12 Chapter

CONIC SECTIONS - 3 (ELLIPSE & HYPERBOLA)

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. An arc of a bridge is semi-elliptical with major axis horizontal. If the length of the base is 9 meter and the highest part of the bridge is 3 meter from the horizontal; the best approximation of the height of the arc 2 meter from the center of the base is

(a) $\frac{11}{4}\,m$ (b) $\frac{8}{3}\,m$

(c) $\frac{7}{2}\,m$ (d) $2\,m$

2. Locus of all such points so that sum of its distances from (2, – 3) and (2, 5) is always 10, is

(a) $\frac{(x-2)^2}{25}+\frac{(y-1)^2}{9}=1$

(b) $\frac{(x-2)^2}{25}+\frac{(y-1)^2}{16}=1$

(c) $\frac{(x-2)^2}{16}+\frac{(y-1)^2}{25}=1$

(d) $\frac{(x-2)^2}{9}+\frac{(y-1)^2}{25}=1$

3. Coordinates of the vertices B and C of a triangle ABC are (2, 0) and (8, 0) respectively. The vertex A is varying in such a way that $4\tan\frac{B}{2}\tan\frac{C}{2}=1$. Then locus of A is

(a) $\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$ (b) $\frac{(x-5)^2}{16}+\frac{y^2}{25}=1$

(c) $\frac{(x-5)^2}{25}+\frac{y^2}{9}=1$ (d) $\frac{(x-5)^2}{9}+\frac{y^2}{25}=1$

4. If the line $lx+my+n=0$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{2}$, then $\frac{a^2l^2+b^2m^2}{n^2}=$

(a) 1 (b) 2

(c) 4 (d) $\frac{3}{2}$

5. A point on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at a distance equal to the arithmetic mean of the lengths of the semi-major axis and semi-minor axis from the centre is

(a) $\left(\pm\frac{2\sqrt{91}}{7},\pm\frac{3\sqrt{105}}{14}\right)$ (b) $\left(\pm\frac{2\sqrt{91}}{7},\pm\frac{3\sqrt{105}}{7}\right)$

(c) $\left(\pm\frac{2\sqrt{105}}{7},\pm\frac{3\sqrt{91}}{14}\right)$ (d) $\left(\pm\frac{2\sqrt{105}}{14},\pm\frac{3\sqrt{91}}{14}\right)$

6. An ellipse is drawn with major and minor axis of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of the circle being outside the ellipse. Then the radius of the circle is .

(a) 4 (b) 5

(c) 2 (d) None of these.

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ				

7. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, then the eccentricity of the ellipse is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

8. PQ and QR are two focal chords of an ellipse and the eccentric angles of P, Q, R are 2α, 2β, 2γ respectively. Then $\tan\beta\ \tan\gamma$ is equal to

(a) $\cot\alpha$ (b) $\cot^2\alpha$

(c) $2\cot\alpha$ (d) None of these

9. The sum of the squares of the perpendiculars on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from two points on the minor axis each at a distance $\sqrt{a^2-b^2}$ from the centre is :

(a) $2a^2$ (b) $2b^2$

(c) a^2+b^2 (d) a^2-b^2

10. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB (O is the origin) is an equilateral triangle, then the eccentricity 'e' of the hyperbola satisfies

(a) $e>\sqrt{3}$ (b) $1<e<\frac{2}{\sqrt{3}}$

(c) $e=\frac{2}{\sqrt{3}}$ (d) $e>\frac{2}{\sqrt{3}}$

11. From any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$. The area cut off by the chord of contact on the asympotes is equal to

(a) $\frac{ab}{2}$ (b) ab

(c) $2ab$ (d) $4ab$

12. Tangents are drawn to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ at ends of laterarecta. The area of quadrilateral so formed is

(a) 27 (b) $\frac{27}{2}$

(c) $\frac{27}{4}$ (d) $\frac{27}{55}$

13. The locus of the point of intersection of tangents to an ellipse at two points, sum of whose ecentric angles is constant is a/an

(a) Parabola (b) Circle

(c) Ellipse (d) Straight line

14. If CF is perpendicular from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent at P and G is the point where the normal at P meets the major axis, then $CF \times PG =$

(a) a^2 (b) b^2

(c) $a^2 b^2$ (d) a^2+b^2

15. The line $2px+y\sqrt{1-p^2}=1$ ($|p|<1$) for different values of p, touches

(a) An ellipse of eccentricity $\frac{2}{\sqrt{3}}$

(b) An ellipse of eccentricity $\frac{\sqrt{3}}{2}$

(c) Hyperbola of eccentricity 2

(d) a hyperbola of eccentricity $\sqrt{2}$

16. A tangent is drawn at the point $(3\sqrt{3}\cos\theta, \sin\theta)$; $0<\theta<\frac{\pi}{2}$ of an ellipse $\frac{x^2}{27}+\frac{y^2}{1}=1$. The least value of the sum of the intercepts on the coordinate axes by this tangent is attained at $\theta =$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

MARK YOUR RESPONSE					
	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ
	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ

17. An ellipse has eccentricity $\frac{1}{2}$ and one focus at $S\left(\frac{1}{2},1\right)$. Its one directrix is the common tangent, (nearer to S) to the circle $x^2+y^2=1$ and $x^2-y^2=1$. The equation of the ellipse in standard form is

(a) $9\left(x-\frac{1}{3}\right)^2+12(y-1)^2=1$

(b) $12\left(x-\frac{1}{3}\right)^2+9(y-1)^2=1$

(c) $\frac{\left(x-\frac{1}{2}\right)^2}{12}+\frac{(y-1)^2}{9}$

(d) $3\left(x+\frac{1}{2}\right)^2+4(y-1)^2=1$

18. The locus of the foot of perpendicular from the centre on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) a circle
(b) a pair of straight lines
(c) another ellipse
(d) None of these

19. The tangent at any point on the ellipse $16x^2+25y^2=400$ meets the tangents at the ends of the major axis at T_1 and T_2. The circle on T_1T_2 as diameter passes through

(a) $(3,0)$
(b) $(0,0)$
(c) $(0,3)$
(d) $(4,0)$

20. The two concentric rectanular hyperbolas, whose axes meet at angles of 45°, cut at

(a) 45°
(b) 90°
(c) nothing can be said
(d) None of these.

21. The point of intersection of two tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, the product of whose slopes is c^2, lies on the curve.

(a) $y^2-b^2=c^2(x^2+a^2)$

(b) $y^2+a^2=c^2(x^2-b^2)$

(c) $y^2+b^2=c^2(x^2-a^2)$

(d) $y^2-a^2=c^2(x^2+b^2)$

22. If $x\cos\alpha+y\sin\alpha=p$, a variable chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle whose radius is equal to

(a) $\sqrt{2}\,a$
(b) $\sqrt{3}\,a$
(c) $2a$
(d) $\sqrt{5}\,a$

23. The centre of a rectangular hyperbola lies on the line $y = 2x$. If one of the asymptotes is $x+y+c=0$. Then the other asymptote is

(a) $x-y-3c=0$
(b) $2x-y+c=0$
(c) $x-y-c=0$
(d) None of these.

24. If the normal at 'θ' on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the transverse axis at G, then AG × A'G is (Where A and A' are the vertices of the hyperbola).

(a) $a^2\sec\theta$
(b) $a^2(e^4\sec^2\theta+1)$
(c) $a^2(e^4\sec^2\theta-1)$
(d) None of these.

25. The area of the rectangle formed by the perpendiculars from the centre of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent and normal at a point whose eccentric angle is $\frac{\pi}{4}$ is $(a>b)$

(a) $\frac{(a^2-b^2)ab}{a^2+b^2}$
(b) $\frac{(a^2+b^2)ab}{a^2-b^2}$
(c) ab
(d) $\frac{a^2b^2}{a^2+b^2}$

26. From any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$

The area cut off by the chord of contact on the region between the asymptotes is equal to

(a) $\frac{ab}{2}$
(b) ab
(c) $2ab$
(d) $4ab$

MARK YOUR RESPONSE					
	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ
	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ

27. If ϕ is the angle between the diameter through any point on a standard ellipse and the normal at that point, then the greatest value of tan ϕ is

(a) $\frac{2ab}{a^2+b^2}$ (b) $\frac{a^2+b^2}{a\,b}$

(c) $\frac{a^2-b^2}{2a\,b}$ (d) $\frac{b^2}{a^2}$

28. In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the length of the perpendiculars from the centre upon all chords, which join the ends of perpendicular diameter, is

(a) $\frac{ab}{\sqrt{a^2+b^2}}$ (b) $\sqrt{a^2+b^2}$

(c) $\sqrt{ab}$ (d) None of these.

29. PQ and RS are two perpendicular chords of the rectangular hyperbola $xy = c^2$. If C is the centre of the rectangular hyperbola. Then the product of the slopes of CP, CQ, CR and CS is equal to .

(a) -1 (b) 1

(c) 0 (d) None of these.

30. Equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2+4y^2=16$, is

(a) $2x^2+2y^2-4x+7=0$

(b) $x^2+y^2-2x+5=0$

(c) $3x^2+3y^2-6x-8=0$

(d) None of these

31. If the normals at P, Q, R on the rectangular hyperbola $xy = c^2$ intersect at a point S on the hyperbola, then centroid of the triangle PQR is at the .... of the hyperbola.

(a) centre (b) focus

(c) vertex (d) director circle

32. If the normal to the rectangular hyperbola $xy = c^2$ at the point $\left(ct, \frac{c}{t}\right)$ meets the curve again at $\left(ct', \frac{c}{t'}\right)$, then

(a) $t^3.t'=1$ (b) $t^3.t'=-1$

(c) $t\ .t'=1$ (d) $t\ .t'=-1$

33. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy = c^2$ to the asymptotes. The locus of the mid-point of MN is a hyperbola with eccentricity

(a) $\sqrt{2}$ (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) $2\sqrt{2}$

34. If the sum of the squares of slopes of the normals from a point P to the hyperbola $xy = c^2$ is equal to λ ($\lambda \in R^+$), then the locus of the point P is

(a) $x^2=\lambda c^2$ (b) $y^2=\lambda c^2$

(c) $xy=\lambda c^2$ (d) $x^2y^2=\lambda c^2$

35. If e is the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and θ is the angle between the asymptotes. Then cos $\frac{\theta}{2}$ is equal to

(a) $\sqrt{e}$ (b) $\frac{e}{1+e}$ (c) $\frac{1}{\sqrt{e}}$ (d) $\frac{1}{e}$

36. Let P (a sec θ, b tan θ) and Q (a sec ϕ, b tan ϕ), where $\theta+\phi=\frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If (h, k) is the intersection point of the normals of P and Q then k is equal to

(a) $\frac{a^2+b^2}{a}$ (b) $-\left(\frac{a^2+b^2}{a}\right)$

(c) $\frac{a^2+b^2}{b}$ (d) $-\left(\frac{a^2+b^2}{b}\right)$

37. If the ellipse $\frac{x^2}{4}+y^2=1$ meets the ellipse $x^2+\frac{y^2}{a^2}=1$ in four distinct points and $a=b^2-5b+7$, then b does not lie in

(a) $[4,5]$ (b) $(-\infty,2)\cup(3,\infty)$

(c) $(-\infty,0)$ (d) $[2,3]$

MARK YOUR RESPONSE					
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ
	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ
	37. ⓐⓑⓒⓓ				

38. With a given point and line as focus and directrix, a series of ellipses are described, the locus of the extremities of their minor axis is
(a) elllipse (b) parabola
(c) hyperbola (d) pair of lines

39. If normal at $P\left(2, \frac{3\sqrt{3}}{2}\right)$ meets the major axis of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at Q and S and S' are *foci* of given ellipse, then $SQ:S'Q$ is
(a) $\frac{8-\sqrt{7}}{8+\sqrt{7}}$ (b) $\frac{2-\sqrt{7}}{2+\sqrt{7}}$
(c) $\frac{6-\sqrt{7}}{6+\sqrt{7}}$ (d) $\frac{4-\sqrt{7}}{4+\sqrt{7}}$

40. The locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ forms a triangle of constant area with the coordinate axes is
(a) a straight line (b) a hyperbola
(c) an ellipse (d) a circle

41. The curve $xy = c (c > 0)$ and the circle $x^2 + y^2 = 1$ touch at two points, then the distance between the points of contacts is
(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) $\sqrt{2}$

42. Portion of asymptote of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (between centre and the tangent at vertex) in the first quadrant is cut by the line $y+\lambda(x-a)=0$ (λ is a parameter) then
(a) $\lambda \in R$ (b) $\lambda \in (0,\infty)$
(c) $\lambda \in (-\infty,0)$ (d) $\lambda \in R-\{0\}$

43. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda(\lambda \in R^+)$, then locus of point P is
(a) $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$
(c) $xy = \lambda c^2$ (d) none of these

44. If a ray of light incident along the line $3x+(5-4\sqrt{2})y=15$, gets reflected from the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, then its reflected ray goes along the line
(a) $x\sqrt{2}-y+5=0$ (b) $\sqrt{2}y-x+5=0$
(c) $\sqrt{2}y-x-5=0$ (d) $3x-y(4\sqrt{2}+5)+15=0$

45. If the circle $(x+c)^2+y^2=a^2$ and ellipse $\frac{(x-h)^2}{b^2}+\frac{y^2}{a^2}=1$ (*a, b, c, h* are positive) have common tangent parallel to x-axis only then
(a) $c > b+a-h$ (b) $c < b+a-h$
(c) $c > b+a$ (d) none of these

46. Equation of chord of contact of pair of tangents drawn to ellipse $4x^2+9y^2=36$ from the point (m, n) where $m.n = m + n$, *m*, *n* being non-zero positive integers, is
(a) $2x+9y=18$ (b) $2x+2y=1$
(c) $4x+9y=18$ (d) $9x+4y=18$

47. The locus of the point of intersection of tangents to an ellipse at two points, sum of whose eccentric angles is constant is a/an
(a) parabola (b) circle
(c) ellipse (d) straight line

48. The locus of a point whose chord of contact with respect to the ellipse $x^2+2y^2=1$ subtends a right angle at the centre of the ellipse is
(a) $x^2+4y^2=3$ (b) $y^2=4x$
(c) $2x^2+y^2=1$ (d) $4x^2+y^2=3$

49. If the equation the family of the ellipse is $\frac{x^2}{\cos^2\alpha}+\frac{y^2}{\sin^2\alpha}=1\left(0<\alpha<\frac{\pi}{4}\right)$ then the locus of the extremities of the latus rectum is
(a) $2y(1-x^2)=1+x^2$
(b) $2y^2(1+x^2)=(1-x^2)^2$
(c) $y^2=(1-x^2)^2$
(d) $2y^2(1-x^2)=(1+x^2)^2$

MARK YOUR RESPONSE					
	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ
	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ
	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ			

50. If eccentric angle of a points lying in the first quadrant on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be θ and line joining centre to the point makes angle ϕ with x-axis then $\theta-\phi$ will be maximum when θ is equal to

(a) $\tan^{-1}\sqrt{\frac{a}{b}}$ (b) $\tan^{-1}\sqrt{\frac{b}{a}}$

(c) $\frac{\pi}{4}$ (d) $\tan^{-1}\left(\frac{b}{a}\right)$

51. Let *P* be any point on any directrix of an ellipse. Then chords of contact of point *P* with respect to the ellipse and its auxiliary circle intersect at

(a) some point on the major axis depending upon the position of point *P*

(b) mid point of the line segment joining the centre to the corresponding focus

(c) corresponding focus

(d) none of these

52. Let *P* be any point on a directrix of an ellipse of eccentricity e. S be the corresponding focus and *C* the centre of the ellipse. The line PC meets the ellipse A. The angle between *PS* and tangent at *A* is α then α, is equal to

(a) $\tan^{-1}e$ (b) $\frac{\pi}{2}$

(c) $\tan^{-1}(1-e^2)$ (d) $\tan^{-1}(1+e^2)$

53. If normals are drawn to the ellipse $x^2+2y^2=2$ from the point (2, 3), then the co-normal points lie on the curve

(a) $xy+2x-3y=0$

(b) $xy+3x-4y=0$

(c) $2xy+3x-4y=0$

(d) none of these

54. The maximum distance of the centre of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ from chord of contact of mutually perpendicular tangents of the ellipse is

(a) $\frac{144}{5}$ (b) $\frac{16}{5}$ (c) $\frac{9}{5}$ (d) 5

55. If a rectangular hyperbola $(x-1)(y-2)=4$ cuts a circle $x^2+y^2+2gx+2fy+c=0$ at points (3, 4), (5, 3), (2, 6) and (–1, 0), then the value of $(g+f)$ is equal to

(a) – 8 (b) – 9 (c) 8 (d) 9

56. The minimum value of $px + qy$ when $xy=r^2$ is (where p, q, r are positive)

(a) $2r\sqrt{pq}$ (b) $2pq\sqrt{r}$

(c) $-2r\sqrt{pq}$ (d) $\sqrt{pq}$

57. A normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$, has equal intercepts on the positive *x* and *y* axes. If this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then a^2+b^2 is equal to

(a) 5 (b) 25 (c) 16 (d) $\frac{25}{3}$

MARK YOUR RESPONSE					
	50. ⓐⓑⓒⓓ	51. ⓐⓑⓒⓓ	52. ⓐⓑⓒⓓ	53. ⓐⓑⓒⓓ	54. ⓐⓑⓒⓓ
	55. ⓐⓑⓒⓓ	56. ⓐⓑⓒⓓ	57. ⓐⓑⓒⓓ		

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Suppose that an ellipse and a circle are respectively given by the equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{....(1)}$$

and $$x^2+y^2+2gx+2fy+c=0 \quad \text{....(2)}$$

The equation,

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)+\lambda(x^2+y^2+2gx+2fy+c)=0 \quad \text{.....(3)}$$

represents a curve which passes through the common points of the ellipse (1) and the circle (2).

