Basic Mathematics and Logarithm

NUMBER SYSTEM

(i) Natural numbers : The counting numbers 1, 2, 3, 4, ... are called Natural Numbers. The set of natural numbers is denoted by N.

Thus N = {1, 2, 3, 4, ...}.

(ii) Whole numbers : Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by W.

Thus W = {0, 1, 2, ...}

(iii) Integers : The numbers ... – 3, – 2, – 1, 0, 1, 2, 3 .... are called integers and the set is denoted by I or Z.

Thus I (or Z) = {.. – 3, – 2, – 1, 0, 1, 2, 3...}

Note :

(a) Positive integers I^+ = {1, 2, 3 ....} = N

(b) Negative integers I^- = {....., –3, –2, –1}.

(c) Non-negative integers (whole numbers) = {0, 1, 2, ...}.

(d) Non-positive integers = {......, –3, –2, –1, 0}.

(iv) Even integers : Integers which are divisible by 2 are called even integers.

e.g. 0, ± 2, ± 4,.......

(v) Odd integers : Integers which are not divisible by 2 are called odd integers.

e.g. ± 1, ± 3, ± 5, ± 7......

(vi) Prime numbers : Natural numbers which are divisible by 1 and itself only are called prime numbers.

e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

(vii) Composite number : Let 'a' be a natural number, 'a' is said to be composite if, it has atleast three distinct factors.

e.g. 4, 6, 8, 9, 10, 12, 14, 15 ....

Note :

(i) 1 is neither a prime number nor a composite number.

(ii) Numbers which are not prime are composite numbers (except 1).

(iii) '4' is the smallest composite number.

(iv) '2' is the only even prime number.

(viii) Co-prime numbers : Two natural numbers (not necessarily prime) are called coprime, if their H.C.F (Highest common factor) is one.

e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.

These numbers are also called as **relatively prime** numbers.

Note :

(a) Two prime number(s) are always co-prime but converse need not be true.

(b) Consecutive natural numbers are always co-prime numbers.

(ix) Twin prime numbers : If the difference between two prime numbers is two, then the numbers are called twin prime numbers.

e.g. {3, 5}, {5, 7}, {11, 13}, {17, 19}, {29, 31}

Note :

Number between twin prime numbers is divisible by 6 (except (3, 5)).

(x) Rational numbers : All the numbers that can be represented in the form p/q, where p and q are integers and q ≠ 0, are called rational numbers and their set is denoted by Q. Thus Q = $\{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\}$. It may be noted that every integer is a rational number since it can be written as p/q. It may be noted that all recurring decimals are rational numbers.

Note :

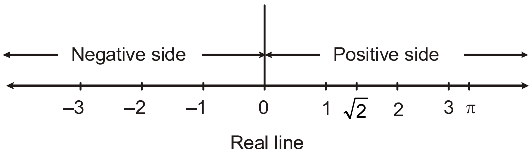
Maximum number of different decimal digits in $\frac{p}{q}$ is equal to q, i.e. $\frac{11}{9}$ will have maximum of 9 different decimal digits.

(xi) Irrational numbers : The numbers which can not be expressed in p/q form where p, q ∈ I and q ≠ 0 i.e. the numbers which are not rational are called irrational numbers and their set is denoted by Q^c. (i.e. complementary set of Q) e..g. $\sqrt{2}$, 1 + $\sqrt{3}$ etc. Irrational numbers can not be expressed as recurring decimals.

Note :

e ≈ 2.71 is called Napier's constant and π ≈ 3.14 are irrational numbers.

(xii) Real numbers : Numbers which can be expressed on number line are called real numbers. The complete set of rational and irrational numbers is the set of real numbers and is denoted by R. Thus R = $Q \cup Q^C$.



All real numbers follow the order property i.e. if there are two distinct real numbers a and b then either $a < b$ or $a > b$.

Note :

(a) Integers are rational numbers, but converse need not be true.

(b) Negative of an irrational number is an irrational number.

(c) Sum of a rational number and an irrational number is always an irrational number

e.g. $2 + \sqrt{3}$

(d) The product of a non zero rational number & an irrational number will always be an irrational number.

(e) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if $a = 0$.

(f) Sum, difference, product and quotient of two irrational numbers need not be a irrational number or we can say, result may be a rational number also.

(xiii) Complex number : A number of the form a + ib is called a complex number, where $a,b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by Z and the set of complex number is represented by C. Thus $C = \{a + ib : a, b \in R$ and $i = \sqrt{-1}\}$

Note : It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

SOLVED EXAMPLE

Example-1

Express the following rational numbers in the form of $\frac{p}{q}$, (where $p, q \in I$)

(i) $0.1\overline{2}$ (ii) $1.5\overline{23}$

Sol. (i) Let $x = = 0.1222.......$

$10x = 1.\overline{2}$ ....(i)

$100x = 12.\overline{2}$ ....(ii)

$\Rightarrow$ $90x = 11 \Rightarrow x = \frac{11}{90}$ (so x is a rational number)

(ii) Let $x = 1.5\overline{23}$

$10x = 15.\overline{23}$

$1000x = 1523.\ \overline{23}$

$990x = 1508$

$\Rightarrow$ $x = \frac{1508}{990} = \frac{754}{495}$ (so x is a rational number)

DIVISIBILITY TEST

(i) A number will be divisible by 2 iff the digit at the unit place is divisible by 2.

(ii) A number will be divisible by 3 iff the sum of its digits of the number is divisible by 3.

(iii) A number will be divisible by 4 iff last two digits of the number together are divisible by 4.

(iv) A number will be divisible by 5 iff digit at the unit place is either 0 or 5.

(v) A number will be divisible by 6 iff the digit at the unit place of the number is divisible by 2 & sum of all digits of the number is divisible by 3.

(vi) A number will be divisible by 8 iff the last 3 digits, all together, is divisible by 8.

(vii) A number will be divisible by 9 iff sum of all it's digits is divisible by 9.

(viii) A number will be divisible by 10 iff it's last digit is 0.

(ix) A number will be divisible by 11 iff the difference between the sum of the digits at even places and sum of the digits at odd places is a multiple of 11.

e.g. 1298, 1221, 123321, 12344321, 1234554321, 123456654321, 795432

Example-2

Prove that the difference $10^{25} - 7$ is divisible by 3.

Sol. Write the given difference in the form $10^{25} - 7 = (10^{25} - 1) - 6$. The number $10^{25} - 1 = \underbrace{99..9}_{25\text{ digits}}$ is divisible by 3(and 9). Since the numbers $(10^{25} - 1)$ and 6 are divisible by 3, the number $10^{25} - 7$, being their difference, is also divisible by 3 without a remainder.

REMAINDER THEOREM

Let p(x) be any polynomial of degree greater than or equal to one and 'a' be any real number. If p(x) is divided by (x – a), then the remainder is equal to p(a).

FACTOR THEOREM

Let p(x) be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that p(a) = 0, then (x – a) is a factor of p(x). Conversely, if (x – a) is a factor of p(x), then p(a) = 0.

SOLVED EXAMPLE

Example-3

Show that (x – 3) is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$.

Sol. Let $p(x) = x^3 - 3x^2 + 4x - 12$ be the given polynomial. By factor theorem, (x – a) is a factor of a polynomial p(x) iff p(a) = 0. Therefore, in order to prove that x – 3 is a factor of p(x), it is sufficient to show that p(3) = 0. Now,

$p(x) = x^3 - 3x^2 + 4x - 12$

$\Rightarrow\ p(3) = 3^3 - 3 \times 3^2 + 4 \times 3 - 12$

$= 27 - 27 + 12 - 12 = 0$

Hence, $(x - 3)$ is a factor of $p(x) = x^3 - 3x^2 + 4x - 12$.

SOLVED EXAMPLE

Example-4

The polynomials $P(x) = kx^3 + 3x^2 - 3$ and $Q(x) = 2x^3 - 5x + k$, when divided by $(x - 4)$ leave the same remainder. The value of k is

Sol. $P(4) = 64k + 48 - 3 = 64k + 45$

$Q(4) = 128 - 20 + k = k + 108$

given $P(4) = Q(4)$

$\therefore\ 64k + 45 = k + 108;$

$\Rightarrow\ 63k = 63 \Rightarrow k = 1$

SOME IMPORTANT IDENTITIES

(1) $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$

(2) $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$

(3) $a^2 - b^2 = (a + b)(a - b)$

(4) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(5) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(6) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$

(7) $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$

(8) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

(9) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

(10) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

(11) $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$

(12) $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

SOLVED EXAMPLE

Example-5

Show that the expression, $(x^2 - yz)^3 + (y^2 - zx)^3 + (z^2 - xy)^3 - 3(x^2 - yz).(y^2 - zx).(z^2 - xy)$ is a perfect square and find its square root.

Sol. $(x^2 - yz)^3 + (y^2 - zx)^3 + (z^2 - xy)^3 - 3(x^2 - yz)(y^2 - zx)(z^2 - xy) = a^3 + b^3 + c^3 - 3abc$

where $a = x^2 - yz,\ b = y^2 - zx,\ c = z^2 - xy$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}(a + b + c)((a - b)^2 + (b - c)^2 + (c - a)^2)$

$= \frac{1}{2}(x^2 + y^2 + z^2 - xy - yz - zx)[(x^2 - yz - y^2 + zx)^2 + (y^2 - zx - z^2 + xy)^2 + (z^2 - xy - x^2 + yz)^2]$

$= \frac{1}{2}(x^2 + y^2 + z^2 - xy - yz - zx)[\{x^2 - y^2 + z(x - y)\}^2 + \{y^2 - z^2 + x(y - z)\}^2 + \{z^2 - x^2 + y(z - x)\}^2]$

$= \frac{1}{2}(x^2 + y^2 + z^2 - xy - yz - zx)(x + y + z)^2[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= (x + y + z)^2(x^2 + y^2 + z^2 - xy - yz - zx)^2 = (x^3 + y^3 + z^3 - 3xyz)^2$

(which is a perfect square) its square roots are

$\pm(x^3 + y^3 + z^3 - 3xyz)$

INDICES

If 'a' is any non zero real or imaginary number and 'm' is the positive integer, then $a^m = a.\ a.\ a.\ ...a$ (m times). Here a is called the base and m is called the index, power or exponent.

Law of indices :

(1) $a^0 = 1,\quad (a \neq 0)$

(2) $a^{-m} = \frac{1}{a^m},\ (a \neq 0)$

(3) $a^{m+n} = a^m . a^n$, where m and n are rational numbers

(4) $a^{m-n} = \frac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$

(5) $(a^m)^n = a^{mn}$

(6) $a^{p/q} = \sqrt[q]{a^p}$

SOLVED EXAMPLE

Example-6

Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$

Sol. $a^{9(1/6)(1/3)4} . a^{9(1/3)(1/6)4} = a^2 . a^2 = a^4$.

RATIO :

(i) If A and B be two quantities of the same kind, then their ratio is A : B; which may be denoted by the fraction $\frac{A}{B}$ (This may be an integer or fraction)

(ii) A ratio may represented in a number of ways e.g. $\frac{a}{b} = \frac{ma}{mb} = \frac{na}{nb} =$...... where m, n,...... are non-zero numbers.

(iii) To compare two or more ratio, reduce them to common denominator.

(iv) Ratio between two ratios may be represented as the ratio of two integers e.g. $\frac{a}{b} : \frac{c}{d} : \frac{a/b}{c/d} = \frac{ad}{bc}$ or ad : bc.

(v) Ratios are compounded by multiplying them together i.e. $\frac{a}{b}\cdot\frac{c}{d}\cdot\frac{e}{f}......=\frac{ace}{bdf}.......$

(vi) If a : b is any ratio then its duplicate ratio is $a^2 : b^2$; triplicate ratio is $a^3 : b^3$..... etc.

(vii) If a : b is any ratio, then its sub-duplicate ratio is $a^{1/2} : b^{1/2}$; sub-triplicate ratio is $a^{1/3} : b^{1/3}$ etc.

PROPORTION :

When two ratios are equal, then the four quantities compositing them are said to be proportional. If $\frac{a}{b}=\frac{c}{d}$, then it is written as a : b = c : d or a : b : : c : d

(i) 'a' and 'd' are known as extremes and 'b and c' are known as means.

(ii) An important property of proportion Product of extremes = product of means.

(iii) If a : b = c : d, then b : a = d : c (Invertando)

(iv) If a : b = c : d, then a : c = b : d (Alternando)

(v) If a : b = c : d, then $\frac{a+b}{b}=\frac{c+d}{d}$ (Componendo)

(vi) If a : b = c : d, then $\frac{a-b}{b}=\frac{c-d}{d}$ (Dividendo)

(vii) If a : b = c : d, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Componendo and dividendo)

(viii) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=.....$, then each $\frac{a+c+e+.....}{b+d+f+......}$ $=\frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

(ix) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=.....$, then each $=\frac{xa+yc+ze+......}{xb+yd+zf+......}$

(x) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=.....$, then each $=\left(\frac{xa^n+yc^n+ze^n}{xb^n+yd^n+zf^n}\right)^{1/n}$

SOLVED EXAMPLE

Example-7

If $\frac{x+y}{2}=\frac{y+z}{3}=\frac{z+x}{4}$, then find x : y : z.

