at$

$\Rightarrow$ $v - u = at$

or we get $\boxed{v = u + at}$

Further we known velocity is given by the rate of change of position w.r.t. time

$$v = \frac{dx}{dt}$$

We get, $vdt = dx$

Now in the above equation v is not independent of t. So we replace the value of v by $v = u + at$ and get

$(u + at)dt = dx$

Now integrating L.H.S. and R.H.S. from limits 0 to t and x_0 to x respectively then we get

$$u\int_0^t dt + a\int_0^t tdt = \int_{x_0}^x dx$$

$\Rightarrow$ $ut + \frac{1}{2}at^2 = (x - x_0)$

or $\boxed{x = x_0 + ut + \frac{1}{2}at^2}$

Again we can write

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$ {Dividing and multiplying by dx}

$\Rightarrow$ $a = v\dfrac{dv}{dx}$

$\Rightarrow$ $adx = vdv$

Integrating L.H.S. and R.H.S. in limit x_0 to x and u to v respectively we get

$$a\int_{x_0}^{x} dx = \int_{u}^{v} vdv$$

$\Rightarrow$ $a(x - x_0) = \dfrac{v^2 - u^2}{2}$

$\Rightarrow$ $\boxed{v^2 - u^2 = 2a(x - x_0)}$

Where $(x - x_0)$ is the displacement of the object.

37. Define relative velocity. Deduce the expression of relative velocity of two objects, and discuss the corresponding cases for zero, positive and negative relative velocities.

Sol. When two objects A and B are moving with different velocities, then the velocity of one object A with respect to another object B is called the relative velocity of object A w.r.t. object B. Consider two objects A and B moving uniformly with uniform velocities V_A and V_B moving along a straight line. Let $x_A(0)$ and $x_B(0)$ be the positions of A and B respectively at $t = 0$ and $x_A(t)$ and $x_B(t)$ be the positions of A and B respectively at time t then

$x_A(t) = x_A(0) + v_At$ …(i)

$x_B(t) = x_B(0) + v_Bt$ …(ii)

Where v_At and v_Bt are the distances moved by A and B in time t respectively.

Subtracting (ii) from (i), we get

$x_A(t) - x_B(t) = (x_A(0) - x_B(0)) + (v_A - v_B)t$ …(iii)

Where $[x_A(t) - x_B(t)] = x$ is the displacement of object A w.r.t. B at time t and $[x_A(0) - x_B(0)] = x_0$ is the initial displacement of object A w.r.t. B

$\Rightarrow$ $x = x_0 + (v_A - v_B)t$

$\Rightarrow$ $\dfrac{(x - x_0)}{t} = (v_A - v_B)$ …(iv)

$(x - x_0)$ is the change in position of A w.r.t. B and $(x - x_0)/t$ gives the time rate of change of position of object A w.r.t. B *i.e.*, the relative velocity of A w.r.t. B, hence

$\boxed{v_{AB} = v_A - v_B}$ …(v)

Similarly by subtracting the equation (i) from (ii), we can get v_{BA} is the relative velocity of B w.r.t. A

$\boxed{v_{BA} = v_B - v_A}$ …(vi)

Equation (iv) and (v) shows

$$v_{BA} = -v_{AB}$$

Now,

If $v_A = v_B$, then $v_A - v_B = 0$ substituting this in equation (iii) we get

$$x_A(t) - x_B(t) = x_A(0) - x_B(0)$$

Therefore, their position-time graphs are straight lines parallel to each other and the relative velocity v_{AB} and v_{BA} is zero. As shown in the graph fig.(a)

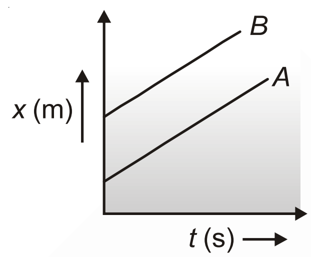


Fig. (a)

If $v_A < v_B$, $v_A - v_B$ is negative.

Substituting this in equation (iv) we get

$$x - x_0 < 0 \Rightarrow x < x_0$$

It means the separation between the two objects will go on decreasing and two objects will meet and object *B* will overtakes object *A* at this time. As shown in fig.(b)

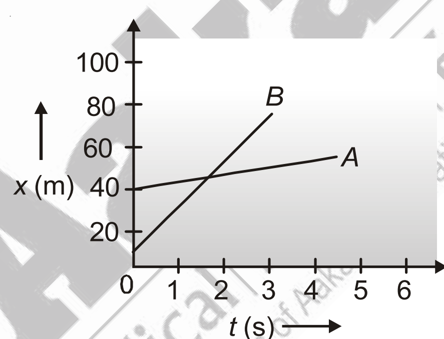


Fig. (b)

If $v_A > v_B$, $v_A - v_B$ is positive, substituting the value in equation (iv).

We get, $x - x_0 > 0$ *i.e.*, $(x - x_0)$ is positive.

It means the separation between the two objects will go on increasing with time. As shown in fig.(c)

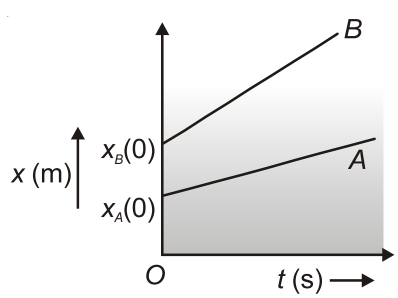
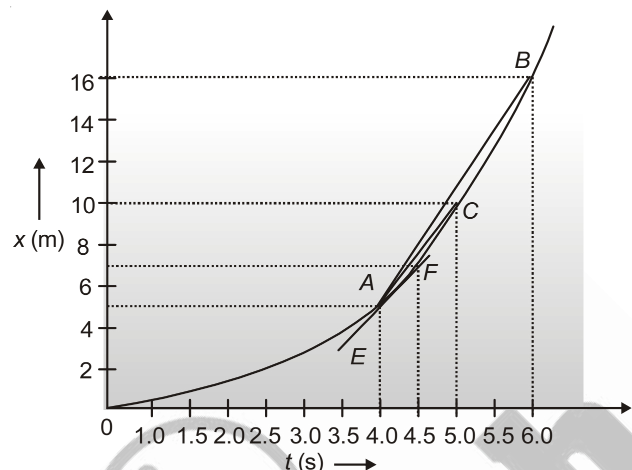


Fig. (c)

38. Define instantaneous velocity. Deduce the expression of instantaneous velocity using the position time graph.

Sol. Instantaneous velocity gives us the velocity of object, at a particular instant in a given interval of time. It is defined as the average velocity as the time interval Δt becomes infinitesimally small.

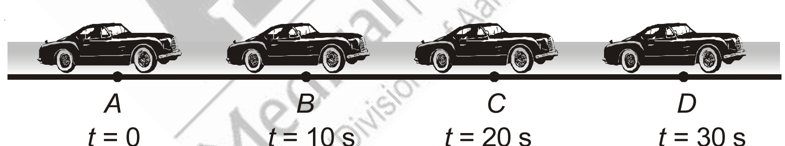
$$v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



Consider the above position-time graph and suppose we have to find the instantaneous velocity at t = 4 s, we know that the average speed over a given time interval is given by the slope of the straight line joining the initial and final point over that time interval. AB is a straight line joining the positions x = 5 m, and x = 16 m. From the graph we can see as long as we keep on decreasing the time interval B point starts approaching A and, as Δt approaches to zero, the line AB becomes a tangent to the given curve at point A and the slope of this tangent with time axis would give the value of instantaneous velocity corresponding to point A.

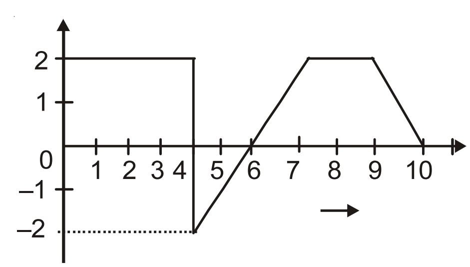
39. What do you understand by the term "reference point" and "frame of reference"? Explain with the help of example how the frame of reference describes the state of motion of an object?

Sol. To describe the position of an object we need a reference point or a set of coordinate axis (x, y and z axis). For example, suppose we are observing a car and at time t = 0, the car is at point A and at t = 10s, t = 20s and t = 30s car is at point B, C and D respectively. As the position of car is changing with time therefore, the car is in motion, and point A is the reference point for this observation. As we started observing its motion when it was at point A.

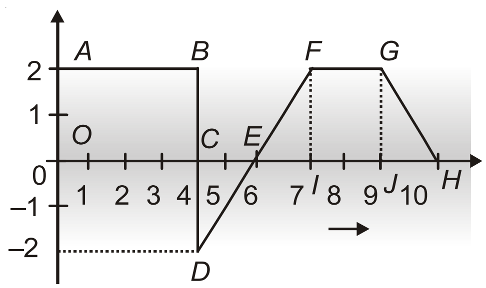


Now, according to us car is moving, but for a person sitting in the car, the car is at rest all the time w.r.t. him and we are moving. So "motion" and "rest" are relative and the state of object depends on the observer's frame of reference.

40. The velocity-time graph of a body moving in a straight line is shown below, find the displacement and the distance travelled by the body in 10 second.



