04

Sequence and Series

Quick Revision

Sequence

A sequence is a succession of numbers or terms formed according to some rule. e.g. 4, 8, 12, …… . A sequence is either finite or infinite according as number of terms in it. A sequence whose range is a subset of the set of real numbers R, is called a real sequence.

Series

If $a_1, a_2, a_3, \ldots, a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + a_4 + \ldots + a_n$ is called series.

Progressions

A sequence whose terms follow a certain pattern or rule, is called progression. e.g. 2, 4, 6, …, 100

Arithmetic Progression (AP)

A sequence in which the difference of two consecutive terms is constant, is called arithmetic progression (AP). This difference is called the common difference and denoted by d. Thus,
$d = a_{n+1} - a_n$

nth Term of an AP

If a is the first term, d is common difference and l is the last term of an AP, then

nth term is given by $a_n = a + (n-1)\,d$.

nth term of an AP from the end is

$$a'_n = a_n - (n-1)\,d \text{ or } l - (n-1)\,d$$

Properties of Arithmetic Progression

(i) If a sequence is an AP, then its nth term is a linear expression in n i.e. its nth term is given by $An + B$, where A and B are constants and A is common difference.

(ii) If a constant is added or subtracted from each term of an AP, then the resulting sequence is an AP with same common difference.

(iii) If each term of an AP is multiplied or divided by a non-zero constant, then the resulting sequence is also an AP.

(iv) Any three terms of an AP can be taken as $(a-d), a, (a+d)$ and any four terms of an AP can be taken as $(a-3d), (a-d), (a+d), (a+3d)$.

Sum of n Terms of an AP

If a is the first term, d is the common difference and l is the last term, then

Sum of n terms, $S_n = \dfrac{n}{2}[2a + (n-1)\,d] = \dfrac{n}{2}(a + l)$

If sum of n terms, S_n of an AP is given, then nth term, $a_n = S_n - S_{n-1}$, where $a_1 = S_1$

Arithmetic Mean

(i) If a, A and b are in AP, then $A = \dfrac{a+b}{2}$ is called the arithmetic mean of a and b.

(ii) If $a, A_1, A_2, A_3, \ldots, A_n, b$ are in AP and $A_1, A_2, A_3, \ldots, A_n$ are n arithmetic mean between a and b, then

$$d = \frac{b-a}{n+1} \text{ and } A_n = a + \frac{n(b-a)}{n+1}$$

(iii) Sum of n arithmetic mean between a and b is $n\left(\dfrac{a+b}{2}\right)$.

i.e. $A_1 + A_2 + A_3 + \ldots + A_n = n\left(\dfrac{a+b}{2}\right)$

Geometric Progression (GP)

A sequence $a_1, a_2, \ldots, a_n$ is called geometric progression, if it follows the relation $\frac{a_{k+1}}{a_k} = r$ (constant) for all $k \in N$ the constant ratio is called common ratio (r) of the GP.

*n*th Term of a GP

If a is the first term and r is the common ratio, then the general term or nth term of GP is

$$a_n = ar^{n-1} \quad \text{or} \quad l = ar^{n-1}$$

where l is last term.

nth term of a GP from the end is

$$a_n' = \frac{l}{r^{n-1}}$$

where, l is last term

Properties of Geometric Progression

(i) If all the terms of GP are multiplied or divided by same non-zero constant, then the resulting sequence is a GP with the same common ratio.

(ii) The reciprocals of the terms of a given GP also form a GP.

(iii) If each term of a GP is raised to some power, the resulting sequence also forms a GP.

(iv) In a finite GP, the product of the terms equidistant from the beginning and from the end is always same and is equal to the product of the first and the last terms.

(v) The resulting sequence formed by taking the product and division of the corresponding terms of two GP's is also a GP.

(vi) Any three terms can be taken in GP as $\frac{a}{r}, a$ and ar and any four terms can be taken in GP as $\frac{a}{r^3}, \frac{a}{r}, ar$ and ar^3.

Sum of *n* Terms of a GP

If a and r are the first term and common ratio of a GP respectively, then

Sum of n terms of a GP,

$$S_n = \begin{cases} a\,\dfrac{(1-r^n)}{1-r}, & \text{if} \quad |r| < 1 \\ a\,\dfrac{(r^n-1)}{r-1}, & \text{if} \quad |r| > 1 \end{cases}$$

Sum of an infinite GP,

$$S_\infty = \begin{cases} \dfrac{a}{1-r}, & |r| < 1 \\ \infty, & |r| \geq 1 \end{cases}$$

Geometric Mean (GM)

(i) If a, G and b are in GP, then G is called the geometric mean of a and b and is given by

$$G = \sqrt{ab}$$

(ii) If $a, G_1, G_2, G_3, \ldots, G_n, b$ are in GP, then $G_1, G_2, G_3, \ldots, G_n$ are n GM's between a and b,

then common ratio, $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ and

$$G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}.$$

Relation between AM and GM

Let A and G be the AM and GM of two positive real numbers a and b, respectively.

$$\Rightarrow \quad A - G = \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0$$

$$\Rightarrow \quad A \geq G$$

Objective Questions

Multiple Choice Questions

1. The first five terms of the sequence, where $a_1 = 3$, $a_n = 3a_{n-1} + 2$ for all $n > 1$ are

(a) 3, 15, 40, 110, 330 (b) 3, 11, 35, 107, 323
(c) 3, 20, 45, 110, 330 (d) 3, 11, 40, 107, 323

2. The first five terms of the sequence $a_n = (-1)^{n-1} 5^{n+1}$ are

(a) 25, –125, 625, –3125, 15625
(b) 25, 125, 625, 3125, 15625
(c) 25, –125, 625, 3125, 15625
(d) 25, –125, 625, –3125, –15625

3. A man starts repaying a loan as first instalment of ₹ 100. If he increases the instalment by ₹ 5 every month, then the amount he will pay in the 30th instalment is

(a) ₹ 241 (b) ₹ 250
(c) ₹ 245 (d) ₹ 265

4. 40 is which term of the sequence 72, 70, 68 …… .

(a) 18th (b) 17th
(c) 15th (d) 10th

5. The number of terms in the AP 20, 25, 30, …… 100 are

(a) 16 (b) 17
(c) 18 (d) 19

6. In an AP, if mth term is n and the nth term is m, where $m \neq n$, then pth term is

(a) $m+n-p$ (b) $m-n+p$
(c) $n-m+p$ (d) $m+n+p$

7. The number of terms in the AP 7, 13, 19, …… 1205 are

(a) 30 (b) 34
(c) 31 (d) 10

8. If the angles of any quadrilateral is in AP and their common difference is 10, then the angles are

(a) 75°, 85°, 95° and 105°
(b) 75°, 80°, 90° and 100°
(c) 75°, 85°, 90° and 105°
(d) 70°, 85°, 95° and 105°

9. 6th term from the end of the sequence 9, 12, 15 ……… 20th term is

(a) 50 (b) 52 (c) 51 (d) 55

10. If the sum and product of three numbers of an AP is 24 and 440 respectively, then the common difference of the AP is

(a) ±1 (b) ±3
(c) ±2 (d) ±5

11. The sum of three consecutive terms of an AP is 15, and their product is 105, then the common difference is

(a) ±1 (b) ±2
(c) ±3 (d) ±4

12. If a, b, c are in arithmetic progression, then the value of $(a + 2b - c)(2b + c - a)(a + 2b + c)$ is

(a) $16abc$ (b) $4abc$
(c) $8abc$ (d) $3abc$

13. In an AP, if Kth term is $5K + 1$. Then, the sum of first n terms is

(a) $\frac{n}{2}(5n+7)$ (b) $\frac{n}{2}(n+7)$
(c) $\frac{n}{2}(n+5)$ (d) $\frac{n}{2}(7n+5)$

14. The rth term of an AP sum of whose first n terms is $2n + 3n^2$ is given by

(a) $6r+1$ (b) $6r-1$
(c) $6r$ (d) $3r-1$

15. If t_n denotes the nth term of the series $2 + 3 + 6 + 11 + 18 + \ldots$, then t_{50} is

[NCERT Exemplar]

(a) $49^2 - 1$ (b) 49^2
(c) $50^2 + 1$ (d) $49^2 + 2$

16. If in an AP, first term is 2 and the sum of first five terms is one-fourth of the next five terms, then the 20th terms is