We can choose λ so that the equation (3) represents a pair of straight lines. In general we get three values of λ, indicating three pair of straight lines can be drawn through the points. Also when

(3) represents a pair of straight lines they are parallel to the lines $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\lambda(x^2+y^2)=0$, which represents a pair of lines equally inclined to axes (the term containing xy is absent). Hence two straight lines through the points of intersection of an ellipse and any circle make equal angles with the axes. Above description can be applied identically for a hyperbola and a circle.

1. The radius of the circle passing through the points of intersection of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2-y^2=0$ is

(a) $\frac{ab}{\sqrt{a^2+b^2}}$ (b) $\frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

(c) $\frac{a^2-b^2}{\sqrt{a^2+b^2}}$ (d) $\frac{a^2+b^2}{\sqrt{a^2-b^2}}$

2. If $\alpha, \beta, \gamma, \delta$ be eccentric angles of the four concyclic points of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\alpha+\beta+\gamma+\delta=$

(a) $(2n+1)\frac{\pi}{2}$ (b) $(2n+1)\pi$

(c) $2n\pi$ (d) $n\pi$ [n is any integer]

3. Suppose two lines are drawn through the common points of intersection of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and circle $x^2+y^2+2gx+2fy+c=0$. If these lines are inclined at angles α and β to x-axis then,

(a) $\alpha=\beta$ (b) $\alpha+\beta=\frac{\pi}{2}$

(c) $\alpha+\beta=\pi$ (d) $\alpha+\beta=2\tan^{-1}\left(\frac{b}{a}\right)$

4. The number of pair of straight lines through points of intersection of rectangular hyperbola $x^2-y^2=1$ and circle $x^2+y^2-4x-5=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

PASSAGE-2

The equation of a curve C is given by

$$C=21x^2-6xy+29y^2+6x-58y-151=0,\ \text{then}$$

5. The curve C is

(a) parabola (b) ellipse

(c) hyperbola (d) pair of straight lines

6. Eccentricity of the conic section C is epual to

(a) $\frac{1}{\sqrt{3}}$ (b) 1

(c) $\frac{5}{\sqrt{3}}$ (d) none of these

7. The centre of the conic C is

(a) $(1,0)$ (b) $(0,0)$

(c) $(0,1)$ (d) none of these

PASSAGE-3

Consider a hyperbola whose centre is at origin. A line $x+y=2$ touches this hyperbola at P(1,1) and intersdects the asymptotes at A and B such that $AB=6\sqrt{2}$ units. (you can use the concept that in case of hyperbola portion of tangent intercepted between asymptotes is bisected at the point of contact).

8. Equation of asymptotes are

(a) $2x^2+2y^2+5xy=0$

(b) $3x^2+4y^2+6xy=0$

(c) $2x^2+2y^2-5xy=0$

(d) none of these

9. Angle subtended by AB at the centre of the hyperbola is

(a) $\sin^{-1}\frac{4}{5}$ (b) $\sin^{-1}\frac{2}{5}$

(c) $\sin^{-1}\frac{3}{5}$ (d) $\tan^{-1}\frac{4}{5}$

10. Equation of the tangent to the hyperbola at $\left(-1,\frac{7}{2}\right)$ is

(a) $5x+2y=2$ (b) $3x+2y=4$

(c) $3x+4y=11$ (d) $x+2y=6$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ

PASSAGE-4

Read the following writeup carefully:

If $ax^2+by^2+2hxy+2gx+2fy+c=0$ represents an ellipse, then $h^2<ab$ and $abc+2fgh-af^2-bg^2-ch^2\neq 0$. If for every point (x_1,y_1) satisfying above equation $(2h-x_1, 2k-y_1)$ also satisty it, then (h,k) is centre of it. The length of semi major axis is the maximum and minimum value of the distances of points lying on the curve from its centre.

Now answer the following question (1-5):

11. For the ellipse $2x^2-2xy+4y^2-(3+\sqrt{2})=0$, the inclination of major axis of it with x-axis is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{8}$

(c) $\frac{3\pi}{8}$ (d) $\frac{5\pi}{8}$

12. The equation of tangent to $2x^2-2xy+4y^2-(3+\sqrt{2})=0$ such that sum of perpendiculars dropped from foci is 2 units, is

(a) $y\cos\frac{3\pi}{4}-x\sin\frac{3\pi}{4}=1$

(b) $y\sin\frac{3\pi}{8}-x\cos\frac{3\pi}{8}=1$

(c) $x\cos\frac{\pi}{8}-y\sin\frac{\pi}{8}=1$

(d) $y\cos\frac{5\pi}{8}+x\sin\frac{5\pi}{8}=1$

13. The product of perpendiculars from the foci to any tangent to above given ellipse is

(a) 4 (b) 2

(c) 1 (d) $\frac{1}{2}$

PASSAGE-5

Read the following concept carefully:

The locus of the mid-points of parallel chords of an ellipse is called diameter of the ellipse. Two diameters are said to be conjugate when bisects all chords parallel to the other. Two diameters $y=m_1x$ and $y=m_2x$, of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, are conjugate if $m_1m_2=-\frac{b^2}{a^2}$.

Now answer the following questions (1-3):

14. If the eccentric angles of the end points P and Q of a pair of conjugate diameters be ϕ_1 and ϕ_2, then $\phi_1-\phi_2$ is equal to

(a) ±45° (b) ±90°

(c) ±135° (d) ±60°

15. If C is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and P and Q are the end points of two conjugate diameters, then CP^2+CQ^2 is equal to

(a) $\frac{b^4+a^4}{a^2+b^2}$ (b) a^2+b^2

(c) $\frac{b^4+a^4}{2(a^2+b^2)}$ (d) $\frac{a^2+b^2}{4}$

16. C is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and P and Q are the end points of a pair of conjugate diameters. If the tangents to the ellipse at P and Q meet at R, then the area of the quadrilateral $CPRQ$ is

(a) $4ab$ (b) $2ab$

(c) ab (d) none of these

PASSAGE-6

An ellipse whose major axis is parallel to x-axis such that the segments of a focal chord are 1 and 3 units. The lines $ax+by+c=0$ are the chords of the ellipse such that a, b, c, are in AP and bisected by the point at which they intersent. The equation of its auxiliary circle is

$x^2+y^2+2\alpha x+2\beta y-2\alpha-1=0$ then.

17. The centre of the ellipse is

(a) $(1,1)$ (b) $(1,2)$

(c) $(1,-2)$ (d) $(-2,1)$

MARK YOUR RESPONSE					
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ			

18. Equation of the director circle is

(a) $x^2 + y^2 - 2x + 4y + 1 = 0$

(b) $x^2 + y^2 + 2x + 2y - 3 = 0$

(c) $x^2 + y^2 + 2x + 4y + 1 = 0$

(d) $x^2 + y^2 - 2x + 4y - 2 = 0$

19. Eccentricity of ellipse is

(a) $\frac{\sqrt{13}}{4}$ (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

PASSAGE-7

A coplanar beam of light emerging from a point source have equation $\lambda x - y + 2(1+\lambda) = 0,\ \lambda \in R.$ the rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having equation $\mu x - y + 2(1-\mu) = 0, \mu \in R.$ Further it is found that the foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0.$ It is given that point source of incident beam and the point of convergence of reflected beams lie on the axis of the ellipse of the cross-section of the reflecting surface by the plane.

Now answer the following questions:

20. The eccentricity of the ellipse is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

21. The area of the largest triangle that an incident ray and the corresponding refleted ray can enclose with the axis of the ellipse is equal to

(a) $4\sqrt{5}$ (b) $2\sqrt{5}$

(c) $\sqrt{5}$ (d) $3\sqrt{5}$

22. Total distance travelled by an incident ray and the corresponding reflected ray is the least if the point of incidence coincides with

(a) an end of the minor axis

(b) an end of the major axis

(c) an end of the latus rectum

(d) none of these

MARK YOUR RESPONSE	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options:

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. Statement-1 : The distance between the foci of an ellipse is always less the sum of focal distances of any point on it.

Statement-2 : The eccentricity of an ellipse is less than 1

2. Statement-1 : The equation $\frac{x^2}{12-p} + \frac{y^2}{p-8} = 1$ represents a real ellipse if $8 < p < 12$

Statement-2 : Eccentricity of an ellipse is less than 1

3. Statement-1 : Number of integral points on the ellipse $\frac{x^2}{5} + \frac{y^2}{4} = 1$ is 4

Statement-2 : The eccentricity of the ellipse is $\frac{\sqrt{5}}{3}$

4. The transverse axis of a hyperbola is given $2a$ and its vertex bisects the distance between the centre and focus.

Statement-1 : The latus rectum of the hyperbola is 6 a

Statement-2 : The eccentricity of the hyperbola is $\sqrt{2}$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. If (5, 12) and (24, 7) are the foci of a conic passing through the origin then the eccentricity of conic is

(a) $\frac{\sqrt{386}}{12}$ (b) $\frac{\sqrt{386}}{38}$

(c) $\frac{\sqrt{386}}{25}$ (d) $\sqrt{2}$

2. If P is any point lying on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS'=\alpha$ and $\angle PS'S=\beta$, then

(a) $PS+PS'=2a$, if $a>b$

(b) $PS+PS'=2b$, if $a<b$

(c) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}$

(d) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}[a-\sqrt{a^2-b^2}]$ when $a>b$

3. If the normals at (x_i, y_i) i = 1, 2, 3, 4 to the rectangular hyperbola $xy = 2$ meet at the point (3, 4), then

(a) $x_1+x_2+x_3+x_4=3$ (b) $y_1+y_2+y_3+y_4=4$

(c) $x_1\,x_2\,x_3\,x_4=-4$ (d) $y_1\,y_2\,y_3\,y_4=4$

4. If the circle $x^2+y^2=1$ cuts the rectangular hyperbola xy = 1 in four points (x_i, y_i) i = 1, 2, 3, 4 then.

(a) $x_1x_2x_3x_4=-1$ (b) $y_1y_2y_3y_4=1$

(c) $x_1+x_2+x_3+x_4=0$ (d) $y_1+y_2+y_3+y_4=0$

5. The equation $\left|\sqrt{x^2+(y-1)^2}-\sqrt{x^2+(y+1)^2}\right|=K$ will represent a hyperbola for

(a) $K\in(0,2)$ (b) $K\in(0,1)$

(c) $K\in(1,\infty)$ (d) $K\in(0,\infty)$

6. If a quadrilateral formed by four tangents to the ellipse $3x^2+4y^2=12$ is a square then

(a) The vertices of the square lie on is $y=\pm x$

(b) The vertices of the square lie on $x^2+y^2=7$

(c) The area of all such squares is constant

(d) Only two such squares are possible

7. If the normal at P to the rectangular hyperbola $x^2-y^2=4$ meets the axis in G and g and C is the centre of the hyperbola, then

(a) $PG=PC$ (b) $Pg=PC$

(c) $\sqrt{2}\,PG=Pg$ (d) $Gg=2PC$

8. The locus of extremities of latus rectum of the family of ellipses $b^2x^2+y^2=a^2b^2$ where b is a parameter ($b^2<1$), is

(a) $x^2+a^2y^2=a^2$

(b) $x^2+ay=a^2$

(c) $x^2-ay=a^2$

(d) $x^2-a^2y=a^2$

9. If the straight line $3x+4y=24$ intersects the axes at A and B and the straight line $4x+3y=24$ at C and D, then points A, B, C, D lies on

(a) circle (b) parabola

(c) ellipse (d) hyperbola

10. If the normals to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the ends of the chord $l_1x+m_1y=1$ and $l_2x+m_2y=1$ are concurrent, then

(a) $l_1l_2=-\frac{1}{a^2}$ (b) $m_1m_2=-\frac{1}{b^2}$

(c) $l_1l_2=\frac{1}{a^2}$ (d) $m_1m_2=\frac{1}{b^2}$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ

11. If two concentric ellipses be such that the foci of one be on the other and their major axes are equal. Let e_1 and e_2 be their eccentricities, then

(a) the quadrilateral formed by joining the foci of the two ellipses is is a parallelogram

(b) the angle θ between their axes is given by, $\cos\theta = \sqrt{\frac{1}{e_1^2}+\frac{1}{e_2^2}-\frac{1}{e_1^2 e_2^2}}$

(c) if $e_1^2+e_2^2=1$ then the angle between the axes of the two ellipses is 90°

(d) area of the quadrilateral $= a^2\sqrt{e_1^2+e_2^2+e_1^2e_2^2-1}$

12. If from $(1,\beta)$ two tangents are drawn on exactly one branch of a hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1,$ then value of β may satisfy

(a) $\left[-\frac{1}{2},1\right]$ (b) $\left[-\frac{1}{4},\frac{1}{4}\right]$

(c) $\left(-\frac{1}{2},\frac{1}{2}\right)$ (d) $\left[-\frac{3}{4},\frac{3}{4}\right]$

MARK YOUR RESPONSE	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ			

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. **Observe the following columns :**

Column-I	Column–II
(A) The locus of mid-points of chords of an ellipse which are drawn through an end of minor axis, is	p. pair of straight lines
(B) The locus of an end of latus rectum of all ellipses having a given major axis is	q. circle
(C) The locus of the foot of perpendicular from a focus of the ellipse on any tangent is	r. parabola
(D) A variable line drawn through a fixed point cuts axes at A and B. The locus of the mid point of AB is	s. ellipse
	t. hyperbola

2. Let the circle $(x-1)^2+(y-2)^2=25$ cuts a rectangular hyperbola with transverse axis along y = x at four points A,B,C and D having coordinates (x_i,y_i), i = 1,2,3,4 respectively. O being the centre of the hypeprbola. Now match the entries from the following two columns :

Column-I	Column–II
(A) $x_1+x_2+x_3+x_4$ is equal to	p. 2
(B) $x_1^2+x_2^2+x_3^2+x_4^2$ is equal to	q. 4
(C) $y_1^2+y_2^2+y_3^2+y_4^2$ is equal to	r. 44
(D) $OA^2+OB^2+OC^2+OD^2$ is equal to	s. 56
	t. 100

MARK YOUR RESPONSE

1.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

2.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

3. Let an ellipse has major axis and minor axis parallel to x-axis and y-axis respectively. Its two foci S and S' are (2, 1), (4, 1) and a line $x+y=9$ is a tangent to this ellipse at point P, let e be the eccentricity of the ellipse.

Column-I		**Column–II**
(A) The value of $\frac{1}{e}$ is equal to	p.	5
(B) Length of major axis axis of ellipse is	q.	$\sqrt{13}$
(C) The latus rectum of ellipse is	r.	$\frac{24}{\sqrt{13}}$
(D) If $x-y+c=0$ be another tangent to to the ellipse meeting the given tangent at D and C is the centre of ellipse then CD is	s.	$2\sqrt{13}$

4.

Column-I		**Column-II**
(A) If $\sqrt{3}bx+ay=2ab$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at a point whose eccentric angle is θ then cosec θ equals	p.	0
(B) Let $e(k)$ be the eccentricity of $(x-3)(y+2)=k^2$, then $e(2)-e(3)$ equals	q.	1
(C) If $x^2+y^2=a^2$ is drawn without intersecting the curve $xy=9$ then integral value of a equals	r.	2
(D) If $xy=1+\sin^2\theta$ (θ being parameter) be a family of rectangular hyperbolas and Δ is the area of triangle formed by any tangent with coodinate axes then Δ can be equal to	s.	3
	t.	4

5.

Column-I		**Column-II**
(A) If P is point on the ellipse $\frac{x^2}{16}+\frac{y^2}{20}=1$ whose foci are S and S', then $PS+PS'$ is	p.	$\frac{4}{5}$
(B) The eccentricity of the ellipse $2x^2+3y^2-4x-12y+13=0$ is	q.	$4\sqrt{5}$
(C) Tangents are drawn from the point on the line $x-y-5=0$ to $x^2+4y^2=4$. Then all the chords of contact pass through a fixed point, whose abscissa is	r.	8
(D) The sum of the distances of any point on the ellipse $3x^2+4y^2=12$ from its directrix is	s.	$\frac{1}{\sqrt{3}}$

MARK YOUR RESPONSE

3.

