

# Electromagnetic Induction

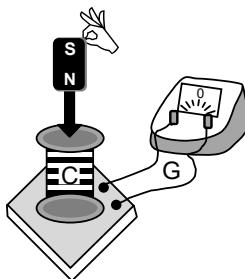
## 1. Experiments of Faraday and Henry

Electromagnetic induction can be understood through a series of experiments carried out by Faraday and Henry.

Some of these experiments are

### Experiment - 1

In this experiment, the coil C is connected to a galvanometer G and a bar magnet is pushed towards the coil.



### Observations

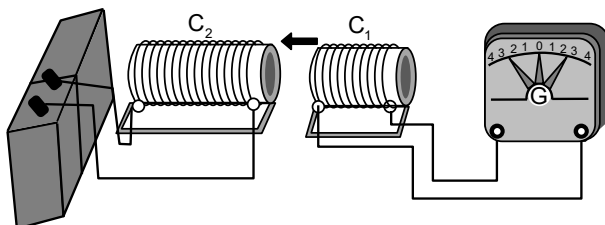
- (a) When the magnet is pushed with North-pole towards the coil as shown above, there is a deflection in the galvanometer which indicates that there is electric current in the coil and when the magnet is pulled away from the coil it is observed that the galvanometer shows deflection in the opposite direction.  
Whereas when the South-pole of the bar magnet is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed with the North-pole for similar movements.
- (b) As long as the bar magnet is in motion, the galvanometer shows deflection and no deflection is observed when the magnet is held stationary.
- (c) The deflection is found to be larger when the magnet is pushed towards or pulled away from the coil faster.
- (d) When the bar magnet is held fixed and the coil C is moved towards or away from the magnet, the same effects are observed.

### Conclusion

The above observations determine that it is the relative motion between the magnet and the coil that is responsible for generation (induction) of electric current in the coil.

### Experiment - 2

In this experiment coil  $C_1$  is connected with galvanometer and a second coil  $C_2$  connected to a battery. The steady current in the coil  $C_2$  produces a steady magnetic field.



### Observations

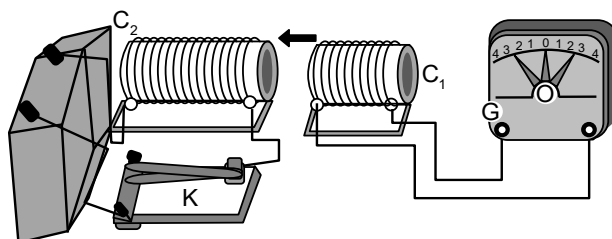
- (a) When the coil  $C_1$  is moved towards the coil  $C_2$ , the galvanometer shows a deflection which indicates that an electric current is induced in the coil  $C_1$ . Whereas, when the coil  $C_1$  is moved away, the galvanometer shows a deflection in the opposite direction.
- (b) There is deflection in the galvanometer as long as coil  $C_1$  is in motion.
- (c) When the coil  $C_1$  is held fixed and  $C_2$  is moved, the same effects are observed.

### Conclusion

The above observations determine that it is the relative motion between the coils that induces the electric current.

### Experiment -3

In this experiment, coil  $C_1$  is connected with galvanometer and the second coil  $C_2$  is connected to a battery through a tapping key K. Two coils are held stationary.



### Observations

- (a) When the tapping key K is pressed, the galvanometer shows a momentary deflection and the pointer in the galvanometer returns to zero immediately. When the key is released, a momentary deflection is observed again, but in the opposite direction.
- (b) If the key is held pressed continuously, there is no deflection in the galvanometer.
- (c) When an iron rod is inserted into the coils along their axis, and the key is tapped, the deflection increases dramatically.

### Conclusion

The above observations determine that relative motion is not an absolute requirement for inducing an electric current.

## 2. Magnetic Flux

- The concept of magnetic lines of field was first proposed by Faraday. In modern day physics the concept of magnetic lines of field is used in visualization or explanation of principles only.
- The tangent drawn at any point on a line of field in a magnetic field shows the direction of magnetic field at that point and the density of lines of field, i.e., the number of lines of field crossing normally a unit area indicates the intensity of magnetic field.
- The lines of field in a uniform magnetic field are parallel straight lines equidistant from each other. Where the lines of field are near each other, B is higher and where the lines of field are far apart, B is lesser.
- The number of lines of field crossing a given surface is called flux from that surface. It generally represented by  $\phi$ . Flux is a property of a vector field. If the vector field is a magnetic field, then the flux is called magnetic flux.

- The magnetic flux crossing a certain area is equal to the scalar product of the vector field ( $\vec{B}$ ) and the vector area ( $d\vec{A}$ ), that is

$$\text{Magnetic flux } d\phi = \vec{B} \cdot d\vec{A} = B dA \cos\theta$$

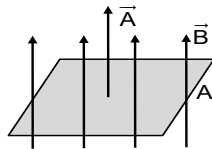
where  $\theta$  is the angle between the vector field and the vector area  $d\vec{A}$ .

$$\phi = \int \vec{B} \cdot d\vec{A}$$

For a uniform magnetic field  $\vec{B}$  and plane surface  $\vec{A}$ ,  $\phi = \vec{B} \cdot \vec{A} = BA \cos\theta$

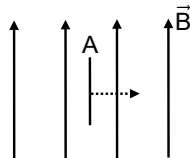
**(Note :** Area is a vector quantity whose direction is in the direction of perpendicular pointing outward from the surface)

- Magnetic flux is a scalar quantity.
- If a plane surface of area  $A$  is imagined in a uniform magnetic field  $\vec{B}$ , then
  - when the normal to the surface makes an angle  $\theta$  with the magnetic field, the magnetic flux is  $\phi = BA \cos\theta$
  - when a surface is perpendicular to the magnetic field, the lines of force crossing that area, i.e., the magnetic flux is



$$\phi = BA \text{ because } \theta = 0, \cos 0^\circ = 1$$

- When the surface is parallel to the field, then

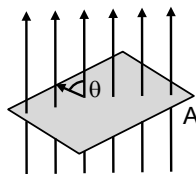


$$\theta = 90^\circ, \cos 90^\circ = 0$$

$$\therefore \phi = BA \cos 90^\circ = 0$$

- If the magnetic field is not uniform and the surface is not plane, then the element  $d\vec{A}$  of the surface may be assumed as plane and magnetic field  $\vec{B}$  may also be assumed as uniform over this element. Thus the magnetic flux coming out from this element is  $d\phi = \vec{B} \cdot d\vec{A}$

Hence magnetic flux coming out from the entire surface



$$\phi = \int_s \vec{B} \cdot d\vec{A}$$

- For a closed surface the vector area element pointing outward is positive and the vector area element pointing inward is negative.
- Magnetic lines of field are closed curves because free magnetic poles do not exist. Thus for a closed surface whatever is the number of the lines of field entering it, the same number of

lines of field come out from it. As a result for a closed curve

$$\phi = \int_S \vec{B} \cdot d\vec{A} = 0$$

Thus the net magnetic flux coming out of a closed surface is equal to zero.

- For a normal plane surface in a magnetic field

$$\phi = BA \quad \text{Hence} \quad B = \frac{\phi}{A}$$

Thus the magnetic flux passing normally from a surface of unit area is equal to magnetic

induction B. Therefore  $\frac{\phi}{A}$  is also called flux density.

- **Unit of magnetic flux** - In S.I. system, the unit of magnetic flux is weber (Wb) and in C.G.S. system unit of magnetic flux is maxwell.

$$1 \text{ weber} = 10^8 \text{ maxwell}$$

The M.K.S unit of flux density or magnetic induction is weber/m<sup>2</sup>. It is also called tesla.

$$1 \text{ tesla} = 1 \text{ weber/m}^2$$

The C.G.S unit of magnetic flux density is gauss.

$$1 \text{ gauss} = 1 \text{ maxwell/cm}^2$$

$$1 \text{ tesla} = 1 \text{ weber/m}^2 = 10^4 \text{ gauss}$$

- **Dimensions of magnetic flux**

$$\phi = BA = \frac{F}{qv} \times A$$

$$[\phi] = \frac{[F]}{[q][v]} [A]$$

$$\frac{(\text{kg} - \text{m} - \text{s}^{-2}) \times \text{m}}{\text{A}} = \text{kg} - \text{m}^2 - \text{s}^{-2} - \text{A}^{-1}$$

$$= \text{M}^1 \text{L}^2 \text{T}^{-2} \text{A}^{-1}$$

### Example 1:

The plane of a coil of area 1 m<sup>2</sup> and having 50 turns is perpendicular to a magnetic field of  $3 \times 10^{-5}$  weber/m<sup>2</sup>. The magnetic flux linked with it will be-

(1)  $1.5 \times 10^{-3}$  weber      (2)  $3 \times 10^{-5}$  weber      (3)  $15 \times 10^{-5}$  weber      (4) 150 weber

**Solution:**  $N = 50, B = 3 \times 10^{-5} \text{ wb/m}^2,$

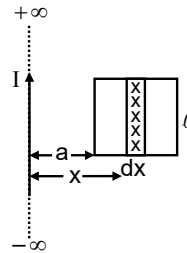
$$A = 1\text{m}^2, \theta = 0 \text{ or}$$

$$\phi = NBA = 50 \times 3 \times 10^{-5} \times 1$$

$$= 150 \times 10^{-5} \text{ weber} = 1.5 \times 10^{-3} \text{ weber}$$

**Example 2:**

Figure shows a long straight wire carrying current  $I$  and a square conducting wire loop of side  $\ell$ , at a distance 'a' from current wire. Both the current wire and loop are in the plane of paper. Find the flux of magnetic field of current wire, passing through the loop.

**Solution:**

Since the field of current wire passing through the loop, is same in direction (normally inward) but not uniform in magnitude. Hence will use integration method for finding the flux.

The small flux through a thin rectangular strip of length  $\ell$  and width  $dx$ , is given by :

$$\begin{aligned} d\phi_B &= \vec{B}_{(x)} \cdot d\vec{A} = B_{(x)} dA \cos 180^\circ = -\frac{\mu_0}{2\pi} \cdot \frac{I}{x} \ell \times dx \\ \therefore \phi_B &= \int d\phi_B = -\int \frac{\mu_0}{2\pi} \cdot \frac{I\ell}{x} dx \\ &= -\frac{\mu_0}{2\pi} I \ell \cdot \left[ \log_e x \right]_{x=a}^{x=a+\ell} = -\frac{\mu_0}{2\pi} I \ell \log_e \frac{a+\ell}{a} \end{aligned}$$

**Concept Builder-1**

- Q.1** A coil has 20 turns and area of each turn is  $0.2 \text{ m}^2$ . If the plane of the coil makes an angle of  $60^\circ$  with the direction of magnetic field of 0.1 tesla, then the magnetic flux associated with the coil will be -  
 (1) 0.4 weber                      (2) 0.346 weber                      (3) 0.2 weber                      (4) 0.02 weber
- Q.2** A loop of wire is placed in a uniform magnetic field. For what orientation of the loop, magnetic flux is maximum? For what orientation of magnetic flux is zero?

**3. Faraday's laws of Electromagnetic Induction :**

- (i) When magnetic flux passing through a loop changes with time or magnetic field lines are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf.

If the circuit is closed then the current flown will be called induced current.

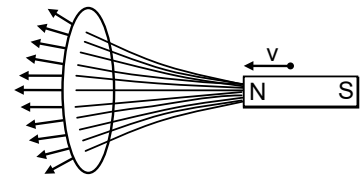
- (ii) The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic field lines are cut by a wire

$$E = -\frac{d\phi}{dt}$$

(-) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.

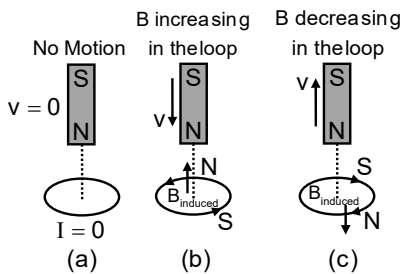
#### 4. Lenz's Law (Based on Energy Conservation)

- According to this law, emf will be induced in such a way that it will oppose the cause which has produced it. Figure shows a magnet approaching a ring with its north pole towards the ring. If the magnet is given some initial velocity towards the coil and is released, it will slow down.

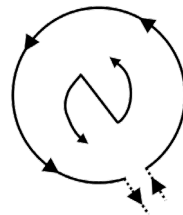


The current induced in the coil will produce heat. From the energy conservation, if heat is produced there must be an equal decrease of energy in some other form, here it is the kinetic energy of the moving magnet. Thus the magnet must slow down. So we can justify that the

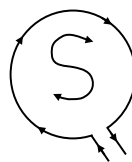
**Lenz's law is conservation of energy principle.**



- If current flowing in a coil appears anti-clockwise, then that plane of coil will behave like a N-pole.

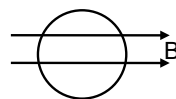


- If current flowing in the coil appears clock-wise, then that plane of coil will behave like a S-pole.



#### Example 3:

A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



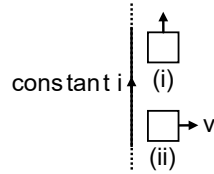
#### Solution:

$\phi = 0$  (always) since area is perpendicular to magnetic field.

$\therefore \text{emf} = 0$

**Example 4:**

Figure shows a long current carrying wire and two rectangular loops moving with velocity  $v$ . Find the direction of current in each loop.

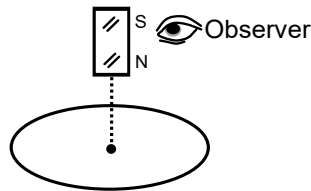
**Solution:**

In loop (i) no emf will be induced because there is no flux change.

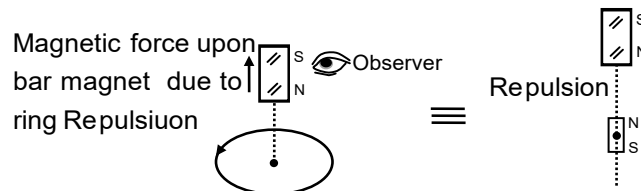
In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

**Example 5:**

A bar magnet is dropped through a horizontal aluminium ring along the axis of the ring. What will be direction of induced current in the loop for the observer shown? What will be the direction of magnetic force experienced by the bar magnet?

**Solution:**

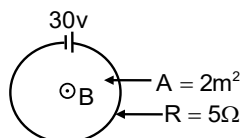
As per Lenz's law, induced current in the loop will be such that it will oppose the cause of change in flux (that is motion of the bar magnet towards itself). Thus, current flows in the loop so that it is equivalent to a bar magnet with its North facing the approaching North pole of the magnet, thereby repelling it. Thus, current for the observer will be anticlockwise.



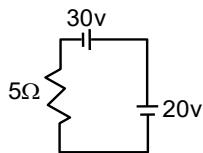
Thus, acceleration of the bar magnet  $a < g$ .

**Example 6:**

Figure shows a coil placed in a magnetic field decreasing at a rate of  $10\text{ T/s}$ . There is also a source of emf  $30\text{ V}$  in the coil. Find the magnitude and direction of the current in the coil.



**Solution:**



$$e = - \frac{AdB}{dt} = 2 \times 10 = +20$$

Induced emf = 20V, equivalent emf = 10 V,  $R = 5\Omega$

$$\text{current} = \frac{10}{5} = 2A ; i = 2A \text{ clockwise}$$

**Example 7:**

A current  $I = 1.5 A$  is flowing through a long solenoid of diameter 3.2 cm, having 220 turns per cm. At its centre, a closely packed coil of 130 turns and diameter 2.1 cm is placed such that the coil is coaxial with the long solenoid. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of emf induced in the coil while the current in the solenoid is changing?

**Solution:**

Initially magnetic flux passing through the coil (one turn)

$$\begin{aligned} \phi_1 &= \vec{B} \cdot \vec{A} = BA \cos 0^\circ = \mu_0 n I \times \frac{\pi d^2}{4} \\ &= 4\pi \times 10^{-7} \times 1.5 \times 22000 \times 3.464 \times 10^{-4} \\ &= 1.44 \times 10^{-5} \text{ Wb} \end{aligned}$$

Finally the flux becomes zero because the current reduces to zero.

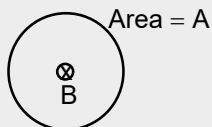
$$\text{Thus, } e = \frac{|\Delta\phi|}{\Delta t} = \frac{1.44 \times 10^{-5}}{25 \times 10^{-3}} = 5.76 \times 10^{-4} \text{ V}$$

$$\text{The total emf} = N \times e = 130 \times 5.76 \times 10^{-4} = 75 \text{ mV}$$

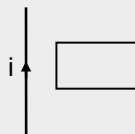
## Concept Builder-2



- Q.1** Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



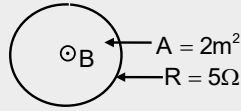
- Q.2** Figure shows a conducting loop placed near a long, straight wire carrying a current  $i$  as shown. If the current increases continuously, find the direction of the induced current in the loop.



