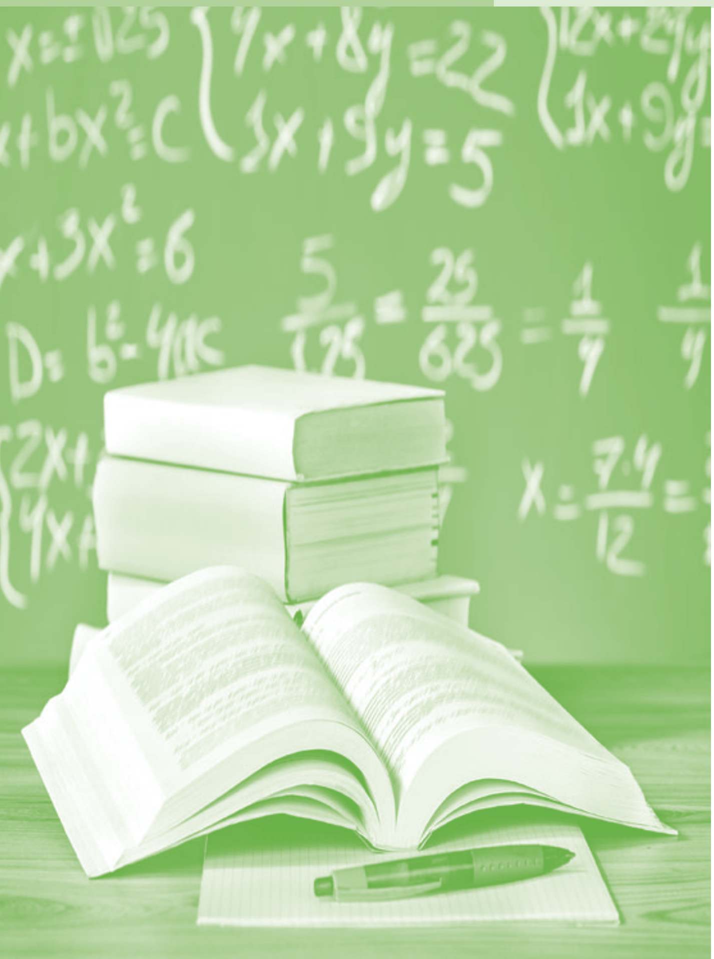
Chapter

3

Polynomials and Square Roots of Algebraic Expressions



REMEMBER

Before beginning this chapter, you should be able to:

- Apply basic operations on polynomials
- Solve basic factorization of polynomials

KEY IDEAS

After completing this chapter, you should be able to:

- State the types of polynomials and operations on polynomials
- Find the factorization of polynomials and obtain HCF and LCM of polynomials
- Understand different methods for finding square roots of algebraic expressions
- Learn about homogeneous, symmetric and cyclic expressions

INTRODUCTION

Before learning the meaning and scope of polynomials, the terms like, constants, variables, algebraic expressions etc., have to be understood.

Constant

A number having a fixed numerical value is called a constant.

Example: 7, $\frac{1}{2}$, 4.7, 16.5, etc.

Variable

A number which can take various numerical values is known as variable.

Example: x, y, z, a, b, c, etc.

A variable raised to any non-zero real number is also a variable.

Example: x^5, $y^{10/3}$, $z^{0.9}$, etc.

A number which is the product of a constant and a variable is also a variable.

Example: $8x^3$, $-7x^5$, $4x^{10}$, etc.

A combination of two or more variables separated by a (+) sign or a (–) sign is also a variable.
Example: $x^2 - y^3 + z$, $x^3 - y^3$, etc.

Algebraic Expression

A combination of constants and variables connected by +, –, × and ÷ signs is known as an algebraic expression.
Example: $8x + 7$, $11x^2 - 13x$, $5x^5 + 8x^2y$, etc.

Terms

The parts of an algebraic expression separated by + or – signs are called the terms of the expression.
Example: In the expression $3x + 4y - 7$, we call $3x$, $4y$ and -7 as terms.

Coefficient of a Term

Consider the term $8x^2$. In this case, 8 is called the numerical coefficient and x^2 is said to be the literal coefficient.

In case of $9xy$, we have the numerical coefficient as 9 and the literal coefficient as xy.

Like Terms

Terms having the same literal coefficients are called like terms.

Examples:
1. $15x^2$, $-19x^2$ and $35x^2$ are all like terms.
2. $8x^2y$, $5x^2y$ and $-7x^2y$ are all like terms.

Unlike Terms

Terms having different literal coefficients are called unlike terms.

Example: $5x^2$, $-10x$ and $15x^3$ are unlike terms.

POLYNOMIAL

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

Example: $5x^2 - 8x + 7$, $3x^3 + 5x^2 - 9$, $3y^2 - 5y + z$, etc.

The expression $3x^5 - 8x + \frac{4}{x} + 11\,x^{5/2}$ is not a polynomial. Since the exponents of x are negative integers and fractions.

A polynomial with one variable is known as a polynomial in that variable.

Example: $5x^4 + 7x^3 + 3x - 9$ is a polynomial in the variable x.

$3y^3 + y^2 + y$ is a polynomial in the variable y.

$4x^2y^2 + 3xy^2 - 7xy$ is a polynomial in variables x and y.

Degree of a Polynomial in One Variable

The highest index of the variable in a polynomial of one variable is called the degree of the polynomial.

Examples:

1. $11x^3 - 7x^2 + 5x + 2$ is a polynomial of degree 3.
2. $15x^6 - 8x + 7$ is a polynomial of degree 6.

Types of Polynomials with Respect to Degree

1. **Linear polynomial:** A polynomial of degree one is called a linear polynomial.

   Example: $11x - 5$, $10y + 7$ and $13z + 4$ are polynomials of degree one and hence they can be called as linear polynomials.

2. **Quadratic polynomial:** A polynomial of degree two is called a quadratic polynomial.

   Example: $5x^2 - 8x + 3$ and $13y^2 - 8y + 3$ are polynomials of degree two and hence can be called as quadratic polynomials.

3. **Cubic polynomial:** A polynomial of degree three is called a cubic polynomial.

   Example: $5x^3 + 6x^2 + 7x + 8$ and $4y^3 - 9y^2 + 3$ are polynomials of degree three and hence can be called as cubic polynomials.

4. **Biquadratic polynomial:** A polynomial of degree four is called a biquadratic polynomial.

   Example: $3x^4 - x^3 + 7x^2 - 2x + 1$ and $5x^4 - 2x + 7$ are polynomials of degree four and hence can be called as biquadratic polynomials.

5. **Constant polynomial:** A polynomial having only one term which is a constant is called a constant polynomial. Degree of a constant polynomial is 0.

   Example: 10, −11 are constant polynomials.

Types of Polynomials with Respect to Number of Terms

1. **Monomial:** An expression containing only one term is called a monomial.

   Example: $8x$, $-11x^2y$, $-15x^2y^3z^2$, etc.

2. **Binomial:** An expression containing two terms is called a binomial.
   Example: $3x - 8y$, $4xy - 5x$, $9x + 5x^2$, etc.

3. **Trinomial:** An expression containing three terms is called a trinomial.
   Example: $5x - 2y + 3z$, $x^2 + 2xy - 5z$, etc.

Addition of Polynomials

The sum of two or more polynomials can be obtained by arranging the terms and then adding the like terms.