Sol. Distance = ar(rectangle $OABC$) + ar(ΔCDE) + ar(ΔEFI) + ar($\square$ $FIJG$) + ar(ΔGJH)



$$= 4\times2+\frac{1}{2}\times2\times2+\frac{1}{2}\times1\times2+2\times2+\frac{1}{2}\times1\times2$$

8 + 2 + 1 + 4 + 1 = 16 m

Displacement = (area of rectangle $OABC$) + $ar(\Delta CDE)$ + $ar(\Delta EFI)$ + $ar(\square\ FIJG)$ + $ar(\Delta GHJ)$

= 4 × 2 – 2 + 1 + 4 + 1

= 8 – 2 + 6 = 12 m.

41. Explain, path length and displacement with illustrations and distinguish them.

Sol. Consider the motion of a boy along a straight line say x-axis and let the origin of x-axis be the reference point *i.e.*, the point from where the boy started moving. A, B and C represent the positions of the boy at different instants of time. At t = 0 the boy was at point A *i.e.*, origin see the figure given below. Now lets consider the two cases of motion in the first case the boy moves from A to B and in the second case he moves back from B to C.

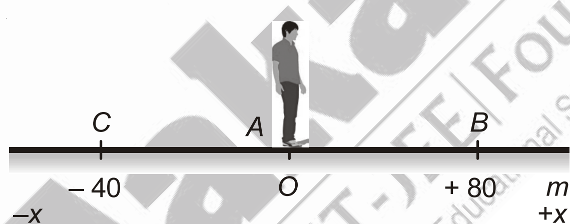


Fig. : x-axis, origin and position of boy at different instants of time

While moving from A to B, the distance covered by the boy is AB *i.e.*, 80 m. In the second case the distance moved by the boy is BA + AC *i.e.*, (80 + 40)m = 120 m. You can see in figure that point C lies on the negative side of x-axis, but while calculating the distance we are considering AC as + 40 m instead of – 40 m this is because the **distance travelled by a body can never be negative.** So in the second case while going from A to C though he is moving in the negative axis side but the distance travelled by him is positive. Distance is a scalar quantity as it has only magnitude and no direction. Now through out his journey the boy moves first from A to B then back from B to C. So the total distance travelled by him is (80 + 120) = 200 m. This is the **path length**. So we can define path length as the actual distance traversed by an object during its motion in a given interval of time. **Path length** is also a **scalar quantity** as it has only magnitude and no direction. The boy started his journey from A and finally reaches point C. This change in position is known as **displacement** which is a vector quantity. The displacement of boy at the end of his journey is –40 m, which has magnitude 40 m and directed towards negative axis.

Hence, we can define the displacement of an object in a given interval as the shortest distance between the initial and final position of the object in a particular direction. The magnitude of displacement is always less than or equals to the total distance *i.e.* the path length traversed by the body *i.e.*,

$$\boxed{|\text{Displacement}| \le \text{Distance}}$$

42. The acceleration 'a' in m/s^2 of a particle is given by $a = 3t^2 + 2t + 2$, where t is the time. If the particle starts with a velocity of 2 m/s at $t = 0$, then find

(i) Velocity at the end of 2 s

(ii) Position at the end of 2 s, if at $t = 0$, $x = 2$ m

Sol. (i) $a = 3t^2 + 2t + 2$

$$a = \frac{dv}{dt}$$

$$v = \int a dt = \int (3t^2 + 2t + 2)dt$$

$$= \frac{3t^3}{3} + \frac{2t^2}{2} + 2t + c$$

at $t = 0$, $v = 2$ m/s

So $c = 2$ m/s

$v = t^3 + t^2 + 2t + 2$

at $t = 2$

$v = 8 + 4 + 4 + 2 = 18$ m/s $\boxed{v = 18 \text{ m/s}}$

(ii) $v = t^3 + t^2 + 2t + 2$

$$v = \frac{dx}{dt}$$

$$x = \int v dt$$

$$= \int t^3 + t^2 + 2t + 2dt$$

$$x = \frac{t^4}{4} + \frac{t^3}{3} + \frac{2t^2}{2} + 2t + c$$

at $t = 0$, $x = 2$ m, so $c = 2$ m

Therefore,

$$x = \frac{t^4}{4} + \frac{t^3}{3} + \frac{2t^2}{2} + 2t + 2$$

at $t = 2$ s

$$x = \frac{16}{4} + \frac{8}{3} + \frac{8}{2} + 4 + 2$$

$$x = 14 + \frac{8}{3} = \frac{42 + 8}{3}$$

$$= \frac{50}{3} = 16.6 \text{ m}$$

$$\boxed{x = 16.6 \text{ m}}$$

43. Define the following terms

(i) Uniform motion

(ii) Average acceleration

(iii) Instantaneous acceleration

(iv) Instantaneous velocity

(v) Relative velocity

Sol. **Uniform motion :** An object moving along a straight line is said to be in uniform motion, if it covers equal distance in equal intervals of time. Otherwise, its motion is non-uniform.

Instantaneous velocity : It is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Average acceleration : The average acceleration $\bar{a}$ over a time interval is defined as the change in velocity divided by the time interval

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration : The instantaneous acceleration at an instant is defined as the acceleration at a particular instant of time and it is given as the limit of the average as the interval Δt approaches to zero

$$a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Relative velocity : The velocity of one object w.r.t. other is known as relative velocity.

44. What do the following represent? [x = position, a = acceleration, v = velocity, s = distance]

(i) Slope of x-t graph

(ii) Slope of v-t graph

(iii) Area of v-t graph

(iv) Area of a-t graph

(v) Slope of s-t graph

Sol. (i) Velocity

(ii) Acceleration

(iii) Displacement

(iv) Change in velocity

(v) Speed

45. Draw the various position-time (x-t) and velocity-time (v-t) graphs for

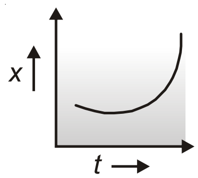
(i) Positive

(ii) Negative

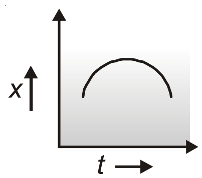
(iii) Zero acceleration

Sol. ***x*-*t* graphs**

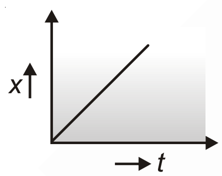
(i) Positive acceleration



(ii) Negative acceleration

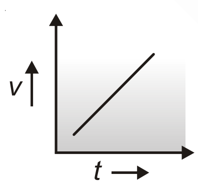


(iii) Zero acceleration

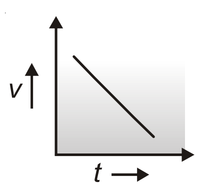


***v*-*t* graphs**

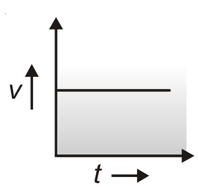
(i) Positive acceleration



(ii) Negative acceleration



(iii) Zero acceleration



SECTION - B

Model Test Paper

Very Short Answer Type Questions :

1. A particle is moving along a circular track of radius r. What is the distance and displacement traversed by the particle after two and half revolutions.

Sol. Distance = $2\pi r + 2\pi r + \pi r = 5\pi r.$

Displacement = $2r$.

2. What does the area under an acceleration-time graph represents?

Sol. The area under the acceleration-time graph gives the change in velocity.

3. What is the velocity of a body when its position-time graph is

(i) a straight line inclined to time axis?

(ii) parallel to time axis?

Sol. (i) Constant

(ii) Zero

4. A body is moving on a curved path with a constant speed. What is the nature of its motion?

Sol. As the velocity is changing at each point, therefore motion is accelerated.

5. What does the slope of the velocity-time graph of an object represent?

Sol. Acceleration of object.

6. Define instantaneous velocity.

Sol. The velocity of an object at a particular instant of time during its course of motion is known as instantaneous velocity.

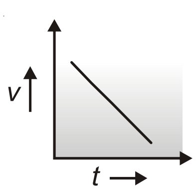
7. The position x (metres) of a particle moving along a straight line at time t (second) is given by $x = 5t$. Find its velocity.

Sol. $x = 5\,t$

$$v = \frac{dx}{dt}\ 5 = \boxed{v = 5\text{ m/s}}$$

8. Draw the velocity-time graph of an object slowing down with time.

Sol.



Short Answer Type Questions :

9. Two persons A and B are walking with speed 5 km/h and 7 km/h respectively in the same direction. Find the relative distance of B w.r.t. A after 2 hours.

Sol. v_A = 5 km/h, v_B = 7 km/h

$v_{BA} = v_B - v_A = 7 - 5 = 2$ km/h

Relative distance of B w.r.t. $A = v_{BA} \times$ time

$= 2 \times 2 = 4$ km

10. Explain that a body can have zero average velocity but not zero average speed

Sol. If a body comes back to its initial position then, its displacement is zero but the distance is non-zero. Therefore the average velocity of object is zero but the average speed is non-zero.

11. The displacement x (in metre) of the body is given by $x = 5t - 3t^2$.

Calculate the velocity and acceleration of the body at time $t = 2$ s

Sol. $x = 5t - 3t^2$

$$v = \frac{dx}{dt} = 5 - 6t = 5 - 6 \times 2 = 5 - 12 = -7 \text{ m/s}$$

$$a = \frac{dv}{dt} = -6 = -6 \text{ m/s}^2$$

12. The acceleration a of an object at time t is given by $a = 2t$. If initial velocity of the object is 2 m/s, then find its velocity after 5 s.

