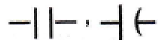
Summary

I. $q \propto V \Rightarrow q = CV$

q: Charge on positive plate of the capacitor
C: Capacitance of capacitor
V: Potential difference between positive and negative plates

II. Representation of capacitor



III. It is a scalar quantity having dimensions

$$[C] = \left[M^{-1}L^{-2}T^{4}A^{2}\right]$$

IV. S.I. Unit is Farad. (F).

V. Energy stored in the capacitor:

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{QV}{2}$$

VI. Energy density

$$= \frac{1}{2}\in_o \in_r E^2 = \frac{1}{2}\in_0 KE^2$$

$K = \in_r =$ Relative permittivity of the medium (Dielectric constant)
For vacuum, energy density

$$= \frac{1}{2}\in_0 E^2$$

VII. Types of Capacitors:

(a) Parallel Plate capacitor

$$C = \frac{\in_0 \in_r A}{d} = K\frac{\in_0 A}{d}$$

A: Area of Plates
d: distance between the plates (<< size of plate)

(b) Spherical Capacitor:

- Capacitance of an isolated spherical Conductor (hollow or solid)

$$C = 4\pi \in_0 \in_r R$$

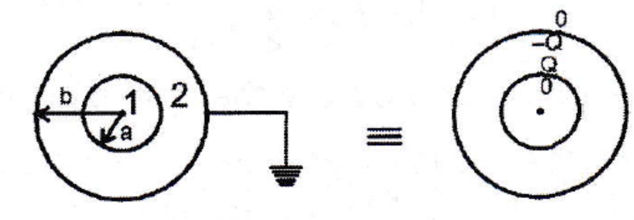
R = Radius of the spherical conductor

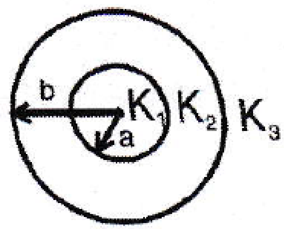
- Capacitance of spherical capacitor

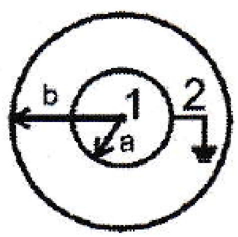
$$C = 4\pi \in_0 \frac{ab}{(b-a)}$$

$$C = \frac{4\pi \in_0 K_2 ab}{(b-a)}$$

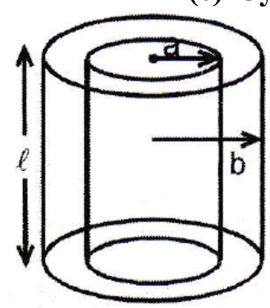
$$C=\frac{4\pi\in_0 b^2}{(b-a)}$$







(c) Cylindrical Capacitor:



$\ell >> \{a, b\}$

Capacitance per unit length $=\dfrac{2\pi\in_0}{\ell n(b/a)} F/m$

VIII. Capacitance of capacitor depends on

(a) Area of plates

(b) Distance between the plates

(c) Dielectric medium between the plates.

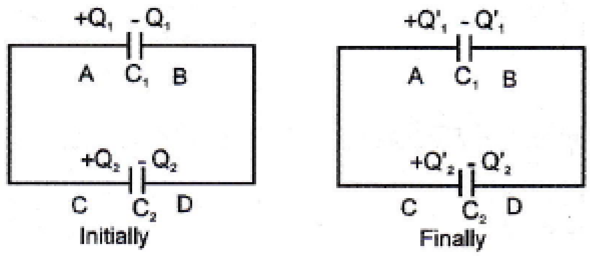
IX. Electric field intensity between the plates of capacitor $E=\dfrac{\sigma}{\in_0}=\dfrac{V}{d}$

σ : Surface change density

X. Force experienced by any plate of capacitor : $F = \dfrac{q^2}{2A \epsilon_0}$

- **Distribution of Charges on Connecting two Charged Capacitors:**

When two capacitors are C_1 and C_2 are connected as shown in figure



$$\text{(a) Common potential} : V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total Capacitance}}$$

$$\text{(b) } Q_1' = C_1V = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)$$

$$Q_2' = C_2V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

(c) Heat loss during redistribution:

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

Note:

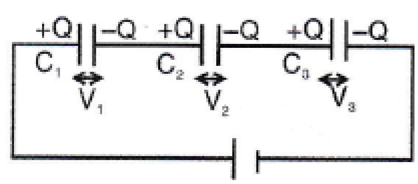
(i) When plates of similar charges are connected with each other (+ with + and - with -) then put all values (Q_1,Q_2, V_1, V_2) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with -) then take charge and potential of one of the plate to be negative.