Sol. Each $=\frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

$=\frac{2(x+y+z)}{9}=\frac{x+y+z}{9/2}$ and therefore each

$$=\frac{(x+y+z)-(y+z)}{\frac{9}{2}-3}=\frac{(x+y+z)-(x+z)}{\frac{9}{2}-4}=\frac{(x+y+z)-(x+y)}{\frac{9}{2}-2}$$

$$=\frac{x}{3/2}=\frac{y}{1/2}=\frac{z}{5/2}\Rightarrow x:y:z=3:1:5$$

INTERVALS :

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers a, b ∈ R such that a < b, we can define four types of intervals as follows :

Open Interval (a, b)

$\{x : a < x < b\}$ i.e. extreme points are not includes

Closed Interval [a, b]

$\{x : a \le x \le b\}$ i.e. extreme points are includes

It can possible when a and b are finite

Semi-Open Interval (a, b]

$\{x : a < x \le b\}$ i.e. a is not include and b is include

Semi-Closed Interval [a, b)

$\{x : a \le x < b\}$ i.e. a is include and b is not include

Note :

(1) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$

(ii) $[a, \infty) = \{x : x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$

(iv) $(\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = \{x : x \in R\}$

(2) $x \in \{1, 2\}$ denotes some particular values of x, i.e. x = 1, 2

(3) If there is no value of x, then we say $x \in \phi$ (null set)

GENERAL METHOD TO SOLVE INEQUALITIES : (Method of intervals (Wavy curve method)

$$\text{Let } g(x)=\left(\frac{(x-b_1)^{k_1}(x-b_2)^{k_2}---(x-b_n)^{k_n}}{(x-a_1)^{r_1}(x-a_2)^{r_2}---(x-a_n)^{r_n}}\right) \text{.... (i)}$$

Where $k_1, k_2 .....k_n$ and $r_1, r_2 ....r_n \in N$ and $b_1, b_2.....b_n$ and $a_1, a_2 ......a_n$ are real numbers.
Then to solve the inequality following steps are taken.

Steps :

Points where numerator becomes zero are called zeros or roots of the function and where denominator becomes zero are called poles of the function.

(i) First we find the zeros and poles of the function.

(ii) Then we mark all the zeros and poles on the real line and put a vertical bar there dividing the real line in many intervals.

(iii) Determine sign of the function in any of the interval and then alternates the sign in the neghbouring interval if the poles or zeros dividing the two interval has appeared odd number of times otherwise retain the sign.

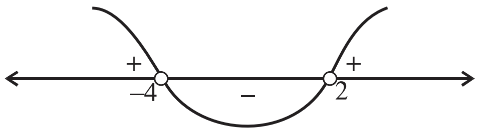
(iv) Thus we consider all the intervals. The solution of the $g(x) > 0$ is the union of the intervals in which we have put the plus sign and the solution of $g(x) < 0$ is the union of all intervals in which we have put the minus sign.

SOLVED EXAMPLE

Example-8

Solution $\dfrac{(x-1)^2(x+4)}{(2-x)} < 0$ is -

Sol. $\dfrac{(x-1)^2(x+4)}{(2-x)} < 0 \Rightarrow \dfrac{(x+4)}{(x-2)} > 0$

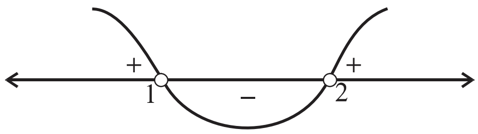


$\Rightarrow (-\infty, -4) \cup (2, \infty)$.

Example-9

The solution of $\dfrac{\sqrt[3]{(x+4)^4}\,(x-1)^3}{(x-2)} > 0$ is-

Sol. $\dfrac{(x+4)^{4/3}(x-1)^3}{(x-2)} > 0 \Rightarrow \dfrac{x-1}{x-2} > 0$



$\Rightarrow (-\infty, 1) \cup (2, \infty)$ Excluding -4.

LOGARITHM FUNCTION :

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N.

This number is designated as $\log_a N$.

Hence $\ell og_a N = x \Leftrightarrow a^x = N, a > 0, a \neq 1 \;\&\; N > 0$

If a = 10, then we write log b rather than $\ell og_{10} b$

a = e, we write ℓn b rather than $\ell og_e b$

The existence and uniqueness of the number $\ell og_a N$ follows from the properties of an exponential functions.

Domain :

The existence and uniqueness of the number $\log_a N$ can be determined with the help of set of conditions, $a > 0 \;\&\; a \neq 1 \;\&\; N > 0$.

The base of the logarithm 'a' must not equal unity otherwise numbers not equal to unity will not have a logarithm and any number will be the logarithm of unity.

FUNDAMENTAL IDENTITY

(i) $\ell og_a 1 = 0 \qquad (a > 0, a \neq 1)$

(ii) $\ell og_a a = 1 \qquad (a > 0, a \neq 1)$

(iii) $\ell og_{1/a} a = -1 \qquad (a > 0, a \neq 1)$

Remember

$\log_{10} 2 \approx 0.3010$; $\log_{10} 3 \approx 0.4771$

$\ell n\, 2 \approx 0.693$; $\ell n\, 10 \approx 2.303$

FUNDAMENTAL LOGARITHMIC IDENTITY :

$a^{\log_a N} = N, a > 0, a \neq 1 \;\&\; N > 0$

Proof :

$\log_a N = x$ ......(1)

$N = (a)^x$ ......(2)

by equation (1) & (2)

$N = (a)^{\log_a N}$

PRINCIPAL PROPERTIES

Let M & N are arbitrary positive numbers, $a > 0, a \neq 1, b > 0, b \neq 1$ and α, β are any real numbers, then :

(i) $\log_a (M.N) = \log_a M + \log_a N$;

Proof :

Let $\log_a M = x$ & $\log_a N = y$

$\Rightarrow M = a^x$ & $N = a^y$

Now, $MN = a^x a^y = a^{x+y} \Rightarrow \log_a MN = x + y$

In general

$\log_a (x_1 x_2 ......x_n) = \log_a x_1 + \log_a x_2 + ...+ \log_a x_n$

(ii) $\log_a (M/N) = \log_a M - \log_a N$

Proof :

Let $\log_a M = x$ & $\log_a N = y$

$\Rightarrow M = a^x$ & $N = a^y$

Now, $M/N = a^x/a^y = a^{x-y} \Rightarrow \log_a M/N = x - y$

(iii) $\log_a M^\alpha = \alpha. \log_a M$

BASE CHANGING THEOREM

It states that ratio of logarithm of two numbers is independent of their common base

Symbolically

$\dfrac{\log_a M}{\log_a b} = \log_b M \;\; (a > 0, M > 0, b > 0)$

Proof :

Let $\log_b M = x$

$\Rightarrow M = b^x$

$\Rightarrow \log_a M = \log_a b^x$

$\Rightarrow \log_a M = x.\log_a b$

$\Rightarrow \dfrac{\log_a M}{\log_a b} = x = \log_b M$

Important results

(i) **Base power formula :** $\log_{a^k} M = \dfrac{1}{k} \log_a M$

Proof :

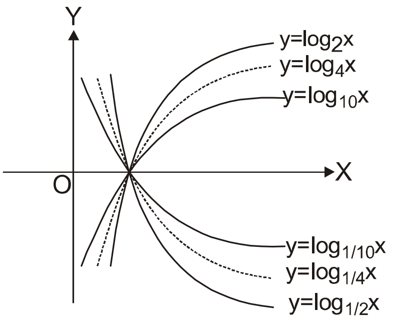
$$\log_{a^k}(m) = \frac{\log_a m}{\log_a a^k} = \frac{\log_a m}{k \log_a a} = \frac{1}{k}\log_a m$$

(ii) $a^{\log_b c} = c^{\log_b a}$

Proof : $a^{\log_b c} = a^{\log_a c . \log_b a} = \left(a^{\log_a c}\right)^{\log_b a} = (c)^{\log_b a}$

GRAPH OF LOGARITHMIC FUNCTIONS :

If $a > 0$, $a \neq 1$, then the function $y = \log_a x$, $x \in R^+$(set of positive real numbers) is called the logarithmic Function with base a.



Domain : R^+
Range : R
Nature : one-one

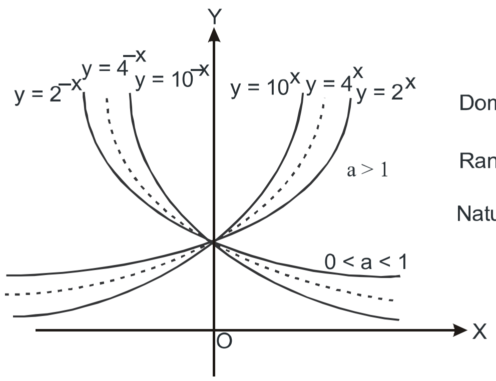
Note :

(i) If the number and the base are on the same side of the unity, then the logarithm is positive.

(ii) If the number and the base are on the opposite sides of unity, then the logarithm is negative.

Exponential Function

If $a > 0$, $a \neq 1$ then the function defined by $f(x) = a^x$, $x \in R$ is called an Exponential Function with base a.



Domain : R
Range : R^+
Nature : one-one

LOGARITHMIC EQUATION :

The equality $\log_a x = \log_a y$ is possible if and only if $x = y$

i.e. $\mathbf{\log_a x = \log_a y \Leftrightarrow x = y}$

Always check validity of given equation, $(x > 0, y > 0, a > 0, a \neq 1)$

LOGARITHMIC INEQUALITY :

Let 'a' is a real number such that

(i) If $a > 1$, then $\log_a x > \log_a y \Rightarrow x > y$

(ii) If $a > 1$, then $\log_a x < \alpha \Rightarrow 0 < x < a^\alpha$

(iii) If $a > 1$, then $\log_a x > \alpha \Rightarrow x > a^\alpha$

(iv) If $0 < a < 1$, then $\log_a x > \log_a y \Rightarrow 0 < x < y$

(v) If $0 < a < 1$, then $\log_a x < \alpha \Rightarrow x > a^\alpha$

Form - I : $f(x) > 0, g(x) > 0, g(x) \neq 1$

Form	**Collection of system**
(a) $\log_{g(x)} f(x) \geq 0 \Leftrightarrow$	$\begin{cases} f(x) \geq 1, & g(x) > 1 \\ 0 < f(x) \leq 1, & 0 < g(x) < 1 \end{cases}$
(b) $\log_{g(x)} f(x) \leq 0 \Leftrightarrow$	$\begin{cases} f(x) \geq 1, & 0 < g(x) < 1 \\ 0 < f(x) \leq 1, & g(x) > 1 \end{cases}$
(c) $\log_{g(x)} f(x) \geq a \Leftrightarrow$	$\begin{cases} f(x) \geq g(x)^a, & g(x) > 1 \\ 0 < f(x) \leq g(x)^a, & 0 < g(x) < 1 \end{cases}$
(d) $\log_{g(x)} f(x) \leq a \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x)^a, & g(x) > 1 \\ f(x) \geq g(x)^a, & 0 < g(x) < 1 \end{cases}$

From - II : When the inequality of the form

Form	**Collection of system**
(a) $\log_{\phi(x)} f(x) \geq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} f(x) \geq g(x), \phi(x) > 1, \\ 0 < f(x) \leq g(x); 0 < \phi(x) < 1 \end{cases}$
(b) $\log_{\phi(x)} f(x) \leq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x), \phi(x) > 1, \\ f(x) \geq g(x) > 0, 0 < \phi(x) < 1 \end{cases}$

COMMON AND NATURAL LOGARITHM :

$\ell og_{10} N$ is referred as a common logarithm and $\ell og_e N$ is called as natural logarithm of N to the base Napierian and is popularly written as ℓn N. Note that e is an irrational quantity lying between 2.7 to 2.8 **Note that** $e^{\ell n x} = x$.

CHARACTERISTIC & MANTISSA :

The common logarithm of a number consists of two parts, integral and fractional, of which the integral part may be zero or an integer (+ve or –ve) and the fractional part a decimal, less than one and always positive.

The integral part is called the characteristic and the decimal part is called the mantissa. It should be noted that, if the characteristic of the logarithm of N is p then number of significant digit in N = p + 1 if p is the non negative characteristic of log N. Number of zeros after decimal before a significant figure start is p – 1

ANTILOGARITHM

The positive real number 'n' is called the antilogarithm of a number 'm' if log n = m

Thus, $\log_a n = m \Leftrightarrow n = \text{antilog}_a m$

So $\text{antilog}_a m = a^m$ $(\because n = a^x)$

e.g. $\text{anti}\log_8\left(\frac{2}{3}\right) = 8^{2/3} = 4$

SOLVED EXAMPLE

Example-10

Compute the value of $\frac{1}{\log_2 36}+\frac{1}{\log_3 36}$.

Sol. $\frac{1}{\log_2 36}+\frac{1}{\log_3 36}$

$=\log_{36}2+\log_{36}3=\log_{36}6=\frac{1}{2}$

Example-11

Find the value of $\log_{100} 0.00001$ by definition.

Sol. Let $y=\log_{100} 0.00001$

$\Rightarrow$ $(100)^y = 0.00001$

$\Rightarrow$ $10^{2y} = 10^{-5}$

$\Rightarrow$ $2y = -5$ $\Rightarrow$ $y = -\frac{5}{2}$

Example-12

How many solutions are there for equation $\log_4 (x-1) = \log_2 (x-3)$?

Sol. $\log_4(x-1)=\log_2(x-3)$

$\Rightarrow$ $\log_{2^2} (x-1) = \log_2 (x-3)$

$\Rightarrow$ $\frac{1}{2}\log_2 (x-1) = \log_2 (x-3)$

$\Rightarrow$ $\log_2 (x-1)^{1/2} = \log_2 (x-3)$

$\Rightarrow$ $(x-1)^{1/2} = (x-3)$

$\Rightarrow$ $x-1 = x^2 - 6x + 9$

$\Rightarrow$ $(x-2)(x-5) = 0 \Rightarrow x = 2, 5$

But $x-1>0$ and $x-3>0$

$x>1$ and $x>3$

So only one solution $x = 5$

Example-13

Solve the logarithmic inequality $\log_{1/5}\frac{4x+6}{x}\ge 0$.

Sol. Since $\log_{1/5} 1 = 0$, the given inequality can be written as.

$\log_{1/5}\frac{4x+6}{x}\ge \log_{1/5} 1$

when the domain of the function is taken into account the inequality is equivalent to the system of ineqaulities.

$$\begin{cases}\frac{4x+6}{x}>0,\\ \frac{4x+6}{x}\le 1\end{cases}$$

Solving the ineqaualities by using method of intervals $x\in\left[-2,\frac{-3}{2}\right)$

Example-14

Solve the inequality $\log_{1/3}(5x-1)>0$.

