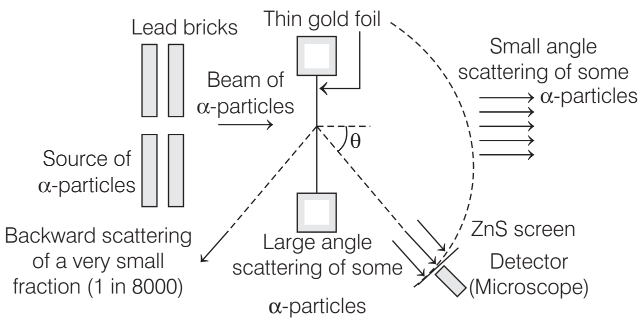
# 29

# Atoms and Nuclei

IN THIS CHAPTER ....



Atoms are the basic building blocks of matter and defining structure of elements. The first model of an atom was proposed by JJ Thomson in 1898. The schematic arrangement of the Rutherford's $\alpha$-particle scattering experiment (also known as Geiger-Marsden experiment) is shown below




He suggested that atoms are just positively charged lumps of matter with electrons embedded in them like resins in a fruit cake. This model is also called the **plum pudding model**.

## $\alpha$-Particle Scattering Experiment

In $\alpha$-particle scattering experiment, following facts are observed

- Most of the $\alpha$-particles pass through the foil without deflection.
- About 0.14% of the incident $\alpha$-particles are scattered by more than 1%.
- A very small number of $\alpha$-particles (about 1 in 8000) practically retraced their paths or suffered deflection of nearly 180°.

### Rutherford's Atomic Model

Based on the above observations, Rutherford proposed a nuclear model of atom. According to this model,

- entire positive charge and most of the mass of the atom is concentrated in a small volume called **nucleus**, with electrons revolving around the nucleus as planets revolve around the sun.
- size of nucleus is to be about $10^{-15}$ m to $10^{-14}$ m. From kinetic theory, the size of an atom is known to be $10^{-10}$ m. Thus, most of an atom has empty space.

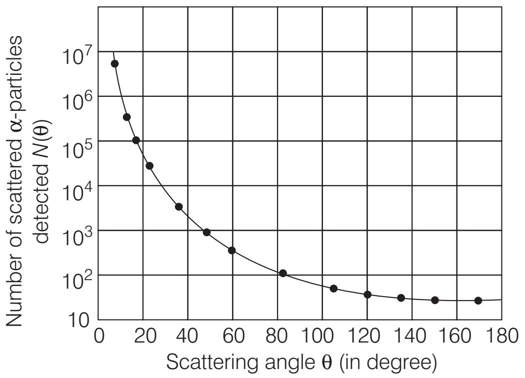
- atomic electrons are so light that they does not affect the path of $\alpha$-particles.

Following are few important points related to Rutherford's model

- The number of $\alpha$-particles scattered per unit area $N(\theta)$ at a scattering angle $\theta$ varies as

$$N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

The following graph represents the variation of $N(\theta)$ with $\theta$



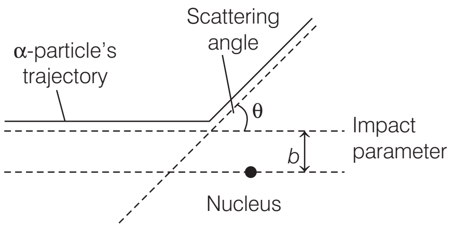

Number of $\alpha$-particle scattered *versus* scattering angle ($\theta$)

- The minimum distance from the nucleus upto which the $\alpha$-particle approaches is called **distance of closest approach**. At this distance, whole KE of $\alpha$-particle converts into electrostatic PE. It is given as

$$r_0 = \frac{2k\,Ze^2}{mv^2} \qquad \left[\text{where, } k = \frac{1}{4\pi\varepsilon_0}\right]$$

- **Impact parameter** ($b$) is the perpendicular distance of the initial velocity vector of the $\alpha$-particle from the centre of the nucleus. Mathematically, it is given as

$$b = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2 \cot\frac{\theta}{2}}{\text{KE}}$$




**Note** Magnitude and direction of the force on the $\alpha$-particle continuously changes as it approaches nucleus and recedes away from it.

***Example 1.*** *The number of particles scattered at 60° is 100 per minute in $\alpha$-particle scattering experiment, using gold foil. Calculate the number of particles per minute scattered at 90°.*

*(a)* 100 particle/min *(b)* 25 particle/min
*(c)* 65 particle/min *(d)* 90 particle/min

***Sol.*** (*b*) Number of $\alpha$-particles scattered at angle $\theta$,

$$N \propto \frac{1}{\sin^4(\theta/2)}$$

*i.e.* $$N = \frac{K}{\sin^4(\theta/2)}$$

where, $K$ is a proportionality constant.

$$\therefore \quad \frac{N_{90^\circ}}{N_{60^\circ}} = \frac{\sin^4(60^\circ/2)}{\sin^4(90^\circ/2)}$$

or $$N_{90^\circ} = \frac{\sin^4 30^\circ}{\sin^4 45^\circ} \times N_{60^\circ} = \left[\frac{1/2}{1/\sqrt{2}}\right]^4 \times 100$$

$$= \frac{100}{4} = 25 \text{ particle min}^{-1}$$

## Drawbacks of Rutherford's Model

According to classical electromagnetic theory, a accelerated charged particle, must radiate energy in the form of electromagnetic radiation.

The energy of accelerating electron should therefore, continuously decrease. Thus, the electron would spiral inward and eventually fall into nucleus. Hence, such an atom can never be stable.

Rutherford's model was unable to explain line spectrum.

# Bohr Model of Hydrogen Atom

Neil Bohr explaning the hydrogen atom spectrum by applying the quantum theory of radiation to Rutherford's atomic model. On the basis of that, he gave his theory in the form of following three postulates

(i) An electron could revolve only in certain circular orbits, called **stationary orbits**, without emitting radiations. Thus there is a definite energy associated with each stable orbit.

(ii) An electron radiates energy only when it makes a transition from one of these orbits to another. The energy is radiated in the form of a photon with energy given by

$$\Delta E = h\nu = E_i - E_f \qquad \text{...(i)}$$

$\therefore$ Frequency of emitted photon is given by

$$\nu = \frac{E_i - E_f}{h}$$

where, $E_i$ and $E_f$ ($E_i > E_f$) are the energies of the initial and final states.

(iii) The electron revolves around the nucleus only in those orbits for which the angular momentum is the integral multiple of $\frac{h}{2\pi}$, where $h$ is Planck's constant ($\simeq 6.6 \times 10^{-34}$ J-s).

$\therefore$ Angular momentum $L$ of the orbiting electron is quantised, *i.e.*

$$L = mvr = \frac{nh}{2\pi}$$

where, $n$ is an integer ($n$ = 1, 2, 3, ...) and is called **principal quantum number.**

Following are few important points related to Bohr's atomic model

For hydrogen like atoms with atomic number $Z$,

- **Orbital radius** of an electron in $n$th orbit is

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2 Z}$$

- **Bohr's radius** is the radius of the first orbit ($n = 1$) of H-atom ($Z = 1$),

$$r_1 = a_0 = \frac{h^2 \varepsilon_0}{\pi m e^2} = 0.53 \text{ Å}$$

Also, $r_n = 0.53 \frac{n^2}{Z} \text{ Å} = \frac{n^2}{Z} r_1$

$\Rightarrow \quad r_n \propto n^2$

- **Orbital velocity** of an electron in $n$th orbit,

$$v_n = \frac{Ze^2}{2\varepsilon_0 nh} \text{ or } v_n = \alpha\left(\frac{cZ}{n}\right)$$

where, $\alpha = \frac{e^2}{2h\varepsilon_0 c}$ is the Summerfield's fine structure constant (a pure number) whose value is equal to $\frac{1}{137}$.

$\Rightarrow \quad v_n \propto \frac{1}{n}$

- **Velocity of electron** in first orbit ($n = 1$) of H-atom ($Z = 1$),

$$v_1 = \frac{e^2}{2h\varepsilon_0} = \frac{c}{137} \simeq 2.19 \times 10^6 \text{ m/s}$$

- **Orbital frequency** of an electron in $n$th orbit,

$$\nu_n = \frac{kZe^2}{nhr_n} \quad \left[\text{where, } k = \frac{1}{4\pi\varepsilon_0}\right]$$

As, $r_n \propto \frac{n^2}{Z} \Rightarrow \nu_n \propto \frac{Z^2}{n^3}$

$\therefore$ Time period $T$ of an electron is

$$T_n = \frac{1}{\nu_n} = \frac{nhr_n}{kZe^2}$$

$\Rightarrow \quad T_n \propto \frac{n^3}{Z^2}$

Also, $\omega_n = 2\pi\nu_n \Rightarrow \omega_n \propto \frac{Z^2}{n^3}$

***Example 2.*** *Calculate the radius of the third Bohr orbit of the electron of the hydrogen atom.*

*(Take, $h = 6.625 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ ms$^{-1}$, $\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$N$^{-1}$m$^{-2}$)*

(a) 2.48 Å (b) 3.68 Å (c) 2.68 Å (d) 4.77 Å

***Sol.*** (*d*) Here, $h = 6.625 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ ms$^{-1}$, $\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$N$^{-1}$m$^{-2}$, $n = 3$

Radius of the $n$th Bohr orbit of the hydrogen atom is given by

$$r_n = 4\pi\varepsilon_0 \cdot \frac{n^2h^2}{4\pi^2 me^2}$$

$$r_3 = 4\pi \times 8.85 \times 10^{-12} \times \frac{(3)^2 \times (6.625 \times 10^{-34})^2}{4\pi^2 \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 4.777 \times 10^{-10} \text{ m}$$

$$= 4.777 \text{Å}$$

***Example 3.*** *A 10 kg satellite circles earth once every 2 h in an orbit having a radius of 8000 km. Assuming that Bohr's angular momentum postulate applies to satellites just as it does to an electron in the hydrogen atom, then the quantum number of the orbit of the satellite is* ***[NCERT]***

(a) $10 \times 10^{40}$ (b) $5.3 \times 10^{45}$

(c) $4 \times 10^{10}$ (d) $3.2 \times 10^{25}$

***Sol.*** (*b*) From Bohr's postulate, we have $mv_n r_n = \frac{nh}{2\pi}$

Given, $m = 10$ kg, $r_n = 8 \times 10^6$ m,

$$T = 2\text{ h} = 2 \times 60 \times 60 = 7200 \text{ s}$$

$$v_n = \frac{2\pi r_n}{T}$$

The quantum number of the orbit of satellite

$$n = (2\pi r_n)^2 \times \frac{m}{T \times h}$$

Putting the values, we have

$$n = (2\pi \times 8 \times 10^6)^2 \times \frac{10}{(7200 \times 6.64 \times 10^{-34})}$$

$\Rightarrow \quad n = 5.3 \times 10^{45}$

***Example 4.*** *If the average life time of an excited state of hydrogen is of the order of $10^{-8}$ s, estimate how many orbits an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$.*

*(Take, Bohr radius, $a_0 = 5.3 \times 10^{-11}$ m)*

(a) $6 \times 10^2$ (b) $8 \times 10^6$

(c) $9 \times 10^8$ (d) $6 \times 10^5$

***Sol.*** (*b*) The angular momentum of an election in $n$th orbit is $nh/2\pi$.

By Bohr's hypothesis, $mvr = nh/2\pi$, where $r$ is the radius of the orbit.

or $\quad v = \frac{nh}{2\pi mr}$ ... (i)

The time period of completing an orbit,

$$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh}$$

or $\quad T = \frac{4\pi^2 mr^2}{nh}$

Since, the radius of the orbit is proportional to $n^2$, hence $r = a_0 n^2$

$$\therefore \quad T = \frac{4\pi^2 m a_0^2 n^4}{nh} = \frac{4\pi^2 m a_0^2 n^3}{h}$$

Number of orbits completed in $10^{-8}$ s

$$= \frac{10^{-8}}{T} = \frac{10^{-8} \times h}{4\pi^2 m a_0^2 n^3}$$

$$= \frac{10^{-8} \times (6.6 \times 10^{-34})}{4(3.14)^2 (9.1 \times 10^{-31}) (5.3 \times 10^{-11})^2 (2)^3}$$

$$= 8 \times 10^6$$

## Energy of an Electron in *n*th Orbit

**Kinetic energy ($K_n$)** of an electron in $n$th orbit,

$$K_n = \frac{1}{2} m v_n^2 = \frac{me^4 Z^2}{8\varepsilon_0^2 n^2 h^2} \qquad \left(\because v_n = \frac{Ze^2}{2\varepsilon_0 nh}\right)$$

In terms of Rydberg's, $K_n = \dfrac{RhcZ^2}{n^2}$

where, $R = me^4 / 8\varepsilon_0^2 ch^3$ = Rydberg's constant

$= 1.09 \times 10^7 \text{ m}^{-1}$

**Potential energy $U_n$** of an electron in $n$th orbit,

$$U_n = -\frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r_n} = -\frac{mc^4 Z^2}{4\varepsilon_0^2 n^2 h^2} \quad \left(\because r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2 Z}\right)$$

So, $$U_n = \frac{-2RhcZ^2}{n^2}$$

The **total energy** $E_n$ is the sum of the kinetic and potential energies.

$$\therefore E_n = K_n + U_n = -\frac{me^4 Z^2}{8\varepsilon_0^2 n^2 h^2}$$

In simplified form, $E_n = \dfrac{-RhcZ^2}{n^2}$

From the above relations, we get

$$|U_n| = 2|K_n|$$

and $E_n = -K_n$

Substituting, values of $m, e, \varepsilon_0$ and $h$ with $n = 1$, we get the least energy of the atom in first orbit, which is −13.6 eV.

Hence, $E_1 = -13.6$ eV

and $E_n = \dfrac{E_1}{n^2} = -\dfrac{13.6}{n^2}$ eV

Substituting, $n = 2, 3, 4 \ldots$, etc., we get energies of electron in different orbits, *i.e.*

$$E_2 = -3.40 \text{ eV}, \; E_3 = -1.51 \text{ eV}, \ldots E_\infty = 0$$

**Note** $Rhc = 1$ Rydberg = 13.6 eV

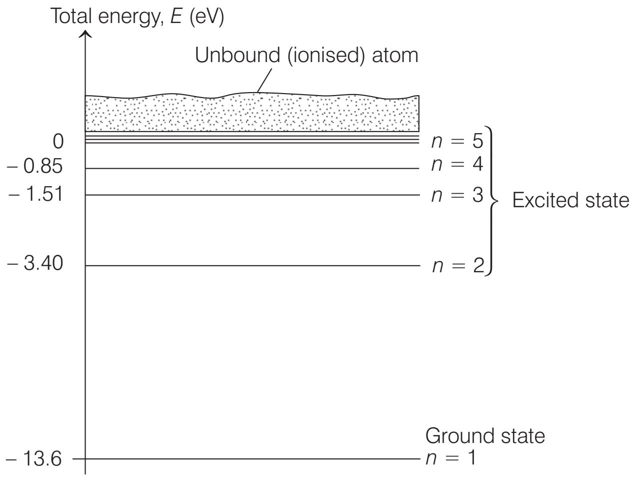
# Energy Levels

The energy of an atom is least, when it is revolving in an orbit closest to the nucleus, *i.e.* for which $n = 1$.

$\therefore$ $E_1 = -13.6$ eV

This lowest state (minimum energy level) of an atom is called the **ground state**.

However, when a hydrogen atom receives energy by processes such as electron collisions, the atom may acquire sufficient energy to raise the electron to higher states, the atom is then said to be in **excited state**.




The energy level diagram for the hydrogen atom

Thus, when an atom makes a transition from the higher energy state with quantum number $n_i$ to the lower energy state with quantum number $n_f$ $(n_f < n_i)$ in hydrogen like atom, the difference in the energy is carried by a photon of frequency $\nu_{if}$ such that,

$$\nu_{if} = \frac{me^4 Z^2}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{ or } \frac{1}{\lambda} = \frac{Z^2 me^4}{8h^2\varepsilon_0^2 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

**Note** The reciprocal of wavelength is called wave number.

$$\Rightarrow \qquad \bar{\nu} = \frac{1}{\lambda}$$

## Ionisation Energy and Potential

It is the energy required to remove the electron completely from its parent atom. In ground state, ($n = 1$) energy of atom is −13.6 eV and energy corresponding to $n = \infty$ is zero. Hence, energy required to remove the electron from ground state is 13.6 eV.

Similarly, the potential difference through which an electron should be accelerated to get ionisation energy is called **ionisation potential**.

$$\therefore \text{Ionisation potential} = \frac{E_{\text{ionisation}}}{e} = \frac{13.6\, Z^2}{n^2} \text{ V}$$

**Note** The energy required to separate the constituents of a nucleus to infinity is called **binding energy.**

***Example 5.*** *A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in terms of Bohr radius $a_0$ will be (Take, $hc = 12500$ eV-Å)* **[JEE Main 2019]**

(a) $4a_0$ (b) $9a_0$ (c) $16a_0$ (d) $25a_0$

**Sol.** (c) We know that, net change in energy of a photon in a transition with wavelength $\lambda$ is $\Delta E = hc/\lambda$.

Here, $hc = 12500$ eV-Å and $\lambda = 980$ Å

$\therefore$ $\Delta E = 12500/980 = 12.76$ eV $\Rightarrow E_n - E_1 = 12.76$ eV

Since, the energy associated with an electron in $n$th Bohr's orbit is given as

$$E_n = \frac{-13.6}{n^2} \text{ eV} \quad \text{...(i)}$$

$$\Rightarrow \quad E_n = E_1 + 12.76 \text{ eV} \quad \text{...(ii)}$$

$$= \frac{-13.6}{(1)^2} + 12.76 = -0.84$$

Putting this value in Eq. (i)

$$\Rightarrow \quad n^2 = \frac{-13.6}{-0.84} = 16 \Rightarrow n = 4$$

and radius of $n^{th}$ orbit, $r_n = n^2 a_0 \Rightarrow r_n = 16\, a_0$

***Example 6.*** *A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantisation conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as* **[JEE Main 2019]**

(a) $r_n \propto n, E_n \propto n$ (b) $r_n \propto n^2, E_n \propto \frac{1}{n^2}$

(c) $r_n \propto \sqrt{n}, E_n \propto n$ (d) $r_n \propto \sqrt{n}, E_n \propto \frac{1}{n}$

**Sol.** (c) As, for conservative fields, $F = -\left(\frac{dU}{dr}\right)$

$\therefore$ Magnitude of force on particle is

$$\Rightarrow \quad F = \frac{dU}{dr} = \frac{d}{dr}\left(\frac{1}{2}kr^2\right) \Rightarrow F = kr$$

This force is acting like centripetal force.

$$\therefore \quad \frac{mv^2}{r} = kr \quad \text{...(i)}$$

So, for $n$th orbit,

$$\Rightarrow \quad m^2 v_n^2 = mkr_n^2$$

$$\Rightarrow \quad \frac{n^2h^2}{4\pi^2 r^2} = mkr_n^2 \quad \left[\because v_n = \frac{nh}{2\pi mr}\right]$$

Therefore, $r_n^4 \propto n^2 \Rightarrow r_n^2 \propto n$

So, $r_n \propto \sqrt{n}$

Energy of particle,

$$E_n = \text{PE} + \text{KE} = \frac{1}{2}kr_n^2 + \frac{1}{2}mv_n^2$$

$$= \frac{1}{2}kr_n^2 + \frac{1}{2}kr_n^2 \quad \text{[using Eq. (i)]}$$

$$= kr_n^2$$

So, energy, $E_n \propto r_n^2 \Rightarrow E_n \propto n$

***Example 7.*** *A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state?*

(a) 20.4 eV (b) 6.2 eV (c) 9 eV (d) 0 eV

**Sol.** (a) Let $K$ be the kinetic energy of the moving hydrogen atom and $K'$ be the kinetic energy of combined mass after collision.

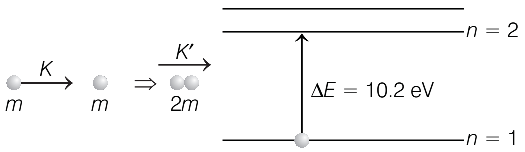
From conservation of linear momentum,

$$p = p'$$

or $\sqrt{2Km} = \sqrt{2K'(2m)}$

or $K = 2K'$ ...(i)

From conservation of energy, $K = K' + \Delta E$ ...(ii)




Solving Eqs. (i) and (ii), we get

$$\Delta E = \frac{K}{2}$$

Now, minimum value of $\Delta E$ for hydrogen atom is 10.2 eV.

or $\Delta E \geq 310.2$ eV

$\therefore$ $\frac{K}{2} \geq 310.2$

$\therefore$ $K \geq 20.4$ eV

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

## Atomic Spectra

When an atomic gas or vapour is excited at low pressure, usually by passing an electric current through it, the emitted radiations has a spectrum containing specific wavelength only. These type of spectrum consists of bright lines on a dark background and is termed as **emission line spectrum**.

When white light passes through a gas and the transmitted light consists of some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission spectrum of gas. This is called **absorption spectrum** of the gas.

**Note** Number of lines in emission spectrum or number of transition possible for an electron in $n$th orbit is $\frac{n(n-1)}{2}$.

# Hydrogen Spectrum

In hydrogen spectrum or line spectra of hydrogen atom there is no resemblance of order or regularity in spectral lines but the spacing between the lines within certain sets of spectrum decreases in regular way. Each of these sets is called a **spectral series**.

The wavelength corresponding to various series of spectra for hydrogen atom are as follows

**Lyman series,** $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2, 3, 4, \ldots$

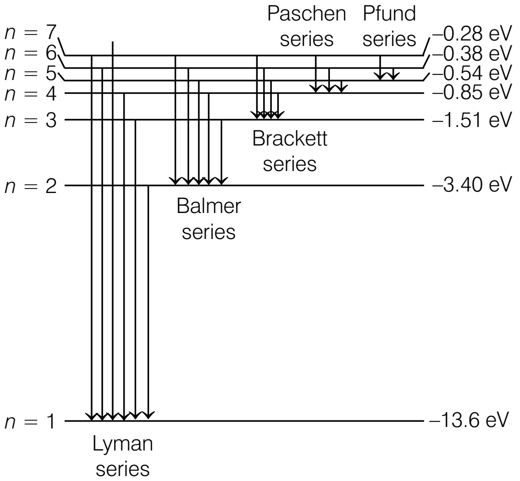
**Balmer series,** $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, $n = 3, 4, 5, \ldots$

**Paschen series,** $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6, \ldots$

**Brackett series,** $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, $n = 5, 6, 7, \ldots$

**Pfund series,** $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, $n = 6, 7, 8, \ldots$

**Note** In Balmer series, the line with largest wavelength 656.3 nm in red is called $H_\alpha$, the next line with wavelength 486.1 nm in blue is called $H_\beta$ and so on.




## Series Limit

The transition corresponding to maximum energy or smallest wavelength of a given series of hydrogen spectrum is called **series limit**. In general, series limit, $\frac{1}{\lambda} = \frac{R}{n^2}$.

***Example 8.*** *Calculate the frequency of the $H_\beta$ line of the Balmer series for hydrogen.*

(a) $4.5 \times 10^{11}$ Hz
(b) $5.2 \times 10^{12}$ Hz
(c) $5.7 \times 10^{13}$ Hz
(d) $6.12 \times 10^{14}$ Hz

***Sol.*** (d) $H_\beta$ line of Balmer series corresponds to the transition from $n_2 = 4$ to $n_1 = 2$ level.

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 0.2056 \times 10^7$$

$$\therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m}$$

$$\therefore \quad f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}}$$

$$= 6.12 \times 10^{14} \text{ Hz}$$

## de-Broglie's Explanation of Bohr's Second Postulate of Quantisation

According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving atom. So, for permissible orbit,

$$2\pi r_n = n\lambda = \frac{nh}{mv_n}$$

or

$$mv_n r_n = \frac{nh}{2\pi}$$

*i.e.* angular momentum of electron revolving in $n$th orbit must be an integral multiple of $\frac{h}{2\pi}$, which is quantum condition proposed by Bohr in second postulate.

***Example 9.*** *Orbits of a particle moving in a circle are such that the perimeter of the orbit equals an integer number of de-Broglie wavelengths of the particle. For a charged particle moving in a plane perpendicular to a magnetic field, the radius of the nth orbital will be proportional to*

(a) $n^2$
(b) $n$
(c) $n^{1/2}$
(d) $n^{1/4}$

***Sol.*** (c) As, $2\pi r = n\lambda$

$$r = \frac{n\lambda}{2\pi}$$

Now, de-Broglie equation $\lambda = \frac{h}{p}$

$$\Rightarrow \quad mv_n = \frac{h}{\lambda} = \frac{h}{\frac{2\pi r_n}{n}} = \frac{nh}{2\pi r_n}$$

Also, for charged particle moving in a magnetic field

$$r_n = \frac{mv_n}{qB} = \frac{nh}{(2\pi r_n)\,qB}$$

$$\Rightarrow \quad r_n^2 = \frac{nh}{2\pi qB}$$

$$\Rightarrow \quad r_n^2 \propto n \Rightarrow r_n \propto n^{1/2}$$

# Composition and Size of Nucleus

The entire positive charge and nearly the entire mass of atom is concentrated in a very small space called the **nucleus** of an atom. It consists of **protons** and **neutrons** and they are together called **nucleons**.

