Chapter - 1
Real Numbers

Multiple Choice Questions

Q: 1 **Which of the following is an irrational number?**

1 $\sqrt{5}$
2 3.14159265359
3 $\frac{\sqrt{4}}{3}$
4 $0.\overline{23517}$

Q: 2 **Which of the following is an irrational number?**

1 $5\sqrt{4}$
2 $\frac{\sqrt{2}}{\sqrt{8}}$
3 $6 + \sqrt{5}$
4 $\sqrt{64} - \sqrt{4}$

Q: 3 **$\frac{63}{p}$ has a terminating decimal expansion.**

Which of these CANNOT be a factor of *p* ?

1 2
2 5
3 13
4 20

Q: 4 **Which of the following have a terminating decimal expansion?**

(Note: You need not evaluate the decimals.)

1 $\frac{1}{3}$
2 $\frac{1}{60}$
3 $\frac{1}{90}$
4 $\frac{1}{625}$

Q: 5 **Which of these is the HCF of 1260 and 1680?**

1 210
2 420
3 630
4 5040

Q: 6 **Which of these is the LCM of 720 and 900?**

1 180
2 1800
3 3600
4 648000

Q: 7 Which of the following is the rationalised form of $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}$?

1 5
2 $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}}$
3 $\sqrt{15} - \sqrt{2}$
4 $\sqrt{15} - \sqrt{10}$

Q: 8 **Which of the following fractions has a terminating decimal expansion?**

1 $\frac{33}{343}$
2 $\frac{19}{49}$
3 $\frac{71}{99}$
4 $\frac{237}{625}$

Q: 9 **Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).**

Assertion (A): Product of HCF and LCM of THREE numbers is equal to the product of those numbers.

Reason (R): Product of HCF and LCM of TWO numbers is equal to the product of those numbers.

1. Both (A) and (R) are true and (R) is the correct explanation for (A).
2. Both (A) and (R) are true and (R) is not the correct explanation for (A).
3. (A) is false but (R) is true.
4. Both (A) and (R) are false.

Free Response Questions

Q: 10 **The prime factorisation of a natural number p is $(5 \times 7 \times t)$ where $t \neq 2, 3$.** **[1]**

What is the prime factorisation of 42 p^2?

Q: 11 **$\sqrt{4} + \sqrt{5}$ is a rational number.** **[1]**

Write true or false and justify your answer.

Q: 12 **Ramesh has two rectangular fields of the same length but different widths. He wants to plant 76 trees in the smaller field and 190 trees in the larger field. In both fields, the trees will be planted in the same number of columns but in different numbers of rows.** **[1]**

What is the most number of columns that can be planted in this arrangement? Show your work.

Q: 13 **Use Euclid's Division Algorithm to find the HCF of 175, 225 and 465. Show your work.** **[2]**

Q: 14 **Given that $\sqrt{3}$ is irrational, show by contradiction that the sum of $\sqrt{3}$ and 2 is irrational. Show your steps.** **[2]**

Q: 15 **M and N are positive integers such that M = $p^5 q^3 r^2$ and N = $p^7 q^5 r$, where p, q, r are prime numbers.** **[2]**
Find LCM(M, N) and HCF(M, N).

Q: 16 **$\sqrt{5}$ is an irrational number. Meera was asked to prove that $(3 + \sqrt{5})$ is an irrational number.** **[2]**

Shown below are the steps of Meera's proof:

Step 1	Let $(3 + \sqrt{5})$ be a rational number. Then $(3 + \sqrt{5})$ can be written as $\frac{p}{q}$, where p and q ($q \neq 0$) are co-primes.
Step 2	Hence, $\sqrt{5} = (\frac{p}{q} - 3)$.
Step 3	Since p and q are integers, $(\frac{p}{q} - 3)$ is also an integer.
Step 4	Since $(\frac{p}{q} - 3)$ is an integer and every integer is a rational number, $(\frac{p}{q} - 3)$ is a rational number. It implies that $\sqrt{5}$ is a rational number.
Step 5	But this contradicts the fact that $\sqrt{5}$ is an irrational number. Hence, $(3 + \sqrt{5})$ is an irrational number.