(a) -140 (b) -100
(c) -112 (d) -138

17. 6 arithmetic means between 3 and 24 are

(a) 6, 9, 12, 15, 18 and 21
(b) 6, 9, 10, 15, 18 and 21
(c) 6, 8, 10, 15, 18 and 21
(d) 6, 9, 12, 13, 18 and 21

18. The nth term of a GP 5, 25, 125, ... is

(a) 5^n (b) 5^{n-1}
(c) 5^{n+1} (d) 5^{n-2}

19. If the third term of GP is 4, then the product of its first 5 terms is ***[NCERT Exemplar]***

(a) 4^3 (b) 4^4
(c) 4^5 (d) None of these

20. In a GP, the 3rd term is 24 and the 6th term is 192. Then, the 10th term is

(a) 1084 (b) 3290
(c) 3072 (d) 2340

21. Common ratio of four numbers of *a* GP in which the third term is greater than the first term by 9 and the second term is greater than 4th by 18 is

(a) 2 (b) -2 (c) 1 (d) -1

22. If the 8th term of a GP is 192 with the common ratio 2, then the 12th term is

(a) 1640 (b) 2084
(c) 3072 (d) 3126

23. 5120 is which term of the GP 5, 10, 20, 40 ……

(a) 11th (b) 10th
(c) 6th (d) 5th

24. The 5th term from the end of the sequence 16, 8, 4, 2, …… $\frac{1}{16}$ is

(a) 1 (b) 2
(c) 3 (d) 4

25. 8th term from the end of the sequence 3, 6, 12, …… 25th term is

(a) 393216 (b) 393206
(c) 313216 (d) 303216

26. The sum of first three terms of a GP is $\frac{13}{12}$ and their product is -1. Then, which of the following statements is incorrect?

(a) Common ratio is $\frac{-3}{4}$ or $\frac{-4}{3}$
(b) First three terms are $\frac{4}{3}, -1, \frac{3}{4}$ for $r = \frac{-3}{4}$
(c) First three terms are $\frac{3}{4}, -1, \frac{4}{3}$ for $r = \frac{-4}{3}$
(d) Common ratio is $\frac{3}{4}$ or $\frac{4}{3}$

27. The sum of first three terms of a GP is $\frac{13}{12}$ and their product is -1 then the common ratio of the GP is

(a) $\frac{-4}{3}$ or $\frac{-3}{4}$ (b) $\frac{3}{4}$ or $\frac{4}{3}$
(c) $\frac{1}{4}$ or $\frac{-1}{4}$ (d) $\frac{5}{3}$ or $\frac{-3}{5}$

28. Sum of the sequence $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}$ …… upto 10 terms is equal to

(a) 14762 (b) 14726
(c) 41762 (d) 12476

29. A person has 2 parents, 4 grandparents, 8 great grandparents and so on. Then, the number of ancestors during the ten generations preceding his own is

(a) 1084 (b) 2046
(c) 2250 (d) 1024

30. If n terms of GP $3, 3^2, 3^3$, ... are needed to give the sum 120, then the value of n is

(a) 2 (b) 3
(c) 4 (d) 5

31. The sum of an infinite GP is $\frac{80}{9}$ and its common ratio is $\frac{-4}{5}$ then its first term is equal to

(a) 10 (b) 14
(c) 15 (d) 16

32. $3^{1/2} \times 3^{1/4} \times 3^{1/8} \times \ldots$ upto infinite terms is equal to

(a) 3^2 (b) 3
(c) 3^3 (d) 3^4

33. The geometric mean of 2 and 8 is

(a) 4 (b) 6
(c) 7 (d) 5

34. 4 gemoteric means between 3 and 96 are

(a) 6, 12, 24, 48 (b) 6, 10, 24, 48
(c) 6, 10, 40, 48 (d) 48, 24, 10, 5

35. The minimum value of $4^x + 4^{1-x}$, $x \in R$ is ***[NCERT Exemplar]***

(a) 2 (b) 4 (c) 1 (d) 0

Asserion-Reasoning MCQs

Directions (Q. Nos. 36-50) Each of these questions contains two statements Assertion (A) and Reason (R). Each of the questions has four alternative choices, any one of the which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation of A.
(b) A is true, R is true; R is not a correct explanation of A.
(c) A is true; R is false
(d) A is false; R is true.

36. If nth term of a sequence is $a_n = 2n^2 - n + 1$.

Assertion (A) First and second terms of same sequence are 2 and 7 respectively.

Reason (R) Third and fourth terms of same sequence are 16 and 29, respectively.

37. **Assertion (A)** If nth term of a sequence is $a_n = \frac{n^2}{2^n}$, then its 7th term is $\frac{49}{128}$.

Reason (R) If nth term of a sequence is $a_n = \frac{n(n-2)}{n+3}$, then its 20th term is $\frac{323}{22}$.

38. **Assertion (A)** The first three terms of the sequence are $\frac{3}{2}$, x, $\frac{21}{2}$ whose nth term is $a_n = \frac{n(n^2+5)}{4}$. Then $x = \frac{9}{2}$

Reason (R) The third term of the sequence whose nth term is $a_n = (-1)^{n-1}5^{n+1}$ is 620.

39. **Assertion (A)** If the nth term of a sequence is $a_n = 4n - 3$. Here, a_{17} and a_{24} are 65 and 93 respectively.

Reason (R) If the nth term of a sequence is $a_n = (-1)^{n-1}n^3$. Here, 9th term is 729.

40. **Assertion (A)** If the sequence of even natural number is 2, 4, 6, 8, …, then nth term of the sequence is a_n given by $a_n = 2n$, where $n \in N$.

Reason (R) If the sequence of odd natural numbers is 1, 3, 5, 7, …, then nth term of the sequence is given by $a_n = 2n - 1$, where $n \in N$.

41. **Assertion (A)** The fourth term of a GP is the square of its second term and the first term is −3, then its 7th term is equal to 2187.

Reason (R) Sum of first 10 terms of the AP 6, 8, 10, ……… is equal to 150.

42. **Assertion (A)** If 5th and 8th term of a GP be 48 and 384 respectively, then the common ratio of GP is 2.

Reason (R) If 18, x, 14 are in AP, then $x = 16$.

43. **Assertion (A)** The sum of first 20 terms of an AP, 4, 8, 12, … is equal to 840.

Reason (R) Sum of first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$, where a = first term and d = common difference.

44. Assertion (A) The sum of the series $\frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} + \sqrt{5} + \ldots\ldots$ 25 terms is $75\sqrt{5}$.

Reason (R) If 27, x, 3 are in GP, then $x = \pm 4$.

45. Assertion (A) The sum of first 23 terms of the AP

16, 11, 6, ... is -897.

Reason (R) The sum of first 22 terms of the AP

$x + y, x - y, x - 3y, \ldots\ldots$ is $22[x - 20y]$.

46. Assertion (A) If the numbers $\frac{-2}{7}, K, \frac{-7}{2}$ are in GP, then $k = \pm 1$.

Reason (R) If a_1, a_2, a_3 are in GP, then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$.

47. Assertion (A) The sum of first 6 terms of the GP 4, 16, 64, … is equal to 5460.

Reason (R) Sum of first n terms of the G. P is given by $S_n = \frac{a(r^n - 1)}{r - 1}$, where $a =$ first term $r =$ common ratio and $|r| > 1$.

48. Assertion (A) If the sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms, then the common ratio is $\frac{1}{4}$.

Reason (R) In an AP 3, 6, 9, 12……… the 10th term is equal to 30.

49. Assertion (A) The sum of first n terms of the series $0.6 + 0.66 + 0.666 + \ldots\ldots$ is $\frac{2}{3}\left[n - \frac{1}{9}\left\{1 - \left(\frac{1}{10}\right)^n\right\}\right]$.

Reason (R) General term of a GP is $T_n = ar^{n-1}$, where $a =$ first term and $r =$ common ratio.

50. Assertion (A) The sum of first n terms of the series

$5 + 55 + 555 + \ldots$ is $\frac{5}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$.

Reason (R) General term of an AP is $T_n = a + (n-1)d$, where $a =$ first term and $d =$ common difference.

Case Based MCQs

51. A company produces 500 computers in the third year and 600 computers in the seventh year. Assuming that the production increases uniformly by a constant number every year.

Based on the above information, answer the following questions.