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

4.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

5.

	p	q	r	s
A	ⓟ	ⓠ	ⓡ	ⓢ
B	ⓟ	ⓠ	ⓡ	ⓢ
C	ⓟ	ⓠ	ⓡ	ⓢ
D	ⓟ	ⓠ	ⓡ	ⓢ

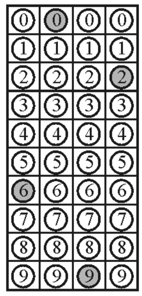
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. Let Δ_1 be the area of a triangle PQR inscribed in an ellipse and Δ_2 be the area of the triangle $P'Q'R'$ whose vertices are the points lying on the auxiliary circle corresponding to the points P, Q, R respectively. If the eccentricity of the ellipse is $\frac{4\sqrt{3}}{7}$ then the ratio $\frac{\Delta_2}{\Delta_1}$ is equal to

2. If p is the length of the perpendicular from a focus upon the tangent at any point P of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and r is the distance of P from the focus, then $\frac{2a}{r}-\frac{b^2}{p^2}$ is equal to

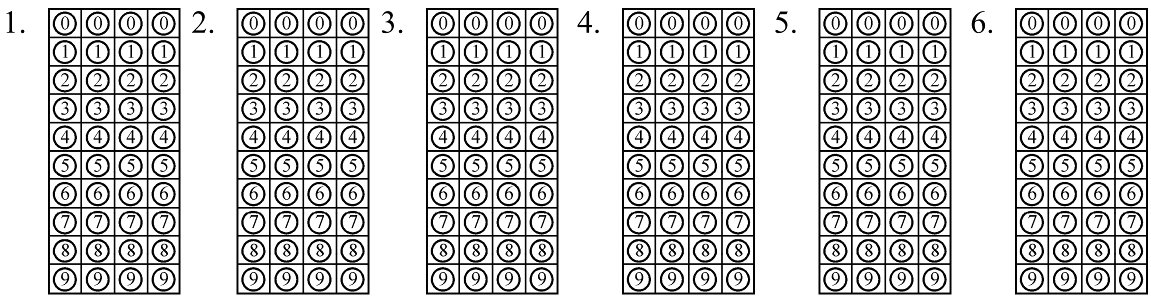
3. The tangent at a point P $(a\cos\phi, b\sin\phi)$ of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, meets its auxillary circle in two points Q and R. If the chord QR subtends right angle at the centre and e be the eccentricity of the ellipse then $e\sqrt{1+\sin^2\phi}$ is equal to

4. If ϕ_1, ϕ_2, ϕ_3, ϕ_4 are the eccentric angles of four concyclic points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\cos(\phi_1+\phi_2+\phi_3+\phi_4)$ is equal to

5. If S_1 and S_2 be the foci of the hyperbola whose transverse axis length is 4 and conjugate axis length is 6, S_3 and S_4 be the foci of the conjugate hyperbola then the area of the quadrilateral $S_1S_3S_2S_4$ is

6. A variable point P on the ellipse of eccentricity e is joined to the foci S and S'. If the locus of the incentre of the triangle PSS' is a conic of eccentricity e_1, then $\frac{2}{e_1^2}-\frac{1}{e}$ equals to

MARK YOUR RESPONSE



7. If the tangent and normal to a rectangular hyperbola at a point cut off intercepts a_1, a_2 on transverse axis and b_1, b_2 on conjugate axis, then $a_1a_2 + b_1b_2$ is equal to

8. A straight line *PQ* touches the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the circle $x^2+y^2=r^2 (3<r<4)$. *RS* is a focal chord of the ellipse which is parallel to *PQ* and meets the circle at point *R* and *S*. Then the length of *RS* is equal to

9. If a chord of hyperbola $xy = c^2$ is normal at point A, subtending an angle α at origin *O*, then the value of $\frac{\sin(\alpha - A)}{\sin(\alpha + A)}$ (where $A = \angle OAB$) is equal to

10. If the normals at the four points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are concurrent, then the value of $(x_1+x_2+x_3+x_4)\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}\right)$ is equal to

11. *C* is the centre of the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$, and '*A*' is any point on it. The tangent at A to the hyperbola meets the line $x-2y=0$ and $x+2y=0$ at *Q* and *R* respectively. The value of *CQ.CR* is equal to

MARK YOUR RESPONSE

7.	8.	9.	10.	11.
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(b)	**11**	(d)	**21**	(c)	**31**	(a)	**41**	(b)	**51**	(c)
2	(d)	**12**	(a)	**22**	(a)	**32**	(b)	**42**	(b)	**52**	(b)
3	(a)	**13**	(d)	**23**	(a)	**33**	(a)	**43**	(a)	**53**	(b)
4	(b)	**14**	(b)	**24**	(c)	**34**	(a)	**44**	(d)	**54**	(b)
5	(a)	**15**	(b)	**25**	(a)	**35**	(d)	**45**	(b)	**55**	(a)
6	(c)	**16**	(a)	**26**	(d)	**36**	(d)	**46**	(c)	**56**	(a)
7	(a)	**17**	(a)	**27**	(c)	**37**	(d)	**47**	(d)	**57**	(d)
8	(b)	**18**	(d)	**28**	(a)	**38**	(b)	**48**	(a)		
9	(a)	**19**	(a)	**29**	(b)	**39**	(a)	**49**	(b)		
10	(d)	**20**	(b)	**30**	(c)	**40**	(b)	**50**	(a)		

B COMPREHENSION TYPE

1	(b)	**5**	(b)	**9**	(c)	**13**	(c)	**17**	(c)	**21**	(b)
2	(c)	**6**	(a)	**10**	(b)	**14**	(b)	**18**	(d)	**22**	(d)
3	(c)	**7**	(c)	**11**	(b)	**15**	(b)	**19**	(b)		
4	(c)	**8**	(a)	**12**	(b)	**16**	(c)	**20**	(c)		

C REASONING TYPE

1	(a)	**2**	(d)	**3**	(b)	**4**	(c)

D MULTIPLE CORRECT CHOICE TYPE

1	(a,b)	**3**	(a,b,c)	**5**	(a,b)	**7**	(a,b,d)	**9**	(a,b,c,d)	**11**	(a, b, c)
2	(a,b,c)	**4**	(b,c,d)	**6**	(b,c)	**8**	(b,c)	**10**	(a, b)	**12**	(b, c)

E MATRIX-MATCH TYPE

1. **A - s; B - r; C - q; D - t**
2. **A - p; B - r; C - s; D - t**
3. **A - q; B - s; C - r; D - p**
4. **A - r; B - p; C - p, q, r, s, t; D - r, s, t**
5. **A - q; B - s; C - p; D - r**

F NUMERIC/INTEGER ANSWER TYPE

1	7	**3**	1	**5**	26	**7**	0	**9**	3	**11**	5
2	1	**4**	1	**6**	1	**8**	6	**10**	4		

Solutions

A SINGLE CORRECT CHOICE TYPE

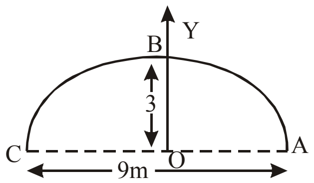
1. **(b)** Let the equation of the semi elliptical arc be,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (y>0)$$

Length of the major axis $=2a=9 \Rightarrow a=\frac{9}{2}$

Length of the semi minor axis $=b=3$.

So, the equation of the arc becomes $\frac{4x^2}{81}+\frac{y^2}{9}=1$



If $x=2$, then $y^2=\frac{65}{9} \Rightarrow y=\frac{1}{3}\sqrt{65}=\frac{8}{3}$ approximately.

2. **(d)** As per the definition, the locus must be an ellipse, with given points as foci and 10 as its major axis. Since the line segment joining (2, – 3) and (2, 5) is parallel to y-axis, therefore, ellipse is veritcal.

$\therefore \quad 2be=8$ and $2b=10 \Rightarrow b=5$ and $e=\frac{4}{5}$

$\therefore \quad a^2=b^2(1-e^2)=9$

and centre of the ellipse is (2, 1)

$\therefore$ Equation of the required ellipse is

$$\frac{(x-2)^2}{9}+\frac{(y-1)^2}{25}=1$$

3. **(a)** $4\tan\frac{B}{2}\tan\frac{C}{2}=1$

$$\Rightarrow \sqrt{\frac{(s-c)(s-a)}{s(s-b)}\times\frac{(s-a)(s-b)}{s(s-c)}}=\frac{1}{4}$$

$$\Rightarrow \frac{s-a}{s}=\frac{1}{4} \Rightarrow \frac{2s-a}{a}=\frac{5}{3}$$

$$\Rightarrow b+c=\frac{5}{3}\times 6=10 \qquad (\because a=BC=6)$$

Thus, sum of distance of variable point A from two given fixed points B and C is always 10, therefore, equation of locus of A is $\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$

(See Q. N. 2)

4. **(b)** Let the points of intersection of the line and the ellipse be $(a\cos\theta, b\sin\theta)$ and $\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$. Since they lie on the given line $lx+my+n=0$,

$la\cos\theta+mb\sin\theta+n=0$

$\Rightarrow la\cos\theta+mb\sin\theta=-n$

and $-la\sin\theta+mb\cos\theta+n=0$

$\Rightarrow la\sin\theta-mb\cos\theta=n.$

Squaring and adding, we get $a^2l^2+b^2m^2=2n^2$

$$\Rightarrow \frac{a^2l^2+b^2m^2}{n^2}=2.$$

5. **(a)** Lengths of semi-major axis and semi-minor axis of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ are 4 and 3 respectively. So that the mean of these lengths is $\frac{7}{2}$. Let the co-ordinates of any point on the ellipse be $P(4\cos\theta, 3\sin\theta)$. If the distance of P from the centre O (0, 0) of the ellipse is $\frac{7}{2}$, then $16\cos^2\theta+9\sin^2\theta=\frac{49}{4}$.

$\Rightarrow 28\cos^2\theta=13$

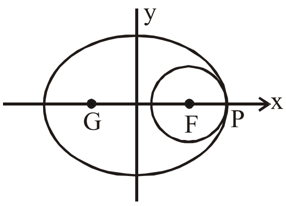
$\Rightarrow \cos\theta=\pm\sqrt{\frac{13}{28}}=\pm\frac{\sqrt{91}}{14}$ and $\sin\theta=\pm\frac{\sqrt{105}}{14}$.

So, the co-ordinates of the required point are $\left(\pm\frac{4\sqrt{91}}{14},\pm\frac{3\sqrt{105}}{14}\right)$ i.e. $\left(\pm\frac{2\sqrt{91}}{7},\pm\frac{3\sqrt{105}}{14}\right)$.

6. **(c)** Let the semimajor and semiminor axes be a and b respectively and the distance of centre to the focus be c. Most students would assume that the circle touches the ellipse in two points, but this is not true! Suppose it were true, and let one of the points of tangency be P. Consider a tangent (to both circle and ellipse) at P, call the centre of the circle F (one focus) and call the other focus G. Consider a ray emanating from F and boucing

off the tangent at P. By reflection properties, the ray must bounce back to F (since it bounces off a tangent to a circle), but it also must bounce directly from P to G (since it bounces off a tangent to the ellipse). This is only possible if P, F and G are collinear.

Thus P must be the end of the major axis? Apparently ellipse are sufficiently rounded at their ends to ensure that a tangent circle actually fits right into that end! The problem is now easy. The radius must equal to $a - c$, which equals $a - \sqrt{a^2 - b^2} = 5 - \sqrt{5^2 - 4^2} = 2$.



7. **(a)** Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Since length of the minor axis BB' is $2b$, the length of the line joining the focus $S(ae, 0)$ to $B(0, b)$ is also $2b$.

$$\Rightarrow \quad a^2e^2 + b^2 = 4b^2 \Rightarrow a^2 - b^2 + b^2 = 4b^2$$

$$\Rightarrow a^2 = 4b^2 \Rightarrow e^2 = \frac{a^2 - b^2}{a^2} = \frac{4b^2 - b^2}{4b^2} = \frac{3}{4}$$

$$\Rightarrow e = \frac{\sqrt{3}}{2}.$$

8. **(b)** Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the points are $P(a\cos 2\alpha, b\sin 2\alpha)$, $Q(a\cos 2\beta, b\sin 2\beta)$, $R(a\cos 2\gamma, b\sin 2\gamma)$.

The equation of the chord PQ is

$$\frac{x}{a}\cos(\alpha + \beta) + \frac{y}{b}\sin(\alpha + \beta) = \cos(\alpha - \beta)$$

If PQ passes through focus$(ae, 0)$, then

$$e = \frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)} \quad \text{...... (i)}$$

Similarly, if the chord PR passes through the focus $(-ae, 0)$, then $-e = \frac{\cos(\alpha - \gamma)}{\cos(\alpha + \gamma)}$ ...... (ii)

Thus from (i) and (ii), we have

$$\frac{\cos(\alpha - \beta)}{\cos(\alpha + \beta)} = -\frac{\cos(\alpha - \gamma)}{\cos(\alpha + \gamma)}$$

Apply componendo and dividendo, we get

$$\frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)} = \frac{\cos(\alpha + \gamma) - \cos(\alpha - \gamma)}{\cos(\alpha + \gamma) + \cos(\alpha - \gamma)}$$

$$\Rightarrow \frac{2\cos\alpha\cos\beta}{2\sin\alpha\sin\beta} = \frac{2\sin\alpha\sin\gamma}{2\cos\alpha\sin\gamma}$$

$$\Rightarrow \tan\beta\ \tan\gamma = \cot^2\alpha$$

9. **(a)** The eccentricity e of the given ellipse is given by

$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow a^2 - b^2 = a^2e^2$. So, the points on the minor axis, i.e. y-axis at a distance $\sqrt{a^2 - b^2}$ from the centre (0, 0) of the ellipse are $(0, \pm ae)$.

The equation of the tangent at any point $(a\cos\theta, b\sin\theta)$ on the ellipse is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

So, the required sum is

$$\left[\frac{\frac{ae\sin\theta}{b} - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right]^2 + \left[\frac{\frac{-ae\sin\theta}{b} - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right]^2$$

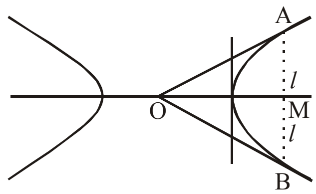
$$= \frac{(ae\sin\theta - b)^2 + (ae\sin\theta + b)^2}{(b^2\cos^2\theta + a^2\sin^2\theta0)} \times a^2$$

$$= \frac{2a^2(a^2e^2\sin^2\theta + b^2)}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{2a^2[(a^2 - b^2)\sin^2\theta + b^2]}{b^2\cos^2\theta + a^2\sin^2\theta} = 2a^2$$

10. **(d)** Let $AB = 2l$, then $AM = l$

Thus, we have $A = \left[\frac{a\sqrt{b^2 + l^2}}{b}, l\right]$



Now, since OAB is an equilateral triangle,

Therefore we have, $OA = 2l$

i.e. $OM^2 + AM^2 = (2l)^2$

i.e. $\frac{a^2(b^2 + l^2)}{b^2} + l^2 = 4l^2$

Gives $l^2 = \frac{a^2b^2}{3b^2 - a^2} > 0$ i.e. $3b^2 - a^2 > 0$

i.e. $3a^2(e^2-1) > a^2$

Gives $e > \frac{2}{\sqrt{3}}$.

ALTERNATE SOLUTION

Let coordinates of A be $(a\sec\theta, b\tan\theta)$

then $\angle AOM = 30°$

$\therefore \tan 30° = \frac{AM}{OM} = \frac{b\tan\theta}{a\sec\theta}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b}{a}\sin\theta \Rightarrow \text{cosec}^2\theta = \frac{3b^2}{a^2} = 3(e^2-1)$

$\because \text{cosec}^2\theta > 1 \Rightarrow 3(e^2-1) > 1 \Rightarrow e^2 > \frac{4}{3}$ or $e > \frac{2}{\sqrt{3}}$

11. (d) Let $P(x_1, y_1)$ be a point on the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$

The chord of contact of tangents from P to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$ is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 2$ .... (i)

The equation of the asympotes are $\frac{x}{a} - \frac{y}{b} = 0$

and $\frac{x}{a} + \frac{y}{b} = 0$

The point of intersection of (i) with the two asymptotes are given by

$$x' = \frac{2a}{\frac{x_1}{a} - \frac{y_1}{b}},\ y' = \frac{2b}{\frac{x_1}{a} - \frac{y_1}{b}},\ x'' = \frac{2a}{\frac{x_1}{a} + \frac{y_1}{b}},$$

$$y'' = \frac{-2b}{\frac{x_1}{a} + \frac{y_1}{b}}$$

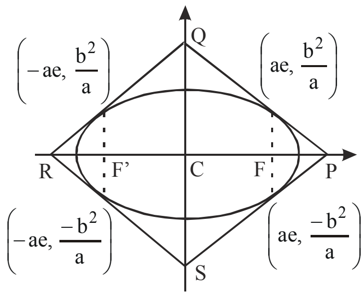
$\therefore$ Area of the triangle $= \frac{1}{2}(x'y'' - x''y')$

$$= \frac{1}{2}\left[\frac{4ab \times 2}{\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}}\right] = 4ab$$

12. (a) $\frac{x^2}{9} + \frac{y^2}{5} = 1 \Rightarrow e^2 = 1 - \frac{5}{9} = \frac{4}{9} \Rightarrow e = \frac{2}{3}$

One end of latusrectum is $\left(2, \frac{5}{3}\right)$

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is $\frac{2x}{9} + \frac{y}{3} = 1$



Area of $\Delta CPQ = \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$

$\therefore$ Area of quadrilateral $PQRS = 4 \times \frac{27}{4} = 27.$

13. (d) The equation of tangents at two points having eccentric angles θ_1 and θ_2 are .

$$\frac{x}{a}\cos\theta_1 + \frac{y}{b}\sin\theta_1 = 1 \quad \text{.... (i)}$$

$$\frac{x}{a}\cos\theta_2 + \frac{y}{b}\sin\theta_2 = 1 \quad \text{.... (ii)}$$

The point of intersection of (i) and (ii) is

$$\left(\frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \frac{a\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}\right),$$

It is given that $(\theta_1 + \theta_2) = K$ = Constant. Therefore, if (x_1, y_1) is the point of intersection of (i) and (ii), then

$$x_1 = \frac{a\cos K}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \text{ and } y_1 = \frac{b\sin K}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$$

$$\Rightarrow \frac{x_1}{y_1} = \frac{a}{b}\cot K \Rightarrow y_1 = \left[\frac{b}{a}\cot K\right]x_1$$

$\Rightarrow (x_1, y_1)$ lies on the straight line $y = \left[\frac{b}{a}\cot K\right]x$

14. (b) Equation of the tangent at $P(a\cos\theta, b\sin\theta)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

Length of the perpendicular from the centre $C(0, 0)$ on this tangent is given by

$$CF = \left|\frac{-1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right|$$

$$= \frac{ab}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}} \quad \text{....(i)}$$

Equation of the normal at P to the ellipse is

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

Which meets the major axis at the point G

$$\left(\frac{(a^2 - b^2)\cos\theta}{a}, 0\right)$$

$\Rightarrow PG$

$$= \sqrt{\left(a\cos\theta - \frac{(a^2 - b^2)\cos\theta}{a}\right)^2 + b^2 \sin^2\theta}$$

$$= \sqrt{\frac{b^4 \cos^2\theta}{a^2} + b^2 \sin^2\theta}$$

$$= \frac{b}{a}\sqrt{b^2 \cos^2\theta + b^2 \sin^2\theta} \quad \text{....(ii)}$$

From (i) and (ii), we get $CF \times PG = b^2$.