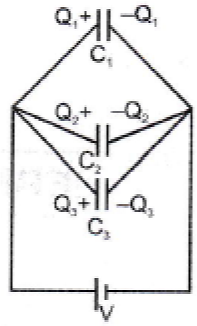
- **Combination of capacitor:**

(i) Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \,;\; V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$



(ii) Parallel Combination:

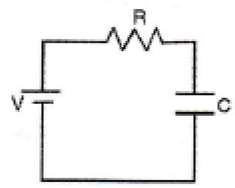


$C_{eq} = C_1 + C_2 + C_3 + ...$

$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

- **Charging and Discharging of a capacitor:**

  (i) Charging of Capacitor (Capacitor initially uncharged):



$q = q_0\left(1 - e^{-t/\tau}\right)$

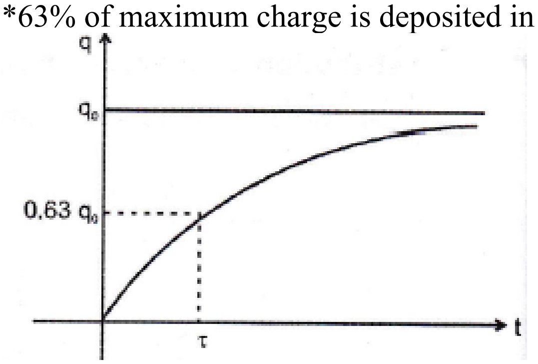
q_0= Charge on the capacitor at steady

State $\Rightarrow q_0 = CV$

τ : Time constant $= CR_{eq}$.

$i = \frac{q_0}{\tau} e^{-t/\tau} = \frac{V}{R} e^{-t/\tau}$

*63% of maximum charge is deposited in one time constant.



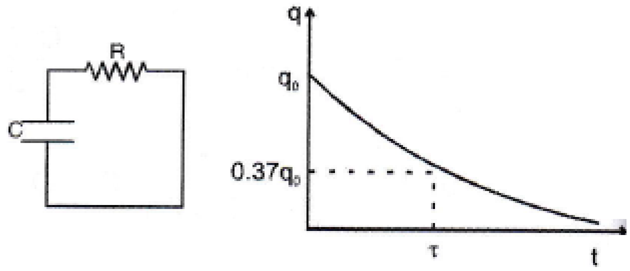
  (ii) Discharging of Capacitor:

$q = q_0 e^{-t/\tau}$

q_0 = initial charge on the capacitor

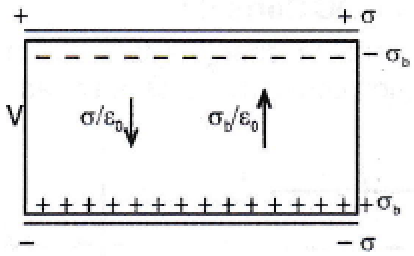
$i = \frac{q_0}{\tau} e^{-t/\tau}$

63% of discharging is complete in one time constant.



- **Capacitor with dielectric:**

  (i) Capacitance in the presence of dielectric



$$C = \frac{K \in_0 A}{d} = KC_0$$

C_0 = Capacitance in the absence of dielectric.:

(ii) If thickness of dielectric slab is t, then its capacitance

$C = \dfrac{\in_0 A}{\left(d - t + t/k\right)}$, where k is the dielectric constant of slab.

It does not depend on the position of the slab.
k = 1 for vacuum or air.
k = ∞ for metals.

