# 5

# Adjoint and Inverse of a Matrix

## BASIC CONCEPTS

1. **Adjoint of a Matrix:** If $A = [a_{ij}]$ is a square matrix of order $n$ and $C_{ij}$ denote the cofactor of $a_{ij}$ in $A$, then the transpose of the matrix of cofactors of elements of $A$ is called the adjoint of $A$ and is denoted by adj $A$.

   *i.e.*, adj $A = [C_{ij}]^T$

   If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then adj $A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$

2. The adjoint of a square matrix of order 2 can be obtained by interchanging the diagonal elements and changing the signs of off-diagonal elements.

   If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

3. If $A$ is a square matrix of order $n$, then $A$ (adj $A$) $= |A| I_n =$ (adj $A$) $A$.

4. **Following are some properties of adjoint of a square matrix:**

   If $A$ and $B$ are square matrices of the same order $n$, then

   (*i*) adj $(AB) =$ (adj $B$) (adj $A$)

   (*ii*) adj $A^T = (\text{adj } A)^T$

   (*iii*) adj (adj $A$) $= |A|^{n-2} A$

   (*iv*) $|\text{adj } A| = |A|^{n-1}$

5. **Invertible Matrix:** A square matrix $A$ of order $n$ is invertible, if there exists a square matrix $B$ of the same order such that $AB = I_n = BA$.

   In such a case, we say that the inverse of matrix $A$ is $B$ and we write $A^{-1} = B$.

   Following are some properties of inverse of a matrix:

   (*i*) Every invertible matrix possesses a unique inverse.

   (*ii*) If $A$ is an invertible matrix, then $(A^{-1})^{-1} = A$.

   (*iii*) A square matrix is invertible, if it is non-singular.

   (*iv*) If $A$ is a non-singular matrix, then $A^{-1} = \frac{1}{|A|}$ (adj $A$).

   (*v*) If $A$ and $B$ are two invertible matrices of the same order, then $(AB)^{-1} = B^{-1} A^{-1}$.

   (*vi*) If $A$ is an invertible matrix, then $(A^T)^{-1} = (A^{-1})^T$.

   (*vii*) The inverse of an invertible symmetric matrix is a symmetric matrix.

   (*viii*) If $A$ is a non-singular matrix, then $|A^{-1}| = \frac{1}{|A|}$.

6. **Elementary Operation of Matrix:** The following are three operations applied on the rows (columns) of a matrix.

   (*i*) Interchange of any two rows (columns) $R_i \leftrightarrow R_j$ $(C_i \leftrightarrow C_j)$.

   (*ii*) Multiplying all elements of a row (column) of a matrix by a non-zero scalar $R_i \to kR_i$ $(C_i \to kC_j)$.

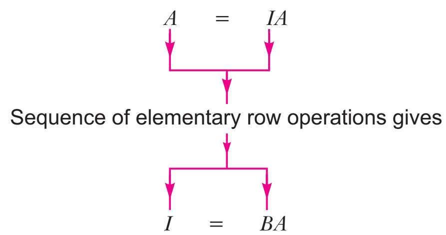
   (*iii*) Adding to the elements of a row (column), the corresponding elements of another row (column) multiplied by any scalar $R_i \to R_i + kR_j$ $(C_i \to C_i + kC_j)$.

7. A matrix obtained from an identity matrix by a single elementary operation is called an elementary matrix.

8. **Elementary Row Operations:** To find the inverse of an invertible matrix '$A$' by elementary row operations, at first we make an equation as

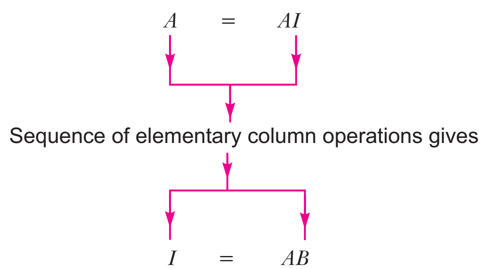
$$A = IA \qquad \ldots(i)$$

On this equation (*i*) we apply sequence of row operations successively on '$A$' on the LHS and on '$I$' on the RHS till we get $I = BA$ like as




$B$ is called inverse of $A$ *i.e.*, $A^{-1} = B$

**Elementary Column Operations:** To find the inverse of $A$ by using elementary column operations we write an equation as




$B$ is called inverse of $A$ *i.e.*, $A^{-1} = B$

9. **System of Linear Equations:** A system of $n$ simultaneous linear equations in $n$ unknowns $x_1, x_2, x_3, \ldots, x_n$ is

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \ldots \quad \ldots \quad \ldots \quad &\ldots \quad \ldots \\ \ldots \quad \ldots \quad \ldots \quad &\ldots \quad \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= b_n \end{aligned}$$

This system of equations can be written in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

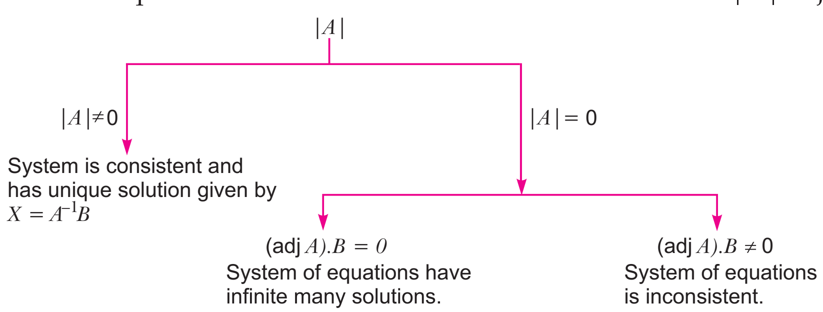
or, $\quad AX = B$

where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

10. A set of values of the variable $x_1, x_2, ..., x_n$ satisfying all the equations simultaneously is called a solution of the system.
11. If a system of equations has one or more solutions, then it is said to be a consistent system of equations, otherwise it is an inconsistent system of equations.
12. A system of equations $AX = B$ is called a homogeneous system, if $B = O$. Otherwise, it is called a non-homogeneous system of equations.
13. System of linear equations may or may not be consistent, if consistent may or may not have unique solution.

    It can be, decided by following flow chart:

    At first system of linear equations is written in matrix form as $AX = B$. Then $|A|$.adj $A$ is obtained




14. A homogeneous system of $n$ linear equations in $n$ unknowns is expressible in the form $AX = O$.

    If $|A| \neq 0$, then $AX = O$ has unique solution $X = 0$ *i.e.*, $x_1 = x_2 = ... = x_n = 0$. This solution is called the trivial solution.

    If $|A| = 0$, then $AX = O$ has infinitely many solutions.