EXAMPLE 3.1

Add $7x^2 - 8x + 5$, $3x^2 - 8x + 5$ and $-6x^2 + 15x - 5$.

SOLUTION

$$\begin{array}{rrr} 7x^2 - & 8x & +5 \\ 3x^2 - & 8x & +5 \\ -6x^2 + & 15x & -5 \\ \hline 4x^2 - & x & +5 \\ \hline \end{array}$$

$\therefore$ The required sum is $4x^2 - x + 5$.

Subtraction of Polynomials

The difference of two polynomials can be obtained by arranging the terms and subtracting the like terms.

EXAMPLE 3.2

Subtract $11x^3 - 7x^2 + 10x$ from $16x^3 + 4x^2 - 11x$.

SOLUTION

$$\begin{array}{rrr} 16x^3 + & 4x^2 & - 11x \\ 11x^3 - & 7x^2 & + 10x \\ - & + & - \\ \hline 5x^3 + & 11x^2 & - 21x \\ \hline \end{array}$$

$\therefore$ The required difference is $5x^3 + 11x^2 - 21x$.

Multiplication of Two Polynomials

The result of multiplication of two polynomials is obtained by multiplying each term of the polynomial by each term of the other polynomial and then taking the algebraic sum of these products.

EXAMPLE 3.3

Multiply $(5x^2 - 8x + 7)$ with $(2x - 5)$.

SOLUTION

$$\begin{array}{l} 5x^2 - 8x + 7 \\ 2x - 5 \\ \hline 10x^3 - 16x^2 + 14x \\ \qquad - 25x^2 + 40x - 35 \\ \hline 10x^3 - 41x^2 + 54x - 35 \\ \hline \end{array}$$

$\therefore$ The required product is $10x^3 - 41x^2 + 54x - 35$.

This is true for all real values of x, such equations are called algebraic identities.

Division of a Polynomial by a Monomial

To divide a polynomial by a monomial, we need to divide each term of the polynomial by the monomial.

EXAMPLE 3.4

Divide $18x^4 - 15x^3 + 24x^2 + 9x$ by $3x$.

SOLUTION

$$\frac{18x^4 - 15x^3 + 24x^2 + 9x}{3x}$$
$$= \frac{18x^4}{3x} - \frac{15x^3}{3x} + \frac{24x^2}{3x} + \frac{9x}{3x}$$
$$= 6x^3 - 5x^2 + 8x + 3$$

$\therefore$ The required result is $6x^3 - 5x^2 + 8x + 3$.

Division of a Polynomial by a Polynomial

Factor Method

In this method, we factorize the polynomial to be divided so that one or more of the factors is equal to the polynomial by which we wish to divide.

EXAMPLE 3.5

Divide $4x^2 + 7x - 15$ by $x + 3$.

SOLUTION

$$4x^2 + 7x - 15 = 4x^2 + 12x - 5x - 15$$
$$4x(x + 3) - 5(x + 3) = (4x - 5)(x + 3)$$
$$\therefore \frac{4x^2 + 7x - 15}{x + 3} = \frac{(4x - 5)(x + 3)}{x + 3} = 4x - 5.$$

Note The factor method for division of polynomials is used only when the remainder is zero.

Long Division Method

Step 1: First arrange the terms of the dividend and the divisor in the descending order of their degrees.

Step 2: Now the first term of the quotient is obtained by dividing the first term of the dividend by the first term of the divisor.

Step 3: Then multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

Step 4: Consider the remainder as new dividend and proceed as before.

Step 5: Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the divisor.

EXAMPLE 3.6

Divide $2x^3 + 9x^2 + 4x - 15$ by $2x + 5$.

SOLUTION

$$
\begin{array}{l}
2x+5\ )\ 2x^3 + 9x^2 + 4x - 15\ (x^2 + 2x - 3 \\
\qquad\quad 2x^3 + 5x^2 \\
\qquad\ \ (-)\quad (-) \\
\hline
\qquad\qquad\quad 4x^2 + 4x \\
\qquad\qquad\quad 4x^2 + 10x \\
\qquad\qquad\ \ (-)\quad (-) \\
\hline
\qquad\qquad\qquad -6x - 15 \\
\qquad\qquad\qquad -6x - 15 \\
\qquad\qquad\qquad (+)\ \ (+) \\
\hline
\qquad\qquad\qquad\quad 0 \\
\hline
\end{array}
$$

$\therefore\ (2x^3 + 9x^2 + 4x - 15) \div (2x + 5) = x^2 + 2x - 3.$

Horner's Method of Synthetic Division

EXAMPLE 3.7

Divide $27x^3 - 81x^2 + 45x + 23$ by $(x - 2)$.

SOLUTION

	27	−81	45	23	
$x = 2$	0	54	−54	−18	
	27	−27	−9	5	remainder

coefficients of quotient

Step 1: We first write the coefficients of the dividend arranging them in descending powers of x with zero as the coefficient for missing power of x.

Step 2: To divide by $x - 2$, we write $x = 2$.

Step 3: Bring down the leading coefficient of the dividend, multiply by 2 and add the second coefficient which results -27.

Step 4: Now multiply -27 by 2 and add this to the third coefficient to get -9.

Step 5: This process is continued until the final sum.

Step 6: Thus, we get the quotient as $27x^2 - 27x - 9$ and the remainder as 5.

Factorization

Factorization is expressing a given polynomial as a product of two or more polynomials.

Example: $x^3 - 15x^2 = x^2(x - 15)$

$\Rightarrow x^2$ and $x - 15$ are the factors of $x^3 - 15x^2$.

1. Factorization of polynomials of the form $x^2 - y^2$.
   $x^2 - y^2 = (x + y)(x - y)$
   $\Rightarrow x + y$ and $x - y$ are the factors of $x^2 - y^2$.

EXAMPLE 3.8

Factorize $81x^2 - 225y^2$

SOLUTION

Let $a = 9x$ and $b = 15y$

$a^2 - b^2 = (a + b)(a - b)$

$\therefore\ 81x^2 - 225y^2$

$= (9x)^2 - (15y)^2$

$= (9x + 15y)(9x - 15y)$

$\therefore\ 9x + 15y$ and $9x - 15y$ are the factors of $81x^2 - 225y^2$.

2. Factorization of polynomials by grouping of terms: In this method we group the terms of the polynomials in such a way that we get a common factor out of them.

EXAMPLE 3.9

(a) Factorize $a^2 - (b - 8)a - 8b$

SOLUTION

$\Rightarrow a^2 - (b - 8)a - 8b$

$= a^2 - ab + 8a - 8b$

$= a(a - b) + 8(a - b)$

$= (a + 8)(a - b)$

$\therefore\ a^2 - (b - 8)a - 8b = (a + 8)(a - b).$

(b) Factorize $4x^3 - 10y^3 - 8x^2y + 5xy^2$

SOLUTION

$4x^3 - 8x^2y + 5xy^2 - 10y^3$

$= 4x^2(x - 2y) + 5y^2(x - 2y)$

$= (4x^2 + 5y^2)(x - 2y).$

3. Factorization of a trinomial that is a perfect square. A trinomial of the form $x^2 \pm 2xy + y^2$ is equivalent to $(x \pm y)^2$. This identity can be used to factorize perfect square trinomials.