(iii) $$E_{in} = E - E_{ind} = \frac{\sigma}{\in_0} - \frac{\sigma_b}{\in_0} = \frac{\sigma}{K \in_0} = \frac{V}{d}$$

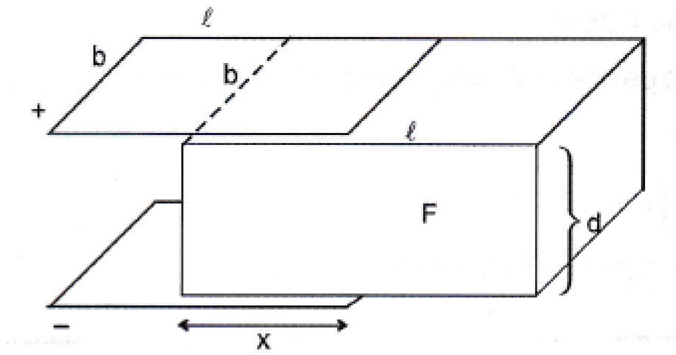
$E = \dfrac{\sigma}{\in_0}$ Electric field in the absence of dielectric

E_{ind}: Induced electric field

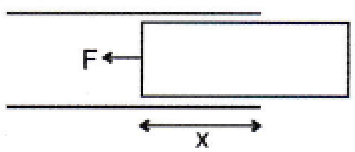
(iv) $\sigma_b = \sigma\left(1 - \dfrac{1}{K}\right)$ (induced charge density)

- **Force on dielectric:**

  (i) When battery is connected $F = \dfrac{\in_0 b\left(K - 1\right) V^2}{2d}$



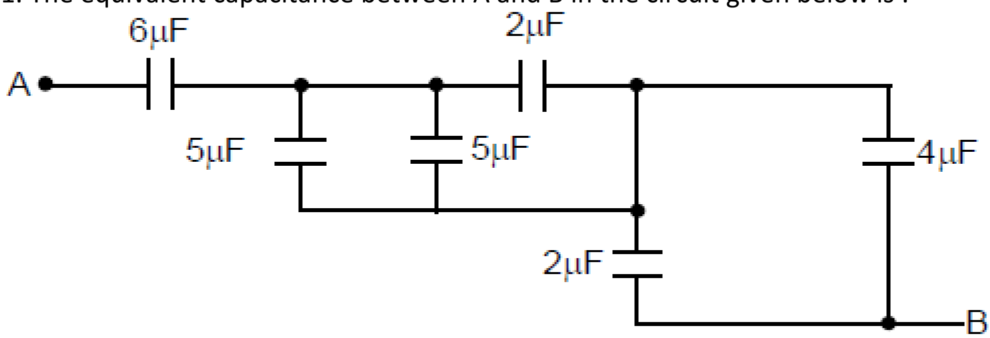
(ii) When battery is not connected $F = \frac{Q^2}{2C^2}\frac{dC}{dx}$



Force on the dielectric will be zero when the dielectric is fully inside.

Practical Questions

1. The equivalent capacitance between A and B in the circuit given below is :

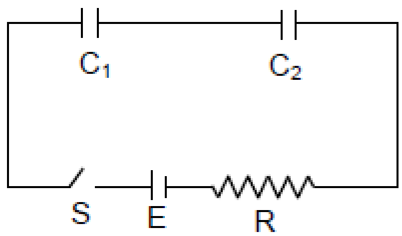


(a) $3.6\mu F$
(b) $2.4\mu F$
(c) $4.9\mu F$
(d) $5.4\mu F$

2. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material constant $K = \frac{5}{3}$ s inserted between the plates, the magnitude of the induced charge will be:

(a) 0.9 n C
(b) 1.2 n C
(c) 0.3 n C
(d) 2.4 n C

3. In the following circuit the switch S is closed at t = 0. The charge on the capacitor C1 as a function of time will be given by $\left(C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \right)$



(a) $C_1 E[1 - exp(-tR / C_1)]$
(b) $C_{eq} E\ exp(-t / RC_{eq})$
(c) $C_{eq} E\ [1 - exp(-t / RC_{eq})]$
(d) $C_2 E\ [1 - \exp(-t / RC_2)]$

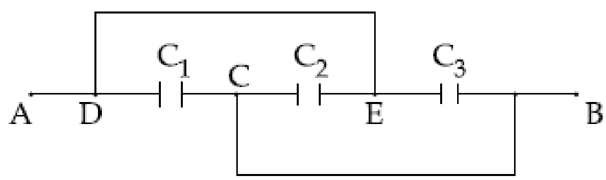
4. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(a) 2
(b) 16
(c) 24
(d) 32