(i) The value of the fixed number by which production is increasing every year is

(a) 25 (b) 20 (c) 10 (d) 30

(ii) The production in first year is

(a) 400 (b) 250
(c) 450 (d) 300

(iii) The total production in 10 years is

(a) 5625 (b) 5265
(c) 2655 (d) 6525

(iv) The number of computers produced in 21st year is

(a) 650 (b) 700
(c) 850 (d) 950

(v) The difference in number of computers produced in 10th year and 8th year is

(a) 25 (b) 50 (c) 100 (d) 75

52. A sequence whose terms increases or decreases by a fixed number, is called an Arithmetic Progression (AP).

In other words, we can say that, a sequence is called an arithmetic progression if the difference of a term and the previous term is always same i.e. $a_{n+1} - a_n = \text{constant}$ for all n.

This constant or same difference is called the common difference of an AP and it is denoted by d.

In an AP, we usually denote the first term by a, common difference by d and the nth term by a_n or T_n defined as

$$T_n = a_n = a + (n-1)\,d$$

Also, $l = a + (n-1)d$, where l is the last term of the sequence.

The sum of n terms, S_n of this AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.

Also, if l be the last term, then the sum of n terms of this AP is $S_n = \frac{n}{2}(a + l)$.

Based on the above information, answer the following questions.

(i) If nth term of an AP is given by $a_n = 2n^2 + 1$, then its 10th term is equal to

(a) 200
(b) 301
(c) 400
(d) Given sequence is not an AP

(ii) 11th term of an AP 11, 18, 25, … is equal to

(a) 80 (b) 81
(c) 71 (d) 70

(iii) If the sum of n terms of an AP is given by $S_n = 3n + 2n^2$, then the common difference of the AP is

(a) 3 (b) 2
(c) 6 (d) 4

(iv) If 9 times the 9th term of an AP is equal to 13 times the 13th term, then the 22nd term of the AP is

(a) 0 (b) 22
(c) 198 (d) 220

(v) Let S_n denote the sum of the first n terms of an AP, if $S_{2n} = 3S_n$, then $S_{3n} : S_n$ is equal to

(a) 4 (b) 6
(c) 8 (d) 10

53. A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant.

Rahul being a plant lover decides to open a nursery and he bought few plants with pots. He wants to place pots in such a way that number of pots in first row is 2, in second row is 4 and in third row is 8 and so on… .

Based on the above information, answer the following questions.

(i) The constant multiple by which the number of pots is increasing in every row is

(a) 2 (b) 4
(c) 8 (d) 1

(ii) The number of pots in 8th row is

(a) 156 (b) 256
(c) 300 (d) 456

(iii) The difference in number of pots placed in 7th row and 5th row is

(a) 86 (b) 50
(c) 90 (d) 96

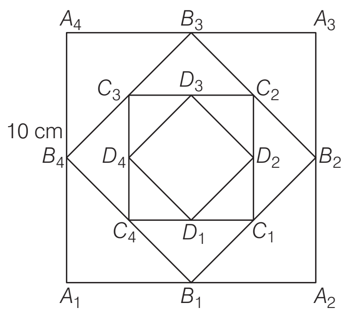
(iv) Total number of pots upto 10th row is

(a) 1046 (b) 2046
(c) 1023 (d) 1024

(v) If Rahul wants to place 510 pots in total, then the total number of rows formed in this arrangement is

(a) 7 (b) 8
(c) 9 (d) 5

54. A student of class XI draw a square of side 10 cm. Another student join the mid-point of this square to form new square. Again, the mid-points of the sides of this new square are joined to form another square by another student. This process is continued indefinitely.



Based on above information, answer the following questions.

(i) The side of fourth square is (in cm)

(a) 5 (b) $\frac{\sqrt{5}}{2}$
(c) $\sqrt{5}$ (d) None of these

(ii) The area of the fifth square is (in sq cm)

(a) $\frac{25}{2}$ (b) 50
(c) 25 (d) $\frac{25}{4}$

(iii) The perimeter of the 7th square is (in cm)

(a) 10 (b) 20
(c) 5 (d) $\frac{5}{2}$

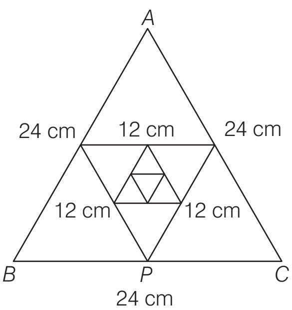
(iv) The sum of areas of all the square formed is (in sq cm)

(a) 150 (b) 200
(c) 250 (d) None of these

(v) The sum of the perimeter of all the square formed is (in cm)

(a) $80+40\sqrt{2}$ (b) $40+40\sqrt{2}$
(c) 40 (d) None of these

55. Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



Based on above information, answer the following questions.

(i) The side of the 5th triangle is (in cm)

(a) 3 (b) 6
(c) 1.5 (d) 0.75

(ii) The sum of perimeter of first 6 triangle is (in cm)

(a) $\frac{569}{4}$ (b) $\frac{567}{4}$ (c) 120 (d) 144

(iii) The area of all the triangle is (in sq cm)

(a) 576 (b) $192\sqrt{3}$
(c) $144\sqrt{3}$ (d) $169\sqrt{3}$

(iv) The sum of perimeter of all triangle is (in cm)

(a) 144 (b) 169
(c) 400 (d) 625

(v) The perimeter of 7th triangle is (in cm)

(a) $\frac{7}{8}$ (b) $\frac{9}{8}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

ANSWERS

Multiple Choice Questions

1. (b)	2. (a)	3. (c)	4. (b)	5. (b)	6. (a)	7. (b)	8. (a)	9. (c)	10. (b)
11. (b)	12. (a)	13. (a)	14. (b)	15. (d)	16. (c)	17. (a)	18. (a)	19. (c)	20. (c)
21. (b)	22. (c)	23. (a)	24. (a)	25. (a)	26. (d)	27. (a)	28. (a)	29. (b)	30. (c)
31. (d)	32. (b)	33. (a)	34. (a)	35. (b)					

Assertion-Reasoning MCQs

36. (b)	37. (c)	38. (c)	39. (b)	40. (b)	41. (d)	42. (b)	43. (a)	44. (c)	45. (b)
46. (a)	47. (a)	48. (b)	49. (b)	50. (b)					

Case Based MCQs

51. (i) - (a); (ii) - (c); (iii) - (a); (iv) - (d); (v) - (b)

52. (i) - (d); (ii) - (b); (iii) - (d); (iv) - (a); (v) - (b)

53. (i) - (a); (ii) - (b); (iii) - (d); (iv) - (b); (v) - (b)

54. (i) - (d); (ii) - (d); (iii) - (c); (iv) - (b); (v) - (a)

55. (i) - (c); (ii) - (b); (iii) - (b); (iv) - (a); (v) - (b)

SOLUTIONS

1. $\because$ $a_1 = 3$ and $a_n = 3a_{n-1} + 2$

Then, $a_2 = 3a_1 + 2 = 3(3) + 2 = 11$

$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$

$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$

$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$

2. We have, $a_n = (-1)^{n-1}\ 5^{n+1}$

On putting $n = 1$, we get

$a_1 = (-1)^{1-1}\ 5^{1+1} = (-1)^0\ 5^2 = 25$

On putting $n = 2$, we get

$a_2 = (-1)^{2-1}\ 5^{2+1} = (-1)^1\ 5^3 = -125$

On putting $n = 3$, we get

$a_3 = (-1)^{3-1}\ 5^{3+1} = (-1)^2\ 5^4 = 625$

On putting $n = 4$, we get

$a_4 = (-1)^{4-1}\ 5^{4+1} = (-1)^3\ 5^5 = -3125$

On putting $n = 5$, we get

$a_5 = (-1)^{5-1}\ 5^{5+1} = (-1)^4\ 5^6 = 15625$

Hence, the first five terms of the given sequence are 25, – 125, 625, – 3125, 15625.

3. Given, $a = 100$, $d = 5$

$\because$ $T_n = a + (n-1)d$

$\therefore$ $T_{30} = 100 + (30-1)\,5 = 100 + 29 \times 5$

$= 100 + 145 = 245$

4. Given seqeunce is 72, 70, 68, 66, …

Clearly, the successive difference of the terms is same. So, the above sequence forms an AP with first term, $a = 72$ and common difference, $d = 70 - 72 = -2$.