EXAMPLE 3.10

(a) Factorize $49x^2 + 9y^2 + 42xy$

SOLUTION

$49x^2 + 9y^2 + 42xy$

$= (7x)^2 + (3y)^2 + 2(7x)(3y)$

$= (7x + 3y)^2.$

(b) Factorize $16x^2 + \dfrac{1}{16x^2} - 2$

SOLUTION

$$16x^2 + \frac{1}{16x^2} - 2$$

$$= (4x)^2 + \left(\frac{1}{4x}\right) - 2(4x)\left(\frac{1}{4x}\right)$$

$$= (4x)^2 - 2(4x)\left(\frac{1}{4x}\right) + \left(\frac{1}{4x}\right)^2$$

$$= \left(4x - \frac{1}{4x}\right)^2$$

$$\therefore 16x^2 + \frac{1}{16x^2} - 2 = \left(4x - \frac{1}{4x}\right)^2.$$

4. Factorization of a polynomial of the form

$x^2 + (a + b)x + ab.$

As we have already seen,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$\therefore\ x^2 + (a + b)x + ab$ can be fatcorized as $(x + a)(x + b)$.

EXAMPLE 3.11

(a) Factorize $x^2 + 25x + 144$

SOLUTION

Here, the constant term is $144 = (16 \times 9)$ and the coefficient of x is $25 = (16 + 9)$

$\therefore\ x^2 + 25x + 144$

$= x^2 + 16x + 9x + 144$

$= x(x + 16) + 9(x + 16)$

$= (x + 16)(x + 9)$.

(b) Factorize $x^2 - 8x + 15$

SOLUTION

Here, the constant term is $15 = (-5)(-3)$ and the coefficient of x is $-8 = -5 - 3$.

$\Rightarrow x^2 - 8x + 15$

$= x^2 - 5x - 3x + 15$

$= x(x - 5) - 3(x - 5)$

$= (x - 3)(x - 5)$

$\therefore\ x^2 - 8x + 15 = (x - 3)(x - 5)$.

(c) Factorize $x^2 - 5x - 14$

SOLUTION

Constant term is $-14 = (-7)(2)$

Coefficient of x is $-5 = -7 + 2$

$\Rightarrow x^2 - 5x - 14$

$= x^2 - 7x + 2x - 14$

$= x(x - 7) + 2(x - 7)$

$= (x + 2)(x - 7)$.

5. Factorization of polynomials of the form $ax^2 + bx + c$.

Step 1: Take the product of the constant term and the coefficient of x^2, i.e., ac.

Step 2: Now this product ac is to split into two factors m and n such that $m + n$ is equal to the coefficient of x, i.e., b.

Step 3: Then we pair one of them, say mx, with ax^2 and the other nx, with c and factorize.

EXAMPLE 3.12

(a) $6x^2 + 19x + 15$

SOLUTION

Here, $6 \times 15 = 90 = 10 \times 9$ and $10 + 9 = 19$

$\therefore\ 6x^2 + 19x + 15$

$= 6x^2 + 10x + 9x + 15$

$= 2x(3x + 5) + 3(3x + 5)$

$= (2x + 3)(3x + 5)$.

(b) Factorize $7 - 17x - 12x^2$

SOLUTION

Here, $(7)(-12) = -84 = (-21)(4)$ and

$-17 = -21 + 4$

$7 - 17x - 12x^2$

$= 7 - 21x + 4x - 12x^2$

$= 7(1 - 3x) + 4x(1 - 3x) = (1 - 3x)(7 + 4x)$.

6. Factorization of expressions of the form $x^3 + y^3$ (or) $x^3 - y^3$.

$x^3 + y^3 = (x + y)\ (x^2 - xy + y^2)$

$\Rightarrow x^3 + y^3$ has factors $(x + y)$ and $(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)\ (x^2 + xy + y^2)$

$\Rightarrow x^3 - y^3$ has factors $(x - y)$ and $(x^2 + xy + y^2)$.

EXAMPLE 3.13

(a) Factorize $27a^3 + 125x^3$

SOLUTION

$27a^3 + 125x^3$

$= (3a)^3 + (5x)^3$

$= (3a + 5x)\{(3a)^2 + (5x)^2 - (3a)(5x)\}$

$= (3a + 5x)(9a^2 + 25x^2 - 15ax)$.

(b) Factorize $216x^3 - 64y^3$

SOLUTION

$216x^3 - 64y^3$

$= (6x)^3 - (4y)^3$

$= (6x - 4y)\{(6x)^2 + (4y)^2 + (6x)(4y)\}$

$= (6x - 4y)(36x^2 + 16y^2 + 24xy)$.

7. Factorization of expressions of the form

$x^3 + y^3 + z^3$ when $x + y + z = 0$.

(Given $x + y + z = 0$)

As $x + y + z = 0$, $z = -(x + y)$

$$
\begin{aligned}
x^3 + y^3 + z^3 &= x^3 + y^3 + \{-(x+y)\}^3 \\
&= x^3 + y^3 - (x+y)^3 \\
&= x^3 + y^3 - \{x^3 + y^3 + 3xy(x+y)\} \\
&= -3xy(x+y) \\
&= -3xy(-z) \ \{\text{Since } x + y = -z\} \\
&= 3xyz
\end{aligned}
$$

$\therefore$ If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

HCF of Given Polynomials

For two given polynomials, $f(x)$ and $g(x)$, $r(x)$ can be taken as the highest common factor, if

1. $r(x)$ is a common factor of $f(x)$ and $g(x)$ and
2. every common factor of $f(x)$ and $g(x)$ is also a factor of $r(x)$.

Highest common factor is generally referred to as HCF.

Method for Finding HCF of the Given Polynomials

Step 1: Express each polynomial as a product of powers of irreducible factors which also requires the numerical factors to be expressed as the product of the powers of primes.

Step 2: If there is no common factor then HCF is 1 and if there are common irreducible factors, we find the least exponent of these irreducible factors in the factorized form of the given polynomials.

Step 3: Raise the common irreducible factors to the smallest or the least exponents found in step 2 and take their product to get the HCF.

EXAMPLE 3.14

(a) Find the HCF of $48x^5y^2$ and $112x^3y$.

SOLUTION

Let $f(x) = 48x^5y^2$ and $g(x) = 112x^3y$

Writing $f(x)$ and $g(x)$ as a product of powers of irreducible factors.

$$f(x) = 2^4 \cdot 3 \cdot x^5 \cdot y^2$$
$$g(x) = 2^4 \cdot 7 \cdot x^3 \cdot y$$

The common factors with the least exponents are 2^4, x^3 and y

$\therefore$ HCF $= 16x^3y$.

(b) Find the HCF of $51x^2(x+3)^3(x-2)^2$ and $34x(x-1)^5(x-2)^3$.

SOLUTION

Let $f(x) = 51x^2(x+3)^3(x-2)^2$ and $g(x) = 34x(x-1)^5(x-2)^3$

Writing $f(x)$ and $g(x)$ as the product of the powers of irreducible factors.

$$f(x) = 17 \cdot 3 \cdot x^2(x+3)^3 \cdot (x-2)^2$$
$$g(x) = 17 \cdot 2 \cdot x(x-1)^5 \cdot (x-2)^3$$

The common factors with the least exponents are 17, x and $(x-2)^2$

∴ The HCF of the given polynomials = $17 \cdot x \cdot (x-2)^2 = 17x(x-2)^2$.