15. (b) $y = \dfrac{-2p}{\sqrt{1-p^2}}\text{x} + \dfrac{1}{\sqrt{1-p^2}}$; $m = \dfrac{-2p}{\sqrt{1-p^2}}$

$$\Rightarrow m^2 = \frac{4p^2}{1-p^2} \Rightarrow m^2 = (4+m^2)p^2$$

$$\Rightarrow p^2 = \frac{m^2}{4+m^2}$$

$$y = mx + \frac{1}{\sqrt{1 - \frac{m^2}{4+m^2}}} \Rightarrow y = mx + \sqrt{\frac{4+m^2}{4}}$$

$$\Rightarrow y = mx + \sqrt{1 + \frac{1}{4}m^2}$$

Which touches the ellipse $\dfrac{x^2}{1/4} + \dfrac{y^2}{1} = 1$

whose eccentricity $e = \sqrt{1 - \dfrac{1}{4}} = \dfrac{\sqrt{3}}{2}$.

16. (a) Equation of tangent is $\dfrac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1$

Sum of intercepts on the axes $l = \dfrac{3\sqrt{3}}{\cos\theta} + \dfrac{1}{\sin\theta}$

$$\frac{dl}{d\theta} = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta = 0$$

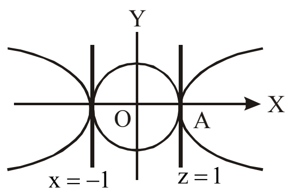
$$\Rightarrow 3\sqrt{3}\tan^3\theta - 1 = 0$$

$$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$

(check for minima by yourself).

17. (a) For the circle $x^2 + y^2 = 1$ and rectangular hyperbola $x^2 - y^2 = 1$, one common tangent is evidently $x = 1$, the other being $x = -1$. There is no other common tangent (verify). We require in standard form the ellipse with focus at S $\left(\frac{1}{2}, 1\right)$ and directrix $x = 1$ which is

$$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = \left(\frac{1}{2}\right)^2 (1-x)^2$$



or $\dfrac{3x^2}{4} - \dfrac{x}{2} + (y-1)^2 = 0$ or $\dfrac{3}{4}\left(x - \dfrac{1}{3}\right)^2 + (y-1)^2$

$= \dfrac{1}{12}$ or $9\left(x - \dfrac{1}{3}\right)^2 + 12(y-1)^2 = 1$

18. (d) Equation of a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is

$$y = mx + \sqrt{a^2 m^2 + b^2} \quad \text{.... (i)}$$

Equation of a perpendicular from the centre (0, 0) on this tangent is $y = -\dfrac{1}{m}x$ .....(ii)

Eliminating m from (i) and (ii), we get the equation of the required locus as $y = -\dfrac{x}{y}.x + \sqrt{a^2 \times \dfrac{x^2}{y^2} + b^2}$

$$\Rightarrow \quad y^2 + x^2 = \sqrt{a^2 x^2 + b^2 y^2}$$

$$\Rightarrow (x^2 + y^2)^2 = a^2 x^2 + b^2 y^2.$$

19. (a) $\dfrac{x^2}{5^2} + \dfrac{y^2}{4^2} = 1.$

Any tangent to the ellipse is $\dfrac{x\cos\theta}{5} + \dfrac{y\sin\theta}{4} = 1$

This meets $x = a = 5$ at $T_1\left\{5, \frac{4}{\sin\theta}(1-\cos\theta)\right\}$

$$= \left(-5, 4\tan\frac{\theta}{2}\right)$$

and meets $x = -a = -5$ at $T_2\left\{-5, \frac{4}{\sin\theta}(1+\cos\theta)\right\}$

$$= \left(-5, 4\cot\frac{\theta}{2}\right)$$

The circle on T_1, T_2, as diameter is

$$(x-5)(x+5) + \left(y - 4\tan\frac{\theta}{2}\right)\left(y - 4\cot\frac{\theta}{2}\right) = 0$$

or $x^2 + y^2 - 4y\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right) - 25 + 16 = 0$

$\Rightarrow x^2 + y^2 - 8y \operatorname{cosec}\theta - 9 = 0$

This is obviously satisfied by (3, 0).

NOTE : It can be verified by taking general equation of an ellipse that such a circle passes through the foci of the ellipse.

20. (b) Let the equation to the rectangular hyperbola be

$x^2 - y^2 = a^2$ .... (i)

As the asymptotes of this are the axes of the other & vice versa, hence the equation of the other hyperbola may be written as $xy = c^2$ .... (ii)

Let (i) and (ii) meet at some point whose coordinates are $(a \sec\alpha, a\tan\alpha)$.

Then the tangent at the point $(a\sec\alpha, a\tan\alpha)$ to (i) is $x - y\sin\alpha = a\cos\alpha$ .... (iii)

and the tangent at the point $(a\sec\alpha, a\tan\alpha)$ to (ii) is

$y + x\sin\alpha = \frac{2c^2}{a}\cos\alpha$ .... (iv)

Clearly, the slopes of the tangents given by (iv) and (iii) are respectively $-\sin\alpha$ and $\frac{1}{\sin\alpha}$, so their product $= -1$. Hence the tangents are at right angle.

21. (c) Let the slopes of the two tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be cm and c/m.

Then the equation of the tangents are

$y = cmx + \sqrt{a^2c^2m^2 - b^2}$ .... (i)

and $my - cx = \sqrt{a^2c^2 - b^2m^2}$ .... (ii)

Squaring and subtracting (ii) from (i), we get $(y - cmx)^2$

$-(my - cx)^2 = a^2c^2m^2 - b^2 - a^2c^2 + b^2m^2$

$\Rightarrow (1-m^2)(y^2 - c^2x^2) = -(1-m^2)(a^2c^2 + b^2)$

$\Rightarrow y^2 + b^2 = c^2(x^2 - a^2)$.

22. (a) Equation of the pair of straight lines joining the origin to the intersection points of the variable chord and the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{2a^2} - \left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2 = 0$

Which will represent a pair of perpendicular lines, if

Coeff. of x^2 + coeff. of $y^2 = 0$

i.e. $\left[\frac{1}{a^2} - \frac{\cos^2\alpha}{p^2}\right] - \left[\frac{1}{2a^2} + \frac{\sin^2\alpha}{p^2}\right] = 0$

i.e. $\frac{1}{a^2} - \frac{1}{2a^2} = \frac{1}{p^2}$

Gives $p = \sqrt{2}\, a$.

Thus the variable chords have a constant perpendicular distance from the origin. Hence, the variable chords are all tangents to the circle centered at origin having radius equal to $\sqrt{2}\, a$.

23. (a) The asymptotes of a rectangular hyperbola are perpendicular to each other.

Given one asymptote, $x + y + c = 0$

Let the other asymptote be $x - y + \lambda = 0$

We also know that the asymptotes pass through centre of the hyperbola. Therefore the line $2x - y = 0$ and the asymptotes must be concurrent.

Thus we have, $\begin{vmatrix} 2 & -1 & 0 \\ 1 & 1 & c \\ 1 & -1 & \lambda \end{vmatrix} = 0$

Gives $\lambda = -3c$.

Hence equation of other asymptote is $x - y - 3c = 0$

24. (c) The equation of the normal at $(a\sec\theta, b\tan\theta)$ to the given hyperbola is $ax\cos\theta + by\cot\theta = (a^2 + b^2)$

This meets the transverse axis i.e. x-axis at G..

So, the coordinates of G are $\left(\frac{a^2+b^2}{a}\sec\theta, 0\right)$

The coordinates of the vertices A and A' are $A\,(a, 0)$ and $A'\,(-a, 0)$ respectively.

$\therefore AG.\, A'G$

$= \left(-a + \frac{a^2+b^2}{a}\sec\theta\right)\left(a + \frac{a^2+b^2}{a}\sec\theta\right)$

$= (-a + ae^2\sec\theta)(a + ae^2\sec\theta)$

$= a^2(e^4\sec^2\theta - 1)$.

25. **(a)** Equation of the tangent at $\frac{\pi}{4}$ is

$$\frac{x\left(\frac{1}{\sqrt{2}}\right)}{a}+\frac{y\left(\frac{1}{\sqrt{2}}\right)}{b}=1$$

i. e., $\frac{x}{a}+\frac{y}{b}-\sqrt{2}=0$ .... (i)

Equation of the normal at $\frac{\pi}{4}$ is

$$\frac{x}{b}-\frac{y}{a}=\frac{a}{b\sqrt{2}}-\frac{b}{a\sqrt{2}} \quad \text{.... (ii)}$$

p_1 = length of the perpendicular from the centre to the tangent $=\left|\frac{-\sqrt{2}}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

p_2 = length of the perpendicular from the centre to the normal $=\left|\frac{\frac{a}{b\sqrt{2}}-\frac{b}{a\sqrt{2}}}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\frac{a^2-b^2}{\sqrt{2}\sqrt{a^2+b^2}}$.

Area of the rectangle $=p_1\,p_2=\frac{ab(a^2-b^2)}{a^2+b^2}$

26. **(d)** Let $P(a\sec\theta, b\tan\theta)$ be any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Equation of the chord of contact of tangents from P to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$ is

$$\frac{xa\sec\theta}{a^2}-\frac{yb\tan\theta}{b^2}=2$$

or $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=2$ ..... (1)

The two hyperbolas have a common set of asymptotes

$$y=\pm\frac{b}{a}x$$

$y=\frac{b}{a}x$ meets the chord of contact of tangents at Q

$$\left(\frac{2a}{\sec\theta-\tan\theta}, \frac{2b}{\sec\theta-\tan\theta}\right)$$

$y=-\frac{b}{a}x$ meets the chord of contact at R

$$\left(\frac{2a}{\sec\theta+\tan\theta}, \frac{-2b}{\sec\theta+\tan\theta}\right)$$

$$\text{Area of } \Delta OQR=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{2a}{\sec\theta-\tan\theta} & \frac{2b}{\sec\theta-\tan\theta} & 1 \\ \frac{2a}{\sec\theta+\tan\theta} & \frac{-2b}{\sec\theta+\tan\theta} & 1 \end{vmatrix}$$

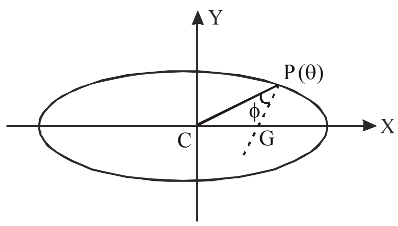
$= 4ab$ square units.

27. **(c)** Any point P on ellipse in $(a\cos\theta, b\sin\theta)$

$\therefore$ Equation of the diameter CP is $y=\left(\frac{b}{a}\tan\theta\right)x$

The normal to ellipse at P is

$$ae\sec\theta-by\,\text{cosec}\,\theta=a^2e^2$$



Slopes of the lines CP and the normal GP are $\frac{b}{a}\tan\theta$ and $\frac{a}{b}\tan\theta$

$$\therefore \quad \tan\phi=\frac{\frac{a}{b}\tan\theta-\frac{b}{a}\tan\theta}{1+\frac{a}{b}\tan\theta.\frac{b}{a}\tan\theta}$$

$$=\frac{a^2-b^2}{ab}\,\frac{\tan\theta}{\sec^2\theta}$$

$$=\frac{a^2-b^2}{ab}\sin\theta\cos\theta$$

$$=\frac{a^2-b^2}{2ab}\sin 2\theta$$

$\therefore$ The greatest value of

$$\tan\phi=\frac{a^2-b^2}{2ab}.1=\frac{a^2-b^2}{2ab}.$$

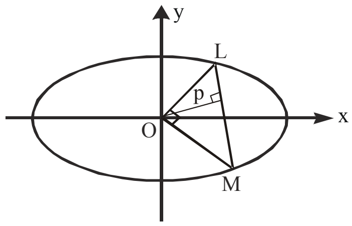
28. **(a)** Let OL and OM be the semi perpendicular diameter of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ..... (i)

Suppose the equation of LM is

$x\cos\alpha + y\sin\alpha = p$ ..... (ii)

So that the perpendicular from centre O upon LM is p.
To find the combined equation of OL & OM,
we have to make (1) homogenous with the help of (2);

So, we get $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2$



or $x^2\left(\frac{1}{a^2} - \frac{\cos^2\alpha}{p^2}\right) - \frac{2xy\sin\alpha\cos\alpha}{p^2}$

$+ y^2\left(\frac{1}{b^2} - \frac{\sin^2\alpha}{p^2}\right) = 0$

As the lines OL and OM are mutually perpendicular, the coefficient of x^2 + coefficient of y^2 must be equal to zero.

So, $\frac{1}{a^2} - \frac{\cos^2\alpha}{p^2} + \frac{1}{b^2} - \frac{\sin^2\alpha}{p^2} = 0$

or $\frac{1}{a^2} + \frac{1}{b^2} = \frac{\sin^2\alpha + \cos^2\alpha}{p^2} = \frac{1}{p^2}$

or $p = \frac{ab}{\sqrt{a^2+b^2}}$.

29. (b) Let t_1, t_2, t_3 and t_4 be the parameters of the point P, Q, R and S respectively. Then, the coordinates of P, Q, R and S are $\left(ct_1, \frac{c}{t_1}\right)$, $\left(ct_2, \frac{c}{t_2}\right)$, $\left(ct_3, \frac{c}{t_3}\right)$ and $\left(ct_4, \frac{c}{t_4}\right)$ respectively.

Now, PQ is perpendicular to RS

$\Rightarrow \frac{\frac{c}{t_2} - \frac{c}{t_1}}{ct_2 - ct_1} \times \frac{\frac{c}{t_4} - \frac{c}{t_3}}{ct_4 - ct_3} = -1$

$\Rightarrow -\frac{1}{t_1t_2} \times -\frac{1}{t_3t_4} = -1$

$\Rightarrow t_1t_2t_3t_4 = -1$ ..... (i)

$\therefore$ Product of the slopes of CP, CQ, CR and CS=

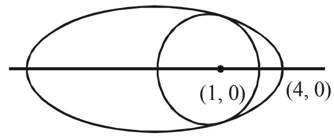
$\frac{1}{t_1^2} \times \frac{1}{t_2^2} \times \frac{1}{t_3^2} \times \frac{1}{t_4^2} = 1$ [Using (i)]

30. (c) Given ellipse is $\frac{x^2}{16} + \frac{y^2}{4} = 1$ ..... (i)

Equation of a circle centered at (1, 0) can be written as
$(x-1)^2 + y^2 = r^2$ ..... (ii)

The abscissae of the intersection points of the circle and the ellipse is given by the equation

$(x-1)^2 + \frac{16-x^2}{4} = r^2$



i.e. $4(x^2 - 2x + 1) + 16 - x^2 = 4r^2$

i.e. $3x^2 - 8x + 20 - 4r^2 = 0$

If the circle lies inside the ellipse, then the roots of the above equation must be imaginary or equal

i.e. $D \le 0$ i.e. $64 + 12(4r^2 - 20) \le 0$

$\Rightarrow r = \le \sqrt{\frac{11}{3}}$

Hence, greatest value of $r = \sqrt{\frac{11}{3}}$ and the equation of required circle is $(x-1)^2 + y^2 = \frac{11}{3}$

i.e. $3(x^2 + y^2) - 6x - 8 = 0$.

31. (a) Equation of the normal at any point $\left(ct, \frac{c}{t}\right)$ to the rectangular hyperbola is $ty - t^3x + ct^4 - c = 0$.

If it passes through a point $\left(ct', \frac{c}{t'}\right)$ on the hyperbola then $t^3t' + 1 = 0$ .... (i)

Which give three value of t say t_1, t_2, t_3 and hence the co-ordinates of three point P, Q, R on the hyperbola the normals at which intersect on the hyperbola.

The coordinates of P, Q, R are $\left(ct_i, \frac{c}{t_i}\right)$ $i = 1, 2, 3$.