She made an error in one step due to which her subsequent steps were incorrect too.

In which step did she make that error? Justify your answer.

Q: 17 **Ajay has a box of length 3.2 m, breadth 2.4 m, and height 1.6 m.** **[2]**
What is the length of the longest ruler that can exactly measure the three dimensions of the box? Show your steps and give valid reasons.

Q: 18 ***m* is a positive integer. HCF of *m* and 450 is 25. HCF of *m* and 490 is 35.** **[2]**

Find the HCF of *m*, 450 and 490. Show your steps.

Q: 19 **Prove that $\sqrt{7}$ is irrational.** **[3]**

Q: 20 **Prove that $\frac{1}{\sqrt{2}}$ is irrational.** **[3]**

Q: 21 **Show that any positive even integer is of the form $(8m)$, $(8m + 2)$, $(8m + 4)$ or $(8m + 6)$, for some positive integer *m*. Show your work.** **[3]**

Q: 22 **Write two rational numbers each between the following pair:** **[3]**

i) $\sqrt{3}$ and $\sqrt{10}$
ii) 7 and $\sqrt{64}$
iii) $\sqrt{15}$ and 6

Q: 23 **The number 3837425721 is divided by a number between 5621 and 5912.** **[3]**

State true or false for the below statements about the remainder and justify your answer.

i) The remainder can be more than 5912.
ii) The remainder cannot be less than 5621.
iii) The remainder is always between 5621 and 5912.

Q: 24 **On the two real numbers $a = 2 + \sqrt{5}$ and $b = 3 - \sqrt{7}$, perform the following operations:** **[5]**

i) Calculate the sum ($a + b$).
ii) Calculate the product (ab).
iii) Find the additive inverse of a.
iv) Rationalise $\frac{1}{b}$.
v) Verify whether the numbers a and b are rational or irrational. Provide a valid reason for your answer.

Q: 25 **i) Find the LCM and HCF of 78, 91, and 195.** **[5]**
ii) Check whether LCM(a, b, c) × HCF(a, b, c) = $a \times b \times c$ where a, b and c are natural numbers.

Show your work.

Case Study

Answer the questions based on the given information.

For the screening of an informational documentary, three schools were selected by the district administration.

Name of the school	No. of students
C.A.V. Public School	78
Bal Vidya Bhawan	117
Bombay Public School	130

♦ During the screening, multiple rooms are used simultaneously, and each room can accommodate an equal number of students.
♦ All students in a particular room belong to the same school.
♦ As a token of appreciation, the district administration has provided an equal number of chocolates to each school.
♦ When distributing these chocolates, each school distributes chocolates equally among its students, ensuring fairness and consistency.

Q: 26 **Find the maximum number of students that can be seated in one room. Show your work.** **[2]**

Q: 27 **What is the minimum number of rooms required? Show your work.** **[2]**

Q: 28 **What is the minimum number of chocolates provided to each school? Show your work.** **[1]**

