Magnetic Effect of Current

SOURCES OF MAGNETIC FIELD Section - 1

Biot Savart Law :

BiotSavart Law gives the magnetic induction due to an infinitesimal current element of length *d* carrying a current *i*. According to BiotSavart Law :

$$d\,\overline{B} = \frac{\mu_0}{4\pi}\,\frac{i\left(d\,\overline{\ell} \times \overline{r}\right)}{r^3}$$

where $d\ell$ vector points in the direction of current *i*. The vector r goes from the current element to the point where the field *d*B is to be calculated. The constant μ_0 is the known as the permeability of the free space and has a value of $4\,\pi \times 10^{-7}$ TmA^{-1}.

If the medium is other than air or vacuum, the magnetic induction is

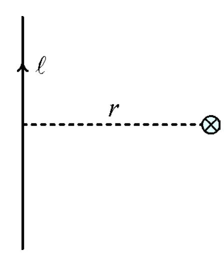
$$d\overline{B} = \frac{\mu_r\,\mu_0}{4\pi}\,\frac{i\left(d\,\overline{\ell} \times \overline{r}\right)}{r^3}$$ where μ_r is the relative permeability of the medium and is a dimensionless quantity.

Magnetic field for some important cases :

1. Field of a straight infinite current wire :

The magnetic field due to a current carrying straight wire of infinite length at a distance r from the wire is

$$B = \frac{\mu_0\,i}{2\,\pi\,r}$$

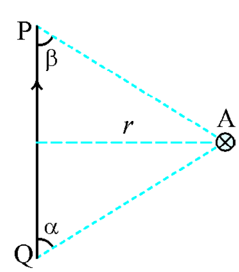


The magnetic lines of force for a long straight wire are concentric circles centred at the wire and lying in the plane perpendicular to the wire. If the wire is gripped in the right hand with the thumb in the direction of current, the curling fingers give the direction of lines of force in circles.

2. Field of a straight current carrying wire of finite length.

Consider a straight wire PQ carrying a current *i* as shown. The magnetic induction at a distance *r* from the wire is given by :

$$B = \frac{\mu_0\,i}{2\,\pi\,r}\left(\frac{\cos\alpha + \cos\beta}{2}\right)$$



where α , β are the angles between the wire and the lines joining the end points of wire to the point A where the field is to be calculated.

3. Field due to a circular current loop

Consider a circular loop of radius r carrying a current i and having N turns.

(a) Field at Centre :

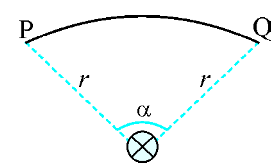
$$B = \frac{\mu_0 i N}{2r}$$

(b) Field at axis :

$$B = \frac{\mu_0 i r^2 N}{2\left(x^2 + r^2\right)^{3/2}}$$

4. Field at the centre of a current arc

The magnetic field at the centre of a wire PQ of length l bent in a shape of a arc of radius r is :

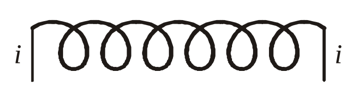


$$B = \frac{\mu_0 i}{2r}\left(\frac{\alpha}{2\pi}\right) = \frac{\mu_0 i}{4\pi r^2} l$$

5. Field inside a long solenoid

Consider a solenoid of length l and radius of cross section r ($r << \ell$) having N turns. If I is the current passing through the solenoid, the magnetic induction inside the solenoid is given as :

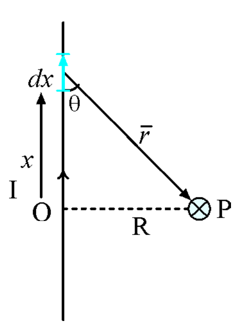
$$B = \frac{\mu_0 I N}{\ell} \quad \text{or} \quad B = \mu_0 In$$



where n is the number of turns per unit length.

Illustration - 1 *Derive an expression for the field due to an infinitely long wire carrying a current I at a point P lying at a distance R from the wire.*

SOLUTION :



Consider an infinitesimal current element of length dx at a distance x from the point O. The angle between dx vector and position vector r is $(\pi - \theta)$.

Using Biot Savart Law, the field due to this element can be written as :

$$dB = \frac{\mu_0 Idx \sin(\pi - \theta)}{4\pi r^2}$$

directed into the plane of paper.

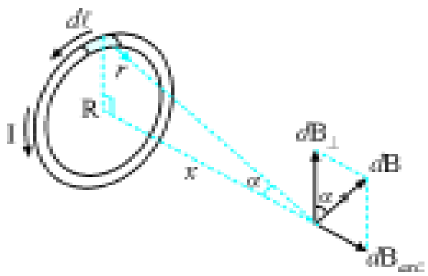
As the field contributions by all elements along the wire are in same direction (inwards at P), the net field is fast the scalar sum of all the $d\bar{B}$ vectors.

$$B = \int dB = \int_{-\infty}^{\infty} \frac{\mu_0 Idx}{4\pi\left(R^2 + x^2\right)} \cdot \frac{R}{\sqrt{R^2 + x^2}}$$

On evaluating the integral, we get : $B = \dfrac{\mu_0 \; I}{2\pi \, R}$

Illustration - 2 *A circular loop of radius R carries a current I. Find the magnetic field along the axis of the loop at a distance z from the center.*

SOLUTION :



The figure shows the infinitesimal contribution to the field dB from an arbitrary current element Id*l*. (Use the right-hand rule to confirm the direction of the field). The field increment dB has components both along and perpendicular to the axis. However, if we consider the contributions of current elements that are diametrically opposite, we see that their compo nents normal to the axis will cancel.

Since $d\bar{l}$ and r are perpendicular, it follows that $|\, d\bar{l} \times \bar{r} \,| = rdl$. Therefore, the component of dB along the axis is :

$$dB_{axis} = dB \sin\alpha = \left(\frac{\mu_0 I \, d\,\ell}{4\pi \, r^2}\right)\left(\frac{R}{r}\right)$$

The total field strength is given by the integral of this expression over all elements. Since the only variable is *l*, the integral reduces to a sum of length elements.

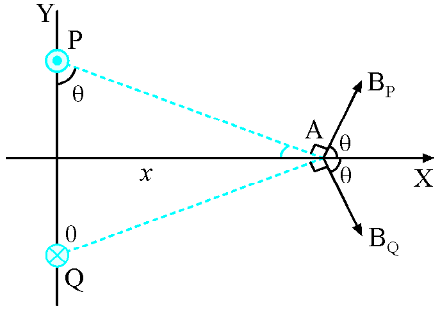
$$B_{axis} = \int dB_{axis} = \frac{\mu_0 I \, R}{4\pi \, r^3} \int_0^{2\pi R} d\, l = \frac{\mu_0 IR^2}{2r^3}$$

$$B = \frac{\mu_0 IR^2}{2\left(x^2 + R^2\right)^{3/2}}$$

Note : Substitute $x = 0$ to get the field at the centre of a circular loop.

Illustration - 3 *Two long wires carrying equal and opposite currents i are placed normal to XY plane and passing through the points P* (0, *l*, 0) *and Q* (0, *l*, 0). *Find the net magnetic induction at the point A* (*x*, 0, 0).

SOLUTION :



The wires are placed parallel to *z* axis.

Let the current in wire through P be towards positive *z* axis.

Field of wire through P at A

$$B_P = \frac{\mu_0 \; i}{2\pi\left(PA\right)} = \frac{\mu_0 \; i}{2\pi\sqrt{x^2 + l^2}}$$

Field of wire through Q at A

$$B_Q = \frac{\mu_0 i}{2\pi (QA)} = \frac{\mu_0 i}{2\pi\sqrt{x^2 + l^2}}$$

Let θ be the angle made by B_P and B_Q with X axis.

$$\Rightarrow \text{ For } \Delta APQ, \cos\theta = \frac{\ell}{\sqrt{l^2 + x^2}}$$

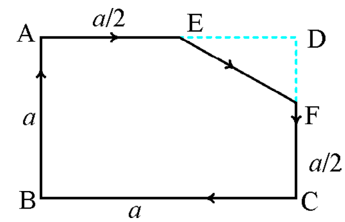
$$B_{net} = B_P \cos\theta + B_Q \cos\theta$$

$$= \frac{2\mu_0 i}{2\pi\sqrt{x^2 + l^2}} \frac{l}{\sqrt{x^2 + l^2}}$$

$$= \frac{\mu_0 i l}{\pi\left(x^2 + l^2\right)} \text{ towards + X -axis}$$

The Y component of B_P and B_Q balance each other.