- Q.3** When a small magnet is moved toward a solenoid, an emf is induced in the coil. Does the induced current in the solenoid depend on the speed of the magnet? Does the induced charge depend on the speed of the magnet?



- Q.4** Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil. The magnetic field is decreasing at a rate of 10T/s. Find out current in magnitude and direction.



## 5. Induced Emf, Current and Charge in a Circuit

- If e.m.f. induced in a circuit is  $E$  and rate of change of magnetic flux is  $d\phi/dt$ , then from Faraday's and Lenz's law

$$E \propto - \left( \frac{d\phi}{dt} \right) \quad \text{or} \quad E = - K \left( \frac{d\phi}{dt} \right)$$

where  $K$  is constant, equal to one.

$$\text{Thus } E = - \left( \frac{d\phi}{dt} \right)$$

- If there are  $N$  turns in the coil, then induced e.m.f. will be

$$E = - N \left( \frac{d\phi}{dt} \right)$$

- If the magnetic flux linked with the circuit changes from  $\phi_1$  to  $\phi_2$ , in time  $t$ , then induced e.m.f. will be

$$E = - N \left( \frac{d\phi}{dt} \right) = - N \left( \frac{\phi_2 - \phi_1}{t} \right)$$

- If the resistance of the circuit is  $R$ , then the current induced in the circuit will be

$$I = \frac{E}{R} = - \frac{N(\phi_2 - \phi_1)}{tR} \text{ ampere}$$

$$= - \frac{N}{R} \left( \frac{d\phi}{dt} \right) \text{ ampere}$$

- Induced current depends upon

(a) the resistance of the circuit  $I \propto \frac{1}{R}$

(b) the rate of change of magnetic flux  $I \propto \left( \frac{d\phi}{dt} \right)$

(c) the number of turns ( $N$ )  $I \propto N$

- If  $R = \infty$ , i.e., the circuit is open, then the current will not flow and if the circuit is closed, then current will flow in the circuit.

- If charge  $dq$  flows in the circuit in time  $dt$ , then the induced current will be

$$I = \left( \frac{dq}{dt} \right) \quad \text{or} \quad dq = I dt$$

$$\text{but } I = \frac{1}{R} \left( \frac{d\phi}{dt} \right) \therefore dq = \frac{1}{R} \left( \frac{d\phi}{dt} \right) dt = \frac{1}{R} d\phi$$

$$\text{or } q = \int \frac{d\phi}{R} = \frac{\phi_2 - \phi_1}{R}$$

If N is the number of turns, then

$$dq = \frac{Nd\phi}{R} \Rightarrow q = \frac{N(\phi_2 - \phi_1)}{R}$$

- Charge flowing due to induction does not depend upon the time but depends upon the total change in the magnetic flux. It does not depend upon the rate or time interval of the change in magnetic flux. Whether the change in magnetic flux be rapid or slow, the charge induced in the circuit will remain same.

Thus  $q \propto d\phi$  or  $q \propto (\phi_2 - \phi_1)$

- Induced charge depends upon the resistance of the circuit, i.e.,  $q \propto 1/R$   
If  $R = \infty$  or circuit is open,  $q = 0$  that is charge will not flow in the circuit.  
If  $R = 0$  or circuit is closed, then  $q \neq 0$ , that is, induced charge will flow in the circuit
- The e.m.f induced in the circuit does not depend upon the resistance of the circuit.
- The e.m.f induced in the circuit depends upon the following factors -
  - (a) Number of turns (N) in the coil
  - (b) Rate of change of magnetic flux
  - (c) Relative motion between the magnet and the coil
  - (d) Cross-sectional area of the coil
  - (e) Magnetic permeability of the magnetic substance or material placed inside the coil

#### Example 8:

Consider a coil (of area A, resistance R and number of turns N) held perpendicular to a uniform magnetic field of strength B. The coil is now turned through  $180^\circ$  in the time  $\Delta t$ . What is

- (i) Average induced emf
- (ii) Average induced current
- (iii) Total charge that flows through a given cross-section of the coil ?

#### Solution:

When plane of coil is perpendicular to the magnetic field,  $\theta = 0^\circ$  and after it is rotated through  $180^\circ$ ,  $\theta = 180^\circ$

$$\Rightarrow \text{Initial flux} = NBA \cos 0^\circ = NBA$$

$$\text{and Final flux} = NBA \cos 180^\circ = -NBA$$

$$\Rightarrow \text{Change in flux} = |\Delta\phi| = NBA - (-NBA) = 2NBA$$

$$(i) \varepsilon = \text{Average induced emf} = \frac{|\Delta\phi|}{\Delta t} = \frac{2NBA}{\Delta t}$$

$$(ii) \text{Average current} = \frac{\varepsilon}{R} = \frac{2NBA}{R\Delta t}$$

$$(iii) \text{Average current} = \frac{\Delta Q}{\Delta t} = \frac{2NBA}{R\Delta t} \quad \Rightarrow \Delta Q = \frac{2NBA}{R}$$

**Example 9:**

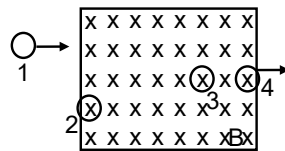
A 25-turn circular coil of wire has diameter 1m. It is placed with its axis along the direction of the earth's magnetic field of 40G, and then in 0.2s it is flipped 180°. An average emf of what magnitude is generated in that coil ?

**Solution:**

$$\begin{aligned}
 e &= -N \frac{BA \cos \theta_2 - BA \cos \theta_1}{\Delta t} \\
 &= NB\pi r^2 \left( \frac{\cos \theta_1 - \cos \theta_2}{\Delta t} \right) \\
 &= 25 \times 40 \times 10^{-4} \times \pi \times (0.5)^2 \times \frac{\cos 0^\circ - \cos 180^\circ}{0.2} \\
 &= 0.75 \text{ V}
 \end{aligned}$$

**Example 10:**

A conducting loop is moving from left to right through a region of uniform magnetic (B) field. Its four positions are shown below. Show the direction of induced current in all four positions.

**Solution:**

In situation 1 and 3 flux is constant and not changing with time, so there will not be any current induced. In situation 2 the loop is gradually getting into the magnetic field so, overlapped area is increasing. So, flux is increasing & hence the induced current will have the tendency to create B field in opposite direction i.e., in outward direction. Hence, anticlockwise current will be induced in the loop in situation 2. In situation 4, the loop is gradually getting out of the field, flux is decreasing, so induced current will support the B field. Current induced will be clockwise.

**Example 11:**

There exists a uniform magnetic field in a region. A circular conducting loop of radius  $r$  and resistance  $R$  is placed with its plane in  $x$ - $y$  plane. Determine the current through the loop and sense of the current.

**Solution:**

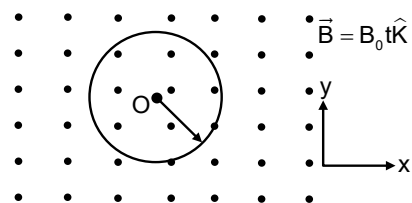
The flux linked with the loop is

$$\phi = \vec{B} \cdot \vec{A} = (B_0 t) (\pi r^2) \cos 0^\circ$$

$$\phi = B_0 \pi r^2 t$$

$$\frac{d\phi}{dt} = B_0 \pi r^2$$

$$\varepsilon = \frac{-d\phi}{dt} = -B_0 \pi r^2 \Rightarrow i = \frac{|\varepsilon|}{R} = \frac{B_0 \pi r^2}{R}$$



By Lenz's law,  $i$  should be clockwise (as seen by an observer standing on  $z$ -axis) so that it can oppose the increase in magnetic field.

## 6. Induced Electric Field

A time varying magnetic field produces induced electric field ( $E_i$ ) which can be calculated from

$$e = \oint \vec{E}_i \cdot d\vec{l} = -\frac{d\phi}{dt}$$

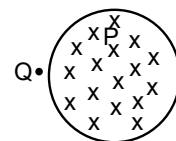
- Induced electric field is non-conservative and non-electrostatic in nature. Its field lines are concentric circular closed curves. Lines integral of electric field in a closed loop is not zero. Also, potential cannot be defined in such a field.
- The changing magnetic field does not need to be in existence at the location of the induced electric field.

### Key Points

- The electric field produced by stationary charges is called electrostatic field and for such a field  $\oint \vec{E} \cdot d\vec{l} = 0$  i.e., electrostatic field is conservative.
- In case of electromagnetic induction, line integral of induced emf  $\vec{E}$  around a closed path is not zero i.e. induced electric field is non-conservative. In such a field, work done in moving a charge round a close path is not zero.
- Just as a changing magnetic field produces an electric field. Similarly, a changing electric field also produces a magnetic field.

### Example 12:

A uniform but increasing with time magnetic field  $B(t)$  exists in a circular region of radius 'a' and is directed into the plane of the paper as shown. Find the magnitude of the induced electric field ( $E_i$ ) at points P and Q as shown.



### Solution:

Due to changing magnetic field, concentric circular field lines of induced electric field are set up everywhere outside as well as inside the cylindrical region.

Consider a point P at a distance  $r (< a)$  as shown.

Induced electric field at point P can be found from

$$\oint \vec{E}_i \cdot d\vec{l} = \frac{d\phi}{dt} = A \frac{dB}{dt}$$

$$E(2\pi r) = \pi r^2 \frac{dB}{dt} \text{ (due to symmetry)}$$

$$\text{or } E = \frac{r}{2} \frac{dB}{dt}$$

i.e., inside the cylindrical region  $E_i \propto r$

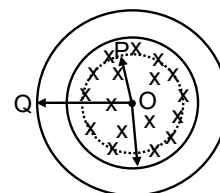
For a point Q at a distance  $r (> a)$ , we get

$$\oint \vec{E}_i \cdot d\vec{l} = \frac{d\phi}{dt} = A \frac{dB}{dt}$$

$$E(2\pi r) = \pi a^2 \frac{dB}{dt} \text{ (due to symmetry)}$$

$$\text{or } E = \frac{a^2}{2r} \times \frac{dB}{dt}$$

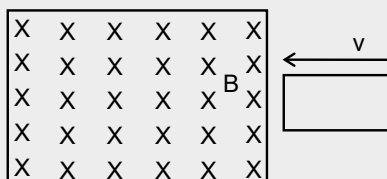
i.e., outside the cylindrical region  $E_i \propto \frac{1}{r}$



### Concept Builder-3



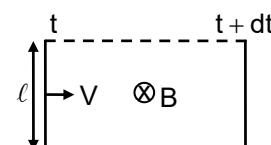
- Q.1** A coil consists of 200 turns of wire. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.8 s, what is the magnitude of the induced emf in the coil while the field is changing ?
- Q.2** A flat loop of wire consisting of a single turn of cross-sectional area  $8 \text{ cm}^2$  is perpendicular to a magnetic field that increases uniformly in magnitude from 0.5 T to 2.5 T in 1s. What is the resulting induced current the loop has a resistance of  $2\Omega$  ?
- Q.3** A 50-turn rectangular coil of dimensions  $5\text{cm} \times 10\text{cm}$  is allowed to fall from a position where  $B = 0$  to a new position where  $B = 0.5 \text{ T}$  and the magnetic field is directed perpendicular to the plane of the coil. Calculate the magnitude of the average emf that is induced in coil if the displacement occurs in 0.25 s.
- Q.4** A small rectangular loop is moving towards left with constant velocity through a uniform B field, as shown. Counting of time  $t$  begins the moment the loop starts entering the field. Plot the variation of flux through the loop with respect to time. Also plot the variation of induced emf w.r.t time  $t$ .



- Q.5** A conducting circular loop is placed in a uniform magnetic field  $B = 0.020 \text{ T}$  with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of  $1.0 \text{ mm s}^{-1}$ . Find the induced emf in the loop at an instant when the radius is 2 cm.
- Q.6** A uniform magnetic field  $B$  exists in a direction perpendicular to the plane of a square frame made of copper wire. The wire has a diameter of 2 mm and a total length of 40 cm. The magnetic field changes with time at a steady rate  $dB/dt = 0.02 \text{ Ts}^{-1}$ . Find the current induced in the frame. Resistivity of copper  $= 1.7 \times 10^{-8} \Omega\text{m}$ .

### 7. Motional Emf

We can find emf induced in a moving rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length  $\ell$  moves with velocity  $v$  in a magnetic field  $B$ , as shown, it will sweep area per unit time equal to  $\ell v$  and hence it will cut  $B \ell v$  lines per unit time.



Hence emf induced between the ends of the rod  $= Bv\ell$

Also,  $\text{emf} = \frac{d\phi}{dt}$ . Here  $\phi$  denotes flux passing through the area, swept by the rod. The rod sweeps an area equal to  $\ell v dt$  in time interval  $dt$ . Flux through this area  $= B\ell v dt$

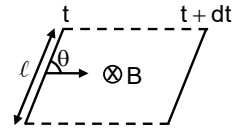
$$\text{Thus } \frac{d\phi}{dt} = \frac{B\ell v dt}{dt} = Bv\ell$$

If the rod is moving as shown in the following figure, it will sweep area per unit time =  $v \ell \sin\theta$  and hence it will cut  $B v \ell \sin\theta$  lines per unit time.

Thus  $\text{emf} = Bv\ell \sin\theta$

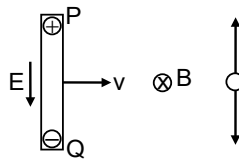
In vector form,

$$E = \vec{\ell} \cdot (\vec{v} \times \vec{B})$$

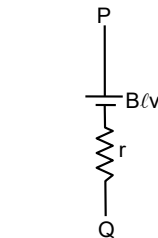


## 7.1 Explanation of Emf Induced in Rod on the Basis of Magnetic Force

If a rod is moving with velocity  $v$  in a magnetic field  $B$ , as shown, the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be

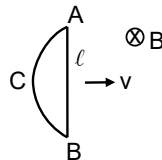


a deficiency of free electrons at upper end and hence a surplus of positive charge at the upper end. These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time, enough charges will accumulate at the ends so that the two forces  $qE$  and  $qvB$  will balance each other. Thus  $E = v B$ .



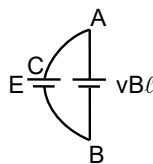
$V_P - V_Q = v B \ell$  The moving rod is equivalent to the following diagram, electrically.

Figure shows a closed coil ABCA moving in a uniform magnetic field  $B$  with a velocity  $v$ . The flux passing through the coil is a constant and therefore the induced emf is zero.



Now consider rod AB, which is a part of the coil. Emf induced in the rod =  $B \ell v$

Suppose the emf induced in part ACB is  $E$ , as shown.



Since the emf in the coil is zero,

$\text{Emf (in ACB)} + \text{Emf (in BA)} = 0$

$$\text{or } -E + vB\ell = 0 \quad \text{or} \quad E = vB\ell$$

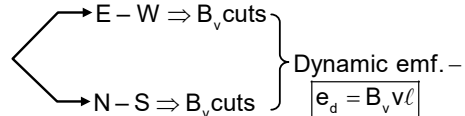
Thus, emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also, the equivalent emf between A and B is  $B \ell v$  (here the two emf's are in parallel).

**Note :** If the velocity of rod is taken as the reference direction, then induced emf  $e = B_{\perp} l_{\perp} v$ , where  $B_{\perp}$  is the component of magnetic field perpendicular to the velocity and  $l_{\perp}$  is the component of the length perpendicular to the velocity.

### Moving Conducting Rod in Earth's Magnetic Field

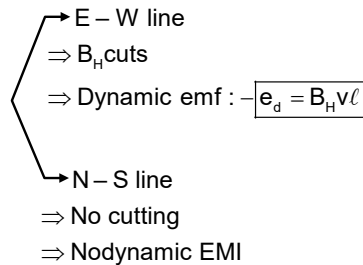
(a) Horizontal rod moving in horizontal plane.

If its ends in



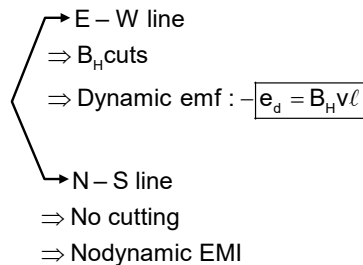
(b) Vertical rod moving in horizontal plane.

If it moves on



(c) Horizontal rod allow to fall under gravity

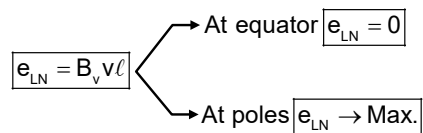
If its ends in



### Applications

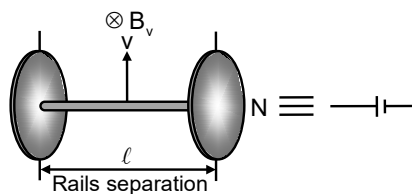
#### (i) Moving Train

Axle of Train cuts  $B_v$ , and Induced emf produced across axle of train is



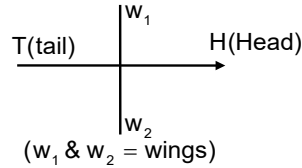
Here  $B_v = B \sin \theta$ ,  $\theta$  angle of dip at place

$v \rightarrow$  Always in m/sec.