Answer key

Q.No	Correct Answers
1	1
2	3
3	3
4	4
5	2
6	3
7	4
8	4
9	3

Q.No	What to look for	Marks
10	Writes the prime factorisation of 42 p^2 as ($2 \times 3 \times 5^2 \times 7^3 \times t^2$).	1
11	Writes False.	0.5
	Justifies the answer. For example, states that $\sqrt{5}$ is irrational as it is the square root of a prime number and sum of a rational and irrational is irrational.	0.5
12	Identifies that the number of columns for the two fields must be HCF of 76 & 190, and applies an appropriate method to find the HCF as 38.	1
13	Finds the HCF of 175, 225 and 465 using Euclid's Division Algorithm as follows: $225 = 175 \times 1 + 50$ $175 = 50 \times 3 + 25$ $50 = 25 \times 2 + 0$ Finds the HCF of 175 and 225 as 25.	1
	$465 = 25 \times 18 + 15$ $25 = 15 \times 1 + 10$ $15 = 10 \times 1 + 5$ $10 = 5 \times 2 + 0$ Finds the HCF of 465 and 25 as 5. Concludes that the HCF of 175, 225 and 465 is 5.	1
14	Assumes that $(2 + \sqrt{3})$ is rational and writes $2 + \sqrt{3} = \frac{p}{q}$, where p and q are co-prime integers and $q \neq 0$.	0.5
	Simplifies the above as $\frac{p}{q} - 2 = \sqrt{3}$.	0.5
	Writes that since p and q ($q \neq 0$) are integers and 2 is a rational, $(\frac{p}{q} - 2)$ is also rational.	0.5
	Writes that since $\sqrt{3}$ is irrational, hence proves by contradiction that the sum of $\sqrt{3}$ and 2 is irrational.	0.5

Q.No	What to look for	Marks
15	Finds LCM(M, N) as $p^7 q^5 r^2$.	1
	Finds HCF(M, N) as $p^5 q^3 r$.	1
16	Identifies that Meera makes an error in step 3.	1
	Writes that if *p* and *q* are integers, ($\frac{p}{q}$ - 3) cannot be an integer since *p* and *q* are co-primes.	1
17	Identifies and reasons that the length of the longest ruler should be equal to the HCF of the three lengths.	0.5
	Finds the HCF of the three numbers as Prime factorization of 32 = 2^5 Prime factorization of 24 = 3×2^3 Prime factorization of 16 = 2^4 Highest Common factor, HCF = 2^3 Mentions the length of the longest ruler as 80 cm or 0.8 m. (Award 0.5 marks if the length is correct but the unit is incorrect).	1.5
18	Writes that the HCF of *m*, 450 and 490 is nothing but the HCF of 25 and 35 and finds the same as: 35 = (25 × 1) + 10 25 = (10 × 2) + 5 10 = (5 × 2) + 0.	1
	Concludes that HCF of *m*, 450 and 490 is 5.	1
19	Assumes $\sqrt{7} = \frac{a}{b}$ where $b \neq 0$, *a* and *b* are co-primes.	0.5
	Writes $b\sqrt{7} = a$ and squares both the sides to get $7b^2 = a^2$.	0.25
	Concludes that *a* is divisible by 7 as a^2 is divisible by 7 because 7 is a prime number.	0.5

Q.No	What to look for	Marks
	Writes $a = 7c$ and squares both the sides to get $a^2 = 49c^2$.	0.25
	Replaces a^2 with $7b^2$ from step 2 to get $7b^2 = 49c^2$ and solves it to get $b^2 = 7c^2$.	0.5
	Concludes that b is divisible by 7 as b^2 is divisible by 7 because 7 is a prime number.	0.5
	Mentions that 7 divides both a and b which contradicts the assumption that a and b are both co-prime and hence $\sqrt{7}$ is irrational.	0.5
20	Assumes $\frac{1}{\sqrt{2}} = \frac{a}{b}$ where $b \neq 0$, a and b are co-primes.	0.5
	Writes $b = a\sqrt{2}$ and squares both the sides to get $b^2 = 2a^2$.	0.25
	Concludes that b is divisible by 2 as b^2 is divisible by 2 because 2 is a prime number.	0.5
	Writes $b = 2c$ and squares to get $b^2 = 4c^2$.	0.25
	Replaces b^2 with $2a^2$ from step 2 to get $2a^2 = 4c^2$ and solves it to get $a^2 = 2c^2$.	0.5
	Concludes that a is divisible by 2 as a^2 is divisible by 2 because 2 is a prime number.	0.5
	Mentions that 2 divides both a and b which contradicts the assumption that a and b are both co-primes and hence $\frac{1}{\sqrt{2}}$ is irrational.	0.5
21	Writes Euclid's Division Lemma for $a = bm + n$, $0 \leq n < b$, where a is a positive integer and substitutes $b = 8$ to get $a = 8m + n$, $0 \leq n < 8$.	0.5
	Mentions that the possible values of n for $a = 8m + n$ are 0, 1, 2, 3, 4, 5, 6, 7.	1
	Writes that a can be $(8m)$, $(8m + 1)$, $(8m + 2)$, $(8m + 3)$, $(8m + 4)$, $(8m + 5)$, $(8m + 6)$ or $(8m + 7)$ where m is the quotient.	0.5

