Chapter 1

Physics and Measurement

UNITS

Measurement of any physical quantity involves its comparison with a certain basic, reference standard called unit.

Measurement = nu

Here, n is numerical value and u is unit. The numerical value is inversely proportional to the size of unit.

$n \times u$ = constant

$$n \propto \frac{1}{u}$$

DIMENSIONS OF PHYSICAL QUANTITIES

All the physical quantities represented by derived units can be expressed in terms of some combination of seven fundamental quantities. These seven fundamental quantities are called seven dimensions of the physical world. They are denoted with square brackets [ ].

S. No.	Physical quantity	Dimensional formula	Useful result
1.	Absolute permittivity (ε_0)	$M^{-1}L^{-3}T^4A^2$	$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$
2.	Absolute permeability (μ_0)	$MLT^{-2}A^{-2}$	$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi r}$
3.	Resistance R	$MLT^{-2}A^{-2}$	$P = I^2R$
4.	Inductance L	$ML^2T^{-2}A^{-2}$	$U = \frac{1}{2}LI^2$
5.	Capacitance C	$M^{-1}L^{-2}T^4A^2$	$U = \frac{q^2}{2C}$
6.	L/R (time constant)	$[M^0L^0T^1]$	–
7.	RC (time constant)	$[M^0L^0T^1]$	–
8.	$\sqrt{LC}$	$M^0L^0T^1$	–
9.	Stress, Pressure, Energy density, $\frac{1}{2}\varepsilon_0E^2, B^2/2\mu_0$	$ML^{-1}T^{-2}$	–
10.	Heat capacity, Boltzmann constant	$ML^2T^{-2}K^{-1}$	–

DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

Principle of Homogeneity of dimensions : It is based on the simple fact that length can be added to length. It states that in a correct equation, the dimensions of each term added or subtracted must be same. If two quantities are being added or subtracted, they must be of same dimensional formula. Every correct equation must have same dimensions on both sides of the equation.

> ***Note :*** *Although torque and work done by a force have same dimensional formula yet they cannot be added as their nature is different.*

Conversion of units : The numerical value of a physical quantity in a system of units can be changed to another system of units using the equation $n[u]$ = constant *i.e.*, $n_1[u_1] = n_2[u_2]$ where n is the numerical value and u is the unit.

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$ where the dimensional formula of the physical quantity is $[M^aL^bT^c]$.

To find a relation among the physical quantities. If one knows the quantities on which a particular physical quantity depends and guesses that this dependence is of product type, method of dimensions are helpful in deducing their relation.

Suppose we want to find the relation between force, mass and acceleration. Let force depends on mass and acceleration as follows.

$F = Km^ba^c$ when K = dimensionless constant, b and c are powers of mass and acceleration.

According to principle of homogeneity,

$[F] = [K]\ [m]^b\ [a]^c$

$\Rightarrow$ $[MLT^{-2}] = [M^0L^0T^0]\ [M]^b\ [LT^{-2}]^c$

$\Rightarrow$ $[MLT^{-2}] = M^bL^c\,T^{-2c}$

Equating the dimension on both sides we get $1 = b$, $1 = c$, $-2c = -2$.

$\Rightarrow$ $b = 1$ and $c = 1$.

ACCURACY AND PRECISION

Accuracy

The closeness of the measured value to the true value of the physical quantity is known as the accuracy of the measurement.

Precision

It is the measure of the extent to which successive measurements of a physical quantity differ from one another.

Suppose the true value of a measurement is 35.75 and two measured values are 35.73 and 35.725. Here 35.73 is closest to 35.75, so its accuracy is more than 35.725 but 35.725 is more precise than 35.73 because 35.725 is measured upto 3 decimal places.

SIGNIFICANT FIGURES

The number of digits in the measured value about the correctness of which we are sure plus one more digit are called significant figures.

Rules for counting the significant figures

Rule I : All non-zero digits are significant.

Rule II : All zeros occurring between the non zero digits are significant. For example 230089 contains six significant figures.

Rule III : All zeros to the left of non zero digit are not significant. For example 0.0023 contains two significant figures.

Rule IV : If a number ends in zeros that are not to the right of a decimal, the zeros are not significant.

For example, number of significant figures in

1500 (Two)

1.5×10^3 (Two)

1.50×10^3 (Three)

1.500×10^3 (Four)

> ***Note :*** *Length of an object may be represented in many ways say 5 m, 5.0 m, 500 cm, 5.00 m, 5×10^2 cm. Here 5.00 m is most precise as it contains 3 significant figures.*

Rules for Arithmetic Operations with Significant Figures

Rule I : In addition or subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.

Rule II : In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

Rounding Off of Uncertain Digits

Rule I : The preceding digit is raised by 1 if the insignificant digit to be removed is more than 5 and is left unchanged if the later is less than 5.