Illustration - 4 *A current i flows through a loop ABCDEF as shown. Find the magnetic induction at the point D if ABCD is a square of side a.*



SOLUTION : $B = \overline{B}_{AB} + \overline{B}_{BC} + \overline{B}_{EF}$

(Note that field due to AE and CF is zero at D because D is collinear with AE and CF)

$$B_{AB} = B_{BC}$$

$$= \frac{\mu_0 i}{4\pi a}\left[\cos 90° + \cos 45°\right] = \frac{\mu_0 i}{4\pi a\sqrt{2}} \text{ inwards}$$

$$B_{EF} = \frac{\mu_0 I}{4\pi\left(\frac{a}{2\sqrt{2}}\right)}\left[\cos 45° + \cos 45°\right]$$

$$= \frac{\mu_0 i}{\pi a} \text{ outwards.}$$

$$B = B_{EF} - (B_{AB} + B_{BC})$$

$$= \frac{\mu_0 i}{\pi a} - 2\left(\frac{\mu_0 i}{4\pi a\sqrt{2}}\right) = \frac{\mu_0 i}{\pi a}\left(1 - \frac{1}{2\sqrt{2}}\right)$$

Illustration - 5 *A thin dielectric disc of radius R has a charge Q uniformly distributed over its surface. The disc is rotated with a constant angular velocity about its axis. Find the magnetic field at the centre of the disc and its magnetic dipole moment.*

SOLUTION :

Let us divide the disc into infinite ring-shaped elements and consider one such element of inner radius x and outer radius $x + dx$.

dq = charge on the element = $Q/\pi R^2$

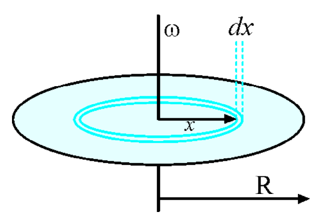
(area of element)

$$= \frac{Q}{\pi R^2}\left[\pi\left(x + dx\right)^2 - \pi x^2\right]$$

$$= \frac{Q(2\pi \times dx)}{\pi R^2} = \frac{2Qxdx}{R^2} \text{ (neglecting } (dx)^2\text{)}$$

This element makes $\omega/2\pi$ revolutions per sec. Hence its motion is equivalent to a circular current of $(dq)\ \omega/2\pi$.

$$\Rightarrow \quad \text{field at centre} = dB = \frac{\mu_0}{2x}\left(\frac{\omega dq}{2\pi}\right)$$



$\Rightarrow$ The net field $= \int dB$

$$B = \int \frac{\mu_0 \omega dq}{2x(2\pi)}$$

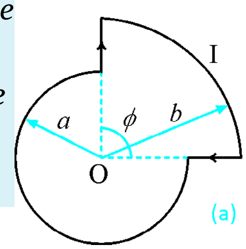
$$B = \int_0^R \frac{\mu_0 \omega}{4\pi x} \frac{2Qxdx}{R^2}$$

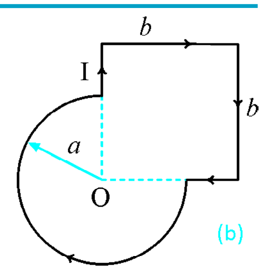
$$B = \frac{\mu_0 \omega Q}{2\pi R}$$

Illustration - 6 *Find the magnetic induction of the field at the point O of a loop with current I, whose shape is illustrated.*

(a) *In figure (a), the radii a and b, as well as the angle φ are known ;*

(b) *In figure (b), the radius a and the side b are known.*





SOLUTION :

(a) $$B = Bc_1 + Bc_2 = \frac{\mu_0 I}{2a} \frac{(2\pi - \phi)}{2\pi} + \frac{\mu_0 I}{2b} \cdot \frac{\phi}{2\pi}$$

(b) $$B = B_{circle\ arc} + B_{square\ portion} = \frac{\mu_0 I}{2a} \frac{3}{4} + 2\left[\frac{\mu_0 I}{2\pi b}\left(\frac{\cos 90^\circ + \cos 45^\circ}{2}\right)\right]$$

IN-CHAPTER EXERCISE - A

1. *Two long straight horizontal parallel wires, one above the other carry equal currents and are separated by a distance*

2a. What is the field in the plane of wires at a point:

(a) *half-way between them* **(b)** *at a distance a above the upper wire ?*

Consider both cases: currents in opposite directions and in same directions.

Find the magnetic induction at the centre of a regular hexagon of side 5 cm and carrying a current of 2A.

2. *Two identical circular coils having a radius 6 cm and 60 turns each, have a common axis and are 16 cm apart. Find the strength of the magnetic induction at a point mid-way between them on their axis when a current of 0.1 A is passed through them in :*

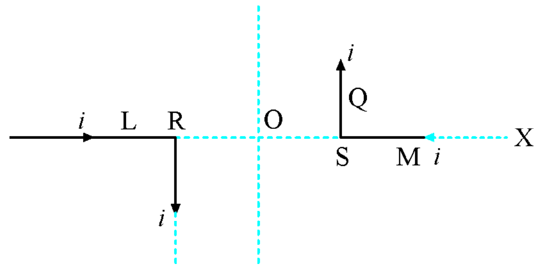
(a) *Opposite direction* **(b)** *Same direction*

(c) *Also find the magnetic induction at centre of each coil.*

3. *You are given a length l of a wire in which a current I may be established. The wire may be bent into a circle and a square. Which of the two shapes yields larger value of B at the centre ?*

4. *Any two points A and B on a uniform circular loop are connected across the terminals of a cell. Show that the total magnetic induction at the centre of the loop is zero.*

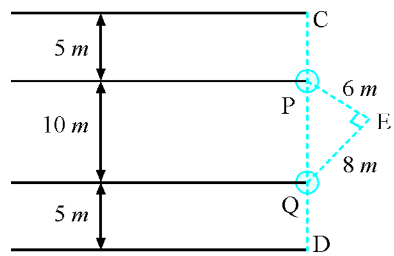
5. *A pair of stationary and infinitely long bent wires are placed in X-Y plane as shown. The wires carry currents of I = 10A each. The segments L and M are along the X-axis. The segments P and Q are parallel to the Y-axis such that OS = OR = 0.02 m. Find the magnitude and direction of the magnetic induction at the origin O.*



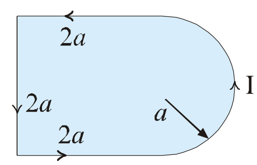
6. *Two long straight parallel wires P and Q are 10 m apart. (The wires are perpendicular to the plane of the paper). The current in P is I_1 = 6A coming not wards.*

(a) *Find the direction and magnitude of current I_2 in Q for a null point at D (i.e., zero field at D)*

(b) *Then find the magnetic induction at C and E.*



7. *The ends of a semicircular loop carrying a current I are connected to three wires that lie along the sides of a square (see figure).What is the field at the centre of the circular section ?*



MAGNETIC FORCE ON MOVING CHARGES — Section - 2

Magnetic field exerts its influence on moving charged particles or current carrying conductors. The charged particles at rest do not experience any force in a magnetic field.

Force on a moving charged particle

The force exerted by a magnetic field of induction B on a charged particle q moving with a velocity *v* is given by

or $\quad \bar{F} = q\left(\bar{v} \times \bar{B}\right)$

$$F = |\, qvB \,| \sin\theta$$

where θ is the angle between the B vector and the velocity of the charge. Note carefully that force on a positive charge is in the direction of $\bar{v} \times \bar{B}$ but the force on a negative charge is opposite to the vector $\bar{v} \times \bar{B}$.

Motion of a charged particle *q* in a uniform magnetic field

CASE : I (Straight line motion)

If a charged particle q is projected into a uniform magnetic field B with a velocity which is parallel to the field lines, the force experienced by the charge is zero and hence it travels in a straight line with uniform velocity.

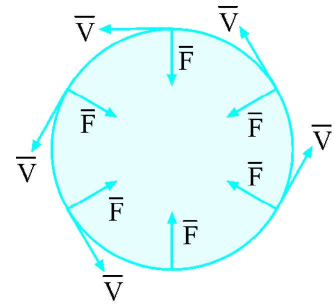
$$\bar{F} = q(\bar{v} \times \bar{B}) = \bar{O} \text{ for } \bar{v} \parallel \bar{B}$$

CASE : II (Uniform circular motion)

If a charged particle q is projected into a uniform magnetic field B with a initial velocity perpendicular to the magnetic field lines, it gets trapped in a circular path.

The force exerted by the field provides the necessary centripetal force.

Mathematically, we have



$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}$$

The time period of revolution is

$$T = \frac{2\pi r}{v} \quad \Rightarrow \quad T = \frac{2\pi m}{qB}$$

Note that the plane of the circular path is perpendicular to the lines of force.

CASE : III (motion in a helical path)

If the charge particle's initial projection velocity makes an angle $\theta(\theta \neq 0, 90°)$ with the magnetic field B, it moves in a helical path. The axis of the helix is parallel to the lines of force.