    Thus, a homogeneous system of equation is always consistent.
15. If $A$ is a square matrix of order $n$, then $|\text{adj } A| = |A|^{n-1}$.

## Selected NCERT Questions

**1. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.**

**Sol.** $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$|AB| = 67 \times 61 - 47 \times 87 = 4087 - 4089 = -2$

$\therefore$ $AB$ is invertible.

Cofactors of elements of determinant $AB$ are,

$$AB_{11} = 61,\ AB_{12} = -47,\ AB_{21} = -87,\ AB_{22} = 67$$

$$\text{Adj}(AB) = \begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix}^T = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|}\text{Adj}(AB) = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1$$

Cofactors of elements of $A$ are

$$A_{11} = 5,\ A_{12} = -2,\ A_{21} = -7,\ A_{22} = 3$$

$$\text{Adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}^T = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2$$

Cofactors of elements of $B$ are

$$B_{11} = 9,\ B_{12} = -7,\ B_{21} = -8,\ B_{22} = 6$$

$$\text{Adj } B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}^T = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\text{Adj } A = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -9 & 8 \\ 7 & -6 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} -9 & 8 \\ 7 & -6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -45-16 & 63+24 \\ 35+12 & -49-18 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$$

Thus, $(AB)^{-1} = B^{-1}A^{-1}$.

**2. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find $A^{-1}$.** **[*HOTS*]**

**Sol.** $$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} . \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \qquad [\because\ A^2 = A.A]$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} . \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \qquad [\because\ A^3 = A.A^2]$$

$$= \begin{bmatrix} 12+5+5 & -10-6-5 & 10+5+6 \\ -6-10-5 & 5+12+5 & -5-10-6 \\ 6+5+10 & -5-6-10 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now, $$A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \qquad \text{(zero matrix)}$$

$$\therefore\quad A^3 - 6A^2 + 9A - 4I = 0$$

Pre-multiplying by $A^{-1}$, we get

$$A^{-1}A.A^2 - 6A^{-1}.A.A + 9A^{-1}.A - 4A^{-1}I = 0$$

$$\Rightarrow \quad A^2 - 6A + 9I - 4A^{-1} = 0 \qquad \Rightarrow \qquad 4A^{-1} = A^2 - 6A + 9I$$

$$4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$

**3. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find $A^{-1}$. Using $A^{-1}$ solve the system of equations.**

**$2x - 3y + 5z = 11$; $3x + 2y - 4z = -5$; $x + y - 2z = -3$.**

**Sol.** $$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4+4) + 3(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0$$

Therefore $A$ is non-singular matrix and $A^{-1}$ exists.

$C_{11} = 0;\ C_{12} = 2;\ C_{13} = 1;\ C_{21} = -1;\ C_{22} = -9;$

$C_{23} = -5;\ C_{31} = 2;\ C_{32} = 23;\ C_{33} = 13$

$$\text{adj } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system can be expressed as $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Now, $AX = B \Rightarrow X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

On equating, we get $x = 1$, $y = 2$ and $z = 3$.

## Multiple Choice Questions [1 mark]

*Choose and write the correct option in the following questions.*

**1. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is** *[CBSE 2020 (65/3/1)]*

(*a*) 64 (*b*) 16 (*c*) 0 (*d*) – 8

2. **The adjoint of matrix $A = [a_{ij}] = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ is**

(*a*) $\begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$ (*b*) $\begin{bmatrix} s & q \\ r & -p \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 0 \\ 0 & q \end{bmatrix}$ (*d*) None of these

3. **If $A$ is square matrix of order 3 × 3 such that $|A| = 2$, then write the value of $|\text{adj}\ (\text{adj}\ A)|$.**

(*a*) –16 (*b*) 16 (*c*) 0 (*d*) 2

4. **If $A$ is a square matrix of order 3, such that $A(\text{adj}\ A) = 10\ I$, then $|\text{adj}\ A|$ is equal to** *[CBSE 2020 (65/5/1)]*

(*a*) 1 (*b*) 10 (*c*) 100 (*d*) 101

5. **If $B.\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ then matrix $B$ is**

(*a*) $\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$ (*b*) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (*d*) $I$

6. **Let $A$ be a 3 × 3 square matrix such that $A\ (\text{adj}\ A) = 2I$, where $I$ is the identity matrix. The value of $|\text{adj}\ A|$ is**

(*a*) 4 (*b*) –4 (*c*) 0 (*d*) None of these

7. **The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is singular matrix, if the value of $b$ is**

(*a*) –3 (*b*) 3 (*c*) 0 (*d*) Any real number

8. **If $x, y, z$ are non-zero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is**

(*a*) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ (*b*) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$ (*c*) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ (*d*) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9. **If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then $A^{-1}$ exists if** *[NCERT Exemplar]*

(*a*) $\lambda = 2$ (*b*) $\lambda \neq 2$ (*c*) $\lambda \neq -2$ (*d*) None of these

10. **If $A^2 - A + I = 0$, then the inverse of $A$ is**

(*a*) $A^{-2}$ (*b*) $I - A$ (*c*) 0 (*d*) $A$

11. **If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then the value of $A^T A^{-1}$ is**

(*a*) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ (*b*) $\begin{bmatrix} \cos x & \sin x \\ 1 & 0 \end{bmatrix}$ (*c*) $A'$ (*d*) Zero matrix

12. **The value of $(A^{-1})^T$ is**

(*a*) $(A^T)^{-1}$ (*b*) $A^{-1}$ (*c*) $I$ (*d*) $A^T$

13. **If $A$ is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to**

(*a*) $\det A$ (*b*) $\frac{1}{\det A}$ (*c*) 1 (*d*) 0

14. **If $A$ is a singular matrix, then $A(\text{adj } A)$ is**

(*a*) Null matrix (*b*) Scalar matrix (*c*) Identity matrix (*d*) None of these

15. **If $A$ is a matrix of order 3 × 3, then the value of $|3A|$ is**

(*a*) $27|A|$ (*b*) $-27|A|$ (*c*) $9|A|$ (*d*) None of these

16. **The sum of the products of elements of any row with the co-factors of corresponding elements is equal to**

(*a*) cofactors sum (*b*) value of the determinant

(*c*) 0 (*d*) Adjoint of matrix

17. **Using matrix method to solve the following system of equations: $5x - 7y = 2$, $7x - 5y = 3$, value of $x, y$ is**

(*a*) $x = \frac{11}{24}, y = \frac{1}{24}$ (*b*) $x = \frac{-11}{24}, y = \frac{1}{24}$ (*c*) $x = 1, y = 1$ (*d*) $x = \frac{10}{24}, y = \frac{1}{24}$