In a nucleus, the number of protons is equal to the atomic number ($Z$) of that element and total number of nucleons is called the mass number ($A$), *i.e.*

Number of protons = Atomic number $Z$

and number of neutrons

= Mass number ($A$) – Atomic number ($Z$) = $A - Z$

Thus, the nucleus of an atom represented as $_ZX^A$.

## Properties of Nucleus

The nuclear properties are described below

- **Nuclear size**
  (i) Size of the nucleus is of the order of fermi (1 fermi = $10^{15}$ m)
  (ii) The radius of the nucleus is given by $R = R_0 A^{1/3}$, where $R_0 = 1.3$ fermi and $A$ is the mass number.
  (iii) The size of the atom is of the order of $10^{-19}$ m.
- **Volume** The volume of nucleus is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3$$

- **Density**
  (i) Density $= \dfrac{\text{Mass of nucleus}}{\text{Volume of the nucleus}}$

$$= \frac{m_p A}{\frac{4}{3}\pi R^3} = \frac{m_p A}{\frac{4}{3}\pi \cdot (R_0 A^{1/3})^3} = \frac{m_p}{\frac{4}{3}\pi R_0^3}$$

  where, $m_p = 1.6 \times 10^{-27}$ kg = mass of proton and $R_0 = 1.3$ fermi.
  (ii) Density of nuclear matter is of the order of $10^{17}$ kg/m$^3$.
  (iii) Density of nuclear matter is independent of the mass number.

## Isotopes, Isobars and Isotones

The atoms of same element which have same atomic number but different mass number are called **isotopes**.

For example,

$$({}_1H^1, {}_1H^2 \text{ and } {}_1H^3), ({}_8O^{16}, {}_8O^{17} \text{ and } {}_8O^{18})$$

and $({}_{10}Ne^{20}, {}_{10}Ne^{21}$ and ${}_{10}Ne^{22})$, etc., are isotopes.

The atoms of different elements which have same mass number but different atomic number are called **isobars**.

For example,

$({}_8O^{17}$ and ${}_9F^{17})$ and $({}_{11}Na^{24}$ and ${}_{12}Mg^{24})$ are isobars.

The atoms of different elements of which the nuclei have the same number of neutrons but different number of protons are called the **isotones**.

For example,

$({}_{11}Na^{23}$ and ${}_{12}Mg^{24})$ and $({}_{19}K^{39}$ and ${}_{20}Ar^{40})$ are isotones.

# Equivalence of Mass and Energy

Einstein suggested that energy mass are equivalent. He predicted that if the energy of a body changes by an amount $E$, its mass changes by an amount $m$ given by the equation, $E = mc^2$

where, $c$ is the speed of light.

This relation is called **mass-energy equivalence** relation.

***Example 10.*** *The energy equivalent of 1g of substance is*

(a) $3 \times 10^{30}$ J (b) $6 \times 10^{13}$ J (c) $3 \times 10^{13}$ J (d) $9 \times 10^{13}$ J

***Sol.*** (d) *From Einstein's mass energy relation, we have*

$$E = mc^2$$

Given, $m = 1\text{g} = 10^{-3}\text{kg}, c = 3 \times 10^8 \text{m/s}$

$\therefore \quad E = mc^2 = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}$ J

## Mass Defect

It is found that mass of the nucleus is less than the total mass of protons and neutrons, this mass difference is called **mass defect.**

Since, number of protons inside the nucleus of ${}_Z X^A$ atom is $Z$ and number of neutrons is $(A - Z)$ hence, if ${}_Z M^A$ is the mass of nucleus, then mass defect

$$\Delta M = [Zm_p + (A - Z)m_n] - {}_Z M^A$$

where, $m_p$ is the mass of proton and $m_n$ is the mass of neutron.

**Note** Energy equivalent to mass defect of 1 amu is equal to 931.5 MeV.

## Nuclear Binding Energy

Binding energy ($E_b$) of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest at infinite distance apart. In other words, it may be defined as the energy equivalence to the mass defect of the nucleus.

$\therefore E_b = \Delta mc^2$, where $\Delta m$ is mass defect.

or $\quad E_b = \Delta m \times 931$ MeV

It can also be written as, $E_b = [Zm_p + (A - Z)m_n - m_x]\, c^2$

where, $m_p$ is mass of proton, $m_n$ is mass of neutron and $m_x$ is mass of nucleus.

**Binding Energy per Nucleon** It is the ratio of binding energy $E_b$ of a nucleus to the number of nucleons $A$ in that nucleus,

$$E_{bn} = \frac{E_b}{A}$$

***Example 11.*** *If the binding energy per nucleon of deuteron is 1.115 MeV, then its mass defect (in amu) is*

(a) 0.056 (b) 0.0072 (c) 0.0024 (d) 0.0017

***Sol.*** (c) Binding energy per nucleon,

$$E_{bn} = \frac{E_b}{A} \quad \text{...(i)}$$

For deuteron, atomic mass $A = 2$

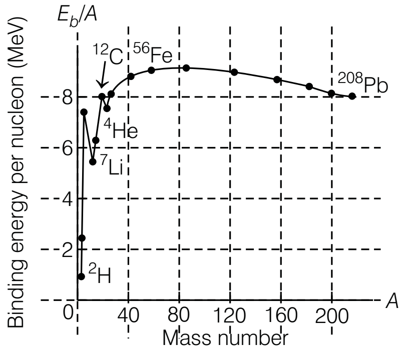
$\therefore$ From Eq. (i), $\quad 1.115 = \dfrac{E_b}{2}$

$\Rightarrow \quad E_b = 2.23$ MeV

$\therefore$ Mass defect (in amu) $= \dfrac{E_b}{931.5} = \dfrac{2.23}{931.5} = 0.0024$

## Binding Energy Curve

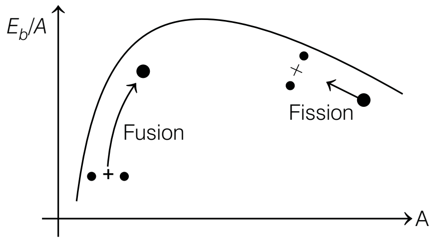
A plot of binding energy per nucleon $E_b/A$ as a function of mass number $A$ for various stable nuclei is shown in figure.




The binding energy per nucleon, $E_b$ / $A$, as a function of the mass number $A$

Following conclusions can be drawn from the above graph

- The greater the binding energy per nucleon, the more stable is the nucleus. The curve reaches a maximum of about 8.75 MeV in the vicinity of $^{56}_{26}\text{Fe}$ and then gradually falls to 7.6 MeV for $^{238}_{92}\text{U}$.
- Force is attractive and sufficiently strong to produce $E_{bn}$ of a few MeV per nucleon.
- The constancy of $E_{bn}$ in the range $30 < A < 170$ is the consequence of the fact that the nuclear force is short-ranged.
- A very heavy nucleus (say $A = 240$) can be split into two lighter nuclei (say $A = 120$) near the flat maximum of the curve, such that $E_{bn}$ will increase. Hence, energy will be released in this process of fission.
- Two very light nuclei ($A \leq 10$) joined together to form a heavier nucleus. $E_{bn}$ of the fused nuclei is more than that of lighter nuclei. Again, energy would be released in such as process of fusion.




# Nuclear Force

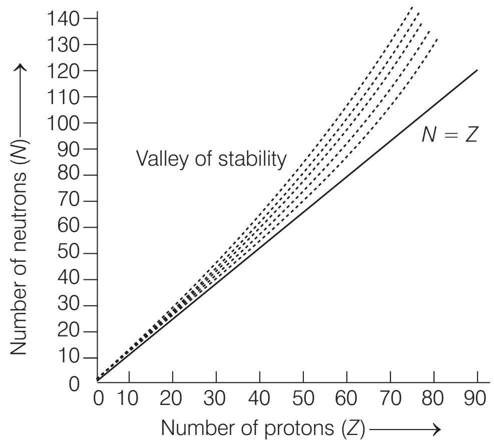
The forces that holds the nucleons together inside the nucleus of an atom are called **nuclear forces.**

Following are few important properties of the nuclear forces

- It is much stronger than the Coulomb force acting between the charges or the gravitational force between masses.
- Nuclear force between two nucleons falls rapidly to zero as their distance is more than few femtometres (fm). This leads to saturation of forces or a large sized nucleus, which is the reason for constancy of binding energy per nucleon $E_{bn}$.
- Nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. It does not depend on the electric charge.

## Nuclear Stability

The stability of nucleus is also determined by its neutron to proton ratio. A plot of number of neutrons and number of protons is shown in the figure below




Graphical representation of number of neutrons *versus* number of protons

In the figure, the solid line shows the nuclei with equal number of protons and neutrons ($N = Z$). Only light nuclei are on this line, *i.e.* they are stable if they contain approximately same number of protons and neutrons.

Heavy nuclei are stable only when they have more neutrons than protons.

# Radioactivity

It is the phenomenon in which an unstable nucleus undergo a decay with emission of some particles ($\alpha$, $\beta$) and electromagnetic radiation ($\gamma$-rays). It is also referred as **radioactive decay**.

Three types of decays are as follows

**$\alpha$-Decay** An $\alpha$-particle is a helium nucleus. Thus, a nucleus emitting an $\alpha$-particle loses two protons and two neutrons. Therefore, the atomic number $Z$ decreases by 2, the mass number $A$ decreases by 4 and the neutron number $N$ decreases by 2. The decay can be written as

$$^{A}_{Z}X = {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He}$$

where, $X$ is the parent nucleus and $Y$ is the daughter nucleus.

**$\beta$-Decay** This decay can involve the emission of either electrons or positrons.

**In** $\beta^-$ **decay**, a neutron in the nucleus is transformed into a proton, an electron and an antineutrino.

$$n \longrightarrow P + e^- + \bar{\nu}$$

Thus, a parent nucleus with atomic number $Z$ and mass number $A$ decays by $\beta^-$ emission into a daughter with atomic number $Z + 1$ and the same mass number $A$.

$${}^{A}_{Z}X \xrightarrow{\beta^-} {}_{Z+1}^{\;\;A}Y$$

$\beta^-$ decay occurs in nuclei that have too many neutrons. An example of $\beta^-$ decay is the decay of carbon 14 into nitrogen,

$${}^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}$$

**In** $\beta^+$ **decay**, a proton changes into a neutron with the emission of a positron (and a neutrino)

$$p \longrightarrow n + e^+ + \nu$$

Positron ($e^+$) emission from a nucleus decreases the atomic number $Z$ by 1, while keeping the same mass number $A$.

$${}^{A}_{Z}X \xrightarrow{\beta^+} {}_{Z-1}^{\;\;A}Y$$

$\beta^+$ decay occurs in nuclei that have two few neutrons.

$\gamma$-**Decay** In gamma decay, a nucleus changes from a higher energy state to a lower energy state through the emission electromagnetic radiations (photons).

So, no change occurs in atomic number and atomic mass of the parent and daughter atoms in gamma decay.

***Example 12.*** *In a radioactive decay chain, the initial nucleus is* ${}^{232}_{90}Th$. *At the end, there are 6* $\alpha$*-particles and 4* $\beta$*-particles which are emitted. If the end nucleus is* ${}^{A}_{Z}X$, *A and Z are given by* **[JEE Main 2019]**

(a) $A = 202, Z = 80$ (b) $A = 208, Z = 82$

(c) $A = 200, Z = 81$ (d) $A = 208, Z = 80$

***Sol.*** (b) The decay of $\alpha$-particle reduces, mass number by 4 and atomic number by 2.

$\therefore$ Decay of 6 $\alpha$-particles results, ${}^{232}_{90}\text{Th} \xrightarrow{6\alpha} {}^{232-24}_{90-12}Y = {}^{208}_{78}Y$

A $\beta$-decay does not produces any change in mass number but it increases atomic number by 1.

$\therefore$ Decay of 4 $\beta$-particles results, ${}^{208}_{78}Y \xrightarrow{4\beta} {}^{208}_{82}X$

$\therefore$ In the end nucleus $A = 208,\ Z = 82$

## Pair Production

When an energetic $\gamma$-ray photon falls on a heavy substance, it is absorbed by some nucleus of the substance and an electron and a positron are produced.

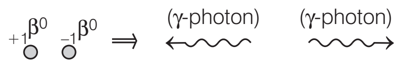
This phenomenon is called **pair production** and may be represented by the equation

$$h\nu \longrightarrow {}_{+1}\beta^0 + {}_{-1}\beta^0$$

## Pair Annihilation

The converse phenomenon of pair production is called **pair annihilation**. When an electron and a positron come very close to each other, they annihilate each other by combining together and two $\gamma$-photons are produced. This phenomenon can be represented by the following equation

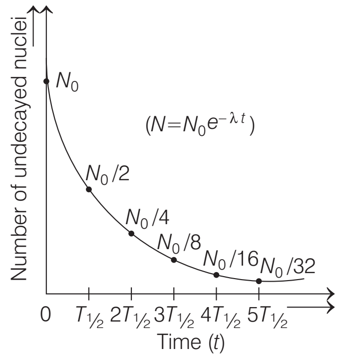
$${}_{+1}\beta^0 + {}_{-1}\beta^0 \longrightarrow h\nu + h\nu$$




# The Radioactive Decay Law

According to this law, the rate of decay of radioactive nuclei at any instant is proportional to the number of nuclei present at that instant. *i.e.* $N = N_0 e^{-\lambda t}$

where, $\lambda$ is called the **decay constant** and $N$ is the number of active nucleus present in the sample at any instant $t$ and $N_0$ is the number of nuclear present in the sample initially.




Exponential decay of a radioactive substance

## Activity of a Radioactive Substance

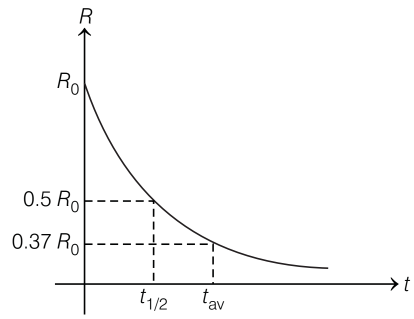
The decay rate $R$ of a radioactive substance is the number of decays per second.

It is given as

$$R = -\frac{dN}{dt} \text{ or } R = R_0\, e^{-\lambda t}$$

where, $R_0 = \lambda N_0$ is the activity of the radioactive substance at time $t = 0$.

The activity *versus* time graph is shown below.



Thus, the number of nuclei and hence the activity of the radioactive substance decrease exponentially with time.

**Units of activity** The SI unit for the decay rate is the becquerel (Bq), but the curie (Ci) and rutherford (Rd) are often used in practice.

$$1\ \text{Bq} = 1\ \text{decay/s},$$
$$1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq}$$
and
$$1\ \text{Rd} = 10^{6}\ \text{Bq}$$

## Half-life

The time required for the number of parent nuclei to fall 50% of its initial value is called **half-life**. It is denoted by $t_{1/2}$. It is given as

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

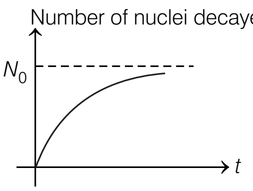
## Mean life

The average or mean life $t_{\text{av}}$ is the reciprocal of the decay constant, *i.e.* $t_{\text{av}} = \frac{1}{\lambda}$

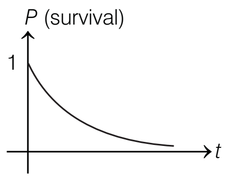
After a time equal to the mean life time, the number of radioactive nuclei and the decay rate have each decreased to 37% of their original values.

## Important Points Related to Radioactivity

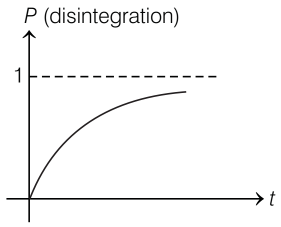
- After $n$ half lives,
  (a) Number of nuclei left $= N_0 \left(\frac{1}{2}\right)^n$
  (b) Fraction of nuclei left $= \left(\frac{1}{2}\right)^n$
  (c) Percentage of nuclei left $= 100 \left(\frac{1}{2}\right)^2$
- Number of nuclei decayed after time $t$,
  $$= N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-\lambda t})$$
  The corresponding graph is shown below




- Probability of a nucleus for survival of time $t$,
  $$P\ (\text{survival}) = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$
  The corresponding graph is shown below




- Probability of a nucleus to disintegrate in time $t$ is
  $$P\ (\text{disintegration}) = 1 - P\ (\text{survival}) = 1 - e^{-\lambda t}$$
  The corresponding graph is shown below




- Half-life and mean life are related to each other by the relation,
  $$t_{1/2} = 0.693\, t_{\text{av}}$$
  or
  $$t_{\text{av}} = 1.44\, t_{1/2}$$

***Example 13.*** *The half-life of $^{238}_{92}U$ undergoing $\alpha$-decay is $4.5 \times 10^9$ yr. The activity of 1g sample of $^{238}_{92}U$ is [Avogadro number $= 6.022 \times 10^{23}$]*

(a) $12.3 \times 10^{10}$ Bq (b) $1.23 \times 10^4$ Bq
(c) $14.2 \times 10^3$ Bq (d) $1.42 \times 10^{17}$ Bq

**Sol.** (b) Given, $t_{1/2} = 4.5 \times 10^9$ yr

$$= 4.5 \times 10^9 \times 3.16 \times 10^7\ \text{s}$$
$$t_{1/2} = 1.42 \times 10^{17}\ \text{s}$$

The number of atoms in 1 g of uranium,

$$N = \frac{6.023 \times 10^{23}}{238}$$

The decay rate (activity) $R$ is

$$R = \lambda N$$
$$= \frac{0.693}{t_{1/2}} N$$
$$= \frac{0.693 \times 6.023 \times 10^{23}}{1.42 \times 10^{17} \times 238}$$
$$= 1.23 \times 10^4\ \text{Bq}$$

***Example 14.*** *In given time t = 0, activity of two radioactive substances A and B are equal. After time t, the ratio of their activities $\frac{R_B}{R_A}$ decreases according to $e^{-3t}$. If the half-life of A is ln 2, the half-life of B will be* **[JEE Main 2019]**

(a) 4ln2 (b) $\frac{\ln 2}{4}$ (c) $\frac{\ln 2}{2}$ (d) 2ln2

**Sol.** (b) Activity of radioactive material is given as

$$R = \lambda N$$

where, $\lambda$ is the decay constant and $N$ is the number of nuclei in the radioactive material.

For substance $A$,

$$R_A = \lambda_A N_A = \lambda_A N_{0A} \quad (\text{initially, } N_A = N_{0A})$$

For substance $B$,

$$R_B = \lambda_B N_B = \lambda_B N_{0B} \quad (\text{initially, } N_B = N_{0B})$$

At $t = 0$, activity is equal.

Therefore, $\lambda_A N_{0A} = \lambda_B N_{0B}$ ...(i)

The half-life is given by $t_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$

So, for substance $A$,

$$(t_{1/2})_A = \frac{\ln 2}{\lambda_A} \Rightarrow \ln 2 = \frac{\ln 2}{\lambda_A}$$

$$\lambda_A = 1 \qquad \text{...(ii)}$$

According to the given question,

at time $t$, $$\frac{R_B}{R_A} = e^{-3t} \qquad \text{...(iii)}$$

Using Eqs. (i), (ii) and (iii), we get

$$\frac{R_B}{R_A} = e^{-3t} = \frac{\lambda_B N_{0B} e^{-\lambda_B t}}{\lambda_A N_{0A} e^{-\lambda_A t}}$$

$\Rightarrow$ $$e^{-3t} = e^{(\lambda_A - \lambda_B)t} \Rightarrow -3 = \lambda_A - \lambda_B$$

$\Rightarrow$ $$\lambda_B = \lambda_A + 3 \Rightarrow \lambda_B = 1 + 3 = 4 \qquad \text{...(iv)}$$

The half-life of substance $B$ is $(t_{1/2})_B = \frac{\ln 2}{\lambda_B} = \frac{\ln 2}{4}$

***Example 15.*** *Half lives of two radioactive nuclei A and B are 10 min and 20 min, respectively. If initially a sample has equal number of nuclei, then after 60 min, the ratio of decayed numbers of nuclei A and B will be*

**[JEE Main 2019]**

(a) 3 : 8 (b) 1 : 8 (c) 8 : 1 (d) 9 : 8

**Sol.** (d) For substance $A$, half-life is 10 min, so it decays as

$$N_{0_A} \xrightarrow{10\text{min}} \frac{N_{0_A}}{2} \xrightarrow{10\text{min}} \frac{N_{0_A}}{4} \xrightarrow{10\text{min}} \frac{N_{0_A}}{8}$$

(Initial number of nuclei at $t = 0$) (Active nuclei remained after 10 min)

$$\xrightarrow{10\text{min}} \frac{N_{0_A}}{16} \xrightarrow{10\text{min}} \frac{N_{0_A}}{32} \xrightarrow{10\text{min}} \frac{N_{0_A}}{64}$$

$\therefore$ For substance $A$, number of nuclei decayed in 60 min is

$$N_{0_A} - \frac{N_{0_A}}{64} = \frac{63 N_{0_A}}{64}$$

Similarly, for substance $B$, half-life is 20 min, so it's decay scheme is

$$N_{0_B} \xrightarrow{20\text{min}} \frac{N_{0_B}}{2} \xrightarrow{20\text{min}} \frac{N_{0_B}}{4} \xrightarrow{20\text{min}} \frac{N_{0_B}}{8}$$

So, number of nuclei of $B$ decayed in 60 min is

$$N_{0_B} - \frac{N_{0_B}}{8} = \frac{7}{8} N_{0_B}$$

Hence, ratio of decayed nuclei of $A$ and $B$ in 60 min is

$$\frac{\frac{63}{64} N_{0_A}}{\frac{7}{8} N_{0_B}} = \frac{9}{8} \qquad [\because N_{0_A} = N_{0_B}]$$

***Example 16.*** *Two radioactive materials A and B have decay constants* $10\lambda$ *and* $\lambda$*, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time* **[JEE Main 2019]**

(a) $\frac{1}{11\lambda}$ (b) $\frac{11}{10\lambda}$ (c) $\frac{1}{9\lambda}$ (d) $\frac{1}{10\lambda}$

**Sol.** (c) Given, $\lambda_A = 10\lambda$ and $\lambda_B = \lambda$

Number of nuclei (at any instant) present in material is

$$N = N_0 e^{-\lambda t}$$

So, for materials $A$ and $B$, we can write

$$\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t} \qquad \text{... (i)}$$

Given, $$\frac{N_A}{N_B} = \frac{1}{e} \qquad \text{... (ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{1}{e} = e^{-(\lambda_A - \lambda_B)t} \Rightarrow e^{-1} = e^{-(\lambda_A - \lambda_B)t}$$

Comparing the power of $e$ on both sides, we get

or $$(\lambda_A - \lambda_B)\, t = 1 \Rightarrow t = \frac{1}{\lambda_A - \lambda_B}$$

By putting values of $\lambda_A$ and $\lambda_B$ in the above equation, we get

$$t = \frac{1}{10\lambda - \lambda} \Rightarrow t = \frac{1}{9\lambda}$$

# Nuclear Reaction

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction.

The general expression for the nuclear reaction is as follows

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C}$$

$$\longrightarrow \underset{\text{(Product nucleus)}}{Y} + \underset{\text{(Daughter nucleus)}}{b} + \underset{\text{energy}}{Q}$$

Here, $X$ and $a$ are the reactants, $Y$ and $b$ are the products and $Q$ is the kinetic energy released in the nuclear reaction.

## *Q*-value

$Q$-value means the difference between the rest mass energy of initial constituents and the rest mass energy of final constituents of a nuclear reaction.

***Example 17.*** *Given the masses of various atomic particles* $m_p = 1.0072\,u, m_n = 1.0087\,u, m_e = 0.000548\,u, m_{\bar{\nu}} = 0,$ $m_d = 2.0141\,u$*, where p = proton, n = neutron, e = electron,* $\bar{\nu}$ *= antineutrino and d = deuteron. Which of the following process is allowed by momentum and energy conservation?*

**[JEE Main 2020]**

(a) $n + n \rightarrow$ *deuterium atom (electron bound to the nucleus)*

(b) $p \rightarrow n + e^+ + \bar{\nu}$

(c) $n + p \rightarrow d + \bar{\nu}$

(d) $e^+ + e^- \rightarrow \gamma$

**Sol.** (c) In nuclear reaction, momentum and energy conservation laws are valid only, if total mass on reactant side is greater than that of the product side.

*i.e.* $\Sigma m_{\text{reactant}} > \Sigma m_{\text{product}}$

because liberated energy appears due to mass defect.

If we check the above condition from given data, then it is found that among all options only option (c) satisfy the condition.

Mass of $n$ + Mass of $p$ = (1.0087 + 1.0072) u
= 2.0159 u

Mass of $d$ + Mass of $\bar{\nu}$ = (2.0141 + 0) u
= 2.0141 u

Hence, only statement in option (c) is true.