## (ii) Moving Aeroplane

Motion of aeroplane can be deal as, motion of two metal rods (H-T) and ( $w_1-w_2$ ), which are perpendicular to each other. For (H-T) conductor  $\vec{\ell} \parallel \vec{v}_{cm}$ , so (H-T) conductor never do cutting process hence no induced emf across (H-T) of aeroplane for its any sort of motion, only ( $w_1-w_2$ ) conductor can do flux cutting process.



### (a) When Aeroplane Flying at a Certain Height Parallel to Earth Surface

Motion in horizontal plane

If wings ( $w_1 - w_2$ )

- (E - W) dir<sup>n</sup>  $\Rightarrow B_v$  cuts
- (N - S) dir<sup>n</sup>  $\Rightarrow B_v$  cuts

Induced emf across wings of aeroplane given as (both cases)

$$e_{w_1 w_2} = B_v \ell_{w_1 w_2} v$$

here  $B_v = B \sin \theta$  [ $\theta$  angle of dip.]

### (b) When Aeroplane Dives Vertically

Motion in vertical plane

If wings ( $w_1 - w_2$ )

- (E - W) dir<sup>n</sup>  $\Rightarrow B_H$  cuts
- (N - S) direction  $\Rightarrow$  No cutting  $\Rightarrow$  No dynamic emi

Induced emf across wings of aeroplane given as (only in one case)

$$e_{w_1 w_2} = B_H \ell_{w_1 w_2} v$$

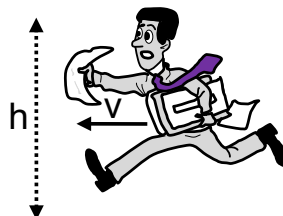
here  $B_H = B \cos \theta$  [ $\theta$  angle of dip.]

### (iii) Human body

A human body of height 'h' moves with constant velocity v then dynamic emf between his head and feet, if it moves along :

→ E - W line  $\Rightarrow B_H$  cuts Dynamic emf  $e_d = B_H v H$

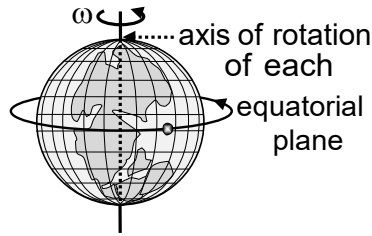
→ N - S line  $\Rightarrow$  No cutting  $\Rightarrow$  No dyn.emi





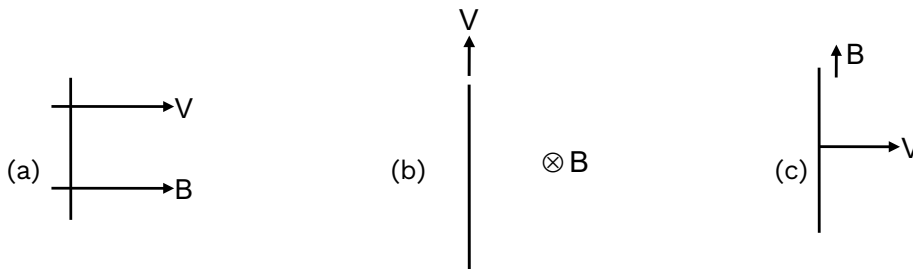
#### (iv) Motion of an Artificial satellite

If a geo-stationary satellite revolves around the earth in equatorial plane  $\Rightarrow$  No cutting  $\Rightarrow$  No dynamic emf



#### Example 13:

Find the emf induced in the rod in the following cases. The figures are self explanatory.



#### Solution:

(a) Here  $\vec{v} \parallel \vec{B}$  so  $\vec{v} \times \vec{B} = 0$

$$\text{emf} = \vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$$

(b) Here  $\vec{v} \parallel \vec{\ell}$

$$\text{so emf} = \vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$$

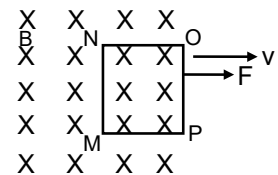
(c) Here  $\vec{B} \parallel \vec{\ell}$

$$\text{so emf} = \vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$$

#### Example 14:

Figure shows a rectangular loop MNOP being pulled out of a magnetic field with a uniform velocity  $v$  by applying an external force  $F$ . The length MN is equal to  $\ell$  and the total resistance of the loop is  $R$ . Find the :

- Current in the loop
- Magnetic force on the loop
- External force  $F$  needed to maintain the velocity
- Power delivered by external force
- Thermal power developed by the loop



#### Solution:

(a) The emf induced in the loop is due to the motion of wire MN. The emf is  $e = vB\ell$  with positive end at N and negative end at M. The current is :

$$i = \frac{e}{R} = \frac{vB\ell}{R}$$

(b) The magnetic force on the wire MN is

$$F_m = i\ell B = \frac{B^2 \ell^2 v}{R}$$

The magnetic forces on the wires NO and PM are equal and opposite and hence gets cancelled.

$$F = \frac{B^2 \ell^2 v}{R}$$

(c) To move the loop at a constant velocity the resultant forces on it is zero. Hence the external force must be equal to magnetic force in magnitude and opposite in direction.

(d) The power delivered by external force is

$$P = Fv = \frac{B^2 \ell^2 v^2}{R}$$

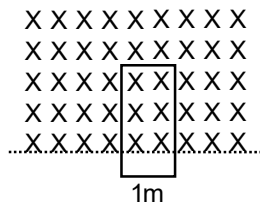
(e) The thermal power developed is

$$P = i^2 R = \frac{B^2 \ell^2 v^2}{R}$$

Thus power delivered by external force is equal to the thermal power developed in the loop.

### Example 15:

Figure shows a horizontal magnetic field 2T which is uniform above the dotted line and is zero below it. A long, rectangular conducting loop of width 1m, mass 5kg and resistance  $2\Omega$  is placed as shown. At what velocity should it be moving downwards so that it continues to fall without any acceleration as long as it is partly in the magnetic field.



### Solution:

Let the velocity with which the loop moves so that it falls without acceleration be  $v$ . Due to the motion of loop an induced emf is generated in the upper arm of the loop and a current will flow in the loop. Due to this current the loop will experience a magnetic force in the upward direction. To fall with zero acceleration this magnetic force will balance weight of the loop. Hence

$$F_m = mg$$

$$I\ell B = mg$$

$$\text{Since } I = \frac{e}{R} = \frac{B\ell v}{R}, \text{ hence } \frac{B^2 \ell^2 v}{R} = mg$$

$$\text{or } v = \frac{mgR}{B^2 \ell^2} = \frac{5 \times 10 \times 2}{2^2 \times 1^2} = 25 \text{ m/s}$$

## 7.2 Motion of Irregular Shaped Body in a Magnetic Field

When a conductor of irregular shape moves in a magnetic field with velocity  $v$ , the induced emf is generated in the conductor according to the same concept discussed above.

This induced emf can be calculated as follows:

Consider a small length element as shown. At equilibrium the electric field in the element will be :

$$\vec{E} = -(\vec{v} \times \vec{B})$$

The induced emf in the element will be

$$de = -\vec{E} \cdot d\vec{\ell}$$

The net emf will be

$$e = -\int \vec{E} \cdot d\vec{\ell} = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

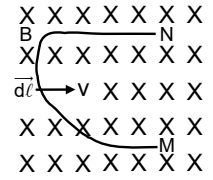
If  $\vec{v}$  and  $\vec{B}$  are constant, then

$$e = -(\vec{v} \times \vec{B}) \cdot \int d\vec{\ell} = -(\vec{v} \times \vec{B}) \cdot \vec{\ell}$$

where  $\vec{\ell}$  is the displacement vector between the ends of conductor. Since mutually perpendicular components  $\vec{v}$ ,  $\vec{B}$  of  $\vec{\ell}$  and are responsible for induced emf, hence the net emf will be :

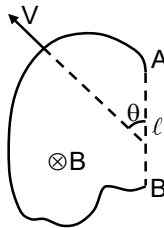
$$e = v_{\perp} B_{\perp} \ell$$

here  $v_{\perp}$  and  $B_{\perp}$  are components  $\vec{v}$  and  $\vec{B}$  of perpendicular to  $\ell$  and are mutually perpendicular.



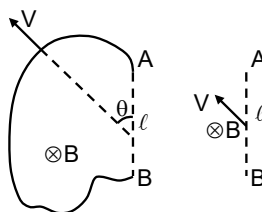
### Example 16:

Figure shows an irregular shaped wire AB moving with velocity  $v$ , as shown. Find the emf induced in the wire.



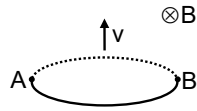
### Solution:

The same emf will be induced in the straight imaginary wire joining A and B, which is  $Bv\ell \sin \theta$ .



**Example 17:**

A circular coil of radius  $R$  is moving in a magnetic field  $\mathbf{B}$  with a velocity  $\mathbf{v}$  as shown in the figure.



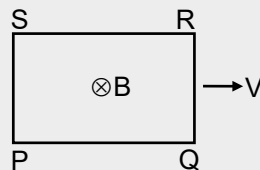
Find the emf across the diametrically opposite points A and B.

**Solution:**

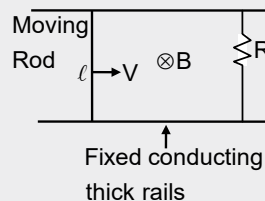
$$\begin{aligned} \text{emf} &= Bv\ell_{\text{effective}} \\ &= 2RvB \end{aligned}$$

**Concept Builder-4**

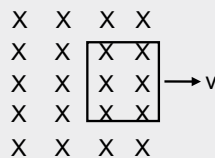
- Q.1** Figure shows a rectangular loop moving in a uniform magnetic field. Show the electrical equivalent of each branch.



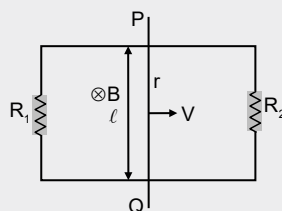
- Q.2** Figure shows a rod of length  $\ell$  and resistance  $r$  moving on two rails shorted by a resistance  $R$ . A uniform magnetic field  $B$  is present normal to the plane of rod and rails. Show the electrical equivalent of each branch.



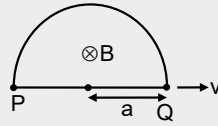
- Q.3** Figure shows a square loop having 200 turns, an area of  $4.9 \times 10^{-3} \text{ m}^2$  and a resistance of  $200\Omega$ . The magnetic field has a magnitude  $B = 0.5\text{T}$ . Find current in the loop where it is pulled out of the field with  $1\text{m/s}$ .



- Q.4** A rod PQ of mass  $m$  and resistance  $r$  is moving on two fixed, resistance-less, smooth conducting rails (closed on both sides by resistances  $R_1$  and  $R_2$ ). Find the current in the rod at the instant its velocity is  $v$ .



- Q.5** Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown. Also draw the electrical equivalent circuit of each branch.

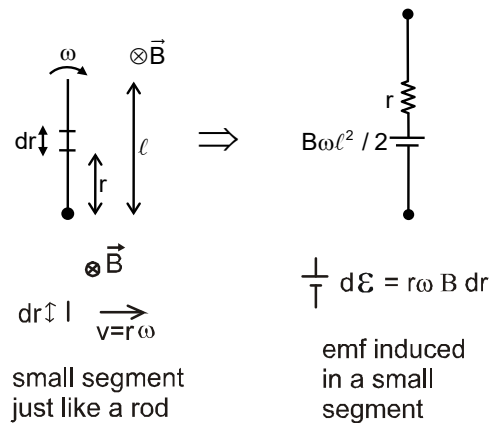


- Q.6** Following situation of the previous example, the magnitude of the force required to move the conducting rod at constant speed 5 cm/s at the same instant  $t = 2\text{ s}$ , is equal to  
 (1) 0.16 N                      (2) 0.12 N                      (3) 0.08 N                      (4) 0.06 N

## 8. Induced Emf due to Rotation

### 8.1 Rotation of Rod

Consider a conducting rod of length  $\ell$  rotating in a uniform magnetic field.



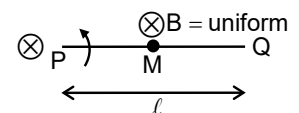
Emf induced in a small segment of length  $dr$ , of the rod  $= v B dr = r\omega B dr$

$\therefore$  emf induced in the rod

$$= \omega B \int_0^\ell r dr = \frac{1}{2} B \omega \ell^2$$

### Example 18:

A rod PQ of length  $\ell$  is rotating about one end P in a uniform magnetic field  $B$  which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V .



### Solution:

$$E_{MQ} + E_{PM} = E_{PQ}$$

$$E_{PQ} \rightarrow \frac{B\omega\ell^2}{2} = 100 ;$$

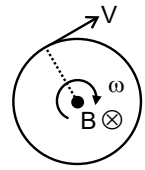
$$\text{hence, } E_{MQ} + \frac{B\omega\left(\frac{\ell}{2}\right)^2}{2} = \frac{B\omega\ell^2}{2}$$

$$\Rightarrow E_{MQ} = \frac{3}{8} B\omega\ell^2 = \frac{3}{4} \times 100 \text{ V} = 75 \text{ V}$$

## 8.2 Rotation of Ring

Consider a ring rotates with angular velocity  $\omega$  about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field  $B$  exists parallel to the axis.

Then flux passing through the ring  $\phi = B.A$  is a constant here, therefore emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.



## 8.3 Rotation of Disc

Consider a disc of radius  $r$  rotating in a magnetic field  $B$ .

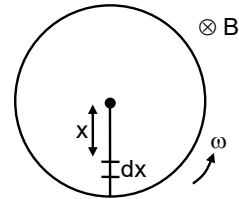
Consider an element  $dx$  at a distance  $x$  from the centre. This element is moving with speed  $v = \omega x$ .

$\therefore$  Induced emf across  $dx$

$$= B(dx) v = Bdx\omega x = B\omega x dx$$

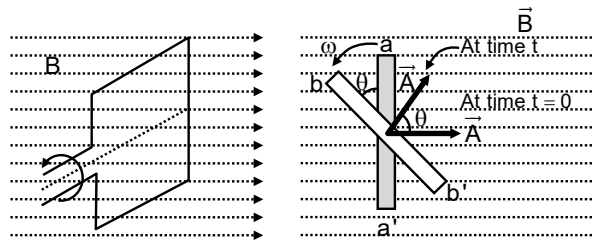
$\therefore$  emf between the centre and the edge of disc.

$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$



## 8.4 Rotation of a rectangular coil

- If the figure a conducting rectangular coil of area  $A$  and turns  $N$  is shown. It is rotated in a uniform magnetic field  $B$  about a horizontal axis perpendicular to the field with an angular velocity  $\omega$ . The magnetic flux linked with the coil is continuously changing due to rotation.



$\theta$  is the angle between the perpendicular to the plane of the coil and the direction of magnetic field.

- The magnetic flux passing through the rectangular coil depends upon the orientation of the plane of the coil about its axis.
- Magnetic flux passing through the coil

$$\phi = \vec{B} \cdot \vec{A} = BA \cos \theta = BA \cos \omega t$$

If there are  $N$  turns in the coil, then the flux linked with the coil  $\phi = BAN \cos \omega t$

- Since  $\phi$  depends upon the time  $t$ , the rate of change of magnetic flux

$$\frac{d\phi}{dt} = -BAN\omega \sin \omega t$$

- According to Faraday's law, the emf induced in the coil ;  $\varepsilon = -\frac{d\phi}{dt}$ ;

$$\text{or } \varepsilon = BAN\omega \sin \omega t$$

$BAN \omega$  is the maximum value of emf induced,

Thus writing

$$BAN\omega = \varepsilon_0$$

$$\therefore \varepsilon = \varepsilon_0 \sin \omega t$$

This equation represents the instantaneous value of emf induced at time t.

- If the total resistance of circuit along with the coil is R, then the induced current due to alternating voltage

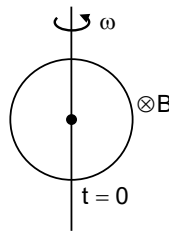
$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0}{R} \sin \omega t \quad \text{or} \quad I = I_0 \sin \omega t$$

where  $I_0 = \frac{\varepsilon_0}{R}$  is the maximum value of current.

- The direction of induced emf in the coil changes during one cycle so it is called alternating emf and current induced due to it is called alternating current. This is the principle of **AC generator**.

### Example 19:

A ring rotates with angular velocity  $\omega$  about an axis passing through its centre in the plane of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.