Sol. by using the basic property of logarithm.

$$\begin{cases}5x-1<1\\ 5x-1>0\end{cases}\Rightarrow\begin{cases}5x<2\\ 5x>1\end{cases}\Rightarrow\begin{matrix}x<\frac{2}{5}\\ x>\frac{1}{5}\end{matrix}$$

$\Rightarrow$ The solution of the inequality is given by $\left(\frac{1}{5},\frac{2}{5}\right)$

Example-15

If $\log_{x-3}(2x-3)$ is a meaningful quantity then find the interval in which x must lie.

Sol. $x-3>0$, $x-3\ne 1$ and $2x-3>0 \Rightarrow x>3$, $x\ne 4$ and $x>3/2 \Rightarrow (3,4)\cup(4,\infty)$

Example-16

The value of $\log_{\sqrt{2}}\left(\text{antilog}_{128}\left(\frac{3}{7}\right)\right)$ equals

(A) 16 (B) 8

(C) 6 (D) 4

Ans. (C)

Sol. $\text{anti}\log_{128}\left(\frac{3}{7}\right) = (128)^{\frac{3}{7}}$

$=\left(2^7\right)^{\frac{3}{7}}=2^3$.

$\log_{\sqrt{2}}(2^3)=\frac{3}{\frac{1}{2}}\log_2 2=6$

Example-17

For $x\ge 0$, what is the smallest possible value of the expression $\log(x^3-4x^2+x+26)-\log(x+2)$?

Sol. $\log\frac{(x^3-4x^2+x+26)}{(x+2)}$

$= \log \dfrac{(x^2-6x^2+13)(x+2)}{(x+2)}$

$= \log(x^2-6x+13) \qquad [\because\ x \neq -2]$

$= \log\{(x-3)^2+4\}$

$\therefore$ Minimum value is log 4 when $x = 3$

Example-18

Given $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \dfrac{2}{s^3+1}$.

Write $\log_2 \dfrac{a^2b^5}{c^4}$ as a function of 's' $(a, b, c > 0,\ c \neq 1)$.

Sol. Given $\log_2 a = s$ ....(1)

$\log_2 b = 2s^2$ ....(2)

$\log_8 c^2 = \dfrac{s^3+1}{2}$ ...(3)

$\Rightarrow \dfrac{2\log c}{3\log 2} = \dfrac{s^3+1}{2} \Rightarrow 4\log_2 c = 3(s^3+1)$ ....(4)

to find $2\log_2 a + 5\log_2 b - 4\log_2 c$

$\Rightarrow 2s + 10s^2 - 3(s^3+1)$

Example-19

Suppose that *a* and *b* are positive real numbers such that $\log_{27}a + \log_9 b = \dfrac{7}{2}$ and $\log_{27}b + \log_9 a = \dfrac{2}{3}$. Find the value of the *ab*.

Sol. $\log_{27}a + \log_9 b = \dfrac{7}{2} \Rightarrow \dfrac{1}{3}\log_3 a + \dfrac{1}{2}\log_3 b = \dfrac{7}{2}$; $\log_{27}b + \log_9 a = \dfrac{2}{3} \Rightarrow \dfrac{1}{3}\log_3 b + \dfrac{1}{2}\log_3 a = \dfrac{2}{3}$

adding the equation

$\dfrac{1}{3}\log_3(ab) + \dfrac{1}{2}\log_3(ab) = \dfrac{7}{2} + \dfrac{2}{3} = \dfrac{25}{6}$

$\dfrac{5}{6}\log_3(ab) = \dfrac{25}{6} \Rightarrow \log_3(ab) = 5 \Rightarrow ab = 3^5 = 243$

Example-20

Let $x = (0.15)^{20}$. Find the characteristic and mantissa in the logarithm of x, to the base 10. Assume $\log_{10}2 = 0.301$ and $\log_{10}3 = 0.477$.

Sol. $\log x = \log(0.15)^{20} = 20\log\left(\dfrac{15}{100}\right)$

$= 20[\log 15 - 2] = 20[\log 3 + \log 5 - 2]$

$= 20[\log 3 + 1 - \log 2 - 2]$

$= 20[-1 + \log 3 - \log 2]$

$= 20[-1 + 0.477 - 0.301]$

$= -20 \times 0.824$

$= -16.48 = \overline{17}.52$

Hence characteristic $= -17$

Mantissa $= 0.52$

Example-21

How many digits are contained in the number 2^{75} ?

Sol. Computing $\log 2^{75}$, we have $\log 2^{75} = 75.\log 2 \approx 75.(0.3010) = 22.5750$.

Consequently, the characteristic of this common logarithm is equal to 22. Therefore, $2^{75} = a.10^{22}$, where $1 \leq a < 10$, a is an integer, and, hence the number 2^{75} has 23 digits.

Example-22

Number of cyphers after decimal before a significant figure starts in $\left(\dfrac{5}{4}\right)^{-100}$ is equal to [**Use:** $\log_{10}2 = 0.3010$]

Sol. Let $N = \left(\dfrac{5}{4}\right)^{-100}$

$\Rightarrow \log_{10}N = -100\log_{10}\left(\dfrac{5}{4}\right) = -100\left(\log_{10}\left(\dfrac{10}{2}\right) - \log_{10}4\right)$

$= -100(1 - 3\log_{10}2)$

$= -100(1 - 3 \times 0.3010)$

$= -100(1 - 0.9030) = -100 \times 0.0970 = -9.7$

$= -10 + 0.3 = \overline{10}.03$

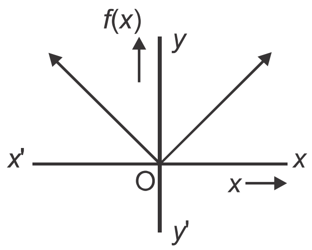
$\therefore$ Number of zeroes = 9 **Ans.**

ABSOLUTE VALUE FUNCTION / MODULUS FUNCTION:

This is also known as absolute value function and denoted by

$f(x) = |x|$ i.e. $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain of this function is set of all real numbers because $f(x)$ exists for all $x \in R$ but $|x| \geq 0$ so range is all non-negative real numbers.



Domain = R; Range = $[0, \infty)$ or $R^+ \cup \{0\}$

Properties of modulus : For any a, b $\in R$

(i) $|a| \geq 0$

(ii) $|a| = |-a|$

(a) $|a|^n = |a^n|$

(b) $|a^n| = a^n$, where n is even and $n \in z$

(iii) $|a| \geq a, |a| \geq -a$

(iv) $|ab| = |a||b|$ (v) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

Note : $|f(x)| + |g(x)| = |f(x) + g(x)|$
$\Rightarrow f(x).g(x) \geq 0$

Example-23

Solve the following linear equation

(i) $x|x| = 4$

(ii) $|x - 3| + 2|x + 1| = 4$

Solution:

(i) $x|x| = 4$

If $x > 0$

$\therefore x^2 = 4 \Rightarrow x = \pm 2$ $\therefore x = 2$ ($\because x \geq 0$)

If $x < 0 \Rightarrow -x^2 = 4$ $\Rightarrow x^2 = -4$ which is not possible

(ii) $|x - 3| + 2|x + 1| = 4$

Case-I If $x \leq -1$

$\therefore -(x - 3) - 2(x + 1) = 4$

$\Rightarrow -x + 3 - 2x - 2 = 4$ $\Rightarrow -3x + 1 = 4$

$\Rightarrow -3x = 3$ $\Rightarrow x = -1$

Case-II If $-1 < x \leq 3$

$\therefore -(x - 3) + 2(x + 1) = 4$

$\Rightarrow -x + 3 + 2x + 2 = 4$

$\Rightarrow x = -1$ which is not possible

Case-III If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4$ $\Rightarrow x = 5/3$ which is not possible

$\therefore x = -1$ **Ans.**

Example-24

The absolute value of sum of real solutions of $\log_2|x^2 + 5x + 4| = \log_2 3 + \log_2|x + 1|$ is

(1) 8 (2) 6 (3) 7 (4) 5

Ans. (3)

Sol. $\log_2 \frac{|(x+1)(x+4)|}{|x+1|} = \log_2 3$

$|x + 4| = 3$

$x + 4 = -3, +3$

$x = -7, -1$ (rejected);

$\Rightarrow x = -7$

Example-25

Number of real solutions of $|x - 1| = |x - 2| + |x - 3|$ is

(1) 0 (2) 1

(3) 2 (4) more than 2

Ans. (3)

Sol. **Case-I:** $x \leq 1$, $1 - x = 2 - x + 3 - x$

$x = 4$ (rejected)

Case-II: $1 < x \leq 2$, $x - 1 = 2 - x + 3 - x$

$x = 2$

Case-III: $2 < x < 3$, $x - 1 = x - 2 + 3 - x$

$x = 2$

Case-IV: $x \geq 3$, $x - 1 = x - 2 + x - 3$

$x = 4$

$\Rightarrow x = 2, 4$

Inequalities Involving Absolute Value

(i) $|x| \leq a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or less than a from zero.

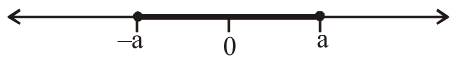


Fig 1.20

$\Rightarrow$ $-a \leq x \leq a$

e.g. $|x| \leq 2 \Rightarrow -2 \leq x \leq 2$

$|x| < 3 \Rightarrow -3 < x < 3$

In general, $|f(x)| \leq a$ (where $a > 0$) $\Rightarrow -a \leq f(x) \leq a$.

(ii) $|x| \geq a$ (where $a > 0$)

It implies those values of z on real number line which are at distance a or more than a from zero

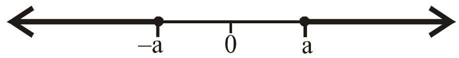


Fig 1.21

$\Rightarrow$ $x \leq -a$ or $x \geq a$

e.g. $|x| \geq 3 \Rightarrow x \leq -3$ or $x \geq 3$

$|x| > 2 \Rightarrow x < -2$ or $x > 2$

In general, $|f(x)| \geq a \Rightarrow f(x) \leq -a$ or $f(x) \geq a$

(iii) $a \leq |x| \leq b$ (where $a, b > 0$)

It implies those value of x on real number line whose distance from zero is equal to a or b or lies between a and b

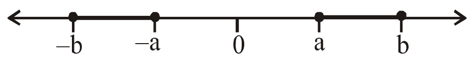


Fig 1.22

$\Rightarrow [-b, -a] \cup [a, b]$

e.g. $2 \leq |x| \leq 4 \Rightarrow x \in [-4, -2] \cup [2, 4]$

(iv) If $|x + y| = |x| + |y|$, $xy \geq 0$

If $|x - y| = |x| + |y|$, $xy \leq 0$

If $|x + y| = ||x| - |y||$, $xy \leq 0$

If $|x - y| = ||x| - |y||$, $xy \geq 0$

Example-26

Solve $x^2 - 4|x| + 3 < 0$.

Sol. $x^2 - 4|x| + 3 < 0$

$\Rightarrow (|x| - 1)(|x| - 3) < 0$

$\Rightarrow 1 < |x| < 3$

$\Rightarrow -3 < x < -1$ or $1 < x < 3$

$\Rightarrow x \in (-3,-1) \cup (1,3)$

Example-27

Solve $1 \le |x - 2| \le 3$

Sol. $1 \le |x - 2| \le 3$

$\Rightarrow -3 \le x - 2 \le -1$ or $1 \le x - 2 \le 3$

$\Rightarrow -1 \le x \le 1$ or $3 \le x \le 5$

$\Rightarrow x \in [-1, 1] \cup [3, 5]$

Example-28

Solve $\left||x-1|-2\right| < 5$

Sol. $\left||x-1|-2\right| < 5$

$\Rightarrow -5 < |x - 1| - 2 < 5$

$\Rightarrow -3 < |x - 1| < 7$

$\Rightarrow |x - 1| < 7$

$\Rightarrow -6 < x < 8$

Example-29

If x satisfies $|x-1| + |x-2| + |x-3| \ge 6$, then

Sol. For $x \le 1$, the given inequation becomes

$1 - x + 2 - x + 3 - x \ge 6 \Rightarrow -3x \ge 0$

$\Rightarrow$ $x \le 0$ and for $x \ge 3$, the given equation becomes

$x - 1 + x - 2 + x - 3 \ge 6 \quad \Rightarrow 3x \ge 12 \Rightarrow x \ge 4$

For $1 < x \le 2$

we get $x - 1 + 2 - x + 3 - x \ge 6$

$\Rightarrow$ $-x + 4 \ge 6$

i.e. $-x \ge 2 \quad \Rightarrow x \le -2$

Not possible

For $2 < x < 3$,

We get $x - 1 + x - 2 + 3 - x \ge 6$

$\Rightarrow$ $x \ge 6$ not possible

Hence solution set is $(-\infty, 0] \cup [4, \infty)$

i.e. $x \le 0$ or $x \ge 4$

GREATEST INTEGER FUNCTION (STEP-UP FUNCTION)

The function $f(x) = [x]$ is called the greatest integer function and is defined as follows:

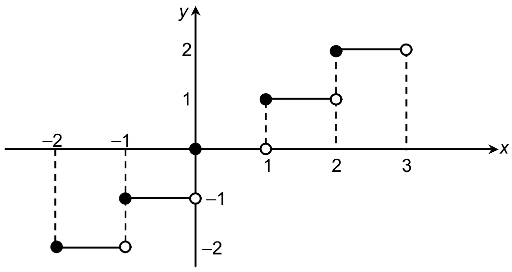
$[x]$ is the greatest integer less than or equal to x.