***Example 18.*** *Consider the nuclear fission*

$$Ne^{20} \longrightarrow 2He^4 + C^{12}$$

*Given that the binding energy/nucleon of $Ne^{20}$, $He^4$ and $C^{12}$ are respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement.*

(a) *Energy of* 9.72 *MeV will be released.*
(b) *Energy of* 12.4 *MeV will be supplied.*
(c) 8.3 *MeV energy will be released.*
(d) *Energy of* 11.9 *MeV has to be supplied.*

***Sol.*** (a) Energy absorbed or released in a nuclear reaction is given by

$\Delta Q$ = Binding energy of products − Binding energy of reactants

If energy is absorbed, $\Delta Q$ is negative and if it is positive, then energy is released.

Also, binding energy = binding energy per nucleon × number of nucleons.

Here, binding energy of products

$$= 2 \times (\text{BE of He}^4) + (\text{BE of } C^{12})$$

$$= 2\ (4 \times 7.07) + (12 \times 7.86) = 150.88 \text{ MeV}$$

and binding energy of reactants

$$= 20 \times 8.03 = 160.6 \text{ MeV}$$

So, $\Delta Q = (\text{BE})_{\text{products}} - (\text{BE})_{\text{reactants}}$
$= 150.88 - 160.6$
$= -9.72$ MeV

As, $\Delta Q$ is negative.

∴ Energy of 9.72 MeV is released in the reaction.

# Nuclear Energy

It is the energy released during the transformation of nuclei with less total binding energy to nuclei with greater binding energy.

Two distinct ways of obtaining energy from nucleus are as follows

(i) Nuclear fission (ii) Nuclear fusion

## Nuclear Fission

It is the process of splitting of a heavy nucleus ( $^{235}_{92}\text{U}$ or $^{239}_{94}\text{U}$) into two lighter nuclei of comparable masses alongwith the release of a large amount of energy after bombarded by slow neutrons.

When a uranium isotope $^{235}_{92}\text{U}$ bombarded with a neutron breakes into two intermediate mass nuclear fragments.

$$^1_0n + ^{235}_{92}\text{U} \longrightarrow ^{236}_{92}\text{U} \longrightarrow ^{144}_{51}\text{Ba} + ^{89}_{36}\text{Kr} + 3^1_0n + Q \text{ (energy released)}$$

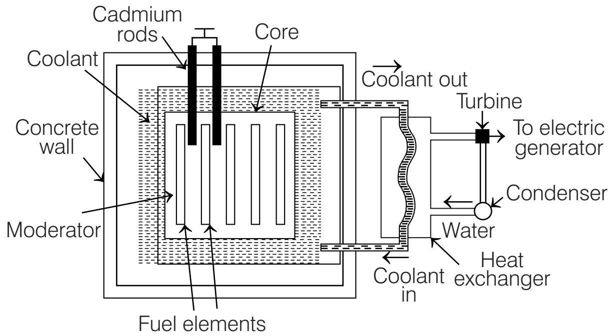
**Chain Reaction** In nuclear fission many neutrons are produced along with the release of large energy. Under favourable conditions these neutrons can cause further fission of other nuclei, producing large number of neutrons.

Thus, a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.

## Nuclear Reactor

It is a device in which nuclear fission can be carried out through a sustained and controlled chain reaction.

It is also called an **atomic pile**. It is thus a source of controlled energy which is utilised for many useful purposes.




Following are the important terms related to reactor

(i) **Core** is the site of nuclear fission. It is usually consists of $^{235}_{92}\text{U}$.
(ii) Core contains a **moderator** which is used to slow down fast moving neutrons, *e.g.* water, heavy water ($D_2O$) and graphite.
(iii) **Reflector** is used to reduce leakage.
(iv) The reactor can be shut down by means of **control rods** (made of cadmium) that have high absorption rate of neutrons.
(v) The **coolant** transfers heat to a working fluid which in turn may produce steam. The steam drives turbines and generates electricity.

**Note** Multiplication factor $k$ is the ratio of the number of fission produced by a given generation of neutrons to the number of fission of the proceeding generation.

***Example 19.*** *What is the power output of a $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel, and if each fission gives 185 MeV of usable energy?*

(a) $2.6 \times 10^3$ (b) $9.26 \times 10^4$
(c) $5.85 \times 10^7$ (d) $6.2 \times 10^6$

***Sol.*** (c) 235 amu of uranium gives 185 MeV energy. Therefore, the energy given by 1 amu of ${}_{92}U^{235}$

$$= \frac{185}{2.35} \text{ MeV}$$

$$= \frac{185}{235} \times 1.6 \times 10^{-13} \text{J}$$

But 1 amu $= 1.66 \times 10^{-27}$ kg. Therefore, energy released by

$$1.66 \times 10^{-27} \text{ kg of } {}_{92}U^{235} = \frac{185 \times 1.6 \times 10^{-13}}{235}$$

Hence, energy released by 2 kg of ${}_{92}U^{235}$

$$W = \frac{185 \times 1.6 \times 10^{-13} \times 2}{235 \times 1.66 \times 10^{-27}} = 1.517 \times 10^{14} \text{J}$$

Therefore, power output of reactor

$$= \frac{W}{t} = \frac{1.517 \times 10^{14}}{30 \text{ days}}$$

$$P = \frac{1.517 \times 10^{14}}{30 \times 24 \times 60 \times 60} = 5.85 \times 10^{7} \text{ W}$$

## Nuclear Fusion

When two lighter nuclei combine to form a heavier nucleus, the process is called **nuclear fusion**. The union of lighter nuclei into heavier nuclei also lead to a transfer of mass and a consequent liberation of energy.

Such a reaction has been achieved in **hydrogen bomb** and it is believed to be the principal source of the sun's energy.

A reaction with heavy hydrogen or deuterium which yields 3.3 MeV per fusion is

$${}_{1}^{2}\text{H} + {}_{1}^{2}\text{H} \rightarrow {}_{2}^{3}\text{He} + {}_{0}^{1}n$$

# X-rays

When cathode rays strike on a heavy metal of high melting point, then a very small fraction of its energy converts into a new type of waves called X-rays. X-rays were discovered by **Roentgen**.

## Properties of X-rays

(i) X-rays are electromagnetic waves of wavelengths ranging from 0.1 Å to 100 Å and frequencies ranging from $10^{16}$ Hz to $10^{18}$ Hz.

(ii) Soft X-rays have greater wavelength and lower frequency.

(iii) Hard X-rays have lower wavelength and higher frequency.

(iv) X-rays are produced in coolidge tube.

(v) Molybdenum and tungsten provide suitable targets. These elements have large atomic number and high melting point for the purpose.

(vi) The intensity of X-rays depends on the heating voltage or filament current.

(vii) The kinetic energy of X-ray photons depends upon the voltage applied across the ends of coolidge tube.

(viii) If total energy of fast moving electron transfer to X-ray photon, then its energy $eV = h\nu = \frac{hc}{\lambda}$.

(ix) Wavelength of emitted X-rays is given by $\lambda = \frac{hc}{eV}$, where $h$ = Planck's constant, $c$ = speed of light, $e$ = electronic charge and $V$ = potential difference applied across the ends of the tube.

(x) Absorption of X-rays $I = I_0 e^{-\mu x}$, where $I_0$ = initial intensity of X-rays, $I$ = final intensity of emergent X-rays, $x$ = thickness of material and $\mu$ = absorption coefficient.

## Continuous X-rays

The continuous X-rays (or bremsstrahlung X-rays) produced at a given accelerating potential $V$ vary in wavelength, but none has a wavelength shorter than a certain value $\lambda_{min}$. This minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy $qV$ or $eV$ of the striking electrons. Thus,

$$\frac{hc}{\lambda_{min}} = eV$$

or

$$\lambda_{min} = \frac{hc}{eV}$$

After substituting values of $h$, $c$ and $e$, we obtain the following simple formula for $\lambda_{min}$.

$$\lambda_{min} \text{ (in Å)} = \frac{12375}{V} \quad \ldots\text{(i)}$$

If $V$ is increased, then $\lambda_{min}$ decreases. This wavelength is also known as the **cut-off wavelength** or the **threshold wavelength**.

## X-rays Spectrum

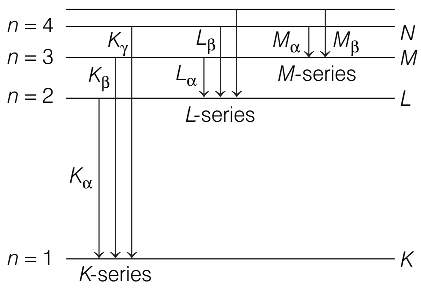
The energy spectrum of X-rays is a line spectrum, containing following series

(i) ***K*-series** When electrons of any higher orbit ($n$ = 2, 3, 4, …) jump to first orbit ($n$ = 1), then $K$-series of X-rays are produced.

(ii) ***L*-series** When electrons of higher orbit ($n$ = 3, 4, 5, …) jump to second orbit ($n$ = 2), then $L$-series of X-rays are produced.

(iii) ***M*-series** When electrons of higher orbit ($n$ = 4, 5, 6, …) jump to third orbit ($n$ = 3), then $M$-series of X-rays are produced.

First lines of these series are called $K_\alpha, L_\alpha, M_\alpha$ and second lines of these series are called $K_\beta, L_\beta, M_\beta$.




The energy of X-ray radiation as

$$\Delta E = Rhc(Z-b)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\nu = Rc(Z-b)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

## Moseley's Law

The frequency of X-rays is given by

$$\nu = a\,(Z-b)^2$$

where, $a$ and $b$ are constants and $Z$ is atomic number of element.

$$\nu \propto Z^2$$

***Example 20.*** *Use Moseley's law with $b=1$ to find the frequency of the $K_\alpha$ X-rays of La ($Z = 57$) if the frequency of the $K_\alpha$ X-rays of Cu ($Z = 29$) is known to be $1.88\times10^{18}$ Hz.*

(a) $7.52\times10^{18}$ *Hz* (b) $3.25\times10^{16}$ *Hz*
(c) $8.51\times10^{19}$ *Hz* (d) $9.1\times10^{15}$ *Hz*

**Sol.** (a) Using the equation,

$$\sqrt{f} = a\,(Z-b) \qquad (b=1)$$

$$\frac{f_{La}}{f_{Cu}} = \left(\frac{Z_{La}-1}{Z_{Cu}-1}\right)^2$$

or

$$f_{La} = f_{Cu}\left(\frac{Z_{La}-1}{Z_{Cu}-1}\right)^2$$

$$= 1.88\times10^{18}\left(\frac{57-1}{29-1}\right)^2$$

$$= 7.52\times10^{18}\text{ Hz}$$

***Example 21.*** *An X-ray tube with a copper target is found to be emitting lines other than those due to copper. The $K_\alpha$ line of copper is known to have a wavelength 1.5405 Å and the other two $K_\alpha$ lines observed have wavelengths 0.7090 Å and 1.6578 Å. Identify the impurities.*

*(a) Nickel* *(b) Molybdenum*
*(c) Both (a) and (b)* *(d) Copper*

**Sol.** (c) According to Moseley's equation for $K_\alpha$ radiation,

$$\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \text{ (}\lambda = \text{wavelength corresponding to Cu)}$$

Let $\lambda_1$ and $\lambda_2$ be the two other unknown wavelengths, then

$$\frac{\lambda_1}{\lambda} = \frac{(Z-1)^2}{(Z_1-1)^2} = \frac{0.7090}{1.5405}$$

For copper $Z = 29$, therefore $(Z_1-1) = 28\sqrt{\dfrac{1.5405}{0.7092}} = 41$

or $Z_1 = 42$ (molybdenum)

Similarly, $\dfrac{\lambda_2}{\lambda} = \dfrac{(28)^2}{(Z_2-1)^2} = \dfrac{1.6578}{1.5405}$

$$(Z_2-1) = 28\sqrt{\left(\frac{1.5405}{1.6578}\right)}$$

or $(Z_2-1) = 27$

or $Z_2 = 28$ (nickel)

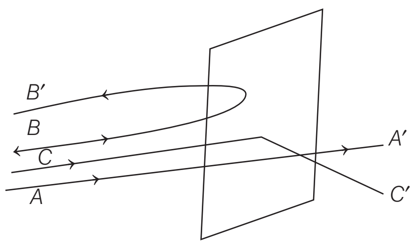
So, the impurities are molybdenum and nickel.

# Practice Exercise

## ROUND I Topically Divided Problems

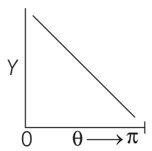
### Rutherford $\alpha$-Particle Scattering Experiment and Bohr's Model

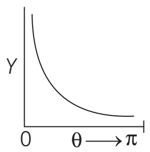
**1.** A beam of fast moving $\alpha$-particles was directed towards a thin gold foil. The parts $A'$, $B'$ and $C'$ of the transmitted and reflected beams corresponding to incident parts $A$, $B$ and $C$ of the beam are as shown in the figure below.

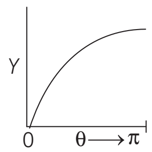


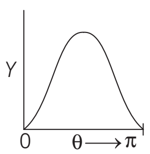

Then, the number of $\alpha$-particles in part
(a) $B'$ will be minimum and $C'$ is maximum
(b) $A'$ will be minimum and $B'$ is maximum
(c) $C'$ will be minimum and $A'$ is maximum
(d) $A'$ will be maximum and $B'$ will be minimum

**2.** The graph which depicts the results of Rutherford gold foil experiment with $\alpha$-particles is
$\theta$ : scattering angle
$Y$ : number of scattered $\alpha$-particles detected
(plots are schematic and not to scale) **[JEE Main 2020]**

(a)



(b)



(c)



(d)



**3.** An $\alpha$-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of
(a) 1 Å (b) $10^{-10}$ cm
(c) $10^{-12}$ cm (d) 10–15 cm

**4.** In some of Rutherford's experiments, a silver foil is used. Let kinetic energy of $\alpha$-particles is 4.8 MeV, mass of $\alpha$-particles is $6.64 \times 10^{-27}$ kg, charge on silver nucleus is $47e$.

For a head-on collision impact parameter for an $\alpha$-particle, which is scattered at an angle of 45° is
(a) $2.8 \times 10^{-14}$ m (b) $3.36 \times 10^{-14}$ m
(c) $1.41 \times 10^{-14}$ m (d) $5.60 \times 10^{-14}$ m

**5.** In $\alpha$-scattering experiment, in a head-on-collision between an $\alpha$-particle and a gold nucleus, the minimum distance of separation or closest approach is $4 \times 10^{-14}$ m. The energy of $\alpha$-particle is (Take, atomic number of gold = 79)
(a) 5.68 MeV (b) 8 MeV
(c) 4.47 MeV (d) 7.24 MeV

**6.** An ionised H-molecule consist of an electron and two protons. One proton are separated by a small distance of the order of angstrom. In the ground state
(a) the electron would not move in circular orbits
(b) the energy would be $(z)^4$ times that of a H-atom
(c) the molecule will soon decay in the proton and a H-atom
(d) None of the above

**7.** For the ground state the electron in the H-atom has an angular momentum = $h$. According to the simple Bohr model, angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true, **[NCERT Exemplar]**
(a) because Bohr model gives incorrect values of angular momentum.
(b) because only one of these would have a minimum energy.
(c) angular momentum must be in the direction of spin of electron.
(d) because electrons go around only in horizontal orbits.

**8.** Taking the Bohr radius as $a_0$ = 53 pm, the radius of $Li^{++}$ ion in its ground state, on the basis of Bohr's model, will be about **[NCERT Exemplar]**
(a) 53 pm (b) 27 pm (c) 18 pm (d) 13 pm

**9.** In the Bohr's model of hydrogen like atom the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the $n$th orbit in terms of the Bohr radius $\left(a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$ is

(a) $r = a_0 n - \beta$ (b) $r = a_0 n^2 + \beta$
(c) $r = a_0 n^2 - \beta$ (d) $r = a_0 n + \beta$

**10.** The electron in a hydrogen atom makes transition from $M$-shell to $L$-shell. The ratio of magnitudes of initial to final centripetal acceleration of electron is

(a) 9 : 4 (b) 81 : 16 (c) 4 : 9 (d) 16 : 81

**11.** Consider an electron in the $n$th orbit of a hydrogen atom in the Bohr model. The circumference of the orbit can be expressed in terms of the de-Broglie wavelength of that electron as

(a) $(0.529)\, n\lambda$ (b) $\sqrt{n}\,\lambda$
(c) $(13.6)\,\lambda$ (d) $n\lambda$

**12.** The time period of revolution of electron in its ground state orbit in a hydrogen atom is $1.6 \times 10^{-16}$ s. The frequency of revolution of the electron in its first excited state (in Hz ) is **[JEE Main 2020]**

(a) $1.6 \times 10^{14}$ (b) $5.6 \times 10^{12}$
(c) $6.2 \times 10^{15}$ (d) $7.8 \times 10^{14}$

**13.** Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is **[JEE Main 2019]**

(a) 3.5 Å (b) 6.6 Å (c) 12.9 Å (d) 9.7 Å

## Energy of Electron and Hydrogen Spectrum

**14.** As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion

(a) its kinetic energy increases but potential energy and total energy decrease
(b) kinetic energy, potential energy and total energy decrease
(c) kinetic energy decreases, potential energy increases but total energy remains same
(d) kinetic energy and total energy decrease but potential energy increases

**15.** In the Bohr model of the hydrogen atom, let $R$, $V$ and $E$ represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantities is proportional to quantum number $n$?

(a) $\frac{R}{E}$ (b) $\frac{E}{V}$ (c) $RE$ (d) $VR$

**16.** Two H-atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is **[NCERT Exemplar]**

(a) 10.20 eV (b) 20.40 eV
(c) 13.6 eV (d) 27.2 eV

**17.** The energy, the magnitude of linear momentum and orbital radius of an electron in a hydrogen atom corresponding to the quantum number $n$ are $E$, $P$ and $r$ respectively. Then according to Bohr's theory of hydrogen atom, choose the incorrect statement.

(a) $Pr$ is proportional to $n$
(b) $P/E$ is proportional to $n$
(c) $Er$ is not constant for all orbits
(d) $EPr$ is proportional to $1/n$

**18.** Which level of the single ionised carbon has the same energy as the ground state energy of hydrogen atom? **[JEE Main 2021]**

(a) 1 (b) 6 (c) 4 (d) 8

**19.** As the electron in Bohr orbit of hydrogen atom passes from state $n = 2$ to $n = 1$, the kinetic energy $K$ and potential energy $U$ changes as

(a) $K$ becomes one-fourth and $U$ becomes four times
(b) $K$ becomes four times, $U$ becomes twice
(c) both $K$ and $U$ becomes four times
(d) both $K$ and $U$ becomes twice

**20.** The energy of an electron in $n$th orbit of the hydrogen atom is given by

$$E_n = \frac{-13.6}{n^2}\,\text{eV}$$

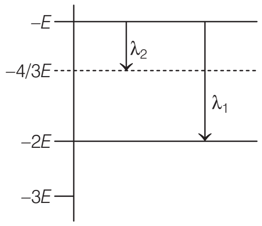
The energy required to raise an electron from the first orbit to the second orbit will be

(a) 10.2 eV (b) 12.1 eV
(c) 13.6 eV (d) 3.4 eV

**21.** An electron jumps from the 4th orbit to 2nd orbit of hydrogen atom. The frequency (in Hz) of the emitted radiation will be (Given, Rydberg's constant, $R = 10^5\,\text{cm}^{-1}$)

(a) $\frac{3}{16} \times 10^5$ (b) $\frac{3}{16} \times 10^{15}$ (c) $\frac{9}{16} \times 10^{15}$ (d) $\frac{3}{4} \times 10^{15}$

**22.** Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by




(a) $r = \frac{2}{3}$ (b) $r = \frac{3}{4}$ (c) $r = \frac{1}{3}$ (d) $r = \frac{4}{3}$

**23.** The ratio of the energies of the hydrogen atom in its first to second excited states is

(a) 9/4 (b) 4/1
(c) 8/1 (d) 1/8

**24.** A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon. **[NCERT]**

(a) $9.7\times10^{-8}$ m and $3.1\times10^{15}$ Hz
(b) $7.6\times10^{-9}$ m and $2.6\times10^{14}$ Hz
(c) $2.9\times10^{-10}$ m and $4.9\times10^{12}$ Hz
(d) $8.6\times10^{-9}$ m and $3.1\times10^{14}$ Hz

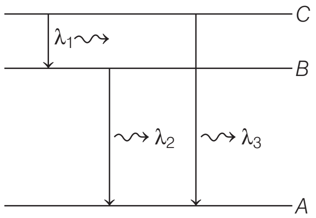
**25.** The wave number of the energy emitted when electron comes from fourth orbit to second orbit in hydrogen is 20397 $cm^{-1}$. The wave number of the energy for the same transition in $He^+$ is

(a) 5099 $cm^{-1}$ (b) 20497 $cm^{-1}$
(c) 14400 Å (d) 81588 $cm^{-1}$

**26.** Hydrogen atom from excited state comes to the ground state by emitting a photon of wavelength $\lambda$. If $R$ is the Rydberg constant, then the principal quantum number $n$ of the excited state is

(a) $\sqrt{\dfrac{\lambda R}{\lambda R-1}}$ (b) $\sqrt{\dfrac{\lambda}{\lambda R-1}}$
(c) $\sqrt{\dfrac{\lambda R^2}{\lambda R-1}}$ (d) $\sqrt{\dfrac{\lambda R}{\lambda-1}}$

**27.** Energy levels $A$, $B$ and $C$ of a certain atom corresponding to increasing values of energy, *i.e.* $E_A < E_B < E_C$. If $\lambda_1$, $\lambda_2$ and $\lambda_3$ are wavelengths of photon corresponding to transitions shown below.




Then,

(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \lambda_1\lambda_2 / \lambda_1 + \lambda_2$
(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

**28.** Imagine an atom made up of proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength $\lambda$ (given in terms of the Rydberg constant $R$ for the hydrogen atom) equal to **(JEE 2000)**

(a) $9/5R$ (b) $36/5R$
(c) $18/5R$ (d) $4/R$

**29.** Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$ respectively for four elements, then approximately which one of the following is correct?

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

**30.** The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths $\lambda_1 / \lambda_2$ of the photons emitted in this process is **[JEE Main 2019]**

(a) 20/7 (b) 27/5
(c) 7/5 (d) 9/7

**31.** In a hydrogen like atom, when an electron jumps from the $M$-shell to the $L$-shell, the wavelength of emitted radiation is $\lambda$. If an electron jumps from $N$-shell to the $L$-shell, the wavelength of emitted radiation will be **[JEE Main 2019]**

(a) $\dfrac{27}{20}\lambda$ (b) $\dfrac{25}{16}\lambda$
(c) $\dfrac{20}{27}\lambda$ (d) $\dfrac{16}{25}\lambda$

**32.** The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state ? **[JEE Main 2020]**

(a) 8.6 nm (b) 24.2 nm
(c) 11.4 nm (d) 35.8 nm

**33.** An excited $He^+$ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number $n$ corresponding to its initial excited state is [for photon of wavelength $\lambda$, energy $E = \dfrac{1240 \text{ eV}}{\lambda \text{ (in nm)}}$ ] **[JEE Main 2019]**

(a) $n = 4$ (b) $n = 5$
(c) $n = 7$ (d) $n = 6$

**34.** Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on $He^+$ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is **[JEE Main 2019]**

(a) $n = 2$ to $n = 3$ (b) $n = 1$ to $n = 4$
(c) $n = 2$ to $n = 5$ (d) $n = 2$ to $n = 4$

**35.** A $He^+$ ion is in its first excited state. Its ionisation energy is **[JEE Main 2019]**

(a) 54.40 eV (b) 13.6 eV
(c) 48.36 eV (d) 6.04 eV

**36.** Imagine that the electron in a hydrogen atom is replaced by a muon ($\mu$). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionisation potential of this hydrogen atom will be **[JEE Main 2021]**

(a) 13.6 eV (b) 2815.2 eV
(c) 331.2 eV (d) 27.2 eV

**37.** A muon is an unstable elementary particle whose mass is 207 $m_e$ and whose charge is either $+e$ or $-e$. A negative muon ($\mu^-$) can be captured by a hydrogen nucleus (or proton) to form a muon atom. Then,

(a) the radius of the first Bohr orbit of this atom is $2.85 \times 10^{-13}$ m
(b) the ionisation energy of the atom is 2.53 keV
(c) radius of first orbit of this atom is $2.86 \times 10^{-10}$ m
(d) Both (a) and (b)

**38.** Ionisation potential of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. The spectral lines emitted by hydrogen atom according to Bohr's theory will be

(a) one (b) two
(c) three (d) four

**39.** If the shortest wavelength in the Lyman series is 911.6 Å, the longest wavelength in the same series will be

(a) 1600 Å (b) 2430 Å (c) 1215 Å (d) $\infty$

**40.** The first line of Balmer series has wavelength 6563 Å. What will be the wavelength of the first member of Lyman series?