Rule II : When the insignificant digit to be removed is 5 and the uncertain digit is even, 5 is simply dropped and if it is odd, then the preceding digit is raised by 1.

ERRORS IN MEASUREMENT

1. **Mean Absolute Error :-** If a_1, a_2, a_3, ... a_n are n measurements then

   $a_m = \dfrac{a_1 + a_2 + \ldots\ldots + a_n}{n}$ is taken as the true value of a quantity, if the same is not known.

   $a_1 = a_m - a_1$

   $a_2 = a_m - a_2$

   ....

   $a_n = a_m - a_n$

   Mean absolute error, $\overline{a} = \dfrac{|\ a_1| + |\ a_2| + \ldots\ldots + |\ a_n|}{n}$

   Final result of measurement may be written as :

   $a = a_m \pm \ \overline{a}$

2. **Relative Error or Fractional Error :** It is given by

   $$\frac{\overline{a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \dfrac{\overline{a}}{a_m} \times 100\%$

4. **Combination of Errors :**

   (i) In Sum : If $Z = A + B$, then maximum absolute error in Z is given by, $Z = \ A + \ B$, maximum fractional error in this case $\dfrac{Z}{Z} = \dfrac{A}{A+B} + \dfrac{B}{A+B}$ *i.e.,* when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(ii) **In Difference :** If $Z = A - B$, then maximum absolute error is $Z = \ A + \ B$ and maximum fractional error in this case $\frac{Z}{Z} = \frac{A}{A-B} + \frac{B}{A-B}$.

(iii) **In Product :** If $Z = AB$, then the maximum fractional error, $\frac{Z}{Z} = \frac{A}{A} + \frac{B}{B}$ where $\frac{Z}{Z}$ is known as fractional error.

(iv) **In Division :** If $Z = A/B$, then maximum fractional error is $\frac{Z}{Z} = \frac{A}{A} + \frac{B}{B}$

(v) **In Power :** If $Z = A^n$ then $\frac{Z}{Z} = n\frac{A}{A}$

In more general form if $Z = \frac{A^p B^q}{C^r}$ then the maximum fractional error in Z is

$$\frac{Z}{Z} = p\frac{A}{A} + q\frac{B}{B} + r\frac{C}{C}$$

Applications :

1. For a simple pendulum, $T \propto l^{1/2}$ $\Rightarrow$ $\frac{T}{T} = \frac{1}{2}\frac{l}{l}$

2. For a sphere, surface area and volume are given by

$$A = 4\pi r^2, V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{A}{A} = 2.\frac{r}{r} \text{ and } \frac{V}{V} = 3.\frac{r}{r}$$

3. When two resistors R_1 and R_2 are connected

(a) In series

$$R_s = R_1 + R_2 \quad \Rightarrow \quad R_s = \ R_1 + \ R_2$$

$$\frac{\Delta R_s}{R_s} = \frac{R_1 + \ R_2}{R_1 + R_2}$$

(b) In parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad \frac{R_p}{R_p^2} = \frac{R_1}{R_1^2} + \frac{R_2}{R_2^2}$$

Also, $$\frac{R_p}{R_p} = \frac{R_1}{R_1} + \frac{R_2}{R_2} - \frac{R_1 + \ R_2}{R_1 + R_2}$$

4. If $x = 2a - 3b$ then, $x = 2\ a + 3\ b$

LEAST COUNT OF MEASURING INSTRUMENTS

The smallest measurement that can be taken by an instrument is equal to least count of the instrument.

For example, a meter scale has smallest division 1 mm. This represents the least count (and also the absolute error) in the measurement.

Let a length measured by the meter scale = 56.0 cm

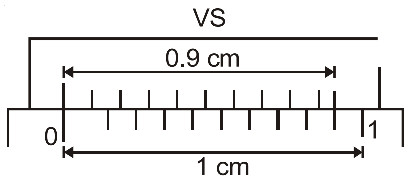
This implies that x = 56.0 cm

Absolute error x = 1 mm = 0.1 cm

Relative error = $\frac{x}{x} = \frac{0.1}{56.0}$

Vernier Callipers

It consists of two scales viz main scale and vernier scale. Vernier scale moves on the main scale. The least count of the instrument is the smallest distance between two consecutive divisions and it is equal to 1 MSD – 1 VSD.



In the figure shown, 1 MSD = 0.1 cm

1 VSD = 0.09 cm

Least count = 1 MSD – 1 VSD = 0.01 cm

Screw Gauge

It contains a main scale and a circular scale. The circular scale is divided into a number of divisions. In other words, the complete rotation of circular scale is divided into a number of parts. The least count of a screw gauge is pitch divided by no. of circular scale divisions.