Sol. $a = 2t$

$$a = \frac{dv}{dt}$$

$dv = adt$

$dv = 2tdt$

Integrating both sides we get

$$v = \frac{2t^2}{2} + c$$

at $t = 0$, $v = 2$ so $c = 2$

$v = t^2 + 2$ at $t = 5$ s

$v = 25 + 2 = 27$ m/s

13. Which of the two, velocity or acceleration decides the direction of motion of a body? Explain, with the help of an example.

Sol. It is the velocity which decides the direction of motion of a body. The acceleration tells the rate at which the velocity is changing. For example, when a body is thrown vertically upwards, its direction of velocity is upwards that is why the body goes upward, whereas its acceleration is downwards.

14. Differentiate between average acceleration and instantaneous acceleration.

Sol. Average acceleration tells us the acceleration of object over time interval of its motion but the instantaneous acceleration gives the acceleration of the object at a particular instant of time during the course of its motion.

15. The distance travelled by a particle moving in a straight line is found to be proportional to the square of the time elapsed. Is it moving with constant speed or constant acceleration? Explain.

Sol. $x \propto t^2$

$$v = \frac{dx}{dt} = 2t$$

$$a = \frac{dv}{dt} = 2$$

Acceleration is independent of time, it is constant and velocity is directly proportional to time.

Short Answer Type Questions :

16. The driver of a car travelling at a velocity v, suddenly sees a broad wall in front of him and applies brakes. If the retardation produced by brakes is a, calculate the stopping distance of the car.

Sol. Using $v^2 - u^2 = 2as$.

Initial velocity u

Final velocity $v = 0$, s = stopping distance

$a = -a$ (Retardation)

So $s = \dfrac{-u^2}{-2a}$

$$\boxed{s = \frac{u^2}{2a}}$$

17. How can we determine the distance and the displacement covered by a uniformly accelerated body from its velocity-time graph?

Sol. The distance and displacement travelled by a body in a given interval of time is equal to the total area of velocity time graph, but in the case of distance we add all the distances without considering the sign while calculating displacement we have to consider the signs.

18. The instantaneous speed is always equal to the magnitude of instantaneous velocity for an object moving along a straight line, why?

Sol. The one dimensional motion *i.e.*, motion along a straight line having uniform acceleration. The direction of velocity does not change with time, only the magnitude of velocity changes with time therefore instantaneous speed is equal to the magnitude of instantaneous velocity for an object moving along a straight line.

19. A ball is dropped from the roof of a tower of height 180 m. What is the distance covered by the ball in the last second? (take g = 10 m/s^2)

Sol. Using

$$s = ut - \frac{1}{2}gt^2$$

$u = 0$

s = 180 m (downward direction)

$$-180 = -\frac{1}{2}\times 10\times t^2$$

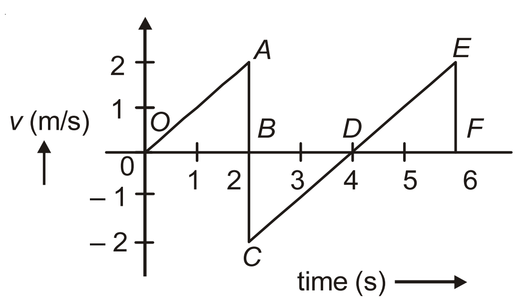
$$t^2 = \frac{360}{10}$$

$t = 6$ s

Distance covered in 5 seconds = $s = -\frac{1}{2}\times 10\times 25 = -125\text{ m}$ (Downward direction).

So, the distance covered in 6th second = 180 – 125 = 55 m.

20. The velocity-time (v-t) graph of a body moving in a straight line is shown below. Find the displacement and the distance travelled by the body in 6 seconds.



Sol. Distance = area under the velocity-time curve

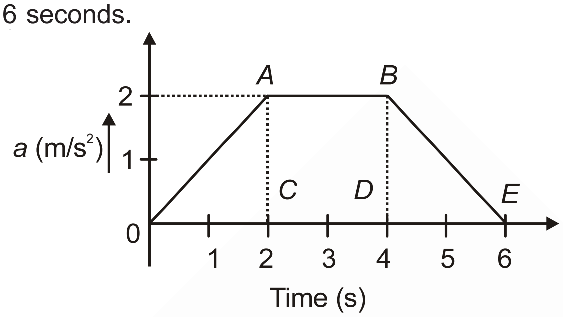
= Area of ΔOAB + area of ΔBCD + area of ΔDEF

$= \frac{1}{2}\times2\times2+\frac{1}{2}\times2\times2+\frac{1}{2}\times2\times2 = 6\text{ m}$

Displacement = area of ΔOAB – area of ΔBCD + area of ΔDEF

$= \frac{1}{2}\times2\times2-\frac{1}{2}\times2\times2+\frac{1}{2}\times2\times2 = 2\text{ m}$

21. The acceleration(a) - time(t) graph of a body in one dimensional motion is shown below. Find the change in the velocity of the object in 6 seconds.



Sol. Change in velocity = area under the acceleration-time graph

$\Rightarrow$ Area of ΔOAC + ar of rectangle $ABCD$ + ar of ΔBDE

$= \frac{1}{2}\times2\times2+2\times2+\frac{1}{2}\times2\times2$

= 8 m/s

Long Answer Type Questions :

22. Deduce the following relations by calculus method for a uniform motion along a straight line, where the symbols have their usual meanings

(i) $v = u + at$

(ii) $s = ut+\frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

OR

Deduce the following relations by graphical method for a uniform motion along a straight line, where the symbols have their usual meaning

(i) $v = u + at$

(ii) $s = ut+\frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

Sol. By the definition of acceleration, it is the rate of change of velocity,

i.e., $a = \frac{dv}{dt}$

$\Rightarrow$ $dv = adt$

Integrating both sides we get

$\int dv = a\int dt$

As acceleration is constant therefore we take it out side the integral. On the velocity we take limit u to v and time from 0 to t

$$\int_u^v dv = a\int_0^t dt$$

$\Rightarrow$ $v\Big|_u^v = at$

$\Rightarrow$ $v - u = at$

or we get $\boxed{v = u + at}$

Further we known velocity is given by the rate of change of position w.r.t. time

$$v = \frac{dx}{dt}$$

We get, $vdt = dx$

Now in the above equation v is not independent of t. So we replace the value of v by $v = u + at$ and get

$(u + at)dt = dx$

Now integrating L.H.S. and R.H.S. from limits 0 to t and x_0 to x respectively then we get

$$u\int_0^t dt + a\int_0^t tdt = \int_{x_0}^x dx$$

$\Rightarrow$ $ut + \frac{1}{2}at^2 = (x - x_0)$

or $\boxed{x = x_0 + ut + \frac{1}{2}at^2}$

Again we can write

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$ {Dividing and multiplying by dx}

$\Rightarrow$ $a = v\frac{dv}{dx}$

$\Rightarrow$ $adx = vdv$

Integrating L.H.S. and R.H.S. in limit x_0 to x and u to v respectively we get

$$a\int_{x_0}^x dx = \int_u^v vdv$$

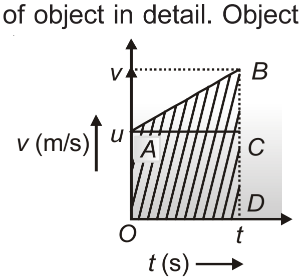
$\Rightarrow$ $a(x - x_0) = \frac{v^2 - u^2}{2}$

$\Rightarrow$ $\boxed{v^2 - u^2 = 2a(x - x_0)}$

Where $(x - x_0)$ is the displacement of the object.

OR

Using the velocity time graph of the object under uniformly accelerated motion, we can derive some simple equations that relate displacement (x), time (t), initial velocity (u), final velocity (v) and acceleration (a). Lets discuss the velocity time graph shown of object in detail. Object velocity changes from u to v in time t



The slope of graph gives acceleration and is given by

$$a = \frac{BC}{AC} = \frac{v-u}{t}$$

$$at = v - u$$

$$\boxed{v = u + at} \quad \ldots\text{(i)}$$

Area under the curve is

Area of ΔABC + area of rectangle $OACD$

Area of ΔABC = $\frac{1}{2} \times (v-u)t$

Area of rectangle $OACD$ = ut

And as explained in the previous section, the area under v-t curve represents the displacement. Therefore, the displacement s of the object is

$$s = \frac{1}{2}(v-u)t + ut$$

But $(v - u) = at$ from equation (i) substituting this value in the above equation we get

$$s = \frac{1}{2}at^2 + ut$$

or $$\boxed{s = ut + \frac{1}{2}at^2} \quad \ldots\text{(ii)}$$

Equation (i) gives us relation of velocity and time

Equation (ii) gives us relation between displacement and time now we can also derive an equation to relate displacement (s) of object and velocity.

s = area of the trapezium $OABD$

= $\frac{1}{2}$ × (sum of the parallel sides) × (perpendicular distance between the parallel sides)

= $\frac{1}{2} \times (OA + BD) \times OD$

= $\frac{1}{2} \times (u \times v) \times t$

From equation (i) we can get

$$t = \frac{v-u}{a}$$

Substituting this value of t in the above equation we get

$$s = \frac{1}{2}(u+v)\frac{(v-u)}{a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$\boxed{v^2 - u^2 = 2as}$$

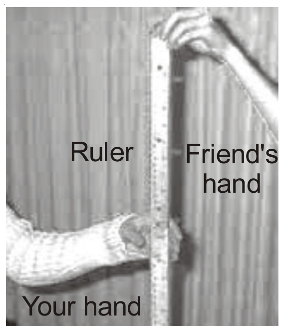
23. What is reaction time? Explain it with the help of an example and deduce the expression for it.