(a) 1215.4 Å (b) 2500 Å (c) 7500 Å (d) 600 Å

**41.** If the series limit frequency of the Lyman series is $\nu_L$, then the series limit frequency of the Pfund series $\nu_P$ is **[JEE Main 2018]**

(a) $25\,\nu_L$ (b) $16\,\nu_L$ (c) $\frac{\nu_L}{16}$ (d) $\frac{\nu_L}{25}$

**42.** $\nu_1$ is the frequency of the series limit of Lyman series, $\nu_2$ is the frequency of the first line of Lyman series and $\nu_3$ is the frequency of the series limit of the Balmer series, then **[Karnataka CET 2010]**

(a) $\frac{1}{\nu} = \frac{1}{\nu_2} + \frac{1}{\nu_3}$ (b) $\frac{1}{\nu_2} = \frac{1}{\nu_1} + \frac{1}{\nu_3}$

(c) $\nu_1 = \nu_2 - \nu_3$ (d) $\nu_1 - \nu_2 = \nu_3$

**43.** In accordance with the Bohr's model the quantum number that characterises the earth's revolution around the sun in an orbit of radius $1.5 \times 10^{11}$ m with orbital speed $3 \times 10^4$ m/s (Mass of earth $= 6.0 \times 10^{24}$ kg). They belong to

(a) Balmer series (b) Lyman series
(c) Humbnery series (d) None of these

**44.** Taking the wavelength of first Balmer line in hydrogen spectrum ($n = 3$ to $n = 2$) as 660 nm, the wavelength of the 2nd Balmer line ($n = 4$ to $n = 2$) will be **[JEE Main 2019]**

(a) 889.2 nm (b) 388.9 nm
(c) 642.7 nm (d) 488.9 nm

**45.** In $Li^{++}$, electron in first Bohr orbit is excited to a level by a radiation of wavelength $\lambda$. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of $\lambda$? [Take, $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms$^{-1}$] **[JEE Main 2019]**

(a) 9.4 nm (b) 12.3 nm (c) 10.8 nm (d) 11.4 nm

## Nucleus, Mass-Energy Relation and Binding Energy

**46.** If Avogadro number is $6 \times 10^{23}$, then number of protons, neutrons and electrons is 14 g of $_6C^{14}$ are respectively

(a) $36 \times 10^{23}$, $48 \times 10^{23}$, $36 \times 10^{23}$
(b) $36 \times 10^{23}$, $36 \times 10^{23}$, $36 \times 10^{23}$
(c) $48 \times 10^{23}$, $36 \times 10^{23}$, $48 \times 10^{23}$
(d) $48 \times 10^{23}$, $48 \times 10^{23}$, $36 \times 10^{23}$

**47.** $r_1$ and $r_2$ are the radii of atomic nuclei of mass numbers 64 and 27, respectively. The ratio $(r_1/r_2)$ is

(a) 64/27 (b) 27/64
(c) 4/3 (d) 1

**48.** The ratio of mass densities of nuclei of $^{40}Ca$ and $^{16}O$ is close to **[JEE Main 2019]**

(a) 5 (b) 2
(c) 0.1 (d) 1

**49.** The radius $R$ of a nucleus of mass number $A$ can be estimated by the formula $R = (1.3 \times 10^{-15})\,A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of $(M_{\text{proton}} \cong M_{\text{neutron}} \cong -1.67 \times 10^{-27}$ kg) **[JEE Main 2020]**

(a) $10^{17}$ kg m$^{-3}$ (b) $10^{24}$ kg m$^{-3}$
(c) $10^{10}$ kg m$^{-3}$ (d) $10^{3}$ kg m$^{-3}$

**50.** Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. The atomic number of nucleus will be

(a) 25 (b) 26
(c) 56 (d) 30

**51.** The curve of binding energy per nucleon as function of atomic mass number has a sharp peak for lithium nucleus. This implies that lithium

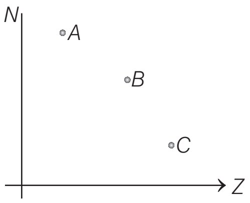
(a) can be used as fissionable material
(b) is highly radioactive
(c) can be easily broken up
(d) is very stable

**52.** Consider the binding energy of $_{17}Cl^{35}$ and $_{15}P^{31}$ are 287.67 MeV and 262.48 MeV, respectively. Then,
(a) $_{17}Cl^{35}$ is more stable than $_{15}P^{31}$
(b) $_{15}P^{31}$ is more stable than $_{17}Cl^{35}$
(c) Stability of both the elements is equivalent
(d) cannot be estimated from the given data

**53.** The binding energies per nucleon of $Li^7$ and $He^4$ are 5.6 MeV and 7.06 MeV respectively, then the energy of the reaction $_3Li^7 + p \rightarrow 2\ [_2He^4]$ will be
(a) 17.28 MeV
(b) 39.2 MeV
(c) 28.24 MeV
(d) 1.46 MeV

**54.** The gravitational force between a H-atom and another particle of mass $m$ will be given by Newton's law : $F = G\dfrac{M \cdot m}{r^2}$, where $r$ is (in km) and **[NCERT Exemplar]**
(a) $M = m_{\text{proton}} + m_{\text{electron}}$
(b) $M = m_{\text{proton}} + m_{\text{electron}} - \dfrac{B}{c^2}$ ($B = 13.6$ eV)
(c) $M$ is not related to the mass of the hydrogen atom.
(d) $M = m_{\text{proton}} + m_{\text{electron}} - \dfrac{|V|}{c^2}$ ($|V| =$ magnitude of the potential energy of electron in the H-atom).

**55.** Consider the plot of $N$ (neutron number) *versus* $Z$ (photon number) for the different nuclei. Let three nuclides $A$, $B$ and $C$ are at the positions as shown in the figure.




The order of their stability may be
(a) $A > B > C$
(b) $A < B < C$
(c) $B > A > C$
(d) $C < A < B$

**56.** Find the binding energy per nucleon for $^{120}_{50}Sn$. Mass of proton $m_p = 1.00783$ u, mass of neutron $m_n = 1.00867$ u and mass of tin nucleus $m_{Sn} = 119.902199$ u. (Take, 1 u = 931 MeV) **[JEE Main 2020]**
(a) 9.0 MeV
(b) 7.5 MeV
(c) 8.0 MeV
(d) 8.5 MeV

**57.** Two nucleons are at a separation of 1 fm. The net force between them is $F_1$, if both are neutrons, $F_2$ if both are protons and $F_3$ if one is a proton and the other is a neutron, then
(a) $F_1 > F_2 > F_3$
(b) $F_2 > F_1 > F_3$
(c) $F_1 = F_3 > F_2$
(d) $F_1 = F_2 > F_3$

**58.** $O_2$ molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms **[NCERT Exemplar]**
(a) is not important because nuclear forces are short-ranged
(b) is as important as electrostatic force for binding the two atoms
(c) cancels the repulsive electrostatic force between the nuclei
(d) is not important because oxygen nucleus have equal number of neutrons and protons

**59.** Binding energy for nuclei $P, Q$ and $R$ are $E_P, E_Q$ and $E_R$, respectively.
In the fusion processes,
$$3P \longrightarrow Q + \text{Energy}(E_1)$$
$$2Q \longrightarrow R + \text{Energy}(E_2)$$
then total energy released in the fusion process
$$6P \longrightarrow R + \text{Energy}(E_3) \text{ is}$$
(a) $E_1 + E_2$
(b) $E_1 - E_2$
(c) $E_1 - 2E_2$
(d) $2E_1 + E_2$

## Radioactivity

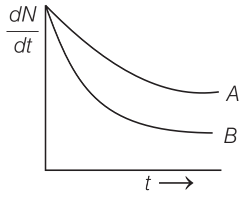
**60.** In the uranium radioactive series, the initial nucleus is $_{92}U^{238}$ and that the final nucleus is $_{82}Pb^{206}$. When uranium nucleus decays to lead, the number of $\alpha$-particle and $\beta$-particles emitted are
(a) $8\alpha, 6\beta$
(b) $6\alpha, 7\beta$
(c) $6\alpha, 8\beta$
(d) $4\alpha, 3\beta$

**61.** For the radioactive nuclei that undergo either $\alpha$ or $\beta$-decay, which one of the following cannot occur?
(a) Isobar of original nucleus
(b) Isotope of original nucleus
(c) Nuclei with higher atomic number than that of the original nucleus
(d) Nuclei with lower atomic number than that of the original nucleus

**62.** The half-life of $Au^{198}$ is 2.7 days. The activity of 1.50 mg of $Au^{198}$ if its atomic weight is 198 g mol$^{-1}$ is, ($N_A = 6 \times 10^{23}$/mol) **[JEE Main 2021]**
(a) 240 Ci
(b) 357 Ci
(c) 535 Ci
(d) 252 Ci

**63.** Tritium is an isotope of hydrogen whose nucleus triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + e^- + \bar{\nu}$. If one of the neutrons in triton decays, it would transform into $He^3$ nucleus. This does not happen. This is because **[NCERT Exemplar]**
(a) triton energy is less than that of a $He^3$ nucleus
(b) the electron created in the beta decay process cannot remain in the nucleus
(c) both the neutrons in trition have to decay simultaneously resulting in a nucleus with 3 protons, which is not a $He^3$ nucleus
(d) because free neutrons decay due to external perturbations which is absent in a triton nucleus

**64.** In the following figure, which of the following, sample $A$ or $B$ has shorter mean life?



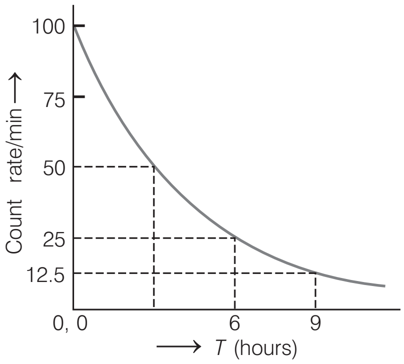

(a) $B$ (b) $A$
(c) Both $A$ and $B$ (d) Neither $A$ nor $B$

**65.** The half-life of radioactive Radon is 3.8 days. The time at the end of which (1/20)th of the Radon sample will remain undecayed is (Given, $\log_{10} e = 0.4343$)
(a) 13.8 days (b) 16.5 days
(c) 33 days (d) 76 days

**66.** Suppose we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half-life of 1 yr. After 1 yr, **[NCERT Exemplar]**
(a) all the containers will have 5000 atoms of the materials
(b) all the containers will contain the same number of atoms of the material but that number will only be approximately 5000
(c) the containers will in general have different numbers of the the atoms of the material but their average will be close to 5000
(d) None of the containers can have more than 5000 atoms

**67.** The half-life period of a radioactive element $x$ is same as the mean life time of another radioactive element $y$. Initially, both of them have the same number of atoms. Then, **(JEE 1999)**
(a) $x$ and $y$ have the same decay rate initially
(b) $x$ and $y$ decay at the same rate always
(c) $y$ will decay at a faster rate than $x$
(d) $x$ will decay at a faster rate than $y$

**68.** The count rate for 10g of radioactive material was measured at different times and this has been shown in figure with scale given. The half-life of the material and the total count in the first half value period, respectively are




(a) 4 h and 9000 (approximately)
(b) 3 h and 14100 (approximately)
(c) 3 h and 235 (approximately)
(d) 10 h and 157 (approximately)

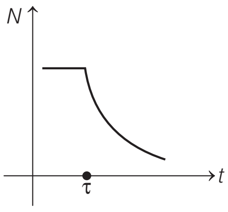
**69.** The half-life period of radium is 1600 yr. The fraction of a sample of radium that would remain after 6400 yr is **[NCERT Exemplar]**
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

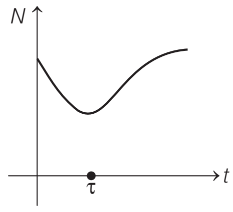
**70.** In a radioactive material, fraction of active material remaining after time $t$ is 9/16. The fraction that was remaining after time $t/2$ is **[JEE Main 2020]**
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{3}{4}$ (d) $\frac{7}{8}$

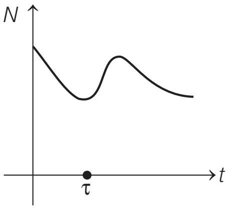
**71.** The activity of a radioactive sample falls from $700 s^{-1}$ to $500 s^{-1}$ in 30 min. Its half-life is close to **[JEE Main 2020]**
(a) 62 min (b) 66 min
(c) 72 min (d) 52 min

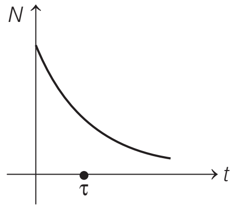
**72.** An accident in a nuclear laboratory resulted in depositions of a certain amount of a radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times the permissible level allowed for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?
(a) 64 (b) 90
(c) 108 (d) 120

**73.** A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life of one species is $\tau$ and that of the other is $5\tau$. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figure best represents the form of this plot?

(a)



(b)



(c)



(d)


**74.** Using a nuclear counter, the count rate of emitted particles from a radioactive source is measured. At $t = 0$, it was 1600 counts per second and $t = 8$ s, it was 100 counts per second. The count rate observed as counts per second at $t = 6$ s is close to **[JEE Main 2019]**

(a) 400 (b) 200
(c) 150 (d) 360

**75.** Samples of two radioactive nuclides $A$ and $B$ are taken. $\lambda_A$ and $\lambda_B$ are the disintegration constants of $A$ and $B$ respectively. In which of the following cases, the two samples can simultaneously have the same decay rate at any time? **[NCERT Exemplar]**

(a) Initial rate of decay of $A$ is twice the initial rate of decay of $B$ and $\lambda_A = \lambda_B$
(b) Initial rate of decay of $A$ is twice the initial rate of decay of $B$ and $\lambda_A > \lambda_B$
(c) Initial rate of decay of $B$ is same as the rate of decay of $A$ at $t = 2$ h and $\lambda_B < \lambda_A$
(d) Both (b) and (c)

**76.** A radioactive nucleus $A$ with a half-life $T$, decays into a nucleus $B$. At $t = 0$, there is no nucleus $B$. After sometime $t$, the ratio of the number of $B$ to that of $A$ is 0.3. Then, $t$ is given by

(a) $t = T\dfrac{\log 1.3}{\log_e 2}$ (b) $t = T \log 1.3$
(c) $t = \dfrac{T}{\log 1.3}$ (d) $t = \dfrac{T \log_e 2}{2 \log 1.3}$

**77.** Half-lives of two radioactive elements $A$ and $B$ are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of $A$ and $B$ nuclei will be

(a) 1 : 16 (b) 4 : 1
(c) 1 : 4 (d) 5 : 4

**78.** A sample of radioactive material $A$, that has an activity of 10 mCi (1 Ci $= 3.7 \times 10^{10}$ decays/s) has twice the number of nuclei as another sample of a different radioactive material $B$ which has an activity of 20 mCi. The correct choices for half-lives of $A$ and $B$ would, then be respectively **[JEE Main 2019]**

(a) 20 days and 10 days
(b) 5 days and 10 days
(c) 10 days and 40 days
(d) 20 days and 5 days

**79.** A radioactive element decays by $\beta$-emission. A detector records $n$ $\beta$-particles in 2 s and in next 2 s it records $0.75\,n$ $\beta$-particles. Find mean life (in s) correct to nearest whole number. (Given, $\ln|2| = 0.6931$ and $\ln|3| = 1.0986$)

(a) 6 (b) 4
(c) 7 (d) 9

**80.** A radioactive nucleus decays by two different processes. The half-life for the first process is 10 s and that for the second is 100 s. The effective half-life of the nucleus is close to **[JEE Main 2020]**

(a) 9 s (b) 6 s
(c) 55 s (d) 12 s

**81.** A radioactive nucleus $X$ decays to a nucleus $Y$ with a decay constant $\lambda_X = 0.1\,s^{-1}$, $Y$ further decays to a stable nucleus $Z$ with a decay constant $\lambda_Y = 1/30\,s^{-1}$. Initially, there are only $X$ nuclei and their number is $N_0 = 10^{20}$. Set up the rate equations for the populations of $X$, $Y$ and $Z$. The population of $Y$ nucleus as a function of time is given by $N_Y(t) = \{N_0 \lambda_X / (\lambda_X - \lambda_Y)\}[\exp(-\lambda_Y t) - \exp(-\lambda_X t)]$. $N_Y$ is maximum

(a) 16.48s (b) 4.26s
(c) 9s (d) 9.28s

**82.** A source contains two phosphorous radio nuclides ${}^{32}_{15}P\,(T_{1/2} = 14.3 \text{ days})$ and ${}^{33}_{15}P\,(T_{1/2} = 25.3 \text{ days})$. Initially, 10% of the decay come from ${}^{33}_{15}P$. How long one must wait until 90% do so?

(a) 250 days (b) 295 days
(c) 305 days (d) 208 days

## Nuclear Reactions, Nuclear Fission and Nuclear Fusion

**83.** $M_x$ and $M_y$ denote the atomic masses of the parent and the daughter nuclei respectively in a radioactive decay. The $Q$-value of a $\beta^-$ decay is $Q_1$ and that for a $\beta^+$ decay is $Q_2$. If $m_e$ denotes the mass of an electron, then which of the following statement is correct? **[NCERT Exemplar]**

(a) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y - 2m_e)c^2$
(b) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y)c^2$
(c) $Q_1 = (M_x - M_y - 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2m_e)c^2$
(d) $Q_1 = (M_x - M_y + 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2m_e)c^2$

**84.** You are given that mass of ${}^{7}_{3}Li = 7.0160$ u,
mass of ${}^{4}_{2}He = 4.0026$ u
and mass of ${}^{1}_{1}H = 1.0079$ u.
When 20g of ${}^{7}_{3}Li$ is converted into ${}^{4}_{2}He$ by proton capture, the energy liberated (in kWh), is
[Mass of nucleon $= 1\,\text{GeV}/c^2$] **[JEE Main 2020]**

(a) $4.5 \times 10^5$ (b) $8 \times 10^6$
(c) $6.82 \times 10^5$ (d) $1.33 \times 10^6$

**85.** The electrostatic energy of $Z$ protons uniformly distributed throughout a spherical nucleus of radius $R$ is given by $E = \dfrac{3}{5}\dfrac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$

The measured masses of a neutron, ${}^{1}_{1}H$, ${}^{15}_{7}N$ and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii

of both the $^{15}_{7}O$ nuclei are the same;
1 u = 931.5 MeV/$c^2$, where $c$ is the speed of light and $e^2/4\pi\varepsilon_0$ = 1.44 MeV-fm. Assuming that, the difference between the binding energies of $^{15}_{7}N$ and $^{15}_{8}O$ is purely due to their electrostatic energy, the radius of either of the nuclei is

(a) 2.85 fm (b) 3.03 fm
(c) 3.42 fm (d) 3.80 fm

**86.** The number of neutrons released during the fission reaction is $^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{133}_{51}Sb + ^{99}_{41}Nb + \text{neutrons}$

(a) 1 (b) 92
(c) 3 (d) 4

**87.** When uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as $_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3X + Q$ (energy) where three particles names $X$ are produced and energy $Q$ is released. What is the name of the particle $X$?

(a) electron (b) $\alpha$-particle
(c) neutron (d) neutrino

**88.** Energy released in the fission of a single nucleus is 200 MeV. The fission rate of a $^{235}_{92}U$ filled reactor operating at a power level of 5W is

(a) $1.56 \times 10^{-10}$ s$^{-1}$ (b) $1.56 \times 10^{11}$ s$^{-1}$
(c) $1.56 \times 10^{-16}$ s$^{-1}$ (d) $1.56 \times 10^{-17}$ s$^{-1}$

**89.** Fusion process, like combining two deuterons to form a He nucleus are impossible at ordinary temperatures and pressure. This reasons for this can be traced to the fact

(a) nuclear forces have short range
(b) the original nuclei must be completely ionized before fussion can take place
(c) the original nuclei must first break up before combining with each other.
(d) All of the above

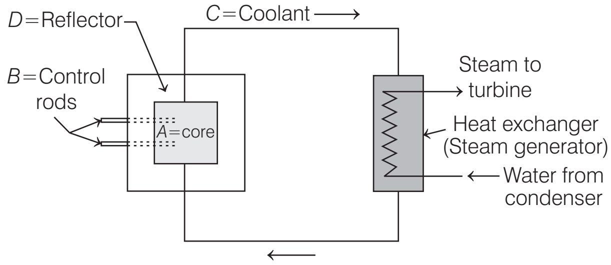
**90.** It is proposed to use the nuclear fusion reaction, $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He$ in a nuclear reactor 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day? (The masses of $^{2}_{1}H$ and $^{4}_{2}He$ are 2.0141 amu and 4.0026 amu, respectively.)

(a) 120.26 g (b) 60.46 g
(c) 80 g (d) 90.26 g

**91.** In a reactor, 2kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that, the Avogadro number, $N = 6.023 \times 10^{26}$ K$^{-1}$ mol$^{-1}$ and 1 eV $= 1.6 \times 10^{-19}$ J. The power output of the reactor is close to **[JEE Main 2020]**

(a) 35 MW (b) 125 MW
(c) 60 MW (d) 54 MW

**92.** Schematic diagram of a nuclear reactor based on thermal neutron fission is as shown below




Here, the correct purpose of the parts specified by $A$, $B$, $C$ and $D$ is given in

(a) $A \rightarrow$ reduces leakage.
(b) $B \rightarrow$ can shut down the reactor
(c) $C \rightarrow$ site of nuclear fission and contains $^{235}_{92}U$
(d) $D \rightarrow$ transfers heat to a working fluid which in turn may produce steam

## X-Rays

**93.** Electrons with de-Broglie wavelength $\lambda$ fall on the target in an X-ray tube. The cut-off wavelength $\lambda_0$ of the emitted X-rays is **[IIT JEE 2007]**

(a) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \dfrac{2h}{mc}$
(c) $\lambda_0 = \dfrac{2m^2c^2\lambda^2}{h^2}$ (d) $\lambda_0 = \lambda$

**94.** If $\lambda_1$ and $\lambda_2$ are the wavelength of characteristic X-rays and $\gamma$-rays respectively, then the relation between them is

(a) $\lambda_1 > \lambda_2$ (b) $\lambda_1 < \lambda_2$
(c) $\lambda_1 = \lambda_2$ (d) $\lambda_1\lambda_2 = 1$

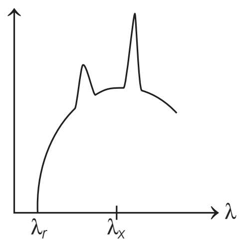
**95.** Which one of the following statement is wrong in the context of X-rays generated from a X-rays tube? **[IIT JEE 2008]**

(a) Wavelength of characteristic X-rays decreases when the atomic number of the target increases
(b) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(c) Intentsity of the characteristic X-rays depends on the electrical power given to the X-rays tube
(d) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

**96.** The binding energy of the innermost electron in tungsten is 40 k eV. To produce characteristic X-rays using a tungsten target in an X-rays tube, the potential difference between the cathode and anti cathode should be

(a) $V < 40$ kV
(b) $V \leq 40$ kV
(c) $V > 40$ kV
(d) $V = 40$ kV

**97.** The intensity of X-rays from a coolidge tube is plotted against wavelength $\lambda$ as shown in figure. The minimum wavelength found is $\lambda_c$ and the wavelength of $K_\alpha$ line is $\lambda_K$. As the accelerating voltage is increased, then

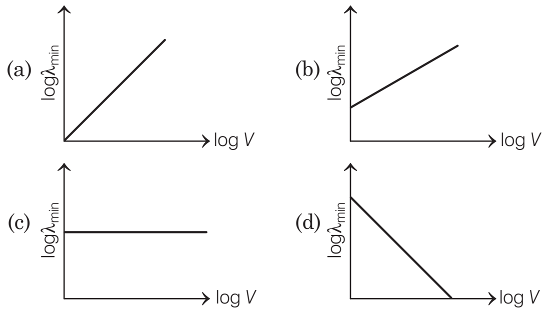



(a) $\lambda_K - \lambda_c$ increases (b) $\lambda_K - \lambda_c$ decreases
(c) $\lambda_K$ increases (d) $\lambda_K$ decreases

**98.** A potential difference of $10^4$ V is applied across an X-ray tube. The ratio of the de-Broglie wavelength of X-rays produced is
($\frac{e}{m}$ for electron= $1.8 \times 10^{11}$ $Ckg^{-1}$)

(a) $\frac{1}{20}$ (b) $\frac{1}{10}$ (c) 1 (d) $\frac{1}{100}$

**99.** An electron beam is accelerated by a potential difference $V$ to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If $\lambda_{min}$ is the smallest possible wavelength of X-rays in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in




**100.** An element with atomic number $Z = 11$ emits $K_\alpha$ X-ray of wavelength $\lambda$. The atomic number of element which emits $K_\alpha$X -ray of wavelength $4\lambda$

**[IIT Screening 2006]**

(a) 6 (b) 4 (c) 11 (d) 44

**101.** Find the constants $a$ and $b$, respectively in Moseley's equation $\sqrt{\nu} = a(Z - b)$ from the following data.