LCM of the Given Polynomials

Least Common Multiple or the Lowest Common Multiple is the product of all the factors (taken once) of the polynomials given with their highest exponents respectively.

Method to Calculate LCM of the Given Polynomials

Step 1: First express each polynomial as a product of powers of irreducible factors.

Step 2: Consider all the irreducible factors (only once) occurring in the given polynomials. For each of these factors, consider the greatest exponent in the factorized form of the given polynomials.

Step 3: Now raise each irreducible factor to the greatest exponent and multiply them to get the LCM.

EXAMPLE 3.15

(a) Find the LCM of $18x^3y^2$ and $45x^5y^2z^3$.

SOLUTION

Let $f(x) = 18x^3y^2$ and

$g(x) = 45x^5y^2z^3$

Writing $f(x)$ and $g(x)$ as the product of the powers of irreducible factors.

$f(x) = 2 \cdot 3^2 \cdot x^3 \cdot y^2$

$g(x) = 3^2 \cdot 5 \cdot x^5 \cdot y^2 \cdot z^3$

Now all the factors (taken only once) with the highest exponents are 2, 3^2, 5, x^5, y^2 and z^3.

∴ The LCM of the given polynomials = $2 \cdot 3^2 \cdot 5 \cdot x^5 \cdot y^2 \cdot z^3 = 90x^5y^2z^3$.

(b) Find the LCM of $51x^2(x+3)^3\,(x-2)^2$ and $34x(x-1)^5\,(x-2)^3$

SOLUTION

Writing $f(x)$ and $g(x)$ as the product of powers of irreducible factors.

$f(x) = 17 \cdot 3x^2(x+3)^3 \cdot (x-2)^2$

$g(x) = 17 \cdot 2(x-1)^5 \cdot (x-2)^3$

Now all the factors (taken only once) with the highest exponents are 2, 3, 17, $x^2(x-1)^5$, $(x-2)^3$ and $(x+3)^3$.

∴ The LCM of the given polynomials = $2 \cdot 3 \cdot 17 \cdot x^2(x-1)^5 \cdot (x-2)^3 \cdot (x+3)^3 = 102x^2(x-2)^3(x-1)^5(x+3)^3$.

Relation between the HCF, the LCM and the Product of Polynomials

If $f(x)$ and $g(x)$ are two polynomials then we have the relation,

(HCF of $f(x)$ and $g(x)$) × (LCM of $f(x)$ and $g(x)$) = $\pm(f(x) \times g(x))$.

Example: Let $f(x) = (x + 5)^2(x - 7)(x + 8)$ and

$g(x) = (x + 5)(x - 7)^2(x - 8)$ be two polynomials.

The common factors with the least exponents are $x + 5$ and $x - 7$.

$\therefore$ HCF = $(x + 5)(x - 7)$

All the factors (taken only once) with the highest exponents are $(x + 5)^2$, $(x - 7)^2$, $(x - 8)$ and $(x + 8)$.

$\Rightarrow$ LCM = $(x + 5)^2(x - 7)^2(x - 8)(x + 8)$

Now $f(x) \cdot g(x) = (x + 5)^2(x - 7)(x + 8)(x + 5)(x - 7)^2(x - 8)$

$= (x + 5)^3(x - 7)^3(x + 8)(x - 8)$

LCM × HCF = $(x + 5)^2(x - 7)^2(x - 8)(x + 8) \times (x + 5)(x - 7) = (x + 5)^3(x - 7)^3(x - 8)(x + 8)$

Thus, we say

(LCM of two polynomials) × (HCF of two polynomials) = Product of the two polynomials.

Concept of Square Roots

If x is any variable, then x^2 is called the square of the variable and for x^2, x is called the square root.

Square root of x^2 can be denoted as $\sqrt{x^2} \cdot x$ and $-x$ can both be considered as the square roots of x^2 because
$(x) \cdot (x) = x^2$ and $(-x)(-x) = x^2$.

In this study we restrict $\sqrt{x^2}$ to x, i.e., positive value of x.

Square Root of Monomials

The square root of a monomial can be directly calculated by finding the square roots of the numerical coefficient and that of the literal coefficients and then multiplying them.

EXAMPLE 3.16

Find the square root of $1296b^4$.

SOLUTION

Now, the given monomial is $1296b^4$.

Square root of

$$1296b^4 = \sqrt{1296b^4} = \sqrt{1296} \times \sqrt{b^4}$$

$$= \sqrt{(36)^2} \times \sqrt{(b^2)} = 36 \times b^2 \Rightarrow \sqrt{1296b^4} = 36b^2.$$

EXAMPLE 3.17

Find the square root of $\frac{81b^2a^4}{36x^2y^6}$.

SOLUTION

Square root of

$$\frac{81b^2a^4}{36x^2y^6} = \sqrt{\frac{81b^2a^4}{36x^2y^6}}$$
$$= \sqrt{\frac{81b^2a^4}{36x^2y^6}}$$
$$= \frac{\sqrt{81} \times \sqrt{b^2a^4}}{\sqrt{36} \times \sqrt{x^2y^6}}$$
$$= \frac{\sqrt{9^2} \times \sqrt{(ba^2)^2}}{\sqrt{6^2} \times \sqrt{(xy^3)^2}}$$
$$= \frac{9ba^2}{6xy^3} = \frac{3ba^2}{2xy^3}$$

$\Rightarrow$ Square root of $\frac{81b^2a^4}{36x^2y^6} = \frac{3ba^2}{2xy^3}$.

Methods of Finding the Square Roots of Algebraic Expressions Other than Monomials

We have four methods to find the square root of an algebraic expression which is not a monomial. They are

1. Method of inspection (using algebraic identities).
2. Method of factorization
3. Method of division
4. Method of undetermined coefficients

Method of Inspection In this method, the square root of the given algebraic expression is found by using relevant basic algebraic identities after proper inspection.

EXAMPLE 3.18

Find the square root of $x^2 + 12xy + 36y^2$.

SOLUTION

$x^2 + 12xy + 36y^2 = (x)^2 + 2(x)(6y) + (6y)^2$

We know that $a^2 + 2ab + b^2 = (a + b)^2$

Now,

$$\sqrt{x^2+12xy+36y^2} = \sqrt{(x)^2+2(x)(6y)+(6y)^2}$$
$$= \sqrt{(x+6y)^2} = (x+6y)$$

$\therefore \sqrt{x^2+12xy+36y^2} = x+6y.$

EXAMPLE 3.19

Find the square root of $a^2x^2 - 2ayx^2 + x^2y^2$.

SOLUTION

$a^2x^2 - 2ayx^2 + x^2y^2 = x^2(a^2 - 2ay + y^2)$

We know that $a^2 - 2ab + b^2 = (a - b)^2$

Now,

$$\sqrt{a^2x^2-2ayx^2+x^2y^2} = \sqrt{x^2(a^2-2ay+y^2)} = \sqrt{x^2(a-y)^2}$$
$$= \sqrt{[x(a-y)]^2} = x(a-y)$$

$\Rightarrow \sqrt{a^2x^2-2ayx^2+x^2y^2} = x(a-y).$

Method of Factorization

EXAMPLE 3.20

(a) Find the square root of $(x^2 - 8x + 15)(2x^2 - 11x + 5)(2x^2 - 7x + 3)$.