18. **If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its cofactor will be**

(*a*) 12 (*b*) 144 (*c*) –12 (*d*) 13

19. **If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then $A^{-1}$ exist if**

(*a*) $\lambda = 2$ (*b*) $\lambda \neq 2$ (*c*) $\lambda \neq -2$ (*d*) None of these

20. **If $A$ and $B$ are invertible matrices, then which of the following is not correct.**

(*a*) $\text{adj } A = |A| . A^{-1}$ (*b*) $\det(A^{-1}) = [\det(A)]^{-1}$

(*c*) $(AB)^{-1} = B^{-1}A^{-1}$ (*d*) $(A + B)^{-1} = B^{-1} + A^{-1}$

## Answers

| | | | | | |
|---|---|---|---|---|---|
| **1.** (*a*) | **2.** (*a*) | **3.** (*b*) | **4.** (*c*) | **5.** (*b*) | **6.** (*a*) |
| **7.** (*d*) | **8.** (*a*) | **9.** (*d*) | **10.** (*b*) | **11.** (*a*) | **12.** (*a*) |
| **13.** (*b*) | **14.** (*a*) | **15.** (*a*) | **16.** (*b*) | **17.** (*a*) | **18.** (*b*) |
| **19.** (*d*) | **20.** (*d*) | | | | |

## Solutions of Selected Multiple Choice Questions

1. We have a matrix $A$ of order 3 × 3 given by

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad \Rightarrow \quad |A| = -2((-2) \times (-2) - 0)$$

$$\Rightarrow \quad |A| = -8$$

$$\therefore \quad |\text{adj } A| = |A|^{n-1} = |A|^{3-1} = (-8)^2 = 64$$

3. We have,

$$|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = 2^{(3-1)^2} \qquad \text{(where } n \text{ is order of matrix } A)$$

$$= 2^4 = 16$$

4. We have,

$$A(\text{adj } A) = |A| I$$

$$\Rightarrow \quad 10 I = |A| I \quad \Rightarrow \quad |A| = 10$$

$\therefore \quad |\text{adj } A| = |A|^{n-1} = |A|^{3-1}$ $(\because n = 3)$

$= |A|^2 = (10)^2 = 100$

**5.** If $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$\therefore$ On comparing , we get

$a + b = 6 \quad \Rightarrow \quad a = 6 - b \quad ...(i)$

$-2a + 4b = 0$

Using $(i)$, we get

$-2(6 - b) + 4b = 0$

$\Rightarrow \quad -12 + 2b + 4b = 0 \quad \Rightarrow \quad 6b = 12 \quad \Rightarrow b = 2$

Using in $(i)$, we get $a = 4$

$c + d = 0 \quad \Rightarrow \quad c = -d \quad ...(ii)$

And $-2c + 4d = 6$

$-2(-d) + 4d = 6 \Rightarrow \quad 6d = 6 \quad \Rightarrow d = 1$

Put in $(ii)$, we get

$c = -1$

$\therefore \quad B = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

**13.** $AA^{-1} = I$

$\Rightarrow \quad |AA^{-1}| = |I| \quad \Rightarrow \quad |A|.|A^{-1}| = 1$

$\Rightarrow \quad |A^{-1}| = \dfrac{1}{|A|} \qquad [\because |A| \neq 0]$

$= \dfrac{1}{|A|}$

**14.** $A\,(\text{adj } A) = |A|\, I$ (Using properties of adjoint of matrix)

As it is singular

$\therefore \quad |A| = 0 \quad \Rightarrow \quad 0 \times I = 0$

So, it is null matrix.

## Fill in the Blanks [1 mark]

**1.** If $A$ is a matrix of order $3 \times 3$, then $(A^2)^{-1} =$ ______________ .

**2.** The value of $k$ for which the system of linear equations:

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$ has a unique solution, is ______________ .

**3.** If $A$ is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, then $(A')^{-1} =$ ______________ .

**4.** If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, then $|\text{adj (adj } A)| =$ ______________ .

## Answers

**1.** $(A^{-1})^2$ **2.** $k \neq 0$ **3.** $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ **4.** 16

## Solutions of Selected Fill in the Blanks

**4.** We have,

$$|A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 2$$

$$\therefore \quad |\text{adj}(\text{adj } A)| = |A|^{(n-1)^2} = |A|^{(3-1)^2} = |A|^4$$

$$= (2)^4 = 16$$

## Very Short Answer Questions [1 mark]

**1. Find adj $A$, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$** **[CBSE 2020 (65/4/1)]**

**Sol.** We have,

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

$\therefore$ Co-factors of matrix $A$ are

$C_{11} = 3, \; C_{12} = -4$

$C_{21} = 1, \; C_{22} = 2$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$

**2. If $A$ is a square matrix of order 3, with $|A| = 9$, then write the value of $|2 \,.\, adj\ A|$.** **[CBSE 2019 (65/4/1)]**

**Sol.** $\because \quad |2 \,.\, \text{adj } A| = 2^3 |\text{adj } A|$ [$\because$ For any matrix $A$ of order $n$, $|kA| = k^n |A|$, where $k$ is some constant]

$= 8 \; |\text{adj } A| = 8 \; |A|^{3-1}$ [$\because$ $|\text{adj } A| = |A|^{n-1}$, for any square matrix of order $n$]

$= 8 \times (9)^2 = 8 \times 81 = 648$

**3. If $A$ is a square matrix of order 2 and $|A| = 4$, then find the value of $|2AA^T|$, where $A^T$ is transpose of the matrix $A$.** **[CBSE 2019 (65/5/1)]**

**Sol.** We have $|A| = 4$ and $A$ is a matrix of order 2.

$\therefore \quad |2AA^T| = 2^2 |AA^T|$ [$\because$ $A$ is a matrix of order 2]

$= 4|AA^T| = 4|A|\,|A^T|$

For any square matrix $A$, $|A^T| = |A|$

$\therefore \quad |2AA^T| = 4 \times 4 \times 4 = 64$

**4. $A$ is a square matrix with $|A| = 4$. Then find the value of $|A.(\text{adj } A)|$** **[CBSE 2019 (65/4/3)]**

**Sol.** $\because \quad |A.(\text{adj } A)| = |A|\,|I_n| = |A| = 4$ [$\because$ $A.\text{ adj } A = |A| I_n$]

**5. If $A = \begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$ then find $|\text{adj } A|$.** **[CBSE 2019 (65/5/3)]**