OR

What is Galileo's law of odd numbers? Prove it.

Sol. Reaction time : When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elapsed before he slams the brakes of the car is the reaction time. Reaction; time depends on complexity of the situation and on an individual.

Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger.



After you catch it see the distance *d* travelled by the ruler, we can estimate the reaction time by using equation

$$s = ut + \frac{1}{2}at^2$$

As the ruler drops under free fall. Therefore, $u = 0$, and a is replaced by (g) *i.e.*, 9.8 m/s^2 and we get

$$s = -\frac{1}{2}gt^2$$

Where t is the reaction time and s is the displacement of the ruler, it is also measured in the downward direction so, it is –ve and we get the expression for reaction time as

$$t = \sqrt{\frac{2s}{g}}$$

By substituting the value of s we can get the reaction time.

OR

Galileo's law of odd numbers : According to this law, the distance traversed by a body falling from rest, during equal intervals of time, stand to one another in the same ratio as the odd numbers beginning with unity [namely 1 : 3 : 5 : 7 ……]. The distance travelled by object during free-fall is related to time by the equation

$s = -\frac{1}{2}gt^2$ [since initial velocity is zero]

If we divide the time interval of motion of object into many equal intervals τ and find out the distance traversed by the object during successive intervals of time

i.e., 0, τ, 2τ, 3τ ……

At $t = 0$, $y = 0$

At $t = \tau, y_0 = -\frac{1}{2}g\tau^2$

At $t = 2\tau, y_0 = -\frac{1}{2}g(2\tau)^2 = -\frac{4}{2}g\tau^2 = 4y_0$

So the distance travelled in the second interval τ is $4y_0 - y_0 = 3y_0$

At $t = 3\tau, y_0 = -\frac{1}{2}g(3\tau)^2 = \frac{-9g\tau^2}{2} = 9y_0$

So the distance travelled in the third interval τ is $9y_0 - 4y_0 = 5y_0$

So we can see that the distance traversed by the object during free fall in successive interval is in the ratio 1 : 3 : 5 : 7 : 9 : 11 …

❑ ❑ ❑

Solutions (Set-2)

Objective Type Questions

(Position, Path length and Displacement, Average Velocity and Average Speed)

1. A particle is moving along a circle such that it completes one revolution in 40 seconds. In 2 minutes 20 seconds, the ratio $\frac{|\text{displacement}|}{\text{distance}}$ is

(1) 0 (2) $\frac{1}{7}$ (3) $\frac{2}{7}$ (4) $\frac{1}{11}$

Sol. Answer (4)

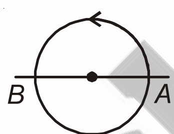
$T = 40$ s

If t = 2 minute 20 second

= 2 × 60 + 20

= 140 s

So, it has completed $3\frac{1}{2}$ revolution.



Distance travelled $= 3 \times 2\pi R + \pi R$

$= 7\pi R$

Displacement = $2R$

$$\frac{|\text{Displacement}|}{\text{Distance}} = \frac{2R}{7\pi R} = \frac{2}{7\times\frac{22}{7}} = \frac{1}{11}$$

2. Consider the motion of the tip of the second hand of a clock. In one minute (R be the length of second hand), its

(1) Displacement is $2\pi R$

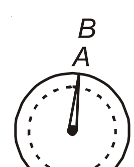
(2) Distance covered is $2R$

(3) Displacement is zero

(4) Distance covered is zero

Sol. Answer (3)

The second hand of the clock in minute covers an angle of 360º and the initial and final positions are same.



So, $\boxed{\text{Displacement} = 0}$

3. The position of a body moving along x-axis at time t is given by $x = (t^2 - 4t + 6)$ m. The distance travelled by body in time interval $t = 0$ to $t = 3$ s is

(1) 5 m (2) 7 m (3) 4 m (4) 3 m

Sol. Answer (1)

$x = t^2 - 4t + 6$

$\frac{dx}{dt} = 2t - 4$

At t = 2 s, particle is at rest and reverses its position so,

$$\left.x\right|_{t=0} = 6\text{ m} \quad \Big] 4\text{ m}$$
$$\left.x\right|_{t=2\text{ s}} = 2\text{ m} \quad \Big] 1\text{ m}$$
$$\left.x\right|_{t=3\text{ s}} = 3\text{ m}$$

Distance = (4 + 1) m = 5 m

Displacement = 3 m

4. A particle moves along x-axis with speed 6 m/s for the first half distance of a journey and the second half distance with a speed 3 m/s. The average speed in the total journey is

(1) 5 m/s (2) 4.5 m/s (3) 4 m/s (4) 2 m/s

Sol. Answer (3)

If a body travels equal distance with speed v_1 and v_2 then average speed is given by

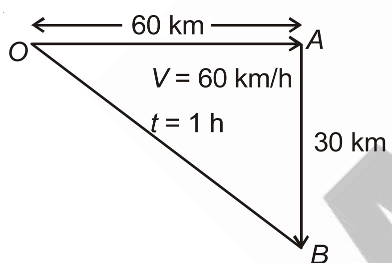
$$v_{av} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times6\times3}{6+3} = 4\text{ ms}^{-1}$$

5. A car moves with speed 60 km/h for 1 hour in east direction and with same speed for 30 min in south direction. The displacement of car from initial position is

(1) 60 km (2) $30\sqrt{3}$ km (3) $30\sqrt{5}$ km (4) $60\sqrt{2}$ km

Sol. Answer (3)

Displacement of car = $\sqrt{60^2+30^2} = 30\sqrt{5}$ km



$$\left[\begin{array}{l}\text{Distance } OA = \text{Speed} \times \text{Time} \\ \Rightarrow 60 \times 1\text{ h} = 60\text{ km}\end{array}\right]$$

6. A person travels along a straight road for the first $\frac{t}{3}$ time with a speed v_1 and for next $\frac{2t}{3}$ time with a speed v_2. Then the mean speed v is given by

(1) $v = \frac{v_1+2v_2}{3}$ (2) $\frac{1}{v} = \frac{1}{3v_1} + \frac{2}{3v_2}$

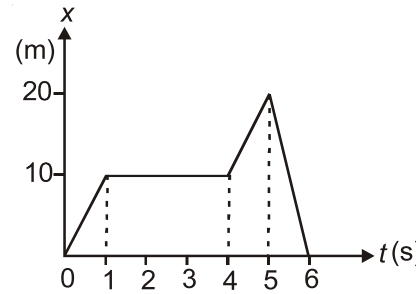
(3) $v = \frac{1}{3}\sqrt{2v_1v_2}$ (4) $v = \sqrt{\frac{3v_2}{2v_1}}$

Sol. Answer (1)

$$v_{av} = \frac{\text{Distance}}{\text{Time}} = \frac{\text{Speed}\times\text{Time}}{\text{Time}} = \frac{v_1\times\frac{t}{3}+v_2\times\frac{2t}{3}}{\frac{t}{3}+\frac{2t}{3}}$$

$$\Rightarrow \quad v_{av} = \frac{\frac{v_1}{3}+\frac{2v_2}{3}}{1} = \frac{v_1+2v_2}{3} \quad \Rightarrow \quad \boxed{v_{av} = \frac{v_1+2v_2}{3}}$$

7. Figure shows the graph of x-coordinate of a particle moving along x-axis as a function of time. Average velocity during t = 0 to 6 s and instantaneous velocity at t = 3 s respectively, will be



(1) 10 m/s, 0 (2) 60 m/s, 0

(3) 0, 0 (4) 0, 10 m/s

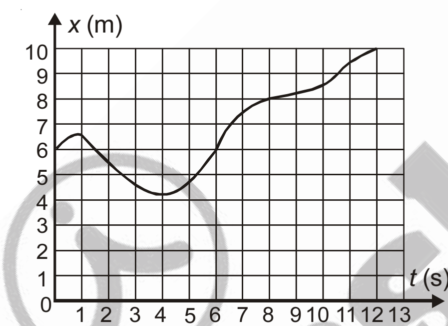
Sol. Answer (3)

From 0 to 6 s $\rightarrow$ Displacement = 0

so, average velocity = 0

at t = 3 s, the displacement = 0, so v = 0

8. Position-time graph for a particle is shown in figure. Starting from t = 0, at what time t, the average velocity is zero?



(1) 1 s (2) 3 s (3) 6 s (4) 7 s

Sol. Answer (3)

If we look at the graph very carefully at t = 0, x = 6 m

The average velocity will be zero if it comes back to the initial position.

It is evident that at t = 6 s, x = 6 m

So, v_{av} at t = 6 s is zero.

(Instantaneous Velocity and Speed, Acceleration)

9. A body in one dimensional motion has zero speed at an instant. At that instant, it must have

(1) Zero velocity (2) Zero acceleration

(3) Non-zero velocity (4) Non-zero acceleration

Sol. Answer (1)

Magnitude of velocity = Speed

So, if the speed is zero then it must have zero velocity also.

10. If a particle is moving along straight line with increasing speed, then

(1) Its acceleration is negative (2) Its acceleration may be decreasing

(3) Its acceleration is positive (4) Both (2) & (3)

Sol. Answer (2)

If the speed of body is increasing then acceleration is in the direction of velocity.

It may be positive or negative.

If acceleration is in negative direction then acceleration is increasing but in negative side, so it will be called as decreasing.