5. The energy stored in the electric field produced by a metal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be: $\left[\text{Take:} \frac{1}{4\pi \in_0} = 9 \times 10^9 N - m^2 / C^2\right]$

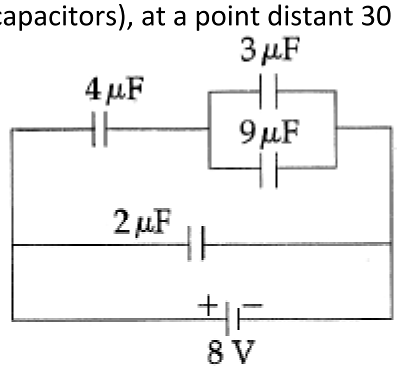
(a) 20 mm
(b) 32 mm
(c) 28 mm
(d) 16 mm

6. A combination of parallel plate capacitors is maintained at a certain potential difference.



When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.
(a) 3
(b) 4
(c) 5
(d) 6

7. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal :

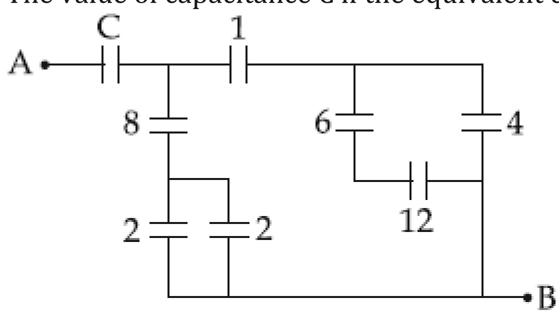


(a) 240 N/C
(b) 360 N/C

(c) 420 N/C
(d) 480 N/C

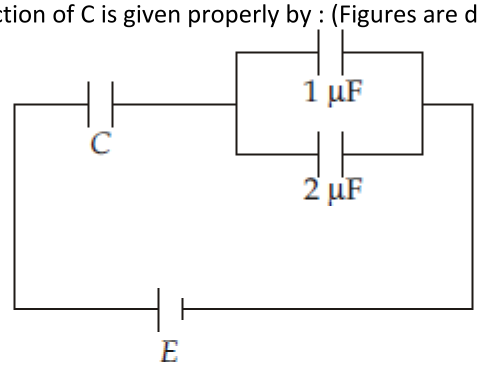
8. Three capacitors each of $4\mu F$ are to be connected in such a way that the effective capacitance is $6\mu F$. This can be done by connecting them :
(a) all in series
(b) two in series and one in parallel
(c) all in parallel
(d) two in parallel and one in series

9. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is :

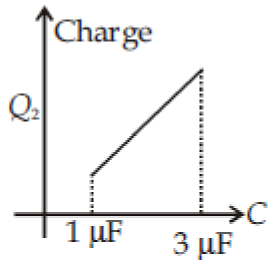


(a) $\frac{31}{23}\mu F$
(b) $\frac{32}{23}\mu F$
(c) $\frac{33}{23}\mu F$
(d) $\frac{34}{23}\mu F$

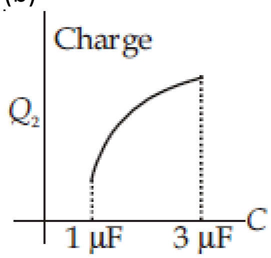
10. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of C is given properly by : (Figures are drawn schematically and are not to scale)



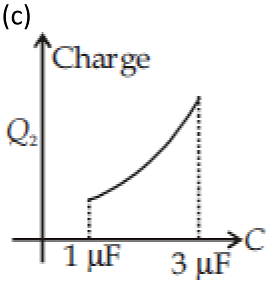
(a)



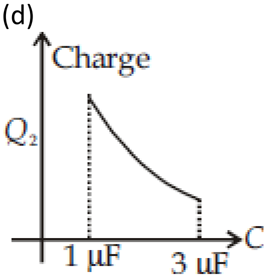
(b)



(c)



(d)