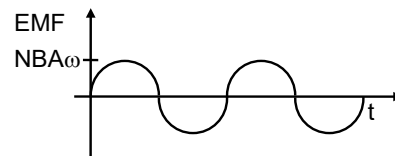


### Solution:

At any time t,  $\phi = BA \cos \theta = BA \cos \omega t$

Now induced emf in the loop;

$$e = \frac{-d\phi}{dt} = BA \omega \sin \omega t$$



If there are N turns,  $\text{emf} = BA\omega N \sin \omega t$ , where  $BA \omega N$  is the amplitude of the emf  $e = e_m \sin \omega t$

$$i = \frac{e}{R} = \frac{e_m}{R} \sin \omega t = i_m \sin \omega t ; \quad i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

## 9. Self Induction

Self induction is induction of emf in a coil due to its own current change. Total flux  $N\phi$  passing through a coil due to its own current is proportional to the current and is given as  $N\phi = Li$ .

Where L is called coefficient of self induction or inductance. The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by  $\Delta I$  in a time interval  $\Delta t$ , the average emf induced in the coil is given as

$$= -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous emf is given as

$$= -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$$

S.I Unit of inductance is wb/amp or Henry(H)

L - self inductance is +ve quantity.

L depends on :

- (1) Geometry of loop
- (2) Medium in which it is kept.

L does not depend upon current.

L is a scalar quantity.

### 9.1 Self Inductance of Solenoid

Let the volume of the solenoid be V, the number of turns per unit length be n.

Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as  $B = \mu_0 nI$ .

The magnetic flux through one turn of solenoid  $\phi = \mu_0 nIA$

The total magnetic flux through the solenoid

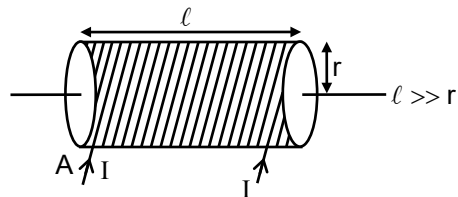
$$= N\phi = N\mu_0 nIA = \mu_0 n^2 IA\ell$$

$$\text{Since, } L = \frac{\phi}{I}$$

$$L = \frac{\phi}{I} = \mu_0 n^2 \pi r^2 \ell = \mu_0 n^2 V = \frac{\mu_0 N^2 A}{\ell}$$

$$\text{Inductance per unit volume} = \mu_0 n^2$$

Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant. Current can not change suddenly in the inductor.



#### Example 20:

- (a) Calculate the inductance of an air core solenoid containing 300 turns, if the length of the solenoid is 25.0 cm and its cross-sectional area is  $4.00 \text{ cm}^2$ .
- (b) Calculate the self induced emf in the solenoid, if the current through it is decreasing at the rate of 50.0 A/s.

#### Solution:

- (a) The inductance of a solenoid is given by,  
Substitution the values, we have

$$L = \frac{(4\pi \times 10^{-7})(300)^2(4.00 \times 10^{-4})}{(25.0 \times 10^{-2})} \text{ H}$$

$$= 1.81 \times 10^{-4} \text{ H}$$



(b)  $e = m$

Here,  $\frac{di}{dt} = -50.0 \text{ A/s}$

$\therefore e = -(1.81 \times 10^{-4}) (-50.0) = 9.05 \text{ mV}$

**Example 21:**

An average induced emf of 0.20 V appears in a coil when the current in it is changed from 5.0A in one direction to 5.0A in the opposite direction in 0.20s. Find the self-inductance of the coil.

**Solution:**

Average  $\frac{di}{dt} = \frac{(-5.0\text{A}) - (5.0\text{A})}{0.20\text{s}} = -50 \text{ A/s}$

Using  $e = -L \frac{di}{dt}$ ,

$0.2\text{V} = L(50 \text{ A/s})$  or,  $L = \frac{0.2\text{V}}{50 \text{ A/s}} = 4.0 \text{ mH}$

**Example 22:**

What will happen to the inductance of a solenoid (a) when the number of turns and the length are doubled keeping the area of cross-section same, (b) when the air inside the solenoid is replaced by iron of relative permeability  $\mu_r$  ?

**Solution:**

In case of a solenoid as

$B = \mu_0 nI, \phi = B(n/A) = \mu_0 n^2 / AI$

hence  $L = \frac{\phi}{I} = \mu_0 n^2 / A = \mu_0 \frac{N^2}{l} A$

$\left[ \text{as } n = \frac{N}{l} \right]$

So (a) when  $N$  and  $l$  are doubled,

$L' = \mu_0 \frac{(2N)^2}{2l} A = 2\mu_0 \frac{N^2}{l} A = 2L$

i.e., inductance of the solenoid will be doubled.

(b) When air is replaced by iron,  $\mu_0$  will change to  $\mu$ , so that

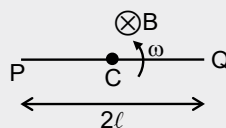
$L' = \mu n^2 / A$

and hence,  $\frac{L'}{L} = \frac{\mu}{\mu_0} = \mu_r$ , i.e.,  $L' = \mu_r L$

**Concept Builder-5**



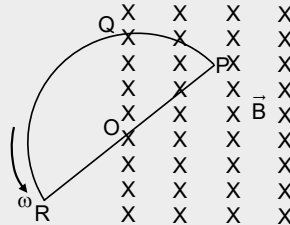
**Q.1** A rod PQ of length  $2\ell$  is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced emf between PQ and PC. Draw the circuit diagram of parts PC and CQ.



**Q.2** The phase difference between the emf induced in the coil rotating in a uniform magnetic field and the magnetic flux associated with it, is

- (1)  $\pi$  (2)  $\pi/2$  (3)  $\pi/3$  (4) zero

**Q.3** A semi-circle conducting loop of radius 'r' with centre at O, is made to rotate with a constant angular velocity  $\omega$ , about an axis passing through O and perpendicular to the plane of paper. A uniform magnetic field exists in a part of the region as shown. Find the induced current in the loop if the resistance is R.



**Q.4** When the current in a coil changes from 8A to 2A in  $3 \times 10^{-2}$ s, the emf induced in the coil is 2V. What is the self-inductance of the coil in mH ?

- (1) 2 mH (2) 5 mH (3) 10 mH (4) 20 mH

**Q.5** Calculate the inductance of a 25 cm long solenoid if it has 1000 turns and radius of its circular cross-section is 5 cm.

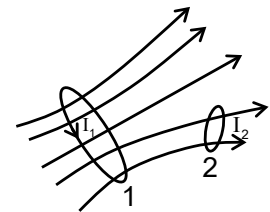
- (1) 0.01 H (2) 0.02 H (3) 0.03 H (4) 0.04 H

**Q.6** The self inductance L of a solenoid of length l and area of cross-section A with a fixed number of turns N increase as

- (1) l and A increase (2) l decreases and A increases  
(3) l increases and A decreases (4) both l and A decrease

## 10. Mutual Inductance

Consider two arbitrary conducting loops 1 and 2. Suppose that  $I_1$  is the instantaneous current flowing around loop 1. This current generates a magnetic field  $B_1$  which links the second circuit, giving rise to a magnetic flux  $\phi_2$  through that circuit. If the current  $I_1$  doubles, then the magnetic field  $B_1$  doubles in strength at all points in space, so the magnetic flux  $\phi_2$  through the second circuit also doubles. Furthermore, it is obvious that the flux through the second circuit is zero whenever the current flowing around the first circuit is zero. It follows that the flux  $\phi_2$  through the second circuit is *directly proportional* to the current  $I_1$  flowing around the first circuit.



Hence, we can write  $\phi_2 = M_{21}I_1$  where the constant of proportionality  $M_{21}$  is called the mutual inductance of circuit 2 with respect to circuit 1. Similarly, the flux  $\phi_1$  through the first circuit due to the instantaneous current  $I_2$  flowing around the second circuit is directly proportional to that current, so we can write  $\phi_1 = M_{12}I_2$  where  $M_{12}$  is the mutual inductance of circuit 1 with respect to circuit 2. It can be shown that  $M_{21} = M_{12}$ .

Note that M is a purely geometric quantity, depending only on the size, number of turns, relative position, and relative orientation of the two circuits. The S.I. unit of mutual inductance is called Henry (H). One henry is equivalent to a volt-second per ampere.

Suppose that the current flowing around circuit 1 changes by an amount  $\Delta I_1$  in a small time

interval  $\Delta t$ . The flux linking circuit 2 changes by an amount  $\Delta\phi_2 = M\Delta I_1$  in the same time interval.

According to Faraday's law, an emf  $\varepsilon_2 = -\frac{\Delta\phi_2}{\Delta t}$  is generated around the second circuit due to the changing magnetic flux linking that circuit. Since,  $\Delta\phi_2 = M\Delta I_1$ , this emf can also be written

$$\varepsilon_2 = -M \frac{\Delta I_1}{\Delta t}.$$

Thus, the emf generated around the second circuit due to the current flowing in the first circuit is directly proportional to the rate at which that current changes. Likewise, if the current  $I_2$  flowing in the second circuit changes by an amount  $\Delta I_2$  in a time interval  $\Delta t$  then the emf

$$\text{generated around the first circuit is } \varepsilon_1 = -M \frac{\Delta I_2}{\Delta t}.$$

- **Unit of M :** In S.I. system unit of mutual inductance is Henry.

$$M = \frac{E_B}{-(dI_A / dt)} = \frac{\phi_B}{I_A}$$

$$\therefore 1 \text{ Henry} = \frac{1 \text{ volt}}{1 \text{ ampere} / \text{s}} = \frac{1 \text{ weber}}{\text{ampere}}$$

$$= \frac{(\text{joule} / \text{coulombs})}{\text{ampere}} = \text{J/A}^2$$

- **Dimensions of M :**

$$M = \frac{\text{J}}{\text{A}^2} = \frac{\text{joule}}{\text{ampere}^2} = \frac{\text{newton} \times \text{metre}}{\text{ampere}^2}$$

$$= \frac{\text{kg} \times \text{metre} \times \text{sec}^{-2} \times \text{metre}}{\text{ampere}^2} = \text{ML}^2\text{T}^{-2}\text{A}^{-2}$$

- Mutual inductance between the coils depends upon the number of turns in the coils, area and the permeability of the core placed inside the coils. Larger is the magnitude of M, more is the emf induced in the secondary coil.
- Out of the two coils coupled magnetically one coil can be taken as primary and the other coil as secondary. Thus, mutual inductance

$$M_{AB} = M_{BA} = M$$

- Mutual inductance between two coaxial solenoids of length  $\ell$  and cross-sectional area A is

$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

where  $N_1$  and  $N_2$  are the number of turns in the two coils respectively.

### Example 23:

Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let  $\ell$  be the length of the core, A is the cross-sectional area of the core,  $N_1$  is the number of turns of first wire wound around the core, and  $N_2$  is the number of turns of second wire wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

### Solution:

If a current  $I_1$  flows around the first wire then a uniform axial magnetic field of strength  $B_1 =$

$\frac{\mu_0 N_1 I_1}{\ell}$  is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is  $B_1 A$ . Thus, the flux linking all  $N_2$  turns of the second wire is

$$\phi_2 = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = M I_1$$

$$\therefore M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

As described previously,  $M$  is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

#### Example 24:

Find the mutual inductance of two concentric coils of radii  $a_1$  and  $a_2$ ,  $a_1$  being very small. If the planes of coils are same.

#### Solution:

Let a current  $i$  flow in coil of radius  $a_2$ .

Since  $a_1$  is very small magnetic field 'B' inside coil 1 can be considered as same everywhere.

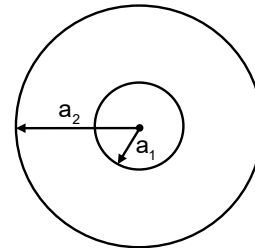
So Magnetic flux linked with first coil

$$= \frac{\mu_0 i}{2a_2} \pi a_1^2$$

Since mutual inductance,

$$M = \frac{\phi}{i} \quad M i = \frac{\mu_0 i}{2a_2} \pi a_1^2$$

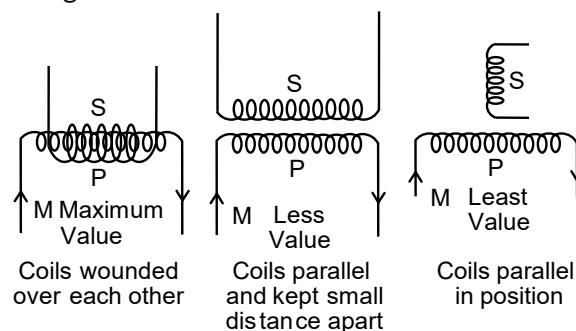
$$\text{or } M = \frac{\mu_0 \pi a_1^2}{2a_2}$$



### 10.1 Coupling/Winding of Coils

- If two coils are wound one over the other, then mutual inductance will be maximum and it will be less in other arrangements.

$M$  and  $L$  have the following relation :



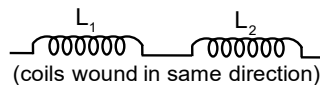
$$M \propto \sqrt{L_1 L_2} ; M = K \sqrt{L_1 L_2}$$

where  $K$  is a coupling constant of coils and its value varies from 0 to 1.

- (a) If  $K = 0$ , then there will be no coupling between the coils, that is magnetic flux produced by the primary coil is not linked with the secondary coil.

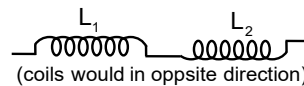
(b) If  $K = 1$ , then both coils are coupled together with maximum transfer to energy, that is, magnetic flux produced by the primary coil is totally linked with the secondary coil.

- If two coils of self inductances  $L_1$  and  $L_2$  are coupled in series such that their windings are in the same sense and mutual inductance between them is  $M$ , then the equivalent inductance will be



$$L = L_1 + L_2 + 2M$$

If two coils are coupled in series such that their windings are in opposite sense then equivalent inductance will be



$$L = L_1 + L_2 - 2M$$

### Example 25:

The coefficients of self induction of two coils are 0.01 H and 0.03 H respectively. If windings are in opposite sense then the resultant self induction will be, if  $M = 0.01$  H.

- (1) 2H                      (2) 0.02H                      (3) 0.02H                      (4) zero

### Solution:

$$\begin{aligned} L &= L_1 + L_2 - 2M \\ &= 0.01 + 0.03 - 2 \times 0.01 = 0.04 - 0.02 \\ &= 0.02 \text{ H} \end{aligned}$$

## Concept Builder-6



- Q.1** A coil of radius 1 cm and 100 turns is placed at the centre of a long solenoid of radius 5 cm and 8 turn/cm. The value of coefficient of mutual induction will be -  
 (1)  $3.15 \times 10^{-5}$  H                      (2)  $6 \times 10^{-5}$  H                      (3)  $9 \times 10^{-5}$  H                      (4) zero
- Q.2** A straight solenoid has 50 turns per cm in primary and 200 turns in the secondary. The area of cross- section of the solenoid is  $4 \text{ cm}^2$ . Calculate the mutual inductance.
- Q.3** A small square loop of wire of side  $\ell$  is placed inside a large square loop of wire of side  $L$  ( $\gg \ell$ ). The loops are coplanar and their centres coincide. What is the mutual inductance of the system?  
 (1)  $\frac{\sqrt{2} \mu_0 \ell^2}{\pi L}$                       (2)  $\frac{2\sqrt{2} \mu_0 \ell^2}{\pi L}$                       (3)  $\frac{\mu_0 \ell^2}{\sqrt{2} \pi L}$                       (4)  $\frac{3\mu_0 \ell^2}{\sqrt{2} \pi L}$
- Q.4** The equivalent inductance of two inductors is 2.4 H when connected in parallel and 10 H when connected in series. What is the value of inductances of the individual inductors?

## 11. Inductors & Their Behaviour

### 11.1 Inductor

A circuit or part of a circuit, that is designed to have a particular inductance is called an **inductor**. The usual symbol for an inductor is,



Thus, an inductor is a circuit element which opposes the change in current in it. It may be a circular coil, solenoid. etc.

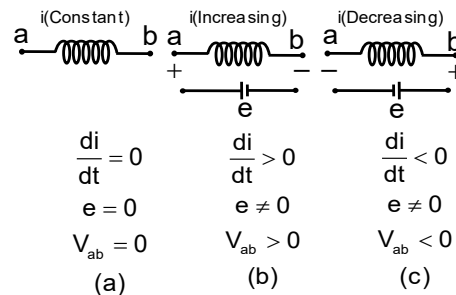
### 11.2 Significance of Self Inductances and Inductor

Like capacitors and resistor, inductors are among the circuit elements of modern electronics. Their purpose is to oppose any variations in the current through the circuit. In a DC circuit, an inductor helps to maintain a steady state current despite fluctuations in the applied emf. In an AC circuit, an inductor tends to suppress variations of the current that are more rapid than desired. An inductor plays a passive role in a circuit so far as current is constant. It becomes active when current changes in the circuit.

Every inductor has some self-inductance which depends on the size, shape and the number of turns, etc. For  $N$  turns close together, it is always proportional to  $N^2$ . It also depends on the magnetic properties of the material enclosed by the circuit. When the current passing through it is changed, an emf of magnitude  $L di/dt$  is induced across it.