11. At any instant, the velocity and acceleration of a particle moving along a straight line are v and a. The speed of the particle is increasing if

(1) $v > 0, a > 0$ (2) $v < 0, a > 0$ (3) $v > 0, a < 0$ (4) $v > 0, a = 0$

Sol. Answer (1)

For increasing speed both velocity (v) and acceleration (a) are in the same direction.

12. If magnitude of average speed and average velocity over a time interval are same, then

(1) The particle must move with zero acceleration

(2) The particle must move with non-zero acceleration

(3) The particle must be at rest

(4) The particle must move in a straight line without turning back

Sol. Answer (4)

The magnitude of average speed and average velocity can only be equal if object moves in a straight line without turning back. In that condition distance will be equal to displacement.

13. If v is the velocity of a body moving along x-axis, then acceleration of body is

(1) $\frac{dv}{dx}$ (2) $v\frac{dv}{dx}$ (3) $x\frac{du}{dx}$ (4) $v\frac{dx}{dv}$

Sol. Answer (2)

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} \qquad \left(\frac{dx}{dt} = \text{velocity}\right)$$

$$\boxed{a = \frac{vdv}{dx}}$$

14. If a body is moving with constant speed, then its acceleration

(1) Must be zero (2) May be variable (3) May be uniform (4) Both (2) & (3)

Sol. Answer (2)

Acceleration is the rate of change of velocity. The magnitude of velocity (*i.e.,* speed) is constant but it may change in direction. So, acceleration may be variable due to change in direction.

15. When the velocity of body is variable, then

(1) Its speed may be constant (2) Its acceleration may be constant

(3) Its average acceleration may be constant (4) All of these

Sol. Answer (4)

If velocity is changing they may change in magnitude or direction or both.

(i) So, if velocity is changing in direction only the magnitude is constant so speed is constant.

(ii) If only direction of velocity is changing and magnitude is constant then acceleration will also be constant in magnitude (in case of uniform circular motion).

(iii) Average acceleration may be constant.

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$$

16. An object is moving with variable speed, then

(1) Its velocity may be zero
(2) Its velocity must be variable
(3) Its acceleration may be zero
(4) Its velocity may be constant

Sol. Answer (2)

If speed is changing then velocity must change.

17. The position of a particle moving along x-axis is given by $x = 10t - 2t^2$. Then the time (t) at which it will momently come to rest is

(1) 0
(2) 2.5 s
(3) 5 s
(4) 10 s

Sol. Answer (2)

$x = 10t - 2t^2$

$v = \frac{dx}{dt} = 10 - 4t$

v = 0, at the time of coming to rest, so

$10 - 4t = 0$

$\boxed{t = 2.5 \text{ s}}$

18. If the displacement of a particle varies with time as $\sqrt{x} = t + 7$, then

(1) Velocity of the particle is inversely proportional to t
(2) Velocity of the particle is proportional to t^2
(3) Velocity of the particle is proportional to $\sqrt{t}$
(4) The particle moves with constant acceleration

Sol. Answer (4)

$\sqrt{x} = t + 7$

$\Rightarrow \quad x = (t+7)^2$

$= t^2 + 49 + 14t$ (squaring)

$\frac{dx}{dt} = 2t + 14$

$\boxed{v = 2t + 14} \Rightarrow \boxed{v \propto t}$

Acceleration :

$a = \frac{dv}{dt}$

$\boxed{a = 2 \text{ ms}^{-2}} \rightarrow$ constant

19. The initial velocity of a particle is u (at t = 0) and the acceleration a is given by $\alpha t^{3/2}$. Which of the following relations is valid?

(1) $v = u + \alpha t^{3/2}$
(2) $v = u + \frac{3\alpha t^3}{2}$
(3) $v = u + \frac{2}{5}\alpha t^{5/2}$
(4) $v = u + \alpha t^{5/2}$

Sol. Answer (3)

$a = \alpha t^{3/2}$ (acceleration is a function of time)

$$\int_u^v dv = \int_0^t a\,dt$$

$$\Rightarrow \int_u^v dv = \int_0^t \alpha t^{3/2} dt$$

$$\Rightarrow v\Big|_u^v = \alpha \left.\frac{t^{3/2+1}}{\frac{3}{2}+1}\right|_0^t$$

$$\Rightarrow (v-u) = \alpha \times \frac{2}{5}\left(t^{5/2} - 0\right)$$

$$\Rightarrow v - u = \frac{2}{5}\alpha t^{5/2}$$

$$\Rightarrow \boxed{v = u + \frac{2}{5}\alpha t^{5/2}}$$

***Note :** The equations of kinematics are valid only for constant acceleration, here a is a function of 't' so we didn't apply those equations.*

20. The position x of particle moving along x-axis varies with time t as $x = A\sin(\omega t)$ where A and ω are positive constants. The acceleration a of particle varies with its position (x) as

(1) $a = Ax$ (2) $a = -\omega^2 x$ (3) $a = A\,\omega\,x$ (4) $a = \omega^2\,x\,A$

Sol. Answer (2)

$x = A\sin\omega t$

$$\frac{dx}{dt} = A\omega\cos\omega t$$

$$\Rightarrow \frac{d^2x}{dt^2} = -A\omega^2\sin\omega t$$

$$\Rightarrow \boxed{a = -\omega^2 x} \quad (\because\ A\sin\omega t = x)$$

21. A body is moving with variable acceleration (a) along a straight line. The average acceleration of body in time interval t_1 to t_2 is

(1) $\dfrac{a[t_2 + t_1]}{2}$ (2) $\dfrac{a[t_2 - t_1]}{2}$ (3) $\dfrac{\int_{t_1}^{t_2} a\,dt}{t_2 + t_1}$ (4) $\dfrac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}$

Sol. Answer (4)

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Time}} \Rightarrow \boxed{a_{av} = \frac{\int_{t_1}^{t_2} a\,dt}{t_2 - t_1}}$$

22. The position of a particle moving along x-axis given by $x = (-2t^3 + 3t^2 + 5)$m. The acceleration of particle at the instant its velocity becomes zero is

(1) 12 m/s^2 (2) –12 m/s^2 (3) –6 m/s^2 (4) Zero

Sol. Answer (3)

$$x = (-2t^3 + 3t^2 + 5)\text{ m}$$

$$\Rightarrow \frac{dx}{dt} = -6t^2 + 6t = v$$

$$\Rightarrow \frac{d^2x}{dt^2} = -12t + 6 \qquad (\text{for } v = 0,\ 6t = 6t^2 \Rightarrow t = 1\text{ s})$$

$$a|_{t=1\text{ s}} = -12 + 6 = -6\text{ ms}^{-2}$$

23. A particle move with velocity v_1 for time t_1 and v_2 for time t_2 along a straight line. The magnitude of its average acceleration is

(1) $\dfrac{v_2 - v_1}{t_1 - t_2}$ (2) $\dfrac{v_2 - v_1}{t_1 + t_2}$ (3) $\dfrac{v_2 - v_1}{t_2 - t_1}$ (4) $\dfrac{v_1 + v_2}{t_1 - t_2}$

Sol. Answer (2)

$$\boxed{a_{avg} = \frac{v_2 - v_1}{t_1 + t_2}} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

(Kinematic Equations for Uniformly Accelerated Motion)

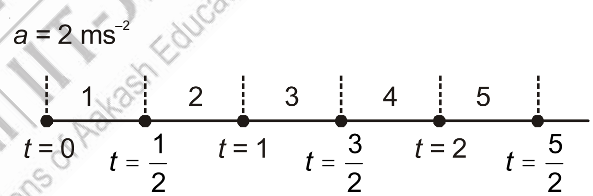
24. A particle starts moving with acceleration 2 m/s^2. Distance travelled by it in 5th half second is

(1) 1.25 m (2) 2.25 m (3) 6.25 m (4) 30.25 m

Sol. Answer (2)

$S_{2.5} - S_2 = ?$ (distance travelled in 5th half second)

$$S_{2.5} = ut + \frac{1}{2}at^2$$



$$\Rightarrow S_{2.5} = \frac{1}{2} \times 2 \times (2.5)^2 = 6.25\text{ m} \quad (\because u = 0)$$

$$S_2 = \frac{1}{2} \times 2 \times 4 = 4\text{ m}$$

So, $\boxed{S_{2.5} - S_2 = 2.25\text{ m}}$

25. The two ends of a train moving with constant acceleration pass a certain point with velocities u and $3u$. The velocity with which the middle point of the train passes the same point is

(1) $2u$ (2) $\dfrac{3}{2}u$ (3) $\sqrt{5}\,u$ (4) $\sqrt{10}\,u$

Sol. Answer (3)

Final velocity

Initial velocity

Velocity at the mid-point = $\sqrt{\dfrac{v^2 + u^2}{2}}$

(When acceleration is constant)

Given, $v = 3u$, $u = u$

So, $v_{mid} = \sqrt{\dfrac{9u^2+u^2}{2}} = \sqrt{\dfrac{10u^2}{2}}$

$v_{mid} = \sqrt{5u^2} = \boxed{\sqrt{5}u = v_{mid}}$

26. A train starts from rest from a station with acceleration 0.2 m/s^2 on a straight track and then comes to rest after attaining maximum speed on another station due to retardation 0.4 m/s^2. If total time spent is half an hour, then distance between two stations is [Neglect length of train]

(1) 216 km (2) 512 km (3) 728 km (4) 1296 km

Sol. Answer (1)

Shortcut : $S = \dfrac{1}{2}\dfrac{\alpha\beta}{\alpha+\beta}T^2$

$\alpha \rightarrow$ Acceleration

$\beta \rightarrow$ Deceleration (magnitude only)

$T \rightarrow$ Time of journey

$S \rightarrow$ Distance travelled

Given, α = 0.2 ms^{-2}

β = 0.4 ms^{-2}

T = half an hour = 30 × 60 s = 1800 s

$S = \dfrac{1}{2}\times\left(\dfrac{0.2\times0.4}{0.2+0.4}\right)\times(1800)^2$

$\Rightarrow$ S = 216000 m

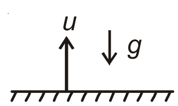
$\Rightarrow$ $\boxed{S = 216\text{ km}}$

27. A body is projected vertically upward direction from the surface of earth. If upward direction is taken as positive, then acceleration of body during its upward and downward journey are respectively

(1) Positive, negative (2) Negative, negative (3) Positive, positive (4) Negative, positive

Sol. Answer (2)

Whether body move upwards or downwards the earth tries to pull it downwards only. Hence during both the motion g will negative.