**Sol.** We have $A = \begin{bmatrix} 8 & 2 \\ 3 & 2 \end{bmatrix}$

$\therefore \quad |A| = 16 - 6 = 10$

Now, $|\text{adj } A| = |A|^{2-1} = (10)^{2-1} = 10$

**6. If $A$ is a matrix of order $3 \times 3$, then find $(A^2)^{-1}$.** *[NCERT Exemplar]*

**Sol.** $(A^2)^{-1} = (A.A)^{-1}$

$= A^{-1} . A^{-1} = (A^{-1})^2 \qquad [\because (AB)^{-1} = B^{-1} A^{-1}]$

## Short Answer Questions-I [2 marks]

**1. Write $A^{-1}$ for $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.** *[CBSE Delhi 2010, 2011]*

**Sol.** For elementary row operations, we write

$$A = IA \Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . A \qquad [\text{Applying } R_1 \leftrightarrow R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A \qquad [\text{Applying } R_2 \to R_2 - 2R_1]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix} A \qquad [\text{Applying } R_1 \to R_1 + 3R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \qquad [\text{Applying } R_2 \to (-1)R_2]$$

$$\Rightarrow I = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \Rightarrow A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

[**Note :** $B$ is called inverse of $A$, if $AB = BA = I$]

**2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write $A^{-1}$ in terms of $A$.** *[CBSE (AI) 2011]*

**Sol.** $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19 \neq 0$

Now, $C_{11} = -2, C_{12} = -5, C_{21} = -3$ and $C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19} A$$

**3. If $A$ is invertible matrix of $3 \times 3$ and $|A| = 7$, then find $|A^{-1}|$.** *[NCERT Exemplar]*

**Sol.** $\because \quad A^{-1} = \frac{1}{|A|} . \text{adj } A$

$$|A^{-1}| = \left| \frac{1}{|A|} \text{adj } A \right| = \left( \frac{1}{|A|} \right)^3 . |\text{adj } A| \qquad [\because |KA| = K^n . |A|, \text{ where } n \text{ is order of } A]$$

$$= \frac{1}{|A|^3} . |A|^{3-1} = \frac{1}{|A|} = \frac{1}{7}.$$

**4. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then find the value of $\lambda$ for which $A^{-1}$ exists.** *[NCERT Exemplar]*

**Sol.** For existence of $A^{-1}$

$$|A| \neq 0 \quad \Rightarrow \quad \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} \neq 0$$

$\Rightarrow \quad 2(6-5) - \lambda(0-5) + (-3)(0-2) \neq 0$ [Expanding along $R_1$]

$\Rightarrow \quad 2 + 5\lambda + 6 \neq 0 \Rightarrow 5\lambda \neq -8 \quad \Rightarrow \quad \lambda \neq -\frac{8}{5}$

Hence, $\lambda$ can have any value other than $-\frac{8}{5}$.

## Short Answer Questions-II [3 marks]

**1. Using elementary transformations, find the inverse of the matrix**

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

*[CBSE Delhi 2011]*

**Sol.** Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

For finding the inverse, using elementary row operation we write

$$A = IA \quad \Rightarrow \quad \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\Rightarrow \quad \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \qquad \left[\text{Applying } R_1 \to R_1 - \frac{1}{3}R_2\right]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -7/9 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/3 & 1/9 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \qquad \left[\text{Applying } R_2 \to \frac{1}{9}R_2\right]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 1/3 \\ 0 & 1 & -7/9 \\ 0 & 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/3 & 1/9 & 0 \\ -1/3 & 5/9 & 1 \end{bmatrix} A \qquad [\text{Applying } R_3 \to R_3 + 5R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/9 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -1/3 & 5/9 & 1 \end{bmatrix} A \qquad [\text{Applying } R_1 \to R_1 - 3R_3, R_2 \to R_2 + 7R_3]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A \qquad [\text{Applying } R_3 \to 9R_3]$$

$$\Rightarrow \qquad I = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$

**2. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.** **[*CBSE Delhi 2015*]**

**Sol.** Given $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$|A'| = 1(-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$$

Hence, $(A')^{-1}$ will exist.

$$A_{11} = \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix} = -1 - 8 = -9; \qquad A_{12} = -\begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix} = -(-2-6) = 8$$

$$A_{13} = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = -8 + 3 = -5; \qquad A_{21} = -\begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix} = -(0+8) = -8$$

$$A_{22} = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 1 + 6 = 7; \qquad A_{23} = -\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = -(4-0) = -4$$

$$A_{31} = \begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix} = 0 - 2 = -2; \qquad A_{32} = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2-4) = 2$$

$$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -1 - 0 = -1$$

$$\text{adj}(A') = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$(A')^{-1} = \frac{1}{1}\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

**3. Find $x, y$ and $z$ if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.** **[*NCERT Exemplar*]**

**Sol.** We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, $A' = A^{-1}$

$$\Rightarrow \qquad AA' = AA^{-1} \qquad [\because AA^{-1} = I]$$

$$\Rightarrow \qquad \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By comparing the corresponding elements, we get

$\Rightarrow \quad 2y^2 - z^2 = 0 \quad \Rightarrow \quad 2y^2 = z^2$

$\Rightarrow \quad 4y^2 + z^2 = 1$

$\Rightarrow \quad 2.z^2 + z^2 = 1 \quad \Rightarrow \quad z = \pm\frac{1}{\sqrt{3}}$

$\therefore \quad y^2 = \frac{z^2}{2} \quad \Rightarrow \quad y = \pm\frac{1}{\sqrt{6}}$

Also, $\quad x^2 + y^2 + z^2 = 1$

$\Rightarrow \quad x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3} = 1 - \frac{3}{6} = \frac{1}{2} \quad \Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}$

$\therefore \quad x = \pm\frac{1}{\sqrt{2}},\ y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$

**4. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less. Using matrix method, find the number of children and the amount distributed by Seema.** **[*CBSE (East) 2016*]**

**Sol.** Let the number of children be $x$ and the amount donated by Seema to each child be ₹ $y$.

$\therefore$ According to question,

$(x-8)(y+10) = xy$ and $(x+16)(y-10) = xy$

$\Rightarrow \quad xy + 10x - 8y - 80 = xy$ and $xy - 10x + 16y - 160 = xy$

$\Rightarrow \quad 5x - 4y = 40 \qquad ...(i)$

and $\quad 5x - 8y = -80 \qquad ...(ii)$

Equation (*i*) and (*ii*) may be written in matrix form as

$AX = B \quad \Rightarrow X = A^{-1}B \qquad ...(iii)$

Where $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}$, $B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Now, $\quad |A| = -40 + 20 = -20$

$\text{Adj } A = \begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$

$\therefore \quad A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$

Putting value of $A^{-1}$ in (*iii*), we get

$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -320 & -320 \\ -200 & -400 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -640 \\ -600 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$

$\Rightarrow \quad x = 32,\ y = 30$

$\Rightarrow$ Number of children = 32 and amount donated to each child = ₹ 30.