### 11.3 Potential difference across on inductor

We can find the direction of self-induced emf across an inductor from Lenz's law.



The induced emf is produced whenever there is a change in the current in the inductor. This emf always acts to oppose this change. Figure shows three cases. Assume that the inductor has negligible resistance, so the PD,  $V_{ab} = V_a - V_b$ , between the inductor terminals a and b is equal in magnitude to the self-induced emf.

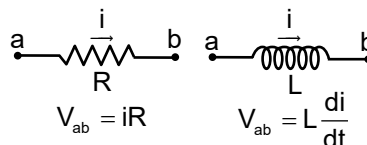
**Refer figure (a)** The current is constant and there is no self-induced emf. Hence,  $V_{ab} = 0$ .

**Refer figure (b)** The current is increasing, so is positive. The induced emf  $e$  must opposes the increasing current, so it must be in the sense from b to a, a becomes the higher potential terminal and  $V_{ab}$  is positive. The direction of the emf is analogous to a battery with a as its positive terminal.

**Refer figure (c)** The current is decreasing and is negative. The self-induced emf  $e$  opposes this decrease and  $V_{ab}$  is negative. This is analogous to a battery with b, as its positive terminal.

In each case, we can write the PD,  $V_{ab}$  as

The circuit behaviour of an inductor is quite different from that of a resistor. While a resistor opposes the current  $i$ , an inductor opposes the change ( $di/dt$ ) in the current.

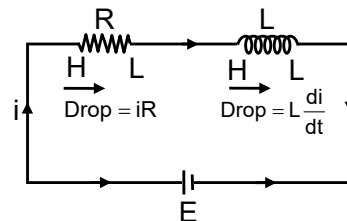


### 11.4 Kirchhoff's Second Law in an Inductor Circuit

In Kirchhoff's second law (Loop rule), when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to  $L \frac{di}{dt}$ , where,  $di/dt$  is to be substituted with sign.

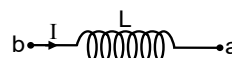
For example, in the loop shown in figure, Kirchhoff's second law gives the equation.

$$E - iR - L \frac{di}{dt} = 0$$



#### Example 26:

The inductor shown in figure has inductance 0.54 H and carries a current in the direction shown that is decreasing at a uniform rate  $\frac{di}{dt} = -0.03 \text{ A/s}$ .



(a) Find the self-induced emf.

(b) Which end of the inductor a or b is at a higher potential ?

#### Solution:

(a) Self-induced emf

$$e = -L \frac{di}{dt} = (-0.54)(-0.03) \text{ V}$$

$$= 1.62 \times 10^{-2} \text{ V}$$

$$(b) V_b - L \frac{di}{dt} = V_a$$

$$V_b - V_a = L \frac{di}{dt} = 0.54 \times -0.03$$

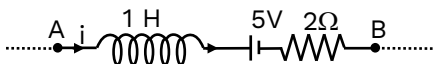
$$= -1.62 \times 10^{-2} \text{ V}$$

$$V_{ba} = L \frac{di}{dt} = -1.62 \times 10^{-2} \text{ V}$$

Since,  $V_{ba} = (V_b - V_a)$  is negative. It implies that  $V_a > V_b$  or a is at higher potential.

#### Example 27:

A B is a part of circuit. Find the potential difference  $V_A - V_B$  if



(i) current  $i = 2 \text{ A}$  and is constant

(ii) current  $i = 2 \text{ A}$  and is increasing at the rate of 1 amp/sec.

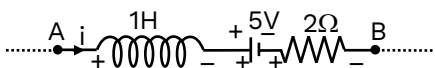
(iii) current  $i = 2 \text{ A}$  and is decreasing at the rate 1 amp/sec.

#### Solution:

$$L \frac{di}{dt} = 1 \frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1 \frac{di}{dt} - 5 - 2i = V_B$$



(i) Put  $i = 2$ ,  $\frac{di}{dt} = 0$

$$V_A - 5 - 4 = V_B$$

$$\therefore V_A - V_B = 9 \text{ volt}$$

(ii) Put  $i = 2$ ,  $\frac{di}{dt} = 1$ ;  $V_A - 1 - 5 - 4 = V_B$  or  $V_A - V_B = 10 \text{ Volt}$

(iii) Put  $i = 2$ ,  $\frac{di}{dt} = -1$ ;  $V_A + 1 - 5 - 2 \times 2 = V_B$

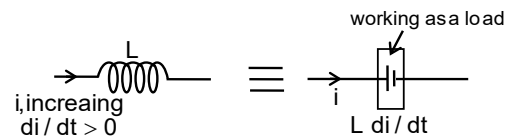
$$\text{or } V_A - V_B = 8 \text{ volt}$$

### 11.5 Energy stored in an inductor

If current in an inductor at an instant is  $i$  and is increasing at the rate  $di/dt$ , the induced emf will oppose the current. Its behaviour is shown in the figure.

$$\text{Power consumed by the inductor} = i L \frac{di}{dt}$$

$$\text{Energy consumed in } dt \text{ time} = i L \frac{di}{dt} dt$$



$$\therefore \text{total energy consumed as the current increases from 0 to } I = \int_0^I i L di = \frac{1}{2} L I^2$$

$$\Rightarrow U = \frac{1}{2} L I^2$$

**Note :** This energy is stored in the magnetic field with energy density for any medium.

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

$$\text{Total magnetic energy } U = \int \frac{B^2}{2\mu_0\mu_r} dV$$

#### Example 28:

What inductance would be needed to store 1.0 kWh of energy in a coil carrying a 200A current? (1kWh =  $3.6 \times 10^6$  J)

**Solution:**

We have,  $i = 200 \text{ A}$

and  $U = 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

$$U = \frac{1}{2} L I^2$$

$$\Rightarrow L = \frac{2U}{i^2} = \frac{2(3.6 \times 10^6)}{(200)^2} = 180 \text{ H}$$

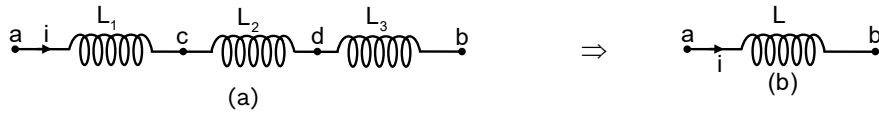


## 12. Inductor Circuits

### 12.1 Combinations of Inductors

#### (i) In Series

If several inductances are in series so that there are no interactions through mutual inductance.



Refer figure (a)

$$V_a - V_c = L_1 \frac{di}{dt}$$

$$V_c - V_d = L_2 \frac{di}{dt} \text{ and } V_d - V_b = L_3 \frac{di}{dt}$$

Adding all these equations, we have

$$V_a - V_b = (L_1 + L_2 + L_3) \frac{di}{dt} \quad \dots(i)$$

Refer figure (b)

$$V_a - V_b = L \frac{di}{dt} \quad \dots(ii)$$

Here,  $L$  = equivalent inductance.

From Eqs.(i) and (ii), we have

$$L = L_1 + L_2 + L_3$$

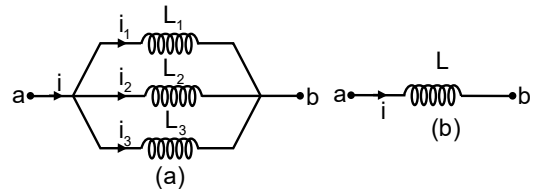
#### (ii) In parallel

Refer figure (a)

$$i = i_1 + i_2 + i_3$$

$$\text{or } \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

$$\text{or } \frac{di}{dt} = \frac{V_a - V_b}{L_1} + \frac{V_a - V_b}{L_2} + \frac{V_a - V_b}{L_3} \quad \dots(i)$$



Refer figure (b)

$$\frac{di}{dt} = \frac{V_a - V_b}{L} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

If the flux from one inductance links another, mutual inductance term becomes important. This mutual interaction may increase or decrease, the flux due to the self induction. The equivalent inductance of the pair of coils in series is,

$$L = L_1 + L_2 \pm 2M$$

## 12.2 Growth of Current in Series L–R Circuit

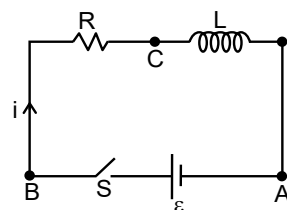
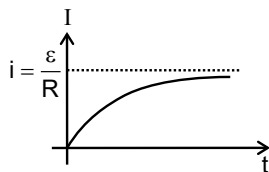
Figure shows a circuit consisting of a cell, an inductor  $L$  and a resistor  $R$ , connected in series. Let the switch  $S$  be closed at  $t=0$ . Suppose at an instant current in the circuit be  $i$  which is increasing at the rate  $di/dt$ .

Writing KVL along the circuit, we have

$$\varepsilon - L \frac{di}{dt} - iR = 0$$

On solving we get,  $i = \frac{\varepsilon}{R}(1 - e^{-\frac{Rt}{L}}) = i_0(1 - e^{-\frac{Rt}{L}})$

The quantity  $L/R$  is called time constant of the circuit and is denoted by  $\tau$ . The variation of current with time is as shown.



**Note :** 1. Final current in the circuit  $= \frac{\varepsilon}{R}$ , which is independent of  $L$ .

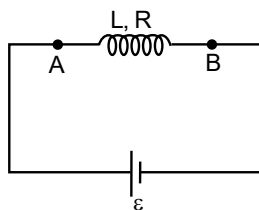
2. After one time constant, current in the circuit  $= 63\%$  of the final current

3. More time constant in the circuit implies slower rate of change of current.

4. If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.  $L_1 i_1 = L_2 i_2$

### Example 29:

An inductor having self inductance  $L$  with its coil resistance  $R$  is connected across a battery of emf  $\varepsilon$ . When the circuit is in steady state at  $t = 0$  an iron rod is inserted into the inductor due to which its inductance becomes  $nL$  ( $n > 1$ ).



(i) After insertion of rod which of the following quantities will change with time ?

(a) Potential difference across terminals  $A$  and  $B$ .

(b) Inductance

(c) Rate of heat produced in coil

(1) Only (a)

(2) (a) & (c)

(3) Only (c)

(4) (a), (b) & (c)

(ii) After insertion of the rod, current in the circuit :

(1) Increases with time

(2) Decreases with time

(3) Remains constant with time

(4) First decreases with time then becomes constant

(iii) When again circuit is in steady state, the current in it is :

(1)  $I < \varepsilon/R$

(2)  $I > \varepsilon/R$

(3)  $I = \varepsilon/R$

(4) None of these

**Solution:**

(i) Ans.(3)

Inductance and potential difference across terminals will not change with time.

(ii) Ans.(1)

Even after insertion of the rod the current in circuit will increase with time till steady state is reached.

(iii) Ans.(3)

At steady state inductor will offer zero resistance and hence  $I = \frac{\mathcal{E}}{R}$ .

**Example 30:**

A coil of resistance  $20\Omega$  and inductance  $0.5\text{H}$  is switched to DC  $200\text{ V}$  supply. Calculate the rate of increase of current.

(a) At the instant of closing the switch

(b) After one time constant

(c) Find the steady state current in the circuit

**Solution:**

(a) This is the case of growth of current in an L-R circuit. Hence, current at time  $t$  is given by,

$$i = i_0 (1 - e^{-t/\tau_L})$$

Rate of increase of current,  $\frac{di}{dt} = \frac{i_0}{\tau_L} e^{-t/\tau_L}$

At  $t = 0$ ,

$$\frac{di}{dt} = \frac{i_0}{\tau_L} = \frac{E/R}{L/R} = \frac{E}{L}$$

Substituting the value, we have  $\frac{di}{dt} = \frac{200}{0.5} = 400\text{ A/s}$

(b) At  $t = \tau_L$ ,

$$\frac{di}{dt} = (400)e^{-1} = (0.37)(400) = 148\text{ A/s}$$

(c) The steady state current in the circuit.

$$i_0 = \frac{E}{R} = \frac{200}{20} = 10\text{ A}$$

**12.3 Decay of Current in the Circuit Containing Resistor & Inductor**

Let the initial current in the circuit be  $I_0$ . At any time  $t$ , let the current be  $i$  and let its rate of

change at this instant be  $\frac{di}{dt}$ .

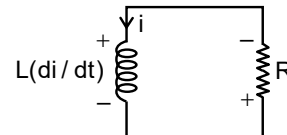
$$L \frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} = -\frac{iR}{L}$$

$$\int_{I_0}^i \frac{di}{i} = \int_0^t \frac{R}{L} -dt \quad ; \quad \ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L} \quad \text{or } i = I_0 e^{-\frac{Rt}{L}}$$

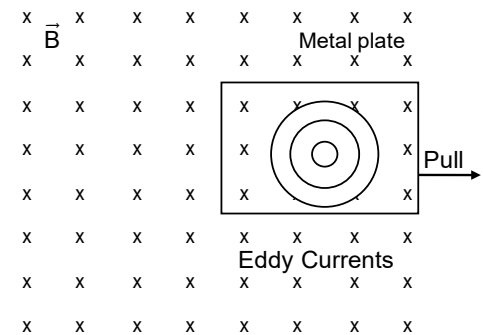
Current after one time constant :  $i = I_0 e^{-1}$

$$\frac{I_0}{e} = 0.37\% \text{ of initial current.}$$



### 13. Eddy Currents

- When a conducting sheet is placed in a changing magnetic field, induced emf is produced in it. As a result local currents are produced in the conducting sheet. These local currents are called eddy currents.
- If a conducting material is moved in a magnetic field, then eddy currents are also produced.
- Eddy currents flows in closed paths.
- There is loss of energy due to eddy currents and it appears in the form of heat.
- In order to minimize the energy loss in the form of heat due to eddy currents, the core of dynamo, motor or transformer is not taken as a single piece of soft iron but in the form of a pack of thin sheets insulated from each other by a layer of insulating varnish, called laminated core. This device increases the resistance for the eddy currents. In this way eddy currents are considerably reduced and loss of energy becomes less.

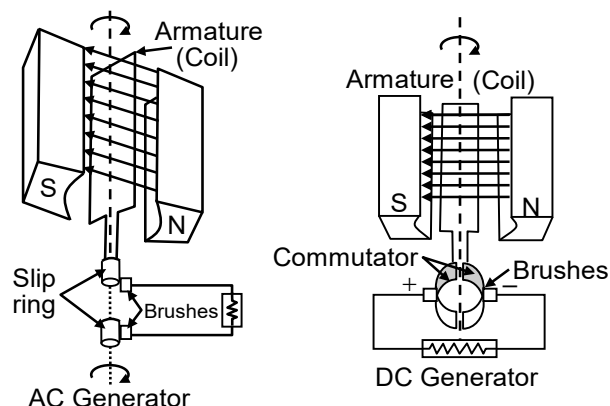


- Uses of eddy currents**

- Moving coil galvanometer
- Induction furnace
- Dead beat galvanometer
- Speedometer
- Electric brakes

### 14. Generator or Dynamo

- Generator or dynamo is an electrical device which converts mechanical energy into electrical energy.
- Working of generators is based on the principle of electromagnetic induction.
- Generators are of two types**
  - A.C. generator : If the current produced by the generator is alternating, then the generator is called A.C. generator.



(b) D.C. generator : If the current produced by the generator is direct current, then the generator is called D.C. generator.

- Generator consists of the following parts.
 

(a) Armature (coil)	(b) Magnet
(c) Slip rings	(d) Brushes

 In D.C. generator, commutator is used in place of slip rings.

- In order to produce the magnetic field in big generators several magnetic poles are used. In these generators the armature coils are kept stationary and magnetic pole pieces are made to rotate around the armature. The frequency of alternating current produced by generator of multi poles is

$$= \frac{\text{number of poles} \times \text{rotational frequency}}{2} = \frac{Nn}{2}$$

- Energy loss in generators :** The loss of energy is due to the following reasons :

- |                         |                       |
|-------------------------|-----------------------|
| (a) Flux leakage        | (b) Copper losses     |
| (c) Eddy current losses | (d) Hysteresis losses |
| (e) Mechanical losses   |                       |

- Efficiency of generator :** Practical efficiency of a generator

$$= \frac{\text{Electrical power generated by the generator}}{\text{Mechanical energy given to the generator}}$$

Practical efficiencies of big generators are about 92% to 95%.

## 15. Transformers

Transformers are based upon mutual induction, which transform an alternating voltage from one to another of greater or smaller value.

A transformer consists of two coils wound on a soft iron core, called primary and secondary coils. Let number of turns in these coils are  $N_p$  and  $N_s$  respectively. The input ac voltage is applied across primary coil whereas output a.c. voltage is across secondary coil.

We consider an ideal transformer in which the primary has negligible resistance and all the flux in the core links both the primary and secondary windings. Let  $\phi$  be the flux linkage through each of primary and secondary coils. Then.