Let nth term, $T_n = 40$

and $T_n = a + (n-1)d$

$\therefore$ $40 = 72 + (n-1)(-2)$

$\Rightarrow$ $40 = 72 - 2n + 2$

$\Rightarrow$ $2n = 72 - 40 + 2$

$\Rightarrow$ $2n = 34$

$\Rightarrow$ $n = 17$

Hence, 17th term of the given sequence is 40.

5. Given AP is 20, 25, 30,..., 100.

Here, $a = 20$, $d = 25 - 20 = 5$ and $l = 100$

$\because$ Last term, $l = 100$

$\Rightarrow$ $a + (n-1)d = 100$

$\Rightarrow$ $20 + (n-1)\,5 = 100$ $[\because a = 20 \text{ and } d = 5]$

$\Rightarrow$ $20 + 5n - 5 = 100$

$\Rightarrow$ $15 + 5n = 100$

$\Rightarrow$ $5n = 100 - 15 = 85 \Rightarrow n = 17$

Hence, there are 17 terms in given AP.

6. We have,

$a_m = a + (m-1)d = n$ …(i)

and $a_n = a + (n-1)d = m$ …(ii)

Solving Eqs. (i) and (ii), we get

$$(m-n)\,d = n-m$$

$\Rightarrow \quad d = -1 \quad$ ...(iii)

Now, $\quad a = n+m-1 \quad$ ...(iv)

Therefore, $\quad a_p = a + (p-1)\,d$

$= n+m-1+(p-1)(-1)$

[using Eqs. (iii) and (iv)]

$= n+m-1-p+1$

$= n+m-p$

7. Given, AP is 7, 13, 19 …… 205.

$\therefore \quad a = 7, d = 13-7 = 6$

Let it has n terms

$\therefore a_n = 205$

Now, $\quad a_n = a+(n-1)d$

$\therefore \quad 205 = 7+(n-1)\times 6$

$\Rightarrow \quad n-1 = \frac{198}{6} = 33$

$\therefore \quad n = 34$

Hence, the given AP has 34 terms.

8. Given, angles of a quadrilateral are in AP and common difference $(d) = 10°$

Let angles of a quadrilateral are a, $a+d$, $a+2d$ and $a+3d$, respectively.

We know that, the sum of all interior angles of a quadrilateral is 360°.

$\therefore \quad a+a+d+a+2d+a+3d = 360°$

$\Rightarrow a+a+10°+a+20°+a+30° = 360°$

$[\because \; d = 10°]$

$\Rightarrow \quad 4a+60° = 360°$

$\Rightarrow \quad 4a = 360°-60° = 300°$

$\Rightarrow \quad a = \frac{300°}{4} = 75°$

$\therefore \quad a+d = 75°+10° = 85°$

$a+2d = 75°+20° = 95°$

and $\quad a+3d = 75°+30° = 105°$

Hence, angles of an quadrilateral are 75°, 85°, 95° and 105°.

9. Given sequence is 9, 12, 15, ..., 20th term.

Clearly, the successive difference of the terms is same. So, the above sequence forms an AP with first term, $a = 9$ and common difference, $d = 12-9 = 3$.

We know that, if a sequence has n term, then mth term from end is equal to $(n-m+1)$th term from the beginning.

Here, $n = 20, m = 6, \; a = 9$ and $d = 3$

Now, 6th term from the end of sequence

$= (20-6+1)$

i.e. 15th term from the beginning.

$\therefore \quad T_{15} = 9+(15-1)\,3 \qquad [\because T_n = a+(n-1)\,d]$

$= 9+(14)\,3$

$= 9+42 = 51$

Hence, the 6th term from the end of the sequence is 51.

10. Let the three number be $(a-d)$, a and $(a+d)$

According to the question,

$(a-d)+a+(a+d) = 24$

$\Rightarrow \quad 3a = 24$

$\Rightarrow \quad a = 8$

and $\quad (8-d)(8)(8+d) = 440$

$\Rightarrow \quad 64-d^2 = 55$

$\Rightarrow \quad d^2 = 9$

$\Rightarrow \quad d = \pm 3$

11. Let three numbers in AP be $a-d$, a and $a+d$.

According to the question,

Sum of three consecutive terms $= 15$

$\therefore \quad (a-d)+a+(a+d) = 15$

$\Rightarrow \quad 3a = 15$

$\Rightarrow \quad a = 5$

and the product of three consecutive terms $= 105$

$\therefore \quad (a-d)\,(a)\,(a+d) = 105$

$\Rightarrow \quad (5-d)\,(5)\,(5+d) = 105 \qquad$ [put $a = 5$]

$\Rightarrow \quad (25-d^2)\,5 = 105$

$[\because \; (A-B)\,(A+B) = A^2-B^2]$

$\Rightarrow \quad 25-d^2 = 21$

[dividing both sides by 5]

$\Rightarrow \quad d^2 = 4$

$\therefore \quad d = \pm 2$

12. Since, $2b = a+c$

Now, $(a+2b-c)\,(2b+c-a)\,(a+2b+c)$

$= (a+a+c-c)\,(a+c+c-a)\,(2b+2b)$

$= 2a\cdot 2c\cdot 4b = 16\,abc$

13. Given, Kth term $(T_K) = 5K+1$

Putting $K = 1, 2$, we get

$T_1 = 5\times 1+1 = 6$

and $\quad T_2 = 5\times 2+1 = 11$

$\Rightarrow \quad a = 6, d = 11-6 = 5$

Now, $$S_n = \frac{n}{2}[2a + (n-1)\ d]$$
$$= \frac{n}{2}[2 \times 6 + (n-1)\ 5]$$
$$= \frac{n}{2}[12 + 5n - 5] = \frac{n}{2}[5n + 7]$$

14. Given that, sum of n terms of an AP,
$$S_n = 2n + 3n^2$$
$$T_n = S_n - S_{n-1}$$
$$= (2n + 3n^2) - [2(n-1) + 3(n-1)^2]$$
$$= (2n + 3n^2) - [2n - 2 + 3(n^2 + 1 - 2n)]$$
$$= (2n + 3n^2) - (2n - 2 + 3n^2 + 3 - 6n)$$
$$= 2n + 3n^2 - 2n + 2 - 3n^2 - 3 + 6n$$
$$= 6n - 1$$
$\therefore$ r th term $T_r = 6r - 1$

15. Let S_n be sum of the series
$2 + 3 + 6 + 11 + 18 + \ldots + t_{50}$.
$$\therefore \quad S_n = 2 + 3 + 6 + 11 + 18 + \ldots + t_{50} \quad \text{...(i)}$$
and
$$S_n = 0 + 2 + 3 + 6 + 11 + 18 + \ldots + t_{49} + t_{50} \quad \text{...(ii)}$$
On subtracting Eq. (ii) from Eq. (i), we get
$$0 = 2 + 1 + 3 + 5 + 7 + \cdots - t_{50}$$
$$\Rightarrow t_{50} = 2 + 1 + 3 + 5 + 7 + \cdots \text{ upto 49 terms}$$
$$\therefore t_{50} = 2 + [1 + 3 + 5 + 7 + \cdots \text{ upto 49 terms}]$$
$$= 2 + \frac{49}{2}[2 \times 1 + 48 \times 2]$$
$$= 2 + \frac{49}{2} \times [2 + 96]$$
$$= 2 + [49 + 49 \times 48]$$
$$= 2 + 49 \times 49$$
$$= 2 + (49)^2$$

16. Let the AP is $a, a + d, a + 2d, a + 3d, \ldots$
Now, given $a = 2$
According to the given condition,
Sum of first five terms $= \frac{1}{4}$ (Sum of next five terms)
$$a + (a + d) + (a + 2d) + (a + 3d) + a + 4d$$
$$= \frac{1}{4}[a + 5d + a + 6d + a + 7d + a + 8d + a + 9d]$$
$$\Rightarrow \quad 5a + 10d = \frac{1}{4}[5a + 35d]$$
$$\Rightarrow \quad 4[5a + 10d] = 5a + 35d$$
$$\Rightarrow \quad 20a + 40d = 5a + 35d$$
$$\Rightarrow \quad 20a - 5a = 35d - 40d$$
$$\Rightarrow \quad 15a = -5d$$
$$\Rightarrow \quad 15 \times 2 = -5d \qquad [\because a = 2]$$
$$\Rightarrow \quad 30 = -5d \Rightarrow d = \frac{-30}{5} = -6$$
Now, $T_n = a + (n-1)d$
$$\Rightarrow \quad T_{20} = 2 + (20-1)(-6) = 2 + 19(-6)$$
$$= 2 - 19 \times 6 = 2 - 114 = -112$$

17. Let A_1, A_2, A_3, A_4, A_5 and A_6 be six arithmetic means between 3 and 24.
Then, $3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ are in AP and number of terms is 8.
$$\therefore \quad a = 3, \text{ and } T_8 = 24$$
$$\Rightarrow \quad a + (8-1)d = 24 \qquad [\because T_n = a + (n-1)d]$$
$$\Rightarrow \quad a + 7d = 24$$
$$\Rightarrow \quad 3 + 7d = 24$$
$$\Rightarrow \quad 7d = 21 \Rightarrow d = 3$$
Now, $A_1 = a + d = 3 + 3 = 6$
$$A_2 = a + 2d = 3 + 2(3) = 9$$
$$A_3 = a + 3d = 3 + 3(3) = 12$$
$$A_4 = a + 4d = 3 + 4(3) = 15$$
$$A_5 = a + 5d = 3 + 5(3) = 18$$
and $A_6 = a + 6d = 3 + 6(3) = 21$
Hence, 6 arithmetic means between 3 and 24 are 6, 9, 12, 15, 18 and 21.