So that the coordinates of the centroid of the triangle PQR are $\left(c\dfrac{(t_1+t_2+t_3)}{3},\ \dfrac{c}{3}\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}+\dfrac{1}{t_3}\right)\right)$

or (0, 0), which is the centre of the hyperbola

(as from (i) $t_1+t_2+t_3=0, \sum t_1 t_2=0, t_1 t_2 t_3=-\dfrac{1}{t'}$)

32. **(b)** Equation of the tangent at $\left(ct, \dfrac{c}{t}\right)$ to the hyperbola

$xy = c^2$ is $x.\dfrac{c}{t} + y.ct = 2c^2$

Slope of the tangent $= -\dfrac{1}{t^2}$

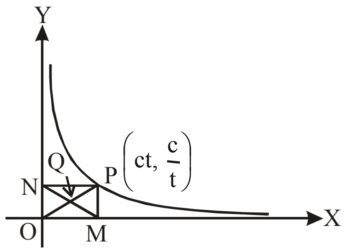
and slope of the normal $= t^2$

Equation of the normal at $\left(ct, \dfrac{c}{t}\right)$ is $y - \dfrac{c}{t} = t^2(x - ct)$

If it passes through $\left(ct', \dfrac{c}{t'}\right)$, then $\dfrac{c}{t'} - \dfrac{c}{t} = t^2$

$(ct' - ct) \Rightarrow (t - t') = t^3 t'(t' - t) \Rightarrow t^3 t' = -1$

33. **(a)** Any point P on the hyperbola is $\left(ct, \dfrac{c}{t}\right)$



Q, the mid - point of $OP = \left(\dfrac{ct}{2}, \dfrac{c}{2t}\right)$

Since $OMPN$ is a rectangle.

[$\because$ mid pt of OP = mid point of MN]

$\therefore$ Locus of Q is $xy = \dfrac{c^2}{4}$ which is a rectangular hyperbola with eccentricity $\sqrt{2}$.

34. **(a)** Equation of normal at any point $\left(ct, \dfrac{c}{t}\right)$ is

$ct^4 - xt^3 + ty - c = 0$ ..... (1)

$\therefore$ Slope of normal $= t^2$

The normal (1) passes through the point $P(h, k)$

$\therefore$ $ct^4 - ht^3 + kt - c = 0$. If the roots of this equation are t_i $i = 1, 2, 3, 4$

then $\sum t_i = \dfrac{h}{c}$ and $\sum t_i t_j = 0$,

$\sum t_i t_j t_k = -\dfrac{k}{c}$ and $t_1 t_2 t_3 t_4 = -1$

Given $\sum t_i^2 = \lambda \Rightarrow \left(\sum t_i\right)^2 - 2\sum t_i t_j = \lambda$

$\Rightarrow \dfrac{h^2}{c^2} - 0 = \lambda \Rightarrow h^2 = \lambda c^2$

$\therefore$ Required locus is $x^2 = \lambda c^2$

35. **(d)** If θ is the angle between the asymptotes, then

$\tan\dfrac{\theta}{2} = \dfrac{b}{a}$

Hence, we have

$\cos\dfrac{\theta}{2} = \dfrac{a}{\sqrt{a^2+b^2}} = \dfrac{1}{\sqrt{1+\dfrac{b^2}{a^2}}} = \dfrac{1}{e}$.

36. **(d)** Equation of the normal to the given hyperbola at $P(\theta)$, is given by

$(a\cos\theta)x + (b\cot\theta)y = a^2 + b^2$ i.e. $x + \left[\dfrac{b}{a\sin\theta}\right]y$

$= \dfrac{a^2+b^2}{a\cos\theta}$ .....(i)

and equation of the normal at Q(ϕ), is given by $x + \left[\dfrac{b}{a\sin\phi}\right]y = \dfrac{a^2+b^2}{a\cos\phi}$ .....(ii)

Subtracting equation (ii) from equation (i) gives the ordinate k of their intersection point as.

$k \times \dfrac{b}{a}\left[\dfrac{1}{\sin\theta} - \dfrac{1}{\sin\phi}\right] = \left[\dfrac{a^2+b^2}{a}\right]\left[\dfrac{1}{\cos\theta} - \dfrac{1}{\cos\phi}\right]$

i.e. $k = \dfrac{a^2+b^2}{b} \times \dfrac{\cos\theta - \cos\phi}{\sin\theta - \sin\phi} \times \tan\theta \,.\, \tan\phi$

$= \dfrac{a^2+b^2}{b} \times \dfrac{2\sin\left(\dfrac{\theta+\phi}{2}\right).\sin\left(\dfrac{\theta-\phi}{2}\right)}{2\cos\left(\dfrac{\theta+\phi}{2}\right).\sin\left(\dfrac{\theta-\phi}{2}\right)} \times 1$

if $\theta + \phi = \dfrac{\pi}{2}$ then $\tan\theta.\tan\phi = 1$

$= -\left[\dfrac{a^2+b^2}{b}\right].\tan\left[\dfrac{\theta+\phi}{2}\right] = -\left[\dfrac{a^2+b^2}{b}\right]$

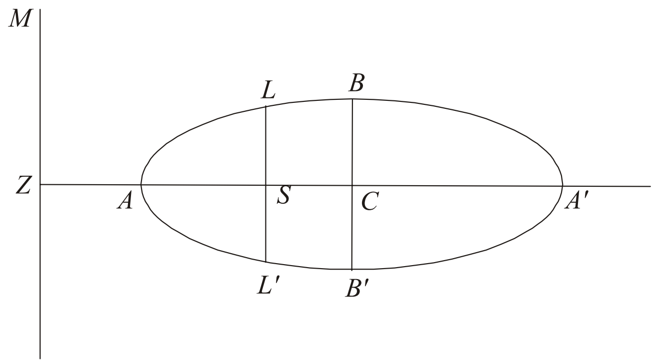
37. **(d)** For the two ellipses to intersect in 4 distinct points, $a > 1$

$\Rightarrow \quad b^2 - 5b + 7 > 1$

$b^2 - 5b + 6 > 0 \Rightarrow b \in (-\infty, 2) \cup (3, \infty)$.

$\therefore \quad b$ does not lie in [2, 3].

38. **(b)** Let S be the given point and ZM be the given line.



Then $SZ = \frac{a}{e} - ae = \frac{a}{e}(1-e^2) = \frac{b^2}{ae} = k$ (say)

$\left[\because b^2 = a^2(1-e^2)\right]$

Now take SC as x-axis and LSL' as y-axis.

Let (x, y) be the coordinates of B w.r.t. these axes then $x = SC = ae, y = CB = b$

hence $\frac{y^2}{x} = \frac{b^2}{ae} = SZ$, which is constant

$\therefore \; y^2 = kx$ is the required locus which is a parabola.

39. **(a)** Normal at P is bisector of angle between focal distances SP and $S'P$. Hence

$$\frac{SQ}{S'Q} = \frac{SP}{S'P}$$

Now the eccentricity of ellipse, $e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

Focal distances are $a + ex$ and $a - ex$, so

$$SP = a - ex = 4 - \frac{\sqrt{7}}{4} \times 2 = \frac{8 - \sqrt{7}}{2}$$

$$S'P = a + ex = 4 + \frac{\sqrt{7}}{4} \times 2 = \frac{8 + \sqrt{7}}{2}$$

$$\therefore \; \frac{SQ}{S'Q} = \frac{8 - \sqrt{7}}{8 + \sqrt{7}}$$

40. **(b)** The chord of contant of tangents from (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$. It meets the axes at the points $\left(\frac{a^2}{x_1}, 0\right)$ and $\left(0, \frac{b^2}{y_1}\right)$.

Area of the triangle $= \frac{1}{2} \cdot \frac{a^2}{x_1} \cdot \frac{b^2}{y_1} = k$ (constant)

$\Rightarrow \; x_1 y_1 = \frac{a^2 b^2}{2k} = c^2$ (c is constant)

So, $xy = c^2$ is the required locus.

41. **(b)** The curve $xy = c$ and the circle $x^2 + y^2 = 1$ touch each other so

$x^2 + \frac{c^2}{x^2} - 1 = 0 \Rightarrow x^4 - x^2 + c^2 = 0$

will have equal roots so $(-1)^2 - 4c^2 = 0 \Rightarrow 4c^2 = 1$

$\Rightarrow \quad c^2 = \frac{1}{4} \Rightarrow c = \frac{1}{2}$.

Roots of the equations $x^4 - x^2 + \frac{1}{4} = 0$ are $x = \pm\frac{1}{\sqrt{2}}$

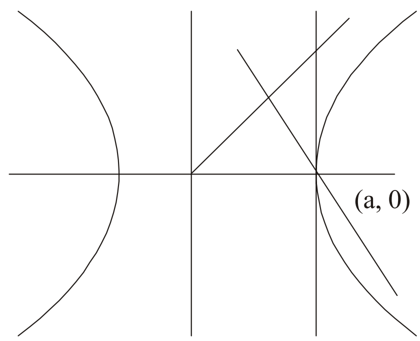
$\Rightarrow \quad y = \mp\frac{1}{\sqrt{2}}$.

Clearly distance between the points of contact =2 units.

42. **(b)** The line $y + \lambda(x - a) = 0$ will intersect the portion of the asymptote in the 1st quadrant only if its slope is negative.

$\Rightarrow \quad -\lambda < 0 \Rightarrow \lambda < 0$

$\therefore \quad \lambda \in (0, \infty)$



43. **(a)** Eqn. of normal at any point $\left(ct, \frac{c}{t}\right)$ is

$ct^4 - xt^3 + ty - c = 0$

$\Rightarrow$ slope of normal $= t^2$

Let P be (h, k) $\Rightarrow$ $ct^4 - ht^3 + tk - c = 0$

$\Rightarrow$ $\sum t_i = \frac{h}{c}$ and $\sum t_i t_j = 0$

$\Rightarrow$ $\sum t_i^2 = \left(\sum t_i\right)^2$

$\Rightarrow$ $h^2 = c^2\lambda$

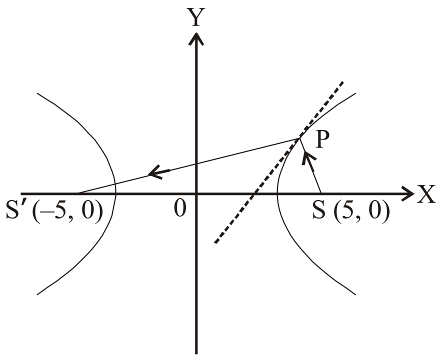
$\Rightarrow$ required locus is $x^2 = \lambda c^2$

44. **(d)** For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1,$

eccentricity, $e = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Coordinates of foci are $\pm(5, 0)$.

We note that the point (5, 0) is satisfying the equation of incident ray $3x + (5 - 4\sqrt{2})y = 15.$ So, the reflected ray must pass through the other focus (–5, 0)



Let the incident ray strikes at point $P(4\sec\theta, 3\tan\theta)$ of hyperbola, then

$$3.4\sec\theta + (5 - 4\sqrt{2}).3\tan\theta = 15$$

$\Rightarrow$ $12(\sec\theta - \sqrt{2}\tan\theta) = 15\,(1 - \tan\theta)$

A simple observation hints that $\theta = \frac{\pi}{4}$ vanishes both the sides hence $\theta = \frac{\pi}{4}$ is the solution (The equation can be solved in usual ways but it is lengthy)

So, the point P is $(4\sqrt{2}, 3)$

Now the equation of the reflected ray PS' is

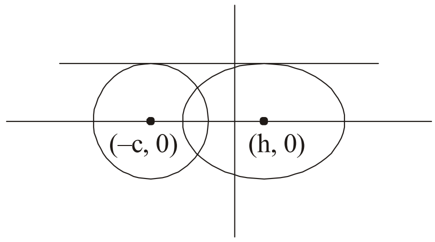
$$y - 0 = \frac{3 - 0}{4\sqrt{2} + 5}(x + 5) \Rightarrow 3x - y(4\sqrt{2} + 5) + 15 = 0$$

45. **(b)** The given circle and ellipse have common tangent parallel to x-axis only.

$\Rightarrow$ the circle and the ellipse intersect at 2 distinct points.

$\Rightarrow$ $h + c < a + b$

$\Rightarrow$ $c < a + b - h.$



46. **(c)** Given $m(n-1) = n$

n is divisible by $n - 1$

$\Rightarrow n = 2 \Rightarrow m = 2$

Hence chord of contact of tangents drawn from (2, 2) to $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $\frac{2x}{9} + \frac{2y}{4} = 1$

$\Rightarrow 4x + 9y = 18.$

47. **(d)** Equations of tangents at two points having eccentric angles θ_1 and θ_2 are

$$\frac{x}{a}\cos\theta_1 + \frac{y}{b}\sin\theta_1 = 1 \quad ...(1)$$

$$\frac{x}{a}\cos\theta_2 + \frac{y}{b}\sin\theta_2 = 1 \quad ...(2)$$

The point of intersection of (1) and (2) is

$$\left(\frac{a\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \frac{b\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}\right)$$

It is given that $\theta_1 + \theta_2 = 2k =$ constant

$\therefore$ point of intersection $A(x_1, y_1)$ is

$$x_1 = \frac{a\cos k}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, y_1 = \frac{b\sin k}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$$

$\Rightarrow$ $\frac{x_1}{y_1} = \frac{a}{b}\cot k$

$\therefore$ Required locus is $\frac{x}{y} = \frac{a}{b}\cot k = \frac{a}{b}\cot\left(\frac{\theta_1+\theta_2}{2}\right)$

which is a straight line.

48. **(a)** Let (h, k) be the point.

Equation of chord of contact with respect to ellipse $x^2 + 2y^2 = 1$ is given by $hx + 2ky = 1$ ...(1)

Making curve $x^2 + 2y^2 = 1$ homogenous with the help of (1) we can write.

$x^2 + 2y^2 = (hx + 2ky)^2$

Now coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad (1-h^2)+(2-4k^2)=0$

$\Rightarrow \quad h^2 + 4k^2 = 3$

so locus of (h,k) is $x^2 + 4y^2 = 3$.

49. (b) Here $a^2 + b^2 = 1 \Rightarrow a^2 + a^2(1-e^2) = 1$

$\Rightarrow \quad 2 - e^2 = \frac{1}{a^2} \Rightarrow e^2 = \frac{2a^2-1}{a^2}$

$\Rightarrow$ Extremities of latus rectum are

$\left(\pm\sqrt{2a^2-1}, \pm\frac{1-a^2}{a}\right)$

on eliminating a, from equations

$x = \pm\sqrt{2a^2-1}$ and

$y = \pm\frac{1-a^2}{a}$

we get

$\Rightarrow 2y^2(1+x^2) = (1-x^2)^2$.

50. (a) $\tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}$

$\tan\phi = \frac{b\sin\theta}{a\cos\theta} = \frac{b}{a}\tan\theta$

$\therefore \tan(\theta-\phi) = \frac{\tan\theta(a-b)}{a+b\tan^2\theta} = \frac{a-b}{a\cot\theta + b\tan\theta}$

$\because \theta \in \left(0, \frac{\pi}{2}\right) \Rightarrow a\cot\theta + b\tan\theta \ge 2\sqrt{ab}$ and equality occurs if $a\cot\theta = b\tan\theta$

$\Rightarrow \quad \tan(\theta-\phi) \le \frac{a-b}{2\sqrt{ab}}$ and maxima occurs if

$\tan^2\theta = \frac{a}{b}$

51. (c) Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Any point on one of the directrix is $P\left(\frac{a}{e}, k\right)$.

Chord of contact of P w.r.t the ellipse is

$\frac{a}{e}\frac{x}{a^2} + \frac{ky}{b^2} = 1$ ...(1)

chord of contact of P w.r.t the auxiliary circle is

$\frac{a}{e}x + ky = a^2$ ...(2)

(1) and (2) intersect at (ae, 0).

52. (b) Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and let $A \equiv (a\cos\theta, b\sin\theta)$.

Equation of AC will be, $y = \frac{b}{a}\tan\theta$.

Solving with $x = \frac{a}{e}$ we get $P \equiv \left(\frac{a}{e}, \frac{b}{e}\tan\theta\right)$.

Slope of tangent at $A = -\frac{b}{a\tan\theta}$

Slope of $PS = \frac{\frac{b}{e}\tan\theta}{\frac{a}{e}-ae} = \frac{b\tan\theta}{a(1-e^2)} = \frac{a}{b}\tan\theta$.

So $\alpha = \frac{\pi}{2}$.

53. (b) Given ellipse is $\frac{x^2}{2}+\frac{y^2}{1}=1$.

Equation of normal to the ellipse at (x_1, y_1) is $\frac{x-x_1}{x_1/2} = \frac{y-y_1}{y_1/2}$.

It passes through (2, 3),

$\therefore \frac{2(2-x_1)}{x_1} = \frac{3-y_1}{y_1}$

$\Rightarrow 4y_1 - 2x_1y_1 = 3x_1 - x_1y_1$

$\therefore$ Locus of (x_1, y_1) is $xy + 3x - 4y = 0$.

54. (b) The mutually perpendicular tangents will be drawn by taking any point on the director circle of given ellipse. Any point on the director circle can be taken as $(5\cos\theta, 5\sin\theta)$. Equation of corresponding chord of contacct is

$\frac{5x}{16}\cos\theta + \frac{5y}{9}\sin\theta - 1 = 0$.

It's distance from the origin is equal to

$$\frac{1}{\sqrt{\frac{25\cos^2\theta}{256}+\frac{25\sin^2\theta}{81}}} = \frac{144}{5}\frac{1}{\sqrt{256-175\cos^2\theta}} \le \frac{144}{5.9}$$

$= \frac{16}{5}$

NOTE : The equation of director circle is $x^2+y^2=a^2+b^2$

55. (a) We know that if a circle cuts a rectangular hyperbola then arithmetic mean of points of intersection is the mid-point of centre of hyperbola and circle.