Induced emf across the primary coil,

$$\varepsilon_p = -N_p \frac{d\phi}{dt} \quad \dots(i)$$

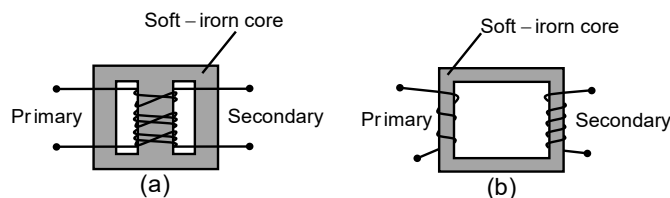
Similarly induced emf across secondary

$$\varepsilon_s = -N_s \frac{d\phi}{dt} \quad \dots(ii)$$

From these equations

$$\frac{\text{AC voltage obtained across secondary}}{\text{AC voltage applied across primary}}$$

$$= \frac{V_s}{V_p} = \frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} \quad \dots(iii)$$



**Fig. :** Two arrangements for winding of primary and secondary coil in a transformer.

- (a) two coils on top of each other,  
 (b) two coils on separate limbs of the core.

**Note :** The above relations are based upon following three assumptions;

- Primary current and resistance are small.
- The same flux links both the primary and secondary coil.
- The secondary current is small.

In a transformer, some energy is always lost. The efficiency of a well designed transformer may be upto 95%. If the transformer is assumed to be 100% efficient (no energy loss)

$$P = I_p V_p = I_s V_s$$

$$\text{Thus } \frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \dots(\text{iv})$$

In this equation, current and voltages can have peak or rms values. Because voltage and current both oscillate with the same frequency.

$$\text{Thus we have } V_s = \left( \frac{N_s}{N_p} \right) V_p \text{ and } I_s = \left( \frac{N_p}{N_s} \right) I_p \quad \dots(\text{v})$$

For step up transformer

$$V_s > V_p \text{ so } N_s > N_p \text{ and } I_s < I_p$$

For step down transformer

$$V_s < V_p \text{ so } N_s < N_p \text{ and } I_s > I_p$$

In actual transformers, small energy losses occur due to following reasons.

- (1) **Flux leakage :** There is always some flux leakage. Not all the flux due to primary winding passes through the secondary winding.
- (2) **Resistance of the windings :** Some energy is lost in the form of heat dissipation. It can be minimised using thick wire in case of high current, low voltage windings.
- (3) **Eddy Currents :** The alternating magnetic flux induces eddy currents in the iron core and causes heating. The loss can be minimized using laminated iron core.
- (4) **Hysteresis :** The magnetisation of core is repeatedly reversed by alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a material when has a low magnetic hysteresis loss.

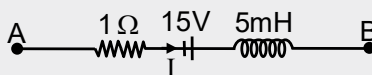
### Use of Transformers in Transmission and distribution of Energy over long Distance

The voltage output of the generator is stepped up, so the current is reduced and consequently, the  $I^2R$  loss is cut down. It is then transmitted over long distances to an area sustain near the consumers. There the voltage is stepped down. It is further stepped down at the distributing substation and utility poles before a power supply of 220 V reaches our homes.

### Concept Builder-7



- Q.1** The network shown in figure is a part of a complete circuit. What is the potential difference  $V_B - V_A$ , when the current  $I$  is 5A and is decreasing at a rate of  $10^3$  (A/s) ?



- (1) 5V                      (2) 10 V                      (3) 15V                      (4) 25 V

- Q.2** Two inductance coils of inductances  $L_1$  and  $L_2$  are kept at sufficiently large distance apart. On connecting them in parallel their equivalent inductance will be :

- (1)  $\frac{L_1 + L_2}{L_1 L_2}$                       (2)  $\frac{L_1 L_2}{L_1 + L_2}$                       (3)  $L_1 + L_2$                       (4)  $\sqrt{L_1 L_2}$

## ANSWER KEY FOR CONCEPT BUILDERS

### CONCEPT BUILDER-1

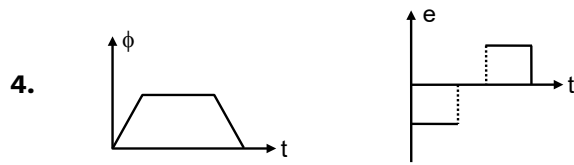
1. (3)                      2. Based on Theory

### CONCEPT BUILDER-2

1. Zero                      2. Based on theory  
 3. If magnet moves faster change in magnetic flux will be more.  
 4.  $i = 20/5 = 4$  amp.  
 Current will be anticlockwise.

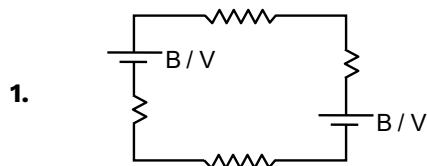
### CONCEPT BUILDER-3

1. 4.05 V                      2. 0.8 mA  
 3. 500 mV

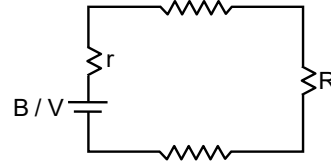


5.  $2.5 \mu\text{V}$   
 6.  $\frac{2 \times 10^{-4} \text{ V}}{2.16 \times 10^{-3} \Omega} = 9.3 \times 10^{-2} \text{ A}.$

### CONCEPT BUILDER-4



2.



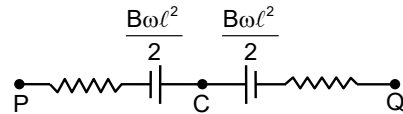
3.  $i = \frac{e}{R} = \frac{7}{200} \text{ A}$

4.  $i = \frac{B\ell V}{r + \frac{R_1 R_2}{R_1 + R_2}}$

5. 0                      6. (3)

### CONCEPT BUILDER-5

1.  $\text{emf PQ} = 0$  ;  $\text{emf PC} = \frac{B\omega\ell^2}{2}$



2. (2)                      3.  $\frac{B\omega r^2}{2R}$   
 4. (3)                      5. (4)  
 6. (2)

### CONCEPT BUILDER-6

1. (1)                      2.  $5.0 \times 10^{-4} \text{ H}$   
 3. (2)                      4.  $L_1 = 6 \text{ H}, L_2 = 4 \text{ H}$

### CONCEPT BUILDER-7

1. (3)                      2. (2)

## Exercise - I

### Flux and Faraday's Laws of Electromagnetic Induction

1. The horizontal component of earth's magnetic field is  $3 \times 10^{-5} \text{ Wb/m}^2$ . The magnetic flux linked with a coil of area  $1 \text{ m}^2$  and having 5 turns, whose plane is normal to the magnetic field, will be -

(1)  $3 \times 10^{-5} \text{ Wb}$                       (2)  $5 \times 10^{-5} \text{ Wb}$   
 (3)  $15 \times 10^{-5} \text{ Wb}$                 (4)  $1 \times 10^{-5} \text{ Wb}$

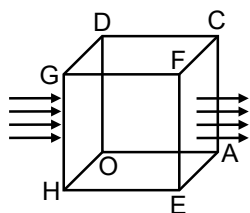
2. Tesla is a unit of -  
 (1) magnetic flux  
 (2) magnetic flux density  
 (3) electric flux  
 (4) self inductance

3. The formula of the induced emf due to rate of change of magnetic flux passing through a coil will be-

(1)  $e = -\frac{d}{dt}(\vec{B} \cdot \vec{A})$                       (2)  $e = \frac{dB}{dt}$   
 (3)  $e = -\vec{A} \cdot \left( \frac{d\vec{B}}{dt} \right)$                       (4)  $e = -\vec{B} \cdot \frac{d\vec{A}}{dt}$

4. Unit of magnetic flux density is -  
 (1) weber/metre                      (2) weber  
 (3) weber/m<sup>2</sup>                      (4) ampere/m

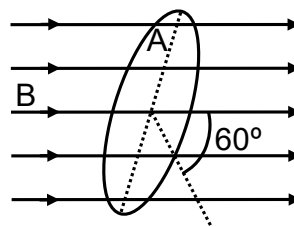
5. A cube of side  $a$  is placed in a magnetic field of intensity  $B$ . The magnetic flux emerging out of the cube will be -



(1)  $Ba^2$                                       (2)  $-Ba^2$   
 (3)  $2Ba^2$                                   (4) zero

6. The figure represents an area  $A = 0.5 \text{ m}^2$  situated in a uniform magnetic field  $B = 2.0 \text{ weber/m}^2$  and making an angle of  $60^\circ$  with respect to the magnetic field. The value of the magnetic flux through

the area would be equal to-



(1) 2.0 weber  
 (2)  $\sqrt{3}$  weber  
 (3)  $\frac{3}{2}$  weber  
 (4) 0.5 weber

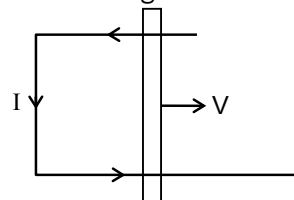
7. A circular loop of radius  $R$ , carrying current  $I$ , lies in x-y plane with its centre at origin. The total magnetic flux through x-y plane is

(1) directly proportional to  $I$   
 (2) directly proportional to  $R$   
 (3) directly proportional to  $R^2$   
 (4) zero

8. When two co-axial coils having same current in same direction are brought to each other, then the value of current in both coils :

(1) increases  
 (2) decreases  
 (3) first increases and then decreases  
 (4) remain same

9. For given arrangement (in horizontal plane) in which induced current is given in anticlockwise direction. The possible direction of magnetic field is :



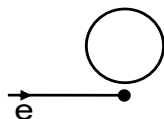
(1) towards right  
 (2) towards left  
 (3) vertically upward  
 (4) vertically downward



10. A magnetic field can be produced by :  
 (1) A moving charge  
 (2) A changing electric field  
 (3) A stationary charge  
 (4) Both (1) and (2)
11. The magnetic flux linked with a coil at any instant  $t$  is given by  $\phi = 5t^3 - 100t + 300$ , the emf induced in the coil at  $t = 2s$  is  
 (1)  $-40V$  (2)  $40V$   
 (3)  $140V$  (4)  $300V$

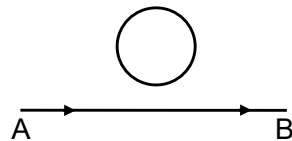
### Lenz's Law

12. Which of the following is proportional to energy density in magnetic field  $B$  :  
 (1)  $1/B$  (2)  $1/B^2$   
 (3)  $B$  (4)  $B^2$
13. Lenz's law gives :  
 (1) the magnitude of the induced emf  
 (2) the direction of the induced current  
 (3) both the magnitude and direction of the induced current  
 (4) the magnitude of the induced current
14. Lenz's law is based on the law of conservation of-  
 (1) charge  
 (2) momentum  
 (3) mass  
 (4) energy
15. A bar magnet is dropped vertically downward through a metal ring held horizontally. The acceleration of falling magnet will be -  
 (1) equal to  $g$   
 (2) greater than  $g$   
 (3) less than  $g$   
 (4) dependent on the radius of the ring
16. An electron is approaching near a ring and approaches to ring, then direction of induced current in ring is :



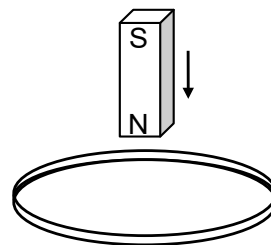
- (1) clockwise (2) anticlockwise  
 (3) both (1) and (2) (4) no current

17. A magnet is taken towards a coil-  
 [a] Rapidly [b] Slowly  
 Then the induced emf is-  
 (1) More in (a)  
 (2) Less in (a)  
 (3) Same in both (a) and (b)  
 (4) More or less depends on radius
18. In the arrangement shown in given figure current from A to B is increasing in magnitude. Induced current in the loop will:



- (1) have clockwise direction  
 (2) have anticlockwise direction  
 (3) be zero  
 (4) oscillate between clockwise and anticlockwise

19. The north pole of a magnet is brought near a metallic ring as shown in fig. The direction of induced current in the ring will be-



- (1) Anticlock wise from magnet side  
 (2) Clock wise from magnet side  
 (3) First anticlock wise and then clock wise from magnet side  
 (4) First clock wise and then anticlock wise from magnet side

20. When a magnet is moved with its north pole towards a coil placed in a closed circuit, then the nearest face of the coil-  
 (1) shows south polarity  
 (2) shows north polarity  
 (3) shows no polarity  
 (4) shows sometimes north and sometimes south polarity

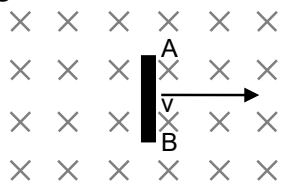
## Induced Emf, Current & Change in a Circuit

21. If  $\phi = 0.02 \cos 100\pi t$  weber/turns and number of turns is 50 in the coil, the maximum induced emf is-  
 (1) 314 volt  
 (2) 100 volt  
 (3) 31.4 volt  
 (4) 6.28 volt
22. The magnetic flux linked with the coil varies with time as  $\phi = 3t^2 + 4t + 9$ . The magnitude of induced emf at  $t = 2$  second is -  
 (1) 4 V  
 (2) 3 V  
 (3) 16 V  
 (4) 9 V
23. A conducting square loop of side  $l$  and resistance  $R$  moves in its plane with a uniform velocity  $v$  perpendicular to one of its sides. A uniform and constant magnetic field  $B$  exists along the perpendicular to the plane of the loop as shown in fig. The current induced in the loop is -
- 
- (1)  $Blv/R$  clockwise  
 (2)  $Blv/R$  anticlockwise  
 (3)  $2Blv/R$  anticlockwise  
 (4) zero
24. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is  
 (1) clockwise  
 (2) anticlockwise  
 (3) zero  
 (4) clockwise or anticlockwise depending on whether the resistance is increased or decreased.

## Motional EMF

25. The unit of mutual inductance of a coil can be expressed as :  
 (1) weber  $\times$  amp  
 (2) weber/amp  
 (3) weber  $\times$  meter  
 (4) weber / meter
26. A wire of length 2m is moving with a velocity of 1 m/s normal to a magnetic field of  $0.5 \text{ Wb/m}^2$ . The emf induced in it will be - ( $\ell \perp$ )  
 (1) 0.5 V  
 (2) 0.1 V  
 (3) 2 V  
 (4) 1 V
27. An aeroplane having a distance of 50 m between the edges of its wings is flying horizontally with a speed of 720 km/hour. If the vertical component of the earth's magnetic field is  $2 \times 10^{-4} \text{ Wb/m}^2$ , then the induced emf will be -  
 (1) 2mV  
 (2) 2V  
 (3) 200V  
 (4) 0.2mV
28. A straight conductor of length 0.4 m is moved in a magnetic field of  $0.9 \text{ weber/m}^2$  with a velocity of 7 m/s. The maximum emf induced in the conductor will be -  
 (1) 2.52 V  
 (2) 25 V  
 (3) 2.8 V  
 (4) 63 V
29. An athlete runs at a velocity of 30 km/hr. towards east with a 3 meter rod. The horizontal component of the earth is  $4 \times 10^{-5} \text{ weber/m}^2$ . If he runs, keeping the rod (i) horizontal and (ii) vertical, the p.d. at the ends of the rod in both the cases, will be-  
 (1) Zero in vertical case and  $1 \times 10^{-3} \text{ V}$  in the horizontal case.  
 (2)  $1 \times 10^{-3} \text{ V}$  in vertical case and zero in the horizontal case.  
 (3) Zero in both the cases.  
 (4)  $1 \times 10^{-3} \text{ V}$  in both the cases.

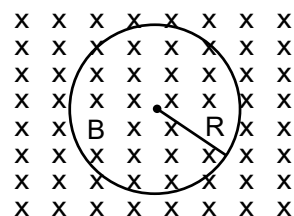
- 30.** A rod AB moves with a uniform velocity  $v$  in a uniform magnetic field as shown in fig.