18. Here, $a = 5$ and $r = 5$
Thus, $a_n = ar^{n-1} = 5 \times (5)^{n-1} = 5^n$

19. It is given that, $T_3 = 4$
Let a and r the first term and common ratio, respectively.
Then, $$ar^2 = 4 \quad \text{...(i)}$$
Product of first 5 terms $= a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$
$$= a^5 r^{10} = (ar^2)^5 = (4)^5$$
[using Eq. (i)]

20. Here, $a_3 = ar^2 = 24$ ...(i)
and $a_6 = ar^5 = 192$ ...(ii)
Dividing Eq. (ii) by Eq. (i), we get
$$r^3 = \frac{192}{24} \Rightarrow r^3 = 8 \Rightarrow r = 2$$
Substituting $r = 2$ in Eq. (i), we get
$$a = 6$$
Hence, $a_{10} = 6(2)^9 = 3072$

21. Let the GP is $a, ar, ar^2, ar^3, \dots$

Given, third term = first term + 9

$\Rightarrow \quad T_3 = a + 9 \Rightarrow ar^2 = a + 9$

$\Rightarrow \quad ar^2 - a = 9 \quad \dots(i)$

Again, second term = fourth term + 18

$T_2 = T_4 + 18$

$\Rightarrow \quad ar = ar^3 + 18$

$\Rightarrow \quad ar - ar^3 = 18 \quad \dots(ii)$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{ar^2 - a}{ar - ar^3} = \frac{9}{18} \Rightarrow \frac{a(r^2 - 1)}{ar(1 - r^2)} = \frac{1}{2}$$

$\Rightarrow \quad \frac{-1}{r}\frac{(1 - r^2)}{(1 - r^2)} = \frac{1}{2} \Rightarrow -\frac{1}{r} = \frac{1}{2} \Rightarrow r = -2$

22. Given, 8th term, $T_8 = 192$

and common ratio $(r) = 2$

$\Rightarrow \quad ar^{8-1} = 192 \quad [\because T_n = ar^{n-1}]$

$\Rightarrow \quad a \times (2)^7 = 192 \Rightarrow a \times 128 = 192$

$\Rightarrow \quad a = \frac{192}{128} \Rightarrow a = \frac{48}{32} = \frac{3}{2}$

Now, $\quad T_{12} = ar^{12-1} = \frac{3}{2} \times (2)^{11}$

$= \frac{3}{2} \times 2^{11} = 3 \times 2^{10}$

$= 3 \times 1024 = 3072$

23. Given GP is 5, 10, 20, 40, …

Here, $a = 5$ and $r = \frac{10}{5} = 2$

Let nth term of given GP = 5120

i.e. $\quad T_n = 5120$

Now, $\quad T_n = ar^{n-1} = 5120$

$\Rightarrow 5(2)^{n-1} = 5120 \quad [\because a = 5, r = 2]$

$\Rightarrow \quad 2^{n-1} = \frac{5120}{5} = 1024$

$\Rightarrow \quad 2^{n-1} = 1024$

$\Rightarrow \quad 2^{n-1} = 2^{10}$

On equating the powers, we get

$n - 1 = 10$

$\Rightarrow \quad n = 10 + 1 = 11$

Hence, 11th term of given GP is 5120.

24. Given sequence is 16, 8, 4, 2, …, $\frac{1}{16}$.

Here, last term $(l) = \frac{1}{16}$,

Common ratio $(r) = \frac{8}{16} = \frac{1}{2}$

We know that, n th term from end in GP

$= \frac{l}{r^{n-1}}$

$\therefore$ 5th term from end $= \frac{1/16}{(1/2)^{5-1}} = \frac{1/16}{(1/2)^4}$

$= \frac{1/16}{1/16} = 1$

[put the values of l, r and n]

Hence, the 5th term from end in GP is 1.

25. We know that if a sequence has n terms, then mth term from end is equal to $(n - m + 1)$th term from the beginning.

Here, $a = 3$, $r = \frac{6}{3} = 2$, $m = 8$ and $n = 25$.

Now, 8th term from the end of sequence is equal to the $(25 - 8 + 1)$. i.e. 18th term from the beginning.

$\because \quad T_n = ar^{n-1}$

$\therefore \quad T_{18} = ar^{18-1} = 3(2)^{18-1} \quad [\because a = 3, r = 2]$

$= 3(2)^{17} = 3 \times 131072 = 393216$

Hence, the 8th term from the end is 393216.

26. Let $\frac{a}{r}, a, ar$ be the three terms of the G.P. .

Then, $\quad \frac{a}{r} + a + ar = \frac{13}{12} \quad \dots(i)$

and $\quad \frac{a}{r} \cdot a \cdot ar = -1 \quad \dots(ii)$

From Eq. (ii), we get

$a^3 = -1$

$\Rightarrow \quad a = -1$

[considering only real roots]

Substituting $a = -1$ in Eq. (i), we have

$$\frac{-1}{r} - 1 - r = \frac{13}{12}$$

$\Rightarrow \quad r^2 + r + 1 + \frac{13r}{12} = 0$

$\Rightarrow \quad 12r^2 + 25r + 12 = 0$

On solving, we get

$$r = \frac{-3}{4} \text{ or } \frac{-4}{3}$$

Thus, the three terms of GP are $\frac{4}{3}, -1, \frac{3}{4}$ for $r = \frac{-3}{4}$ and $\frac{3}{4}, -1, \frac{4}{3}$ for $r = \frac{-4}{3}$.

27. Let the first three terms of the GP be $\frac{a}{r}, a, ar$.

Now, according to the question,

$$\frac{a}{a} + a + ar = \frac{13}{12} \quad \ldots(\text{i})$$

and $$\frac{a}{r}.r.ar = -1 \quad \ldots(\text{ii})$$

From Eq. (ii),

$$a^3 = -1 \Rightarrow a = -1$$

Put $a = -1$ in Eq. (i), we get

$$\left(\frac{-1}{r} - 1 - r\right) = \frac{13}{12}$$

$$\Rightarrow \quad \frac{13}{12} + r + 1 + \frac{1}{r} = 0$$

$$\Rightarrow \quad \frac{25}{12} + r + \frac{1}{r} = 0$$

$$\Rightarrow \quad 12r^2 + 25r + 12 = 0$$

$$\Rightarrow \quad 12r^2 + 16r + 9r + 12 = 0$$

$$\Rightarrow \quad 4r(3r+4) + 3(3r+4) = 0$$

$$\Rightarrow \quad (3r+4)(4r+3) = 0$$

$$\Rightarrow \quad r = \frac{-3}{4} \text{ or } \frac{-4}{3}$$

28. We have, sequence $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \ldots$

Clearly, the successive ratio of the terms is same.