Examples: $[3] = 3$, $[2.7] = 2$, $[-7.8] = -8$, $[0.8] = 0$

In other words if we list all the integers less then or equal to x, then the integer greatest among them is called greatest integer of x. Greater integer of x is also called integral part of x.

$y = f(x) = [x]$

Domain : **R** ; Range : **I** ;



Remarks

(i) $[x] \le x < [x] + 1$

(ii) $[x + m] = [x] + m$; $m \in I$

(iii) $[x] + [-x] = \begin{cases} 0 & ; x \in I \\ -1 & ; x \notin I \end{cases}$

FRACTIONAL PART FUNCTION

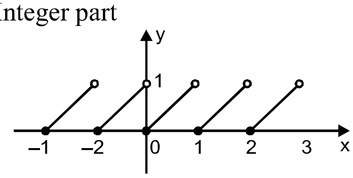
$y = f(x) = \{x\} = x - [x]$

Domain : $x \in R$; Range : $[0, 1)$

Eg. $2.3 = 2 + 0.3 \rightarrow$ fractional part

$\downarrow$

Integer part



Remarks

(i) Fractional part of any integer is zero.

(ii) $\{x + n\} = \{x\}, n \in I$

(iii) $\{x\} + \{-x\} = \begin{cases} 0 ; & x \in I \\ 1 ; & \text{otherwise} \end{cases}$

EXERCISE-I

Basic concepts and Number System

Q.1 Let $x \in Q$, $y \in Q^c$, Which of the following statement is always WRONG ?
(1) $xy \in Q^c$
(2) $y/x \in Q$, whenever defined
(3) $\sqrt{2}x + y \in Q$
(4) $x/y \in Q^c$, whenever defined

Q.2 If x and y are two rational numbers such that $(x + y) + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$, then :
(1) x = 1, y = 1
(2) x = 2, y = 1
(3) x = 5, y = 1
(4) x and y can take infinitely many values

Q.3 Which of the following statement is incorrect :
(1) rational number + rational number = rational number
(2) irrational number + rational number = irrational number
(3) integer + rational number = rational number
(4) irrational number + irrational number = Irrational number

Q.4 The number of real roots of the equation $(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$ is :
(1) 0 (2) 1 (3) 2 (4) 3

Q.5 If $x - a$ is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to
(1) 0 (2) 2 (3) –2 (4) 1

Q.6 Every irrational number can be expressed on the number line. This statement is
(1) always true
(2) never true
(3) true subject to some condition
(4) None of these

Q.7 The multiplication of a rational number 'x' and an irrational number 'y' is
(1) always rational
(2) rational except when $y = \pi$
(3) always irrational
(4) irrational except when x = 0

Q.8 If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is
(1) 2 (2) 4 (3) 6 (4) 2 or 4 or 6

Q.9 If a, b, c are real, then $a(a - b) + b(b - c) + c(c - a) = 0$, only if
(1) a + b + c = 0
(2) a = b = c
(3) a = b or b = c or c = a
(4) a – b – c = 0

Q.10 If $2x^3 - 5x^2 + x + 2 = (x - 2)(ax^2 - bx - 1)$, then a & b are respectively
(1) 2, 1 (2) 2, – 1 (3) 1, 2 (4) –1, 1/2

Q.11 The value of $[e] - [-\pi]$ is, where [.] denotes greatest integer function.
(1) 5 (2) 6 (3) 7 (4) 8

Q.12 Number of real solution (x) of the equation $|x - 3|^{3x^2 - 10x + 3} = 1$ is
(1) exactly four (2) exactly three
(3) exactly two (4) exactly one

Logarithm and its principle properties

Q.13 $\dfrac{1}{\log_{\sqrt{bc}} abc} + \dfrac{1}{\log_{\sqrt{ca}} abc} + \dfrac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to
(1) 1/2 (2) 1 (3) 2 (4) 4

Q.14 If $\log_x \log_{18}(\sqrt{2} + \sqrt{8}) = \dfrac{1}{3}$. Then the value of 1000 x is equal to
(1) 8 (2) 1/8 (3) 1/125 (4) 125

Q.15 Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :
(1) none (2) exactly 1
(3) exactly 2 (4) 4

Q.16 Greatest integer less than or equal to the number $\log_2 15 \cdot \log_{1/6} 2 \cdot \log_3 1/6$ is
(1) 4 (2) 3 (3) 2 (4) 1

Q.17 The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(1) $a^2 - a - 1$ (2) $a^2 + a - 1$
(3) $a^2 - a + 1$ (4) $a^2 + a + 1$

Q.18 If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(1) zero (2) 1 (3) 2 (4) more than 2

Q.19 The number $\log_2 7$ is

(1) an integer (2) a rational number
(3) an irrational number (4) a prime number

Q.20 Anti logarithm of 0.75 to the base 16 has the value equal to

(1) 4 (2) 6 (3) 8 (4) 12

Inequalities

Q.21 If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 (\sqrt{x^2+5} + x) > 0$ contains 'n' integral values, then n equals to

(1) 7 (2) 8 (3) 6 (4) 10

Q.22 If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval

(1) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$
(2) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$
(3) $(\sqrt{5}, 3\sqrt{5})$
(4) ϕ

Q.23 The set of all the solutions of the inequality $\log_{1-x}(x-2) \geq -1$ is

(1) $(-\infty, 0)$ (2) $(2, \infty)$
(3) $(-\infty, 1)$ (4) ϕ

Q.24 Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is :

(1) $[-4, 1]$
(2) $[-4, -3) \cup (0, 1]$
(3) $(-\infty, -3) \cup (1, \infty)$
(4) $(-\infty, -4) \cup [1, \infty)$

Modulus function

Q.25 Solutions of $|4x + 3| + |3x - 4| = 12$ are

(1) $x = -\frac{7}{3}, \frac{3}{7}$ (2) $x = -\frac{5}{2}, \frac{2}{5}$
(3) $x = -\frac{11}{7}, \frac{13}{7}$ (4) $x = -\frac{3}{7}, \frac{7}{5}$

Q.26 If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$, then the set of all real values of x is

(1) $[1, 4] \cup \{-2\}$ (2) $[1, 4]$
(3) $[-2, 1] \cup [4, \infty)$ (4) $(-\infty, -2] \cup [1, 4]$

Q.27 The complete set of real 'x' satisfying $||x - 1| - 1| \leq 1$ is:

(1) $[0, 2]$ (2) $[-1, 3]$
(3) $[-1, 1]$ (4) $[1, 3]$

Q.28 The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

(1) 1 (2) 2 (3) 3 (4) 4

EXERCISE-II

Q.1 If A & B are two rational numbers and AB, A + B and A – B are rational numbers, then A/B is

(1) always rational
(2) never rational
(3) rational when $B \neq 0$
(4) rational when $A \neq 0$

Q.2 If a, b, c are real and distinct numbers, then the value of

$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$ is

(1) 1 (2) a b c
(3) 2 (4) 3

Q.3 The polynomials $P(x) = kx^3 + 3x^2 - 3$ and $Q(x) = 2x^3 - 5x + k$, when divided by $(x - 4)$ leave the same remainder. The value of k is

(1) 2 (2) 1 (3) 0 (4) –1

Q.4 The remainder obtained when the polynomial $1 + x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by x – 1 is

(1) 3 (2) 5 (3) 7 (4) 11

Q.5 If $x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$, then x =

(1) 1 (2) –1 (3) 0 (4) 2

Q.17 The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(1) $a^2 - a - 1$ (2) $a^2 + a - 1$
(3) $a^2 - a + 1$ (4) $a^2 + a + 1$

Q.18 If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is
(1) zero (2) 1 (3) 2 (4) more than 2

Q.19 The number $\log_2 7$ is
(1) an integer (2) a rational number
(3) an irrational number (4) a prime number

Q.20 Anti logarithm of 0.75 to the base 16 has the value equal to
(1) 4 (2) 6 (3) 8 (4) 12

Inequalities

Q.21 If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 (\sqrt{x^2+5+x}) > 0$ contains 'n' integral values, then n equals to
(1) 7 (2) 8 (3) 6 (4) 10

Q.22 If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval
(1) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$
(2) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3\sqrt{5})$
(3) $(\sqrt{5}, 3\sqrt{5})$
(4) ϕ

Q.23 The set of all the solutions of the inequality $\log_{1-x}(x-2) \geq -1$ is
(1) $(-\infty, 0)$ (2) $(2, \infty)$
(3) $(-\infty, 1)$ (4) ϕ

Q.24 Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is :
(1) $[-4, 1]$
(2) $[-4, -3) \cup (0, 1]$
(3) $(-\infty, -3) \cup (1, \infty)$
(4) $(-\infty, -4) \cup [1, \infty)$

Modulus function

Q.25 Solutions of $|4x + 3| + |3x - 4| = 12$ are
(1) $x = -\frac{7}{3}, \frac{3}{7}$ (2) $x = -\frac{5}{2}, \frac{2}{5}$
(3) $x = -\frac{11}{7}, \frac{13}{7}$ (4) $x = -\frac{3}{7}, \frac{7}{5}$

Q.26 If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3\,|x + 2|$, then the set of all real values of x is
(1) $[1, 4] \cup \{-2\}$ (2) $[1, 4]$
(3) $[-2, 1] \cup [4, \infty)$ (4) $(-\infty, -2] \cup [1, 4]$

Q.27 The complete set of real 'x' satisfying $||x - 1| - 1| \leq 1$ is:
(1) $[0, 2]$ (2) $[-1, 3]$
(3) $[-1, 1]$ (4) $[1, 3]$

Q.28 The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is
(1) 1 (2) 2 (3) 3 (4) 4

EXERCISE-II

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(1) always rational
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(3) rational when $B \neq 0$
(4) rational when $A \neq 0$

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$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)} \text{ is}$$

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(3) 2 (4) 3

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(1) 2 (2) 1 (3) 0 (4) –1

Q.4 The remainder obtained when the polynomial $1 + x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by x – 1 is
(1) 3 (2) 5 (3) 7 (4) 11

Q.5 If $x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$, then x =
(1) 1 (2) –1 (3) 0 (4) 2

Q.6 The equation $4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$ has the solution

(1) $x = \pm 1$ (2) $x = 10$

(3) $x = \pm\sqrt{2}$ (4) $x = \sqrt{3}$

Q.7 Logarithm of $32\sqrt[5]{4}$ to the base $2\sqrt{2}$ is

(1) 3.6 (2) 5

(3) 5.6 (4) 10

Q.8 The number $\log_2 7$ is

(1) an integer (2) a rational number

(3) an irrational number (4) a prime number

Q.9 The value of $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$ is

(1) 1 (2) 2

(3) 3 (4) 4

Q.10 $\frac{1}{1+\log_b a+\log_b c}+\frac{1}{1+\log_c a+\log_c b}+\frac{1}{1+\log_a b+\log_a c}$ has the value equal to

(1) abc (2) $\frac{1}{abc}$ (3) 0 (4) 1

Q.11 If $\log_e\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log_e a+\log_e b)$, then relation between *a* and *b* will be

(1) $a = b$ (2) $a = \frac{b}{2}$

(3) $2a = b$ (4) $a = \frac{b}{3}$

Q.12 $\log_7 \log_7 \sqrt{7(\sqrt{7\sqrt{7}})} =$

(1) $3 \log_2 7$ (2) $1 - 3 \log_3 7$

(3) $1 - 3\log_7 2$ (4) $1 - 10 \log_2 7$

Q.13 The value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$ is equal to

(1) 49 (2) 625

(3) 216 (4) 890

Q.14 If $\log_{10} x = y$, then $\log_{1000} x^2$ is equal to

(1) y^2 (2) 2y

(3) 3y/2 (4) 2y/3

Q.15 If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$, then which of the following is equal to 1

(1) $x + y + z$

(2) $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1}$

(3) xyz

(4) $x + y - z$

Q.16 If $a = \log_{24}12$, $b = \log_{36}24$ and $c = \log_{48} 36$, then $1+abc$ is equal to

(1) 2ab (2) 2ac

(3) 2bc (4) 0

Q.17 The solution of the equation $\log_7 \log_5\left(\sqrt{x^2+5+x}\right) = 0$.

(1) $x = 2$ (2) $x = 3$

(3) $x = 4$ (4) $x = -2$

Q.18 The value of $(0.05)^{\log_{\sqrt{20}}(0.1+0.01+0.001+........)}$ is

(1) 81 (2) $\frac{1}{81}$

(3) 20 (4) $\frac{1}{20}$

Q.19 The value of $\log_2 .\log_3 ....\log_{100} 100^{99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}}$ is

(1) 0 (2) 1

(3) 2 (4) 100!