Resolving the velocity of particle, parallel and perpendicular to the field direction,

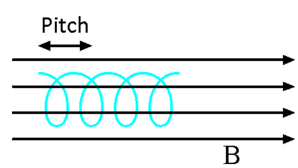
$$v_{11} = v \cos\theta \quad \text{and} \quad v_{\perp} = v \sin\theta \qquad \begin{bmatrix} v_{||}: & \text{Velocity parallel to field} \\ v_{\perp}: & \text{Velocity perpendicular to field} \end{bmatrix}$$

The $v \sin\theta$ component is responsible for rotation of particle and hence,

(a) the radius of the helix is

$$r = \frac{mv\perp}{qB}$$

(b) the time period of revolution is



$$T = \frac{2\pi m}{qB}$$

(c) the pitch of the helix (the displacement parallel to axis during one circular revolution)

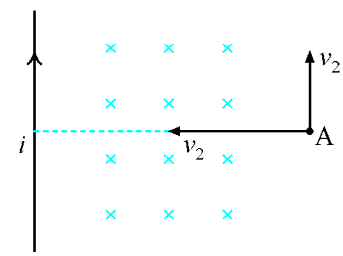
$$P = v_{||} T \qquad P = v_{||} \frac{2\pi m}{qB}$$

Illustration - 7 *A long straight wire carries a current of 50 A. An electron moving at 10^7 m/s is 5 cm from the wire. Find the force acting on the electron if its velocity is directed*

(i) *towards the wire* **(ii)** *parallel to the wire*

(iii) *perpendicular to directions defined by (i) and (ii).*

SOLUTION :



Magnetic Induction at the point A,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{2 \times 10^{-7} \times 50}{0.05}$$

$$= 2 \times 10^{-4} \text{ T}$$

The field is directed inwards.

(i) Force on electron = $qv_1 B \sin\theta$

$= (1.6 \times 10^{-19} \times 10^7 \times 2 \times 10^{-4}) \sin 90°$

$= 3.2 \times 10^{-16}$ N

The direction of force is opposite to $\bar{v}_1 \times B$ i.e. parallel to wire and in direction of current.

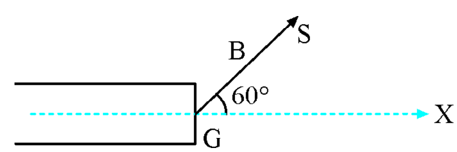
(ii) Magnitude of force = $qv_2 B \sin 90°$

$= 3.2 \times 10^{-16}$ N

This force is directed away from the wire.

(iii) Force = 0 because velocity is collinear with field and hence $\bar{v}_3 \times \bar{B} = \bar{0}$.

Illustration - 8 *An electron gun G emits electrons of energy 2 keV travelling in the positive X direction. The electrons are required to hit the spot S where GS = 0.1 m and the line GS makes an angle of 60° with the X-axis. A uniform magnetic field B parallel to GS exists in the region outside the gun. Find the minimum value of B needed to make the electron hit S.*



SOLUTION :

Let u be the speed of electron

$\Rightarrow \quad 1/2\, mu^2 = 2000\, e$

$\Rightarrow \quad u = \sqrt{4000 \frac{e}{m}}$

The electrons move in a helix as they come out.

To reach S, pitch of helix = GS = l

$V_{||}\ T = l$

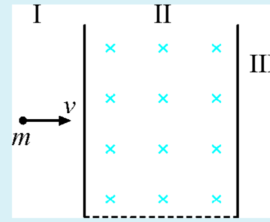
$$u \cos 60° \left(\frac{2\pi m}{eB}\right) = l$$

$$B = \frac{2\pi m}{eB} = \frac{\pi}{l}\sqrt{4000 \frac{m}{e}}$$

$$B = \frac{\pi}{0.1}\sqrt{\frac{4000 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}}$$

$$= 4.738 \times 10^{-3} \text{ T}$$

Illustration - 9 *A charged particle of mass m and charge q moves from a region I and to another region II crossing the interface normally. There is a uniform magnetic field B (inwards) present in the region II. There is no magnetic field in the region I and III.*

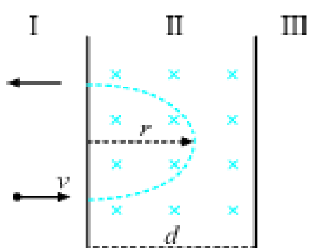


(a) *What is the maximum speed v_{max} of the charged particle so that it is able to return back in the region I ?*

(b) *Analyse the motion of particle if it is projected into the field as in (a) with a velocity $v = 2\,v_{max}$?*

SOLUTION :

(a)



The charged particle starts moving in a uniform circular motion as it enters the region II. It can come back to the region I if it follows a semi-circular path as shown. Hence the radius of path should be less than the width d of the field.

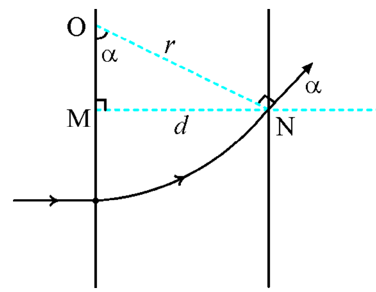
$$\Rightarrow \quad r < d$$

$$\frac{mv}{qB} < d$$

$$v < \frac{qBd}{m}$$

$$v_{max} = \frac{qBd}{m}$$

(b)



For $v > v_{max}$, the particle is able to penetrate through the field over to the region III following a circular trajectory in region II. Let α be the angle through which the particle is deviated. Let O be the centre of the circular path. For Δ OMN,

$$\sin\alpha = \frac{d}{r} \quad \text{where } r = \text{radius}$$

$$r = \frac{mv}{qB} \quad \Rightarrow \quad \sin\alpha = \frac{dBq}{mv}$$

$$\text{Angle of deviation} = \alpha = \sin^{-1}\frac{dBq}{mv}$$

$$= \sin^{-1}\left[\frac{q\,B\,d}{m\left(2v_{max}\right)}\right] = \frac{\pi}{6}$$

Illustration - 10 *A particle of mass* 1×10^{-26} *kg and charge* $+1.6 \times 10^{-19}$ *C travelling with a velocity* 1.28×10^{6} *m/s in the +X direction enters a region in which a uniform electric field E and a magnetic field of induction B are present such that* $E_x = 0$, $E_z = -102.4$ *kV/m and* $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$ *Wb/m*^{2}. *The particle enters this region at the origin at time t* = 0. *Determine the location* (*x, y, z coordinates*) *of the particle at t* = 5×10^{-6} *s. If the electric field is switched off at this point or instant* (*with the magnetic field still present*) *what will be the position of the particle at* $t = 7.45 \times 10^{-6}$s.

SOLUTION :

$$F = q\overline{E} + q\left(\overline{v} \times \overline{B}\right) = q\left[E + \overline{v} \times \overline{B}\right]$$

$$F = q\left[-102.4 \times 10^3 + 1.28 \times 8 \times 10^4\right]\hat{k} = \overline{0}$$

Hence particle moves in straight line with constant velocity.

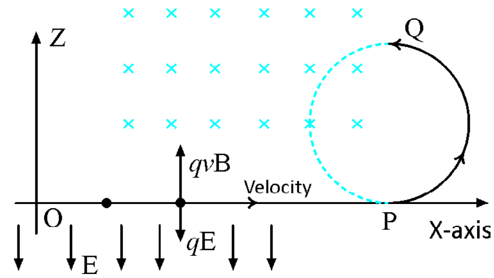
At $t_1 = 5\ \mu s$, it is at P.

$\Rightarrow$ OP = $vt_1 = 6.4\ m$.

After $t_1 = 5\mu s$, E = 0

Hence it moves in circle in ZX plane (perpendicular to B).

$$T = \text{Period} = \frac{2\pi m}{qB} = 4.9\mu s$$



At $t_2 = 7.45\ \mu s$, the particle is at Q.

$$\text{angular disp.} = \frac{t_2 - t_1}{T} \times 2\pi = \frac{(t_2 - t_1)qB}{m}$$

$\Rightarrow$ the charge is at Q where PQ = 2r

where r = radius = $\frac{mv}{qB} = 1.0\,m$.

$\Rightarrow$ its coordinate are (6.4 , 0, 2).

MAGNETIC FORCE ON CURRENTS — Section - 3

1. Force on a current element in a magnetic field

The force exerted on a current element of length *dl* carrying current *I* placed in a magnetic field at a point where the magnetic induction is *B* is given by :

$$d\bar{F} = I\left(d\bar{l} \times \bar{B}\right)$$

The force experienced by a current carrying conductor in a magnetic field is calculated by dividing the conductor into infinite current elements

$$\bar{F} = \int I\left(d\bar{l} \times \bar{B}\right)$$

2. Force on a straight current carrying wire in uniform field

Consider a straight wire of length *l* carrying a current I. The wire is placed in a uniform magnetic field B. The force acting on the wire is

$$\bar{F} = \int I\left(d\bar{l} \times \bar{B}\right) = I\left[\int d\bar{l}\right] \times \bar{B}$$

$$\bar{F} = I\left(\bar{l} \times \bar{B}\right)$$

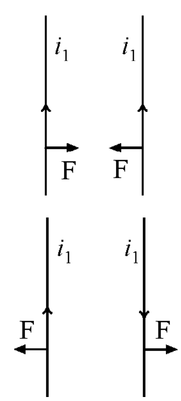
where $\bar{l}$ is the vector whose magnitude is *l* and the direction is same as that of current.