So, the above sequence forms a GP, with first term, $a = \frac{1}{2}$ and common ratio, $r = \frac{3}{2} \div \frac{1}{2} = 3$.

$$\therefore S_{10} = \frac{\frac{1}{2}[3^{10} - 1]}{3 - 1} \quad \left[\because S_n = \frac{a(r^n - 1)}{r - 1} \text{ as } r > 1\right]$$

$$= \frac{1}{2}\frac{(3^{10} - 1)}{2}$$

$$= \frac{1}{4}[3^{10} - 1]$$

$$= 14762$$

29. Here, $a = 2, r = 2$ and $n = 10$

Since, $$S_n = \frac{a(r^n - 1)}{r - 1}$$

We have, $$S_{10} = \frac{2(2^{10} - 1)}{2 - 1}$$

$$= 2(1024 - 1)$$

$$= 2 \times 1023$$

$$= 2046$$

Hence, the number of ancestors preceding the person is 2046.

30. Here, $a = 3, r = 3, S_n = 120$

$$\because \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad [\because r > 1]$$

$$\Rightarrow \quad 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$\Rightarrow \quad 120 = \frac{3(3^n - 1)}{2}$$

$$\Rightarrow \quad 120 \times 2 = 3(3^n - 1)$$

$$\Rightarrow \quad \frac{240}{3} = 3^n - 1$$

$$\Rightarrow \quad 3^n - 1 = 80$$

$$\Rightarrow \quad 3^n = 80 + 1$$

$$\Rightarrow \quad 3^n = 81$$

$$\Rightarrow \quad 3^n = 3^4$$

On comparing the power of 3 both sides, we get

$$n = 4$$

31. Common ratio i.e $r = \frac{-4}{5}$

and $$|r| = \left|\frac{-4}{5}\right| = \frac{4}{5} < 1$$

Given, Sum of this in finite GP is $S = \frac{80}{9}$

Now, Let a be the first term of the given infinte GP.

$$\therefore \quad S = \frac{a}{1 - r}$$

$$\Rightarrow \quad \frac{80}{9} = \frac{a}{\left(1 + \frac{4}{5}\right)}$$

$$\Rightarrow \quad \frac{80}{9} = \frac{5a}{9}$$

$$\Rightarrow \quad a = 16$$

32. $3^{1/2} \times 3^{1/4} \times 3^{1/8} \ldots$

$$= 3^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots}$$

$$= 3^{\frac{1}{2}\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\ldots\right)}$$

$$= 3^{\frac{1}{2}\times\left(\frac{1}{1-\frac{1}{2}}\right)} \qquad \left[\because a = 1 \text{ and } r = \frac{1}{2}\right]$$

$$= 3^{\frac{1}{2}\times\frac{2}{1}} = 3$$

33. The geometric mean of 2 and 8 is $\sqrt{16}$ i.e 4.

34. Let G_1, G_2, G_3 and G_4 be the required GM's.
Then, 3, G_1, G_2, G_3, G_4, 96 are in GP.
Let r be the common ratio. Here, 96 is the 6th term.

$\therefore \quad 96 = ar^{6-1} = 3r^5$

$\Rightarrow \quad 32 = r^5 \Rightarrow (2)^5 = r^5 \Rightarrow r = 2$

$\therefore \quad G_1 = ar = 3 \cdot 2 = 6$

$G_2 = ar^2 = 3 \cdot 2^2 = 12$

$G_3 = ar^3 = 3 \cdot 2^3 = 24$

and $\quad G_4 = ar^4 = 3 \cdot 2^4 = 48$

35. We know that, $\quad$ AM $\geq$ GM

$\Rightarrow \quad \dfrac{4^x + 4^{1-x}}{2} \geq \sqrt{4^x \cdot 4^{1-x}}$

$\Rightarrow \quad 4^x + 4^{1-x} \geq 2\sqrt{4}$

$\Rightarrow \quad 4^x + 4^{1-x} \geq 2 \cdot 2$

$\Rightarrow \quad 4^x + 4^{1-x} \geq 4$

36. We have, $a_n = 2n^2 - n + 1$

Assertion Putting $n = 1$, we get

$a_1 = 2(1)^2 - 1 + 1 = 2 - 1 + 1 = 2$

Putting $n = 2$, we get

$a_2 = 2(2)^2 - 2 + 1 = 8 - 2 + 1 = 7$

Reason Putting $n = 3$, we get

$a_3 = 2(3)^2 - 3 + 1 = 18 - 3 + 1 = 16$

Putting $n = 4$, we get

$a_4 = 2(4)^2 - 4 + 1 = 32 - 4 + 1 = 29$

Hence, Assertion and Reason both are true but Reason is not the correct explanation of Assertion.

37. **Assertion** We have, $a_n = \dfrac{n^2}{2^n}$

Putting $n = 7$, $\qquad a_7 = \dfrac{7^2}{2^7} = \dfrac{49}{128}$

Reason We have, $a_n = \dfrac{n(n-2)}{n+3}$

Putting $n = 20$, $\qquad a_{20} = \dfrac{20(20-2)}{20+3}$

$\Rightarrow \qquad a_{20} = \dfrac{20 \times 18}{23}$

$= \dfrac{360}{23}$

Hence, Assertion is true and Reason is false.

38. **Assertion** We have, $a_n = \dfrac{n(n^2+5)}{4}$

Now, we need to find x which is second term of the sequence, so put $n = 2$ in a_n.

$\therefore \quad a_2 = \dfrac{2(4+5)}{4} = \dfrac{18}{4} = \dfrac{9}{2}$

Reason We have, $a_n = (-1)^{n-1} 5^{n+1}$

$\therefore \quad a_3 = (-1)^{3-1} 5^4 = 625$

Hence, Assertion is true and Reason is false.

39. **Assertion** $a_n = 4n - 3$

Then, $\quad a_{17} = 4(17) - 3 = 65$

and $\quad a_{24} = 4(24) - 3 = 93$

Reason $\quad a_n = (-1)^{n-1} \cdot n^3$

Then, $\quad a_9 = (-1)^{9-1} \cdot (9)^3$

$= (-1)^8 \cdot 729$

$= 729$

Hence, Assertion and Reason both are true but Reason is not the correct explanation of Assertion.

40. **Assertion** It is given that

$a_1 = 2 = 2 \times 1$

$a_2 = 4 = 2 \times 2$

$a_3 = 6 = 2 \times 3$

$a_4 = 8 = 2 \times 4$

$\vdots$

Hence, the nth term of this sequence is $a_n = 2n$, where $n \in N$.

Reason It is given that

$$a_1 = 1 = 2 - 1$$
$$a_2 = 3 = 2 \times 2 - 1$$
$$a_3 = 5 = 2 \times 3 - 1$$
$$a_4 = 7 = 2 \times 4 - 1$$
$$\vdots$$

Hence, the nth term of this sequence is $a_n = 2n - 1$, where $n \in N$.

Hence, Assertion and Reason both are true but Reason is not the correct explanation of Assertion.

41. Assertion Let a be the first term and r be the common ratio of the given GP.

According to the question,

$$T_4 = (T_2)^2 \text{ and } a = -3$$

$\because \quad T_4 = (T_2)^2$

$\therefore \quad ar^3 = (ar)^2$

$\Rightarrow \quad -3r^3 = (-3)^2 r^2 \qquad [\because a = -3]$

$\Rightarrow \quad r = -3$

Now, $T_7 = ar^6 = -3(-3)^6 = -3 \times 729 = -2187$

Reason Given AP is 6, 8, 10, …

$\therefore a = 6, \; d = 8 - 6 = 2$

$\therefore \quad S_{10} = \frac{10}{2}[2 \times 6 + (10 - 1) \times 2]$

$= 5[12 + 18]$

$= 5 \times 30 = 150$

Hence, Assertion is false and Reason is true.

42. Assertion Let a be the first term and r be the common ratio of the given GP.

According to the question,

$T_5 = 48 \Rightarrow ar^4 = 48 \qquad \text{…(i)}$

and $\quad T_8 = 384 \Rightarrow ar^7 = 384 \qquad \text{…(ii)}$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{ar^7}{ar^4} = \frac{384}{48}$$

$\Rightarrow \quad r^3 = 8$

$\Rightarrow \quad r = 2$

Reason 18, x, 14 are in AP.

$\Rightarrow \quad x - 18 = 14 - x$

$\Rightarrow \quad 2x = 32$

$\Rightarrow \quad x = 16$

Hence, Assertion and Reason both are true but Reason is not the correct explanation of Assertion.