Q.20 The number of solution of $\log_2(x + 5) = 6 - x$ is

(1) 2 (2) 0

(3) 3 (4) 1

Q.21 If $\log_{10}2 = 0.30103$, $\log_{10}3 = 0.47712$, the number of digits in $3^{12} \times 2^8$ is

(1) 7 (2) 8

(3) 9 (4) 10

Q.22 Exhaustive set of values of x satisfying $\log_{|x|}(x^2 + x + 1) \geq 0$ is
(1) $(-1, 0)$
(2) $(-\infty, -1) \cup (1, \infty)$
(3) $(-\infty, \infty) - \{-1, 0, 1\}$
(4) $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

Q.23 The set of real values of x satisfying $\log_{1/2}\left(x^2 - 6x + 12\right) \geq -2$ is
(1) $(-\infty, 2]$ (2) $[2, 4]$
(3) $[4, +\infty)$ (4) $[3, 8]$

Q.24 If $\log_{0.04}(x - 1) \geq \log_{0.2}(x - 1)$ then x belongs to the interval
(1) $(1, 2]$ (2) $(-\infty, 2]$
(3) $[2, +\infty)$ (4) $[2, 2]$

Q.25 If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the interval
(1) $(2, \infty)$ (2) $(-2, -1)$
(3) $(1, 2)$ (4) $(-2, 2)$

Q.26 The minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ is equal to
(1) 1 (2) 2 (3) 3 (4) 0

Q.27 The set of real value(s) of p for which the equation $|2x + 3| + |2x - 3| = px + 6$ has more than two solutions is :
(1) $[0, 4)$ (2) $(-4, 4)$
(3) $R - \{4, -4, 0\}$ (4) $\{0\}$

Q.28 Number of solutions of the equation $[2x] - 3\{2x\} = 1$ (where [.] and { . } denotes greatest integer and fractional part function respectively)
(1) 1 (2) 2 (3) 3 (4) 0

EXERCISE-III

MCQ/COMPREHENSION/MATCHING/NUMERICAL

Q.1 If x & y are real numbers and $\frac{y}{x} = x$, then 'y' cannot take the value(s)
(A) -1 (B) 0
(C) 1 (D) 2

Q.2 Indicate all correct alternatives, where base of the log is 2.
The equation $x^{(3/4)(\log x)^2 + \log x - (5/4)} = \sqrt{2}$ has :
(A) at least one real solution
(B) exactly three real solutions
(C) exactly one irrational solution
(D) Imaginary roots

Q.3 Values of x satisfying the equation
$\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are
(A) 1 (B) 5
(C) $\frac{1}{25}$ (D) 3

Q.4 Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is
(A) a natural number (B) a prime number
(C) a rational number (D) an integer

Q.5 The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is
(A) (6, 3) (B) (3, 6)
(C) (6, 12) (D) (12, 6)

Q.6 The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has
(A) one irrational solution
(B) no prime solution
(C) two real solutions
(D) one integral solution

Q.7 The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has
(A) exactly three real solution
(B) at least one real solution
(C) exactly one irrational solution
(D) complex roots

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S

Q.1 The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ is equal to :

[JEE Main - 2019 (April)]

(1) 4 (2) 9
(3) 10 (4) 12

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 The value of

$$6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$$

is **[IIT JEE-2012]**

Q.2 If $3^x = 4^{x-1}$, then x = **[JEE Advanced-2013]**

(A) $\dfrac{2\log_3 2}{2\log_3 2-1}$

(B) $\dfrac{2}{2-\log_2 3}$

(C) $\dfrac{1}{1-\log_4 3}$

(D) $\dfrac{2\log_2 3}{2\log_2 3-1}$

Q.3 The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}}\times(\sqrt{7})^{\frac{1}{\log_4 7}}$ is ______

[JEE Advanced-2018]

Q.22 Exhaustive set of values of x satisfying $\log_{|x|}(x^2 + x + 1) \geq 0$ is
(1) $(-1, 0)$
(2) $(-\infty, -1) \cup (1, \infty)$
(3) $(-\infty, \infty) - \{-1, 0, 1\}$
(4) $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

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(1) $\left(-\infty, 2\right]$ (2) $[2, 4]$
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EXERCISE-III

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(A) exactly three real solution
(B) at least one real solution
(C) exactly one irrational solution
(D) complex roots

Q.8 Solution set of the inequality

$$(\log_2 x)^4 - \left(\log_{1/2}\frac{x^3}{8}\right)^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4(\log_{1/2} x)^2$$

is (a, b) ∪ (c, d) then the correct statement is

(A) a = 2b and d = 2c

(B) b = 2a and d = 2c

(C) $\log_c d = \log_b a$

(D) there are 4 integers in (c, d)

Q.9 If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(A) maximum value of x is $\frac{1}{\sqrt{10}}$

(B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(C) minimum value of x is $\frac{1}{10}$

(D) minimum value of x is $\frac{1}{100}$

Q.10 If |x – 5| + |x + 5| = 10, then

(A) the number of integral solutions is 10

(B) the number of integral solutions is 11

(C) the sum of all the integral solutions is 0

(D) all the solutions of the equation are rational numbers

Comprehension # 02 (Q. no. 11 to 13)

A denotes the product xyz where x, y and z satisfy

$\log_3 x = \log 5 - \log 7$

$\log_5 y = \log 7 - \log 3$

$\log_7 z = \log 3 - \log 5$

B denotes the sum of square of solution of the equation

$\log_2(\log_2 x^6 - 3) - \log_2(\log_2 x^4 - 5) = \log_2 3$

C denotes characterstic of logarithm $\log_2(\log_2 3) - \log_2(\log_4 3) + \log_2(\log_4 5) - \log_2(\log_6 5) + \log_2(\log_6 7) - \log_2(\log_8 7)$

Q.11 Find value of A + B + C

(A) 18 (B) 34 (C) 32 (D) 24

Q.12 Find $\log_2 A + \log_2 B + \log_2 C$

(A) 5 (B) 6 (C) 7 (D) 4

Q.13 Find |A – B + C|

(A) – 30 (B) 32 (C) 28 (D) 30

Paragraph for question nos. 14 to 16

Let α and β are the solutions of the equation

$\left(\sqrt{x}\right)^{\log_5 x - 1} = 5$ where α ∈ I and β ∈ Q. Then

[**Use:** $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$]

Q.14 The number of significant digits before decimal in $(\alpha)^{10}$ is

(A) 13 (B) 14

(C) 15 (D) none

Q.15 Number of zeroes after decimal before a significant digit in $(b)^{10}$ is

(A) 5 (B) 7

(C) 8 (D) 6

Q.16 The value of $(\beta)^{\log_{25} 9}$ is

(A) $\frac{1}{3}$ (B) 5 (C) $\frac{1}{5}$ (D) 9

Q.17 **Match the Column:**

Column-I	**Column-II**
(A) The value(s) of x, which does not satisfy the equation $\log_2^2(x^2 - x) - 4\log_2(x - 1)\log_2 x = 1$, is (are)	(P) 2
(B) The value of x satisfying the equation	(Q) 3
$2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}} = 13$, is	(R) 4
(C) The number $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right)$ is less than	(S) 5
(D) Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1 + 9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l + m)$ is divisible by	(T) 6

Q.18 **Match the Column:**

Column-I	Column-II
(A) Anti logarithm of $(0.\overline{6})$ to the base 27 has the value equal to	(P) 5
(B) Characteristic of the logarithm of 2008 to the base 2 is	
(C) The value of b satisfying the equation, $\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is	(Q) 7
(D) Number of naughts after decimal before a significant figure	(R) 9
comes in the number $\left(\frac{5}{6}\right)^{100}$, is	(S) 10

NUMERICAL BASED QUESTIONS

Q.19 Find the number of integral solution of the equation

$\log_{\sqrt{x}}(x+|x-2|) = \log_x(5x-6+5|x-2|)$.

Q.20 If $\log_{3x}45 = \log_{4x} 40\sqrt{3}$ then find the characteristic of x^3 to the base 7.

Q.21 Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16}$

$-\log_2 12 \cdot \log_2 48 + 10$.

Find $y \in N$.

Q.22 If a, b are co-prime numbers and satisfying

$$\left(2+\sqrt{3}\right)^{\frac{1}{\log_a\left(2-\sqrt{3}\right)}+\frac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}} = \frac{1}{12},$$

then (a + b) can be is equal to

Q.23 Let $N = 10^{3\log 2 - 2\log\left(\log 10^3\right) + \log\left(\left(\log 10^6\right)^2\right)}$ where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is equal to

Q.24 If p is the smallest value of x satisfying the equation

$2^x + \frac{15}{2^x} = 8$ then the value of 4^p is equal to

Q.25 Positive numbers x, y and z satisfy $xyz = 10^{81}$ and $(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$.

Find the value of

$\sqrt{(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2}$

Q.26 If 'x' and 'y' are real numbers such that,

$2\log(2y-3x) = \log x + \log y$, find $\frac{y}{x}$.

Q.27 If $\log_{3x}45 = \log_{4x} 40\sqrt{3}$ then find the characteristic of x^3 to the base 7.

Q.28 If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1$,

then find the value of $\log_{30}(x_1 y_1 x_2 y_2)$.

Answer Key

EXERCISE-I

Q.1 (2) **Q.2** (2) **Q.3** (4) **Q.4** (1) **Q.5** (3) **Q.6** (1) **Q.7** (4) **Q.8** (2) **Q.9** (2) **Q.10** (1)
Q.11 **(2)** **Q.12** (2) **Q.13** (2) **Q.14** (4) **Q.15** (3) **Q.16** (3) **Q.17** (4) **Q.18** (2) **Q.19** (3) **Q.20** (3)
Q.21 (2) **Q.22** (1) **Q.23** (4) **Q.24** (2) **Q.25** (3) **Q.26** (1) **Q.27** (2) **Q.28** (4)

EXERCISE-II

Q.1 (3) **Q.2** (4) **Q.3** (2) **Q.4** (3) **Q.5** (1) **Q.6** (1) **Q.7** (1) **Q.8** (3) **Q.9** (2) **Q.10** (4)
Q.11 (1) **Q.12** (3) **Q.13** (4) **Q.14** (4) **Q.15** (2) **Q.16** (3) **Q.17** (3) **Q.18** (1) **Q.19** (2) **Q.20** (4)
Q.21 (3) **Q.22** (4) **Q.23** (2) **Q.24** (3) **Q.25** (1) **Q.26** (2) **Q.27** (4) **Q.28** (3)

EXERCISE-III

MCQ/COMPREHENSION/MATCHING/NUMERICAL

Q.1 (A,B) **Q.2** (A, B, C) **Q.3** (A, B, C) **Q.4** (A,B,C,D) **Q.5** (A,B)
Q.6 (A,B,C,D) **Q.7** (A,B,C,D) **Q.8** (B, C) **Q.9** (A, B, D) **Q.10** (B, C)
Q.11 (B) **Q.12** (A) **Q.13** (D) **Q.14** (B) **Q.15** (D)
Q.16 (A)
Q.17 **[**(A) Q, R, S, T; (B) P; (C) Q, R, S, T; (D) P, R, S**]**
Q.18 **[**(A) R; (B) S; (C) P; (D) Q]
Q.19 [0001] **Q.20** [0002] **Q.21** [0006] **Q.22** [0007] **Q.23** [0003]
Q.24 [0009] **Q.25** [5625] **Q.26** [2.25] **Q.27** [2] **Q.28** [12]

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S
Q.1 (3)

JEE-ADVANCED
PREVIOUS YEAR'S
Q.1 [4] **Q.2 (A, B, C)** **Q.3 (8)**

EXERCISE (Solution)

EXERCISE-I

Q.1 (2)
It is a property

Q.2 (2)
x and y are rational and
$(x + y) + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$
Since real part of both sides are equal and coefficient of irrational parts should be zero in both sides
$\therefore\ x - 2y = 0 \Rightarrow x = 2y$
and $x - y - 1 = 0 \Rightarrow y = 1 \therefore x = 2$
$x = 2, y = 1$

Q.3 (4)
It is a property

Q.4 (1)
Here $(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$
Here sum of three + ve number can not be equal to zero then all three number independently be zero.
$\therefore$ There are no any real root.

Q.5 (3)
Let $P(x) = x^3 - a^2x + x + 2$ be the given polynomial
Then by factor theorem, $(x - a)$ is a factor of P(x) iff P(A) = 0
$\Rightarrow a^3 - a^2 . a + a + 2 = 0$
$\Rightarrow a + 2 = 0 \Rightarrow a = -2$

Q.6 (1)
Every irrational number can be expressed on the number line. This statement is always true.

Q.7 (4)
$\underset{x}{(\text{rational})} \times \underset{y}{(\text{irrational})} = \text{irrational except when } x = 0$

Q.8 (2)
$2x^2 - 4xy + xy - 2y^2 = 7$
$2x(x - 2y) + y(x - 2y) = 7$
$(x - 2y)(2x + y) = 7$
x, y are integers $\Rightarrow x - 2y, 2x + y$ are also integers
Four cases are possible
Case I: $x - 2y = 1, 2x + y = 7 \Rightarrow x = 3, y = 1$
$|x + y| = 4$
Case II : $x - 2y = 7, 2x + y = 1 \Rightarrow x = \frac{9}{5}$ rejected
Case III : $x - 2y = -1, 2x + y = -7$
$\Rightarrow x = -3, y = -1$
$|x + y| = 4$
Case IV : $x - 2y = -7, 2x + y = -1 \Rightarrow x = -\frac{9}{5}$ rejected
Hence $|x + y| = 4$

Q.9 (2)
$a(a - b) + b(b - c) + c(c - a) = 0$
$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$
Multiplying & deviding by 2,
$\frac{1}{2}[(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ac)] = 0$
$\Rightarrow \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$
$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0 \Rightarrow a = b = c$

Q.10 (1)
$2x^3 - 5x^2 + x + 2$
$= ax^3 + (-b - 2a)x^2 + (a - 1 + 2b)x + 2$
On comparing coefficient of x^3 & x, we get
$a = 2$ & $b = 1$

Q.11 **(2)**
$[e] - [-\pi]$
here $e \approx 2.7$
$\pi \approx 3.14$
$\therefore\ [2.7] - [-3.14]$
$= 2 - (-4) = 2 + 4 = 6$

Q.12 (2)
Taking log on both sides,
$(3x^2 - 10x + 3) \log|x - 3| = 0$
$\log|x - 3| = 0$ or $3x^2 - 10x + 3 = 0$
$x - 3\ ^{1}\ 0;\ |x - 3| = 1$ or $(x - 3)(3x - 1) = 0$
$x \neq 3;\ (x - 3) = \pm 1$ or $x = 3;\ x = \frac{1}{3}$
$\therefore\ x = 2, 4$ or $x =$ $[\because x \neq 3]$
Hence, three real solutions

Q.13 (2)
Applying base change theorem,
$= \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab}$
$= \log_{abc}\sqrt{bc} \cdot \sqrt{ca} \cdot \sqrt{ab} = \log_{abc} abc = 1$

Q.14 (4)

Given, $\log_x \log_{18}(\sqrt{2} + \sqrt{8}) = \frac{1}{3}$

$\Rightarrow \log_{18}(\sqrt{2} + 2\sqrt{2}) = x^{1/3} \Rightarrow \log_{18} 3\sqrt{2} = x^{1/3}$