SOLUTION

Step 1: Factorize each expression in the given product, i.e.,

$$\begin{aligned}
x^2 - 8x + 15 &= x^2 - 5x - 3x + 15\\
&= x(x-5) - 3(x-5)\\
&= (x-5)(x-3)\\
2x^2 - 11x + 5 &= 2x^2 - 10x - x + 5\\
&= 2x(x-5) - 1(x-5)\\
&= (2x-1)(x-5)\\
2x^2 - 7x + 3 &= 2x^2 - 6x - x + 3\\
&= 2x(x-3) - 1(x-3)\\
&= (x-3)(2x-1)
\end{aligned}$$

Step 2: Write all the factors in a row.

$\therefore$ The given expression is $(x-5)(x-3)(2x-1)(x-5)(x-3)(2x-1)$, i.e.,

$(x-3)^2(x-5)^2(2x-1)^2$

Step 3: Evaluate the square root.

Hence, the square root of the given expression is $(x-3)(x-5)(2x-1)$.

(b) Find the square root of $(x - 1)(x - 2)(x - 3)(x - 4) + 1$.

SOLUTION

Step 1: First we select the terms in such a way that, they have a common expression in their product. In the given expression, we consider terms $(x - 1)$ and $(x - 4)$ and $(x - 2)(x - 3)$.

Step 2: Find their product, i.e.,

$$x^2 - 5x + 4 \text{ and } x^2 - 5x + 6 = (x^2 - 5x + 4)(x^2 - 5x + 6)$$

Step 3: Take the common expression in the products as a $(= x^2 - 5x)$ and substitute in the given expression.

$\therefore$ The given expression becomes $(a + 4)(a + 6) + 1$

Step 4: Factorize the resultant expression and find its square root, i.e.,

$$a^2 + 10a + 25 = (a + 5)^2$$

$$\therefore \sqrt{(a+5)^2} = a + 5.$$

Step 5: Resubstitute the value of a, which is the required square root.

$\therefore$ The square root of the given expression is $x^2 - 5x + 5$.

Method of Division We discuss the method of division to find the square root of an algebraic expression using the following example.

EXAMPLE 3.21

Find the square root of $x^2 - 18x + 81$.

SOLUTION

$x - 9$

x	$x^2 - 18x + 81$	(x)
	x^2	
$2x - 9$	$-18x + 81$	(-9)
	$-18x + 81$	
	0	

$$\therefore \sqrt{x^2 - 18x + 81} = x - 9.$$

Step 1: First the given expression is arranged in the descending powers of x.

Step 2: Then the square root of the first term in the expression is calculated. In the above problem first term is x^2 whose square root is x. This is now the first term of the square root of the expression.

Step 3: Then the square of x, i.e., x^2 is written below the first term of the expression and subtracted. The difference is zero. Then the next two terms in the expression $-18x + 81$ are brought down as the dividend for the next step. Double the first term of the square root and put it down as the first term of the next divisor, i.e., $2(x) = 2x$ is to be written as the first term of the next divisor. Now the first term $-18x$ of the dividend $-18x + 81$ is to be divided by the first term $2x$ (of the new divisor). Here we get -9 which is the second term of the square root of the given expression and the second term of the new divisor.

Step 4: Thus the new divisor becomes $2x - 9$. Multiply $(2x - 9)$ by (-9) and the product $-18x + 81$ is to be brought down under the second dividend $-18x + 81$ and subtracted where we get 0.

Step 5: Thus $x - 9$ is the square root of the given expression $x^2 - 18x + 81$.

EXAMPLE 3.22

Find the square root of $4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4$.

SOLUTION

Follow the steps indicated in the previous example.

$$
\begin{array}{r|l l}
 & 2x^3 - 3x^2 + 2 & \\
\hline
2x^3 & 4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4 & (2x^3) \\
 & 4x^6 & \\
 & - & \\
\hline
4x^3 - 3x^2 & \quad -12x^5 + 9x^4 & (-3x^2) \\
 & \quad -12x^5 + 9x^4 & \\
 & \quad + \qquad - & \\
\hline
4x^3 - 6x^2 + 2 & \qquad\qquad\qquad 8x^3 - 12x^2 + 4 & (+2) \\
 & \qquad\qquad\qquad 8x^3 - 12x^2 + 4 & \\
 & \qquad\qquad\qquad\quad + \qquad - & \\
\hline
 & \qquad 0 &
\end{array}
$$

$\therefore \sqrt{4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4} = 2x^3 - 3x^2 + 2.$

Method of Undetermined Coefficients The method of undetermined coefficients to find the square root of an algebraic expression is explained in the following examples.

EXAMPLE 3.23

(a) Find the square root of $x^4 + 4x^3 + 10x^2 + 12x + 9$.

SOLUTION

The degree of the given expression is 4, its square root will hence be an expression of degree 2. Let us assume the square root to be $ax^2 + bx + c$.

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9 = (ax^2 + bx + c)^2$

We know that $(p + q + r)^2 = p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

Here, $p = ax^2$, $q = bx$, $r = c$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9$

$= (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9 = a^2x^4 + b^2x^2 + 2abx^3 + 2cax^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equality sign, we have

$x^4 = a^2x^4$

$\Rightarrow a^2 = 1 \Rightarrow a = 1$

$4x^3 = 2abx^3 \Rightarrow 2ab = 4$

$\Rightarrow ab = 2$, but $a = 1 \Rightarrow b = 2$

$b^2 + 2ca = 10 \Rightarrow 2^2 + 2c = 10$

$\Rightarrow 2c = 6 \Rightarrow c = 3$

$\therefore$ The square root of the given expression is $ax^2 + bx + c$, i.e., $x^2 + 2x + 3$.

(b) Find the square root of $4x^4 - 4x^3 + 5x^2 - 2x + 1$.

SOLUTION

The degree of the given expression is 4, its square root will hence be an expression in degree 2.

Let $\sqrt{4x^4 - 4x^3 + 5x^2 - 2x + 1} = ax^2 + bx + c$

$\Rightarrow (4x^4 - 4x^3 + 5x^2 - 2x + 1) = (ax^2 + bx + c)^2$

$\Rightarrow 4x^4 - 4x^3 + 5x^2 - 2x + 1$

$= (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow 4x^4 - 4x^3 + 5x^2 - 2x + 1 = a^2x^4 + 2abx^3 + (b^2 + 2ac)x^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equation, we have

$4x^4 = a^2x^4 \Rightarrow a^2 = 4 \Rightarrow a = 2$

$c^2 = 1 \Rightarrow c = 1$

$2bcx = -2x \Rightarrow 2bc = -2 \Rightarrow bc = -1$

$\Rightarrow b = \dfrac{-1}{c} \Rightarrow b = -1$ ($\because c = 1$)

$\therefore$ The square root of the given expression is $ax^2 + bx + c$, i.e., $2x^2 - x + 1$.

Rational Integral Function of *x*

A polynomial in x, the exponents in powers of x are non-negative integers and the coefficients of the various powers of x are integers.

Example:

$11x^2 - 8x + 3$, $4x^2 - 5x + 1$, $8x^5 - 7x^3 + 8x^2 + 4x + 5$, etc.

Remainder Theorem

$q(x)$ is a rational integral function of x.