11. If the capacitance of a nanocapacitor is measured in terms of a unit 'u' made by combining the electronic charge 'e', Bohr radius 'a_0', Planck's constant 'h' and speed of light 'c' then:

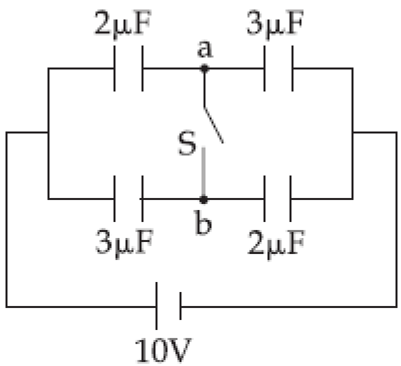
(a) $u = \frac{e^2 c}{h a_0}$

(b) $u = \frac{e^2 c}{c a_0}$

(c) $u = \frac{e^2 a_0}{hc}$

(d) $u = \frac{hc}{e^2 a_0}$

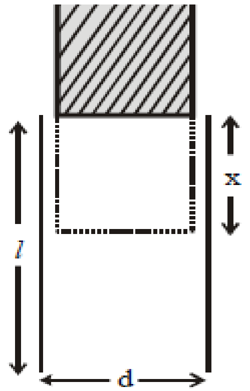
12. In figure is shown a system of four capacitors connected across a 10 V battery. Charge that will flow from switch S when it is closed is :



(a) 5 μC from b to a
(b) 20 μC from a to b
(c) 5 μC from a to b
(d) zero

13. Three capacitances, each of 3 μF, are provided. These cannot be combined to provide the resultant capacitance of :
(a) 4.5 μF
(b) 1 μF
(c) 2 μF
(d) 6 μF

14. parallel plate capacitor is made of two plates of length l, width ω and separated by distance d. A dielectric slab (dielectric constant K) that fits exactly between the plates is held near the edge of the plates. It is pulled into the capacitor by a force $F = -\frac{\partial U}{\partial x}$ where U is the enerey of the capacitor when dielectric is inside the capacitor up to distance x (See figure). If the charge on the capacitor is Q then the force on the dielectric when it is near the edge is



(a) $\frac{Q^2 d}{2\omega l^2 \varepsilon_0} K$

(b) $\frac{Q^2 \omega}{2dl^2 \varepsilon_0} (K-1)$

(c) $\frac{Q^2 d}{2\omega l^2 \varepsilon_0} (K-1)$

(d) $\frac{Q^2 \omega}{2dl^2 \varepsilon_0} K$

15. The gap between the plates of a parallel plate capacitor of area A and distance between plates d, is filled with a dielectric whose permittivity varies linearly from $\in_1$ at one plate to $\in_2$ at the other. The capacitance of capacitor is :

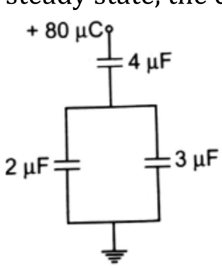
(a) $\in_0 (\in_2 + \in_1) A / 2d$

(b) $\in_0 A / [d \ln(\in_2 / \in_1)]$

(c) $\in_0 (\in_1 + \in_2)] A / d$

(d) $\in_0 (\in_2 - \in_1)] A / [d \ln(\in_2 / \in_1)]$

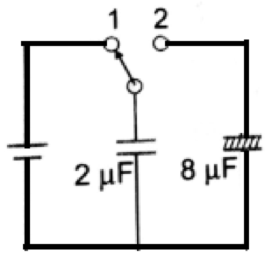
16. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



(a) + 32 μC
(b) + 40 μC

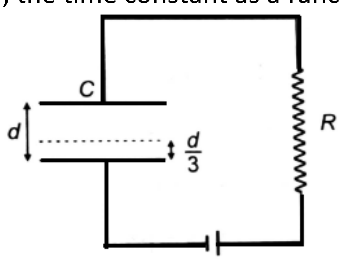
(c) + 48 μC
(d) + 80 μC

17. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



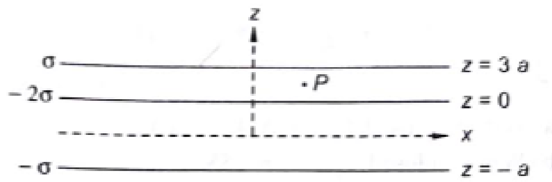
(a) 0%
(b) 20%
(c) 75%
(d) 80%

18. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is d / 3 initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time t is