Least count of spherometer and Screw Gauge = $\frac{\text{Pitch}}{\text{No. of CSD}}$

Total reading of screw gauge = Main scale reading + [(Circular scale reading) × Least count]

Table : SI Units and Dimensions of Some Important Physical Quantities

S.No.	Quantity	SI Unit	Dimensional Formula
1.	Volume	m^3	$[M^0L^3T^0]$
2.	Density	$kg\ m^{-3}$	$[M^1L^{-3}T^0]$
3.	Velocity	ms^{-1}	$[M^0L^1T^{-1}]$
4.	Acceleration	ms^{-2}	$[M^0L^1T^{-2}]$
5.	Angular Velocity	rad s^{-1}	$[M^0L^0T^{-1}]$
6.	Frequency	s^{-1} or hertz (Hz)	$[M^0L^0T^{-1}]$
7.	Momentum	$kg\ ms^{-1}$	$[M^1L^1T^{-1}]$
8.	Force	$kg\ ms^{-2}$ or newton (N)	$[M^1L^1T^{-2}]$
9.	Work, Energy	$kg\ m^2s^{-2}$ or joule (J)	$[M^1L^2T^{-2}]$
10.	Power	$kg\ m^2\ s^{-3}$ or Js^{-1} or watt (W)	$[M^1L^2T^{-3}]$
11.	Pressure, Stress	Nm^{-2} or pascal (Pa)	$[M^1L^{-1}T^{-2}]$
12.	Modulus of Elasticity	Nm^{-2}	$[M^1L^{-1}T^{-2}]$
13.	Moment of Inertia	$kg\ m^2$	$[M^1L^2T^0]$
14.	Torque	Nm	$[M^1L^2T^{-2}]$
15.	Angular Momentum	$kg\ m^2\ s^{-1}$ or J.s	$[M^1L^2T^{-1}]$
16.	Impulse	Ns	$[M^1L^1T^{-1}]$
17.	Coefficient of Viscosity	$kg\ m^{-1}\ s^{-1}$	$[M^1L^{-1}T^{-1}]$
18.	Surface Tension	Nm^{-1}	$[M^1L^0T^{-2}]$

19.	Universal Gravitational Constant	$Nm^{2}\,kg^{-2}$	$[M^{-1}L^{3}T^{-2}]$
20.	Latent Heat	$J\,kg^{-1}$	$[M^{0}L^{2}T^{-2}]$
21.	Specific Heat	$J\,kg^{-1}\,K^{-1}$	$[M^{0}L^{2}T^{-2}K^{-1}]$
22.	Thermal Conductivity	$J\,m^{-1}\,s^{-1}\,K^{-1}$	$[M^{1}L^{1}T^{-3}K^{-1}]$
23.	Electric Charge	Coulomb (C) or A.s	$[M^{0}L^{0}T^{1}A^{1}]$
24.	Electric Potential	JC^{-1} or volt (V)	$[M^{1}L^{2}T^{-3}A^{-1}]$
25.	Electric Resistance	VA^{-1} or ohm ( )	$[M^{1}L^{2}T^{-3}A^{-2}]$
26.	Electric Resistivity	m	$[M^{1}L^{3}T^{-3}A^{-2}]$
27.	Electric Conductance	$^{-1}$ or siemen (S)	$[M^{-1}L^{-2}T^{3}A^{2}]$
28.	Electric Conductivity	$^{-1}\,m^{-1}$ or $S\,m^{-1}$	$[M^{-1}L^{-3}T^{3}A^{2}]$
29.	Capacitance	CV^{-1} or farad (F)	$[M^{-1}L^{-2}T^{4}A^{2}]$
30.	Inductance	$Vs\,A^{-1}$ or henry (H)	$[M^{1}L^{2}T^{-2}A^{-2}]$
31.	Electric field	NC^{-1} or Vm^{-1}	$[M^{1}L^{1}T^{-3}A^{-1}]$
32.	Magnetic Induction	$NA^{-1}\,m^{-1}$ or tesla (T)	$[M^{1}L^{0}T^{-2}A^{-1}]$
33.	Magnetic Flux	Tm^{2} or weber (Wb)	$[M^{1}L^{2}T^{-2}A^{-1}]$
34.	Permittivity	$C^{2}\,N^{-1}\,m^{-2}$	$[M^{-1}L^{-3}T^{4}A^{2}]$
35.	Permeability	$Tm\,A^{-1}$ or $Wb\,A^{-1}\,m^{-1}$	$[M^{1}L^{1}T^{-2}A^{-2}]$
36.	Planck's Constant	Js	$[M^{1}L^{2}T^{-1}]$
37.	Boltzman Constant	JK^{-1}	$[M^{1}L^{2}T^{-2}K^{-1}]$
38.	Stefan's Constant	$W\,m^{-2}K^{-4}$	$[M^{1}L^{0}T^{-3}K^{-4}]$

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