**5. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust.** *[CBSE (Central) 2016]*

**Sol.** Let ₹ $x$ and ₹ $y$ be the amount of money invested in 1st and 2nd bond respectively.

According to information given in question

$$x \times \frac{10}{100} + y \times \frac{12}{100} = 2800$$

$$x \times \frac{12}{100} + y \times \frac{10}{100} = 2700$$

$\Rightarrow \quad 10x + 12y = 280000 \qquad \ldots(i)$

and $\quad 12x + 10y = 270000 \qquad \ldots(ii)$

Above system of equation can be written in matrix form as

$AX = B$, where

$$A = \begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \end{bmatrix}, \; B = \begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$$

$AX = B \quad \Rightarrow \quad X = A^{-1}B \qquad \ldots(iii)$

Now, $\quad |A| = \begin{vmatrix} 10 & 12 \\ 12 & 10 \end{vmatrix} = 100 - 144 = -44$

$$A^{-1} = \frac{1}{|A|}.\text{adj } A = \frac{1}{|A|}.\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}^T = \frac{1}{-44}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}$$

Putting the value of $X$, $A^{-1}$ and $B$ in (*iii*), we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-44}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}\begin{bmatrix} 280000 \\ 270000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{44}\begin{bmatrix} 2800000 - 3240000 \\ -3360000 + 2700000 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{44}\begin{bmatrix} -440000 \\ -660000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$$

$\Rightarrow \quad x = 10000, \; y = 15000.$

Hence, invested amount in 1st and 2nd bonds are ₹10000 and ₹15000.

## Long Answer Questions [5 marks]

**1. Using elementary row transformations, find the inverse of the matrix**

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

*[CBSE 2019 (65/4/1)]*

**Sol.** $$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

We know that $A = IA$

*i.e.*, $$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}A$$

Applying $R_1 \to R_1 - R_3$, we have

$$\Rightarrow \begin{bmatrix} 1 & -4 & 7 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Apply $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - R_1$, we have

$$\Rightarrow \begin{bmatrix} 1 & -4 & 7 \\ 0 & 14 & -25 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

Apply $R_2 \to -(R_2 - 3R_2)$, we have

$$\Rightarrow \begin{bmatrix} 1 & -4 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

Apply $R_1 \to R_1 + 4R_2$ and $R_3 \to R_3 - 5R_2$, we have

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 11 \\ 0 & -1 & 3 \\ -1 & 5 & -13 \end{bmatrix} A$$

Apply $R_1 \to R_1 + R_3$ and $R_2 \to R_2 + 2R_3$, we have

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} A$$

$$\therefore \qquad A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

**2. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.** **[*CBSE (F) 2012*]**

**Sol.** For $B^{-1}$

$$|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3-0) - 2(-1-0) - 2(2-0) = 3 + 2 - 4 = 1 \neq 0$$

*i.e.*, $B$ is invertible matrix $\Rightarrow$ $B^{-1}$ exists and have unique solution.

Now, $C_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3 - 0 = 3$; $C_{12} = (-1)^{1+2}\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$

$C_{13} = (-1)^{1+3}\begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2$; $C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -(2-4) = 2$

$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$; $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -(-2-0) = 2$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = 0 + 6 = 6$; $C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = -(0-2) = 2$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 3 + 2 = 5$

$$\therefore \qquad \text{adj}\,B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\Rightarrow \qquad B^{-1} = \frac{1}{|B|}(\text{adj}\, B) = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1}.A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} . \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

**3. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj $A$ and verify that $A(\text{adj}\, A) = (\text{adj}\, A)A = |A|\, I_3$. [*CBSE (F) 2016*]**

**Sol.** Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because \quad A_{11} = \begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos\alpha; \quad A_{12} = -\begin{vmatrix} \sin\alpha & 0 \\ 0 & 1 \end{vmatrix} = -\sin\alpha; \; A_{13} = 0$

$A_{21} = -\begin{vmatrix} -\sin\alpha & 0 \\ 0 & 1 \end{vmatrix} = \sin\alpha; \quad A_{22} = \begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos\alpha; \quad A_{23} = -\begin{vmatrix} \cos\alpha & \sin\alpha \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = \begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & 0 \end{vmatrix} = 0; \quad A_{32} = -\begin{vmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \end{vmatrix} = 0; \quad A_{33} = \begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = 1$

$$\therefore \quad \text{adj} A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now $A.\,\text{adj}\, A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha.\sin\alpha - \sin\alpha.\cos\alpha & 0 \\ \sin\alpha.\cos\alpha - \sin\alpha.\cos\alpha & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A.\text{adj}\, A = |A| I_3 \qquad \ldots(i) \qquad \left[\because \; |A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1\right]$

Again, $\text{adj}\, A.A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\sin\alpha.\cos\alpha + \sin\alpha.\cos\alpha & 0 \\ -\sin\alpha.\cos\alpha + \sin\alpha.\cos\alpha & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \qquad \text{adj}\, A.A = |A|\; I_3 \qquad \ldots(ii)$

From (*i*) and (*ii*), we get

$A\, .\, \text{adj}\, A = \text{adj}\, A\, .\, A = |A|\, I_3.$

**4. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:**

**$x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 2$** **[*CBSE (F) 2011*]**

**OR**

**Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations**

**$x + 3z = 9,\ -x + 2y - 2z = 4,\ 2x - 3y + 4z = -3.$** **[*CBSE Delhi 2017*]**

**Sol.** Given system of equations are

$$x - y + 2z = 1,\quad 2y - 3z = 1,\quad 3x - 2y + 4z = 2$$

Above system of equations can be written in matrix form

as $\quad AX = B \quad \Rightarrow X = A^{-1} B$

where, $\quad A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Let $C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now, $\quad AC = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad AC = I \quad \Rightarrow \quad A^{-1}(AC) = A^{-1} I$ [Pre-multiply by $A^{-1}$]

$\Rightarrow \quad (A^{-1} A)C = A^{-1}$ [By Associativity]

$\Rightarrow \quad IC = A^{-1} \Rightarrow A^{-1} = C$

$\Rightarrow \quad A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Putting $X$, $A^{-1}$ and $B$ in $X = A^{-1}B$, we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \quad \Rightarrow \quad x = 0, y = 5 \text{ and } z = 3$$

