INDEFINITE INTEGRATION

1. If f & g are functions of x such that $g'(x) = f(x)$ then,

$$\int f(x)\,dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$$, where c is called the **constant of integration**.

2. Standard Formula:

(i) $\int (ax + b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\,\ell n\,(ax + b) + c$

(iii) $\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$

(iv) $\int a^{px+q}\,dx = \frac{1}{p}\,\frac{a^{px+q}}{\ell n\,a} + c;\ a > 0$

(v) $\int \sin(ax + b)\,dx = -\frac{1}{a}\cos(ax + b) + c$

(vi) $\int \cos(ax + b)\,dx = \frac{1}{a}\sin(ax + b) + c$

(vii) $\int \tan(ax + b)\,dx = \frac{1}{a}\,\ell n\,\sec(ax + b) + c$

(viii) $\int \cot(ax + b)\,dx = \frac{1}{a}\,\ell n\,\sin(ax + b) + c$

(ix) $\int \sec^2(ax + b)\,dx = \frac{1}{a}\tan(ax + b) + c$

(x) $\int \text{cosec}^2(ax + b)\,dx = -\frac{1}{a}\cot(ax + b) + c$

(xi) $\int \sec x\, dx = \ell n (\sec x + \tan x) + c$ **OR** $\ell n \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

(xii) $\int \operatorname{cosec} x\, dx = \ell n (\operatorname{cosec} x - \cot x) + c$

OR $\ell n \tan \frac{x}{2} + c$ **OR** $- \ell n (\operatorname{cosec} x + \cot x) + c$

(xiii) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

(xiv) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

(xv) $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ell n \left[x + \sqrt{x^2 + a^2} \right] + c$

(xvii) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ell n \left[x + \sqrt{x^2 - a^2} \right] + c$

(xviii) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ell n \left| \frac{a+x}{a-x} \right| + c$

(xix) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ell n \left| \frac{x-a}{x+a} \right| + c$

(xx) $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

(xxi) $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \ell n \left(\frac{x + \sqrt{x^2 + a^2}}{a} \right) + c$

(xxii) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \ell n \left(\frac{x + \sqrt{x^2 - a^2}}{a} \right) + c$

3.

If we subsitute f(x) = t, then f ′(x) dx = dt

4. Integration by Part :

$$\int (f(x)\, g(x))\, dx = f(x) \int (g(x))\, dx - \int \left(\frac{d}{dx}(f(x)) \int (g(x))\, dx \right) dx$$

5. Integration of type $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\, dx$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type

$$\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx, \int (px+q)\sqrt{ax^2+bx+c}\, dx$$

Make the substitution x + $\frac{b}{2a}$ = t , then split the integral as some of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions

(i) $\int \frac{dx}{a + b\sin^2 x}$ **OR** $\int \frac{dx}{a + b\cos^2 x}$

OR $\int \frac{dx}{a\sin^2 x + b\sin x \cos x + c\cos^2 x}$ **put tan x = t.**

(ii) $\int \frac{dx}{a + b\sin x}$ **OR** $\int \frac{dx}{a + b\cos x}$

OR $\int \frac{dx}{a + b\sin x + c\cos x}$ **put tan** $\frac{x}{2}$ **= t**

(iii) $\int \frac{a.\cos x + b.\sin x + c}{\ell.\cos x + m.\sin x + n}$ **dx. Express Nr** $\equiv$ **A(Dr) + B** $\frac{d}{dx}$ **(Dr) + c & proceed.**

8.

$$\int \frac{x^2+1}{x^4+Kx^2+1}\,dx$$ where K is any constant.

Divide Nr & Dr by x² & put $x \mp \frac{1}{x}$ = t.

9. Integration of type

$$\int \frac{dx}{(ax+b)\sqrt{px+q}} \quad \textbf{OR} \quad \int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$$; put px + q = t^2.

10. Integration of type

$$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$$, put ax + b = $\frac{1}{t}$;

$$\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$$ **, put x =** $\frac{1}{t}$