So, negative, negative

28. A particle start moving from rest state along a straight line under the action of a constant force and travel distance x in first 5 seconds. The distance travelled by it in next five seconds will be

(1) x (2) 2 x (3) 3 x (4) 4 x

Sol. Answer (3)

Body starts from rest and moves with a constant acceleration, then the distance travelled in equal time intervals will be in the ratio of odd number. (Galileo's law of odd number)

$x_1 : x_2 \Rightarrow 1 : 3$

$x : x_2 \Rightarrow 1 : 3$

$\Rightarrow \quad \dfrac{x}{x_2} = \dfrac{1}{3}$

$\Rightarrow \quad \boxed{x_2 = 3x}$

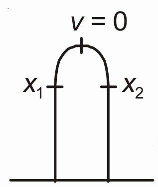
29. A body is projected vertically upward with speed 40 m/s. The distance travelled by body in the last second of upward journey is [take g = 9.8 m/s^2 and neglect effect of air resistance]

(1) 4.9 m (2) 9.8 m (3) 12.4 m (4) 19.6 m

Sol. Answer (1)

As the motion under gravity is symmetric, so distance travelled in last second of ascent is equal to first second of descent.

$t = 1$ s (1st second)



$-x_2 = ut - \dfrac{1}{2} g \times 1^2$

$x_2 = \dfrac{1}{2} \times 9.8 \times 1^2 \quad (\because \ u = 0)$

$\Rightarrow \quad \boxed{x_2 = 4.9 \text{ m}}$

This distance is constant for every body thrown with any speed.

30. A body is projected vertically upward with speed 10 m/s and other at same time with same speed in downward direction from the top of a tower. The magnitude of acceleration of first body w.r.t. second is {take g = 10 m/s^2}

(1) Zero (2) 10 m/s^2 (3) 5 m/s^2 (4) 20 m/s^2

Sol. Answer (1)

The acceleration of first body

$a_1 = 10 \text{ ms}^{-2}$

$a_2 = 10 \text{ ms}^{-2}$

$a_{rel} = a_1 - a_2 = 10 \text{ ms}^{-2} - 10 \text{ ms}^{-2} = 0$

31. A car travelling at a speed of 30 km/h is brought to rest in a distance of 8 m by applying brakes. If the same car is moving at a speed of 60 km/h then it can be brought to rest with same brakes in

(1) 64 m (2) 32 m (3) 16 m (4) 4 m

Sol. Answer (2)

$d_s = \dfrac{u^2}{2a} \Rightarrow d_s \propto u^2$

$u' = 2u$

$\dfrac{d'}{d} = \dfrac{(2u)^2}{u^2}$

$\Rightarrow \quad \dfrac{d'}{8} = 4$

$\Rightarrow \quad d' = 32$

32. A particle is thrown with any velocity vertically upward, the distance travelled by the particle in first second of its decent is

(1) g (2) $\frac{g}{2}$ (3) $\frac{g}{4}$ (4) Cannot be calculated

Sol. Answer (2)

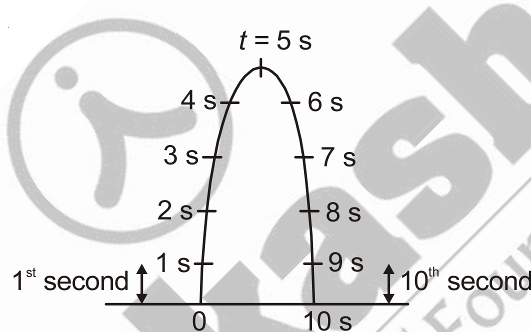
$$s = \frac{1}{2}g \times 1^2 \Rightarrow \boxed{s = \frac{g}{2}}$$

33. A body is thrown vertically upwards and takes 5 seconds to reach maximum height. The distance travelled by the body will be same in

(1) 1st and 10th second (2) 2nd and 8th second (3) 4th and 6th second (4) Both (2) & (3)

Sol. Answer (1)

The motion under gravity is a symmetric motion and the time taken to go up is same as time taken to come back to the initial position.



So, clearly the distance travelled in 1st second is same as that travelled in 10th second.

34. A ball is dropped from a bridge of 122.5 metre above a river. After the ball has been falling for two seconds, a second ball is thrown straight down after it. Initial velocity of second ball so that both hit the water at the same time is

(1) 49 m/s (2) 55.5 m/s (3) 26.1 m/s (4) 9.8 m/s

Sol. Answer (3)

$-h = -\frac{1}{2}gt^2$ 1st ball

$\Rightarrow \quad 122.5 = \frac{1}{2} \times 9.8 t^2$

$\Rightarrow t^2 = 25 \Rightarrow \boxed{t = 5\text{ s}}$

Another ball is dropped after 2 second so it took only (5 – 2) = 3 s

$-122.5 = -u(3) - \frac{1}{2} \times 9.8 \times 3^2$

$\Rightarrow$ 122.5 = 3u + 4.9 × 9

$\Rightarrow$ 3u = 78.4

$\Rightarrow \boxed{u = 26.1\text{ s}}$

35. A balloon starts rising from ground from rest with an upward acceleration 2 m/s². Just after 1 s, a stone is dropped from it. The time taken by stone to strike the ground is nearly

(1) 0.3 s (2) 0.7 s (3) 1 s (4) 1.4 s

Sol. Answer (2)

$u = 0,\ a = 2\ \text{ms}^{-2}$

The velocity of object after one second

$v = u + at$

$\Rightarrow \boxed{v = 2\ \text{ms}^{-1}}$

$s = \frac{1}{2}\times 2\times 1^2 = 1\ \text{m}$

Now after separating from the balloon it will move under the effect of gravity alone.

$-h = vt - \frac{1}{2}\times 9.8\times t^2$

$\Rightarrow -1 = 2t - 4.9t^2$

$\Rightarrow 4.9t^2 - 2t - 1 = 0$

$\Rightarrow \boxed{t = 0.7\ \text{s}}$

36. A boy throws balls into air at regular interval of 2 second. The next ball is thrown when the velocity of first ball is zero. How high do the ball rise above his hand? [Take g = 9.8 m/s²]

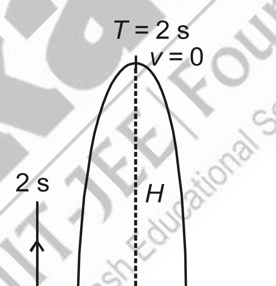
(1) 4.9 m (2) 9.8 m (3) 19.6 m (4) 29.4 m

Sol. Answer (3)

$2T = \frac{2u}{g} \Rightarrow 2 = \frac{u}{9.8} \Rightarrow u = 19.6$

$H = \frac{u^2}{2g} = \frac{19.6\times 19.6}{2\times 9.8}$

$\Rightarrow \boxed{H = 19.6\ \text{m}}$



37. A ball projected from ground vertically upward is at same height at time t_1 and t_2. The speed of projection of ball is [Neglect the effect of air resistance]

(1) $g[t_2 - t_1]$ (2) $\frac{g[t_1 + t_2]}{2}$ (3) $\frac{g[t_2 - t_1]}{2}$ (4) $g[t_1 + t_2]$

Sol. Answer (2)

$t_1 + t_2$ = total time of flight

$t_1 + t_2 = 2T$

$T = \frac{t_1 + t_2}{2}$, also $T = \frac{u}{g}$

$\frac{u}{g} = \frac{t_1 + t_2}{2} \Rightarrow \boxed{u = \frac{1}{2}g(t_1 + t_2)}$

38. Two balls are projected upward simultaneously with speeds 40 m/s and 60 m/s. Relative position (x) of second ball w.r.t. first ball at time t = 5 s is [Neglect air resistance].