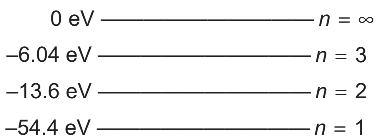
| Element | $Z$ | Wavelength of $K_\alpha$ X-ray |
|---|---|---|
| Mo | 42 | 71 pm |
| Co | 27 | 178.5 pm |

(a) 1.37, $5 \times 107$ (b) 6.27, 13.4
(c) $5 \times 10^7$, 1.37 (d) 9.26, $7 \times 10^7$

## ROUND II Mixed Bag

### Only One correct Option

**1.** The energy level diagram for an hydrogen like atom is shown in the figure. The radius of its first Bohr orbit is




(a) 0.265 Å
(b) 0.53 Å
(c) 0.132 Å
(d) None of the above

**2.** A hydrogen atom emits a photon corresponding to an electron transition from $n = 5$ to $n = 1$. The recoil speed of hydrogen atom is almost (mass of proton $\approx 1.6 \times 10^{-27}$ kg)

(a) 10 $ms^{-1}$
(b) $2 \times 10^2$ $ms^{-1}$
(c) 4 $ms^{-1}$
(d) $8 \times 10^2$ $ms^{-1}$

**3.** A hydrogen atom and a $Li^{2+}$ ion are both in second excited state. If $L_H$ and $L_{Li}$ are their respective electronic angular momenta and $E_H$ and $E_{Li}$ their respective energies, then

(a) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$
(b) $L_H = L_{Li}$ and $|E_H| < |E_{Li}|$
(c) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$
(d) $l_H > l_{Li}$ and $|E_H| << |E_{Li}|$

**4.** A sample of an element is 10.38 g. If half-life of element is 3.8 days, then after 19 days, how much quantity of element remains?

(a) 0.151 g (b) 0.32 g
(c) 1.51 g (d) 0.16 g

**5.** The radioactivity of a given sample of whisky due to tritium (half-life 12.3 yr) was found to be only 3% of that measured in a recently purchased bottle marked 7 yr old. The sample must have been prepared about

(a) 220 yr back (b) 300 yr back
(c) 400 yr back (d) 70 yr back

**6.** The ratio of molecular mass of two radioactive substances is 3/2 and the ratio of their decay constants is 4/3. Then, the ratio of their initial activity per mole will be

(a) 2 (b) 4/3 (c) 8/9 (d) 9/8

**7.** A radioactive substance of half-life 6 min is placed near a Geiger counter which is found to register 1024 particles per minute. How many particles per minute will it register after 42 min?

(a) 4 per min (b) 8 per min
(c) 5 per min (d) 7 per min

**8.** The half-life for the $\alpha$-decay of uranium $_{92}U^{238}$ is $4.47 \times 10^9$ yr. If a rock contains sixty percent of its original $_{92}U^{238}$ atoms, its age is
[log 6 = 0.778; log 2 = 0.3]

(a) $3.3 \times 10^9$ yr (b) $6.6 \times 10^9$ yr
(c) $1.2 \times 10^8$ yr (d) $5.4 \times 10^7$ yr

**9.** The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where $n_1$ and $n_2$ are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of electron in the initial state is 8 times that in the final state. The possible values of $n_1$ and $n_2$ are

(a) $n_1 = 6, n_2 = 3$ (b) $n_1 = 8, n_2 = 2$
(c) $n_1 = n_2 = 1$ (d) $n_1 = 8, n_2 = 1$

**10.** There are two radioactive substances $A$ and $B$. Decay constant of $B$ is two times that of $A$. Initially, both have equal number of nuclei. After $n$ half-lives of $A$, rate of disintegration of both are equal. The value of $n$ is

(a) 4 (b) 2 (c) 1 (d) 5

**11.** The binding energy of an electron in the ground state of He is equal to 24.6 eV. The energy required to remove both the electrons is

(a) 49.2 eV (b) 24.6 eV
(c) 38.2 eV (d) 79.0 eV

**12.** Calculate the time interval between 33% decay and 67% decay, if half-life of a substance is 20 minutes.

(a) 60 minutes (b) 20 minutes
(c) 40 minutes (d) 13 minutes

**13.** The highly excited states ($n >> 1$) for hydrogen-like atoms (also called Rydberg states) with nuclear charge $Ze$ are defined by their principal quantum number $n$. Which of the following statement(s) is/are false?

(a) The relative change in the radii of two consecutive orbitals does not depend on $Z$.
(b) The relative change in the radii of two consecutive orbitals varies as $1/n$.
(c) The relative change in the energy of two consecutive orbitals varies as $1/n^3$.
(d) The relative change in the angular momentum of two consecutive orbitals varies as $1/n$.

**14.** Let $E_n = \frac{-1}{8\varepsilon_0^2 n^2 h^2}$ be the energy of the $n$th level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $(E_2 - E_1)/h$ falls on it then

(a) it will not be absorbed at all
(b) some of atoms will move to the first excited state
(c) all atoms will be excited to the $n = 2$ state
(d) None of the above

**15.** In a hydrogen atom, electron makes a transition from $(n + 1)$th level to the $n$th level. If $n >> 1$, the frequency of radiation emitted is proportional to **[JEE Main 2020]**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^3}$ (c) $\frac{1}{n^2}$ (d) $\frac{1}{n^4}$

**16.** An electron in hydrogen atom first jumps from second excited state to first excited state and then from first excited state to ground state. Let the ratio of wavelength, momentum and energy of photons emitted in these two cases be $a$, $b$ and $c$ respectively. Then,

(a) $a = \frac{9}{4}$ (b) $b = \frac{6}{27}$ (c) $c = \frac{7}{27}$ (d) $c = \frac{1}{a}$

**17.** After absorbing a slowly moving neutron of mass $n_N$ (momentum ~0) a nucleus of mass $M$ breaks into two nuclei of masses $m_1$ and $5m_1$ $(6m_1 = M + m_N)$, respectively. If the de-Broglie wavelength of the nucleus with mass $m_1$ is $\lambda$, then de-Broglie wavelength of the other nucleus will be **[AIEEE 2011]**

(a) $25\lambda$ (b) $5\lambda$ (c) $\lambda/5$ (d) $\lambda$

**18.** The normal activity of living carbon containing matter is found to be about 15 decays/min for every gram of carbon. This activity arises from the small proportion of radioactive $^{14}_{6}C$ present with the stable carbon isotope $^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 yr) of $^{14}_{6}C$ and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays/min$^{-1}$g$^{-1}$ of carbon. Estimate the approximate age of the Indus-Valley civilisation.

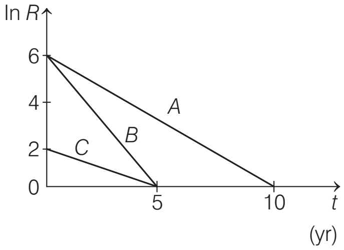
(a) 5224 yr (b) 4224 yr (c) 8264 yr (d) 6268 yr

**19.** A radioactive sample disintegrates via two independent decay processes having half lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$, respectively. The effective half-life $T_{1/2}$ of the nuclei is **[JEE Main 2021]**

(a) $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$ (b) $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$

(c) $T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$ (d) None of these

**20.** Activities of three radioactive substances $A$, $B$ and $C$ are represented by the curves $A$, $B$ and $C$ in the figure. Then, their half-lives $T_{1/2}(A): T_{1/2}(B): T_{1/2}(C)$ are in the ratio **[JEE Main 2020]**




(a) 4 : 3 : 1 (b) 3 : 2 : 1
(c) 2 : 1 : 1 (d) 2 : 1 : 3

**21.** An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let $\lambda_n$, $\lambda_g$ be the de-Broglie wavelength of the electron in the $n$th state and the ground state, respectively. Let $\Lambda_n$ be the wavelength of the emitted photon in the transition from the $n$th state to the ground state. For large $n$, ($A$, $B$ are constants)

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n^2$
(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$

**22.** Photoelectrons are emitted when 400 nm radiation is incident on a surface of work-function 1.9 eV. These photoelectrons pass through a region containing $\alpha$-particles. A maximum energy electron combines with an $\alpha$-particle to form a $He^+$ ion, emitting a single photon in this process. $He^+$ ions thus formed are in their fourth excited state. Find the energies (in eV) of the photons lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination.
[Take, $h = 4.14 \times 10^{-15}$ eV-s]

(a) 3.4 eV, 3.86 eV (b) 3.86 eV, 3.4 eV
(c) 2.4 eV, 6 eV (d) 9.6 eV, 2.4 eV

## Numerical Value Questions

**23.** The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. 5 min later, the rate becomes 2700 $\text{min}^{-1}$. Then, the half-life of the sample (in min) is ......... .

**24.** In the fusion reaction ${}_1^2H + {}_1^2H \longrightarrow {}_2^3He + {}_0^1n$, the masses of deuteron, helium and neutron expressed in amu are 2.015, 3.017 and 1.009, respectively. If 1 g of deuterium undergoes complete fusion, then the amount of total energy released is $9 \times 10^n$ kJ. Find the value of $n$.

**25.** In the chemical analysis of a rock the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are $4 \times 10^9$ yr and $2 \times 10^9$ yr, respectively. If it is assumed that at the time of formation the atoms of both the isotopes were in equal proportion, the age of the rock $p \times 10^{10}$ yr, where the value of $p$ is ......... . (Take, ratio of the atomic weights of the two isotopes is 1.02 : 1 .)

**26.** A hydrogen-like atom of atomic number $Z$ is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. (Ground state energy of hydrogen atom is 13.6 eV .)

**27.** In an experiment on two radioactive isotopes of an element (which do not decay into one another), their mass ratio at a given instant was formed to be 3. The decaying isotope has a larger mass and activity of 1.0 curie initially. The half-lives of the two radioactive isotopes are known to be 12 h and 16 h. Activity of the each isotope and their mass ratio after 2 days was studied. If the ratio of number of atoms of first isotope to that of the other isotope is found to be $3/x$, then the value of $x$ is ......... .

**28.** The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is ......... . **[JEE Main 2020]**

**29.** In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 Å. The corresponding difference for the Paschan series (in Å) is ......... . **[JEE Main 2020]**

**30.** A particle of mass 200 MeV/$c^2$ collides with a hydrogen atom at rest. Soon after the collision, the particle comes to rest and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of $N$ is ......... . (Given, the mass of the hydrogen atom to be 1 GeV/$c^2$) **[JEE Main 2020]**

**31.** An electron in hydrogen-like atom makes a transition from $n$th orbit and emits radiation corresponding to Lyman series. If de-Broglie wavelength of electron in $n$th orbit is equal to the wavelength of radiation emitted, find the value of $n$. The atomic number of atom is 11.

**32.** A particle of mass $m$ moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals $r_n$ vary with $n^{1/\alpha}$, where $\alpha$ is ...... . **[JEE Main 2021]**

# Answers

## Round I

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (d) | **2.** (b) | **3.** (c) | **4.** (b) | **5.** (a) | **6.** (a) | **7.** (a) | **8.** (c) | **9.** (c) | **10.** (d) |
| **11.** (d) | **12.** (d) | **13.** (d) | **14.** (a) | **15.** (d) | **16.** (a) | **17.** (c) | **18.** (b) | **19.** (c) | **20.** (a) |
| **21.** (c) | **22.** (c) | **23.** (a) | **24.** (a) | **25.** (d) | **26.** (a) | **27.** (b) | **28.** (c) | **29.** (c) | **30.** (a) |
| **31.** (c) | **32.** (c) | **33.** (b) | **34.** (d) | **35.** (b) | **36.** (b) | **37.** (a) | **38.** (c) | **39.** (c) | **40.** (a) |
| **41.** (d) | **42.** (d) | **43.** (b) | **44.** (d) | **45.** (c) | **46.** (a) | **47.** (c) | **48.** (d) | **49.** (a) | **50.** (b) |
| **51.** (d) | **52.** (b) | **53.** (a) | **54.** (b) | **55.** (c) | **56.** (d) | **57.** (c) | **58.** (a) | **59.** (d) | **60.** (a) |
| **61.** (b) | **62.** (a) | **63.** (a) | **64.** (b) | **65.** (b) | **66.** (c) | **67.** (c) | **68.** (b) | **69.** (d) | **70.** (c) |
| **71.** (a) | **72.** (c) | **73.** (d) | **74.** (b) | **75.** (c) | **76.** (a) | **77.** (d) | **78.** (d) | **79.** (c) | **80.** (a) |
| **81.** (a) | **82.** (d) | **83.** (a) | **84.** (d) | **85.** (c) | **86.** (d) | **87.** (c) | **88.** (b) | **89.** (a) | **90.** (a) |
| **91.** (c) | **92.** (b) | **93.** (a) | **94.** (a) | **95.** (a) | **96.** (c) | **97.** (a) | **98.** (b) | **99.** (d) | **100.** (a) |
| **101.** (a) | | | | | | | | | |

## Round II

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (a) | **2.** (c) | **3.** (b) | **4.** (b) | **5.** (d) | **6.** (b) | **7.** (b) | **8.** (a) | **9.** (a) | **10.** (c) |
| **11.** (d) | **12.** (b) | **13.** (c) | **14.** (b) | **15.** (b) | **16.** (d) | **17.** (d) | **18.** (b) | **19.** (c) | **20.** (d) |
| **21.** (a) | **22.** (a) | **23.** 6.132 | **24.** 9 | **25.** 1.834 | **26.** 10.58 | **27.** 2 | **28.** 486 | **29.** 10553 | **30.** 51 |
| **31.** 25 | **32.** 3 | | | | | | | | |

# Solutions

## Round I

**1.** In $\alpha$-scattering experiment,

(i) most of the $\alpha$-particles pass through the foil, which can corresponds to part $AA'$ in the figure.

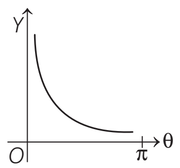
(ii) only 0.14% are scatter by more than 1°, which can corresponds to part $CC'$ in the figure.

(iii) 1 out of 8000 is deflect by more than 90°, which can corresponds to part $BB'$ in the figure.

Thus, in the given figure, the number of $\alpha$-particles in part $A'$ will be maximum and part $B'$ will be minimum.

**2.** In Rutherford's experiment, number of particles scattered at large angles is very less and most of the particles are scattered at small angles.

Hence, graph of $Y$ = number of $\alpha$- particles and $\theta$ = scattering angle is as shown in the figure below.




**3.** As, $r_0 = \dfrac{(Ze)(2e)}{4\pi\varepsilon_0(E)} = \dfrac{2\times 92\,(1.6\times10^{-19})^2\times 9\times10^9}{5\times10^6\times1.6\times10^{-19}}$

$= 5.3\times10^{-14}$ m $\approx 10^{-12}$ cm

**4.** Impact parameter is

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\times\frac{1}{2K}\times(\cot\theta/2)$$

For a head on collision, distance of closest approach

$$d_0 = \frac{Z_1 Z_2 e^2}{4\pi\,\varepsilon_0}\times\frac{1}{K}$$

So, from above expressions, we have

$$b = d_0\times\frac{1}{2\tan(\theta/2)}$$

Now, $d_0 = \dfrac{1}{4\pi\varepsilon_0}\times Z_1 Z_2\times e^2\times\dfrac{1}{K}$

$$= \frac{9\times10^9\times2\times47\times(1.6\times10^{-19})^2}{4.8\times10^6\times1.6\times10^{-19}}$$

$= 2.8\times10^{-14}$ m

$\therefore$ $b = \dfrac{2.8\times10^{-14}}{2\times(\sqrt{2}-1)} = 3.36\times10^{-14}$ m

**5.** At distance of minimum separation,

Kinetic Energy [KE] = Potential Energy [PE]

$\Rightarrow$ $\text{KE} = \dfrac{k\,Ze\cdot2e}{r}$

Here, $k = 9\times10^9$ N-m$^2$C$^{-2}$

$Z = 79$, $e = 1.6\times10^{-19}$ C

and $r = 4\times10^{-14}$ m

$\therefore$ $\text{KE} = \dfrac{9\times10^9\times79\times2\times(1.6\times10^{-19})^2}{4\times10^{-14}}$ J

$= \dfrac{9\times10^9\times79\times2\,(1.6\times10^{-19})^2}{4\times10^{-14}\times1.6\times10^{-19}}$ eV

$= 5.68$ MeV

**6.** In an ionised hydrogen molecule, as there are two protons and one electron, therefore, electron's orbit would go around the two protons separated by a small distance (~ Å). This orbit shall not be a circular orbit.

**7.** Simple Bohr model, infact does not give correct values of angular momentum of revolving electron. It gives only the magnitude of angular momentum, which is a vector.

**8.** On the basis of Bohr's model, $r = \dfrac{\varepsilon_0 n^2 h^2}{\pi m Z e^2} = a_0 \dfrac{n^2}{Z}$

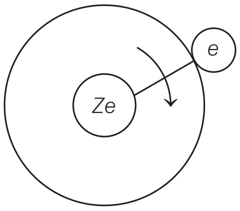
Let $Li^{++}$ ion, $Z = 3$; $n = 1$ for ground state

Given, $a_0 = 53$ pm

$\therefore \quad r = \dfrac{53 \times 1^2}{3} = 18$ pm

**9.** As, force between nucleus and electron is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$




where, $r$ = atomic radius

and $e$ = electronic charge.

From question,

$$F_e = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right), \text{ where } \beta = \text{constant.}$$

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$$

$$\Rightarrow \quad \frac{m\left(\frac{nh}{2\pi mr}\right)^2}{r} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$$

$$\left[\because mvr = \frac{nh}{2\pi} \text{ or } v = \frac{nh}{2\pi mr}\right]$$

$$\Rightarrow \quad \frac{1}{r^2} + \frac{\beta}{r^3} = \frac{mn^2h^2 \cdot 4\pi\varepsilon_0}{4\pi^2 m^2 e^2 r^3} = a_0 \frac{n^2}{r^3}$$

$$\Rightarrow \quad \frac{a_0 n^2}{r^3} = \frac{1}{r^2} + \frac{\beta}{r^3}$$

$$r = a_0 n^2 - \beta$$

**10.** For an electron in $n$th state of H-atom,

$$v_n = 2.2 \times 10^6 \frac{Z}{n} \text{ ms}^{-1}$$

$$r_n = 0.53 \times \frac{n^2}{Z} \text{ Å}$$

So, centripetal acceleration of electron $= v_n^2/r_n$

$$= \frac{(2.2 \times 10^6 \; Z/n)^2}{0.53 \times 10^{-10}\left(\frac{n^2}{Z}\right)}$$

$$= \frac{4.84 \times 10^{12}}{0.53 \times 10^{-10}} \cdot \left(\frac{Z^3}{n^4}\right)$$

$$= 9.1 \times 10^{22} \cdot \left(\frac{Z^3}{n^4}\right)$$

So, ratio of centripetal acceleration of 3rd and 2nd orbits is

$$\frac{a_3}{a_2} = \frac{9.1 \times 10^{22} \times \frac{Z^3}{3^4}}{9.1 \times 10^{22} \times \frac{Z^3}{2^4}}$$

$$\Rightarrow \quad \frac{a_3}{a_2} = \frac{2^4}{3^4} = \frac{16}{81}.$$

**11.** According to de-Broglie hypothesis,

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow \quad 2\pi r = n\left(\frac{h}{mv}\right)$$

$$\Rightarrow \quad 2\pi r = n\lambda$$

$\Rightarrow$ Circumference $= n\lambda$

**12.** Time period of revolution of an electron in its $n$th state (or $n$th orbit) of H-atom,

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi r_1 \cdot n^2}{(v_1/n)}$$

where, $r_1$ and $v_1$ are radius and velocity of electron in ground state.

So, $\quad T_n = \dfrac{2\pi r_1}{v_1} \cdot n^3$ or $T_n = T_1 \cdot n^3$

where, $T_1$ = time period of revolution in ground state.

Now, in first excited state $n = 2$, so time period,

$$T_2 = T_1 \times 2^3 = 8 \times 1.6 \times 10^{-16} \text{ s}$$

As, frequency $= \dfrac{1}{\text{time period}}$

So, frequency of revolution of electron in first excited state,

$$f_2 = \frac{1}{T_2} = \frac{1}{8 \times 1.6 \times 10^{-16}} = 7.8 \times 10^{14} \text{ Hz}$$

**13.** By Bohr's second postulate, for revolving electron,

Angular momentum $= \dfrac{nh}{2\pi}$

$$\Rightarrow \quad mvr_n = \frac{nh}{2\pi}$$

$\Rightarrow$ Momentum of electron, $p = mv = \dfrac{nh}{2\pi r_n}$

de-Broglie wavelength associated with electron is

$$\lambda_n = \frac{h}{p} = \frac{2\pi r_n}{n}$$

Given, $\quad n = 3$, $r_n = 4.65$ Å

$$\therefore \quad \lambda_n = \frac{(2 \times \pi \times 4.65)}{3} \approx 9.7 \text{ Å}$$

**14.** As, we know that, kinetic energy of an electron is

$$KE \propto \left(\frac{Z}{n}\right)^2$$

When the electron makes transition from an excited state to the ground state, then $n$ decreases and KE increases. We know that, PE is lowest for ground state. As, TE = – KE. Hence, TE also decreases.

**15.** As, $R \propto n^2; V \propto \frac{1}{n}$ and $E \propto \frac{1}{n^2}$

$\therefore \quad VR \propto \left(\frac{1}{n} \times n^2\right)$, *i.e.* $VR \propto n$

**16.** Initial KE of each of two H-atoms in ground state = 13.6 eV

$\therefore$ KE of both H-atoms before collision = 2 × 13.6 eV = 27.2 eV

As, the collision is inelastic, linear momentum is conserved, but some KE is lost.

If one H-atom goes over to first excited state and other remains in ground state, then their combined

KE after collision $= \left(\frac{13.6}{2^2}\right) + \left(\frac{13.6}{1^2}\right) = 17.0$ eV

$\therefore$ Reduction in combined KE = (27.2 – 17.0) eV = 10.2 eV

**17.** We know that, $E \propto \frac{1}{n^2}; P \propto \frac{1}{n}$ and $r \propto n^2$

$$Pr \propto \frac{1}{n}(n^2), \text{ i.e. } Pr \propto n$$

$$\frac{P}{E} \propto \frac{1/n}{1/n^2}, \text{ i.e. } \frac{P}{E} \propto n$$

$$Er \propto \frac{1}{n^2} \times n^2, \text{ i.e. } Er = \text{constant for all orbits.}$$

$$EPr \propto \frac{1}{n^2} \cdot \frac{1}{n} \cdot n^2, \text{ i.e. } EPr \text{ is proportional to } 1/n.$$

**18.** Energy of H-atom is $E = -13.6\, Z^2/n^2$

for H-atom $Z = 1$ and for ground state, $n = 1$.

$\Rightarrow \quad E = -13.6 \times \frac{1^2}{1^2} = -13.6$ eV

Now for carbon atom (single ionised), $Z = 6$

$$E = -13.6\frac{Z^2}{n^2} = -13.6 \text{ (given)}$$

$\Rightarrow \quad n^2 = 6^2 \Rightarrow n = 6$

**19.** As, we know that, potential energy $U = 2E$, kinetic energy $K = -E$

and total energy of electron, $E = -\frac{13.6}{n^2}$

For state $n = 1$, $E_1 = \frac{-13.6}{(1)^2} = -13.6$

For state $n = 2$, $E_2 = \frac{-13.6}{(2)^2} = \frac{E_1}{4} \Rightarrow E_1 = 4E_2$

But $\quad K = -E$, so $K_1 = 4K_2$

Similarly, for state $n = 1$, potential energy,

$$U_1 = 2E_1 = 2 \times \frac{-13.6}{(1)^2} = -2 \times 13.6$$

For state $n = 2$, potential energy,

$$U_2 = 2E_2 = 2 \times \frac{-13.6}{(2)^2} = \frac{-2 \times 13.6}{4}$$

$$U_2 = \frac{U_1}{4}$$

$\Rightarrow \quad U_1 = 4U_2$

Thus, both kinetic energy $K$ and potential energy $U$ becomes four times.

**20.** As, $E = E_2 - E_1 = -\frac{13.6}{2^2} - \left(-\frac{13.6}{1^2}\right) = 10.2$ eV

**21.** As, $\upsilon = \frac{c}{\lambda} = c \cdot R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$= 3 \times 10^8 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{9}{16} \times 10^{15} \text{ Hz}$$

**22.** We have, $\lambda = \frac{hc}{\Delta E}$

$\therefore$ So, ratio of wavelengths

$$\frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

**23.** Ist excited state corresponds to $n_1 = 2$

2nd excited state corresponds to $n_2 = 3$

As, $\quad E \propto \frac{1}{n^2}$

$$\frac{E_1}{E_2} = \frac{n_2^2}{n_1^2} = \frac{3^2}{2^2} = \frac{9}{4}$$

**24.** For ground state $n_1 = 1$ to $n_2 = 4$

Energy absorbed by photon, $E = E_2 - E_1$

$$= +\,13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \times 1.6 \times 10^{-19} \text{ J}$$

$$= 13.6\left(\frac{1}{1} - \frac{1}{4^2}\right) \times 1.6 \times 10^{-19}$$

$$= 13.6 \times 1.6 \times 10^{-19}\left(\frac{15}{16}\right)$$

$$= 20.4 \times 10^{-19}$$

or $\quad E = h\nu = 20.4 \times 10^{-19}$

Frequency, $\nu = \frac{20.4 \times 10^{-19}}{h} = \frac{20.4 \times 10^{-19}}{6.63 \times 10^{-34}}$

$$= 3.076 \times 10^{15}$$

$$\simeq 3.1 \times 10^{15} \text{ Hz}$$

Wavelength of photon, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.076 \times 10^{15}}$

$$= 9.74 \times 10^{-8} \text{ m}$$

$$\simeq 9.7 \times 10^{-8} \text{ m}$$

**25.** $\bar{\nu} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R}{4} = 20397 \text{ cm}^{-1}$

For the same transition in $He^+$-atom ($Z = 2$)

$$\bar{\nu} = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R \times 2^2}{4}$$

$$= 20397 \times 4$$

$$= 81588 \text{ cm}^{-1}$$

**26.** According to Rydberg's formula, $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Here, $n_f = 1$, $n_i = n$

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right) \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{R}{n^2} = R - \frac{1}{\lambda}$$

$$\Rightarrow \quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

**27.** Energy at $A$, $B$ and $C$ state is $E_A$, $E_B$ and $E_C$, then

$(E_C - E_B) + (E_B - E_A) = E_C - E_A$

or $\quad \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3}$

$$\Rightarrow \quad \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$$

$$\Rightarrow \quad \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

**28.** In hydrogen atom, $E_n = -\frac{Rhc}{n^2}$

Also, $\quad E_n \propto m$

where, $m$ is the mass of the electron.