- (1) The rod becomes electrically charged  
 (2) The end A becomes positively charged  
 (3) The end B become positively charged  
 (4) The rod becomes hot because of Joule heating
- 31.** The distance between the ends of wings of an aeroplane is 5m. The aeroplane is moving with velocity of 200 km/sec in a magnetic field of 10T. The emf induced across the ends of wings will be:  
 (1)  $10^7$  volt (2) 10 volt  
 (3)  $10^6$  volt (4) none of these
- 32.** An aeroplane having a wing space of 35m flies due north with the speed of 90m/s given  $B = 4 \times 10^{-5}$  tesla. The potential difference between the tips of the wings will be :  
 (1) 0.013 V (2) 1.26 V  
 (3) 12.6 V (4) 0.126 V
- 33.** A straight conductor of length 4m moves at a speed of 10m/s. When the conductor makes an angle of  $30^\circ$  with the direction of magnetic field of induction of 0.1 wb. per  $m^2$  then induced emf is :  
 (1) 8V (2) 4V  
 (3) 1V (4) 2V
- 34.** Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon :  
 (1) the rates at which currents are changing in the two coils  
 (2) relative position and orientation of the two coils  
 (3) the materials of the wires of the coils  
 (4) the currents in the two coils

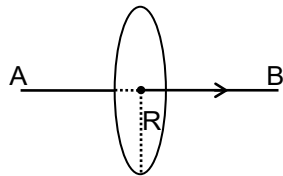
### Induced Emf in a Fixed Loop Due to Time Varying Magnetic Field

- 35.** The resistance of a coil is 5 ohm and a current of 0.2 A is induced in it due to a varying magnetic field. The rate of change of magnetic flux in it will be -  
 (1) 0.5 Wb/s (2) 0.05 Wb/s  
 (3) 1 Wb/s (4) 20 Wb/s
- 36.** A coil of 100 turns having an average area of  $100 \text{ cm}^2$  for each turn is held in a uniform field of 50 gauss, the direction of the field being at right angle to the plane of the coil. If the field is removed in 0.01 sec, then average e.m.f induced in coil is -  
 (1) 0.5 V (2) 10 V  
 (3) 20 V (4) 50 V
- 37.** A coil is placed in transverse magnetic field of 0.02 T. This coil starts shrinking at a rate of 1 mm/sec. When its radius 4 cm, then what is the value of induced emf-  
 (1) 2  $\mu\text{V}$  (2) 2.5  $\mu\text{V}$   
 (3) 5  $\mu\text{V}$  (4) 8  $\mu\text{V}$
- 38.** A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time 't', as  $R = R_0 + t$ . The e.m.f induced in the loop is



- (1)  $2\pi(R_0 + t)B$  clockwise  
 (2)  $\pi(R_0 + t)B$  clockwise  
 (3)  $2\pi(R_0 + t)B$  anticlockwise  
 (4) zero

39. A long conductor AB lies along the axis of a circular loop of radius  $R$ . If the current in the conductor AB varies at the rate of  $I$  ampere/second, then the induced emf in the loop is



- (1)  $\frac{\mu_0 IR}{2}$  (2)  $\frac{\mu_0 IR}{4}$   
 (3)  $\frac{\mu_0 \pi IR}{2}$  (4) zero

### Induced Emf Due to Rotation

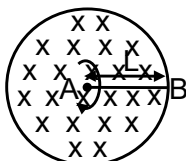
40. A conductor of length  $L$  is rotated about an axis passing through one end of it with an angular velocity  $\omega$  in a normal and uniform magnetic field  $B$ . The emf induced between its end will be-

- (1)  $\omega BL^2$  (2)  $\frac{\omega BL^2}{2}$   
 (3)  $2 \omega BL^2$  (4)  $\omega BL$

41. A metal rod of length  $L$  is placed normal to a magnetic field and rotated in a circular path with frequency  $f$ . The potential difference between the ends will be -

- (1)  $\pi L^2 B f$  (2)  $BL/f$   
 (3)  $\pi L^2 B/f$  (4)  $fBL$

42. A copper rod AB of length  $L$ , pivoted at one end A, rotates at constant angular velocity  $\omega$ , at right angles to a uniform magnetic field of induction  $B$ .



The e.m.f developed between the mid point C of the rod and end B is

- (1)  $\frac{B\omega L^2}{4}$  (2)  $\frac{B\omega L^2}{2}$   
 (3)  $\frac{3B\omega L^2}{4}$  (4)  $\frac{3B\omega L^2}{8}$

43. A coil is placed in a uniform magnetic field such that its plane is parallel to the magnetic field. In time interval  $\Delta t$  its plane becomes perpendicular to the magnetic field, then induced charge  $q$  in coil depends on the time interval  $\Delta t$  as-

- (1)  $q \propto \Delta t$  (2)  $q \propto \frac{1}{\Delta t}$   
 (3)  $q \propto (\Delta t)^0$  (4)  $q \propto (\Delta t)^2$

44. A metallic conductor of 1 m length is rotated vertically about its one end at an angular velocity of 5 rad/sec. If the horizontal component of earth's field is  $0.2 \times 10^{-4}$  T, the voltage generated at both ends of the conductor will be-

- (1) 5 mV (2)  $5 \times 10^{-4}$  V  
 (3) 50 mV (4) 50  $\mu$  V

45. A rectangular coil has 60 turns and its length and width is 20 cm and 10 cm respectively. The coil rotates at a speed of 1800 rotation per minute in a uniform magnetic field of 0.5 tesla. Then the maximum induced emf will be-

- (1) 98 V (2) 110 V  
 (3) 113 V (4) 118 V

### Self-Induction

46. If the length and area of cross-section of an inductor remain same but the number of turns is doubled, its self-inductance will become -

- (1) half (2) four times  
 (3) double (4) one-fourth

47. Dimensions of coefficient of self-induction are-

- (1)  $MLT^{-2}A^{-2}$  (2)  $ML^{-2}T^{-2}A^{-2}$   
 (3)  $ML^2T^{-2}A^{-2}$  (4)  $M^2LT^{-2}A^{-2}$

48. Self-inductance of a solenoid depend on-

- (1) the number of turns  $N$  of the coil  
 (2) the area of cross-section  $A$  and length  $\ell$  of the coil  
 (3) the permeability of the core of the coil  
 (4) all the above

49. Equivalent unit of self-inductance is -

- (1)  $\frac{\text{volt} \times \text{ampere}}{\text{second}}$
- (2)  $\frac{\text{volt} \times \text{second}}{\text{ampere}}$
- (3)  $\frac{\text{ampere}}{\text{volt} \times \text{second}}$
- (4)  $\frac{\text{ampere} \times \text{second}}{\text{volt}}$

50. When current flowing in a coil changes from 3A to 2A in one millisecond, 5 volt emf is induced in it. The self-inductance of the coil will be -

- (1) zero
- (2) 5 kH
- (3) 5H
- (4) 5 mH

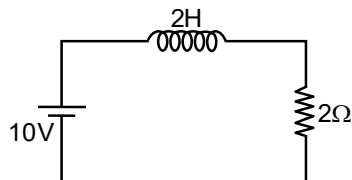
51. The equivalent inductance of two inductance is 2.4 henry when connected in parallel and 10 henry, which connected in series. The difference between the two inductance is -

- (1) 2 henry
- (2) 3 henry
- (3) 4 henry
- (4) 5 henry

52. The value of self inductance of a coil is 5 henry. The value of current changes from 1 ampere to 2 amperes in 5 seconds. The value of induced emf in it is-

- (1) 10 Volt
- (2) 0.10 Volt
- (3) 1.0 Volt
- (4) 100 Volt

53. In the figure magnetic energy stored in the coil is (in steady state)



- (1) 0
- (2)  $\infty$
- (3) 25 joules
- (4) none of these

54. The self inductance of a solenoid of length L, area of cross section A and having N turns is-

- (1)  $\frac{\mu_0 N^2 A}{L}$
- (2)  $\frac{\mu_0 N A}{L}$
- (3)  $\mu_0 N^2 L A$
- (4)  $\mu_0 N A L$

55. A long solenoid has 200 turns per cm. and carries a current of 2.5 amps. The magnetic field at its centre is-

$$[\mu_0 = 4\pi \times 10^{-7} \text{ Weber/m}^2]$$

- (1)  $3.14 \times 10^{-2} \text{ Weber/m}^2$
- (2)  $6.28 \times 10^{-2} \text{ Weber/m}^2$
- (3)  $9.42 \times 10^{-2} \text{ Weber/m}^2$
- (4)  $12.56 \times 10^{-2} \text{ Weber/m}^2$

56. Energy is stored in the choke coil in the form of-

- (1) Heat
- (2) Electric energy
- (3) Magnetic energy
- (4) Electro-magnetic energy

57. For a inductor coil L = 0.04 H then work done by source to establish a current of 5A in it is-

- (1) 0.5 J
- (2) 1.00 J
- (3) 100 J
- (4) 20 J

58. Average energy stored in a pure inductance L, when a current i flow through it-

- (1)  $Li^2$
- (2)  $2Li^2$
- (3)  $\frac{Li^2}{4}$
- (4)  $\frac{1}{2} Li^2$

59. Current passing through a coil is changing at the rate of 1.5 ampere per second. If it induces emf of 45 volt, then the self inductance of the coil will be

- (1) 30 H
- (2) 67.5 H
- (3) 60 H
- (4) 33.3 H

60. When the current changes from +2A to -2A in 0.05s, an emf of 8V is induced in a coil. The coefficient of self induction of the coil is :

- (1) 0.2 H
- (2) 0.4H
- (3) 0.8H
- (4) 0.1H

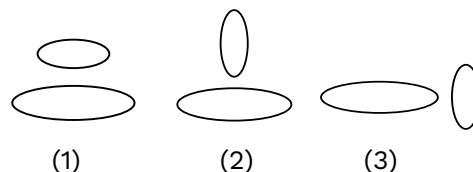
### Mutual Inductance

61. The unit of mutual inductance is -

- (1) volt
- (2) weber
- (3) tesla
- (4) henry

62. The self-inductances of two identical coils are 0.1 H. They are wound over each other. Mutual inductance will be -  
 (1) 0.1 H (2) 0.2 H  
 (3) 0.01 H (4) 0.05 H
63. Two conducting loops of radi  $R_1$  and  $R_2$  are concentric and are placed in the same plane. If  $R_1 \gg R_2$ , the mutual inductance  $M$  between them will be directly proportional to-  
 (1)  $\frac{R_1}{R_2}$  (2)  $\frac{R_2}{R_1}$   
 (3)  $\frac{R_1^2}{R_2}$  (4)  $\frac{R_2^2}{R_1}$
64. The mutual inductance between a primary and secondary circuits is 0.5H. The resistance of the primary and the secondary circuits are  $20\ \Omega$  and  $5\ \Omega$  respectively. To generate a current of 0.4 A in the secondary, current in the primary must be changed at the rate of-  
 (1) 4.0 amp./sec. (2) 16.0 amp./sec.  
 (3) 1.6 amp./sec. (4) 8.0 amp./sec.
65. Two coils A and B having turns are placed near each other, on a passing a current of 3.0 A in A, the flux linked with A is  $1.2 \times 10^{-4}$  weber and with B it is  $9.0 \times 10^{-5}$  weber. The mutual inductance of the system is-  
 (1)  $2 \times 10^{-5}$  henry  
 (2)  $3 \times 10^{-5}$  henry  
 (3)  $4 \times 10^{-5}$  henry  
 (4)  $6 \times 10^{-5}$  henry
66. A long straight wire is placed along the axis of a circular ring of radius  $R$ . The mutual inductance of this system is  
 (1)  $\frac{\mu_0 R}{2}$   
 (2)  $\frac{\mu_0 \pi R}{2}$   
 (3)  $\frac{\mu_0}{2}$   
 (4) 0

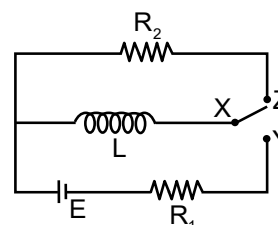
67. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be:



- (1) maximum in situation (1)  
 (2) maximum in situation (2)  
 (3) maximum in situation (3)  
 (4) the same in all situations

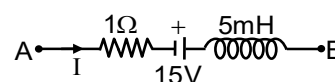
### Inductors & Their Behaviour and Induced Circuits

68. In the circuit shown, X is joined to Y for a long time, and then X is joined to Z. The total heat produced in  $R_2$  is :



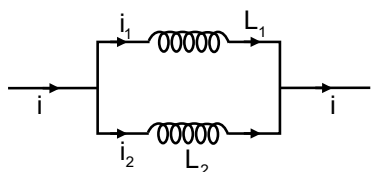
- (1)  $\frac{LE^2}{2R_1^2}$  (2)  $\frac{LE^2}{2R_2^2}$   
 (3)  $\frac{LE^2}{2R_1 R_2}$  (4)  $\frac{LE^2 R_2}{2R_1^2}$

69. The network shown in the figure is part of a complete circuit. If at a certain instant, the current  $I$  is 5A and it is decreasing at a rate of  $10^3\text{ As}^{-1}$  then  $V_B - V_A$  equals



- (1) 20 V (2) 15 V  
 (3) 10 V (4) 5 V

- 70.** Two inductors  $L_1$  and  $L_2$  are connected in parallel and a time varying current  $i$  flows as shown. The ratio of currents  $i_1/i_2$  at any time  $t$  is



- (1)  $L_1/L_2$  (2)  $L_2/L_1$   
 (3)  $\frac{L_1^2}{(L_1 + L_2)^2}$  (4)  $\frac{L_2^2}{(L_1 + L_2)^2}$

- 71.** An LR circuit with a battery is connected at  $t = 0$ . Which of the following quantities is not zero just after the connection?  
 (1) current in the circuit  
 (2) magnetic field energy in the inductor

- (3) power delivered by the battery  
 (4) emf induced in the inductor

- 72.** A coil of inductance 300 mH and resistance  $2\Omega$  is connected to a source of voltage 2V. The current reaches half of its steady state value in  
 (1) 0.05 s (2) 0.1 s  
 (3) 0.15s (4) 0.3 s

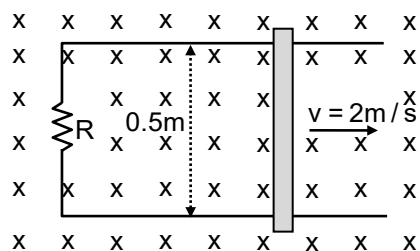
- 73.** A coil of resistance  $R$  and inductance  $L$  is connected to a battery of emf  $E$  volt. The final current in the coil is :  
 (1)  $E/R$  (2)  $E/L$   
 (3)  $\sqrt{E / (R^2 + L^2)}$  (4)  $\sqrt{\frac{EL}{(R^2 + L^2)}}$

### ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Ans.	3	2	1	3	1	4	4	2	4	4	2	4	2	4	3	2	1	1	1	2	1	3	4	3	2
Que.	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
Ans.	4	2	1	2	2	1	4	4	2	3	1	3	3	4	2	1	4	3	4	3	2	3	4	2	4
Que.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73		
Ans.	1	3	3	1	2	3	2	4	1	4	4	1	4	1	2	4	1	1	2	2	4	2	1		

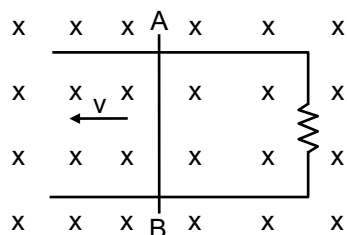
## Exercise - II

1. A metallic rod completes its circuit as shown in the figure. The circuit is normal to a magnetic field of  $B = 0.15$  tesla. If the resistance of the rod is  $3\Omega$  the force required to move the rod with a constant velocity of  $2 \text{ m/sec}$  is-



- (1)  $3.75 \times 10^{-3} \text{ N}$       (2)  $3.75 \times 10^{-2} \text{ N}$   
 (3)  $3.75 \times 10^2 \text{ N}$       (4)  $3.75 \times 10^{-4} \text{ N}$

2. Consider the situation shown in the figure. The wire AB is slide on the fixed rails with a constant velocity. If the wire AB is replaced by a semicircular wire, the magnitude of the induced current will-



- (1) Increase  
 (2) Remain the same  
 (3) Decrease  
 (4) Increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

3. A thin wire of length  $2\text{m}$  is perpendicular to the  $xy$  plane. It is moved with velocity  $\vec{v} = (2\hat{i} + 3\hat{j} + \hat{k}) \text{ m/s}$  through a region of magnetic induction  $\vec{B} = (\hat{i} + 2\hat{j}) \text{ Wb/m}^2$ . Then potential difference induced between the ends of the wire :
- (1) 2 volts      (2) 4 volts  
 (3) 0 volts      (4) none of these

4. Two identical coaxial circular loops carry a current  $i$  each circulating in the same direction. If the loops approach each other

- (1) the current in each loop will decrease  
 (2) the current in each loop will increase

(3) the current in each loop will remain the same

(4) the current in one loop will increase and in the other loop will decrease

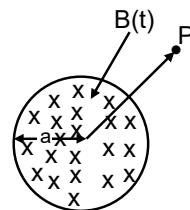
5. A conducting rod is moved with a constant velocity  $\vec{v}$  in a magnetic field. A potential difference appears across the two ends

- (1) if  $\vec{v} \parallel \vec{\ell}$       (2) if  $\vec{v} \parallel \vec{B}$   
 (3) if  $\vec{v} \parallel \vec{B}$       (4) none of these

6. A rod of length  $\ell$  rotates with a uniform angular velocity  $\omega$  about its perpendicular bisector. A uniform magnetic field  $B$  exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

- (1) zero      (2)  $\frac{1}{2} \omega B^2$   
 (3)  $B\omega^2$       (4)  $2B\omega^2$

7. A uniform but time-varying magnetic field  $B(t)$  exists in a circular region of radius  $a$  and is directed into the plane of the paper, as shown fig. The magnitude of the induced electric field at point P at a distance  $r$  from the centre of the circular region.