$\Rightarrow (3\sqrt{2})^2 = \left(18^{x^{1/3}}\right)^2 \Rightarrow 18 = 18^{2x^{1/3}}$

$2x^{1/3} = 1 \Rightarrow x^{1/3} = \frac{1}{2} \Rightarrow x = \frac{1}{8} \Rightarrow 1000x = 125$

Q.15 (3)

$\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$

$\Rightarrow x < 0$

$\sqrt{\log_{10}(-x)} = \log_{10}(-x)$

$\Rightarrow \log^2_{10}(-x) - \log_{10}(-x) = 0$

$\Rightarrow \log_{10}(-x) = 0, 1 \Rightarrow x = -1, -10$

two solution

Q.16 (3)

$\log_2 15 \,.\, \log_{1/6} 2 \,.\, \log_3 1/6$

$= \frac{\log_e 15}{\log_e 2} \times \frac{\log_e 2}{\log_e 1/6} \times \frac{\log_e 1/6}{\log_e 3} = \frac{\log_e 15}{\log_e 3} =$

$\frac{\log_e(3 \times 5)}{\log_e 3} = 1 + \log_3 5$

$\therefore \ [1 + \log_3 5] = 2$

Q.17 (4)

$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$

$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$

Q.18 (2)

$3^{\log_3(x^2)} - 2x - 3 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = -1, 3, (x = -1 \text{ reject } \because x > 0)$

number of values of x is one

Q.19 (3)

$\log_2 7 \Rightarrow \log_2 4 < \log_2 7 < \log_2 8$

$\Rightarrow 2 < \log_2 7 < 3$ i.e. not integer

Let $\log_2 7 = \frac{p}{q}$ (where p and q are coprime)

$\Rightarrow 2^{p/q} = 7 \Rightarrow 2^p = 7^q$

which is not possible so $\log_2 7$ is an irratoinal number

Q.20 (3)

$\Rightarrow \text{antilog}_{16}\, 0.75 = (16)^{0.75}$

$= (16)^{3/4} = (2^4)^{3/4} = 2^3 = 8$

Q.21 (2)

$\log_{\sqrt{0.9}} \log_5\left(\sqrt{x^2 + 5 + x}\right) > 0$

$\log_5\left(\sqrt{x^2 + 5 + x}\right) < 1$

$(x^2 + 5 + x)^{1/2} < 5$ and $x^2 + x + 5 > 0$

$\Rightarrow x^2 + 5 + x < 25$

$\Rightarrow x^2 + x - 20 < 0$

$\Rightarrow (x+5)(x-4) < 0$

$\Rightarrow x \in (-5, 4)$

$\therefore n = 8$

Q.22 (1)

$x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

$\log_5(x^2 - 4) > 0 \Rightarrow x^2 - 4 > 1 \Rightarrow x^2 - 5 > 0$

$\Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

Now

$\log_{0.5} \log_5(x^2 - 4) > \log_{0.5} 1 \Rightarrow \log_5(x^2 - 4) < 1$

$x^2 - 4 < 5 \Rightarrow x^2 - 9 < 0$, $x \in (-3, 3)$

$\therefore$ **Ans. :** $\left(-3, -\sqrt{5}\right) \cup \left(\sqrt{5}, 3\right)$

Q.23 (4)

$\log_{1-x}(x - 2) \geq -1$

$1 - x > 0 \Rightarrow 1 > x \Rightarrow x \in (-\infty, 1) - \{0\}$

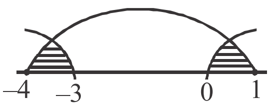
$x - 2 > 0 \Rightarrow x > 2$ No solution.

Q.24 (2)

$2 - \log_2(x^2 + 3x) \geq 0$

$\log_2(x^2 + 3x) \leq 2$

$\Rightarrow x^2 + 3x \leq 4$



$\Rightarrow x^2 + 3x - 4 \leq 0$

$\Rightarrow (x+4)(x-1) \leq 0$

$\Rightarrow x \in [-4, 1]$

and $x^2 + 3x > 0 \Rightarrow x \in (-\infty, -3) \cup (0, \infty)$

Ans. : $[-4, -3) \cup (0, 1]$

Q.25 (3)

When $x \leq -\frac{3}{4}$ $-4x - 3 - 3x + 4 = 12 - 11 = 7x$

$\Rightarrow x = -\frac{11}{7}$

When $-\frac{3}{4} < x \leq \frac{4}{3}$; $4x + 3 - 3x + 4 = 12$

x = 5 will not satisfy

When $x > \frac{4}{3}$ $4x + 3 + 3x - 4 = 12$

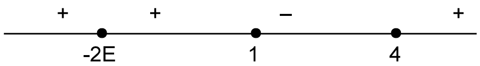
$7x = 13 \Rightarrow x = \frac{13}{7}$

Q.26 (1)

Since $(x^2+x-2)-(x^2-2x-8)=3x+6=3(x+2)$

$\therefore\ (x^2-2x-8)(x^2+x-2)\le 0$

i.e. $(x-4)(x+2)(x+2)(x-1)\le 0$



$\therefore$ Solution set is $[1,4]\cup\{-2\}$

Q.27 (2)

$||x-1|-1|\le 1$

$\Rightarrow -1\le |x-1|-1\le 1$

$\Rightarrow 0\le |x-1|\le 2$

$\Rightarrow -2\le x-1\le 2$

$\Rightarrow -1\le x\le 3$

$\therefore$ **Ans. :** $x\in[-1,3]$

Q.28 (4)

$|x|^2-3|x|+2=0 \Rightarrow (|x|-2)(|x|-1)=0$

$\Rightarrow |x|=1,2 \Rightarrow x=\pm1,\pm2$

$\therefore$ number of real roots is 4.

EXERCISE-II

Q.1 (3)

A & B are two rational number then $\frac{A}{B}$ is

Also rational number if $B\ne 0$.

Q.2 (4)

$$\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}=?$$

Let $a-b=A,\ (b-c)=B,\ (c-a)=C$

$[\because A+B+C=0 \Rightarrow A^3+B^3+C^3=3ABC]$

$$\therefore\ \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}=\frac{3ABC}{ABC}=3$$

Q.3 (2)

$P(4)=k4^3+3\cdot4^2-3$ & $Q(4)=2\cdot4^3-5\cdot4+k$

remainder is same

$P(4)=Q(4) \Rightarrow 64k+48-3=128-20+k$

$\Rightarrow 63k=108-45 \Rightarrow k=\frac{63}{63}=1$

Q.4 **(3)**

Putting $x=1$, remainder $=7$

Q.5 **(1)**

$$x^{x.x^{1/3}}=\left(x.x^{1/3}\right)^x \Rightarrow x^{x^{1+\frac{1}{3}}}=\left(x^{1+\frac{1}{3}}\right)^x$$

$$\Rightarrow x^{x^{4/3}}=\left(x^{4/3}\right)^x=x^{x^{4/3}}=x^{\frac{4}{3}x} \Rightarrow x^{4/3}=\frac{4}{3}x;$$

Also is an obvious solution.

Q.6 **(1)**

$$4^{\left(x^2+2\right)}-9.2^{\left(x^2+2\right)}+8=0$$

$$\Rightarrow\left(2^{\left(x^2+2\right)}\right)^2-9.2^{\left(x^2+2\right)}+8=0$$

Put $2^{\left(x^2+2\right)^2}=y$. Then $y^2-9y+8=0$, which gives $y=8,\ y=1$.

when $y=8 \Rightarrow 2^{x^2+2}=8 \Rightarrow 2^{x^2+2}=2^3 \Rightarrow x^2+2=3$

$\Rightarrow x^2=1 \Rightarrow x=1,-1$.

when $y=1 \Rightarrow 2^{x^2+2}=1 \Rightarrow 2^{x^2+2}=2^{\circ}$

$\Rightarrow x^2+2=0 \Rightarrow x^2=-2$, which is not

Q.7 **(1)**

Let x be the required logarithm , then by definition

$\log_{2\sqrt{2}} 32\sqrt[5]{4}=x$

$\left(2\sqrt{2}\right)^x=32\sqrt[5]{4} \Rightarrow \left(2.2^{1/2}\right)^x=2^5.2^{2/5}$; $\therefore\ 2^{\frac{3x}{2}}=2^{5+\frac{2}{5}}$

Here, by equating the indices, $\frac{3}{2}x=\frac{27}{5}$

$\therefore x=\frac{18}{5}=3.6$.

Q.8 (3)

$\log_2 7 \Rightarrow \log_2 4<\log_2 7<\log_2 8$

$\Rightarrow 2<\log_2 7<3$ i.e. not integer

Let $\log_2 7=\frac{p}{q}$ (where p and q are coprime)

$\Rightarrow 2^{p/q}=7 \Rightarrow 2^p=7^q$

which is not possible so $\log_2 7$ is an irratoinal number

Q.9 **(2)**

$\log_3 4.\ \log_4 5.\ \log_5 6.\ \log_6 7.\ \log_7 8.\ \log_8 9$

$$=\frac{\log 4}{\log 3}.\frac{\log 5}{\log 4}.\frac{\log 6}{\log 5}.\frac{\log 7}{\log 6}.\frac{\log 8}{\log 7}.\frac{\log 9}{\log 8}=\frac{\log 9}{\log 3}$$

$=\log_3 9=\log_3 3^2=2.$

Q.10 (4)

$$\frac{1}{\log_b{}^b+\log_b{}^a+\log_b{}^c}+\frac{1}{\log_c{}^c+\log_c{}^a+\log_c{}^b}+\frac{1}{\log_a{}^a+\log_a{}^b+\log_a{}^c}$$

$$=\frac{1}{\log_b abc}+\frac{1}{\log_c abc}+\frac{1}{\log_a abc}$$

$=\log_{abc} b+\log_{abc} c+\log_{abc} a=\log_{abc} abc=1$

Q.11 **(1)**

$$\log_e\left(\frac{a+b}{2}\right)=\frac{1}{2}\left(\log_e a+\log_e b\right)$$

$$=\frac{1}{2}\log_e(ab)=\log_e\sqrt{ab}$$

$$\Rightarrow \frac{a+b}{2}=\sqrt{ab}\Rightarrow a+b=2\sqrt{ab}$$

$$\Rightarrow\left(\sqrt{a}-\sqrt{b}\right)^2=0\Rightarrow\sqrt{a}-\sqrt{b}=0\Rightarrow a=b$$

Q.12 **(3)**

$$\log_7\log_7\sqrt{7\sqrt{7\sqrt{7}}}=\log_7\log_7 7^{7/8}=\log_7(7/8)$$

$= \log_7 7 - \log_7 8 = 1 - \log_7 2^3 = 1 - 3\log_7 2.$

Q.13 **(4)**

$$81^{(1/\log_5 3)}+27^{\log_9 36}+3^{4/\log_7 9}$$

$$=3^{4\log_3 5}+3^{3\cdot\frac{1}{2}\log_3 36}+3^{4\log_9 7}$$

$$=3^{\log_3 5^4}+3^{\log_3 36^{3/2}}+3^{\log_3 7^{4/2}}$$

$$=5^4+36^{3/2}+7^2=890$$

Q.14 **(4)**

$$\log_{1000}x^2=\log_{10^3}x^2=2\log_{10^3}x=\frac{2}{3}\log_{10}x=\frac{2}{3}y$$

Q.15 **(2)**

$x = \log_a bc \Rightarrow 1 + x = \log_a a + \log_a bc = \log_a abc$

$\therefore (1 + x)^{-1} = \log_{abc} a$

$\therefore (1+x)^{-1}+(1+y)^{-1}+(1+z)^{-1}=\log_{abc}a+\log_{abc}b+\log_{abc}c=\log_{abc}abc=1.$

Q.16 **(3)**

$$a=\log_{24}12=\frac{\log 12}{\log 24}=\frac{2\log 2+\log 3}{3\log 2+\log 3}$$

$$b=\log_{36}24=\frac{3\log 2+\log 3}{2(\log 2+\log 3)}$$

$$c=\log_{48}36=\frac{2(\log 2+\log 3)}{4\log 2+\log 3}$$

$$\therefore abc=\frac{2\log 2+\log 3}{4\log 2+\log 3}$$

$$\Rightarrow 1+abc=\frac{6\log 2+2\log 3}{4\log 2+\log 3}=2.\frac{3\log 2+\log 3}{4\log 2+\log 3}$$

$= 2bc.$

Q.17 **(3)**

$$\log_7\log_5\left(\sqrt{x^2+5+x}\right)=0=\log_7 1$$

$$\Rightarrow\log_5\left(x^2+5+x\right)^{1/2}=1=\log_5 5$$

$\Rightarrow (x^2 + 5 + x)^{1/2} = 5$

$\Rightarrow (x^2 + x + 5) = 25 \Rightarrow x^2 + x - 20 = 0$

$\Rightarrow (x - 4)(x + 5) = 0 \Rightarrow x = 4, -5 \Rightarrow x = 4$

Q.18 **(1)**

$$(0.05)^{\log_{\sqrt{20}}(0.1+0.01+\ldots\ldots)}=\left(\frac{1}{20}\right)^{2\log_{20}\left(\frac{0.1}{1-0.1}\right)}$$

$$=20^{-2\log_{20}(1/9)}=20^{2\log_{20}9}=20^{\log_{20}9^2}=9^2=81$$

Q.19 **(2)**

$$\log_2.\log_3\ldots.\log_{99}\ \log_{100}100^{99^{98^{\cdot^{\cdot^{2^1}}}}}=\log_2\log_3\ldots.\log_{99}99^{98^{\cdot^{\cdot^{2^1}}}}$$

$[\log_{100}100 = 1]$

$$=\log_2.\log_3\ldots.\log_{98}98^{97^{\cdot^{\cdot^{2^1}}}}$$

$$=\log_2.\log_3\ldots.\log_{97}97^{96^{\cdot^{\cdot^{2^1}}}}=\log_2\log_3 3^{2^1}$$

$= \log_2 2' \log_3 3 = \log_2 2 = 1.$

Q.20 **(4)**

$\log_2(x + 5) = 6 - x \Rightarrow x + 5 = 2^{6-x}$

$\Rightarrow x + 5 = 64.2^{-x}$

Let $y = x + 5$, $y = 64.2^{-x}$ will intersect at one point.