**OR**

Solution is similar as above. Ans.: $x = 0$, $y = 5$ and $z = 3$

**5. Using matrices, solve the following system of equations:**

**$4x + 3y + 2z = 60;\ x + 2y + 3z = 45;\ 6x + 2y + 3z = 70$** **[*CBSE (AI) 2011*]**

**Sol.** The system can be written as $\quad AX = B \quad \Rightarrow \quad X = A^{-1}B \quad$ ...(*i*)

where $A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$

$$|A| = 4(6-6) - 3(3-18) + 2(2-12) = 0 + 45 - 20 \neq 0$$

For adj $A$

$A_{11} = 6 - 6 = 0$ $\quad$ $A_{21} = -(9-4) = -5$ $\quad$ $A_{31} = (9-4) = 5$

$A_{12} = -(3-18) = 15$ $\quad$ $A_{22} = (12-12) = 0$ $\quad$ $A_{32} = -(12-2) = -10$

$A_{13} = (2-12) = -10$ $\quad$ $A_{23} = -(8-18) = 10$ $\quad$ $A_{33} = (8-3) = 5$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 0 & 15 & -10 \\ -5 & 0 & 10 \\ 5 & -10 & 5 \end{bmatrix}^T = \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix} = \frac{5}{25}\begin{bmatrix} 0 & -1 & 1 \\ 3 & 0 & -2 \\ -2 & 2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 0 & -1 & 1 \\ 3 & 0 & -2 \\ -2 & 2 & 1 \end{bmatrix}$$

Now putting values in (*i*), we get

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 0 & -1 & 1 \\ 3 & 0 & -2 \\ -2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 0 - 45 + 70 \\ 180 + 0 - 140 \\ -120 + 90 + 70 \end{bmatrix} \qquad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 25 \\ 40 \\ 40 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Hence, $x = 5,\ y = 8,\ z = 8$.

**6. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$** **[*CBSE (AI) 2017*]**

**Sol.**

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & 4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I$$

$$\Rightarrow \quad \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = I \qquad \Rightarrow \quad \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Given system of equation can be written in matrix form as

$$AX = B \Rightarrow X = A^{-1}B, \qquad \ldots(i)$$

where, $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

We have $A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

Now from (*i*)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \quad \Rightarrow \quad x = 3, y = -2, z = -1$$

**7. A shopkeeper has 3 varieties of pens '*A*', '*B*' and '*C*'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of '*A*' variety, 3 pens of 'B' variety and 2 pens of '*C*' variety for ₹ 60. While Shikha purchased 6 pens of '*A*' variety, 2 pens of '*B*' variety and 3 pens of '*C*' variety for ₹ 70. Using matrix method, find cost of each variety of pen. [*CBSE (Central) 2016*]**

**Sol.** Let the cost of varieties of pens *A*, *B* and *C* be ₹ $x$, ₹ $y$, and ₹ $z$ respectively.

From question

$$x + y + z = 21, \quad 4x + 3y + 2z = 60, \quad 6x + 2y + 3z = 70$$

The given system of linear equation in matrix equation is as follows

$$AX = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\because \quad AX = B \quad \Rightarrow X = A^{-1}B \qquad \ldots(i)$$

Now $$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4) - 1(12-12) + 1(8-18)) = 5 - 0 - 10 = -5 \neq 0$$

$$A_{11} = (9-4) = 5 \qquad A_{21} = -(3-2) = -1 \qquad A_{31} = (2-3) = -1$$

$$A_{12} = -(12-12) = 0 \qquad A_{22} = (3-6) = -3 \qquad A_{32} = -(2-4) = 2$$

$$A_{13} = (8-18) = -10 \qquad A_{23} = -(2-6) = 4 \qquad A_{33} = (3-4) = -1$$

$$\therefore \quad \text{Adj } A = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\text{adj} A = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Now from ($i$) $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5, y = 8, z = 8$$

$\Rightarrow$ Cost of pen $A$ = ₹ 5; cost of pen $B$ = ₹ 8 and cost of pen $C$ = ₹ 8

**8. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find $A^{-1}$ and hence solve the following system of equations:**

**$3x + 4y + 7z = 14, 2x - y + 3z = 4, x + 2y - 3z = 0$**

**Sol.** Given $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$

$$|A| = 3(3-6) + (-2)(-12-14) + 1(12+7) = 62 \neq 0$$

Hence, $A^{-1}$ exists. Let $C_{ij}$ be the cofactor of $a_{ij}$ ($ij^{\text{th}}$ element of $A$)

Now, $C_{11} = (3-6) = -3 \qquad C_{21} = -(-6-3) = 9 \qquad C_{31} = (4+1) = 5$

$C_{12} = -(-12-14) = 26 \qquad C_{22} = -9-7 = -16 \qquad C_{32} = -(6-4) = -2$

$C_{13} = (12 + 7) = 19 \qquad C_{23} = -(9 - 14) = 5 \qquad C_{33} = (-3 - 8) = -11$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}' = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

The given system of equations can be written in matrix form as

$$A'X = B, \quad \text{where } A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \; B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad X = (A')^{-1}B \quad \Rightarrow X = (A^{-1})'B \qquad [\because (A')^{-1} = (A^{-1})']$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}'\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow \quad x = 1, y = 1, z = 1$

**9. A mixture is to be made of three foods *A, B, C*. The three foods *A, B, C* contain nutrients *P, Q, R* as shown below:** **[*HOTS*]**

| Food | Grams per kg of nutrient | | |
|---|---|---|---|
| | ***P*** | ***Q*** | ***R*** |
| ***A*** | **1** | **2** | **5** |
| ***B*** | **3** | **1** | **1** |
| ***C*** | **4** | **2** | **1** |

**How to form a mixture which will have 8 grams of *P*, 5 grams of *Q* and 7 grams of *R*?**

**Sol.** Let food needed be $x$ kg of $A$, $y$ kg of $B$ and $z$ kg of $C$. Therefore $x$ kg of $A$ contains 1 gram of nutrient $P$. So, $x$ kg of $A$ will contain $x$ grams of nutrient $P$. Similarly, the amount of nutrient $P$ in $y$ kg of food $B$ and $z$ kg of food $C$ are $3y$ and $4z$ grams respectively. So, total quantity of nutrient $P$ in $x$ kg of food $A$, $y$ kg of food $B$ and $z$ kg of food $C$ is $x + 3y + 4z$ grams.

$x + 3y + 4z = 8$

Similarly, $2x + y + 2z = 5$ [For $Q$]

and $5x + y + z = 7$ [For $R$]

The above system of simultaneous linear equations can be written in matrix form as $AX = B$.

$$\text{or,} \quad \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

$$\text{Now,} \quad |A| = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{vmatrix} = 1(1-2) - 3(2-10) + 4(2-5)) = -1 + 24 - 12 = 11 \neq 0$$

So, $A^{-1}$ exists and system have unique solution.