If $q(x)$ is divided by $x - a$, then the remainder is $q(a)$.

EXAMPLE 3.24

Find the remainder when $x^3 - 8x^2 + 5x + 1$ is divided by $x - 1$.

SOLUTION

Let $q(x) = x^3 - 8x^2 + 5x + 1$

If $q(x)$ is divided by $x - 1$, then the remainder is $q(1)$.

$\therefore\ q(1) = (1)^3 - 8(1)^2 + 5(1) + 1 = 1 - 8 + 5 + 1$

$q(1) = -1.$

Note If $q(x)$ is divided by $ax - b$, then the remainder is $q\left(\frac{b}{a}\right)$.

EXAMPLE 3.25

Find the remainder when $x^2 - 8x + 6$ is divided by $2x - 1$.

SOLUTION

Let $q(x) = x^2 - 8x + 6$

$\therefore$ Remainder $= q\left(\frac{1}{2}\right)$

i.e., $q\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 6$

$= \frac{1}{4} - 4 + 6 = \frac{1}{4} + 2$

$\therefore\ q\left(\frac{1}{2}\right) = \frac{9}{4}.$

Factor Theorem

$q(x)$ is a rational integral function of x and if $q(\alpha) = 0$, then $x - \alpha$ is the factor of $q(x)$.

EXAMPLE 3.26

(a) Is $x - 2$ a factor of $x^3 + x^2 - 4x - 4$?

SOLUTION

Let $q(x) = x^3 + x^2 - 4x - 4$

$q(2) = 8 + 4 - 8 - 4 = 0$

$\therefore\ x - 2$ is a factor of $q(x)$.

(b) Find the value of m, if $x + 2$ is a factor of $x^3 - 4x^2 + 3x - 5m$.

SOLUTION

Let $q(x) = x^3 - 4x^2 + 3x - 5m$.

Given $x + 2$ is a factor of $q(x)$.

$\therefore q(-2) = 0$

$$\Rightarrow (-2)^3 - 4(-2)^2 + 3(-2) - 5m = 0$$

$$-8 - 16 - 6 - 5m = 0$$

$$-5m = 30$$

$$m = -6.$$

(c) Factorize $x^3 - 2x^2 - 5x + 6$.

SOLUTION

Let $f(x) = x^3 - 2x^2 - 5x + 6$

Step 1: First we find one of the factors of $f(x)$ by substituting the value of x as ±1, ±2 and so on till the remainder is zero.

Here,

$$f(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$
$$= 1 - 2 - 5 + 6$$
$$f(1) = 0$$

$\therefore$ $x - 1$ is a factor of $f(x)$.

Step 2: To find the other two factors, we use synthetic division.

	1	−2	−5	6
$x = 1$	0	1	−1	−6
	1	−1	−6	0

$\therefore$ The other two factors are $x^2 - x - 6$, i.e., $(x - 3)(x + 2)$.

$\therefore$ The factorization of the given expression is $(x - 1)(x - 3)(x + 2)$.

(d) Factorize $x^4 - x^3 - 11x^2 + 9x + 18$.

SOLUTION

Let $f(x) = x^4 - x^3 - 11x^2 + 9x + 18$

$$f(-1) = (-1)^4 - (-1)^3 - 11(-1)^2 + 9(-1) + 18$$
$$= 1 + 1 - 11 - 9 + 18 = 20 - 20 = 0$$

$\therefore$ $x + 1$ is a factor of $f(x)$.

Now,

$$f(2) = (2)^4 - 2^3 - 11(2)^2 + 9(2) + 18$$
$$= 16 - 8 - 44 + 18 + 18 = 52 - 52 = 0$$

$\therefore$ $x - 2$ is a factor of $f(x)$.

	1	−1	−11	9	18
$x = -1$	0	−1	2	9	−18
$x = 2$	1	−2	−9	18	0
	0	2	0	−18	
	1	0	−9	0	

∴ The other two factors of $f(x)$ are $(x + 3)$ and $(x - 3)$.

Hence, the factorization of the given expression, is $(x + 1)(x - 2)(x - 3)(x + 3)$.

Homogeneous Expression

An algebraic expression in which, the degree of all the terms is equal is a homogeneous expression.

Example: $bx + ay$ is a first degree homogeneous expression.

$ax^2 + bxy + cy^2$ is a second degree homogeneous expression.

Notes

1. A homogeneous expression is complete if it contains all the possible terms in it.
2. The product of two homogeneous expressions is a homogeneous expression.
3. The degree of the product of two or more homogeneous expressions is the sum of degrees of all the expressions involved in product.

Symmetric Expressions

$f(x, y)$ is an expression in variables x and y.

If $f(x, y) = f(y, x)$, then $f(x, y)$, is called a symmetric expression.

i.e., If an expression remains same after interchanging the variables x and y is said to be a symmetric expression.

EXAMPLE 3.27

Consider the expressions given below and find if the expressions are symmetric or not:

(a) $ax + ay + b$

(b) $ax^2 + bxy + ay^2$

SOLUTION

(a) Let $f(x, y) = ax + ay + b$

$f(y, x) = ay + ax + b$

$= ax + ay + b$

$\Rightarrow f(y, x) = f(x, y)$

∴ $ax + ay + b$ is symmetric.

(b) $f(x, y) = ax^2 + bxy + ay^2$

$f(y, x) = ay^2 + byx + ax^2$

$= ax^2 + bxy + ay^2$

$\therefore f(y, x) = f(x, y)$

Hence, $ax^2 + bxy + ay^2$ is symmetric.

Notes

1. An expression which is homogeneous and symmetric is called a homogeneous symmetric expression.

   Example: $ax + ay$, $ax^2 + bxy + ay^2$

2. The sum, difference, product and quotient of two symmetric expressions is always symmetric.

Cyclic Expressions

$f(x, y, z)$ is an expression in variables x, y and z.

If $f(x, y, z) = f(y, z, x)$, then $f(x, y, z)$ is cyclic.

Example:

$a^2(a - b) + b^2(b - c) + c^2(c - a)$

Let $f(a, b, c) = a^2(a - b) + b^2(b - c) + c^2(c - a)$

Now, $f(b, c, a) = b^2(b - c) + c^2(c - a) + a^2(a - b)$

$= a^2(a - b) + b^2(b - c) + c^2(c - a)$

$f(b, c, a) = f(a, b, c)$

$\therefore f$ is cyclic.

Cyclic expressions are lengthy to write, so we use symbols Σ (read as sigma) and π(pi) to abbreviate them.

Σ is used for sum of terms and π is used for product of terms.

Example:

$x^2(y^2 - z^2) + y^2(z^2 - x^2) + z^2(x^2 - y^2)$ can be represented as $\sum_{x, y, z} x^2(y^2 - z^2)$

$\therefore \Sigma x^2(y^2 - z^2) = x^2(y^2 - z^2) + y^2(z^2 - x^2) + z^2(x^2 - y^2)$

Example:

$(x^2 + y^3)(y^2 + z^3)(z^2 + x^3)$ can be represented as $\sum_{x, y, z} (x^2 + y^3) = (x^2 + y^3)(y^2 + z^3)(z^2 + x^3)$

EXAMPLE 3.28

Factorize $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

SOLUTION

Let $f(a) = a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

$f(b) = b(b^2 - c^2) + b(c^2 - b^2) + c(b^2 - b^2)$

$= b(b^2 - c^2) + b(c^2 - b^2)$

$f(b) = 0$

$\therefore$ By remainder theorem, $(a - b)$ is a factor of the given expression.