Number of solutions = 1.

Q.21 **(3)**

$y = 3^{12} \times 2^8 \Rightarrow \log_{10}y = 12\log_{10}3 + 8\log_{10}2$

$= 12 \times 0.47712 + 8 \times 0.30103$

$= 5.72544 + 2.40824 = 8.13368$

$\therefore$ Number of digits in y = 8 + 1 = 9.

Q.22 (4)

$\log_{|x|}(x^2+x+1)\geq 0$

$D: |x| \neq 0, 1$

case-I : if $|x|<1$

$x^2+x+1\leq 1\Rightarrow x(x+1)\leq 0$

$x\in(-1,0)$

case-II : if $|x|>1$

$x^2+x+1>1\Rightarrow x(x+1)\geq 0$

$\Rightarrow\ x\in(-\infty,-1)\cup(0,\infty)$

$\therefore\ x\in(-\infty,-1)\cup(1,\infty)$

Q.23 **(2)**

$\log_{1/2}\left(x^2 - 6x + 12\right) \ge -2$ .....(i)

For log to be defined, $x^2 - 6x + 12 > 0$

$\Rightarrow (x - 3)^2 + 3 > 0$, which is true $\forall\ x \in R$.

From (i), $x^2 - 6x + 12 \le \left(\frac{1}{2}\right)^{-2}$

$\Rightarrow\ x^2 - 6x + 12 \le 4\ \Rightarrow x^2 - 6x + 8 \le 0$

$\Rightarrow (x-2)\,(x-4) \le 0 \Rightarrow\ 2 \le x \le 4$

$\therefore\ x \in [2, 4]$.

Q.24 **(3)**

$\log_{0.04}(x - 1) \ge \log_{0.2}(x - 1)$ ....(i)

For log to be defined $x - 1 > 0 \Rightarrow x > 1$

From (i), $\log_{(0.2)^2}(x-1) \ge \log_{0.2}(x-1)$

$\Rightarrow \frac{1}{2}\log_{0.2}(x-1) \ge \log_{0.2}(x-1) \Rightarrow \sqrt{x-1} \le (x-1)$

$\Rightarrow \sqrt{x-1}(1-\sqrt{x-1}) \le 0 \Rightarrow 1-\sqrt{x-1} \le 0$

$\Rightarrow\ \sqrt{x-1} \ge 1 \Rightarrow\ x \ge 2\ \therefore\ x \in [2,\infty)$

Q.25 **(1)**

$\log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

$\log_{0.3}(x-1) < \frac{1}{2}\log_{(0.3)}(x-1)$

$\log_{0.3}(x-1) < \log_{(0.3)}(x-1)^{1/2}$

here base is less than 1, therefore the inequality is reversed

$(x-1) > (x-1)^{1/2}$

$(x-1)^2 > (x-1)$

$x^2\ 1 - 2x - x + 1 > 0$

$x^2 - 3x + 2 > 0$

$x^2 - 2x - x + 2 > 0$

$x\,(x-2) - 1\,(x-2) > 0$

$(x-1)\,(x-2) > 0$

+ – +

1 2

$x \in (-\infty, 1) \cup (2, \infty)$

$\because\ x - 1 > 0 \Rightarrow x > 1$

than $x \in (2, \infty)$

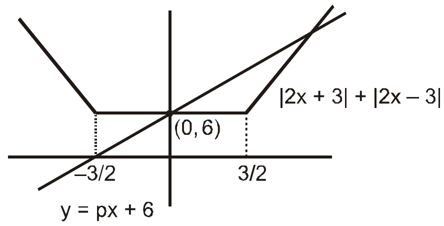
Q.26 (2)

$f(x) = |x - 1| + |x - 2| + |x - 3|$

$$= \begin{cases} -x+1-x+2-x+3 = 6-3x, & x \le 1 \\ x-1-x+2-x+3 = 4-x & 1 < x \le 2 \\ x-1+x-2-x+3 = x & 2 < x \le 3 \\ x-1+x-2+x-3 = 3x-6 & x > 3 \end{cases}$$

min $f(x) = 2$.

Q.27 (4)



For more than 2 solution p = 0

Q.28 (3)

$[2x] - 3\,\{2x\} = 1$

$\{2x\} = \frac{[2x]-1}{3}$

$0 \le \{2x\} < 1$

$\Rightarrow\ 1 \le [2x] < 4$

$\Rightarrow\ [2x] = 1, 2, 3$

$[2x] = 1,\ \{2x\} = 0$

$2x = 1 + 0$

$x = \frac{1}{2}$

$[2x] = 2\ ;\ \{2x\} = \frac{1}{3}$

$2x = 2 + \frac{1}{3}$

$2x = \frac{7}{3}$

$x = \frac{7}{6}$

$[2x] = 3,\ \ \{2x\} = \frac{2}{3},\ \ 2x = 3 + \frac{2}{3}$ we get three value of x

EXERCISE-III

Q.1 (A,B)

$\frac{y}{x} = x$

$\Rightarrow y = x^2\ \ [\because\ x \ne 0]$

$\Rightarrow y \ne 0$

$\because\ x^2 > 0 \Rightarrow y > 0\ \ \therefore\ y \ne -1$

Q.2 (A, B, C)

$x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$

$\left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right]\ \log_2 x = \log_2 \sqrt{2} = \frac{1}{2}$

Let $\log_2 x = t$

$(3t^2 + 4t - 5)\, t = 2 \Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0$

$t = 1, -2, -\frac{1}{3} = \log_2 x$

$x = 2, \frac{1}{4}, 2^{-\frac{1}{3}}$

Q.3 (A, B, C)

$(\log_5 x)^2 + \log_{5x}\frac{5}{x} = 1$

$\Rightarrow (\log_5 x)^2 + \log_{5x}5 - \log_{5x}x = 1$

$\Rightarrow (\log_5 x)^2 + \frac{\log_5 5}{\log_5 5 + \log_5 x} - \frac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow (\log_5 x)^2 + \frac{1}{1 + \log_5 x} - \frac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore\ t^2 + \frac{1}{1+t} - \frac{t}{1+t} = 1$

$\Rightarrow \frac{t^2(1+t) + 1 - t}{1+t} = 1$

$\Rightarrow t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$

$t(t^2 + t - 2) = 0$

$t(t-1)(t+2) = 0$

$t = 0, 1, -2$

$\therefore\ \log_5 x = 0, 1, -2$

$\therefore\ x = 1, 5, \frac{1}{25}$

Q.4 (A,B,C,D)

$= \log_3 135 \log_3 15 - \log_3 5 \log_3 405$

$= \log_3(5 \times 3^3) \cdot \log(5 \times 3) - \log_3 5 . \log_3 (5 \times 3^4)$

$= (\log_3 5 + \log_3 3^3)(\log_3 5 + \log_3 3) - \log_3 5 (\log_3 5 + \log_3 3^4)$

$= (x + 3)(x + 1) - x(x + 4)$ {Let $\log_3 5 = x$}

$= x^2 + 4x + 3 - x^2 - 4x = 3$

which is Prime, rational Integer and natural number

Q.5 (A,B)

$\Rightarrow \log_3 xy = 2\log_3 3 + \log_3 2$

$\Rightarrow \log_3 xy = \log_3(2 \times 9) \Rightarrow xy = 18$ .....(i)

and $\log_{27}(x + y) = \frac{2}{3}$ $\Rightarrow x + y = 27^{2/3}$

$\Rightarrow x + y = 3^2$ $\Rightarrow x + y = 9$ ......(ii)

from equation (i) & (ii)

$\therefore x^2 - 9x + 18 = 0 \Rightarrow (x - 6)(x - 3) = 0$

$\Rightarrow x = 6, 3$ so $(x, y) \equiv (6, 3) \equiv (3, 6)$

Q.6 (A,B,C,D)

$\frac{4}{2}\log_x 2 + \frac{\log_x 64}{\log_x 2x}$

$\Rightarrow \frac{2x\log_x 2}{x} + \frac{6\log_x 2}{1 + \log_x 2} = 3$

Let $\alpha = \log_x 2$

$2\alpha + \frac{6\alpha}{1+\alpha} = 3$

$2\alpha + 2\alpha^2 + 6\alpha - 3 - 3\alpha = 0$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0$

$\Rightarrow (\alpha + 3)(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore\ \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \frac{1}{2} \Rightarrow x = 4$ (Integer)

Q.7 (A,B,C,D)

Taking $\log_3$ on both sides

$[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5](\log_3 x) = \frac{3}{2}$

Let $\log_3 x = \alpha$

$\Rightarrow \frac{(2\alpha^3 - 9\alpha^2 + 10\alpha)}{2} = \frac{3}{2}$

$\Rightarrow 2\alpha^3 - 9\alpha^2 + 10\alpha - 3 = 0$

$\Rightarrow (\alpha - 1)(2\alpha^2 - \alpha + 3) = 0$

$\Rightarrow (\alpha - 1)(\alpha - 3)(2\alpha - 1) = 0 \Rightarrow \alpha = 1, 3, \frac{1}{2}$

$\therefore\ \log_3 x = 1; \log_3 x = 3; \log_3 x = \frac{1}{2}$

$\Rightarrow x = 3;\ x = 3^3 = 27;\ x = \sqrt{3}$

Exactly three solution, one is irrational solution and every real number is also complex.

Q.8 (B,C)

$$(\log_2 x)^4 - \left(\log_2\left(\frac{x}{2}\right)^3\right)^2 + 9\left[\log_2 32 - \log_2 x^2\right] < 4(\log_2 x)^2$$

(note $x > 0$)

$(\log_2 x)^4 - (3\log_2 x - 3)^2 + 45 - 18\log_2 x < 4(\log_2 x)^2$

let $\log_2 x = t$

$t^4 - (3t - 3)^2 + 45 - 18t < 4t^2$

$\Rightarrow t^4 - (9t^2 + 9 - 18t) - 18t + 45 < 4t^2$

$\Rightarrow t^4 - 13t^2 + 36 < 0$

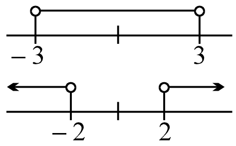
$\Rightarrow (t^2 - 4)(t^2 - 9) < 0$

$\Rightarrow 4 < t^2 < 9$

$t^2 < 9 \Rightarrow -3 < t < 3$

and $t^2 > 4 \Rightarrow t > 2$ or $t < -2$

hence, $t \in (-3, -2) \cup (2, 3)$



$x \in \left(\frac{1}{8}, \frac{1}{4}\right) \cup (4, 8) \quad \Rightarrow \quad$ **B, C**

Q.9 (A, B, D)

$$\frac{1}{2} \le \log_{0.1}x \le 2 \Rightarrow \left(\frac{1}{10}\right)^{1/2} \ge x \ge \left(\frac{1}{10}\right)^2$$

Q.10 (B,C)

$|x-5| + |x+5| = 10$

Case-I: $x \ge 5$, the equation becomes

$(x-5) + (x+5) = 10$

$\Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = 5$ which satisfies the case, therefore accepted.

Case-II: $-5 < x < 5$ The above equation becomes

$-(x-5) + (x+5) = 10$

$\Rightarrow -x + 5 + x + 5 = 10$

$\Rightarrow 10 = 10$ which is true.

So, the solution is $x \in (-5, 5)$

Case-III: $x \le -5$, The above equation becomes

$-(x-5) - (x+5) = 10$

$\Rightarrow -x + 5 - x - 5 = 10$

$\Rightarrow -2x = 10$

$\Rightarrow x = -5$ which satisfies the above case so, accepted.

$\therefore$ final answer is $x \in [-5, 5]$

Q.11 (B)

Q.12 (A)

Q.13 (D)

$x = 3^{\log 5 - \log 7}$

$y = 5^{\log 7 - \log 3}$

$z = 7^{\log 3 - \log 5}$

$\therefore \quad x \cdot y \cdot z = 1$

$\therefore$ **A = 1**

$\log_2 (6 \log_2 |x| - 3) - \log_2 (4 \log |x| - 5) = \log_2 3$

$$\frac{6\log_2|x|-3}{4\log_2|x|-5} = 3$$

let $\log_2 |x| = 1$

$$\therefore \quad \frac{6t-3}{4t-5} = 3$$

$6t - 3 = 12t - 15, \; 6t = 12$

$\therefore \; t = 2, \; \log_2 |x| = 2, \qquad |x| = 4$

$\therefore \; x = \pm 4$

B = 16 + 16 = 32

$\log_2 (\log_2 3) + \log_2 (\log_3 4) + \log_2 (\log_4 5) + \log_2 (\log_5 6) + \log_2 (\log_6 7) + \log_2 (\log_7 8)$

$= \log_2 (\log_2 8) = \log_2 3$

$\therefore$ **C = 1**

Q.14 (B)

Q.15 (D)

Q.16 (A)

$$\left(\sqrt{x}\right)^{\log_5 x - 1} = 5 \Rightarrow (\log_5 x - 1)\,\frac{1}{2}\log_5 x = 1$$

$\Rightarrow \log_5 x\,(\log_5 x - 1) = 2$

Put $\log_5 x = y$

$\Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0$

$\Rightarrow y = 2$ or $-1 \Rightarrow \log_5 x = 2$ or -1

$x = 25$ or $x = \frac{1}{5}$

$\alpha = 25$ and $\beta = \frac{1}{5}$

(A) Now, let $N = (25)^{10}$

$\log_{10} N = 10 \cdot 2\log_{10} 2 = 20\,[1 - 0.3010] = 20\,[0.6990] = 13.980 = 13.98$

$\Rightarrow$ N is a 14 digit number.