So, $\frac{3+5+2+(-1)}{4}=\frac{-g+1}{2},\ \frac{4+3+6+0}{4}=\frac{-f+2}{2}$

$\Rightarrow\quad g+f=\left(-\frac{7}{2}\right)+\left(-\frac{9}{2}\right)=-8.$

56. (a) $f(x)=px+qy \Rightarrow f(x)=px+\frac{qr^2}{x}$

$f'(x)=p-\frac{qr^2}{x}=0 \Rightarrow x=\pm r\sqrt{\frac{q}{p}}$

$f''(x)>0$ for $x=r\sqrt{\frac{q}{p}}$

$\therefore\ f(x)_{\min}=pr\sqrt{\frac{q}{p}}+\frac{qr^2}{r\sqrt{\frac{q}{p}}}$

$=\sqrt{pq}.r+\sqrt{pq}.r=2r\sqrt{pq}$

ALTERNATE SOLUTION

Any point on $xy=r^2$ is $\left(rt,\frac{r}{t}\right)$

$\therefore\ f(x)=px+qy=prt+q\left(\frac{r}{t}\right)$

If $t>0$ then $\frac{prt+\frac{qr}{t}}{2}\ge\sqrt{prt.\frac{qr}{t}}$

$\therefore\ prt+\frac{qr}{t}\ge 2r\sqrt{pq}$

$\therefore\ f(x)\ge 2r\sqrt{pq}$

57. (d) The equation of the normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ at $(2\sec\theta,\tan\theta)$ is

$2x\cos\theta+y\cot\theta=5$ ...(1)

Slope of the normal $=-2\sin\theta=-1 \Rightarrow \sin\theta=\frac{1}{2}$

$\Rightarrow\quad \theta=\frac{\pi}{6}.$

Y–intercept of the normal $=\frac{5}{\cot\theta}=\frac{5}{\sqrt{3}}$

Since it touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore\ \left(\frac{5}{\sqrt{3}}\right)^2=a^2(-1)^2+b^2 \Rightarrow a^2+b^2=\frac{25}{3}$

B COMPREHENSION TYPE

1. (b) Two curves are symmetrical about both axes and intersect in four points, so, the circle through their points of intersection will have centre at origin.

Solving $x^2-y^2=0$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get

$x^2=y^2=\frac{a^2b^2}{a^2+b^2}$

Therefore radius of circle $=\sqrt{\frac{2a^2b^2}{a^2+b^2}}=\frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

2. (c) The equations of chords of the ellipse are

$\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$

and $\frac{x}{a}\cos\frac{\gamma+\delta}{2}+\frac{y}{b}\sin\frac{\gamma+\delta}{2}=\cos\frac{\gamma-\delta}{2}$

If $\alpha,\beta,\gamma,\delta$ are concyclic points then these lines will make equal angles with axes and therefore,

$\tan\frac{\alpha+\beta}{2}=-\tan\frac{\gamma+\delta}{2}$

$\Rightarrow \frac{\alpha+\beta}{2}+\frac{\gamma+\delta}{2}=n\pi \Rightarrow \alpha+\beta+\gamma+\delta=2n\pi,\ n\in I$

3. (c) As the lines joining common points of intersection must be equally inclined to axes, so,

$\tan\alpha=-\tan\beta \Rightarrow \alpha+\beta=\pi$

4. (c) Any curve through the points of intersection of given curves is

$x^2+y^2-4x-5+\lambda(x^2+y^2-1)=0$

$\Rightarrow (1+\lambda)x^2+(1-\lambda)y^2-4x-5-\lambda=0$

It represents a pair of straight lines, when

$(1+\lambda)(1-\lambda)(-5-\lambda)+0-(1+\lambda)0$
$-(1-\lambda).(-2)^2+(5+\lambda).0=0$

$\Rightarrow (\lambda-1)\{(\lambda+1)(\lambda+5)+4\}=0$

$\Rightarrow (\lambda-1)(\lambda^2+6\lambda+9)=0$

$\Rightarrow (\lambda-1)(\lambda+3)^2=0 \Rightarrow \lambda=1$ or -3

Hence, two pairs of straight lines can be drawn.

7. **(c)** $21x^2-6xy+29y^2+6x-58y-151=0$

$\Rightarrow 3(x-3y+3)^2+2(3x+y-1)^2=180$

$$\Rightarrow \frac{(x-3y+3)^2}{60}+\frac{(3x+y-1)^2}{90}=1$$

$$\left(\frac{x-3y+3}{\sqrt{1+3^2}\sqrt{6}}\right)^2+\left(\frac{3x+y-1}{\sqrt{1+3^2}.3}\right)^2=1$$

$$\frac{X^2}{6}+\frac{Y^2}{9}=1$$

Thus C is an ellipse whose lengths of axes are $6, 2\sqrt{6}$.

The major and the minor axes are $x-3y+3=0$ and $3y+y-1=0$ respectively. Their point of intersection gives the centre of the conic.

$\therefore$ centre $=(0,1)$.

Eccentricity $e=\sqrt{1-\left(\frac{2\sqrt{6}}{6}\right)^2}=\frac{1}{\sqrt{3}}$

8. **(a)** Equation of tangent in parametric form

$$\frac{x-1}{-1/\sqrt{2}}=\frac{y-1}{1/\sqrt{2}}=\pm 3\sqrt{2}$$

$\Rightarrow A\equiv(4,-2), B\equiv(-2,4)$

Equation of asymptotes (OA and OB) are

$y+2=\frac{-2}{4}(x-4) \Rightarrow 2y+4+x-4=0$

$2y+x=0$

and $y-4=\frac{4}{-2}(x+2) \Rightarrow y-4=-2x-4$

$2x+y=0$

$(2x+y)(x+2y)=0 \Rightarrow 2x^2+2y^2+5xy=0.$

9. **(c)** $m_{OA}=-\frac{1}{2}, m_{OB}=-2$

$\therefore \tan\theta=\left|\frac{-1/2+2}{1+1}\right|=\frac{3}{4}$

$\sin\theta=\frac{3}{5} \Rightarrow \theta=\sin^{-1}\frac{3}{5}.$

10. **(b)** Equation of the hyperbola is $2x^2+2y^2+5xy+\lambda=0.$

It passes through $(1, 1)$

So, $2+2+5+\lambda=0 \Rightarrow \lambda=-9$

Hyperbola $2x^2+2y^2+5xy=9$ so, the tangent at $\left(-1,\frac{7}{2}\right)$ is

$$2x(-1)+2y\left(\frac{7}{2}\right)+5\frac{x.7/2+(-1)y}{2}=9.$$

$3x+2y=4.$

11. **(b)** Clearly centre of the ellipse is origin. Taking point $(r\cos\theta, r\sin\theta)$, we get

$$r^2=\frac{3+\sqrt{2}}{3-\sqrt{2}\sin\left(2\theta+\frac{\pi}{4}\right)}$$

$$r_{\max}=\sqrt{\frac{3+\sqrt{2}}{3-\sqrt{2}}}, r_{\min}=1$$

max exist when $2\theta+\frac{\pi}{4}=\frac{\pi}{2} \Rightarrow \theta=\frac{\pi}{8}.$

12. **(b)** Length of semi minor axis $=1$ unit $\Rightarrow$ tangent is at the end of minor axis. so its equation can be

$x\sin\frac{\pi}{8}-y\cos\frac{\pi}{8}=\pm 1. \Rightarrow x\cos\frac{3\pi}{8}-y\sin\frac{3\pi}{8}=\pm 1$

13. **(c)** Product of length of perpendiculars from foci $=b^2$ (b is length of semi minor axis) $=1$.

14. **(b)** If C is the centre of the ellipse then slope of CP, where $C(0,0)$ and $P(a\cos\phi_1, b\sin\phi_1)$ is

$$m_1=\frac{b\sin\phi_1}{a\cos\phi_1}=\frac{b}{a}\tan\phi_1.$$

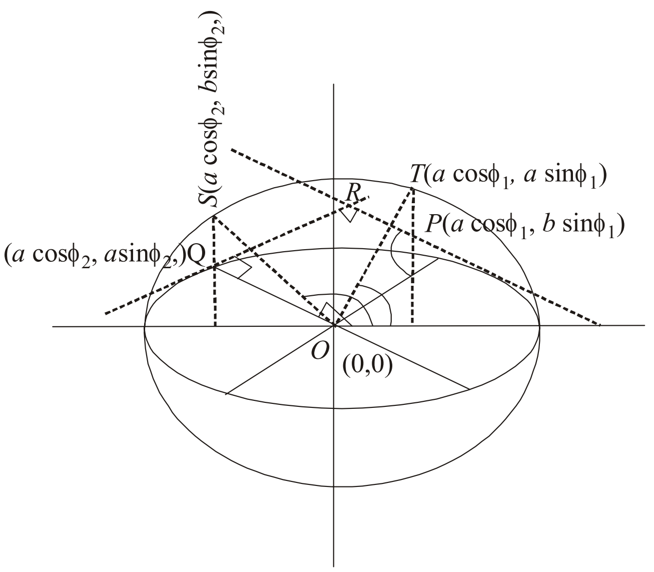
Similarly slope of CQ is $\frac{b}{a}\tan\phi_2$.

Since CP and CQ are conjugate,

$\frac{b^2}{a^2}\tan\phi_1\tan\phi_2=-\frac{b^2}{a^2} \Rightarrow \tan\phi_1\tan\phi_2=-1$

$\Rightarrow \sin\phi_1\sin\phi_2+\cos\phi_1\cos\phi_2=0$

$\Rightarrow \cos(\phi_1\sim\phi_2)=0 \Rightarrow \phi_1\sim\phi_2=\pm 90°.$



15. (b) If the point P is $(a\cos\phi, b\sin\phi)$, then Q is $(-a\sin\phi, b\cos\phi)$, and hence

$$CP^2 + CQ^2 = a^2\cos^2\phi + b^2\sin\phi$$
$$+a^2\sin^2\phi + b^2\cos^2\phi$$

$= a^2 + b^2.$

16. (c) By definition the tangent at P is parallel to CQ and tangent at Q is parallel to CP

$\Rightarrow$ $CPRQ$ is a parallelogram. Also

P and Q are respectively P $(a\cos\phi, b\sin\phi)$, and $Q(-a\sin\phi, b\cos\phi)$. Equation of the tangent at P is $\frac{x\cos\phi}{a} + \frac{y\sin\phi}{b} = 1$. The length of the perpendicular from C to this tangent

i.e. $PR = \frac{ab}{\sqrt{b^2\cos^2\phi + a^2\sin\phi}} = \frac{ab}{CQ}$

$\Rightarrow$ area of parallelogram $CPRQ = ab$.

(17-19) $a - 2b + c = 0$

$\Rightarrow$ $ax + by + c = 0$

passes through $(1, -2)$

So, the centre of the ellipse is $(1, -2)$, which is also the centre of the auxiliary circle, so, $-\alpha = 1, -\beta = -2$

$\Rightarrow$ $\alpha = -1, \beta = 2$.

Radius of auxiliary circle $= \sqrt{1+4-1} = 2$

$\therefore$ Major axis of ellipse = 4. Also if segments of focal chord are l_1 and l_2 then $\frac{1}{l_1} + \frac{1}{l_2} = \frac{2}{(b^2/a)}$

$\therefore$ $\frac{1}{1} + \frac{1}{3} = \frac{4}{b^2}$ $\Rightarrow b^2 = 3$

$\therefore$ Equation of director circle is

$(x-1)^2 + (y+2)^2 = 4 + 3$

Eccentricity $= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

(20-22) Incident beam have equation $\lambda x - y + 2(1+\lambda) = 0$

$\Rightarrow$ $\lambda(x+2) + (-y+2) = 0$

$\Rightarrow$ the rays are coming from (–2, 2) Reflected beam follow the equation $\mu x - y + 2(1-\mu) = 0$

$\Rightarrow$ $\mu(x-2) + (-y+2) = 0$

$\Rightarrow$ the rays converge at (2, 2)

So, the *foci* of the ellipse are $S_1(2,2)$ and $S_2(-2,2)$

The centre is $C(0,2)$. Further the given circle must be auxiliary circle. So its radius = 3 = semi-major axis (a)

20. $S_1S_2 = 4 = 2al.$ Thus $e = \frac{2}{3}$

21. The triangle has largest area if the point of incidence coincedes with an end of major axis

$\therefore$ Largest area $= bae = a\sqrt{1-e^2}\,ae$

$= 3\sqrt{1 - \frac{4}{9}} \times 2 = 2\sqrt{5}$

22. Since for every position of the point P on the ellipse $PS_1 + PS_2 =$ constant $(= 2a = 6)$. So, no such point exists.

C REASONING TYPE

1. (a) Distance between foci = 2 ae

Sum of focal distance = 2 $a >$ 2 ae ($\because$ $e < 1$)

2. (d) The equation represents an ellipse if

$p \in (8,12), p \neq 10.$ for $p = 10,$ it is a circle.

3. (b) Any point on ellipse is $(3\cos\theta, \sin\theta)$. Clearly only integral points are $(\pm 3, 0), (0, \pm 2)$

.

4. **(c)** Let the centre be origin and a vertex be $(a, 0)$ then focus is $(2a, 0) \Rightarrow e = 2$

$\therefore \quad b = a\sqrt{4-1} = \sqrt{3}\,a$

$\therefore$ Equation of hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{3a^2} = 1$

$\Rightarrow \quad 3x^2 - y^2 = 3a^2$

$\therefore$ Latus rectum $= \dfrac{2b^2}{a} = 6a$

D MULTIPLE CORRECT CHOICE TYPE

1. **(a,b)** If two foci be $S\,(5, 12)$ and $S'(24, 7)$ and it passes through origin O.

Then $SO = \sqrt{25+144} = 13$; $S'O = \sqrt{576+49} = 25$

and $SS' = \sqrt{386}$

If conic be an ellipse, then $SO + S'O = 2a$ and $SS' = 2ae$

$\therefore \quad e = \dfrac{SS'}{SO + S'O} = \dfrac{\sqrt{386}}{38}$

If conic be a hyperbola, then $S'O - SO = 2a$ and $SS' = 2ae$

$\therefore \quad e = \dfrac{SS'}{S'O + SO} = \dfrac{\sqrt{386}}{12}$

2. **(a,b,c)** $PS + PS' = ePZ + ePZ' = e\,(PZ + PZ') = e.\,ZZ' = e.\dfrac{2a}{e}$

$= 2a$, if $a > b$ $= e.\dfrac{2b}{e} = 2b$, if $a < b$

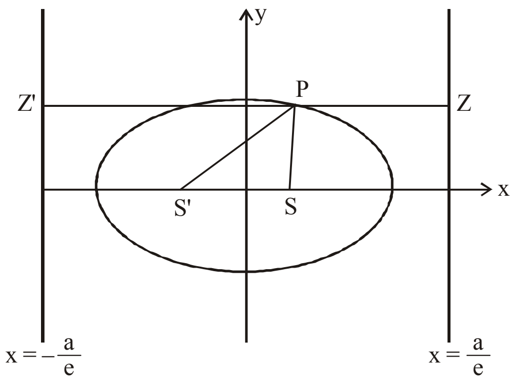
If co-ordinates of P be (x, y) then

$PS + PS' + SS' = 2a + 2ae = 2a\,(1 + e)$

$\therefore\ s = a\,(1 + e)$, semi perimeter of the $\Delta PSS'$ and $s - SS' = a\,(1 - e)$

Now, $\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)(s-a)(s-b)}{s(s-b)\,s(s-c)}}$

$= \dfrac{s-a}{s} = \dfrac{1-e}{1+e}$ $\quad [a = SS', PS' = c, PS = b)$



$= \dfrac{a - \sqrt{a^2-b^2}}{a + \sqrt{a^2-b^2}} = \dfrac{a^2 + a^2 - b^2 - 2a\sqrt{a^2-b^2}}{b^2}$

$= \dfrac{2a^2 - b^2 - 2a\sqrt{a^2-b^2}}{b^2}$.

3. **(a,b,c)** Let $(x_i, y_i) = \left(\sqrt{2}t_i, \dfrac{\sqrt{2}}{t_i}\right)$

$i = 1, 2, 3, 4.$

The equation of the normal at $\left(\sqrt{2}t, \dfrac{\sqrt{2}}{t}\right)$ to $xy = 2$ is

$y - \dfrac{\sqrt{2}}{t} = t^2(x - \sqrt{2}t)$

[$\because$ Equation of the tangent at $\left(\sqrt{2}t, \dfrac{\sqrt{2}}{t}\right)$ to $xy = 2$ is

$\sqrt{2}\,ty + \dfrac{\sqrt{2}}{t}x = 4$. Slope of the tangent is $-\dfrac{1}{t^2}$]

$\Rightarrow \quad ty = t^3x + \sqrt{2} - \sqrt{2}t^4$. If it passes through (3, 4) then

$4t = 3t^3 + \sqrt{2} - \sqrt{2}t^4$ or $\sqrt{2}t^4 - 3t^3 + 4t - \sqrt{2} = 0$

If the roots of this equation are t_1, t_2, t_3, t_4 then

$t_1 + t_2 + t_3 + t_4 = \dfrac{3}{\sqrt{2}} \Rightarrow x_1 + x_2 + x_3 + x_4 = 3$

$\sum t_1t_2 = 0$ and $\sum t_1 t_2 t_3 = -\dfrac{4}{\sqrt{2}}$

and $\quad t_1 t_2 t_3 t_4 = -1 \Rightarrow y_1 y_2 y_3 y_4$

$= x_1 x_2 x_3 x_4 = -(\sqrt{2})^4 = -4$

and $\quad y_1 + y_2 + y_3 + y_4 = \sqrt{2}\left[\dfrac{1}{t_1} + \dfrac{1}{t_2} + \dfrac{1}{t_3} + \dfrac{1}{t_4}\right]$

$= \dfrac{\sqrt{2}\sum t_1t_2t_3}{t_1t_2t_3t_4} = 4.$

4. **(b,c,d)** Let $(x_i, y_i) = \left(t_i, \frac{1}{t_i}\right)$ $i = 1, 2, 3, 4.$

Any point on the rectangular hyperbola $xy = 1$ is $\left(t, \frac{1}{t}\right)$ which lies on the circle

$x^2 + y^2 = 1$ if $t^2 + \frac{1}{t^2} = 1 \Rightarrow t^4 - t^2 + 1 = 0$

The roots of this equation are t_1, t_2, t_3, t_4 where

$t_1 + t_2 + t_3 + t_4 = 0 \Rightarrow x_1 + x_2 + x_3 + x_4 = 0$

$$\sum t_1 t_2 = -1, \sum t_1 t_2 t_3 = 0$$

$t_1\, t_2\, t_3\, t_4 = 1 \Rightarrow x_1 x_2 x_3 x_4 = y_1 y_2 y_3 y_4 = 1$

and $y_1 + y_2 + y_3 + y_4 = \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4}$

$$= \frac{\sum t_1 t_2 t_3}{t_1 t_2 t_3 t_4} = 0.$$

5. **(a,b)** We have $\left|\sqrt{x^2 + (y-1)^2} - \sqrt{x^2 + (y+1)^2}\right| = K$

Which is equivalent to $|S_1P - S_2P| =$ Const.