The given expression is cyclic, so the other two factors will also be cyclic.

$\therefore$ The other two factors are $(b - c)$ and $(c - a)$.

The given expression may have a constant factor which is non-zero. Let it be m.

$\therefore a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = m(a - b)(b - c)(c - a)$

Put $a = 0$, $b = 1$, $c = -1$ in the above equation.

i.e., $0(1^2 - (-1)^2) + 1((-1)^2 - 0) + (-1)(0 - 1^2)$

$= m(0 - 1)(1 - (-1))(-1 - 0)$

$\Rightarrow 0 + 1 + 1 = m(-1)(2)(-1)$

$\Rightarrow m = 1$

$\therefore$ The factorization of the given cyclic expressions is $(a - b)(b - c)(c - a)$.

EXAMPLE 3.29

Factorize $a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

SOLUTION

Let $f(a) = a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

$f(b) = b(b^3 - c^3) + b(c^3 - b^3) + c(b^3 - b^3)$

$= b(b^3 - c^3) - b(b^3 - c^3) + 0$

$f(b) = 0$

$\therefore$ By remainder theorem, $a - b$ is the factor of the given expression.

The given expression is cyclic, so the other two factors are also cyclic.

$\therefore$ The factors are $(a - b)(b - c)(c - a)$.

The given expression is of degree 4 but the degree of factor is 3, hence a first degree cyclic expression is the another factor.

Let $m(a + b + c)$ be the factor $(m \neq 0)$.

$\therefore a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

$= m(a + b + c)(a - b)(b - c)(c - a)$

Put $a = 0$, $b = 1$ and $c = 2$ in the above equation.

$0(1^3 - (2)^3) + 1((2)^3 - 0) + 2(0 - 1^3) = m(0 + 1 + 2)(0 - 1)(1 - 2)(2 - 0)$

$\Rightarrow 0 + 1(8) + 2(-1) = m(3)(-1)(-1)(2)$

$6 = 6m$

$\Rightarrow m = 1$

$\therefore$ The factorization of the given cyclic expression is $(a - b)(b - c)(c - a)(a + b + c)$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $11x^2 - 88x^3 + 14x^4$ is called a _____ polynomial.
2. The degree of the polynomial $7x^3y^{10}z^2$ is _____.
3. The expression is a polynomial. (True/False)
4. If $A = 3x^2 + 5x - 3$ and $B = 5x^2 - 7$, then $2A - B$ is _____.
5. If $a + b + c = 0$, then $a^3 + b^3 + c^3 =$ _____.
6. Factors of $x^6 - y^6$ is _____.
7. The LCM of $\sqrt{2}x, \sqrt{8}x^7y^2$ is _____.
8. The HCF of $44a^3$ and $66b^pa^4$ is $22a^3$, then p can be _____.
9. One of the factor of $x^3 - x^2 + x - 1$ is _____ .
10. The quotient of $8x^3 - 7x^2 + 5x + 8$ when divided by $2x$ is _____.
11. The remainder obtained when $80x^3 + 55x^2 + 20x + 172$ is divided by $x + 2$ is _____.
12. Factorize $6x^2 + x - 2$.
13. Find the LCM and HCF of the polynomials $15x^2y^3z$, $3x^3yz^2$.
14. Find the remainder when x^{15} is divided by $x - 2$.
15. Find the remainder if $x^5 - 3x^3 + 5x + 1$ is divided by $2x - 1$.
16. $\sqrt{a + b - 2\sqrt{ab}}$ is _____ where $\sqrt{a} > \sqrt{b}$.
17. The product of two symmetric expressions is a/an _____ expression.
18. The square root of $a^{m^2} \cdot b^{n^2}$ is _____.
19. The value of a if $x^3 - 8x^2 + 2x + a$ is divisible by $x - 2$ is _____.
20. Factorize $a^5b - ab^5$.
21. The degree of a polynomial A is 7 and that of polynomial AB is 56, then find the degree of polynomial B.
22. If $A = x^3$, $B = 4x^2 + x - 1$, then find AB.
23. Factorize $m^7 + m^4$.
24. Factorize $\frac{1}{6}a^2 - a + \frac{4}{3}$.
25. If $3x^2 + 8ax + 3$ is a perfect square, then find the value of a.
26. The factors of $a^3 + b^3 + c^3 - 3abc$ are _____.
27. The HCF of $(a^2 + 1)(a + 11)$ and $(a^2 + 1)^2 (a + 11)^2$ is _____.
28. The value of $81^3 - 100^3 + 19^3$ is _____.
29. If $A = 4x^3 - 8x^2$, $B = 7x^3 - 5x + 3$ and $C = 3x^3 + x - 11$, then find $(A + C) - B$.
30. $8x^2 + 11xy + by^2$ is a symmetric expression, then b = _____.

Short Answer Type Questions

31. The HCF of $(a - 1)(a^3 + m)$ and $(a + 1)(a^3 - n)$ and $(a + 1)(a^2 - n)$ is $a^2 - 1$, then the values of m and n are _____.
32. Expand $\underset{a,b,c}{\pi}\, a^2(b + c)$.
33. The factors of $(a - b)^3 + (b - c)^3 + (c - a)^3$ is _____.
34. Expand $\sum c^2(a^2 - b^2)$.
35. If $A = 4x^3 - 8x^2$, $B = 7x^3 - 5x + 3$ and $C = 3x^3 + x - 11$, then find $2A - 3B + 4C$.
36. If $A = x^3$, $B = 4x^2 + x - 1$, $C = x + 1$, then find $(A - B)(A - C)$.
37. Find the quotient and remainder when $x^4 + 4x^3 - 31x^2 - 94x + 120$ is divided by $x^2 + 3x - 4$.
38. Factorize $a^3 + \frac{3ax}{8} + \frac{x^3}{64} - \frac{1}{8}$.
39. Find the LCM and HCF of the following polynomials.
    $36(x + 2)^2 (x - 1)^3 (x + 3)^5$, $45(x + 2)^5 (x - 1)^2 (x + 3)^5$ and $63(x - 1)^5 (x + 2)^5 (x + 3)^4$.
40. The LCM of the polynomials $(x^2 + x - 2)(x^2 + x - a)$ and $(x^2 + x - b)(x^2 + 5x + a)$ is $(x - 1)(x + 2)^2 (x + 3)$, then find the values of a and b.
41. Find the remainder when x^{23} is divided by $x^2 - 3x + 2$.