43. Assertion Given AP is 4, 8, 12, …

$\therefore \; a = 4, d = 8 - 4 = 4$

Now, $S_{20} = \frac{20}{2}[2 \times 4 + (20 - 1) \times 4]$

$= 10[8 + 76]$

$= 840$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

44. Assertion

Let $S_n = \frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} + \sqrt{5} + \ldots$ 25th terms

$\Rightarrow \quad S_n = \frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} + \frac{5}{\sqrt{5}} + \ldots$ 25th terms

Clearly, the successive difference of the terms is same. So, RHS of the above series forms an AP, with first term, $a = \frac{3}{\sqrt{5}}$ and common difference, $d = \frac{4}{\sqrt{5}} - \frac{3}{\sqrt{5}} = \frac{1}{\sqrt{5}}$.

$\therefore \quad S_{25} = \frac{25}{2}\left[2 \times \frac{3}{\sqrt{5}} + (25 - 1)\frac{1}{\sqrt{5}}\right]$

$= 25\left[\frac{3}{\sqrt{5}} + \frac{12}{\sqrt{5}}\right]$

$= 25 \times \frac{15}{5} \times \sqrt{5} = 75\sqrt{5}$

Reason Given, 27, x, 3 are in GP.

$\therefore \quad \frac{x}{27} = \frac{3}{x}$

$\Rightarrow \quad x^2 = 81 \Rightarrow x = \pm 9$

Hence, Assertion is true and Reason is false.

45. Assertion Given AP is 16, 11, 6, …

Here, $a = 16, d = 11 - 16 = -5$

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{23} = \frac{23}{2}[2 \times 16 + (23 - 1)(-5)]$

$= \frac{23}{2}[32 + (22)(-5)] = \frac{23}{2}[32 - 110]$

$= \frac{23}{2}[-78] = -897$

Reason Given AP is $x + y, x - y, x - 3y, \ldots$

Here, $\quad a = x + y$

$d = (x - y) - (x + y) = -2y$

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)\, d]$

$$\therefore \quad S_{22} = \frac{22}{2}[2\times(x+y)+(22-1)(-2y)]$$
$$= 11[2x+2y+(21)(-2y)]$$
$$= 11[2x+2y-42y]$$
$$= 11[2x-40y]$$
$$= 22[x-20y]$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

46. **Assertion** If $-\frac{2}{7}, k, -\frac{7}{2}$ are in GP.

Then, $\frac{a_2}{a_1} = \frac{a_3}{a_2}$

$$\left[\because \text{common ratio } (r) = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots\right]$$

$$\therefore \quad \frac{k}{-2/7} = \frac{-7/2}{k}$$
$$\Rightarrow \quad \frac{7}{-2}k = \frac{-7}{2}\times\frac{1}{k}$$
$$\Rightarrow \quad 7k\times 2k = -7\times(-2)$$
$$\Rightarrow \quad 14k^2 = 14$$
$$\Rightarrow \quad k^2 = 1 \Rightarrow k = \pm 1$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

47. **Assertion** Given GP 4, 16, 64, …

$\therefore\ a = 4, r = \frac{16}{4} = 4 > 1$

$$\therefore\ S_6 = \frac{4((4)^6-1)}{4-1} = \frac{4(4095)}{3} = 5460$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

48. **Assertion** Let a be the first term and $r\,(|r|<1)$ be the common ratio of the GP.

$\therefore$ The GP is $a, ar, ar^2, \dots$

According to the question,

$$T_1 + T_2 = 5 \Rightarrow a + ar = 5 \Rightarrow a(1+r) = 5$$

and $\quad T_n = 3(T_{n+1} + T_{n+2} + T_{n+3} + \dots)$

$$\Rightarrow \quad ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + \dots)$$
$$\Rightarrow \quad ar^{n-1} = 3ar^n(1 + r + r^2 + \dots)$$
$$\Rightarrow \quad 1 = 3r\left(\frac{1}{1-r}\right)$$
$$\Rightarrow \quad 1 - r = 3r$$
$$\Rightarrow \quad r = \frac{1}{4}$$

Reason Given, 3, 6, 9, 12 …

Here, $a = 3, d = 6 - 3 = 3$

$$\therefore \quad T_{10} = a + (10-1)d$$
$$= 3 + 9\times 3$$
$$= 3 + 27 = 30$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

49. **Assertion** Let

$S = 0.6 + 0.66 + 0.666 + \dots$ upto n terms

$S = 6(0.1 + 0.11 + 0.111 + \dots$ upto n retms)

$$= \frac{6}{9}(0.9 + 0.99 + 0.999 + \dots \text{ upto } n \text{ terms})$$
$$= \frac{2}{3}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \text{ upto } n \text{ terms}\right]$$
$$= \frac{2}{3}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + \left(1-\frac{1}{1000}\right) + \dots \text{ upto } n \text{ terms}\right]$$
$$= \frac{2}{3}\left[(1+1+1+\dots \text{ upto } n \text{ terms}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{ upto } n \text{ terms}\right)\right]$$
$$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left\{1-\left(\frac{1}{10}\right)^n\right\}}{1-\frac{1}{10}}\right]$$

$$\left[\because \text{sum of GP} = \frac{a(1-r^n)}{1-r},\ r<1\right]$$

$$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left\{1-\left(\frac{1}{10}\right)^n\right\}}{\frac{9}{10}}\right] = \frac{2}{3}\left[n - \frac{1}{9}\left\{1-\left(\frac{1}{10}\right)^n\right\}\right]$$

Hence, Assertion and Reason both are true but Reason is not the correct explanation of Assertion.

50. **Assertion**

Let $S = 5 + 55 + 555 + \dots$ upto n terms

$$= 5(1 + 11 + 111 + \dots \text{ upto } n \text{ terms})$$
$$= \frac{5}{9}(9 + 99 + 999 + \dots \text{ upto } n \text{ terms})$$

$$= \frac{5}{9}[(10-1)+(100-1)+(1000-1) + \ldots \text{ upto } n \text{ terms}]$$

$$= \frac{5}{9}[(10+100+1000+\ldots \text{ upto } n \text{ terms} - (1+1+1+\ldots \text{ upto } n \text{ terms})]$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$\left[\because \text{ sum of GP } = \frac{a(r^n-1)}{r-1}, r>1 \text{ and } \Sigma 1 = n\right]$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{9} - n\right]$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

51. (i) Since, it is given that, production increases uniformly by a constant number, hence number of production every year forms an AP.

$\therefore a_3 = 500 \Rightarrow a + 2d = 500$ ...(i)

$a_7 = 600 \Rightarrow a + 6d = 600$ ...(ii)

Now, subtracting Eq. (i) from Eq. (ii), we get

$$4d = 100 \Rightarrow d = 25$$

(ii) Put $d = 25$ in Eq. (i), we get

$$a + 50 = 500 \Rightarrow a = 450$$

(iii) The total production in 10 years = S_{10}

$$\therefore S_{10} = \frac{10}{2}[2\times 450 + 9 \times 25]$$

$$= 5[900+225] = 5625$$

(iv) The number of computers produced in 21st year = a_{21}

$$\therefore a_{21} = 450 + 20 \times 25 = 450 + 500 = 950$$

(v) $a_{10} - a_8 = (a+9d)-(a+7d)$

$$= 2d = 2 \times 25 = 50$$

52. (i) We have, $a_n = 2n^2 + 1$

On replacing n by $n+1$, we get

$$a_{n+1} = 2(n+1)^2 + 1 = 2(n^2+1+2n)+1$$

$$= 2n^2 + 2 + 4n + 1 = 2n^2 + 4n + 3$$

Here, $a_{n+1} - a_n = (2n^2+4n+3)-(2n^2+1)$

$$= 2n^2 + 4n + 3 - 2n^2 - 1$$

$$= 4n + 2$$

It is not independent of n. So, given sequence is not an AP.

(ii) Given, AP 11, 18, 25, ...

Here, $a = 11$, $d = 18 - 11 = 7$

$\therefore \quad a_{11} = 11 + 10 \times 7 = 11 + 70 = 81$

(iii) Given, $S_n = 3n + 2n^2$

First term of an AP,

$$\therefore \quad T_1 = 3 \times 1 + 2(1)^2$$

$$= 3 + 2 = 5$$

and $T_2 = S_2 - S_1$

$$= [3\times 2 + 2\times(2)^2] - [3\times 1 + 2 \times (1)^2]$$

$$= [6+8]-[3+2]$$

$$= 14 - 5 = 9$$

$\therefore$ Common difference $(d) = T_2 - T_1$

$$= 9 - 5 = 4$$

(iv) According to the question,

$$9 \cdot T_9 = 13 \cdot T_{13}$$

$\Rightarrow 9(a+8d) = 13(a+12d)$

$\Rightarrow 9a + 72d = 13a + 156d$

$\Rightarrow (9a - 13a) = 156d - 72d$

$\Rightarrow -4a = 84d$

$\Rightarrow a = -21d$

$\Rightarrow a + 21d = 0$ ...(i)

$\therefore$ 22nd term i.e. $T_{22} = [a + 21d]$

$T_{22} = 0$ [using Eq. (i)]

(v) Let first term be a and common difference be d.