(1) 20 m (2) 80 m (3) 100 m (4) 120 m

Sol. Answer (3)

$$s_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$$

$\Rightarrow$ s_{rel} = (60 – 40) 5 (a_{rel} = 0)

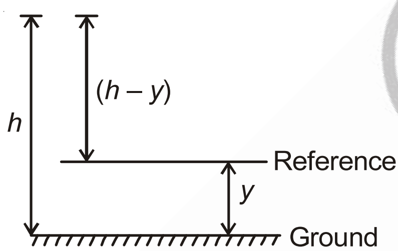
$\Rightarrow$ $\boxed{s_{rel} = 100 \text{ m}}$

39. A ball is dropped from a height h above ground. Neglect the air resistance, its velocity (v) varies with its height above the ground as

(1) $\sqrt{2g(h-y)}$ (2) $\sqrt{2gh}$

(3) $\sqrt{2gy}$ (4) $\sqrt{2g(h+y)}$

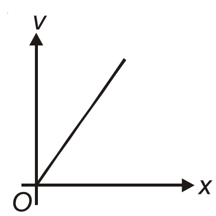
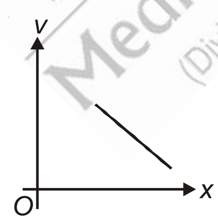
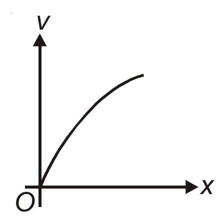
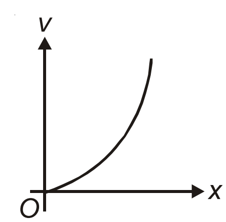
Sol. Answer (1)



$\boxed{v = \sqrt{2g(h-y)}}$

(Graphs)

40. For a body moving with uniform acceleration along straight line, the variation of its velocity (v) with position (x) is best represented by

(1) 
(2) 
(3) 
(4) 

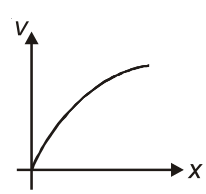
Sol. Answer (3)

For uniform acceleration, $a \rightarrow$ constant

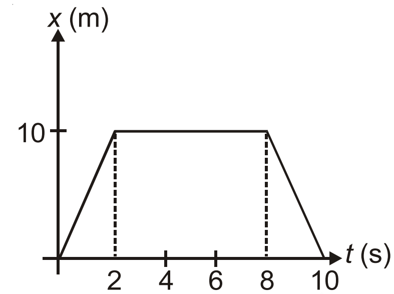
$v^2 = u^2 + 2as$

Taking u = 0

$\Rightarrow$ $\boxed{v^2 \propto x}$



41. The position-time graph for a particle moving along a straight line is shown in figure. The total distance travelled by it in time $t = 0$ to $t = 10$ s is



(1) Zero (2) 10 m (3) 20 m (4) 80 m

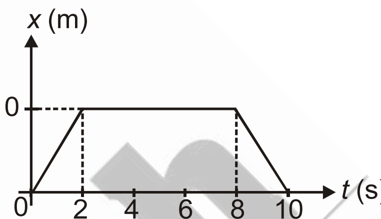
Sol. Answer (3)

The total distance travelled from 0 to 2 s is 10 m

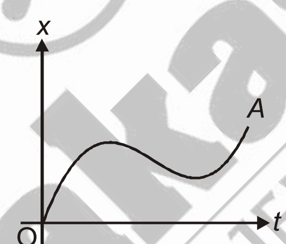
2 s to 8 s $\rightarrow$ Zero distance

and from 8 s to 10 s $\rightarrow$ 10 m

So, distance = 10 + 0 + 10 = 20 m



42. The position-time graph for a body moving along a straight line between O and A is shown in figure. During its motion between O and A, how many times body comes to rest?



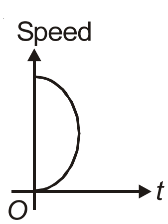
(1) Zero (2) 1 time (3) 2 times (4) 3 times

Sol. Answer (3)

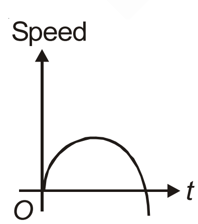
As there are two extremes in the graph one is maxima and other is minima. At both maxima and minima the slope is zero. So, it comes to rest twice.

43. Which one of the following graph for a body moving along a straight line is possible?

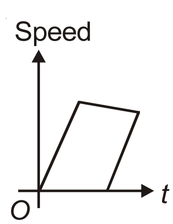
(1)



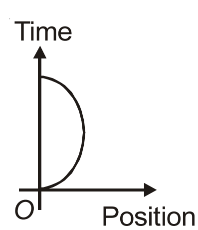
(2)



(3)



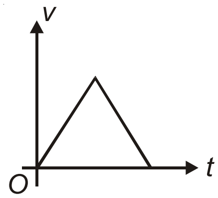
(4)

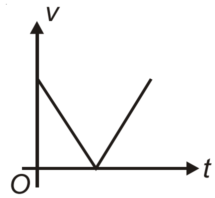


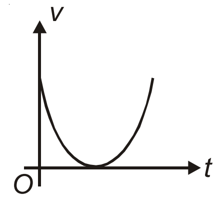
Sol. Answer (4)

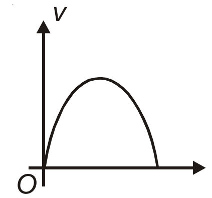
This graph is possible.

44. A body is projected vertically upward from ground. If we neglect the effect of air, then which one of the following is the best representation of variation of speed (v) with time (t)?

(1) 

(2) 

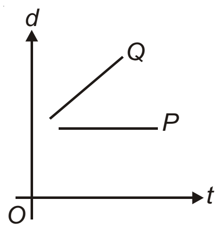
(3) 

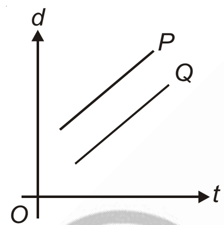
(4) 

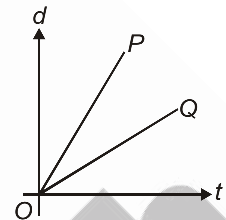
Sol. Answer (2)

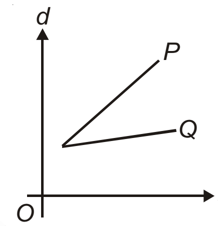
The speed of an object is directly proportional to time $\boxed{v \propto t}$.

45. Which one of the following time-displacement graph represents two moving objects P and Q with zero relative velocity?

(1) 

(2) 

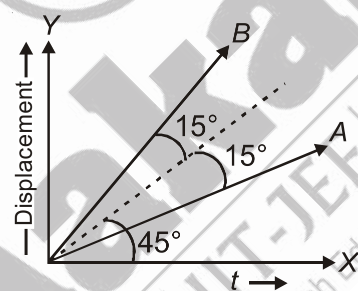
(3) 

(4) 

Sol. Answer (2)

Zero relative velocity means that both of them have same slope.

46. The displacement-time graph for two particles A and B is as follows. The ratio $\frac{v_A}{v_B}$ is

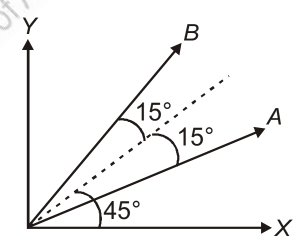


(1) 1 : 2 (2) $1:\sqrt{3}$ (3) $\sqrt{3}:1$ (4) 1 : 3

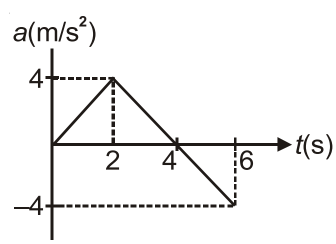
Sol. Answer (4)

The slope of line A is tan30º and B = tan60º

$$\frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3} \Rightarrow \boxed{v_A : v_B = 1:3}$$



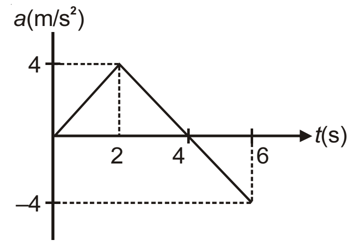
47. For the acceleration-time (a-t) graph shown in figure, the change in velocity of particle from t = 0 to t = 6 s is



(1) 10 m/s (2) 4 m/s (3) 12 m/s (4) 8 m/s

Sol. Answer (2)

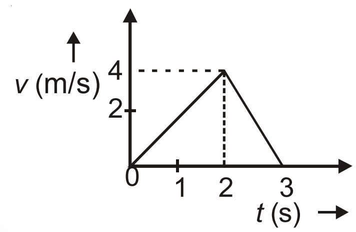
Area under *a-t* graph gives change in velocity.