Here, the electron has been replaced by a particle whose mass is double of an electron. Therefore, for this hypothetical atom energy in $n$th orbit will be given by

$$E_n = -\frac{2Rhc}{n^2}$$

The longest wavelength $\lambda_{max}$ (or minimum energy) photon will correspond to the transition of particle from $n = 3$ to $n = 2$.

$$\therefore \quad \frac{hc}{\lambda_{max}} = E_3 - E_2 = 2Rhc\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

This gives, $\quad \lambda_{max} = 18/5R$

**29.** For hydrogen atom, we get

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda_1 = \frac{4}{3R} \quad \text{...(i)}$$

For deuterium ($Z = 1$), we get

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda_2 = \frac{4}{3R} \quad \text{...(ii)}$$

For singly ionised helium ($Z = 2$), we get

$$\frac{1}{\lambda_3} = R(2)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda_3 = \frac{1}{4} \cdot \frac{4}{3R} = \frac{\lambda_1}{4}$$

$$\Rightarrow \quad \lambda_1 = 4\lambda_3 \quad \text{...(iii)}$$

For doubly ionised lithium ($Z = 3$), we get

$$\frac{1}{\lambda_4} = R(3)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda_4 = \frac{1}{9} \cdot \frac{4}{3R} = \frac{\lambda_1}{9}$$

$$\Rightarrow \quad \lambda_1 = 9\lambda_4 \quad \text{...(iv)}$$

From Eqs. (i), (ii), (iii) and (iv), we get

$$\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

**30.** When an electron in a hydrogen atom jumps from an initial energy level $n_i$ to some final energy level $n_f$, then if $\lambda$ be the wavelength of emitted photon, then by Balmer's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where, $R$ = Rydberg constant.

In transition from $n = 4$ to $n = 3$, we have

$$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{7}{9 \times 16}\right) \quad \text{...(i)}$$

In transition from $n = 3$ to $n = 2$, we have

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{5}{9 \times 4}\right) \quad \text{...(ii)}$$

So, from Eqs. (i) and (ii), the ratio of $\frac{\lambda_1}{\lambda_2}$ is

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{9 \times 16}{7R}\right)}{\left(\frac{9 \times 4}{5R}\right)} = \frac{20}{7}$$

**31.** For hydrogen or hydrogen like atoms, we know that

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \text{...(i)}$$

where, $R$ is Rydberg constant and $Z$ is atomic number. When electron jumps from $M$ - shell to the $L$ - shell, then

$n_1 = 2$ (for $L$-shell)

$n_2 = 3$ (for $M$-shell)

$\therefore$ Eq (i) becomes,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}RZ^2 \quad \text{...(ii)}$$

Now, electron jumps from $N$-shell to the $L$ - shell, for this

$$n_1 = 2 \quad \text{(for } L \text{ - shell)}$$
$$n_2 = 4 \quad \text{(for } N \text{ - shell)}$$

$\therefore$ Eq (i) becomes,

$$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}RZ^2 \quad \text{...(iii)}$$

Now, we divide Eq. (ii) by Eq. (iii), we get

$$\frac{\lambda'}{\lambda} = \left(\frac{5}{36}RZ^2\right) \div \left(\frac{3}{16}RZ^2\right) = \frac{20}{27}$$

or $\lambda' = \frac{20}{27}\lambda$

***32.*** As, ionisation energy of ground state

$$= -\text{(ground state energy)}$$

$\Rightarrow$ $9 \times 13.6 = -E_1$ ($\because$ 1 Rydberg = 13.6 eV)

or $E_1 = -9 \times 13.6$ eV

and energy of $n$th state for the given atom will be

$$E_n = \frac{E_1}{n^2} = \frac{-9 \times 13.6}{n^2} \text{ eV}$$

In transition from second excited state ($n = 3$) to ground state ($n = 1$), energy released will be

$$\Delta E = E(n = 3) - E(n = 1)$$
$$= \left(\frac{-9 \times 13.6}{3^2}\right) - \left(\frac{-9 \times 13.6}{1^2}\right)$$

or $\Delta E = 9 \times 13.6\left(1 - \frac{1}{9}\right) = 8 \times 13.6$ eV

So, wavelength of radiation emitted will be

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ (eV-nm)}}{\Delta E \text{ (eV)}}$$

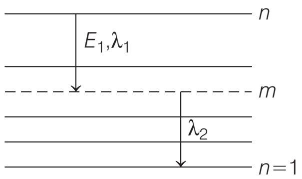
$\Rightarrow$ $\lambda = \frac{1240}{8 \times 13.6} = 11.39 \text{ nm} \approx 11.4 \text{ nm}$

***33.*** Change in energy in transition from $n$ to $m$ stage is given by ($n > m$),

$$E_n = -\frac{E_0 Z^2}{n^2}$$

Here, $Z = 2$

$$\Delta E_n = +13.6 \times 4\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = \frac{hc}{\lambda} \quad \text{...(i)}$$




Let it start from $n$ to $m$ and then $m$ to ground.

So, in first case,

$$13.6 \times 4 \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right) = \frac{hc}{108.5 \text{ nm}} \quad \text{...(ii)}$$

and in second case,

$$13.6 \times 4 \times \left(\frac{1}{1^2} - \frac{1}{m^2}\right) = \frac{hc}{30.4 \text{ nm}}$$

$\Rightarrow$ $\left(1 - \frac{1}{m^2}\right) = \frac{1240 \text{ eV}}{30.4 \times 13.6 \times 4}$ $\left(\because \text{given, } E = \frac{1240 \text{ eV}}{\lambda(\text{in nm})}\right)$

$\Rightarrow$ $\left(1 - \frac{1}{m^2}\right) = 0.74980 \approx 0.75$

or $\frac{1}{m^2} = 1 - 0.75 = 0.25 \Rightarrow m^2 = \frac{1}{0.25} = 4$

Hence, $m = 2$

So, by putting the value of $m$ in Eq. (ii), we get

$$13.6 \times 4 \times \left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{1240}{108.5} \text{ eV}$$

$\Rightarrow$ $\left(\frac{1}{4} - \frac{1}{n^2}\right) = \frac{1240}{108.5 \times 13.6 \times 4}$

$\Rightarrow$ $\frac{1}{4} - \frac{1}{n^2} = 0.21$

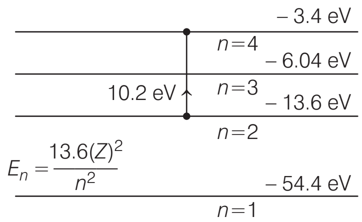
or $\frac{1}{n^2} = 0.25 - 0.21 = 0.04$

$$n^2 = \frac{1}{0.04} = 25 \Rightarrow n^2 = 25 \Rightarrow n = 5$$

***34.*** De-excitation energy of hydrogen electron in transition $n = 2$ to $n = 1$ is

$$E = 13.6 \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{eV} = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$
$$= 10.2 \text{ eV}$$

Now, energy levels of helium ion's ($He^+$) electron are (For helium, $Z = 2$)




So, a photon of energy 10.2 eV can cause a transition $n = 2$ to $n = 4$ in a $He^+$ ion.

***35.*** Energy of a hydrogen atom like ion by Bohr's model is $E_n = -13.6\frac{Z^2}{n^2}$

where, $Z$ = atomic number

and $n$ = principal quantum number.

For a $He^+$ ion in first excited state,

$$n = 2, Z = 2$$

$\therefore$ $E_2 = -13.6 \times \frac{4}{4} = -13.6$ eV

$\therefore$ Ionisation energy of $He^+$ ion in first excited state

$$E = -E_2 = -(-13.6 \text{ eV}) = 13.6 \text{ eV}$$

**36.** $E \propto \frac{1}{r}, \ r \propto \frac{1}{m}, \ E \propto m$

$\therefore$ Ionisation potential $= 13.6 \times \frac{(\text{Mass}_\mu) \text{ eV}}{(\text{Mass}_e)}$

$= 13.6 \times 207$ eV

$= 2815.2$ eV

**37.** Here, mass of muon, $m = 207 m_e$

and mass of proton, $M = 1836 m_e$; so the reduced mass is

$$\mu = \frac{mM}{m + M}$$

$$= \frac{(207m_e)(1836m_e)}{207m_e + 1836m_e} = 186 m_e$$

According to equation $\lambda = h/p$, the orbit radius corresponding to $n = 1$ is $r_1 = \frac{h^2 \varepsilon_0}{\pi m_e e^2} = 5.29 \times 10^{-11}$ m

Hence, the radius $r'$ that corresponds to the reduced mass $\mu$ is

$$r_1' = \left(\frac{m}{\mu}\right) r_1 = \left(\frac{m_e}{186 m_e}\right) r_1$$

$$= \frac{5.29 \times 10^{-11}}{186}$$

$$= 2.85 \times 10^{-13} \text{ m}$$

The muon is 186 times closer to the proton than an electron would be in the same orbit.

We have, $E_1 = -13.6$ eV (for $n = 1$)

$$E_1' = \left(\frac{\mu}{m}\right) E_1 = 186 \times (-13.6) \text{ eV}$$

$\Rightarrow$ $E_1 = -2.53 \times 10^3 \text{ eV} = -2.53$ keV

The ionisation energy is, therefore, 2.53 keV, 186 times that for an ordinary hydrogen atom.

**38.** Total energy of electron in excited state $= -13.6 + 12.1 = -1.5$ eV, which corresponds to third orbit. The possible spectral lines are when electron jumps from orbit 3rd to 2nd; 3rd to 1st and 2nd to 1st.

**39.** As,
$$\frac{\lambda_l}{\lambda_s} = \frac{R\left(\frac{1}{1^2} - \frac{1}{\infty}\right)}{R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = \frac{4}{3}$$

$$\lambda_l = \frac{4}{3}\lambda_s = \frac{4}{3} \times 911.6$$

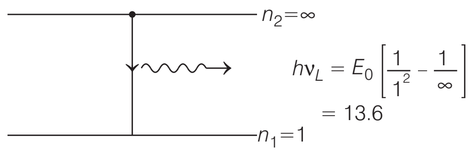
$$= 1215.4 \text{ Å}$$

$$\simeq 1215 \text{Å}$$

**40.** As,
$$\frac{\lambda_B}{\lambda_L} = \frac{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = \frac{\frac{3}{4}}{\frac{5}{36}} = \frac{27}{5}$$

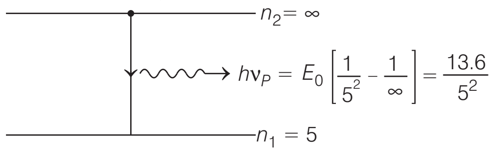
$$\lambda_L = \frac{5}{27}\lambda_B = \frac{5}{27} \times 6563 = 1215.4 \text{ Å}$$

**41.** Series limit occurs in the transition $n_2 = \infty$ to $n_1 = 1$ in Lyman series and $n_2 = \infty$ to $n_1 = 5$ in Pfund series. For Lyman series,




$$h\nu_L = 13.6 \quad \text{...(i)}$$

For Pfund series,




$$h\nu_P = \frac{13.6}{5^2} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$25 h\nu_P = h\nu_L \Rightarrow \nu_P = \frac{\nu_L}{25}$$

**42.** As, $\nu = RC\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$, we get

$$\nu_1 = RC\left(\frac{1}{1} - \frac{1}{\infty}\right) = RC,$$

$$\nu_2 = RC\left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4}RC$$

and $$\nu_3 = RC\left(\frac{1}{4} - \frac{1}{\infty}\right) = \frac{1}{4}RC$$

Clearly, $\nu_1 - \nu_2 = \frac{RC}{4} = \nu_3$, *i.e.* $\nu_1 - \nu_2 = \nu_3$

**43.** Given, radius of orbit, $r = 1.5 \times 10^{11}$ m

Orbital speed, $v = 3 \times 10^4$ m/s

Mass of earth, $m = 6 \times 10^{24}$ kg

Angular momentum, $mvr = \frac{nh}{2\pi}$

or $$n = \frac{2\pi vrm}{h}$$

[where, $n$ = quantum number of the orbit]

$$= \frac{2 \times 3.14 \times 3 \times 10^4 \times 1.5 \times 10^{11} \times 6 \times 10^{24}}{6.63 \times 10^{-34}}$$

$$= 2.57 \times 10^{74}$$

or $n = 2.6 \times 10^{74}$

Thus, the quantum number is $2.6 \times 10^{74}$ which is too large.

The electron would jump from $n = 1$ to $n = 3$.

$$E_3 = \frac{-13.6}{3^2} = -1.5 \text{ eV}$$

So, they belong to Lyman series.

**44.** Expression for the energy of the hydrogenic electron states for atoms of atomic number $Z$ is given by

$$E = h\nu = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \qquad \text{(Here, } m < n\text{)}$$

or $$\frac{hc}{\lambda} = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto \left(\frac{1}{m^2} - \frac{1}{n^2}\right) Z^2$$

For first case, $\lambda = 660$ nm, $m = 2$ and $n = 3$

$$\therefore \quad \frac{1}{660 \text{ nm}} \propto \left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] Z^2$$

$$\Rightarrow \quad \frac{1}{660 \text{ nm}} \propto \left(\frac{1}{4} - \frac{1}{9}\right) Z^2 \text{ or } \frac{5}{36} Z^2 \qquad \text{...(i)}$$

For second case, transition is from $n = 4$ to $n = 2$, *i.e.* $m = 2$ and $n = 4$

$$\therefore \quad \frac{1}{\lambda} \propto \left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right) Z^2$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto \left(\frac{1}{4} - \frac{1}{16}\right) Z^2$$

or $$\frac{1}{\lambda} \propto \frac{3}{16} Z^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda}{660 \text{ nm}} = \frac{5}{36} \times \frac{16}{3}$$

$$\Rightarrow \quad \lambda = \frac{80}{108} \times 660 \text{ nm} = 488.9 \text{ nm}$$

**45.** Number of spectral lines produced as an excited electron falls to ground state ($n = 1$) is

$$N = \frac{n(n-1)}{2}$$

In given case, $N = 6$

$$\therefore \quad 6 = \frac{n(n-1)}{2} \Rightarrow n = 4$$

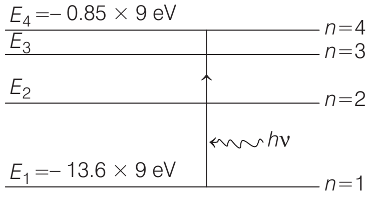
So, $L^{++}$ electron is in it's 3rd excited state.

Now, using the expression of energy of an electron in $n$th energy level,

$$E_n = -\frac{13.6 Z^2}{n^2} \text{ eV}$$

where, $Z$ is the atomic number.

$\therefore$ Energy levels of $L^{++}$ electron are as shown




So, energy absorbed by electron from incident photon of wavelength $\lambda$ is

$$\Delta E = \frac{hc}{\lambda}$$

$$\Rightarrow (13.6 \times 9 - 0.85 \times 9) = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{9(13.6 - 0.85)}$$

$$\Rightarrow \quad \lambda = \frac{1240 \text{ eV-nm}}{9 \times 12.75 \text{ eV}} = 10.8 \text{ nm}$$

**46.** Each atom of ${}_6C^{14}$ contains 6 $p$, 6 $e$ and 8 $n$.

$\therefore$ In 14 g of ${}_6C^{14}$,

$$p = 6 \times 6 \times 10^{23} = 36 \times 10^{23}$$

$$n = 8 \times 6 \times 10^{23} = 48 \times 10^{23}$$

$$e = p = 36 \times 10^{23}$$

**47.** As, $\frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}$

**48.** Mass density of nuclear matter is a constant quantity for all elements. It does not depends on element's mass number or atomic radius.

$\therefore$ The ratio of mass densities of ${}^{40}Ca$ and ${}^{16}O$ is 1 : 1.

**49.** Mass of nucleus = Mass number × Mass of a nucleon

$$= A \times 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (1.3 \times 10^{-15} \times A^{1/3})^3$$

$$= \frac{4}{3}\pi \times (1.3)^3 \times 10^{-45} \times A$$

$$\therefore \text{Density of nucleus} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi \times (1.3)^3 \times 10^{-45} \times A}$$

$$\approx 10^{17} \text{ kg m}^{-3}$$

**50.** As, $$R = R_0 A^{1/3}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\Rightarrow \quad \frac{R}{R_{He}} = \left(\frac{A}{4}\right)^{1/3}$$

$$\Rightarrow \quad (14)^{1/3} = \left(\frac{A}{4}\right)^{1/3}$$

$$\Rightarrow \quad A = 56$$

or $$Z = A - N$$

$$\therefore \quad Z = 56 - 30 = 26$$

**51.** The elements higher on the BE *versus* mass number plot, *i.e.* representing the high peaks on the plot are very tightly bound and hence are stable. However, the elements on lower end of this plot, are less tightly bound due to less binding energy per nucleon and hence are unstable.

As, lithium nucleus shows a peak on this plot and having large value of binding energy per nucleon, so it implies that it is very stable.

**52.** Given, $E_{bn}$ of chlorine $= 287.67$ MeV

and $E_{bn}$ of phosphorus $= 262.48$ MeV

Binding energy per nucleon of ${}_{17}Cl^{35}$

$$= \frac{\text{Binding energy}}{\text{Mass number}}$$

$$= \frac{287.67}{35} = 8.22 \text{ MeV}$$

Similarly,

Binding energy per nucleon of ${}_{15}P^{31} = \frac{262.48}{31}$

$$= 8.47 \text{ MeV}$$

Since, BE per nucleon of ${}_{15}P^{31}$ is more than BE per nucleon of ${}_{17}Cl^{35}$, hence ${}_{15}P^{31}$ is more stable.

**53.** The reaction is ${}_3Li^7 + {}_1p^1 \longrightarrow 2\,({}_2He^4)$

$\therefore \quad E_p = 2E\,({}_2He^4) - E_{(Li)}$

$= 2\,(4 \times 7.06) - 7 \times 5.6$

$= 56.48 - 39.2 = 17.28$ MeV

**54.** During formation of H-atom, some mass of nucleons convert into energy by $E = mc^2$, this energy is used to bind the nucleons along with nucleus.

$\therefore \quad E = B = mc^2$

$\Rightarrow \quad m = \frac{B}{c^2}$

So, mass of atom becomes slightly less than sum of actual masses of nucleons and electrons. Actual mass ($m'$) of H-atom

$$m' = m_{\text{proton}} + m_{\text{electron}} - \frac{B}{c^2}$$

where, $B = 13.6$ eV (binding energy) per atom.

**55.** Most stable, $N = Z\,(B)$. Nucleus with more neutron number ($N$) is slightly more stable than nucleus which have more atomic number ($Z$).

So, order of stability is $B > A > C$.

**56.** $m_{\text{theoretical}} = Zm_p + (A - Z)m_n$

$= 50(1.00783 \text{ u}) + (120 - 50)\,(1.00867 \text{ u})$

$= 50.391500 \text{ u} + 70.606900 \text{ u}$

$= 120.998400$ u

$m_{\text{experimental}} = 119.902199$ u

So, mass defect, $\Delta m = m_{\text{theoretical}} - m_{\text{experimental}}$

$= 120.998400 \text{ u} - 119.902199 \text{ u}$

$= 1.096201$ u

Now, total binding energy, $\text{BE} = \Delta m \times 931$ MeV

$= 1.096201 \times 931$ MeV

$= 1020.563131$ MeV

Therefore, binding energy per nucleon,

$$(\text{BE})_{\text{per nucleon}} = \frac{\text{BE}}{\text{Total number of nucleons}}$$

$$= \frac{1020.563131}{120}$$

$= 8.50469275833$ MeV

$\simeq 8.5$ MeV

**57.** Nuclear force of attraction between any two nucleons ($n$-$n$, $p$-$n$, $p$-$n$) is same. The difference comes up only due to electrostatic force of repulsion between two protons.

$\therefore \quad F_1 = F_3 \neq F_2$

Since, nuclear force is greater than electrostatic force, hence

$\therefore \quad F_1 = F_3 > F_2$

**58.** In the given oxygen molecule, nuclear force between the nuclei of two atoms is not important because nuclear forces being short-ranged are confined only within one particular nucleus.The distance between the nuclei of two atoms may be large. So the nuclear forces between two nuclei may not be operative/effective.

**59.** (d) As, $\quad 3P \longrightarrow Q + E_1$

$\Rightarrow \quad Q - 3P = E_1$

$\Rightarrow \quad E_1 = E_Q - 3E_P \quad \ldots\text{(i)}$

Aslo, $\quad 2Q \longrightarrow R + E_2$

$\Rightarrow \quad E_2 = E_R - 2E_Q \quad \ldots\text{(ii)}$

And $\quad R - 6P = E_3$

$\Rightarrow \quad E_R - 6\,E_P = E_3 \quad \ldots\text{(iii)}$

Now, $\quad 2E_1 + E_2 = 2(E_Q - 3E_P) + (E_R - 2E_Q)$

$= E_R - 6\,E_P = E_3$

**60.** Let number of α-particle emitted be $x$ and number of β-particles emitted be $y$.

Difference in mass number $4\,x = 238 - 206 = 32$

$x = 8$

Difference in charge number $2\,x - 1\,y = 92 - 82 = 10$

$16 - y = 10 \Rightarrow y = 6$

**61.** Isotopes are the atoms of same element having same number of protons but different number of neutrons, *i.e.* $Z$ remains same.

However, in α or β-decay, the nuclei hence formed always have $Z$ either decreased by 2 or increased by 1. Thus, isotopes of the original nuclei cannot be occured in these decay.

**62.** $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{2.7 \times 24 \times 60 \times 60}$

$\lambda = 2.9 \times 10^{-6}\ \text{s}^{-1}$

$N = 6 \times 10^{23} \times \frac{1.0 \text{ mg}}{198 \text{ g}} = 3.03 \times 10^{18}$

Activity $A = \lambda N$

$= 2.9 \times 10^{-6} \times 3.03 \times 10^{18}$

$= 8.8 \times 10^{12}$ disintegration/s

$= \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} = 240$ Ci

**63.** Tritium $\rightarrow {}_1H^3$. The nucleus contains 1 proton and 2 neutrons. If one neutron decays $n \rightarrow p + e^- + \bar{\nu}$, the nucleus may have 2 protons and one neutron, *i.e.* tritium will transform into ${}_2He^3$ (with 2 protons and one neutron). But this does not happen because triton energy is less than that of ${}_2He^3$ nucleus, *i.e.* transformation is not allowed energetically.

**64.** $A$ has shorter mean life as $\lambda$ is greater for $B$.

**65.** $t_{1/2} = 3.8$ days

$$\therefore \quad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{3.8} = 0.182$$

We know that, $t = \dfrac{2.303}{\lambda} \log \dfrac{N_0}{N} = \dfrac{2.303}{0.182} \log \dfrac{N_0}{N_0/20}$

$$= \frac{2.303}{0.182} \log 20 = 16.46$$

$$\simeq 16.5 \text{ days}$$

**66.** Radioactivity is spontaneous self disruptive activity of the radioactive material. In $t = 1$ yr = half-life of material; on the average, half the number of atoms will decay. Therefore the containers will in general have different number of atoms of the material, but their average will be close to 5000.

**67.** $(t_{1/2})_x = (t_{\text{mean}})_y$

or $$\frac{0.693}{\lambda_x} = \frac{1}{\lambda_y}$$

$$\therefore \quad \lambda_x = 0.693\, \lambda_y$$

$$\lambda_x < \lambda_y$$

or rate of decay = $\lambda N$

Initially, number of atoms ($N$) of both are equal but since $\lambda_y > \lambda_x$, therefore $y$ will decay at a faster rate than $x$.

**68.** Taking average count per minute in the first half value period as $(100 + 50)/2$, *i.e.* 75

Total number of counts during this period

$$= 75 \times 3 \times 60 = 13500$$

which is closest to the given result (14100).

**69.** Fraction $= \dfrac{N}{N_0} = \left(\dfrac{1}{2}\right) \Rightarrow \dfrac{6400}{1600} = 4$

Fraction of radium after 4 half-life is

$$= \frac{1}{2^4} = \frac{1}{16}$$

**70.** By radioactive disintegration (Rutherford and Soddy) law, amount of active (undecayed) radioactive material remained in a sample after time $t$ is given by

$$M(t) = M_0 e^{-\lambda t}$$

where, $M_0$ = initial sample of mass taken at $t = 0$ and $\lambda$ = disintegration constant.