Let $C_{ij}$ be the cofactor of $a_{ij}$ in $A = [a_{ij}]$. Then,

$$C_{11} = -1; \quad C_{12} = 8; \quad C_{13} = -3$$
$$C_{21} = 1; \quad C_{22} = -19; \quad C_{23} = 14$$
$$C_{31} = 2; \quad C_{32} = 6; \quad C_{33} = -5$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} -1 & 8 & -3 \\ 1 & -19 & 14 \\ 2 & 6 & -5 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{11}\begin{bmatrix} 1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

Putting value of $X$, $A^{-1}$ and $B$ in $X = A^{-1}B$, we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+5+14 \\ 64-95+42 \\ -24+70-35 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 11 \\ 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow \quad x = 1, y = 1$ and $z = 1$.

Thus, the mixture is formed by mixing 1 kg of each of the food $A$, $B$ and $C$.

## PROFICIENCY EXERCISE

■ **Objective Type Questions:** **[1 mark each]**

**1. Choose and write the correct option in each of the following questions.**

(*i*) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then $k$ equals

(*a*) 19 (*b*) $\frac{1}{19}$ (*c*) –19 (*d*) $\frac{-1}{19}$

(*ii*) If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

(*a*) $a = 1, b = 1$ (*b*) $a = \cos 2\theta, b = \sin 2\theta$

(*c*) $a = \sin 2\theta, b = \cos 2\theta$ (*d*) none of these

(*iii*) If $A$ is an invertible matrix of order 3 and $|A| = 5$, then $|\text{adj } A| =$

(*a*) 5 (*b*) 125 (*c*) 25 (*d*) none of these

(*iv*) If $A$ satisfies the equation $x^3 - 5x^2 + 4x + \lambda = 0$, then $A^{-1}$ exists if

(*a*) $\lambda \neq 1$ (*b*) $\lambda \neq 2$ (*c*) $\lambda \neq -1$ (*d*) all of them

(*v*) For any 2 × 2 matrix, if $A(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ is equal to

(*a*) 20 (*b*) 10 (*c*) 0 (*d*) 100

**2. Fill in the blanks.**

(*i*) If adj.$A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj $B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, then adj $(AB) =$ ______________ .

(*ii*) If $A$ is a non-singular sequence matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, than $(A^T)^{-1} =$ ______________ .

(*iii*) If $A = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}$ then adj $A =$ ______________ .

## ■ Very Short Answer Questions: **[1 mark each]**

**3.** If $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$ and $A$ (adj $A$) = $\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then find the value of $k$.

**4.** $A$ is a square matrix of order 3 and $|A| = 7$. Write the value of $|\text{adj } A|$. *[CBSE (AI) 2010]*

**5.** If $A$ is a square matrix of order 3 such that $|\text{adj } A| = 64$, find $|A|$.

**6.** If $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ then find adj ($AB$).

## ■ Short Answer Questions–I: **[2 marks each]**

**7.** If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$, show that $A^{-1}=\frac{1}{19}A$

**8.** Find the adjoint of the matrix $\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}$

**9.** Solve the following system of equations by matrix method

$3x + y = 19$

$3x - y = 23$

**10.** Using elementary transformations, find the inverse of the following matrix

$\begin{bmatrix}2 & 1\\ 7 & 4\end{bmatrix}$

## ■ Short Answer Questions–II: **[3 marks each]**

**11.** A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately. *[CBSE (North) 2016]*

**12.** If $A$, $B$ are square matrices of the same order, then prove that adj ($AB$) = (*adj* $B$) (*adj* $A$).

**13.** Using elementary transformations, find the inverse of the matrix $\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$. *[CBSE (F) 2011]*

**14.** Let $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}$. Then compute $AB$. Hence, solve the following system of equations :

$2x + y = 4,\ 3x + 2y = 1$ *[CBSE Sample Paper 2016]*

## ■ Long Answer Questions: **[5 marks each]**

**15.** If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ find $A^{-1}$. Use it to solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$ *[CBSE 2018]*

**16.** Using elementary row transformations, find the inverse of the matrix

$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 7\\ -2 & -4 & -5\end{bmatrix}$ *[CBSE 2018]*

17. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$. Find $A^{-1}$. Hence, solve the system of equations

$x + y + z = 6, x + 2z = 7, 3x + y + z = 12$. [CBSE 2019 (65/1/1)]

18. Using matrix, solve the following system of equations:

$2x - 3y + 5z = 11, \quad 3x + 2y - 4z = -5, \quad x + y - 2z = -3$ [CBSE Delhi 2009]

19. Obtain the inverse of the following matrix, using elementary operations:

$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$ [CBSE (AI) 2009]

20. Using elementary operations, find the inverse of the following matrix:

$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ [CBSE Delhi 2011]

21. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I = 0$. [CBSE Delhi 2008]

22. Using elementary transformations, find the inverse of the following matrix:

$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}$ [CBSE (AI) 2008]

23. Using matrices, solve the following system of linear equations:

$3x - 2y + 3z = 8, \quad 2x + y - z = 1, \quad 4x - 3y + 2z = 4$ [CBSE (F) 2009]

24. Using matrices, solve the following system of equations:

$x + 2y + z = 7; \quad x + 3z = 11$ and $2x - 3y = 1$ [CBSE (AI) 2011]

25. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix $X$ such that $A^2 - 5A + 4I + X = 0$.

[CBSE Delhi 2015]

26. Using elementary row operations (transformations), find the inverse of the following matrix:

$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$ [CBSE Ajmer 2015]

27. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence, show that $A.(adj\ A) = |A| I_3$.