The magnitude of force is F = BI*l* *sin*θ, where θ is the angle between the *B* vector and the direction of current in the wire.

(a) If the wire is placed perpendicular to the field direction, the force is $F = BIl$.

(b) If the wire is placed parallel to the field direction, no force is experienced by the wire.

3. Force between two parallel current carrying wires



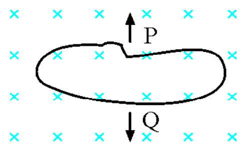
Consider two long straight wires 1 and 2 carrying currents i_1 and i_2 respectively and placed parallel to each other with a distance *r* between them.

The magnetic field of wire 1 exerts a force on wire 2 and that of wire 2 exerts an equal and opposite force on wire 1. This mutual force of interaction is attractive if the currents are in same direction and repulsive if the currents are opposite. The magnitude of force experienced by each wire on a unit length is given as force per unit length

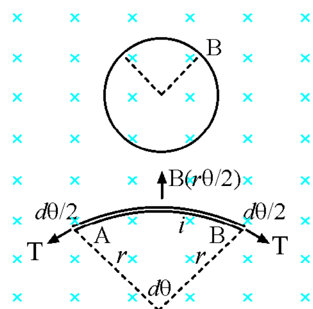
$$= \frac{\mu_0 \, i_1 \, i_2}{2\pi r}.$$

Illustration - 11 *A loop of flexible conducting wire of length* 0.5 *m lies in a perpendicular magnetic field of* 1.0 *T perpendicular going inwards to the plane of the loop. Show that when a clockwise current is passed through the loop, it opens into a circle. Also calculate the tension developed in the wire if the current is* 1.57 *amp.*

SOLUTION :



When current passes through the loop, any small portion (like P and Q) of the loop experiences magnetic force. Hence there is tendency to expand and loop opens up into a circle.



Now we consider an infinitesimal portion AB of the circular loop subtending an angle $d\theta$ at the centre. We isolate this portion and draw forces acting on it.

In the force diagram, the resultant of two tensions (at A and B) balances the outward magnetic force $Bi(rd\,\theta)$.

$$2T\left(\frac{d\,\theta}{2}\right) = Bi\,(rd\,\theta)$$

$$2T\left(\frac{d\,\theta}{2}\right) = B\,ird\theta \text{ using } \left(\sin\frac{d\,\theta}{2} \approx \frac{d\,\theta}{2}\right)$$

$$T = B\,ir$$

$$= 1 \times 1.57 \times 0.5/2\pi = 0.125\ N$$

Illustration - 12 *Two horizontal parallel conductors are suspended by light vertical threads* 0.75 *m long. Each conductor has a mass* 4×10^{-2} *k g/m. In the absence of any current, the conductors are parallel and are* 0.5 *cm apart. Equal currents in the two wires result in the separation of* 1.5 *cm between them. Find the magnitude and the direction of the currents.*

SOLUTION :

Let θ be the angle between the threads and vertical.

$$2\,l\sin\theta + 0.5 = 1.5$$

$$\sin\theta = 1/150$$

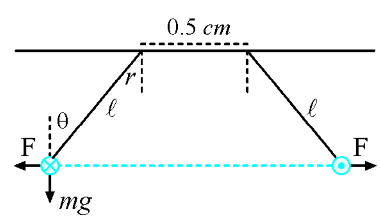
(taking $\tan\theta \approx \sin\theta$ for small θ)

If F is the force per unit length and m is the mass per unit length, then

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = F$$

$$\Rightarrow F = mg\tan\theta$$

$$\Rightarrow \frac{\mu_0 i^2}{2\pi r} = m\,g\sin\theta$$

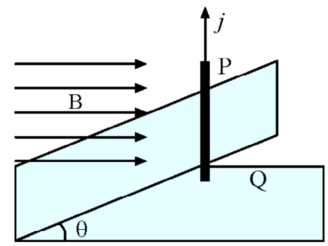


$$i = \sqrt{\frac{2\pi\,rmg\sin\theta}{\mu_0}}$$

$$= \sqrt{\frac{2\pi\left(1.5\times10^{-2}\right)\left(4\times10^{-2}g\right)}{150\left(4\pi\times10^{-7}\right)}} = 14A$$

As the force is repulsive, currents should be in opposite directions.

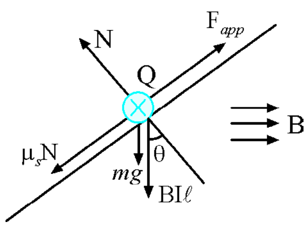
Illustration - 13 *A straight conductor PQ of weight 1 N and length 0.4 m, located in a plane making an angle 30° with the horizontal, so that it is perpendicular to a uniform horizontal magnetic field of induction B = 0.109 T. Given that the conductor carries a current I = 10A and the coefficient of static friction = 0.1. Find the force needed to be applied parallel to the plane to sustain the conductor at rest.*



SOLUTION :

The magnetic force on the wire is BI*l* directed vertically downwards.

Maximum value of force (wire tends to slip up)

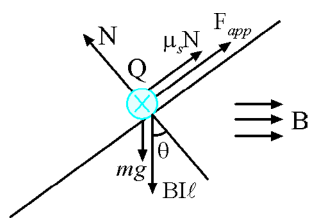


$$F_{app} = \mu_s N + (mg + BIl)\sin\theta$$

$$N = (mg + BIl)\cos\theta$$

$$F_{app} = (mg + BIl)(\mu_s\cos\theta + \sin\theta).$$

Minimum value of force (wire PQ tends to slip down).



$$F_{app} + \mu_s N = (mg + BIl)\sin\theta$$

$$N = (mg + BIl)\cos\theta\,]$$

$$F_{app} = (mg + BIl)(\sin\theta - \mu_s\cos\theta)$$

$$(mg + BIl)(\sin\theta - \mu_s\cos\theta) \le F$$

$$\le (mg + BIl)(\sin\theta + \mu_s\cos\theta).$$

Illustration - 14 *A thin conducting rod of mass 0.1 kg and length 0.25 m lies on a horizontal rough table with its length normal to a uniform horizontal field of induction 0.2 T. If a current of 10 A is passed through the rod, find :*

(a) *The horizontal force required to be applied perpendicular to the length of the rod to keep it in uniform velocity over the table.*

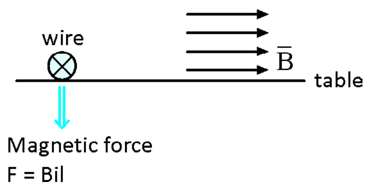
(b) *If the magnetic field is vertically downwards, find the current I in the rod so that it moves with uniform velocity over the table. Take $\mu = 0.1$ $\mu = 0.1$*

SOLUTION :

(a) The magnetic force can be up or down depending on the direction of *i* and *B*.

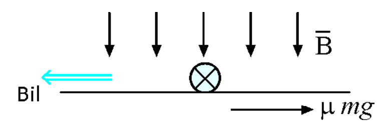
$$N = mg \pm Bil$$

$$F_{app.} = \mu N = \mu(mg \pm Bil)$$



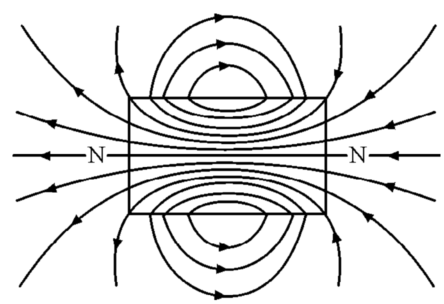
(b) $Bil = \mu mg$

$$\Rightarrow \quad i = \frac{\mu mg}{B\ell}$$



MAGNETIC DIPOLE Section - 4

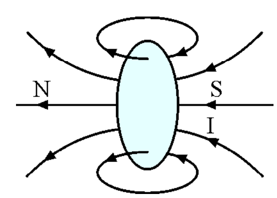
A closed current loop, a solenoid or a bar magnet have similar pattern of lines of force. The lines of force originate from one face and enter into another face. Such arrangements are known as magnetic dipoles. The face from which the lines of force comes out is known as North pole and the face into which lines of force enter is known as South Pole.



Let us consider a closed loop of N turns and area A carrying a current i. The magnetic dipole moment of this magnetic dipole is defined as

$$\bar{m} = i\,\bar{A}N$$

The direction of dipole moment is normal to the plane of loop. If we curl the fingers of right hand along the current in loop, the direction of thumb gives the direction of dipole moment.