- (1) is zero  
 (2) decreases as  $1/r$   
 (3) increases as  $r$   
 (4) decreases as  $1/r^2$



8. An inductor coil stores energy  $U$  when a current  $i$  is passed through it and dissipates heat energy at the rate of  $P$ . The time constant of the circuit when this coil is connected across a battery of zero internal resistance is :

(1)  $\frac{4U}{P}$  (2)  $\frac{U}{P}$   
 (3)  $\frac{2U}{P}$  (4)  $\frac{2P}{U}$

9.  $L$ ,  $C$  and  $R$  represent the physical quantities inductance, capacitance and resistance. Which of the following combinations have dimensions of time:

(1)  $\frac{1}{RC}$  (2)  $\frac{R}{L}$   
 (3)  $\frac{1}{\sqrt{LC}}$  (4)  $\sqrt{LC}$

10. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate  $15.0 \text{ A/s}$  the e.m.f. is coil 1 is  $25.0 \text{ mV}$ , when coil 2 has no current and coil 1 has a current of  $3.6 \text{ A}$ , flux linkage in coil 2 is

(1)  $16 \text{ mWb}$  (2)  $10 \text{ mWb}$   
 (3)  $4.00 \text{ mWb}$  (4)  $6.00 \text{ mWb}$

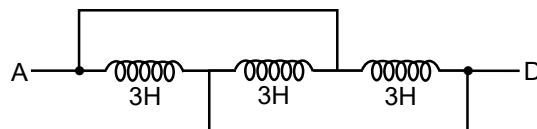
11. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B

- (1) remains stationary  
 (2) is attracted by the loop-A  
 (3) is repelled by the loop-A  
 (4) rotates about its CM, with CM fixed

12. An induction coil stores  $32 \text{ joules}$  of magnetic energy and dissipates energy as heat at the rate of  $320 \text{ watts}$  when a current of  $4 \text{ amperes}$  is passed through it. Find the time constant of the circuit when the coil is joined across a battery.

(1)  $0.2 \text{ s}$  (2)  $0.1 \text{ s}$   
 (3)  $0.3 \text{ s}$  (4)  $0.4 \text{ s}$

13. The inductance between A and D is :



(1)  $3.66 \text{ H}$  (2)  $9 \text{ H}$   
 (3)  $0.66 \text{ H}$  (4)  $1 \text{ H}$

14. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon :

- (1) the rates at which currents are changing in the two coils  
 (2) relative position and orientation of the two coils  
 (3) the materials of the wires of the coils  
 (4) the currents in the two coils

15. When the current changes from  $+2 \text{ A}$  to  $-2 \text{ A}$  in  $0.05 \text{ second}$ , an emf of  $8 \text{ V}$  is induced in a coil. The coefficient of self-induction of the coil is :

(1)  $0.2 \text{ H}$  (2)  $0.4 \text{ H}$   
 (3)  $0.8 \text{ H}$  (4)  $0.1 \text{ H}$

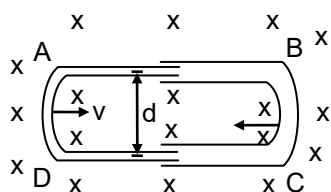
16. A coil having  $n$  turns and resistance  $R \Omega$  is connected with a galvanometer of resistance  $4R \Omega$ . This combination is moved in time  $t$  seconds from a magnetic field  $W_1$  Weber to  $W_2$  Weber. The induced current in the circuit is :

(1)  $\frac{(W_2 - W_1)A}{5Rnt}$  (2)  $-\frac{n(W_2 - W_1)A}{5Rt}$   
 (3)  $-\frac{(W_2 - W_1)A}{Rnt}$  (4)  $-\frac{n(W_2 - W_1)A}{Rt}$

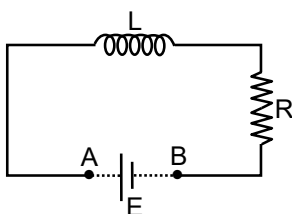
17. A metal conductor of length  $1 \text{ m}$  rotates vertically about one of its ends at angular velocity  $5 \text{ radians per second}$ . If the horizontal component of earth's magnetic field is  $0.2 \times 10^{-4} \text{ T}$ , then the emf developed between the two ends of the conductor is :

(1)  $5 \mu\text{V}$  (2)  $50 \mu\text{V}$   
 (3)  $5 \text{ mV}$  (4)  $50 \text{ mV}$

18. One conducting u tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field  $B$  is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed  $v$ , then the emf induced in the circuit in terms of  $B$ ,  $\ell$  and  $v$ , where  $\ell$  is the width of each tube, will be :



- (1)  $B\ell v$  (2)  $-B\ell v$   
 (3) zero (4)  $2B\ell v$
19. An inductor ( $L = 100 \text{ mH}$ ), a resistor ( $R = 100 \Omega$ ) and a battery ( $E = 100 \text{ V}$ ) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit, 1 ms after the short circuit is :

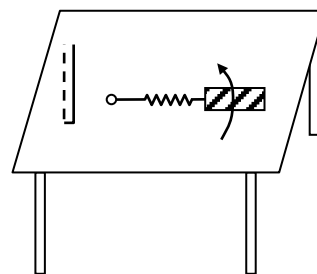


- (1) 1 A (2)  $1/e \text{ A}$   
 (3)  $e \text{ A}$  (4) 0.1 A
20. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area  $A = 10 \text{ cm}^2$  and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ( $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$ ) :
- (1)  $4.8\pi \times 10^{-4} \text{ H}$  (2)  $4.8\pi \times 10^{-5} \text{ H}$   
 (3)  $2.4\pi \times 10^{-4} \text{ H}$  (4)  $2.4\pi \times 10^{-5} \text{ H}$
21. A horizontal straight wire 20 m long extending from to east to west falling with a speed of 5.0 m/s, at right angles to the horizontal component of the

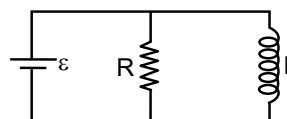
earth's magnetic field  $0.30 \times 10^{-4} \text{ Wb/m}^2$ . The instantaneous Value of the e.m.f. induced in the wire will be :

- (1) 3 mV (2) 4.5 mV  
 (3) 1.5 mV (4) 6.0 mV

22. A metallic rod of length ' $\ell$ ' is tied to a string of length  $2\ell$  and made to rotate with angular speed  $\omega$  on a horizontal table with one end of the string fixed. If there is a vertical magnetic field ' $B$ ' in the region, the e.m.f. induced across the ends of the rod is:



- (1)  $\frac{2B\omega\ell^2}{2}$  (2)  $\frac{3B\omega\ell^2}{2}$   
 (3)  $\frac{4B\omega\ell^2}{2}$  (4)  $\frac{5B\omega\ell^2}{2}$
23. An infinitely long cylindrical conducting rod is kept along + Z direction. A constant magnetic field is also present in + Z direction. Then current induced will be
- (1) 0  
 (2) along +z direction  
 (3) along clockwise as seen from + Z  
 (4) along anticlockwise as seen from + Z
24. The battery shown in the figure is ideal. The values are  $\varepsilon = 10 \text{ V}$ ,  $R = 5 \Omega$ ,  $L = 2 \text{ H}$ . Initially the current in the inductor is zero. The current through the battery at  $t = 2 \text{ s}$  is :



- (1) 12 A (2) 7 A  
 (3) 3 A (4) none of these

ANSWER KEY																								
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Ans.	1	2	1	1	4	1	2	3	4	4	3	1	4	2	4	2	2	4	2	3	1	4	1	1

### Exercise – III (Previous Year Question)

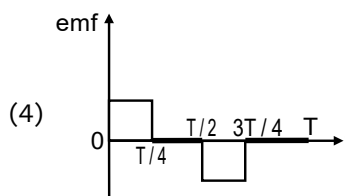
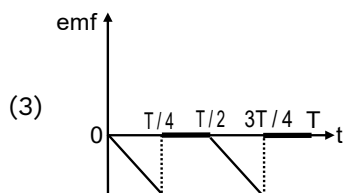
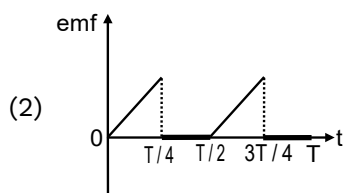
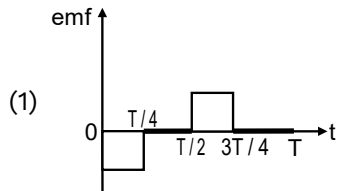
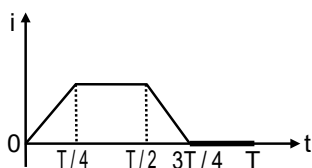
1. A 220 volt input is supplied to a transformer. The output circuit draws a current of 2.0 ampere at 440 volts. If the efficiency of the transformer is 80%, the current drawn by the primary windings of the transformer is:

**[AIPMT Pre-2010]**

- (1) 3.6 ampere                      (2) 2.8 ampere  
(3) 2.5 ampere                      (4) 5.0 ampere

2. The current  $i$  in a coil varies with time as shown in the figure. The variation of induced emf with time would be:

**[AIPMT (Screening) 2011]**

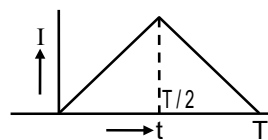


3. A coil of resistance  $400\Omega$  is placed in a magnetic field. If the magnetic flux  $\phi$  (wb) linked with the coil varies with time  $t$  (sec) as  $\phi = 50t^2 + 4$ . The current in the coil at  $t = 2$  sec is:

**[AIPMT\_Pre\_2012]**

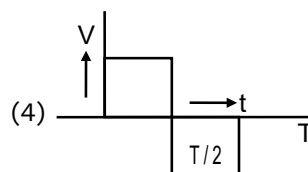
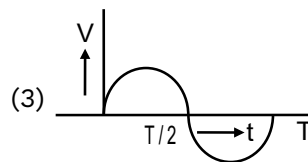
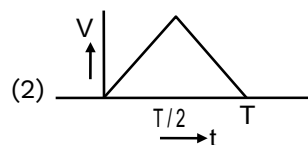
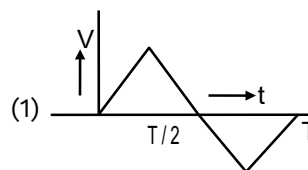
- (1) 0.5A                              (2) 0.1 A  
(3) 2 A                                (4) 1 A

4. The current ( $I$ ) in the inductance is varying with time according to the plot shown in figure.

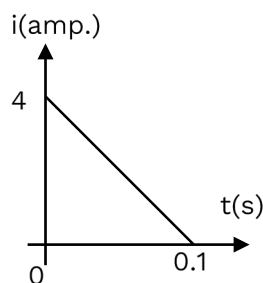


Which one of the following is the correct variation of voltage with time in the coil?

**[AIPMT\_Pre\_2012]**



5. In a coil of resistance  $10\ \Omega$ , the induced current developed by changing magnetic flux through it, is shown in figure as a function of time. The magnitude of change in flux through the coil in Weber is : **[AIPMT 2012 (Mains)]**

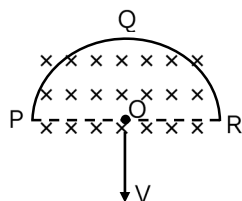


- (1) 8 (2) 2  
(3) 6 (4) 4

6. A wire loop is rotated in a magnetic field. The frequency of change of direction of the induced e.m.f. is: **[NEET\_2013]**

- (1) twice per revolution  
(2) four times per revolution  
(3) six times per revolution  
(4) once per revolution

7. A thin semicircular conducting ring (PQR) of radius 'r' is falling with its plane vertical in a horizontal magnetic field B, as shown in figure. The potential difference developed across the ring when its speed is v, is: **[NEET-2014]**

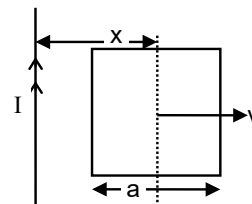


- (1)  $\pi r B v$  and R is at higher potential  
(2)  $2r B v$  and R is at higher potential  
(3) Zero  
(4)  $B v \pi r^2$  and P is at higher potential

8. A transformer having efficiency of 90% is working on 200 V and 3kW power supply. If the current in the secondary coil is 6 A, the voltage across the secondary coil and the current in the primary coil respectively are : **[NEET-2014]**

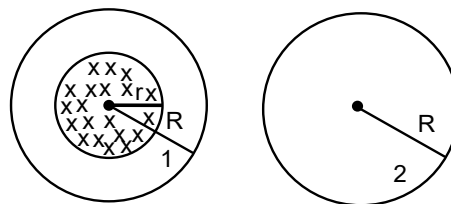
- (1) 450 V, 13.5 A (2) 600 V, 15 A  
(3) 300 V, 15 A (4) 450 V, 15 A

9. A conducting square frame of side 'a' and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity V. The emf induced in the frame will be proportional to : **[NEET-2015]**



- (1)  $\frac{1}{(2x-a)^2}$  (2)  $\frac{1}{(2x+a)^2}$   
(3)  $\frac{1}{(2x-a)(2x+a)}$  (4)  $\frac{1}{x^2}$

10. A uniform magnetic field is restricted with a region of radius r. The magnetic field changes with time at a rate  $\frac{d\vec{B}}{dt}$ . Loop 1 of radius  $R > r$  encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure below. Then the e.m.f. generated is:- **[NEET-2016]**



- (1)  $-\frac{d\vec{B}}{dt} \pi R^2$  in loop 1 and zero in loop 2  
 (2)  $-\frac{d\vec{B}}{dt} \pi R^2$  in loop 1 and zero in loop 2  
 (3) Zero in loop 1 and zero in loop 2  
 (4)  $-\frac{d\vec{B}}{dt} \pi R^2$  in loop 1 and  $-\frac{d\vec{B}}{dt} \pi R^2$  in loop 2

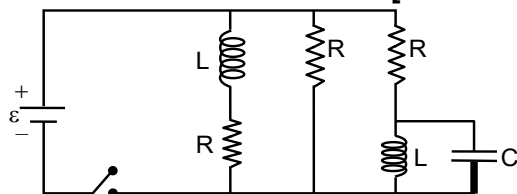
11. A long solenoid has 1000 turns. When a current of 4A flows through it, the magnetic flux linked with each turn of the solenoid is  $4 \times 10^{-3}$  Wb. The self inductance of the solenoid is:

[NEET - 2016]

- (1) 4 H (2) 3 H  
 (3) 2 H (4) 1 H

12. Figure shows a circuit that contains three identical resistors with resistance  $R = 9.0 \Omega$  each, two identical inductors with inductance  $L = 2.0$  mH each, and an ideal battery with emf = 18 V. The current 'i' through the battery just after the switch closed is :

[NEET - 2017]



- (1) 2 mA (2) 4 A  
 (3) 2 A (4) 0 Ampere

13. A long solenoid of diameter 0.1 m has  $2 \times 10^4$  turns per meter. At the centre of the solenoid, a coil of 100 turns and radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduced at a constant rate to 0 A from 4 A in 0.05 s. If the resistance of the coil is  $10 \pi^2 \Omega$ . the total charge flowing through the coil during this times is :

[NEET - 2017]

- (1)  $32 \pi \mu C$  (2)  $16 \pi \mu C$   
 (3)  $32 \mu C$  (4)  $16 \mu C$

14. The magnetic potential energy stored in a certain inductor is 25 mJ, when the current in the inductor is 60 mA. this inductor is of inductance [NEET - 2018]

- (1) 0.138 H (2) 138.88 H  
 (3) 1.389 H (4) 13.89 H

15. The magnetic flux linked with a coil (in Wb) is given by the equation

$$\phi = 5t^2 + 3t + 16$$

The magnitude of induced emf in the coil at the fourth second will be

[NEET-Covid-2020]

- (1) 33 V (2) 43 V  
 (3) 108 V (4) 10 V

16. A wheel with 20 metallic spokes each 1 m long is rotated with a speed of 120 rpm in a plane perpendicular to a magnetic field of 0.4 G. The induced emf between the axle and rim of the wheel will be, ( $1 \text{ G} = 10^{-4} \text{ T}$ )

[NEET-Covid-2020]

- (1)  $2.51 \times 10^{-4} \text{ V}$  (2)  $2.51 \times 10^{-5} \text{ V}$   
 (3)  $4.0 \times 10^{-5} \text{ V}$  (4) 2.51 V

17. Two conducting circular loops of radii  $R_1$  and  $R_2$  are placed in the same plane with their centres coinciding. If  $R_1 \gg R_2$ , the mutual inductance M between them will be directly proportional to: [NEET-2021]

- (1)  $\frac{R_1}{R_2}$  (2)  $\frac{R_2}{R_1}$   
 (3)  $\frac{R_1^2}{R_2}$  (4)  $\frac{R_2^2}{R_1}$

18. A square loop of side 1 m and resistance  $1\Omega$  is placed in a magnetic field of 0.5 T. If the plane of loop is perpendicular to the direction of magnetic field, the magnetic flux through the loop is:

[NEET-2022]

- (1) 2 weber (2) 0.5 weber  
 (3) 1 weber (4) zero weber

### ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	4	1	1	4	2	1	2	4	3	2	4	2	3	4	2	1	4	2