(a) $\dfrac{6\varepsilon_0 R}{5d + 3vt}$
(b) $\dfrac{(15d + 9vt)\varepsilon_0 R}{2d^2 - 3dvt - 9v^2t^2}$
(c) $\dfrac{6\varepsilon_0 R}{5d - 3vt}$
(d) $\dfrac{(15d - 9vt)\varepsilon_0 R}{2d^2 + 3dvt - 9v^2t^2}$

19. Three infinitely long charge sheets are placed as shown in figure. The electric field at point P is



(a) $\dfrac{2\sigma}{\varepsilon_0}\hat{k}$

(b) $-\dfrac{2\sigma}{\varepsilon_0}\hat{k}$

(c) $\dfrac{4\sigma}{\varepsilon_0}\hat{k}$

(d) $-\dfrac{4\sigma}{\varepsilon_0}\hat{k}$

20. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. Likely charged plates are then connected. Then, the decrease in energy of the combined system is

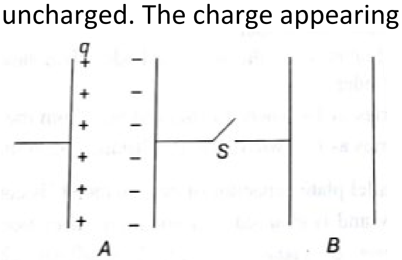
(a) $\dfrac{1}{4}C(V_1^2 - V_2^2)$

(b) $\dfrac{1}{4}C(V_1^2 + V_2^2)$

(c) $\dfrac{1}{4}C(V_1 - V_2)^2$

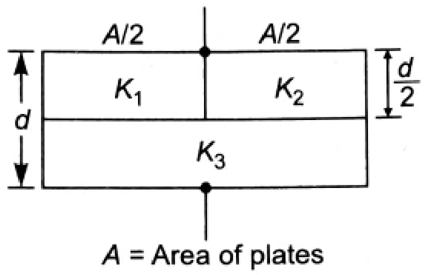
(d) $\dfrac{1}{4}C(V_1 + V_2)^2$

21. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(a) zero
(b) q/2
(c) q
(d) 2q

22. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor then its dielectric constant K is given by



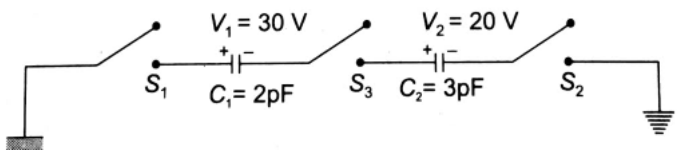
(a) $\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{2K_3}$

(b) $\frac{1}{K}=\frac{1}{K_1+K_2}+\frac{1}{2K_3}$

(c) $\frac{1}{K}=\frac{K_1K_2}{K_1+K_2}+2K_3$

(d) $\frac{1}{K}=\frac{K_1K_3}{K_1+K_3}+\frac{K_2K_3}{K_2+K_3}$

23. For the circuit shown, which of the following statements is true?



(a) With S_1 closed, V_1 = 15V, V_2 = 20 V
(b) With S_3 closed, V_1 = V_2 = 25 V
(c) With S_1 and S_2 closed V_1 = V_2 = 0
(d) With S_3 closed, V_1 = 30V, V_2 = 20 V