Dipole in a uniform magnetic field

When a magnetic dipole is placed in a uniform magnetic field B, it experience a torque given by,

$$\bar{\tau} = \bar{m} \times \bar{B} \quad \Rightarrow \quad \tau = mB\sin\theta$$

where θ is the angle between the dipole moment vector $\bar{m}$ and the vector $\bar{B}$.

The force on the dipole is zero in a uniform magnetic field. The torque also becomes zero for the two positions of the dipole. These positions are the equilibrium positions of the dipole.

(a) Stable equilibrium

When the dipole moment vector $\bar{m}$ is in the direction of the magnetic lines of force, $\theta = 0°$ and hence $\tau = 0$. The dipole is in the position of stable equilibrium.

(b) Unstable equilibrium

When the dipole moment vector $\bar{m}$ is opposite to the direction of magnetic field, $\theta = 180°$ and hence $\tau = 0$. This is the unstable equilibrium position.

Potential Energy of the dipole

When placed in the uniform magnetic field, the dipole possesses potential energy given by

$$U = mB\cos\theta = -\bar{m} \cdot \bar{B}$$

where θ is the angle between $\bar{m}$ and $\bar{B}$. The potential energy is minimum when the dipole is in stable equilibrium ($\theta = 0°$). The potential energy is maximum when the dipole is in unstable equilibrium ($\theta = 180°$).

Illustration - 15 *For the rotating disc of illustration* 5, *calculate the magnetic dipole moment.*

SOLUTION :

The net dipole moment is

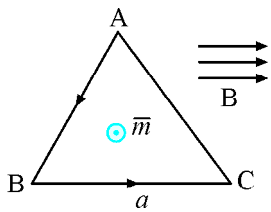
$$M = \int dm = \int \frac{\omega\, dq}{2\pi} \pi\, x^2$$

$$M = \int_0^R \frac{\omega x^2}{2} \frac{2Qxdx}{R^2}$$

$$= \frac{\omega Q}{R^2} \frac{R^4}{4} = \frac{Q\omega R^2}{4}$$

Illustration - 16 *A coil in the shape of an equilateral triangle of side* 0.02 *m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanant magnet producing a permanent magnetic field of* 5×10^{-2} *T. Find the couple acting on the coil when a current of* 0.1 *A is passed through it and magnetic field is parallel to its plane.*

SOLUTION :



Magnetic moment of the loop is $m = iA$

Substituting the area of the equilateral triangle, we get :

$$\vec{m} = i\frac{\sqrt{3}}{4}a^2$$

The vector $\vec{m}$ is directed outwards in the figure.

Torque acting on the loop is

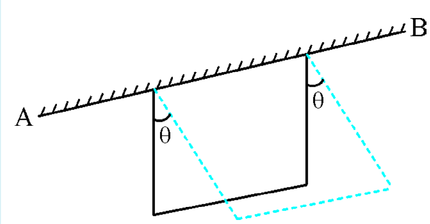
$$\tau = m\,B\,\sin\theta$$

$$= \left(i\frac{\sqrt{3}a^2}{4}\right)B\sin 90° = \frac{\sqrt{3}}{4}ia^2B$$

$$= \frac{\sqrt{3}}{4}(0.1)(0.02)^2\,5\times10^{-2}\ Nm$$

$$= 5\sqrt{3}\times10^{-7}\ Nm = 8.65\times10^{-7}\ Nm$$

Illustration - 17 *A cooper wire with cross-sectional area A is bent to make a square loop. One side of the square is fixed along a horizontal line AB and the loop can rotate about AB. The system is kept in a vertically upwards uniform magnetic field B. Find the angle of the plane of the loop with vertical in equilibrium position if the loop carries a current I.*



SOLUTION :

Let θ = angle of loop with vertical.

$\Rightarrow$ $90° - \theta$ = angle of normal (to the loop) with vertical.

Torque acting of loop due to magnetic forces is

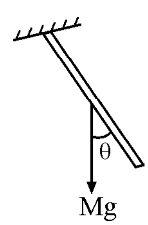
$\tau_1 = mB\sin(90 - \theta)$ (m : magnetic moment)

$= B\,I\,a^2\cos\theta$ (a : side length of loop)

Torque due to gravity about AB

$\tau_2 = (Mg\sin\theta)\,a/2$ where M is the mass of loop.

$= (4Aa\,\rho)\,g\sin\theta\,a/2$



In equilibrium, $\tau_1 = \tau_2$

$$BIa^2\cos\theta = 2a^2A\rho g\sin\theta$$

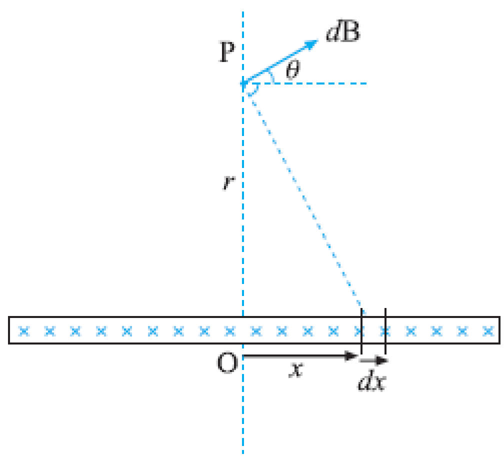
$$B = \frac{2A\rho g\tan\theta}{I}$$

Illustration - 18 *A current I flows through an infinitely long conducting strip of width b. Calculate the magnetic induction at a point located symmetrically at a perpendicular distance r from the strip.*

SOLUTION :

Let us divide the current strip into a collection of infinite long wires. We consider one such wire located at coordinate *x* from mid point O and having a width *dx*. The current passing through this element is

$$dI = \frac{I}{b}\,dx$$



Magnetic induction dB due to this element at P is directed as shown and has the magnitude

$$dB = \frac{\mu_0 dI}{2\pi\sqrt{r^2 + x^2}}$$

It can be easily seen by symmetry or by direct integration of $dB \sin\theta$ that the net vertical field is zero.

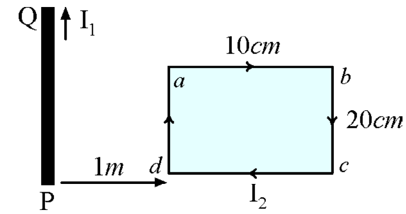
$$B = \int dB\cos\theta = \int \frac{\mu_0 dI}{2\pi\sqrt{r^2+x^2}} \cdot \frac{r}{\sqrt{r^2+x^2}}$$

$$= \int_{-b/2}^{b/2} \frac{\mu_0\, I\, r\, dx}{2\pi b\left(r^2+x^2\right)} = \frac{\mu_0 I}{\pi b}\tan^{-1}\frac{b}{2r}$$

in horizontal direction (parallel to the strip)

SUBJECTIVE SOLVED EXAMPLES

Example - 1 *The long straight wire PQ in the figure carries a current $I_1 = 20$ A. A rectangular loop abcd 'whose longer sides are parallel to PQ, carries a current $I_2 = 10$ A. Find the magnitude and direction of the force on ab, bc, cd, da. Hence find the net force on the loop .*



SOLUTION :

$$F_{ad} = \frac{\mu_0 i_1 i_2}{2\pi(1)} \times 0.2 = 8\times10^{-6} N \text{ towards left.}$$

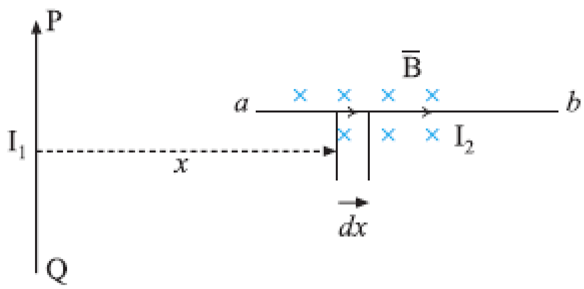
$$F_{bc} = \frac{\mu_0 i_1 i_2}{2\pi(1.1)}(0.2) = 7.27\times10^{-6} N$$

towards right.

Force on *ab* and *cd*

$$\bar{F}_{ab} = \int d\bar{F}$$

Where dF is the force on an infierimal element of length *dx* at a distance *x* form PQ.



$dF = B(x)\, I_2\, dx.$ (*B* is inwards)

$$= \frac{\mu_0 I_1}{2\pi x} I_2\, dx \quad \text{(upwards)}$$

$$F_{ab} = \int_{x_a}^{x_b} \frac{\mu_0 I_1 I_2\, dx}{2\pi x} = \frac{\mu_0 I_1 I_2}{2\pi} \ell og \frac{x_b}{x_a}$$

(upwards)

$$F_{ab} = 2\times10^{-7} \times (20\times10)\, log \frac{1.1}{1} = 3.8\times10^{-6} N$$

$$F_{cd} = -\bar{F}_{ab}$$

because cd carries equal and opposite current and is located in a similar position.