Where $S_1 \equiv (0, 1)$, $S_2 \equiv (0, -1)$ and $P \equiv (x, y)$.

Using properties of a hyperbola, the above equation represents a hyperbola, then we have.

$2a = K$

[where 2a is the transverse axis and e is the eccentricity]

and $2ae = S_1\, S_2 = 2$

Dividing, we have $e = \frac{2}{K}$

Since, $e > 1$ for a hyperbola, therefore $K < 2$.

Also, K must be a positive quantity. Hence, we have, $K \in (0, 2)$.

6. **(b,c)** Clearly the vertices of the squares will lie on the director circle, i.e. on $x^2 + y^2 = 4 + 3$ and hence the area of the squares is $2\,(4 + 3) = 14$. Only one such square is possible

7. **(a,b,d)**

Let $P(x_1, y_1)$ be any point on the hyperbola $x^2 - y^2 = 4$, then equation of the normal at P is

$y - y_1 = -\frac{y_1}{x_1}(x - x_1) \Rightarrow x_1 y + y_1 x = 2x_1 y_1$.

Then coordinates of G are $(2x_1, 0)$ and of g are $(0, 2y_1)$

So that $PG = \sqrt{(2x_1 - x_1)^2 + y_1^2} = \sqrt{x_1^2 + y_1^2} = PC$

$Pg = \sqrt{x_1^2 + (2y_1 - y_1)^2} = \sqrt{x_1^2 + y_1^2} = PC$

and $Gg = \sqrt{(2x_1)^2 + (2y_1)^2} = 2\sqrt{x_1^2 + y_1^2} = 2PC$

8. **(b,c)** The given ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2\, b^2} = 1$ has eccentricity $e = \sqrt{1 - b^2}$.

Thus if $P\,(h, k)$ be the ends of the latus rectum of the ellipse then we have

$h = \pm\, a\, e = \pm\, a\sqrt{1 - b^2}$, $k = \pm\frac{a^2\, b^2}{a} = \pm\, a\, b^2$

Eliminating b from equations (i) and (ii), we have

$1 - \frac{h^2}{a^2} = \pm\frac{k}{a}$ i.e. $h^2 \pm ak = a^2$

Hence, equation of the required lotus is $x^2 \pm a\, y = a^2$

9. **(a, b, c, d)**

Equation of the curve passing through all four points A, B, C, D can be written as

$(3x + 4y - 24)(4x + 3y - 24) + \lambda xy = 0.$

$\Rightarrow 12x^2 + 12y^2 + (25 + \lambda)xy - 168x - 168y + 576 = 0$

Clearly for $\lambda = -25$, it represents a circle for different values of λ, it can represent other curves

10. **(a, b)**

Suppose the chords $l_1 x + m_1 y = 1$ and $l_2 x + m_2 y = 1$ cut the ellipse at P and Q and R and S respectively, such that the normals to ellipse at P, Q, R and S pass through $T\,(h, k)$. Let (α, β) be one of the points P, Q, R or S. Then equation of normal at (α, β) is

$$\frac{a^2 x}{\alpha} - \frac{b^2 y}{\beta} = a^2 - b^2$$

it passes through $T\,(h, k)$. Therefore

$$\frac{a^2 h}{\alpha} - \frac{b^2 k}{\beta} = a^2 - b^2$$

$\therefore$ locus of (α, β) is $\frac{a^2 h}{x} - \frac{b^2 k}{y} = a^2 - b^2$

$\Rightarrow a^2 hy - b^2 kx = (a^2 - b^2)xy$ ...(1)

The equation of the curve passing through the intersection of the elllipse and one of the chords $l_1 x + m_1 y = 1$ or $l_2 x + m_2 y = 1$ is

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)+\lambda(l_1x+m_1y-1)(l_2x+m_2y-1)=0$$

...(2)

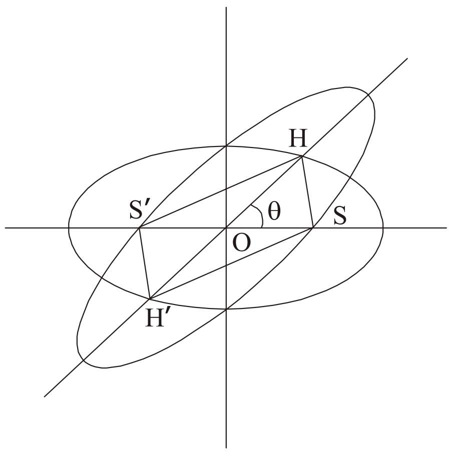
(1) and (2) both represents the same curve.

$$\therefore \frac{1}{a^2}+\lambda l_1 l_2=0, \frac{1}{b^2}+\lambda m_1 m_2=0 \text{ and } -1+\lambda=0$$

$$\Rightarrow \frac{1}{a^2}+l_1 l_2=0, \frac{1}{b^2}+m_1 m_2=0$$

$$\Rightarrow l_1 l_2=-\frac{1}{a^2}, m_1 m_2=-\frac{1}{b^2}.$$

11. (a, b, c)



Clearly O is the point of SS' and HH'

$\Rightarrow$ diagonals of quadrilateral $HSH'S'$ bisect each other so, it is a parallelogram.

Let $H'OH=2r \Rightarrow OH=r=ae_2$

H lies on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (suppose)

$$\therefore \frac{r^2\cos^2\theta}{a^2}+\frac{r^2\sin^2\theta}{b^2}=1$$

$$e_2^2\cos^2\theta+\frac{e_2^2\sin^2\theta}{1-e_1^2}=1 \qquad \left(\because b^2=a^2(1-e_1^2)\right)$$

$$\Rightarrow e_2^2\cos^2\theta+\frac{e_2^2\cos^2\theta}{1-e_1^2}=1-\frac{e_2^2}{1-e_1^2}$$

$$\Rightarrow \cos^2\theta=\frac{1}{e_1^2}+\frac{1}{e_2^2}+\frac{1}{e_1^2e_2^2}$$

If $\theta=90°$

$$\frac{e_1^2+e_2^2}{e_1^2e_2^2}=\frac{1}{e_1^2e_2^2} \Rightarrow e_1^2+e_2^2=1.$$

Also, area of parallelogram

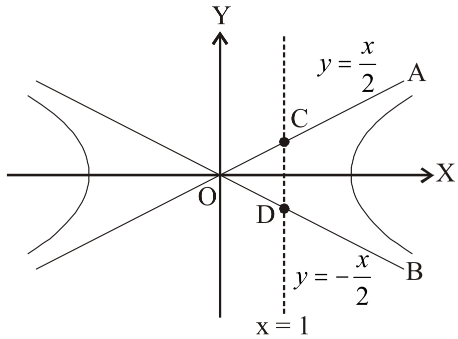
$$=\frac{1}{2}HH'SS'\sin\theta$$

$$=\frac{1}{2}(2ae_2)(2ae_1)\sqrt{1-\frac{1}{e_1^2}-\frac{1}{e_2^2}+\frac{1}{e_1^2e_2^2}}$$

$$=2a\sqrt{e_1^2e_2^2-e_1^2-e_2^2+1}$$

$$=2a\sqrt{(1-e_1^2)(1-e_2^2)}$$

12. (b, c)



Two tangents can be drawn if the point lies between the asymptotes OA and OB. The asymptotes are

$$y=\pm\frac{x}{2}$$

$x=1$ intersects these at $C\left(1,\frac{1}{2}\right)$ and $D\left(1,-\frac{1}{2}\right)$

$$\therefore \beta\in\left(-\frac{1}{2},\frac{1}{2}\right)$$

E MATRIX-MATCH TYPE

1. A - s; B - r; C - q; D - t

(A) The chord with mid-point (h, k) is

$\frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$, which passes through $(0, b)$.

$$\therefore \frac{k}{b}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

$\Rightarrow$ Locus is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{y}{b}$, which is an ellipse.

(B) If (h, k) be an end of latus-rectum, then $h=ae$ and $k=a(1-e^2)$. Eliminating e, we get $h^2=-a(k-a)$

$\Rightarrow$ Locus is $x^2=-a(y-a)$, a parabola.

(C) Any tangent is $y - mx = \sqrt{a^2m^2 + b^2}$

Perpendicular to it from $(ae, 0)$ is $my + x = ae$

Squaring and adding we get $x^2 + y^2 = a^2$, which is a circle.

(D) If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the fixed point

(α, β) then $\frac{\alpha}{a} + \frac{\beta}{b} = 1$. The mid point (h, k) of AB is

given by $h = \frac{a}{2},\ k = \frac{b}{2} \Rightarrow \frac{\alpha}{h} + \frac{\beta}{k} = 2$

$\therefore$ Locus is $\frac{\alpha}{x} + \frac{\beta}{y} = 2 \Rightarrow \left(x - \frac{\alpha}{2}\right)\left(y - \frac{\beta}{2}\right) = \frac{\alpha\beta}{4}$.

Which is a rectangular hyperbola.

2. A - p; B - r; C - s; D - t

The circle is $x^2 + y^2 - 2x - 4y - 20 = 0$ and let the hyperbola be $xy = c^2$. If $\left(ct, \frac{c}{t}\right)$ be the points of intersection then

$$c^2t^2 + \frac{c^2}{t^2} - 2ct - \frac{4c}{t} - 20 = 0$$

$$\Rightarrow c^2t^4 - 2ct^3 - 20t^2 - 4ct + c^2 = 0$$

If t_1, t_2, t_3, t_4 be its roots then,

$$\sum t_1 = \frac{2}{c}; \sum t_1t_2 = -\frac{20}{c^2};$$

$$\sum t_1t_2t_3 = \frac{4}{c} \text{ and } t_1t_2t_3t_4 = 1$$

(A) $x_1 + x_2 + x_3 + x_4 = ct_1 + ct_2 + ct_3 + ct_4 = 2$

(B) $\Sigma x_1x_2 = c^2\Sigma t_1t_2 = -20$

$\therefore \Sigma x_1^2 = (\Sigma x_1)^2 - 2\Sigma x_1x_2 = 44$

(C) $\Sigma y_1 = \Sigma \frac{c}{t_1} = c\frac{\Sigma t_1t_2t_3}{t_1t_2t_3t_4} = 4$

$\Sigma y_1y_2 = c^2\Sigma \frac{1}{t_1t_2} = -20$

$\therefore \Sigma y_1^2 = (\Sigma y_1)^2 - 2\Sigma y_1y_2 = 56$

(D) $OA^2 + OB^2 + OC^2 + OD^2 = \Sigma(x_1^2 + y_1^2) = 100$

3. A - q; B - s; C - r; D - p

Product of length of perpendiculars from foci upon any tangent $= b^2$

$$\Rightarrow b^2 = \left|\frac{2+1-9}{\sqrt{2}}\right|\left|\frac{4+1-9}{\sqrt{2}}\right| = 12$$

$SS' = 2ae \Rightarrow 2 = 2ae \Rightarrow ae = 1$

Now $b^2 = a^2 - a^2e^2 \Rightarrow a^2 = 13$ and $e = \frac{1}{\sqrt{13}}$

Latus rectum $= \frac{2b^2}{a} = \frac{24}{\sqrt{13}}$

Two tangents are perpendicular, so they intersect on the director circle. Thus CD = radius of director cirle

$= \sqrt{a^2 + b^2} = 5$

4. A - r; B - p; C - p, q, r, s, t; D - r, s, t

(A) Equation of tangent is $\frac{x}{a}\frac{\sqrt{3}}{2} + \frac{y}{b}\frac{1}{2} = 1$ and equation of tangent at the point $(a\cos\phi, b\sin\phi)$ is

$\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$. Both are same i.e.

$\cos\phi = \frac{\sqrt{3}}{2},\ \sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$.

(B) Since eccentricity of conic is independent of k, hence $e(2) - e(3) = 0$.

Also, for all k the equation represents rectangular hyperbola, so, $e(k) = \sqrt{2}\ \forall k \neq 0$

(C) If the circle touches the curve $xy = 9$

$\Rightarrow x^2 + \frac{81}{x^2} = a^2$ should have equal roots

$\Rightarrow a = 3\sqrt{2}$.

For $a = 0, 1, 2, 3, 4,$ the circle does not intersect the curve $xy = 9$

(D) If hyperbola is $xy = c^2$. Tangent at point (x_1, y_1) on the curve is

$\frac{xy_1 + x_1y}{2} = c^2 \Rightarrow \frac{x}{x_1} + \frac{y}{y_1} = 2 \qquad (\because x_1y_1 = c^2)$

$\therefore$ Area of triangle,

$$\Delta = \frac{1}{2}(2x_1)(2y_1) = 2x_1y_1 = 2c^2$$

$\therefore \quad \Delta = 2(1 + \sin^2\theta)$

$\Rightarrow \quad 2 \le \Delta \le 4$

5. A - q; B - s; C - p; D - r

(A) Here $a^2 = 16, b^2 = 20$

$\Rightarrow a^2 < b^2$

$\therefore PS + PS' = 2b = 2\sqrt{20} = 4\sqrt{5}$.

(B) Given ellipse is $2x^2+3y^2-4x-12y+13=0$

$$\Rightarrow \frac{(x-1)^2}{\frac{1}{2}}+\frac{(y-2)^2}{\frac{1}{3}}=1. \text{ Here } a^2=\frac{1}{2}, b^2=\frac{1}{3}$$

$$\therefore \quad e=\sqrt{1-\frac{1}{3}\times 2}=\frac{1}{\sqrt{3}}.$$

(C) Any point on the line $x-y-5=0$ will be of the form $(t, t-5)$. Chord of contact of this point with respect to curve $x^2+4y^2=4$ is

$$tx+4(t-5)y-4=0$$

or $(-20y-4)+t(x+4y)=0$

Which is a family of straight lines, each member of this family passes through point of intersection of strainght lines $-20y-4=0$ and $x+4y=0$. So the point is

$$\left(\frac{4}{5},-\frac{1}{5}\right)$$

(D) $\frac{x^2}{4}+\frac{y^2}{3}=1 \Rightarrow a=2, b=\sqrt{3}$ and $e=\frac{1}{2}$

Sum of distances $=\frac{2a}{e}=\frac{4}{1/2}=8.$

F NUMERIC/INTEGER ANSWER TYPE

1. Ans : 7

Let $P(a\cos\theta_1, b\sin\theta_1)$, $Q(a\cos\theta_2, b\sin\theta_2)$ and $R(a\cos\theta_3, b\sin\theta_3)$ be the vertices of the triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The points on the auxiliary circle corresponding to these points are $P'(a\cos\theta_1, a\sin\theta_1)$, $Q'(a\cos\theta_2, a\sin\theta_2)$ and $R'(a\cos\theta_3, a\sin\theta_3)$.

$$\therefore \quad \Delta_1 = \text{Area of } \Delta PQR = \frac{1}{2}\begin{vmatrix} a\cos\theta_1 & b\sin\theta_1 & 1 \\ a\cos\theta_2 & b\sin\theta_2 & 1 \\ a\cos\theta_3 & b\sin\theta_3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}ab\begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 1 \\ \cos\theta_2 & \sin\theta_2 & 1 \\ \cos\theta_3 & \sin\theta_3 & 1 \end{vmatrix}$$

and $\Delta_2 = \text{Area of } \Delta P'Q'R' = \frac{1}{2}$

$$\begin{vmatrix} a\cos\theta_1 & a\sin\theta_1 & 1 \\ a\cos\theta_2 & a\sin\theta_2 & 1 \\ a\cos\theta_3 & a\sin\theta_3 & 1 \end{vmatrix} = \frac{1}{2}a^2\begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 1 \\ \cos\theta_2 & \sin\theta_2 & 1 \\ \cos\theta_3 & \sin\theta_3 & 1 \end{vmatrix}$$

Clearly, $\frac{\Delta_1}{\Delta_2}=\frac{b}{a}=\sqrt{1-e^2}=\frac{1}{7}$

2. Ans : 1

The equation of the tangent at $P(a\cos\theta, b\sin\theta)$ on the given ellipse is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$.