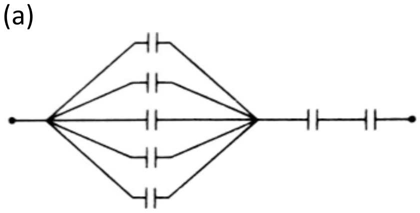
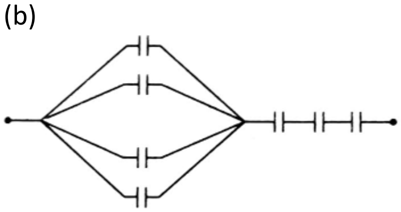
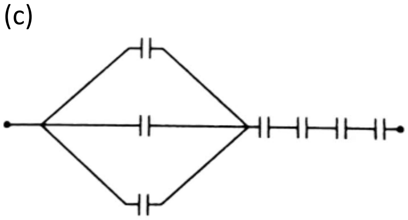
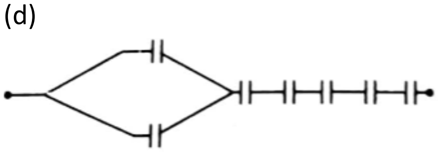
24. Two identical metal plates are given positive charges Q_1 and Q_2 (< Q_1) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is
(a) $(Q_1 + Q_2) / 2C$
(b) $(Q_1 + Q_2) / C$
(c) $(Q_1 - Q_2) / C$
(d) $(Q_1 - Q_2) / 2C$

25. A parallel combination of 0.1 MΩ resistor and a 10μF capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged upto 0.75 V is approximately (in second)
(a) infinite
(b) $\log_e 2$
(c) $\log_{10} 2$
(d) zero

26. The magnitude of electric field E in the annular region of a charged cylindrical capacitor

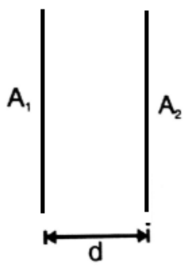
(a) is same throughout
(b) is higher near the outer cylinder than near the inner cylinder
(c) varies as 1/r where r is the distance from the axis
(d) varies as $1/r^2$ where r is the distance from the axis

27. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charged to a potential difference 2V. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is
(a) zero
(b) $\frac{3}{2}CV^2$
(c) $\frac{25}{6}CV^2$
(d) $\frac{9}{2}CV^2$

28. Seven capacitors each of capacitance 2μF are connected in a configuration to obtain an effective capacitance $\frac{10}{11}\mu F$ .Which of the following combination will achieve the desired result ?
(a)

(b)

(c)

(d)


29. Metallic sphere of radius R is charged to potential V. Then charge q is proportional to
(a) V
(b) R
(c) Both
(d) None

30. The capacitance of the capacitors of plate areas A_1 and A_2 ($A_1 < A_2$) at a distance d is

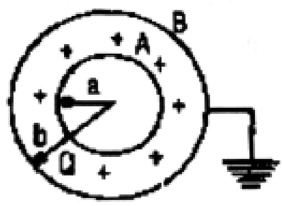


(a) $\frac{\varepsilon_0 A_1}{d}$
(b) $\frac{\varepsilon_0 A_2}{d}$
(c) $\frac{\varepsilon_0 (A_1 + A_2)}{2d}$
(d) $\frac{\varepsilon_0 \sqrt{A_1 A_2}}{d}$

31. Two capacitors, 3μF and 4μF, are individually charged across a 6V battery. After being disconnected from the battery, they are connected together with the negative plate of one attached to the positive plate of the other. What is the common potential ?
(a) 6V
(b) $\left(\frac{6}{7}\right)V$
(c) 2V
(d) $\left(\frac{3}{2}\right)V$

32. A capacitor of capacitance 10 μF is charged to a potential of 100 V. Now connecting it in parallel with an uncharged capacitor, the resultant potential difference becomes 40 volt. The capacitance of this capacitor
(a) 2.5 μF
(b) 5 μF
(c) 10 μF
(d) 15 μF

33. Two spherical conductors A and B of radii a and b (b > a) are placed concentrically in air. A is given a charge +Q while B is earthed. Then the equivalent capacitance of the system is



(a) $4\pi\varepsilon_0 \frac{ab}{b-a}$

(b) $4\pi\varepsilon_0 (b+a)$

(c) $4\pi\varepsilon_0 b$

(d) $\frac{4\pi\varepsilon_0 b^2}{b-a}$

34. A parallel plate air capacitor is charged to a potential difference of V volts. After disconnecting the charging battery the distance between the plates of the capacitor is increased using an insulating handle. As a result the potential difference between the plates:
(a) decreases
(b) does not change
(c) becomes zero
(d) increases

35. Force acting upon a charged particle kept between the plates of a charged condenser is F. If one of the plates of the condenser is removed, force acting on the same particle becomes
(a) 0
(b) F/2
(c) F
(d) 2F