(B) Let $x = \left(\frac{1}{5}\right)^{10}$

$\log x = 10\,(-\log 5) = -10\,(1 - \log 2) = -10\,[0.699] = -6.99$

Hence, there are 6 zeroes after decimal before a significant figure.

(C) $\left(\frac{1}{5}\right)^{\log_{25} 9} = \left(\frac{1}{5}\right)^{\log_5 3} = \frac{1}{5^{\log_5 3}} = \frac{1}{3}$. **Ans.**]

Q.17 (A) Q, R, S, T; (B) P; (C) Q, R, S, T; (D) P, R, S]

(A) Let $\log_2(x-1) = A$ & $\log_2 x = B$ then

$\Rightarrow \quad (A+B)^2 - 4AB = 1$

$\Rightarrow \quad (A-B)^2 = 1$

$\Rightarrow \quad (A-B) = \pm 1$

$\therefore \quad \log_2\left(\frac{x-1}{x}\right) = 1$ or $2^{-1} \quad \therefore \; x = -1$ or 2

Only 2 is solution.

(B) $(8x-3) = 13 \qquad [a^{\log_a b} = b]$

$(8x = 16)$

$\therefore x = 2$

(C) $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right) \Rightarrow (\log_\pi 2 + \log_\pi 6)$

$\Rightarrow \log_\pi 12 >$ $\quad\quad \therefore \pi^2 = 10$

$\Rightarrow \log_\pi \pi^2 < \log_\pi 12 < \log_\pi \pi^3$ $\quad\quad \pi^3 < 28$

$\Rightarrow 2 < \log_\pi 12 < 3$

(D) $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$

$m = 0.8\ (1+9^{\log_3 8})^{\log_{65} 5}$

$\therefore \quad l = (A+B)^2 - (A-B)^2 = 4AB = 4\log_3 4 \cdot \log_2 9 \Rightarrow$

$4 \cdot \dfrac{\log 4}{\log 3} \times \dfrac{\log 9}{\log 2}$ Þ 16

$l = 16$

$m = 0.8\ (1+3^{\log_3 8^2})^{\log_{65} 5}$

$\Rightarrow \quad (0.8)\ (1+64)^{\log_{65} 15} \quad\quad \Rightarrow \quad (0.8)$

$(65)^{\log_{65}(5)} \quad \Rightarrow \quad (0.8)(5) = 4$

$\therefore \quad m = 4$

so, $\ l + m = 16 + 4 = 20$

Q.18 (A) R; (B) S; (C) P; (D) Q]

(A) Antilog $_{27}(2/3) = x \Rightarrow (2/3) = \log_{27} x$

$\Rightarrow x = (27)^{2/3} = 9$ **Ans**

(B) We have to find char of $\log_2 2008$

we know , $\log_2 1024 = 10$ and $\log_2 2048 = 11$

$\therefore\ 10 < \log_2 2008 < 11 \quad\quad \therefore$ it has char = 10 **Ans.**

(C) $\dfrac{\log 2}{\log e} \times \dfrac{\log 625}{\log b} = \dfrac{4\log 2}{\log 10} \times \dfrac{\log 10}{\log e}$

$\Rightarrow \log 5 = \log b \Rightarrow b = 5$ **Ans.**

(D) $y = \left(\dfrac{5}{6}\right)^{100} \Rightarrow \log_{10} y = 100(\log_{10} 5 - \log_{10} 6)$

$= 100(1 - \log_{10} 2 - \log_{10} 3 - \log_{10} 2)$

$\Rightarrow \quad -7.91 \Rightarrow -8 + 0.09$

since char is –8. Hence number of zeros after decimal = 7 **Ans.**

INTEGER TYPE

Q.19 **[1]**

Let $x + |x-2| = y$

$\therefore$ Equation becomes

$\log_x y^2 = \log_x (5y - 6)$

$\Rightarrow\ y^2 = 5y - 6 \quad\quad \Rightarrow\ y^2 - 5y + 6 = 0 \quad \Rightarrow\ y = 2$ or 3

If $y = 2$

then $x + |x-2| = 2 \ \Rightarrow 0 < x < 1 \cup 1 < x \le 2$

If $y = 3$

then $x + |x-2| = 3 \ \Rightarrow x = \dfrac{5}{2}$ only

Hence number of integral solutions is 1.

Q.20 **[2]**

Let $\log_{3x} 45 = \log_{4x} 40\sqrt{3} = k$

$45 = (3x)^k,\ 40\sqrt{3} = (4k)^k$

$\dfrac{45}{40\sqrt{3}} = \left(\dfrac{3k}{4x}\right)^k \Rightarrow \dfrac{3\sqrt{3}}{8} = \left(\dfrac{3}{4}\right)^k$

$\Rightarrow \left(\dfrac{3}{4}\right)^{\frac{3}{2}} = \left(\dfrac{3}{4}\right)^k \Rightarrow k = \dfrac{3}{2}$

$\log_{3x} 45 = \dfrac{3}{2}$

$45 = (3x)^{\frac{3}{2}}$

squaring both sides

$45 \times 45 = (3x)^3$

$x^3 = 75$

$\log_7 x^3 = \log_7 75$

$\Rightarrow \quad \log_7 49 < \log_7 75 < \log_7 343$

$\Rightarrow \quad 2 < \log_7 75 < 3$;

Characteristic = 2.

Q.21 **0006**

$y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16}$

$- \log_2 12 \cdot \log_2 48 + 10$

$= \sqrt{\log_2 3 \cdot (2+\log_2 3)(4+\log_2 3)(6+\log_2 3) + 16} -$

$(2+\log_2 3)(4+\log_2 3) + 10$

Let us put $\log_2 3 = x$

$= \sqrt{x(2+x)(4+x)(6+x)+16} - (2+x)(4+x) + 10$

$= \sqrt{(x^2+6x)(x^2+6x+8)+16} - (x^2+6x+8) + 10$

Put again $x^2 + 6x = \alpha$

$= \sqrt{\alpha(\alpha+8)+16} - (\alpha+8) + 10$

$= \sqrt{\alpha^2 + 8\alpha + 16} - (\alpha+8) + 10$

$= \sqrt{(\alpha+4)^2} - (\alpha+8) + 10$

$= (\alpha+4) - (\alpha+8) + 10 = y = 6.$

Q.22 **0007**

As, $\dfrac{1}{\log_a(2-\sqrt{3})} + \dfrac{1}{\log_b\left(\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)}$

$= \log_{2-\sqrt{3}} a + \log_{\frac{\sqrt{3}-1}{\sqrt{3}+1}} b$

$= \log_{2-\sqrt{3}} a + \log_{2-\sqrt{3}} b = \log_{2-\sqrt{3}}(ab)$

Now, $\left(2+\sqrt{3}\right)^{\log_{2-\sqrt{3}}(ab)} = \frac{1}{12}$

$\Rightarrow \left(2-\sqrt{3}\right)^{\log_{2-\sqrt{3}}\left(\frac{1}{ab}\right)} = \frac{1}{12}$

$\Rightarrow \frac{1}{ab} = \frac{1}{12} \Rightarrow ab = 12$

As a, b are co-prime numbers, so either a = 4, b = 3 or a = 3, b = 4.

Hence , (a + b) = 7. Ans.

Q.23 **0003**

$N = 10^p$; $p = \log_{10}8 - \log_{10}9 + 2\log_{10}6$

$p = \log\left(\frac{8\cdot 36}{9}\right) = \log_{10}32$

$\therefore N = 10^{\log_{10}32} = 32$

Hence characteristic of $\log_3 32$ is 3.

Q.24 **0009**

We have,

$2^{2x} - 8\cdot 2^x + 15 = 0 \Rightarrow (2^x - 3)(2^x - 5) = 0 \Rightarrow 2^x = 3$ or $2^x = 5$

Hence smallest x is obtained by equating $2^x = 3$

$\Rightarrow x = \log_2 3$

So, $p = \log_2 3$

Hence, $4^p = 2^{2\log_2 3} = 2^{\log_2 9} = 9$.

Q.25 5625

Let $\log_{10}x = a$; $\log_{10}y = b$ and $\log_{10}z = c$

Here $xyz = 10^{81}$

$\Rightarrow \log_{10}x + \log_{10}y + \log_{10}z = 81$

i.e. $a + b + c = 81$ ....(1)

Also $a(b + c) + bc = 468$

$ab + bc + ca = 468$ ....(2)

Now $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\sum ab = (81)^2 - (2)(468) = 6561 - 936 = 5625$ **Ans.**

Q.26 2.25

$\log(2y - 3x)^2 = \log xy$

$(2y - 3x)^2 = xy$

$4y^2 - 12xy + 9x^2 + xy$

Dividing the equation by y^2

$$9\left(\frac{x}{y}\right)^2 - 13\frac{x}{y} + 4 = 0$$

$$\left(\frac{x}{y} - 1\right)\left(\frac{9x}{y} - 4\right) = 0$$

$$\frac{x}{y} = 1,\ \frac{x}{y} = \frac{4}{9}.$$

x = y disregarded as for x = y, 2y – 3x is negative.

Hence $\frac{y}{x} = \frac{9}{4}$.

Q.27 2

Let $\log_{3x}45 = \log_{4x} 40\sqrt{3} = k$

$45 = (3x)^k$, $40\sqrt{3} = (4k)^k$

$$\frac{45}{40\sqrt{3}} = \left(\frac{3k}{4x}\right)^k \Rightarrow \frac{3\sqrt{3}}{8} = \left(\frac{3}{4}\right)^k$$

$$\Rightarrow \left(\frac{3}{4}\right)^{\frac{3}{2}} = \left(\frac{3}{4}\right)^k \Rightarrow k = \frac{3}{2}$$

$$\log_{3x} 45 = \frac{3}{2}$$

$45 = (3x)^{\frac{3}{2}}$

squaring both sides

$45 \times 45 = (3x)^3$

$x^3 = 75$

$$\log_7 x^3 = \log_7 75 \Rightarrow$$

$$\log_7 49 < \log_7 75 < \log_7 343$$

$2 < \log_7 75 < 3$; Characteristic = 2.

Q.28 12

$\log_{225}x + \log_{64}y = 4$

$\log_x 225 - \log_y 64 = 1$

Let us put $\log_{225}x = a$, $\log_{64}y = b$

$a + b = 4$

$\frac{1}{a} - \frac{1}{b} = 1$

$b - (4 - b) = (4 - b)\, b$

$2b - 4 = 4b - b^2$

$b^2 - 2b - 4 = 0$, Let b_1, b_2 be the roots

$b_1 + b_2 = 2$

$\log_{64}y_1 + \log_{64}y_2 = 2$

$\log_{64}y_1y_2 = 64^2 = 2^{12}$

Similarly $a_1 + a_2 = (4 - b_1) + (4 - b_2)$

$= 8 - (b_1 + b_2) = 8 - 2$

$a_1 + a_2 = 6$

$\log_{225}x_1 + \log_{225}x_2 = 6$

$x_1x_2 = 225^6 = 15^{12}$.

Now, $\log_{30}x_1x_2y_1y_2 = \log_{30}2^{12} \times 15^{12}$

$= 12\log_{30}30 = 12$. **Ans.**

EXERCISE-IV

JEE-Main
PREVIOUS YEAR'S

Q.1 **(3)**

$$\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$$

$$|\sqrt{x}-2|+\left(\sqrt{x}\right)^2-4\sqrt{x}+2=0$$

$$|\sqrt{x}-2|^2+\left|\sqrt{x}-2\right|-2=0$$

$$|\sqrt{x}-2|=-2 \text{ (not possible) or } |\sqrt{x}-2|=1$$

$$\sqrt{x}-2=1,-1$$

$$\sqrt{x}=3,1$$

$x=9, 1$

Sum $=10$

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 **4**

Let $\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}}\text{.......}}=t \Rightarrow \sqrt{4-\frac{1}{3\sqrt{2}}t}=$

$t \Rightarrow 4-\frac{1}{3\sqrt{2}}t=t^2 \Rightarrow$

$t^2+\frac{1}{3\sqrt{2}}t-4=0 \Rightarrow 3\sqrt{2}\,t^2+t-12\sqrt{2}=0$

$$\Rightarrow t=\frac{-1\pm\sqrt{1+4\times3\sqrt{2}\times12\sqrt{2}}}{2\times3\sqrt{2}}=\frac{-1\pm17}{2\times3\sqrt{2}}$$

$t=\frac{16}{6\sqrt{2}},\frac{-18}{6\sqrt{2}} \Rightarrow t=\frac{8}{3\sqrt{2}},\frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\times\frac{8}{3\sqrt{2}}\right)=6+\log_{3/2}\left(\frac{4}{9}\right)$

$$=6+\log_{3/2}\left(\left(\frac{2}{3}\right)^2\right)=6-2=4$$

Q.2 **(A, B, C)**

$3^x=4^{x-1} \Rightarrow x=(x-1)\log_3 4$

$\Rightarrow x(1-2\log_3 2)=-2\log_3 2$

$x=\frac{2\log_3 2}{2\log_3 2-1}$ **Ans. (A)**

Again $x\log_2 3=(x-1)\cdot 2$

$\Rightarrow x(\log_2 3-2)=-2 \Rightarrow x=\frac{2}{2-\log_2 3}$ **Ans. (B)**

$x=\frac{1}{1-\frac{1}{2}\log_2 3}=\frac{1}{1-\log_4 3}$ **Ans.(C)**

Q.3 **(8)**

$$\log_2 9^{\frac{2}{\log_2(\log_2 9)}}\times 7^{\frac{1/2}{\log_4 7}}$$

$$=(\log_2 9)^{2\log^{2}_{\log 2^9}}\times 7^{\frac{1}{2}\log_7 4}$$

$=4\times2=8$