So, $\Delta v = \frac{1}{2}\times 4\times 4 - \frac{1}{2}\times 2\times 4 = 8-4$

$\boxed{\Delta v = 4 \text{ ms}^{-1}}$

48. The velocity versus time graph of a body moving in a straight line is as shown in the figure below



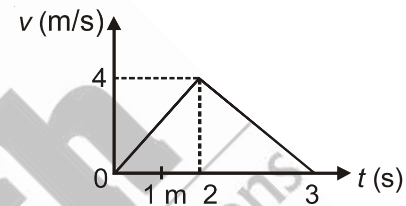
(1) The distance covered by the body in 0 to 2 s is 8 m

(2) The acceleration of the body in 0 to 2 s is 4 ms^{-2}

(3) The acceleration of the body in 2 to 3 s is 4 ms^{-2}

(4) The distance moved by the body during 0 to 3 s is 6 m

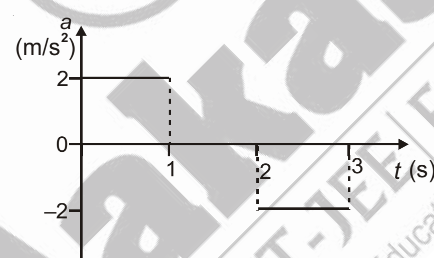
Sol. Answer (4)



Distance covered = Area under *v-t* graph = $\frac{1}{2}\times 3\times 4 = 6$ m

$\text{Acceleration}\Big|_{t=0 \text{ to } 2\text{ s}} = \frac{4-0}{2} = 2 \text{ ms}^{-2}$

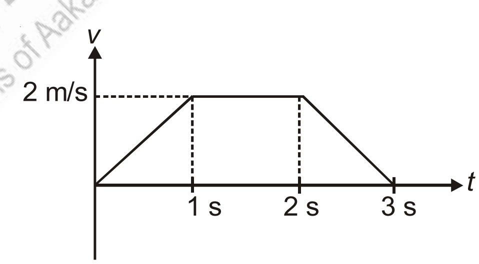
49. Acceleration-time graph for a particle is given in figure. If it starts motion at t = 0, distance travelled in 3 s will be



(1) 4 m (2) 2 m (3) 0 (4) 6 m

Sol. Answer (1)

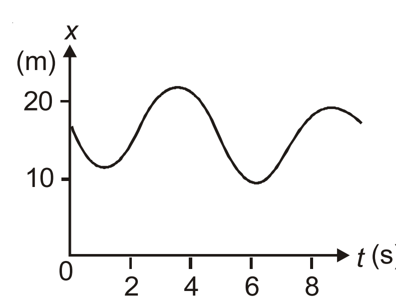
Draw the *v-t* graph from *a-t* graph.



Area under *v-t* graph = $\frac{1}{2}\times 2\times(3+1)$

$= 4$ m

50. Figure shows the position of a particle moving on the *x*-axis as a function of time



(1) The particle has come to rest 4 times

(2) The velocity at t = 8 s is negative

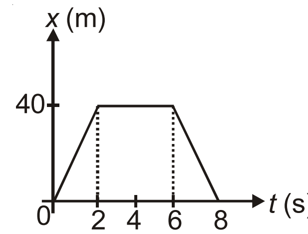
(3) The velocity remains positive for t = 2 s to t = 6 s

(4) The particle moves with a constant velocity

Sol. Answer (1)

The particle has come to rest four times.

51. The position (x) of a particle moving along x-axis varies with time (t) as shown in figure. The average acceleration of particle in time interval $t = 0$ to $t = 8$ s is



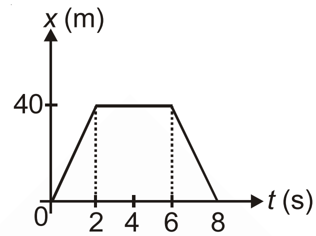
(1) 3 m/s^2 (2) –5 m/s^2

(3) – 4 m/s^2 (4) 2.5 m/s^2

Sol. Answer (2)

$t = 0$ to $t = 2$ $t = 6$ to $t = 8$

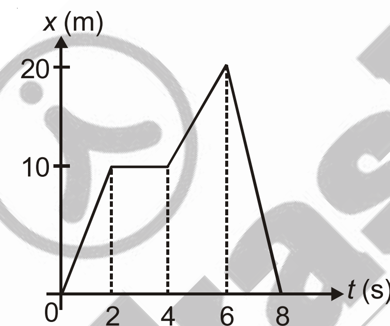
$v = 20$ m/s $v = -20$ m/s



$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{-20-20}{8} = \frac{-40}{8} = -5\ \text{ms}^{-2}$$

$$\boxed{a_{avg} = -5\ \text{ms}^{-2}}$$

52. The position (x)-time (t) graph for a particle moving along a straight line is shown in figure. The average speed of particle in time interval $t = 0$ to $t = 8$ s is



(1) Zero (2) 5 m/s (3) 7.5 m/s (4) 9.7 m/s

Sol. Answer (2)

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{40}{8} = 5\ \text{ms}^{-1}$$

(Relative Motion)

53. A boat covers certain distance between two spots in a river taking t_1 hrs going downstream and t_2 hrs going upstream. What time will be taken by boat to cover same distance in still water?

(1) $\frac{t_1+t_2}{2}$ (2) $2(t_2 - t_1)$ (3) $\frac{2t_1t_2}{t_1+t_2}$ (4) $\sqrt{t_1t_2}$

Sol. Answer (3)

For upstream, Speed $\Rightarrow v - u$ (where $v \rightarrow$ man and $u \rightarrow$ water)

For downstream, Speed $\Rightarrow v + u$

$$t_{up} = \frac{d}{v-u}$$

$$t_2 = \frac{d}{v-u}$$

$$\Rightarrow \quad d = (v-u)t_2 \qquad \text{...(i)}$$

$$t_{down} = \frac{d}{v+u}$$

$$t_1 = \frac{d}{v+u}$$

$$\Rightarrow \quad d = (v+u)t_1 \qquad \text{...(ii)}$$

$$t_{still} = \frac{d}{v}$$

$$\boxed{t_{still} = \frac{2t_1t_2}{t_1+t_2}}$$

On equating (i) and (ii)

$(v - u)\, t_2 = (v + u)\, t_1$

$\Rightarrow \quad vt_2 - ut_2 = vt_1 + ut_1$

$\Rightarrow \quad v(t_2 - t_1) = u(t_1 + t_2)$

$$\Rightarrow \quad u = \frac{v(t_2 - t_1)}{t_2 + t_1}$$

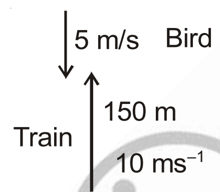
$$\text{So, } d = \left(v - \frac{v(t_2 - t_1)}{t_1 + t_2}\right) t_2 = vt_2 \left(\frac{t_1 + t_2 - t_2 + t_1}{t_1 + t_2}\right)$$

$\boxed{\dfrac{d}{v} = \dfrac{2t_1t_2}{t_1 + t_2}}$ $\rightarrow$ **Remember as shortcut**

54. A train of 150 m length is going towards North at a speed of 10 m/s. A bird is flying at 5 m/s parallel to the track towards South. The time taken by the bird to cross the train is

(1) 10 s (2) 15 s (3) 30 s (4) 12 s

Sol. Answer (1)



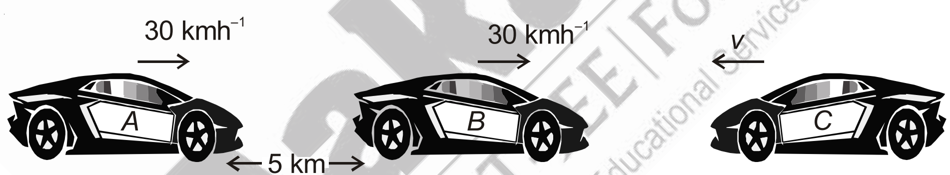
$$\text{Time} = \frac{150}{10+5} = \frac{150}{15}$$

$\Rightarrow \quad \boxed{T = 10 \text{ s}}$

55. Two cars are moving in the same direction with a speed of 30 km/h. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. The speed of the third car is

(1) 30 km/h (2) 25 km/h (3) 40 km/h (4) 45 km/h

Sol. Answer (4)



The distance of 5 km is in between *A* and *B* is covered by *C* in 4 minute with relative velocity (v + 30).

So, $d_{rel} = v_{rel} \times t$

$$\Rightarrow \quad 5 \text{ km} = (v + 30) \times \frac{4}{60}$$

$\Rightarrow \quad 75 \text{ kmh}^{-1} = v + 30$

$\Rightarrow \quad \boxed{v = 45 \text{ kmh}^{-1}}$

56. Two cars *A* and *B* are moving in same direction with velocities 30 m/s and 20 m/s. When car *A* is at a distance *d* behind the car *B*, the driver of the car *A* applies brakes producing uniform retardation of 2 m/s**²**. There will be no collision when

(1) d < 2.5 m (2) d > 125 m (3) d > 25 m (4) d < 125 m

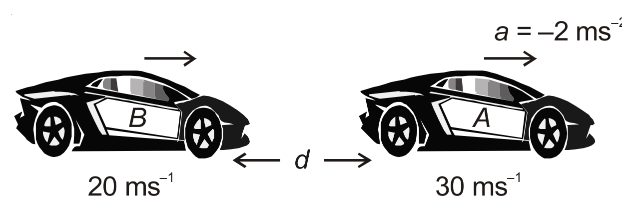
Sol. Answer (3)

$v^2 = u^2 + 2ad$

$\Rightarrow \quad 0 = (10)^2 - 2 \times 2 \times d_{rel}$

$$\Rightarrow \quad \frac{100}{4} \le d_{rel}$$

$\Rightarrow \quad \boxed{d_{rel} \ge 25 \text{ m}}$



57. Two trains each of length 100 m moving parallel towards each other at speed 72 km/h and 36 km/h respectively. In how much time will they cross each other?

(1) 4.5 s (2) 6.67 s (3) 3.5 s (4) 7.25 s

Sol. Answer (2)

When two trains are moving in opposite direction then

$v_{rel} = (20 + 10) = 30 \text{ ms}^{-1}$

$t = \dfrac{200}{30} = 6.67 \text{ s}$

58. A ball is dropped from the top of a building of height 80 m. At same instant another ball is thrown upwards with speed 50 m/s from the bottom of the building. The time at which balls will meet is

(1) 1.6 s (2) 5 s (3) 8 s (4) 10 s

Sol. Answer (1)

$t = \dfrac{h}{v_{rel}} = \dfrac{80}{50}$

$\Rightarrow \boxed{t = 1.6 \text{ s}}$

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