$F_{cd} = 3.8 \times 10^{-6}$ N downwards.

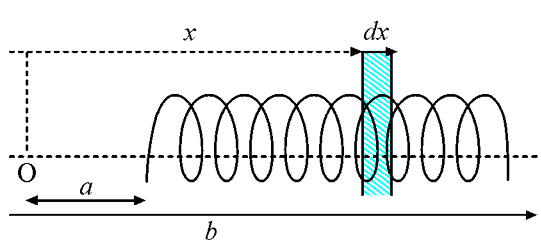
New force on loo $= \bar{F}_{ad} + \bar{F}_{bc}$

$F_{net} = 8 \times 10^{-6}$ N – 7.22×10^{-6} N

$= 0.73 \times 10^{-6}$ N towards PQ.

Example - 2 *Derive an expression for the magnetic field at a point on the axis of a solenoid having on N turns & length l carrying a current I. The radius of the cross-section of solenoid is r. What is the field at mid-point of axis and at the ends ?*

SOLUTION :



Let us Calculate the field at a point *O* which lies on the axis at distances *a* and *b* from the ends.

dB = field due to an element of length *dx* located at a distance *x* from *O*

$$= \frac{\mu_0 Ir^2}{2\left(x^2 + r^2\right)^{3/2}} \times \text{no. of turns.}$$

The element can be considered as a current loop of turns dN = (no. of turns per unit length) $dx = \frac{Ndx}{l}$

$$\Rightarrow \; dB = \frac{\mu_0 I r^2 (N\,d\,x)}{2\left(x^2 + r^2\right)^{3/2} l} \text{ directed along the axis.}$$

$$B = \int_a^b \frac{\mu_0 I r^2 N\,d\,x}{2l\left(x^2 + r^2\right)^{3/2}}$$

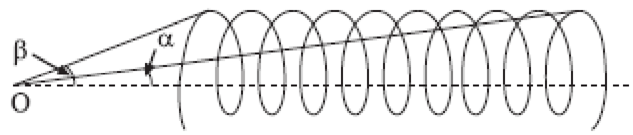
$$= \frac{\mu_0 I r^2 N}{2l} \left| \frac{1}{r^2} \frac{x}{\sqrt{x^2 + r^2}} \right|_a^b$$

$$= \frac{\mu_0 I N}{2l} \left\{ \frac{b}{\sqrt{b^2 + r^2}} - \frac{a}{\sqrt{a^2 + r^2}} \right\}$$

$$B = \frac{\mu_0 I\ N}{l} \left[\frac{\cos\alpha - \cos\beta}{2} \right]$$

where α, β are the angles subtended by the end-radii on the point O. Note that for points inside the coil, the angle β will be obtuse while α will be acute.

At mid point of axis



$$\cos\alpha = \frac{l/2}{\sqrt{\frac{l^2}{4} + r^2}},$$

$$\cos\beta = -\frac{l/2}{\sqrt{\frac{l^2}{4} + r^2}} \Rightarrow B = \frac{\mu_0 IN}{\sqrt{l^2 + 4r^2}}$$

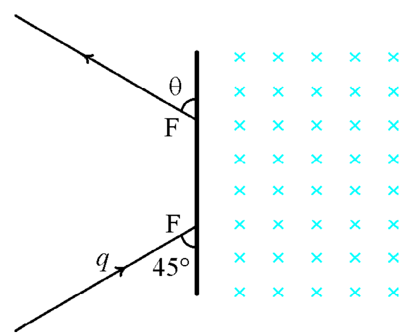
At the ends $\beta = 90°$,

$$\cos\alpha = \frac{l}{\sqrt{l^2 + r^2}} \Rightarrow B = \frac{\mu_0\ I\ N}{2\sqrt{l^2 + 4r^2}}$$

Example - 3 *A particle of mass m = 1.67 × 10^{-27} kg and charge q = 1.6 × 10^{-19} C enters a region of uniform magnetic field of strength 1.0 T along the direction shown in the figure. The speed of the particle is 10^7 m/s.*

(a) *The magnetic field is directed along the inward normal to the plane of the paper. The particle leaves the region at point F. Find EF*

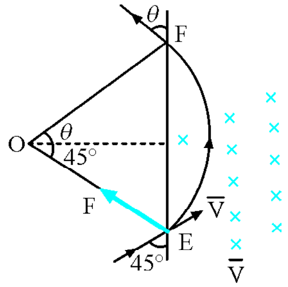
(b) *If the direction of the magnetic field is along the outward normal to the plane of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.*



SOLUTION :

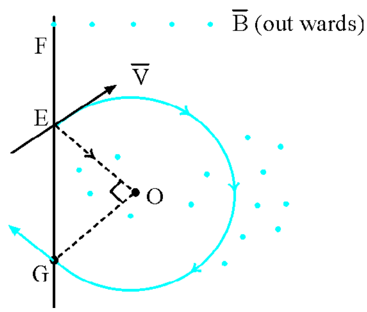
(a) We have to calculate EF and θ. The particle moves along a circular arc and comes out of F. Let O be the centre of the circle. From the geometry of figure we can see that $\theta = 45°$

$$EF = 2r\sin 45^\circ = 2\left(\frac{mv}{qB}\right)\sin 45^\circ$$

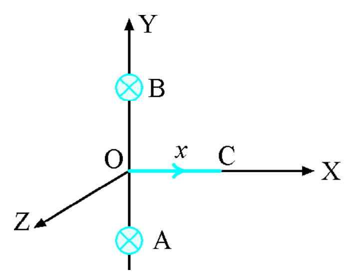


(b) The circle descried now is clockwise

$$\Delta t = \frac{3T}{4} = \frac{3}{4}\left(\frac{2\pi m}{qB}\right).$$



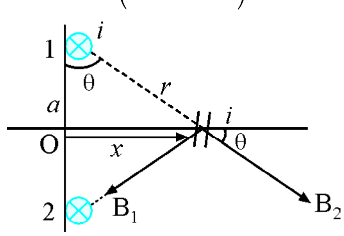
Example - 4 *A straight segment OC (of length 1 cm) of a circuit carrying a current 1A is placed along the X-axis. Two infinitely long straight wires A and B, are fixed at y = + a y = – a respectively as shown in the figure. If the wires A and B are each carrying a current of 1 A into the plane of paper, obtain the expression for the force acting on the segment OC. What will be the force on OC if the current in the wire B is reversed ?*



SOLUTION :

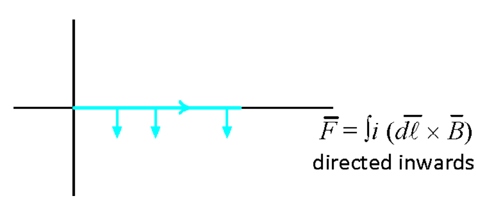
$$B_{net}(x) = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos\theta$$

$$= \frac{\mu_0 i}{\pi}\frac{a}{r^2} = \frac{\mu_0 ia}{\pi\left(a^2 + x^2\right)} \text{ down words.}$$



Force on $OC = \int dF = \int B(x).\, i\, dx\, \sin 90^\circ$ directed into the paper.

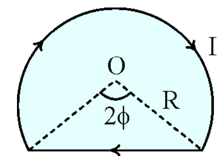
$$F = \frac{\mu_0 i^2 a}{\pi}\frac{1}{a}\ln\left(\frac{a^2 + l^2}{a^2}\right)$$



If current in $\overline{B}$ is reversed, magnetic magnetic becomes parallel to x-axis.

Hence no force acts on wire OC because. Current is also along x-axis.

Example - 5 *A current I = 5.0 A flows along a thin wire, shaped as shown. The radius of a curved part of the wire is equal to r = 120 mm, the angle ϕ is 30°. Find the magnetic induction of the field at the point O.*



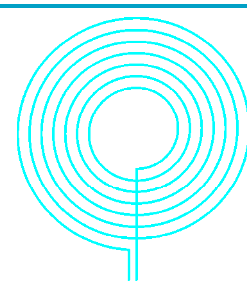
SOLUTION :

$$B_{net} = B_{curve} + B_{segment} = \frac{\mu_0 I}{2R}\left(\frac{2\pi - 2\phi}{2\pi}\right) + \frac{\mu_0 I}{2\pi\ R\cos\phi}\left\{\cos\left(\frac{\pi}{2} - \phi\right)\right\}$$

Example - 6 *A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 mA. The radii of inside and outside turns are equal to a = 50 mm and b = 100 mm. Find :*

(*a*) *the magnetic induction at the centre of the spiral ;*

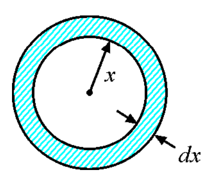
(*b*) *the magnetic moment of the spiral with a given current.*



SOLUTION :