Fraction of active material remained after time $t$ is

$$\frac{M(t)}{M_0} = e^{-\lambda t} \quad \text{...(i)}$$

Given that, after time $t$, $\dfrac{M(t)}{M_0} = \dfrac{9}{16}$

So, $$e^{-\lambda t} = \frac{9}{16} \quad \text{[from Eq. (i)]}$$

Similarly, fraction of active material remained after time $t/2$ is

$$\frac{M\left(\frac{t}{2}\right)}{M_0} = e^{-\lambda \frac{t}{2}} = (e^{-\lambda t})^{\frac{1}{2}} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

**71.** Activity of a sample after time $t$ is related to initial activity as

$$A = A_0 \left(\frac{1}{2}\right)^n \quad \text{... (i)}$$

where, $A$ = present activity
and $A_0$ = initial activity.

Also, $$n = t/t_{1/2} \quad \text{... (ii)}$$

where, $t$ = time interval
and $t_{1/2}$ = half-life time.

Here, $A = 500\ s^{-1}$, $A_0 = 700\ s^{-1}$, $t = 30$ min

So, from Eq. (i), we get

$$500 = 700 \left(\frac{1}{2}\right)^n \Rightarrow 2^n = 1.4 \approx \sqrt{2}$$

So, $$n = \frac{1}{2}$$

Now, from Eq. (ii), we get

$$\frac{t}{t_{1/2}} = \frac{1}{2} \Rightarrow t_{1/2} = 2 \times t = 60 \text{ min}$$

which is close to 62 min.

**72.** Activity, $A = A_0 \left(\dfrac{1}{2}\right)^{t/T}$

or $$\frac{A_0}{64} = A_0 \left(\frac{1}{2}\right)^{t/T}$$

or $$\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/\,18 \text{ days}}$$

$$\Rightarrow \quad 6 = \frac{t}{18} \Rightarrow t = 108 \text{ days.}$$

**73.** The total number of atoms can neither remain constant (as in option a) nor can ever increase (as in options b and c). They will continuously decrease with time. Therefore, (d) is the appropriate option.

**74.** Here given, at $t = 0$, count rate or initial activity is $A_0 = 1600\ s^{-1}$

At $t = 8$ s, count rate or activity is $A = 100\ s^{-1}$

So, decay scheme for given sample is

$$1600 \xrightarrow{t_{1/2}} 800 \xrightarrow{t_{1/2}} 400 \xrightarrow{t_{1/2}} 200 \xrightarrow{t_{1/2}} 100$$

So, $$8\text{s} = 4\, t_{1/2}$$

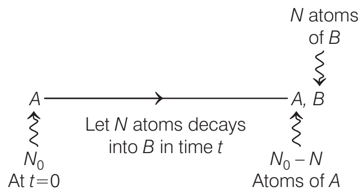
where, $t_{1/2}$ = half-life time.

$$\Rightarrow \quad t_{1/2} = 2 \text{ s}$$

$\therefore$ From above decay scheme, we see that activity after 6 s is 200 counts per second.

**75.** The two samples of two radioactive nuclides $A$ and $B$ can simultaneously have the same decay rate at any time if initial rate of decay of $A$ is twice the initial rate of decay of $B$ and $\lambda_A > \lambda_B$. Also, when initial rate of decay of $B$ is same as rate of decay of $A$ at $t = 2$ h and $\lambda_B < \lambda_A$.

**76.** Decay scheme is




Given, $\frac{N_B}{N_A} = 0.3 = \frac{3}{10}$

$\Rightarrow \quad \frac{N_B}{N_A} = \frac{30}{100}$

So, $\quad N_0 = 100 + 30 = 130$ atoms

By using $\quad N = N_0 e^{-\lambda t}$, we have

$$100 = 130\, e^{-\lambda t}$$

$$\Rightarrow \quad \frac{1}{1.3} = e^{-\lambda t}$$

$$\Rightarrow \quad \log 1.3 = \lambda t$$

If $T$ is half-life, then $\lambda = \frac{\log_e 2}{T}$

$$\Rightarrow \quad \log 1.3 = \frac{\log_e 2}{T} \cdot t$$

$$\therefore \quad t = \frac{T \cdot \log (1.3)}{\log_e 2}$$

**77.** Given, 80 min = 4 half-lives of $A$ = 2 half-lives of $B$.

Let the initial number of nuclei in each sample be $N$.

For radioactive element $A$,

$$N_A \text{ after 80 min} = \frac{N}{2^4}$$

$\Rightarrow$ Number of $A$ nuclides decayed $= N - \frac{N}{16} = \frac{15}{16} N$

For radioactive element $B$,

$N_B$ after 80 min $= \frac{N}{2^2}$

$\Rightarrow$ Number of $B$ nuclides decayed

$$= N - \frac{N}{4} = \frac{3}{4} N$$

$\therefore$ Ratio of decayed numbers of $A$ and $B$ nuclei will be

$$\frac{(15/16)N}{(3/4)N} = \frac{5}{4}$$

**78.** Activity of a radioactive material is given as

$$R = \lambda N$$

where, $\lambda$ is the decay constant and $N$ is the number of nuclei in the radioactive material.

For substance $A$,

$$R_A = \lambda_A N_A = 10 \text{ mCi}$$

For substance $B$,

$$R_B = \lambda_B N_B = 20 \text{ mCi} \quad \text{...(i)}$$

As given in the question,

$$N_A = 2N_B \Rightarrow R_A = \lambda_A (2N_B) = 10 \text{ mCi} \quad \text{...(ii)}$$

$\therefore$ Dividing Eq. (ii) by Eq.(i), we get

$$\frac{R_A}{R_B} = \frac{\lambda_A (2N_B)}{\lambda_B (N_B)} = \frac{10}{20} \text{ or } \frac{\lambda_A}{\lambda_B} = \frac{1}{4} \quad \text{...(iii)}$$

As, half-life of a radioactive material is given as

$$T_{1/2} = \frac{0.693}{\lambda}$$

$\therefore$ For material $A$ and $B$, we can write

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{\frac{0.693}{\lambda_A}}{\frac{0.693}{\lambda_B}} = \frac{\lambda_B}{\lambda_A}$$

Using Eq. (iii), we get

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{4}{1}$$

Hence, from the given options, only option (d) satisfies this ratio.

Therefore, $(T_{1/2})_A = 20$ days and $(T_{1/2})_B = 5$ days.

**79.** Let $n_0$ be the number of radioactive nuclei at time $t = 0$. Number of nuclei decayed in time $t$ are given by $n_0\,(1 - e^{-\lambda t})$, which is also equal to the number of $\beta$-particles emitted during the same interval of time.

For the given condition,

$$n = n_0\,(1 - e^{-2\lambda}) \quad \text{...(i)}$$

$$(n + 0.75n) = n_0\,(1 - e^{-4\lambda}) \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$1.75 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$$

or $\quad 1.75 - 1.75\, e^{-2\lambda} = 1 - e^{-4\lambda}$

$\therefore \quad 1.75\, e^{-2\lambda} - e^{-4\lambda} = 0.75 \quad$ ...(iii)

Let us take $e^{-2\lambda} = x \quad$ ...(iv)

Then, the above equation is

$$x^2 - 1.75\, x + 0.75 = 0$$

or $\quad x = \frac{1.75 \pm \sqrt{(1.75)^2 - (4)\,(0.75)}}{2}$

or $\quad x = 1$ and $\frac{3}{4}$

$\therefore$ From Eq. (iv) either

$$e^{-2\lambda} = 1$$

or $\quad e^{-2\lambda} = \frac{3}{4}$

but $e^{-2\lambda} = 1$ is not acceptable because which means $\lambda = 0$.

Hence, $e^{-2\lambda} = \frac{3}{4}$

or $-2\lambda \ln (e) = \ln (3) - \ln (4) = \ln (3) - 2 \ln (2)$

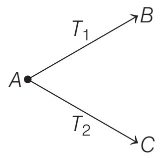
$\therefore$ $\lambda = \ln (2) - \frac{1}{2} \ln (3)$

Substituting the given values,

$$\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395 \text{ s}^{-1}$$

$\therefore$ Mean life, $t_{\text{mean}} = \frac{1}{\lambda} = 6.947 \text{ s} \simeq 7 \text{ s}$

**80.** If a radioactive sample undergoes two different processes simultaneously, then




$$\frac{dN}{dT} = -\lambda_1 N - \lambda_2 N \Rightarrow \frac{dN}{dT} = -(\lambda_1 + \lambda_2)N$$

$$\frac{dN}{dT} = -\lambda_{\text{eff}} N$$

Here, $\lambda_{\text{eff}} = \lambda_1 + \lambda_2$

Let $T_{\text{eff}}$ be the effective half-life of the nucleus.

$$\Rightarrow \frac{\ln 2}{T_{\text{eff}}} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2}$$

$$\frac{1}{T_{\text{eff}}} = \frac{1}{T_1} + \frac{1}{T_2} = \frac{1}{10} + \frac{1}{100} = \frac{11}{100}$$

$$\Rightarrow T_{\text{eff}} = \frac{100}{11} = 9.09 \text{ s} \approx 9 \text{ s}$$

**81.** Let at time $t = t$, number of nuclei of $Y$ and $Z$ are $N_Y$ and $N_Z$. Then, rate equations of the populations of $X$, $Y$ and $Z$ are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \quad \text{...(i)}$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \quad \text{...(ii)}$$

and $$\left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \quad \text{...(iii)}$$

Given, $N_Y(t) = \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y} [e^{-\lambda_Y t} - e^{-\lambda_X t}]$

For $N_Y$ to be maximum

$$\frac{dN_Y(t)}{dt} = 0$$

*i.e.* $\lambda_X N_X = \lambda_Y N_Y$ ...(iv) [from Eq. (ii)]

or $$\lambda_X (N_0 e^{-\lambda_X t}) = \lambda_Y \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y} [e^{-\lambda_Y t} - e^{-\lambda_X t}]$$

or $$\frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1$$

$$\frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

or $(\lambda_X - \lambda_Y)\, t \ln (e) = \ln \left(\frac{\lambda_X}{\lambda_Y}\right)$

or $$t = \frac{1}{\lambda_X - \lambda_Y} \ln \left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of $\lambda_X$ and $\lambda_Y$, we have

$$t = \frac{1}{(0.1 - 1/30)} \ln \left(\frac{0.1}{1/30}\right) = 15 \ln (3)$$

or $t = 16.48$ s

**82.** Initially, the source have 90% of $^{32}_{15}\text{P}$ and 10% of $^{33}_{15}\text{P}$.

Let $x$ g be initial number of $^{32}\text{P}$ nuclides and $9x$ g be initial number of $^{33}\text{P}$.

After $t$ days, the source has 90% of $^{33}\text{P}$ and 10% of $^{32}\text{P}$, *i.e.* of $^{33}\text{P}$ and $9y$ g of $^{32}\text{P}$.

Using the equation,

$$\frac{N}{N_0} = e^{-\lambda t} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

For $\text{P}^{33}$, $y = 9x \cdot 2^{-t/14.3}$ ...(i)

For $\text{P}^{32}$, $9y = x \cdot 2^{-t/25.3}$ ...(ii)

Dividing Eq. (i) by Eq. (ii),

$$\frac{y}{9y} = \frac{9x}{x} \cdot \frac{2^{-t/14.3}}{2^{-t/25.3}}$$

or $$\frac{1}{9} = 9 \times 2^{(t/25.3 - t/14.3)}$$

or $$\frac{1}{81} = 2^{-11t/25.3 \times 14.3}$$

Taking log on both the sides,

$$\log 1 - \log 81 = -\frac{11t}{25.3 \times 14.3} \log 2$$

or $$-1.9085 = \frac{-11 \times t}{25.3 \times 14.3} \times 0.3010$$

or $$t = \frac{25.3 \times 14.3 \times 1.9085}{11 \times 0.3010}$$

$$= 208.5 \simeq 208 \text{ days}$$

So, we must wait for 208 days to do so.

**83.** $\beta^-$ decay is represented as

$$_Z X^A \rightarrow {}_{Z+1}A^Y + {}_{-1}e^0 + \bar{\nu} + Q_1$$

$$\therefore Q_1 = [m_N({}_Z X^A) - m_N({}_{Z+1}Y^A) - m_e]c^2$$
$$= [m_N({}_Z X^A) + Zm_e - m_N({}_{Z+1}Y^A) - (Z+1)m_e]c^2$$
$$= [m({}_Z X^A) - m({}_{Z+1}Y^A)]\, c^2 = (M_x - M_y)c^2$$

$\beta^+$ decay is represented as

$$_Z X^A = {}_{Z-1}Y^A + {}_{+1}e^0 + \bar{\nu} + Q_2$$

$$\therefore Q_2 = [m_N({}_Z X^A) - m_N({}_{Z-1}Y^A) - m_e]c^2$$
$$= [m_N({}_Z X^A) + Zm_e - m_N({}_{Z+1}Y^A) - (Z-1)m_e - 2m_e]c^2$$
$$= [m({}_Z X^A) - m({}_{Z-1}Y^A) - 2m_e]\, c^2$$
$$= (M_x - M_y - 2m_e)c^2$$

**84.** Given that, $m({}^{7}_{3}\text{Li}) = 7.0160$ u

$$m({}^{4}_{2}\text{He}) = 4.0026 \text{ u}$$
$$m({}^{1}_{1}\text{H}) = 1.0079 \text{ u}$$

Nuclear reaction, ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{H} \longrightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He}$

Energy released in one nuclear reaction,

$$Q = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$$
$$= (7.0160 + 1.0079 - 2 \times 4.0026) \times 931$$
$$= 17.41 \text{ MeV}$$

Number of atoms in 20 g of Li

$$= \frac{20}{7} \times 6.023 \times 10^{23} = 1.72 \times 10^{24}$$

$\therefore$ Total energy liberated

$$= Q \times \text{Total number of Li atoms}$$
$$= (17.41 \text{ MeV}) \times (1.72 \times 10^{24})$$
$$= 4.79 \times 10^{12} \text{ J}$$

$$\text{Energy (in kWh)} = \frac{4.79 \times 10^{12}}{3.6 \times 10^6} \quad (\because 1 \text{ kWh} = 3.6 \times 10^6 \text{ J})$$
$$= 1.33 \times 10^6 \text{ kWh}$$

**85.** Consider the reaction, ${}^{1}_{0}n + {}^{15}_{8}\text{O} \rightarrow {}^{15}_{7}\text{N} + {}^{1}_{1}\text{H} + Q$

So, $Q = (\Delta M)c^2 = (M_n + M_\text{O} - M_\text{N} - M_\text{H})c^2$

$$= 0.003796 \times 931.5 = 3.5359 \text{ MeV} \quad \ldots\text{(i)}$$

From the given relation,

$$\Delta Q = \Delta E = E_{{}^{15}_{8}\text{O}} - E_{{}^{15}_{7}\text{N}}$$
$$= \frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}(8 \times 7 - 7 \times 6)$$
$$= \frac{3}{5} \times 1.44 \text{ MeV-fm} \times \frac{14}{R} \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$R = 3.42 \text{ fm}$$

**86.** ${}_0n^1 + {}_{92}\text{U}^{235} \longrightarrow {}_{51}\text{Sb}^{133} + {}_{41}\text{Nb}^{99}$ + neutrons

Charge number is conserved $(92 = 51 + 41)$

Applying principle of conservation of mass number

$$133 + 99 + x = 235 + 1$$
$$x = 236 - 232 = 4$$

$\therefore$ Number of neutrons $({}_0n^1) = 4$

**87.** The fission of ${}_{92}\text{U}^{235}$ is represented by

$${}_{92}\text{U}^{235} + {}_0n^1 \rightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3\,{}_0n^1 + Q$$

The name of the particle $X$ is neutron $({}_0n^1)$.

**88.** $$\text{Fission rate} = \frac{\text{Total power}}{\text{Energy / Fission}}$$
$$= \frac{5}{200 \times 1.6 \times 10^{-13}} = 1.56 \times 10^{11} \text{ s}^{-1}$$

**89.** Nuclear forces are short range forces because they arise from small separation of charges inside neutral atom.

**90.** Mass defect in the given nuclear reaction,

$$\Delta m = 2\,(\text{mass of deuterium}) - (\text{mass of helium})$$
$$= 2\,(2.0141) - (4.0026) = 0.0256$$

Therefore, energy released

$$\Delta E = (\Delta m)\,(931.48) \text{ MeV} = 23.85 \text{ MeV}$$
$$= 23.85 \times 1.6 \times 10^{-13} \text{ J} = 3.82 \times 10^{-12} \text{ J}$$

Efficiency is only 25%, therefore

$$25\% \text{ of } \Delta E = \left(\frac{25}{100}\right)(3.82 \times 10^{-12}) \text{ J}$$
$$= 9.55 \times 10^{-13} \text{ J}$$

*i.e.* by the fusion of two deuterium nuclei, $9.55 \times 10^{-13}$ J energy is available to the nuclear reactor.

Total energy required in one day to run the reactor with a given power of 200 MW,

$$E_\text{Total} = 200 \times 10^6 \times 24 \times 3600 = 1.728 \times 10^{13} \text{ J}$$

$\therefore$ Total number of deuterium nuclei required for this purpose,

$$n = \frac{E_\text{Total}}{\Delta E/2} = \frac{2 \times 1.728 \times 10^{13}}{9.55 \times 10^{-13}}$$
$$= 0.362 \times 10^{26}$$

$\therefore$ Mass of deuterium required = (Number of g-moles of deuterium required) × 2 g

$$= \left(\frac{0.362 \times 10^{26}}{6.02 \times 10^{23}}\right) \times 2 = 120.26 \text{ g}$$

**91.** Number of moles of $\text{U}^{235}$ in given sample,

$$n = \frac{\text{Sample mass}}{\text{Molar mass}} = \frac{2}{235} \text{ (kilomoles)}$$

Number of atoms of $\text{U}^{235}$ in sample

$$= \text{Number of moles} \times N = \frac{2}{235} \times 6.023 \times 10^{26}$$

Total energy obtained

$$= \text{Number of atoms} \times \text{Energy from one atom}$$
$$= \frac{2 \times 6.023 \times 10^{26}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$
$$= \frac{2 \times 6.023 \times 10^{26} \times 200 \times 10^6 \times 1.6 \times 10^{-19}}{235 \times 30 \times 24 \times 3600}\left(\frac{\text{J}}{\text{s}}\right)$$
$$= 63.2 \times 10^6 \text{ W} = 63.2 \text{ MW}$$

It is close to 60 MW.

**92.** Here,

$A \rightarrow$ is the core of the reactor, which is the site of nuclear fission. It contains elements in suitably fabricated form. The fuel may be enriched uranium say ${}^{235}_{92}\text{U}$.

$D \rightarrow$ is a reflector surrounding the core which is used to reduce the leakage.

$C \rightarrow$ is coolant which helps in removing the energy (heat) released in fission. It transfers heat to working fluid which in turn may produce steam.

$B \rightarrow$ is control rods that shut down the reactor as they have high absorption of neutrons.

Hence, only option (b) shows correct purpose of given part.

**93.** Let $E_k$ be the KE of the incident electron. Its linear momentum, $p = \sqrt{2mE_k}$

de-Broglie wavelength related to electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} \quad \text{or} \quad E_k = \frac{h^2}{2m\lambda^2}$$

The cut-off wavelength of the emitted X-rays is related to the KE of the incident electron as

$$\frac{hv}{\lambda_0} = E_k = \frac{h^2}{2m\lambda^2}$$

or $\quad \lambda_0 = \dfrac{2mc\lambda^2}{h}$

**94.** In general, X-rays have larger wavelength than that of $\gamma$-rays.

**95.** In X-ray tube, the cut-off wavelength is given by $\lambda_{\min} = \dfrac{hc}{eV}$ · The cut-off wavelength depends on the energy $eV$ of the accelerated electrons and is independent of the atomic number of target.

**96.** Applied potential difference ($V$) must be greater than binding energy.

*i.e.* $\quad V > 40\,\text{kV}$

**97.** As accelerating voltage $V$ across X-rays tube increases, the value of minimum wavelength of X-rays, $\lambda_c = \dfrac{hc}{eV}$; decreases ; So, the separation between $\lambda_K$ and $\lambda_c$ increases.

**98.** de-Broglie wavelength, $\lambda_1 = \dfrac{h}{\sqrt{2meV}}$

X-ray wavelengths, $\lambda_2 = \dfrac{hc}{eV}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{eV}{c\sqrt{2meV}} = \frac{1}{c}\sqrt{\frac{1}{2}\left(\frac{e}{m}\right)V}$$

$$= \frac{1}{3 \times 10^8}\sqrt{\frac{1}{2} \times 1.8 \times 10^{11} \times 10^4} = \frac{1}{10}$$

**99.** $\lambda_{\min} = \dfrac{hc}{eV}$

$$\log(\lambda_{\min}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - mx$$

So, the required graph is given in option (d).

**100.** According to Moseley's law, $\sqrt{\nu} = a\,(Z - b)$

or $\quad \nu = a^2(Z - b)^2$ or $\dfrac{c}{\lambda} = a^2(Z - b)^2$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{(Z_2 - 1)^2}{(Z_1 - 1)^2}$$

Here, $\lambda_1 = \lambda, \lambda_2 = 4\lambda, Z_1 = 11$ and $Z_2 = ?$

$$\therefore \quad \frac{\lambda}{4\lambda} = \frac{(Z_2 - 1)^2}{(11 - 1)^2}$$

or $\quad (Z_2 - 1)^2 = 25$ or $Z_2 = 6$

**101.** Moseley's equation is $\sqrt{\nu} = a(Z - b)$

Thus, $\quad \sqrt{\dfrac{c}{\lambda_1}} = a(Z_1 - b) \quad$ ...(i)

and $\quad \sqrt{\dfrac{c}{\lambda_2}} = a(Z_2 - b) \quad$ ...(ii)

From Eqs. (i) and (ii), $\sqrt{c}\left(\dfrac{1}{\sqrt{\lambda_1}} - \dfrac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$

or $\quad a = \dfrac{\sqrt{c}}{(Z_1 - Z_2)}\left(\dfrac{1}{\sqrt{\lambda_1}} - \dfrac{1}{\sqrt{\lambda_2}}\right)$

$$= \frac{(3 \times 10^8)^{1/2}}{42 - 27}\left[\frac{1}{(71 \times 10^{-12})^{1/2}} - \frac{1}{(178.5 \times 10^{-12})^{1/2}}\right]$$

$$= 5.0 \times 10^7\,(\text{Hz})^{1/2}$$

Dividing Eq. (i) by Eq. (ii),

$$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$$

or $\quad \sqrt{\dfrac{178.5}{71}} = \dfrac{42 - b}{27 - b}$ or $b = 1.37$

## *Round II*

**1.** We know that, $E_n = -13.6\dfrac{Z^2}{n^2}$ eV and $r_n = 0.53\dfrac{n^2}{Z}$ (Å)

Here, for $n = 1$, $E_1 = -54.4$ eV

Therefore, $\quad -54.4 = -13.6\dfrac{Z^2}{1^2} \Rightarrow Z = 2$

Hence, radius of first Bohr orbit $r = \dfrac{0.53\,(1)^2}{2} = 0.265$ Å

**2.** The hydrogen atom before the transition was at rest. Therefore, from conservation of momentum,

$$p_{\text{H-atom}} = p_{\text{photon}} = \frac{E_{\text{radiated}}}{c} = \frac{13.6\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)\text{eV}}{c}$$

$$1.6 \times 10^{-27} \times v = \frac{13.6\left(\dfrac{1}{1^2} - \dfrac{1}{5^2}\right) \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

$$\Rightarrow \quad v = 4.352 \text{ m/s} \approx 4 \text{ m/s}$$

**3.** In the second excited state, $n = 3$

$$\therefore \quad L_{\text{H}} = L_{\text{Li}} = 3\left(\frac{h}{2\pi}\right)$$

As $E \propto Z^2$ and $Z_{\text{H}} = 1$ and $Z_{\text{Li}} = 3$

$$\therefore \quad |E_{\text{Li}}| = 9|E_{\text{H}}| \text{ or } |E_{\text{H}}| < |E_{\text{Li}}|$$

**4.** $\quad \dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{t/T}$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{19/3.8} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$\Rightarrow \quad N = \frac{N_0}{32} = \frac{10.38}{32} = 0.32 \text{ g}$$

**5.** After one half-life period, the activity of tritium becomes 50%.

After 2 half-life period 25%

After 3 half-life period 12.5%

After 4 half-life period 6.25%

After 5 half-life period 3.12% = 3%

It is $5 \times 12.5$ yr + 7 yr, *i.e.* approximately 70 yr back only.