Q.No	What to look for	Marks
	Writes that out of the above expressions only (8 *m*), (8 *m* + 2), (8 *m* + 4) and (8 *m* + 6) are even and concludes that any positive even integer is of the form (8 *m*), (8 *m* + 2), (8 *m* + 4) or (8 *m* + 6).	**1**
22	**i) Writes any 2 rational numbers between √3 and √10. For example, 2 and 2.1.**	**1**
	ii) Writes any 2 rational numbers between 7 and √64. For example, 7.22 and 7.5.	**1**
	iii) Writes any 2 rational numbers between √15 and 6. For example, 4 and 5.	**1**
23	**i) Writes false and justifies the answer. For example, writes that Euclid's Division Lemma states that the remainder is always less than the divisor and all the divisors are less than 5912.**	**1**
	ii) Writes false and justifies the answer. For example, the remainder is always less than the divisor and the numbers from 0 to the divisor are all possible remainders.	**1**
	iii) Writes false and justifies the answer. For example, writes that Euclid's Division Lemma states that the remainder always lies between 0 and the divisor.	**1**
24	**i) Calculates the sum correctly as 5 + √5 - √7.**	**1**
	ii) Calculates the product correctly as 6 - 2√7 + 3√5 - √35.	**1**
	iii) Calculates the additive inverse of *a* correctly as (-2 - √5).	**1**
	iv) Calculates the rationalised form of $\frac{1}{b}$ correctly as $\frac{(3+\sqrt{7})}{2}$.	**1**
	v) Verifies both *a* and *b* are irrational because they are the sum of rational and irrational numbers.	**1**

Q.No	What to look for	Marks
25	**i) Finds the LCM and HCF of 78, 91, and 195 as 2730 and 13 respectively. The working may look as follows:** **Prime factorization of:** $78 = 2^1 \times 3^1 \times 13^1$ $91 = 7^1 \times 13^1$ $195 = 3^1 \times 5^1 \times 13^1$ **LCM = 2 x 3 x 5 x 7 x 13 = 2730** **HCF = 13**	3
	ii) Considers a, b and c as 78, 91 and 195 respectively. **Finds LCM(a, b, c) × HCF(a, b, c) as 2730 × 13 = 35,490.**	1
	Finds the product of a, b, and c as 78 × 91 × 195 = 13,84,110. **Concludes that LCM(a, b, c) × HCF(a, b, c) $\neq a \times b \times c$.**	1
26	**Identifies that to find the required number, HCF of 78, 117, and 130 is needed and finds the HCF of 78, 117, and 130 as:** **Prime factorization of 78, 117, and 130-** $78 = 2^1 \times 3^1 \times 13^1$ $117 = 3^2 \times 13^1$ $130 = 2^1 \times 5^1 \times 13^1$ **Concludes that the maximum number of students to be seated in a room = HCF(78, 117, 130) = 13.**	2
27	**Finds the total number of students as 78 + 117 + 130 = 325.**	1
	Divides the total number of students by 13 to obtain the minimum number of rooms required as 25.	1
28	**Identifies that LCM of 78, 117, and 130 is the minimum number of chocolates received by each school and uses the prime factorization used earlier to find the LCM of 78, 117, and 130 as:** **LCM = 2 × 3 × 3 × 5 × 13 = 1170.** ***(Note: Award full marks if the student performs prime factorization.)***	1