Then, $S_n = \frac{n}{2}[2a + (n-1)d]$ ...(i)

$\therefore \quad S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$

$S_{2n} = n[2a + (2n-1)d]$ ...(ii)

$S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$ ...(iii)

According to the question,

$$S_{2n} = 3S_n$$

$\Rightarrow n[2a + (2n-1)d] = 3\frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 4a + (4n-2)d = 6a + (3n-3)d$

$\Rightarrow -2a + (4n - 2 - 3n + 3)d = 0$

$\Rightarrow -2a + (n+1)d = 0$

$\Rightarrow d = \frac{2a}{n+1}$ ...(iv)

Now, $\dfrac{S_{3n}}{S_n} = \dfrac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]}$

$$= \dfrac{6a + (9n-3)\dfrac{2a}{n+1}}{2a + (n-1)\dfrac{2a}{n+1}}$$

$$= \dfrac{6an + 6a + 18an - 6a}{2an + 2a + 2an - 2a} = \dfrac{24\,an}{4\,an}$$

$\Rightarrow \quad \dfrac{S_{3n}}{S_n} = 6$

53. (i) The number of pots in each row is 2, 4, 8, …. .

$\therefore$ This forms a geometric progression,

where $a = 2,\ r = \dfrac{4}{2} = 2$

Hence, the constant multiple by which the number of pots is increasing in every row is 2.

(ii) Number of pots in 8th row $= a_8$

$$a_8 = ar^{8-1} = 2(2)^7 = 2^8 = 256$$

(iii) Number of pots in 7th row,

$$a_7 = 2(2)^{7-1} = 2\cdot 2^6 = 2^7 = 128$$

Number of pots in 5th row,

$$a_5 = 2(2)^{5-1} = 2\cdot 2^4 = 2^5 = 32$$

$\therefore$ Required answer $= 128 - 32 = 96$

(iv) Total number of pots upto 10th row $= S_{10}$

$$\therefore\ S_{10} = \frac{a(r^{10}-1)}{r-1} = \frac{2(2^{10}-1)}{2-1}$$

$$= \frac{2(1024-1)}{1} = 2046$$

(v) Let there be n number of rows.

$\therefore \quad S_n = 510 = \dfrac{2(2^n-1)}{2-1}$

$\Rightarrow \quad \dfrac{510}{2} = 2^n - 1$

$\Rightarrow \quad 255 = 2^n - 1$

$\Rightarrow \quad 256 = 2^n \Rightarrow 2^8 = 2^n \Rightarrow n = 8$

54. Let A_1, A_2, A_3, A_4 be the vertices of the first square with each side equal to 10 cm. Let B_1, B_2, B_3, B_4 be the mid-point of its side.

Then, $\quad B_1B_2 = \sqrt{A_2B_1^2 + A_2B_2^2}$

$$= \sqrt{5^2 + 5^2} = \sqrt{25+25} = 5\sqrt{2}$$

$\therefore \quad C_1B_2 = B_2C_2 = \dfrac{5\sqrt{2}}{2}$

Similarly, $C_1C_2 = \sqrt{B_2C_2^2 + B_2C_1^2}$

$$= \sqrt{\left(\frac{5\sqrt{2}}{2}\right)^2 + \left(\frac{5\sqrt{2}}{2}\right)^2}$$

$$= \sqrt{\frac{25}{2} + \frac{25}{2}} = \sqrt{25} = 5 \text{ cm}$$

Similarly, the side of fourth square is $\dfrac{5}{\sqrt{2}}$ cm.

$\therefore$ Sides of each square are

$10, 5\sqrt{2}, 5, \dfrac{5}{\sqrt{2}}, \dfrac{5}{2}, \dfrac{5}{2\sqrt{2}}, \dfrac{5}{4}, \ldots\ldots\ldots$ respectively

which form a GP with $a = 10$ and $r = \dfrac{1}{\sqrt{2}}$.

(i) Side of fourth square $= ar^3 = 10\left(\dfrac{1}{\sqrt{2}}\right)^3$

$$= \frac{10}{2\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ cm}$$

(ii) Side of fifth square $= ar^4 = 10\left(\dfrac{1}{\sqrt{2}}\right)^4$

$$= \frac{10}{4} = \frac{5}{2}$$

$\therefore$ Area of fifth square $= \left(\dfrac{5}{2}\right)^2 = \dfrac{25}{4}\text{cm}^2$

(iii) The side of 7th square $= ar^6 = 10\left(\dfrac{1}{\sqrt{2}}\right)^6$

$$= \frac{10}{8} = \frac{5}{4}$$

$\therefore$ Perimeter of 7th square $= \dfrac{5}{4} \times 4 = 5$ cm

(iv) Sum of areas of all square formed is

$$10^2 + (5\sqrt{2})^2 + (5)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 \ldots.$$

$$= 100 + 50 + 25 + \frac{25}{2} + \ldots\ldots$$

Here, $a = 100,\ r = \dfrac{50}{100} = \dfrac{1}{2}$, which is an infinite GP.

$$= \frac{a}{1-r} = \frac{100}{1-\frac{1}{2}} = 200 \text{ cm}^2$$

(v) Sum of perimeter of all square is

$$4\left(10 + 5\sqrt{2} + 5 + \frac{5}{\sqrt{2}} \dots\right)$$

$$= 4 \times \frac{10}{1 - \frac{1}{\sqrt{2}}} = \frac{40\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{40\sqrt{2}(\sqrt{2}+1)}{(2-1)} = 80 + 40\sqrt{2}$$

55. (i) Side of first triangle is 24.

Side of second triangle is $\frac{24}{2} = 12$

Similarly, side of second triangle is $\frac{12}{2} = 6$

$\therefore \quad a = 24, r = \frac{12}{24} = \frac{1}{2}$

$\therefore$ Side of the fifth triangle,

$$a_5 = ar^4 = 24 \times \left(\frac{1}{2}\right)^4$$

$$= \frac{24}{16} = \frac{3}{2} = 1.5 \text{ cm}$$

(ii) Perimeter of first triangle $= 24 \times 3 = 72$

Perimeter of second triangle $= \frac{72}{2} = 36$

Similarly, perimeter of third triangle $= \frac{36}{2}$

$= 18$

$\therefore \quad a = 72, \; r = \frac{36}{72} = \frac{1}{2}$

$\therefore$ Sum of perimeter of first 6 triangle

$$= S_6 = \frac{a(1 - r^6)}{1 - r}$$

$$= \frac{72\left(1 - \left(\frac{1}{2}\right)^6\right)}{1 - \frac{1}{2}} = \frac{72 \times 63 \times 2}{2^6}$$

$$= \frac{567}{4} \text{ cm}$$

(iii) Area of first triangle is $\frac{\sqrt{3}}{4}(24)^2$

Area of second triangle $= \frac{\sqrt{3}}{4}\left(\frac{24}{2}\right)^2$

$$= \frac{\sqrt{3}}{4}(24)^2 \times \frac{1}{4}$$

$\therefore \quad a = \frac{\sqrt{3}}{4}(24)^2, \; r = \frac{1}{4}$

$\therefore$ Sum of area of all triangles

$$= \frac{a}{1-r} = \frac{\sqrt{3}}{4} \frac{(24)^2}{1 - \frac{1}{4}}$$

$$= \frac{\sqrt{3} \times (24)^2}{3} = 192\sqrt{3} \text{ cm}^2$$

(iv) The sum of perimeter of all triangle

$3(24 + 12 + 6 + \dots)$ is

$$3\left(\frac{24}{1 - \frac{1}{2}}\right) = 144 \text{ cm} \qquad \left[\because a = 24, \; r = \frac{1}{2}\right]$$

(v) Here, $a = 72, r = \frac{1}{2}$

$$a_7 = (72)\left(\frac{1}{2}\right)^6$$

$$= \frac{72}{64} = \frac{9}{8} \text{ cm}$$