Consider a ring shaked element of inner radius x and outer radius $x + dx$.
The no. of turns contained in this ring are :

dN = (no. of turns per unit radial distance) dx



$$= \frac{N}{b-a}\,dx\,.$$

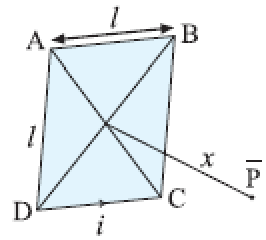
$$m = \int dm = \int I\left(\pi x^2\right) dN = \int_a^b I\left(\pi x^2\right)\frac{Ndx}{b-a}$$

$$m = \frac{NI\pi}{3}(a^2 + b^2 + ab)$$

$$B_{centre} = \int dB = \int \frac{\mu_0 I}{2x}\,dN = \int_a^b \frac{\mu_0 I\ N\,dx}{2x\,(b-a)}$$

$$B = \frac{\mu_0 IN}{2(b-a)}\log\frac{b}{a}$$

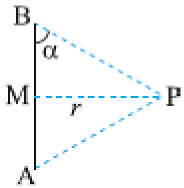
Example - 7 *A wire is formed into the shape of square of edge length l. When the current in the loop is I, find the magnetic field at point P a distance x from the centre of the square along its axis.*



SOLUTION :

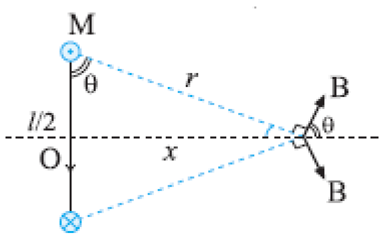
Field due to one side = B

$$= \frac{\mu_0 I}{2\pi r}\left[\frac{\cos\alpha + \cos\alpha}{2}\right]$$



If $ABCD$ is the square, let M be the mid point of AB.

$$\Rightarrow \cos\alpha = \frac{l/2}{\sqrt{l^2/4 + r^2}} = \frac{l/2}{\sqrt{\frac{l^2}{4} + \frac{l^2}{4} + x^2}}$$



Net field at P is : $\left[\cos\alpha = \frac{l}{\left(\sqrt{l^2 + 2x^2}\right)\left(\sqrt{2}\right)}\right]$

$$B_{net} = 4B\cos\theta = 4B\,\frac{l/2}{r}$$

$$4B\,\frac{l/2}{\sqrt{l^2/4 + x^2}} = \frac{4Bl}{\sqrt{l^2 + 4x^2}}$$

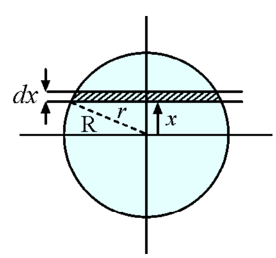
$$= 4\left[\frac{\mu_0 I}{2\pi\sqrt{l^2/4 + x^2}}\,\frac{l}{\sqrt{l^2 + 2x^2}}\,\frac{1}{\sqrt{2}}\right]\cdot\frac{l}{\sqrt{l^2 + 4x^2}}$$

$$= \frac{2\sqrt{2}\,\mu_0 I l^2}{\pi\left(l^2 + 4x^2\right)\sqrt{l^2 + 2x^2}}$$

Example - 8 *A charge q is uniformly distributed over the volume of a uniform ball of mass m and radius R which rotates with an angular velocity ω about the axis passing through its centre. Find the respective magnetic moment and its ratio to the mechanical moment. (Angular momentum)*

SOLUTION :

In example -15, we proved that magnetic moment of a rotating disc of charge Q is $\frac{Q wR^2}{4}$. Charge in a thin disc of thickness dx at a distance x above the centre of the sphere is $dq = \frac{QV}{4/3\pi R^3}$



$$dq = \frac{3Q}{4\pi R^3}\cdot\pi\left(R^2 - x^2\right)dx \text{ (where radius of disc} = r\ \sqrt{R^2 - x^2}\text{)}$$

magnetic moment of this element is $dm = \frac{(dq)\omega r^2}{4} = \frac{3Q\pi\left(R^2 - x^2\right)dx}{4\pi R^3}\cdot\frac{\omega}{4}\cdot\left(R^2 - x^2\right)$

$$m = \int dm = \int_{-R}^{R} \frac{3Q\omega}{16R^3}\left(R^2 - x^2\right)^2 dx = \frac{Q\omega R^2}{5}$$

we have $L = \frac{2}{5} MR^2\omega$

$$ratio = \frac{M}{L} = \frac{Q}{2M}$$

THINGS TO REMEMBER

Magnetic Effect of Current

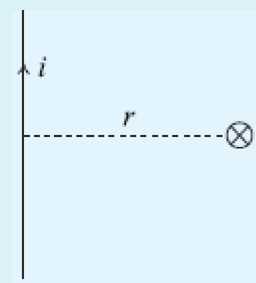
1. Biot Savart Law :

BiotSavart Law gives the magnetic induction due to an infinitesimal current element of length *d* carrying a current *i*.

According to BiotSavart Law :

$$d\,\overline{B} = \frac{\mu_0}{4\pi}\frac{i(d\overline{\ell} \times \overline{r})}{r^3}$$

2. Field of a straight infinite current wire :

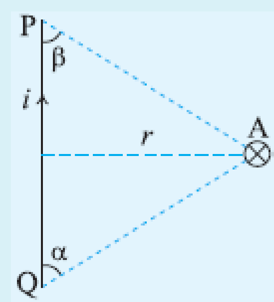


The magnetic field due to a current carrying straight wire of infinite length at a distance r from the wire is $B = \frac{\mu_0 i}{2\pi r}$

Field of a straight current carrying wire of finite length

The magnetic induction at a distance *r* from the wire is given by :

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{\cos\alpha + \cos\beta}{2}\right)$$



Where α, β are the angles between the wire and the lines joining the end points of wire to the point A where the field is to be calculated.

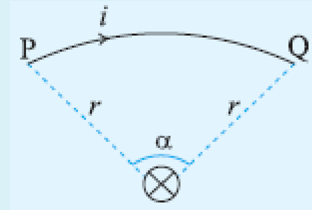
3. Field due to a circular current loop

Consider a circular loop of radius *r* carrying a current *i* and having N turns.

(a) **Field at Centre :** $B = \frac{\mu_0 i N}{2r}$

(b) **Field at axis :** $B = \frac{\mu_0 i r^2 N}{2\left(x^2 + r^2\right)^{3/2}}$

4. Field at the centre of a current arc



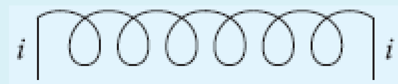
The magnetic field at the centre of a wire PQ of length l bent in a shape of a arc of radius r is :

$$B = \frac{\mu_0 i}{2r}\left(\frac{\alpha}{2\pi}\right) = \frac{\mu_o i}{4\pi r^2} l$$

5. Field inside a long solenoid

Consider a solenoid of length l and radius of cross section $r(r << \ell)$ having N turns. If I is the current passing through the solenoid, the magnetic induction inside the solenoid is given as :

$$B = \frac{\mu_0 IN}{\ell} \quad or \quad B = \mu_0 \text{ In}$$



Where n is the number of turns per unit length.

6. Force on a moving charged particle

The force exerted by a magnetic field of induction B on a charged particle q moving with a velocity v is given by

or $$\overline{F} = q\left(\overline{v} \times \overline{B}\right)$$

$$F = |qvB| \sin\theta$$

where θ is the angle between the B vector and the velocity of the charge. Note carefully that force on a positive charge is in the direction of $\overline{v} \times \overline{B}$ but the force on a negative charge is opposite to the vector $\overline{v} \times \overline{B}$.

7. Motion of a charged particle *q* in a uniform magnetic field

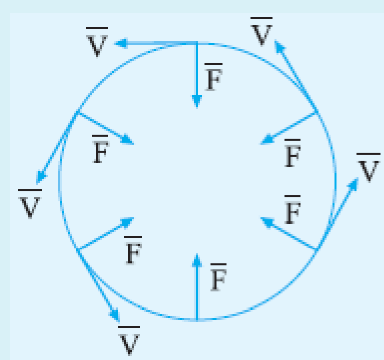
CASE : I (Straight line motion)

If a charged particle q is projected into a uniform magnetic field B with a velocity which is parallel to the field lines, the force experienced by the charge is zero and hence it travels in a straight line with uniform velocity.

$$\vec{F} = q(\overline{v} \times \overline{B}) = \overline{O} \text{ for } \overline{v} \,||\, \overline{B}$$

CASE : II (Uniform circular motion)

If a charged particle q is projected into a uniform magnetic field B with a initial velocity perpendicular to the lines of force, it gets trapped in a circular path. The force exerted by the field provides the necessary centripetal force.



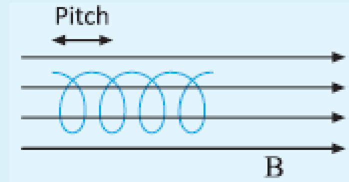
$$qvB = \frac{mv^2}{r}$$

$$T = \frac{2\pi m}{qB}$$

Note that the plane of the circular path is perpendicular to the lines of force.