[CBSE Allahabad 2015]

28. Find the inverse of matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and hence show that $A^{-1}. A = I$. [CBSE Chennai 2015]

29. Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of linear equations:

$8x + 4y + 3z = 19; \; 2x + y + z = 5$ and $x + 2y + 2z = 7$ [CBSE Delhi 2016]

**30.** Using matrices, solve the following system of equations:

$x - y + z = 4;\ 2x + y - 3z = 0;\ x + y + z = 2$ [CBSE (F) 2012]

**31.** Using elementary row operations, find the inverse of the following matrix:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$ [CBSE (South) 2016]

**32.** Using matrices, solve the following system of equations:

$2x + 3y + 3z = 5,\quad x - 2y + z = -4,\quad 3x - y - 2z = 3$ [CBSE (AI) 2012]

**33.** Using matrices, solve the following system of equations:

$x + y - z = 3;\ 2x + 3y + z = 10;\ 3x - y - 7z = 1$ [CBSE (AI) 2012]

**34.** Solve the following system of equations by matrix method:

$x - y + 2z = 7$

$2x - y + 3z = 12$

$3x + 2y - z = 5$ [CBSE 2020 (65/3/1)]

## Answers

**1.** (*i*) (*b*) (*ii*) (*b*) (*iii*) (*c*) (*iv*) (*d*) (*v*) (*b*)

**2.** (*i*) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$ (*iii*) $\begin{bmatrix} 0 & 3 \\ -2 & 1 \end{bmatrix}$

**3.** 1 **4.** 49 **5.** $\pm 8$ **6.** $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

**9.** $x = 7, y = -2$ **10.** $\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$ **11.** ₹ 10 and ₹ 15 respectively

**13.** $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ **14.** $AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, x = 7, y = -10$

**15.** $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix};\ x = 1, y = 2, z = 3$ **16.** $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

**17.** $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix};\ x = 3, y = 1, z = 2$ **18.** $x = 1,\ y = 2,\ z = 3$

**19.** $\begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$ **20.** $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$ **22.** $A^{-1} = \frac{1}{21}\begin{bmatrix} 2 & 8 & -7 \\ -5 & 1 & 7 \\ 14 & -7 & -7 \end{bmatrix}$

**23.** $x = 1,\ y = 2,\ z = 3$ **24.** $x = 2,\ y = 1,\ z = 3$ **25.** $\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}, X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

**26.** $A^{-1}=\begin{bmatrix}3 & -2 & 1\\ -9 & 6 & -2\\ 5 & -3 & 1\end{bmatrix}$ **27.** adj $A=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$ **28.** $A^{-1}=\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$

**29.** $A^{-1}=\begin{bmatrix}0 & \frac{2}{3} & -\frac{1}{3}\\ 1 & -\frac{13}{3} & \frac{2}{3}\\ -1 & 4 & 0\end{bmatrix}$; $x=1, y=2$ and $z=1$ **30.** $x=2,\ y=-1,\ z=1$

**31.** $A^{-1}=\begin{bmatrix}-7 & -9 & 10\\ -12 & -15 & 17\\ 1 & 1 & -1\end{bmatrix}$ **32.** $x=1,\ y=2,\ z=-1$ **33.** $x=3,\ y=1,\ z=1$

**34.** $x=2,\ y=1$ and $z=3$

## SELF-ASSESSMENT TEST

**Time allowed: 1 hour** **Max. marks: 30**

**1. Choose and write the correct option in the following questions.** **(4 × 1 = 4)**

(*i*) The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12. Using matrices, then the numbers are

(*a*) $x=3, y=1, z=2$ (*b*) $x=-3, y=1, z=2$
(*c*) $x=3, y=-1, z=1$ (*d*) $x=3, y=2, z=5$

(*ii*) The matrix $A$ satisfying the equation $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ is

(*a*) $\begin{bmatrix}4 & 1\\ 1 & 0\end{bmatrix}$ (*b*) $\begin{bmatrix}1 & 4\\ 1 & 0\end{bmatrix}$ (*c*) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ (*d*) zero matrix

(*iii*) If $A=\begin{bmatrix}2 & 1\\ 7 & 5\end{bmatrix}$, then $|A\,.\,\text{adj}\,A|$ is equal to

(*a*) 3 (*b*) 9 (*c*) 27 (*d*) 81

(*iv*) If $A=\begin{bmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{bmatrix}$, then $|\text{adj}\,(\text{adj}A)|$ is equal to

(*a*) 14 (*b*) $14^2$ (*c*) $14^3$ (*d*) $14^4$

**2. Fill in the blanks.** **(2 × 1 = 2)**

(*i*) If $A$ is an invertible matrix of order 3 and $|A|=4$, then $|\text{adj}\,A|$ = ______________ .

(*ii*) If $A$ is an invertible matrix of order $n$ then $|\text{adj}\,(\text{adj}\,A)|$ = ______________ .

■ **Solve the following questions.** **(2 × 1 = 2)**

**3.** Write the adjoint of the following matrix: $\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$ [*CBSE (AI) 2010*]

**4.** If $A=\begin{bmatrix}a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & a\end{bmatrix}$, find the value of $|adj\ A|$.

■ **Solve the following questions.** **(4 × 2 = 8)**

**5.** If $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$, then write $\boldsymbol{A}^{-1}$.

**6.** Find the inverse of the matrix $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$, using elementary transformations.

**7.** If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19}A$

**8.** Solve the following system of equations by matrix method

$3x + y = 19$

$3x - y = 23$

■ **Solve the following questions.** **(3 × 3 = 9)**

**9.** In a survey of 20 richest persons of three residential society *A*, *B*, *C* it is found that in society *A*, 5 believe in honesty, 10 in hard work and 5 in unfair means while in *B*, 5 believe in honesty, 8 in hard work and 7 in unfair means and in *C*, 6 believe in honesty, 8 in hard work and 6 in unfair means. If the per day income of 20 richest persons of society *A*, *B*, *C* are ₹ 32,500, ₹ 30,500, ₹ 31,000 respectively, then find the per day income of each type of people by matrix method.

**10.** A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types.

**11.** Using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and use it to solve the following system of linear equations:

$8x + 4y + 3z = 19;\ 2x + y + z = 5;\ \ x + 2y + 2z = 7$

■ **Solve the following question.** **(1 × 5 = 5)**

**12.** Two Trusts *A* and *B* receive ₹ 70000 and ₹ 55000 respectively from central government to award prize to persons of a district in three fields agriculture, education and social service. Trust ***A*** awarded 10, 5 and 15 persons in the field of agriculture, education and social service respectively while trust ***B*** awarded 15, 10 and 5 persons respectively. If all three prizes together amount to ₹6000, then find the amount of each prize by matrix method.

# Answers

**1.** (*i*) (*a*) (*ii*) (*c*) (*iii*) (*b*) (*iv*) (*d*) **2.** (*i*) 16 (*ii*) $|A|^{(n-1)^2}$

**3.** adj $A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ **4.** $a^6$ **5.** $\begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$ **6.** $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ **8.** $x = 7, y = -2$

**9.** Per day income who believe in honesty, hard work, and unfair means are ₹1500, ₹2000, ₹ 1000 respectively.

**10.** Fee for rich children and poor children are ₹ 1000 and ₹ 200.

**11.** $A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$, $x = 1, y = 2$ and $z = 1$

**12.** Prizes in the field of agriculture, education and social service are ₹ 2000, ₹ 1000 and ₹ 3000 respectively.