Length of perpendicular from the focus $(ae, 0)$

$$p=\frac{e\cos\theta-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}=\frac{ab(e\cos\theta-1)}{\sqrt{b^2\cos^2\theta+a^2(1-\cos^2\theta)}}$$

$$=\frac{ab(e\cos\theta-1)}{\sqrt{a^2-a^2e^2\cos^2\theta}}=-b\sqrt{\frac{1-e\cos\theta}{1+e\cos\theta}}$$

$$\Rightarrow \frac{b^2}{p^2}=\frac{1+e\cos\theta}{1-e\cos\theta}$$

Now $r^2=(ae-a\cos\theta)^2+b^2\sin^2\theta$

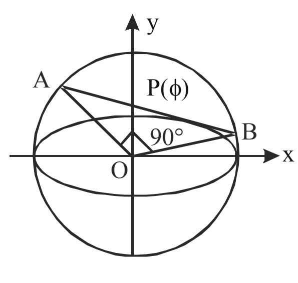
$=a^2[(e-\cos\theta)^2+(1-e^2)\sin^2\theta)]$

$=a^2[e^2\cos^2\theta-2e\cos\theta+1]=a^2(1-e\cos\theta)^2$

$\Rightarrow r=a(1-e\cos\theta)$

Now $\frac{2a}{r}-\frac{b^2}{P^2}=\frac{2}{1-e\cos\theta}-\frac{1+e\cos\theta}{1-e\cos\theta}=1$

3. Ans : 1



Equation of the auxillary circle is $x^2+y^2=a^2$ .... (i)

Equation of tangent at a point $P(a\cos\phi, b\sin\phi)$ is $\left(\frac{x}{a}\right)$

$$\cos\phi + \left(\frac{y}{b}\right)\sin\phi = 1 \quad \text{....(ii)}$$

Which meets the auxillary circle at point A and B.

$\therefore$ Equation of pair of lines OA and OB is obtained by making homogenous (i) with the help of (ii) as

$$x^2 + y^2 = a^2\left(\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi\right)^2$$

$$\Rightarrow (1-\cos^2\phi)x^2 - \frac{2xy\, a\sin\phi\cos\phi}{b} + \left[1 - \frac{a^2}{b}\sin^2\phi\right]y^2 = 0$$

But $\angle AOB = 90°$

$\therefore$ Coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow 1-\cos^2\phi + 1 - \frac{a^2}{b^2}\sin^2\phi = 0$$

$$\Rightarrow \sin^2\phi\left(1-\frac{a^2}{b^2}\right) + 1 = 0 \Rightarrow (a^2-b^2)\sin^2\phi = b^2$$

$$\Rightarrow a^2e^2\sin^2\phi = a^2(1-e^2)$$

$$\Rightarrow (1+\sin^2\phi)e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{1+\sin^2\phi}}$$

$$\Rightarrow e\sqrt{1+\sin^2\phi} = 1$$

4. Ans : 1

Let a circle be $x^2+y^2+2gx+2fy+c=0$, which intersects the ellipse at $(a\cos\phi, b\sin\phi)$, then

$$a^2\cos^2\phi + b^2\sin^2\phi + 2ga\cos\phi + 2fb\sin\phi + c = 0$$

$$\Rightarrow a^2\left[\frac{1-\tan^2\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right]^2 + b^2\left[\frac{2\tan\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right]^2$$

$$+2ga\left[\frac{1-\tan^2\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right] + 2fb\left[\frac{2\tan\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right] + c = 0$$

$$\Rightarrow a^2\left(1-2\tan^2\frac{\phi}{2}+\tan^4\frac{\phi}{2}\right) + 4b^2\tan^2\frac{\phi}{2}$$

$$+2ga\left(1-\tan^4\frac{\phi}{2}\right) + 2fb\left(2\tan\frac{\phi}{2}+2\tan^3\frac{\phi}{2}\right)$$

$$+c\left(1+2\tan^2\frac{\phi}{2}+\tan^4\frac{\phi}{2}\right) = 0$$

$$\Rightarrow (a^2-2ga+c)\tan^4\frac{\phi}{2} + 4fb\tan^3\frac{\phi}{2}$$

$$+(4b^2-2a^2+2c)\tan^2\frac{\phi}{2} + 4fb\tan\frac{\phi}{2} + (a^2+2ga+c) = 0 \quad \text{...(1)}$$

Clearly $\phi_1, \phi_2, \phi_3, \phi_4$ are the four values of ϕ obtained from equation (1)

So, $\sum \tan\frac{\phi_1}{2} = -\frac{4fb}{a^2-2ga+c}$,

$$\sum \tan\frac{\phi_1}{2}+\tan\frac{\phi_2}{2} = \frac{4b^2-2a^2+2c}{a^2-2ga+c}$$

$$\sum \tan\frac{\phi_1}{2}\tan\frac{\phi_2}{2}\tan\frac{\phi_3}{2} = -\frac{4fb}{a^2-2ga+c}$$

and $\tan\frac{\phi_1}{2}\tan\frac{\phi_2}{2}\tan\frac{\phi_3}{2}\tan\frac{\phi_4}{2} = \frac{a^2+2ga+c}{a^2-2ga+c}$

Now,

$$\tan\left(\frac{\phi_1}{2}+\frac{\phi_2}{2}+\frac{\phi_3}{2}+\frac{\phi_4}{2}\right) =$$

$$\frac{\sum\tan\frac{\phi_1}{2} - \sum\tan\frac{\phi_1}{2}\tan\frac{\phi_2}{2}\tan\frac{\phi_3}{2}}{1-\sum\tan\frac{\phi_1}{2}\tan\frac{\phi_2}{2}+\tan\frac{\phi_1}{2}\tan\frac{\phi_2}{2}\tan\frac{\phi_3}{2}\tan\frac{\phi_4}{2}} = 0$$

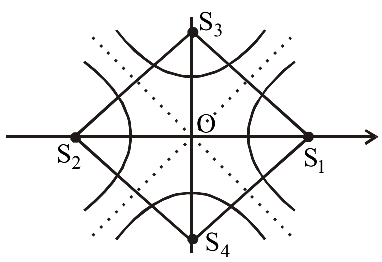
$$\frac{\phi_1}{2}+\frac{\phi_2}{2}+\frac{\phi_3}{2}+\frac{\phi_4}{2} = n\pi$$

$$\therefore \phi_1+\phi_2+\phi_3+\phi_4 = 2n\pi \Rightarrow \cos(\phi_1+\phi_2+\phi_3+\phi_4) = 1$$

5. Ans : 26

Due to summitry the desired area $= 4 \times$ area of ΔS_1OS_3

$$= 4\times\frac{1}{2}ae\times be_1$$



Where e_1 is eccentricity of conjugate hyperbola

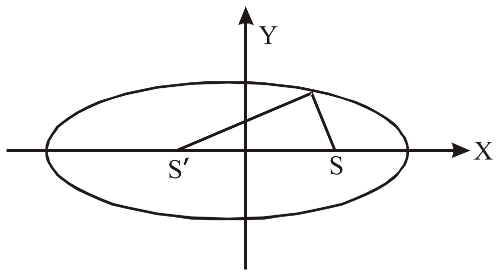
$= 2 \times 2e \times 3e_1 = 12ee_1$

Now $b^2 = a^2(e^2 - 1) \Rightarrow e^2 = 13/4$

and $\frac{1}{e^2} + \frac{1}{e_1^2} = 1 \Rightarrow e_1^2 = \frac{13}{9}$

$\therefore$ Required area $= 12 \times \frac{\sqrt{13}}{2} \times \frac{\sqrt{13}}{3} = 26$

6. **Ans : 1**



Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, Foci are $S(ae, 0)$ and $S'(-ae, 0)$

Let $P(a\cos\theta, b\sin\theta)$ be any point on the ellipse.

Then $SP = a - e(a\cos\theta) = a(1 - e\cos\theta)$ and

$S'P = a + e(a\cos\theta) = a(1 + e\cos\theta)$.

Also, $SS' = 2ae$

Let (h, k) be the in centre of the $\Delta\ PSS'$, then

$$h = \frac{-ae.\,a(1-e\cos\theta) + ae.\,a(1+e\cos\theta) + a\cos\theta.\,2ae}{a(1-e\cos\theta) + a(1+e\cos\theta) + 2ae}$$

$= ae\cos\theta$.

$$k = \frac{b\sin\theta.\,2ae}{a(1-e\cos\theta) + a(1+e\cos\theta) + 2ae} = \frac{b\sin\theta.e}{1+e}$$

$\therefore \cos\theta = \frac{h}{ae}, \sin\theta = \frac{(1+e)k}{eb}$

$$\Rightarrow \frac{h^2}{a^2e^2} + \frac{(1+e)^2k^2}{e^2b^2} = 1$$

Hence locus of (h, k) is $\frac{x^2}{a^2e^2} + \frac{y^2}{e^2b^2/(1+e)^2} = 1$,

Which is an ellipse,

Its eccentricity $e_1^2 = 1 - \frac{e^2b^2}{e^2a^2(1+e)^2}$

$$= 1 - \frac{a^2(1-e^2)}{a^2(1+e)^2} = \frac{2e}{1+e} \Rightarrow \frac{2}{e_1^2} - \frac{1}{e} = 1$$

7. **Ans : 0**

Equation of a rectangular hyperbola can be chosen as $x^2 - y^2 = a^2$

Any point on the above hyperbola can be chosen as $P = (a\sec\theta, a\tan\theta)$

For the given hyperbola, we have $\frac{dy}{dx} = \frac{x}{y}$

Thus, slope of the tangent at $P = \frac{a\sec\theta}{a\tan\theta} = \frac{1}{\sin\theta}$

and slope of the normal at $P = -\sin\theta$

Thus, equations of the tangent and the normal at P are

$$y - a\tan\theta = \frac{1}{\sin\theta}(x - a\sec\theta)$$

and $y - a\tan\theta = -\sin\theta(x - a\sec\theta)$ respectively.

The x and y intercepts of the tangent and the normal are given by

$a_1 = a\sec\theta - a\sin\theta\tan\theta = a\cos\theta$ and

$$a_2 = a\sec\theta + \frac{a\tan\theta}{\sin\theta} = \frac{2a}{\cos\theta}$$

and $b_1 = a\tan\theta - \frac{a\sec\theta}{\sin\theta} = -\frac{a\cos\theta}{\sin\theta}$ and

$$b_2 = a\tan\theta + a\sin\theta\sec\theta = \frac{2a\sin\theta}{\cos\theta}$$

Hence, we have $a_1a_2 + b_1b_2 = 2a^2 - 2a^2 = 0$

8. **Ans : 6**

$y = mx + \sqrt{a^2m^2 + b^2}$ is tangent to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

This tangent also touches the circle $x^2 + y^2 = r^2$

$$\Rightarrow \pm r = \frac{0 - 0 + \sqrt{a^2m^2 + b^2}}{\sqrt{1+m^2}}$$

$$m = \sqrt{\frac{r^2 - b^2}{a^2 - r^2}} \quad (\because b < r < a) \qquad ...(1)$$

RS passes through $(ae, 0)$ and parallel to PQ

$\Rightarrow$ Equation of RS is $y - 0 = m(x - ae)$

$\Rightarrow mx - y - ame = 0$

Let T be foot of perpendicular dropped from origin on RS, then

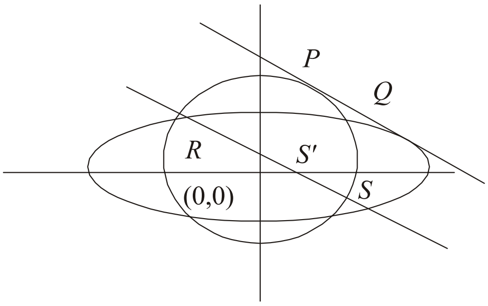
$RT^2 = OR^2 - OT^2$

$\Rightarrow \quad RT^2 = r^2 - \dfrac{a^2m^2e^2}{1+m^2}$

$= \dfrac{r^2 + r^2m^2 - a^2m^2e^2}{1+m^2} = \dfrac{a^2m^2 + b^2 - a^2m^2e^2}{1+m^2}$

$= \dfrac{m^2b^2 + b^2}{1+m^2} = b^2$ (from (1))

$\therefore \quad RS = 2b \Rightarrow RS = 6 \quad (\because \text{ given } b = 3)$



9. Ans : 3

Let $A\left(cp, \dfrac{c}{p}\right)$ be a point on hyperbola $xy = c^2$.

Normal at point A, again cuts the hyperbola at $B\left(cp', \dfrac{c}{p'}\right)$

where $p' = -\dfrac{1}{p^3}$

Slope of $OA = \dfrac{1}{p^2}$

Slope of $OB = p^6$

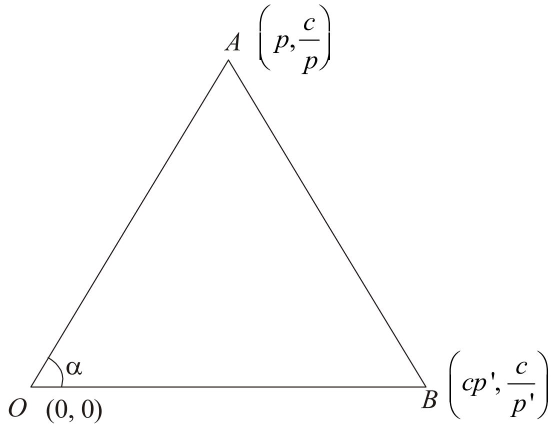
$\Rightarrow \quad \tan\alpha = \dfrac{\dfrac{1}{p^2} - p^6}{1+p^4} = \dfrac{1-p^4}{p^2}$

$\tan(\angle OAB) = \dfrac{p^2 - \dfrac{1}{p^2}}{1+1} = \dfrac{p^4-1}{2p^2}$

$\Rightarrow \quad \dfrac{\tan\alpha}{\tan OAB} = -2 \Rightarrow \dfrac{\tan\alpha}{\tan A} = -2 \Rightarrow \dfrac{\sin\alpha\cos A}{\cos\alpha\sin A} = -2$

$\Rightarrow \quad \dfrac{\sin(\alpha+A) + \sin(\alpha-A)}{\sin(\alpha+A) - \sin(\alpha-A)} = -2$

$\Rightarrow \quad \dfrac{\sin(\alpha - A)}{\sin(\alpha + A)} = 3$



10. Ans : 4

Let the point of concurrency be (h, k).

Equation of normal at (x', y') is $\dfrac{x - x'}{\dfrac{x'}{a^2}} = \dfrac{y - y'}{\dfrac{y'}{b^2}}$.

It passes through (h, k), then

$y'^2\{a^2(h - x') + b^2x'\}^2 = b^4k^2x'^2$ ...(1)

$\dfrac{x'^2}{a^2} + \dfrac{y'^2}{b^2} = 1 \quad \Rightarrow \quad y'^2 = \dfrac{b^2}{a^2}(a^2 - x'^2)$

Putting in (1),

$\dfrac{b^2}{a^2}(a^2 - x'^2)\{a^2h + (b^2 - a^2)x'\}^2 = b^4k^2x'^2$

$\Rightarrow \quad \dfrac{b^2}{a^2}(a^2 - x'^2)\{a^4h^2 + (b^2 - a^2)^2x'^2 + 2a^2hx'$

$(b^2 - a^2)\} = b^4k^2x'^2$

Rearranging above equation as a 4$^{\text{th}}$ degree equation in x', we get

$-(a^2 - b^2)^2 x'^4 + 2ha^2(a^2 - b^2)x'^3 +$

$\left[a^2(b^2 - a^2)^2 - a^4h^2 - a^2b^2k^2\right]x'^2$

$+ 2a^4h(b^2 - a^2)x' + a^6h^2 = 0$

The roots of the above equation are x_1, x_2, x_3, x_4

$\therefore \quad (x_1 + x_2 + x_3 + x_4)\left(\dfrac{1}{x_1} + \dfrac{1}{x_2} + \dfrac{1}{x_3} + \dfrac{1}{x_4}\right)$

$$= \frac{2ha^2}{a^2 - b^2} \times \frac{2a^4 h\left(a^2 - b^2\right)}{a^6 h^2} = 4$$

11. Ans : 5

The tangent at any point $A(2\sec\theta, \tan\theta)$ is given by

$$\frac{x\sec\theta}{2} - \frac{y\tan\theta}{1} = 1.$$

It meets the line $x - 2y = 0$

$$\Rightarrow \quad \frac{x\sec\theta}{2} - \frac{x\tan\theta}{2} = 1 \quad \Rightarrow \quad x = \frac{2}{\sec\theta - \tan\theta}$$

$$\Rightarrow \quad Q \equiv \left(\frac{2}{\sec\theta - \tan\theta}, \frac{1}{\sec\theta - \tan\theta}\right) \qquad ...(1)$$

Also, the tangent meets the line $x + 2y = 0$ at R, so

$$\Rightarrow \quad \frac{x}{2}\sec\theta + \frac{x}{2}\tan\theta = 1 \quad \Rightarrow \quad x = \frac{2}{\sec\theta + \tan\theta}$$

$$\Rightarrow \quad R \equiv \left(\frac{2}{\sec\theta + \tan\theta}, \frac{-1}{\sec\theta + \tan\theta}\right) \qquad ...(2)$$

Now, $CQ.CR = \sqrt{\frac{2^2 + 1^2}{(\sec\theta - \tan\theta)^2}} \sqrt{\frac{2^2 + 1^2}{(\sec\theta + \tan\theta)^2}}$

$$= 2^2 + 1^2$$

$$\Rightarrow \quad CQ.CR = 5$$

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