CASE : III (motion in a helical path)

If the charge particle's initial projection velocity makes an angle $\theta(\theta \neq 0, 90°)$ with the magnetic field B, it moves in a helical path. The axis of the helix is parallel to the lines of force.



(a) the time period of revolution is $T = \dfrac{2\pi m}{qB}$

(b) the pitch of the helix (the displacement parallel to axis during one circular revolution) $P = v_{||}T = P = \dfrac{2\pi m}{qB} v_{||}$

8. Force on a current element in a magnetic field

The force exerted on a current element of length dl carrying current I placed in a magnetic field at a point where the magnetic induction is B is given by :

$$d\overline{F} = I(d\overline{I} \times \overline{B})$$

The force experienced by a current carrying conductor in a magnetic field is calculated by dividing the conductor into infinite current elements

$$\overline{F} = \int I(d\overline{I} \times \overline{B})$$

9. Force on a straight current carrying wire in uniform field

Consider a straight wire of length l carrying a current I. The wire is placed in a uniform magnetic field B. The force acting on the wire is

$$\overline{F} \int I(d\overline{l} \times \overline{B}) = I\left[\int d\overline{l}\right] \times \overline{B}$$

$$\overline{F} = I(\overline{l} \times \overline{B})$$

where $\overline{l}$ is the vector whose magnitude is l and the direction is same as that of current.

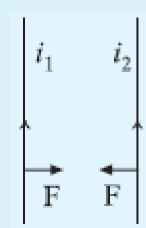
The magnitude of force is $F = BIl\sin\theta$, where θ is the angle between the B vector and the direction of current in the wire.

(a) If the wire is placed perpendicular to the field direction, the force is $F = BIl.$

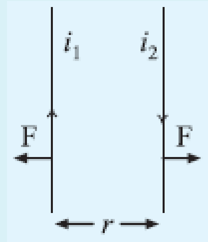
(b) If the wire is placed parallel to the field direction, no force is experienced by the wire.

10. Force between two parallel current carrying wires

Consider two long straight wires 1 and 2 carrying currents i_1 and i_2 respectively and placed parallel to each other with a distance r between them.



The magnitude of force experienced by each wire on a unit length is given as force per unit length $= \dfrac{\mu_0 i_1 i_2}{2\pi r}$.



11. Dipole in a uniform magnetic field

When a magnetic dipole is placed in a uniform magnetic field B, it experience a torque given by,

$$\bar{\tau} = \bar{m} \times \bar{B} \qquad \Rightarrow \qquad \tau = mB\sin\theta$$

Where θ is the angle between the dipole moment vector m and the vector $\bar{B}$.

The force on the dipole is zero in a uniform magnetic field. The torque also becomes zero for the two positions of the dipole.

These positions are the equilibrium positions of the dipole.

(a) Stable equilibrium

When the dipole moment vector $\bar{m}$ is in the direction of the magnetic lines force, $\theta = 0°$ and hence $\tau = 0$. The dipole is in the position of stable equilibrium.

(b) Unstable equilibrium

When the dipole moment vector $\bar{m}$ is opposite to the direction of magnetic field, $\theta = 180°$ and hence $\tau = 0$. This is the unstable equilibrium position.

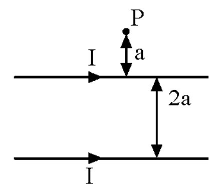
Potential Energy of the dipole

When placed in the uniform magnetic field, the dipole possesses potential energy given by

$U = mB\cos\theta = -\bar{m}\cdot\bar{B}$ where θ is the angle between $\bar{m}$ *and* $\bar{B}$. The potential energy is minimum when the dipole is the stable equilibrium $(\theta = 0°)$. The potential energy is maximum when the dipole is in unstable equilibrium $(\theta = 180°)$.

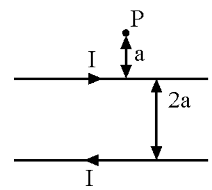
SOLUTIONS - IN-CHAPTER EXERCISE-A

1. Same direction of current : **(a)** $B = \dfrac{\mu_0 I}{2\pi a} - \dfrac{\mu_0 I}{2\pi a} = 0$



(b) $B = \dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{a} + \dfrac{1}{3a}\right)$ (Outwards)

Opposite direction of current : **(a)** $B = 2\left(\dfrac{\mu_0 I}{2\pi a}\right)$ (Inwards)



(b) $B = \dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{a} - \dfrac{1}{3a}\right)$ (Outwards)

For hexagon : $B = \left\{\dfrac{6\mu_0 I}{2\pi(a\sin 60^\circ)}\left(\dfrac{\cos 60^\circ + \cos 60^\circ}{2}\right)\right\}$

2. **(a)** $B = 0$

(b) $B = 2\dfrac{\mu_0 I r^2 N}{2\left(r^2 + x^2\right)^{3/2}}$ where $r = 0.06\ m : x = 0.08m$; $N = 60$; $I = 0.1A$

(c) For Ist case : $B = \left(\dfrac{\mu_0 I}{2r}\right) N - \dfrac{\mu_0 I r^2 N}{2\left(r^2 + (2x)^2\right)^{3/2}}$,

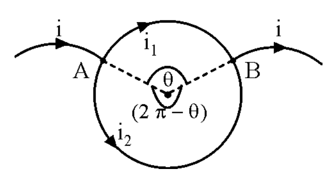
For IInd case : $B = \left(\dfrac{\mu_0 I}{2r}\right) N + \dfrac{\mu_0 I r^2 N}{2\left(r^2 + (2x)^2\right)^{3/2}}$

3. $B_{\text{circle}} = \dfrac{\mu_0 I}{2r} = \dfrac{\mu_0 I}{2(\ell / 2\pi)} = \dfrac{\mu_0 I \pi}{\ell}$ $\quad B_{\text{Square}} = 4.\dfrac{\mu_0 I}{2\pi\left(\dfrac{a}{2}\right)}\left(\dfrac{\cos 45^\circ + \cos 45^\circ}{2}\right)$ where $a = \dfrac{\ell}{4}$

$\Rightarrow \quad B_{\text{Square}} > B_{\text{Circle}}$

4. Let ℓ_1, ℓ_2 be the lengths of the two parts of circular loop. These two parts are connected in parallel for current i.

$$\Rightarrow \quad i_1\left(\frac{\rho\ell_1}{A}\right) = i_2\left(\frac{\rho\ell_2}{A}\right) \quad \Rightarrow \quad i_1\left(\frac{\rho(r\theta)}{A}\right) = i_2\left(\frac{\rho(r(2\pi - \theta))}{A}\right)$$



$$\vec{B}_{\text{at centre}} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 i_1}{2r}\left(\frac{\theta}{2\pi}\right) - \frac{\mu_0 i_2}{2r}\left(\frac{2\pi - \theta}{2\pi}\right) = 0$$

5. $B_0 = B_1 + B_2 = \left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{1+0}{2}\right)+\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{1+0}{2}\right)=\frac{\mu_0 i}{2\pi r} = 10^{-4}T$

6. $B_P = B_Q$ for null point at D. [Current in P is coming outwards]

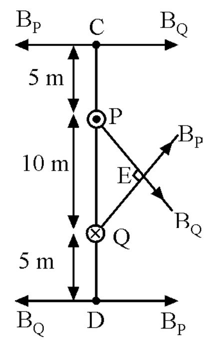
$\Rightarrow$ Current in Q should be inwards if current in P is outwards

$\Rightarrow \quad \frac{\mu_0 i_P}{2\pi(15)} = \frac{\mu_0 i_Q}{2\pi(5)} \quad \Rightarrow \quad i_Q = \frac{i_P}{3} = 2A\text{ (Inwards)}$

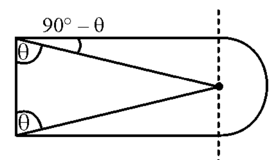
At $C: B = B_P - B_Q = \frac{\mu_0 \times 6}{2\pi(5)} - \frac{\mu_0 \times 2}{2\pi(15)} = \frac{8\mu_0}{15\pi}$

At $E:\ B = \sqrt{B_P^2 + B_Q^2} = \sqrt{\frac{\mu_0^2}{4\pi^2}\left(\frac{36}{36}+\frac{4}{64}\right)}$ at $\tan^{-1}4$ with PE.

[*E* is a point in the plane of paper]



7. $B=2\left[\frac{\mu_0 I}{2\pi a}\left(\frac{\cos 90° + \cos\left(90^\circ - \theta\right)}{2}\right)\right] + \frac{\mu_0 I}{2\pi(2a)}\left[\frac{\cos\theta+\cos\theta}{2}\right] + \frac{1}{2}\left(\frac{\mu_0 I}{2a}\right)$



$\sin\theta = \frac{2}{\sqrt{5}} \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$