**6.** Activity, $A = \frac{-N}{dt} = \lambda N$

As the number of nuclei ($N$) per mole are equal for both the substances, irrespective of their molecular mass, therefore, $A \propto \lambda$

$$\frac{A_1}{A_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

**7.** $n = \frac{t}{T} = \frac{42}{6} = 7$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$$

As rate of disintegration $\propto N$

$$\therefore \quad \frac{R}{R_0} = \frac{1}{128}; R = \frac{R_0}{128} = \frac{1024}{128} = 8\,\text{min}^{-1}$$

**8.** Here, $T = 4.47 \times 10^9$ yr

$$\frac{N}{N_0} = \frac{60}{100} = \left(\frac{1}{2}\right)^n \text{ or } 2^n = \frac{10}{6}$$

$$n \log 2 = \log 10 - \log 6 = 1 - 0.778 = 0.222$$

$$n = \frac{0.222}{\log 2} = \frac{0.222}{0.3} = 0.74$$

$$t = nT = 0.74 \times 4.47 \times 10^9 \text{ yr}$$

$$= 3.3 \times 10^9 \text{ yr}$$

**9.** As, $T = \frac{2\pi r}{V}$ or $V = \frac{rh}{2\pi mr}$

$$\therefore \quad T = \frac{2\pi r}{nh/2\pi mr} = \frac{mr^2}{nh} \propto \frac{r^2}{n}$$

But $r \propto n^2 \therefore T \propto n^3$

$$\text{or} \quad \frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$$

As, $T_1 = 8T_2$

$$\therefore \quad \left(\frac{n_1}{n_2}\right)^3 = 8, \frac{n_1}{n_2} = 2$$

Therefore, in given values $n_1 = 6, n_2 = 3$

**10.** Let $\lambda_A = \lambda$

$$\therefore \quad \lambda_B = 2\lambda$$

If $N_0$ is total number of atoms in $A$ and $B$ at $t = 0$, then initial rate of disintegration of $A = \lambda N_0$ and initial rate of disintegration of $B = 2\lambda N_0$

$$\text{As,} \quad \lambda_B = 2\lambda A$$

$$\therefore \quad T_B = \frac{1}{2} T_A$$

*i.e.* half-life of $B$ is half the half-life of $A$.

After one half-life of $A$

$$\left(-\frac{dN}{dt}\right)_A = \frac{\lambda N_0}{2}$$

Equivalently, after two half lives of $B$

$$\left(-\frac{dN}{dt}\right)_B = \frac{2\lambda N_0}{4} = \frac{\lambda N_0}{2}$$

$$\text{Clearly,} \quad \left(-\frac{dN}{dt}\right)_A = -\left(\frac{dN}{dt}\right)_B$$

After $n = 1$, *i.e.* one half-life of $A$.

**11.** Helium atom has 2 electrons. When one electron is removed, the remaining atom is hydrogen like atom, whose energy in first orbit is

$$E_1 = -(2)^2 (13.6 \text{ eV}) = -54.4 \text{ eV}$$

Therefore, to remove the second electron from the atom, the additional energy of 54.4 eV is required. Hence, total energy required to remove both the electrons

$$= 24.6 + 54.4 = 79.0 \text{ eV}$$

**12.** $N_1 = N_0 e^{-\lambda t_1}$

$$\frac{N_1}{N_0} = e^{-\lambda t_1}$$

$$0.67 = e^{-\lambda t_1}$$

$$\ln(0.67) = -\lambda t_1 \quad \ldots\text{(i)}$$

$$N_2 = N_0 e^{-\lambda t_2}$$

$$\frac{N_2}{N_0} = e^{-\lambda t_2}$$

$$0.33 = e^{-\lambda t_2}$$

$$\ln(0.33) = -\lambda t_2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\ln(0.67) - \ln(0.33) = \lambda t_1 - \lambda t_2$$

$$\lambda(t_1 - t_2) = \ln\left(\frac{0.67}{0.33}\right)$$

$$\lambda(t_1 - t_2) \cong \ln 2$$

$$t_1 - t_2 \simeq \frac{\ln 2}{\lambda} = t_{1/2}$$

Half life = $t_{1/2}$ = 20 minutes.

**13.** As, radius of $n$th orbit,

$$r = r_0 \frac{n^2}{Z}$$

$$\Rightarrow \quad r_n \propto \frac{n^2}{Z}$$

$$\therefore \quad \frac{\Delta r_n}{r_n} = \frac{r_{n+1} - r_n}{r_n} \propto \frac{\frac{(n+1)^2}{Z} - \frac{n^2}{Z}}{\frac{n^2}{Z}}$$

$$= \frac{2n+1}{n^2} = \frac{2}{n} + \frac{1}{n^2} \simeq \frac{2}{n} \quad \left[n >> 1, \frac{1}{n^2} \to 0\right]$$

$$\Rightarrow \quad \frac{\Delta r_n}{r_n} \propto \frac{1}{n}$$

Angular momentum, $L = \frac{nh}{2\pi}$

$$\therefore \qquad \frac{\Delta L}{L} = \frac{(n+1)\frac{h}{2\pi} - n\frac{h}{2\pi}}{n\frac{h}{2\pi}} = \frac{1}{n}$$

$$\therefore \qquad \frac{\Delta L}{L} \propto \frac{1}{n}$$

Total energy, $E_n = -\frac{13.6\,Z^2}{n^2}$

$$\Rightarrow \qquad \frac{\Delta E_n}{E_n} = \frac{E_{n+1} - E_n}{E_n} \propto \frac{\frac{Z^2}{(n+1)^2} - \frac{Z^2}{n^2}}{\frac{Z^2}{n^2}}$$

$$\Rightarrow \qquad \frac{\Delta E_n}{E_n} \propto \frac{2n+1}{n^2} \propto \frac{2}{n} + \frac{1}{n^2} \simeq \frac{1}{n} \qquad [\text{As, } n >> 1]$$

**14.** Here, $E_n = \frac{-me^4}{8\varepsilon_0^2 n^2 h^2}$

is the energy of $n$th level of hydrogen atom. If all the H-atom are in ground state, ($n = 1$), then the radiation of frequency $(E_2 - E_1)/h$ falling on it may be absorbed by some of the atoms and move them to the first excited state ($n = 2$). All atoms may not be excited to $n = 2$ state. Further, as $(E_2 - E_1)/h$ is sufficient only to take the atom from $n = 1$ state to $n = 2$ state, no atoms shall make a tansition to $n = 3$ state.

**15.** Frequency of emitted radiation during transition from $(n+1)$th level to the $n$th level in hydrogen atom is

$$f = \frac{\Delta E}{h} \qquad (\because \Delta E = hf)$$

$$= \frac{1}{h}[E_{n+1} - E_n]$$

$$= \frac{1}{h}\left(\frac{-13.6}{(n+1)^2} - \left(\frac{-13.6}{n^2}\right)\right)$$

$$= \frac{1}{h} \times 13.6 \times \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$= \frac{13.6}{h}\left\{\frac{2n+1}{n^2(n+1)^2}\right\}$$

$$\Rightarrow \qquad f = \frac{13.6}{h}\left\{\frac{n\left(2 + \frac{1}{n}\right)}{n^4\left(1 + \frac{1}{n^2}\right)^2}\right\} \qquad \ldots\text{(i)}$$

If $n >> 1$, $\frac{1}{n}$ and $\frac{1}{n^2} \approx 0$

So, neglecting $\frac{1}{n}$ and $\frac{1}{n^2}$ in Eq (i), we have

$$f = \frac{13.6}{h} \times \left(\frac{2n}{n^4}\right) = \frac{2 \times 13.6}{h} \times \frac{1}{n^3}$$

$$\Rightarrow \qquad f \propto \frac{1}{n^3}$$

**16.** First transition is from $n = 3$ to $n = 2$. Second transition is from $n = 2$ to $n = 1$

$$\therefore \qquad \frac{E_1}{E_2} = c = \frac{1/2^2 - 1/3^2}{1/1^2 - 1/2^2}$$

As, $\quad p = \frac{E}{c}$, therefore,

$$\frac{p_1}{p_2} = b = \frac{E_1}{E_2} = c$$

*i.e.* $\quad b = c = \frac{5}{27}$

As, $\quad E = \frac{hc}{\lambda}$

$$\therefore \qquad \lambda \propto \frac{1}{E}$$

or $\quad a = \frac{\lambda_1}{\lambda_2} = \frac{E_2}{E_1} = \frac{27}{5} = \frac{1}{c}$ or $c = \frac{1}{a}$

**17.** de-Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{p}$

where, $p$ = momentum.

By conservation of momentum, $\mathbf{p}_1 + \mathbf{p}_2 = 0$

or $\quad p_1 = p_2$

$\therefore \quad \lambda_1 = \lambda_2 = \lambda$

**18.** Given, normal activity, $A_0 = 15$ decays/min

Present activity, $A = 9$ decays/min

$$T_{1/2} = 5730 \text{ yr}$$

Using the formula, $\frac{A}{A_0} = e^{-\lambda t}$

$$\frac{9}{15} = e^{-\lambda t}$$

or $\quad \frac{3}{5} = e^{-\lambda t}$ or $e^{\lambda t} = \frac{5}{3}$

Taking log on both the sides, we get

$$\lambda t \log_e e = \log_e 5 - \log_e 3$$

or $\quad \lambda t = 2.303\,(0.69 - 0.47)$

$$\lambda t = 0.5109 \qquad \left(\because \lambda = \frac{0.693}{T_{1/2}}\right)$$

$$t = \frac{0.5066 \times T_{1/2}}{0.693}$$

$$= \frac{0.5066 \times 5730}{0.693}$$

$$= 4224.47 \text{ yr}$$

Thus, the approximate age of Indus-Valley civilisation is 4224 yr.

**19.** $$\lambda_{eq.} = \lambda_1 + \lambda_2$$

$$\frac{1}{T_{1/2}} = \frac{1}{T_{1/2}^{(1)}} + \frac{1}{T_{1/2}^{(2)}}$$

$$T_{1/2} = \frac{T_{1/2}^{(1)}\,T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$$

**20.** According to law of disintegration for radioactive substances,

$$R = R_0 e^{-\lambda t} \quad \Rightarrow \ln R = -\lambda t + \ln R_0$$

Slope of ln $R$ *versus* $t$ graph will give $\lambda$.

Also, decay constant, $\lambda = \dfrac{\ln 2}{T_{1/2}} \Rightarrow T_{1/2} = \dfrac{1}{\lambda} \ln 2$

$\therefore$ Slope of curve $A = \lambda_A = \dfrac{6}{10} \Rightarrow T_{1/2}(A) = \dfrac{10}{6} \ln 2$

Similarly, slope of curve $B = \lambda_B = \dfrac{6}{5}$

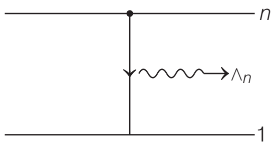
$$\Rightarrow \qquad T_{1/2}(B) = \frac{5}{6} \ln 2$$

and slope of curve $C = \lambda_C = \dfrac{2}{5} \Rightarrow T_{1/2}(C) = \dfrac{5}{2} \ln 2$

So, $T_{1/2}(A) : T_{1/2}(B) : T_{1/2}(C) = \dfrac{10}{6} : \dfrac{5}{6} : \dfrac{5}{2}$

$$= 20 : 10 : 30 = 2 : 1 : 3$$

**21.** If wavelength of emitted photon in de-excitation is $\Lambda_n$;




Then, $$\frac{hc}{\Lambda_n} = E_n - E_g \qquad \left[\because E = \frac{p^2}{2m}\right]$$

$$\frac{hc}{\Lambda_n} = \frac{p_n^2}{2m} - \frac{p_g^2}{2m}$$

As energies are negative, we get

$$\frac{hc}{\Lambda_n} = \frac{p_g^2}{2m} - \frac{p_n^2}{2m}$$

$$\Rightarrow \quad = \frac{p_g^2}{2m}\left(1 - \left(\frac{p_n}{p_g}\right)^2\right) = \frac{h^2}{2m\lambda_g^2}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)$$

$$\left[\because p \propto \lambda^{-1}, p = \frac{h}{\lambda}\right]$$

$$\Rightarrow \quad \Lambda_n = \frac{2m\lambda_g^2 c}{h}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)^{-1}$$

$$\Rightarrow \quad \Lambda_n = \frac{2m\lambda_g^2 c}{h}\left(1 + \frac{\lambda_g^2}{\lambda_n^2}\right) \qquad [\because (1-x)^{-n} = 1 + nx]$$

$$\Rightarrow \quad \Lambda_n \simeq A + \frac{B}{\lambda_n^2}$$

where, $A = \left[\dfrac{2mc\lambda_g^2}{h}\right]$ and $B = \left[\dfrac{2mc\lambda_g^4}{h}\right]$ are constants.

**22.** Given, work-function, $W = 1.9$ eV

Wavelength of incident light, $\lambda = 400$ nm

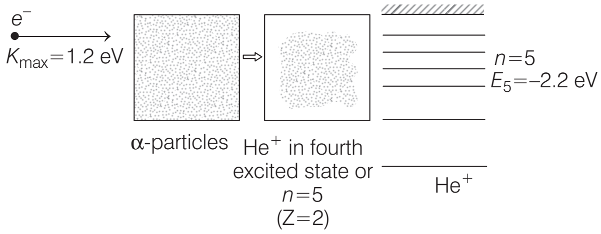
$\therefore$ Energy of incident light, $E = \dfrac{hc}{\lambda} = 3.1$ eV

(Substituting the values of $h$, $c$ and $\lambda$)

Therefore, maximum kinetic energy of photoelectrons

$$K_{max} = E - W = (3.1 - 1.9) = 1.2 \text{ eV}$$

Now, the situation is as shown in figure.




Energy of electron in 4th excited state of $He^+$ ($n = 5$) will be

$$E_5 = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

$$\Rightarrow \quad E_5 = -(13.6)\frac{(2)^2}{(5)^2} = -2.2 \text{ eV}$$

Therefore, energy released during the combination

$$= 1.2 - (-2.2) = 3.4 \text{ eV}$$

Similarly, energies in other energy states of $He^+$ will be

$$E_4 = -13.6 \frac{(2)^2}{(4)^2} = -3.4 \text{ eV}$$

$$E_3 = -13.6 \frac{(2)^2}{(3)^2} = -6.04 \text{ eV}$$

$$E_2 = -13.6 \frac{(2)^2}{(2)^2} = -13.6 \text{ eV}$$

The possible transitions are

$$\Delta E_{5\to 4} = E_5 - E_4 = 1.2 \text{ eV} < 2 \text{ eV}$$

$$\Delta E_{5\to 3} = E_5 - E_3 = 3.84 \text{ eV}$$

$$\Delta E_{5\to 2} = E_5 - E_2 = 11.4 \text{ eV} > 4 \text{ eV}$$

$$\Delta E_{4\to 3} = E_4 - E_3 = 2.64 \text{ eV}$$

$$\Delta E_{4\to 2} = E_4 - E_2 = 10.2 \text{ eV} > 4 \text{ eV}$$

Hence, the energy of emitted photons in the range of 2 eV and 4 eV are 3.4 eV during combination and 3.84 eV and 2.64 after combination.

**23.** From the relation,

$$R = R_0 e^{-\lambda t} \qquad \text{(Here, } R = \text{activity of sample)}$$

Substituting the values, we have

$$2700 = 4750 e^{-5\lambda}$$

$$\therefore \quad 5\lambda = \ln\left(\frac{4750}{2700}\right) = 0.56$$

$$\therefore \quad \lambda = 0.113 \text{ min}^{-1}$$

Half-life of the sample,

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.113} = 6.132 \text{ min}$$

**24.** $\Delta m = 2\,(2.015) - (3.017 + 1.009) = 0.004$ amu

$\therefore$ Energy released $= (30.004 \times 931.5)$ MeV

$$= 3.726 \text{ MeV}$$

Energy released per deuteron

$$= \frac{3.726}{2} = 1.863 \text{ MeV}$$

Number of deuterons in 1g

$$= \frac{6.02 \times 10^{23}}{2} = 3.01 \times 10^{23}$$

$\therefore$ Energy released per g of deuterium fusion

$$= (3.01 \times 10^{23} \times 1.863)$$
$$= 5.6 \times 10^{23} \text{ MeV}$$
$$\approx 9.0 \times 10^{10} \text{ J}$$
$$\simeq 9 \times 10^{9} \text{ kJ}$$
$$\simeq 9 \times 10^{n} \text{ kJ (given)}$$

$\therefore \quad n = 9$

***25.*** At the time of observation ($t = t$),

$$\frac{m_1}{m_2} = \frac{100}{1} \quad \text{(given)}$$

Further, it is given that $\frac{A_1}{A_2} = \frac{1.02}{1}$

Number of atoms $N = \frac{m}{A}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{m_1}{m_2} \times \frac{A_2}{A_1} = \frac{100}{1.02} \quad \text{...(i)}$$

Let $N_0$ be the number of atoms of both the isotopes at the time of formation, then

$$\frac{N_1}{N_2} = \frac{N_0\, e^{-\lambda_1 t}}{N_0\, e^{-\lambda_2 t}} = e^{(\lambda_2 - \lambda_1)\, t} \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we have

$$e^{(\lambda_2 - \lambda_1)\, t} = \frac{100}{1.02}$$

or $\quad (\lambda_2 - \lambda_1)t = \ln (100) - \ln (1.02)$

$$\therefore \quad t = \frac{\ln (100) - \ln (1.02)}{\left(\frac{1}{2 \times 10^9} - \frac{1}{4 \times 10^9}\right)}$$

Substituting the values, we have

$$t = 1.834 \times 10^{10} \text{ yr}$$

***26.*** Given, $\quad E_{2n} - E_1 = 204$ eV

$$\therefore \quad (13.6)Z^2\left(1 - \frac{1}{4n^2}\right) = 204 \quad \text{...(i)}$$

$$E_{2n} - E_n = 40.8 \text{ eV}$$

$$\therefore \quad 13.6\, Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) = 40.8 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$n = 2 \quad \text{and} \quad Z = 4$$

$$E_1 = (-13.6)\, Z^2 \text{ eV}$$
$$= (-13.6)\,(4)^2 \text{ eV} = -217.6 \text{ eV}$$

During de-excitation, minimum energy emitted is

$$E_{\min} = E_{2n} - E_{2n-1} = E_4 - E_3$$
$$= \frac{-217.6}{4^2} - \left(\frac{-217.6}{3^2}\right)$$
$$= 10.58 \text{ eV}$$

***27.*** As, $A_1 = \lambda_1 N_1$ and $A_2 = \lambda_2 N_2$

$$\lambda_1 N_1 = \lambda_2 N_2 \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{N_1}{N_2} = 3 \quad (\because N_1 > N_2)$$

$$T_1 = 12 \text{ h and } T_2 = 16 \text{ h}$$

After integer, *i.e.* $2 \times 24 = 48$ h

$$N_1' = N_1 \left(\frac{1}{2}\right)^{t/T_1} \quad \left(\text{But, } \frac{t}{T_1} = \frac{48}{12} = 4\right)$$

$$\therefore \quad N_1' = N_1 \left(\frac{1}{2}\right)^4 \quad \text{...(ii)}$$

and $$N_2' = N_2 \left(\frac{1}{2}\right)^{t/T_2} \quad \left(\text{But, } \frac{t}{T_2} = \frac{48}{16} = 3\right)$$

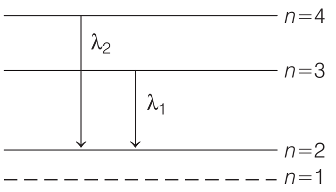
$$\therefore \quad N_2' = N_2 \left(\frac{1}{2}\right)^3 \quad \text{...(iii)}$$

Now, dividing Eq. (ii) by Eq. (iii), we get

$$\frac{N_1'}{N_2'} = \frac{N_1}{N_2} \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{3}\right)^3} = 3 \times \frac{8}{16} = \frac{3}{2} = \frac{3}{x} \text{ (given)}$$

$\therefore \quad x = 2$

***28.*** Balmer series of H-atom spectrum occurs when an excited electron jumps to second level or $n = 2$ state.




Emitted wavelengths are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where, $n = 3, 4, 5 \ldots \infty$.

First member is obtained in transition $n = 3$ to $n = 2$.

$$\Rightarrow \quad \frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \quad \text{...(i)}$$

Second member is obtained in transition $n = 4$ to $n = 2$.

$$\Rightarrow \quad \frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{2^2} - \frac{1}{4^2}\right)} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{1}{4} - \frac{1}{9}\right)}{\left(\frac{1}{4} - \frac{1}{16}\right)} = \frac{5}{36} \times \frac{64}{12}$$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{5}{9} \times \frac{4}{3} = \frac{20}{27} \Rightarrow \lambda_2 = \lambda_1 \times \frac{20}{27}$$

$$= 6561 \times \frac{20}{27} = 4860 \text{ Å} = 486 \text{ nm}$$

**29.** As, $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\Rightarrow \quad \lambda = \frac{1}{R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)} \quad \ldots(i)$$

For Lyman series,

Largest wavelength ($n_i = 2$ and $n_f = 1$)

$$\lambda_1 = \frac{1}{R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = \frac{1}{R\left(\frac{3}{4}\right)} = \frac{4}{3R}$$

Shortest wavelength ($n_i = \infty$ and $n_f = 1$)

$$\lambda_2 = \frac{1}{R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)} = \frac{1}{R(1)} = \frac{1}{R}$$

Now, it is given that the difference between the largest and the shortest wavelengths of the Lyman series is 304 Å.

So, $\Delta\lambda = \lambda_1 - \lambda_2$

$$304 = \frac{4}{3R} - \frac{1}{R} \Rightarrow 304 = \frac{1}{3R}$$

$$R = \frac{1}{3 \times 304} \Rightarrow R = \frac{1}{912} \quad \ldots(ii)$$

For Paschan series,

Largest wavelength ($n_i = 4$ and $n_f = 3$)

$$\lambda_3 = \frac{1}{R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)} = \frac{1}{R\left(\frac{7}{144}\right)} = \frac{144}{7R}$$

Shortest wavelength ($n_i = \infty$ and $n_f = 3$)

$$\lambda_4 = \frac{1}{R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)} = \frac{1}{R\left(\frac{1}{9}\right)} = \frac{9}{R}$$

Now, the difference between the largest and the shortest wavelengths of the Paschan series,

$$\Delta\lambda = \lambda_3 - \lambda_4$$

$$= \frac{144}{7R} - \frac{9}{R} = \frac{144 - 63}{7R} = \frac{81}{7R} = \frac{81}{7\left(\frac{1}{912}\right)}$$

[from Eq. (i)]

$$= \frac{81 \times 912}{7} = 10553.14 \text{ Å} \simeq 10553 \text{ Å}$$

**30.** Here, mass of hydrogen atom is 5 times mass of colliding particle.

Using law of conservation of linear momentum,

$$mv + 0 = 0 + 5mv' \Rightarrow v' = \frac{v}{5}$$

When the atom goes to its first excited state,

$$\text{Energy dissipated} = \left[\frac{-13.6}{(2)^2}\right] \text{eV} - \left[\frac{-13.6}{(1)^2}\right] \text{eV}$$

$$= -3.4 + 13.6 = 10.2 \text{ eV} \quad \ldots(i)$$

$$\text{Loss of kinetic energy} = \frac{1}{2}mv^2 - \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2$$

$$= \frac{1}{2}mv^2\left(1 - \frac{1}{5}\right) \text{s} = \frac{4}{5}\left(\frac{mv^2}{2}\right) = \frac{4}{5}K \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{4}{5}K = 10.2 \text{ eV}$$

$$\Rightarrow \quad K = \frac{10.2 \times 5}{4} \text{ eV} = \frac{51.0}{4} \text{ eV}$$

$$\Rightarrow \quad \frac{N}{4} = \frac{51}{4} \text{ eV} \Rightarrow N = 51$$

**31.** If $\lambda$ is de-Broglie wavelength, then for $n$th stationary orbit

$$2\pi r_n = n\lambda$$

where $r_n$ is radius of $n$th orbit.

$$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

$$\therefore \quad 2\pi\left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right) = n\lambda \Rightarrow \frac{1}{\lambda} = \frac{mZe^2}{2\varepsilon_0 h^2 n} \quad \ldots(i)$$

For Lyman series of hydrogen like atom, wavelength is given as,

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad \ldots(ii)$$

From Eqs. (i) and (ii), $Z^2R\left(1 - \frac{1}{n^2}\right) = \frac{mZe^2}{2\varepsilon_0 h^2 n}$

Rydberg constant, $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$

$$\therefore \quad \frac{Z^2 me^4}{8\varepsilon_0^2 ch^3}\left(1 - \frac{1}{n^2}\right) = \frac{mZe^2}{2\varepsilon_0 h^2 n}$$

$$\text{or} \quad \left(1 - \frac{1}{n^2}\right) = \frac{4\varepsilon_0 ch}{m^2 Z}$$

Substituting given values, we get

$$= \frac{4 \times (8.85 \times 10^{-12}) \times (3 \times 10^8) \times (6.62 \times 10^{-34})}{n \times (1.6 \times 10^{-19})^2 \times 11} = \frac{25}{n}$$

$$\therefore \quad n^2 - 1 = 25n$$

$$\text{or} \quad n^2 - 25n - 1 = 0$$

$$n - \frac{25 \pm \sqrt{(-25)^2 + 4 \times 1 \times 1}}{2} \approx \frac{25 \pm \sqrt{625}}{2} \approx 25$$

As negative $n$ is not possible, $n \approx 25$.

**32.** $F = \frac{-dU}{dr} = -4U_0 r^3 = \frac{mv^2}{r}$

$$\therefore \quad mv^2 = 4U_0 r^4$$

$$\text{or} \quad v \propto r^2$$

we have $mvr = \frac{nh}{2\pi}$

$$r^3 \propto n$$

$$r \propto n^{1/3}$$

$$\therefore \quad \alpha = 3$$