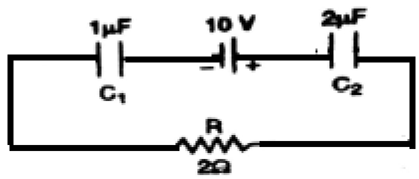
36. A capacitor of 6 μF is charged to such an extent that the potential difference between the plates becomes 50 V. The work done in this process will be
(a) 7.5×10^{-2} J
(b) 7.5×10^{-3} J
(c) 3×10^{-6} J
(d) 3×10^{-3} J

37. What fraction of the energy drawn from the charging battery is stored in a capacitor
(a) 100 %
(b) 75%
(c) 50%
(d) 25%

38. How do adjust three capacitors to get high energy on same potential
(a) Two parallel and one in series

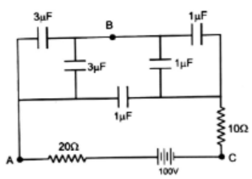
(b) three in series
(c) three in parallel
(d) none of these

39. A 10 V battery is connected to two capacitors C_1 and C_2 and a resistor R. When the capacitors are fully charged, then the potential difference across the resistor will be



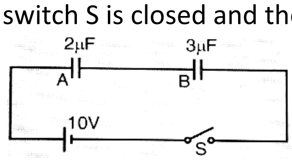
(a) 10 V
(b) 5 V
(c) 3.33 V
(d) zero

40. In the given figure, the potential difference between A and B in steady state will be



(a) 20 V
(b) 25 V
(c) 75 V
(d) 100 V

41. The capacitors A and B are connected in series with a battery as shown in the figure. When the switch S is closed and the two capacitors get charged fully, then -



(a) The potential difference across the plates of A is 4V and across the plates of B is 6V
(b) The potential difference across the plates of A is 6V and across the plates of B is 4V
(c) The ratio of electric energies stored in A and B is 2 : 3
(d) The ratio of charges on A and B is 3 : 2

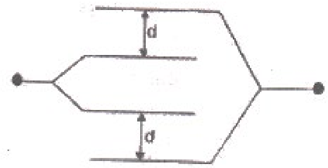
42. Two capacitors of 2μF and 3μF are connected in series. The potential at point A is 1000 volt and the other plate is 3μF capacitor is earthed. The potential at point B is

(a) 300 volt
(b) 500 volt
(c) 600 volt

(d) 400 volt

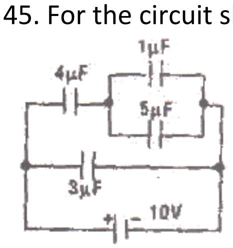
43. Capacity of a parallel plate condenser is 10μF when the distance between its plates is 8 cm. If the distance between the plates is reduced to 4 cm, its capacity will be:
(a) 10μF
(b) 15μF
(c) 20μF
(d) 40μF

44. Four plates are arranged as shown in the diagram. If area of each plate is A and the distance between two neighbouring parallel plates is d, then the capacitance of this system will be



(a) $\frac{4 \in_0 A}{d}$
(b) $\frac{3 \in_0 A}{d}$
(c) $\frac{2 \in_0 A}{d}$
(d) $\frac{\in_0 A}{d}$

45. For the circuit shown in figure, the equivalent capacitance of the combination is



(a) 13μF
(b) $\left(\frac{30}{13}\right)\mu F$
(c) $\left(\frac{54}{10}\right)\mu F$
(d) 1μF

Answer Keys

1. (b) 2. (b) 3. (c) 4. (d) 5. (c) 6. (c) 7. (c) 8. (b) 9. (b) 10. (b) 11. (c) 12. (a) 13. (d) 14. (c) 15. (d) 16. (c) 17. (d) 18. (a) 19. (b) 20. (c) 21. (a) 22. (d) 23. (d) 24. (d) 25. (d) 26. (c) 27. (b) 28. (a) 29. (c) 30. (a) 31. (b) 32. (d) 33. (a) 34. (d) 35. (b) 36. (b) 37. (c) 38. (c) 39. (d) 40. (b) 41. (b) 42. (d) 43. (c) 44. (c) 45. (c)

"Detail solutions are mentioned in the content library"