42. If $lmx^2 + mnx + ln$ is a perfect square then prove that, $4l^2 = mn$.

43. Find the value of $\sqrt{(a+b+c)^2 + (a+b-c)^2 + 2(c^2 - a^2 - b^2 - 2ab)}$.

44. If $\sqrt{\dfrac{125a^6b^4c^2}{5a^4b^2}} = x$, then find $\dfrac{x^2}{abc}$.

45. Find the square root of $(x^2 + 6x + 8)(x^2 + 5x + 6)(x^2 + 7x + 12)$.

Essay Type Questions

46. Factorize $6x^4 - 5x^3 - 38x^2 - 5x + 6$.

47. For what values of p and q, the expression, $x^4 - 14x^3 + 71x^2 + px + q$ is a perfect square?

48. Find the square root of $16x^6 - 24x^5 + 25x^4 - 20x^3 + 10x^2 - 4x + 1$ by the method of division.

49. Find the quadratic polynomial when divided by x, $x - 1$ and $x - 2$ leaves remainders 1, 2 and 9 respectively.

50. Find the factors of $a^2(b + c) + b^2(c + a) + c^2(a + b) - 2b^2c$.

CONCEPT APPLICATION

Level 1

1. If the degree of a polynomial AB is 15 and the degree of polynomial B is 5, then the degree of polynomial A is

(a) 3 (b) 8
(c) 4 (d) 10

2. The expression $21x^2 + 11x - 2$ equals to

(a) $(x - 2)(7x + 1)$ (b) $(7x + 1)(3x - 2)$
(c) $(7x - 1)(3x - 2)$ (d) $(7x - 1)(3x + 2)$

3. If the LCM and HCF of two polynomials are $90\ m^5a^6b^3x^2$ and $m^3\ a^5$ respectively and also one of the monomial is $18\ m^5a^6x^2$, then the other monomial is

(a) $5\ m^3a^5b^3$ (b) $15\ m^5a^3b^2$
(c) $5\ m^5a^3b^5$ (d) $15\ m^3a^5b^4$

4. The remainder when $x^3 - 3x^2 + 5x - 1$ is divided by $x + 1$ is ______.

(a) −8 (b) −12
(c) −10 (d) −9

5. Which of the following is a homogeneous expression?

(a) $4x^2 - 5xy + 5x^2y + 10y^2$
(b) $5x + 10y + 100$
(c) $14x^3 + 15x^2y + 16y^2x + 24y^3$
(d) $x^2 + y^2 + x + y + 1$

6. $\Sigma x(y^3 - z^3) =$ ______.

(a) $(x - y)(y - z)(z - x)(x + y + z)$
(b) $(x - y)(y - z)(x - z)(x - y - z)$
(c) $(x + y)(y + z)(z + x)(x + y + z)$
(d) $(x + y)(y + z)(z + z)(z - y - z)$

7. The remainder when $f(x) = 4x^3 - 3x^2 + 2x - 1$ is divided by $2x + 1$ is ______.

(a) 1 (b) $\dfrac{-3}{4}$
(c) $\dfrac{-13}{4}$ (d) $\dfrac{-7}{4}$

8. The HCF of the polynomials $12a^3b^4c^2$, $18a^4b^3c^3$ and $24a^6b^2c^4$ is ______.

(a) $12a^3b^2c^2$ (b) $6a^6b^4c^4$
(c) $6a^3b^2c^2$ (d) $48a^6b^4c^4$

9. Find the value of a, if $(x + 2)$ is a factor of the polynomial $f(x) = x^3 + 13x^2 + ax + 20$.

(a) −15 (b) 20
(c) 25 (d) 32

10. The polynomial $x^3 - 4x^2 + x - 4$ on factorization gives

(a) $(x - 4)(x^2 - 1)$
(b) $(x - 4)(x^2 + 4)$
(c) $(x + 4)(x^2 + 1)$
(d) $(x - 4)(x^2 + 1)$

11. If the expression $ax^3 + 2x^2y - bxy^2 - 2y^3$ is symmetric, then $(a, b) =$

(a) (2, 2) (b) (−2, 2)

(c) (−2, −2) (d) (2, −2)

12. The square root of $y^2 + \frac{1}{y^2} + 2$ is

(a) $y + \frac{1}{y}$ (b) $y - \frac{1}{y}$

(c) $y^2 + \frac{1}{y^2}$ (d) $y^2 - \frac{1}{y^2}$

13. The product of the polynomials $2x^3 - 3x^2 + 6$ and $x^2 - x$ is ______.

(a) $2x^6 - 5x^4 + 3x^3 + 6x^2 - 6x$

(b) $2x^5 - x^4 + 3x^3 - 6x^2 + 6x$

(c) $2x^5 - 5x^4 + 3x^3 + 6x^2 - 6x$

(d) None of these

14. The LCM of $x^2 - 16$ and $2x^2 - 9x + 4$ is

(a) $(2x + 1)(x + 4)(x - 4)$

(b) $(x^2 + 16)(2x + 1)$

(c) $2(1 - 2x)(x + 4)(x - 4)$

(d) $(2x - 1)(x + 4)(x - 4)$

15. If $P = 3x^3 - 5x + 9$, $Q = 4x^3 + 5x^2 - 11$ and $R = 5x^3 + 4x^2 - 3x + 7$, then $P - 2Q + R$ is

(a) $2(3x^2 + 4x - 19)$

(b) $-6x^2 - 5x + 38$

(c) $-2(3x^2 + 4x + 19)$

(d) $-2(3x^2 + 4x - 19)$

16. If $g(x) = 3a^x + 7a^2b - 13ab^2 + 9b^y$ is a homogeneous expression in terms of a and b, then the values of x and y respectively are ______.

(a) 2, 2 (b) 2, 1

(c) 3, 2 (d) 3, 3

17. The polynomial $11a^2 - 12\sqrt{2}\,a + 2$ on factorization gives

(a) $(11a + \sqrt{2})(a - \sqrt{2})$

(b) $(a - \sqrt{2})(11a - \sqrt{2})$

(c) $(a + 11)(a + \sqrt{2})$

(d) $(11a - \sqrt{2})(a + \sqrt{2})$

18. If $x^n + 1$ is divisible by $x + 1$, n must be

(a) any natural number

(b) an odd natural number

(c) an even natural number

(d) None of these

19. What is the first degree expression to be subtracted from $x^6 + 8x^4 + 2x^3 + 16x^2 + 4x + 5$ in order to make it a perfect square?

(a) $-4x - 4$ (b) $4x + 4$

(c) $4x - 4$ (d) $-4x + 4$

20. Find the square root of $\frac{m^{n^2} n^{m^2} a^{(m+n)}}{(m+n)^{(m+n)^2}}$.

(a) $m^n n^m a^{\frac{m+n}{2}}$ (b) $\frac{m^{\frac{n^2}{2}} n^{\frac{m^2}{2}} a^{\frac{m+n}{2}}}{(m+n)^{\frac{(m+n)^2}{2}}}$

(c) $\frac{m^n n^m a^{\sqrt{m+n}}}{(m+n)^{(m+n)}}$ (d) None of these

21. What is the first degree expression to be added to $16x^6 + 8x^4 - 2x^3 + x^2 + 2x + 1$ in order to make it a perfect square?

(a) $\frac{5}{2}x + \frac{15}{16}$ (b) $-\frac{5}{2}x - \frac{15}{16}$

(c) $-\frac{5}{2}x + \frac{15}{16}$ (d) $+\frac{2}{2}x - \frac{15}{16}$

22. Factorize the polynomial $8x^3 - \frac{1}{64}$.

(a) $\left(2x - \frac{1}{4}\right)\left(4x^2 - \frac{x}{2} + \frac{1}{16}\right)$

(b) $\left(2x - \frac{1}{8}\right)\left(4x^2 + \frac{x}{2} - 16\right)$

(c) $\left(2x - \frac{1}{4}\right)\left(4x^2 + \frac{1}{16} + \frac{x}{2}\right)$

(d) $\left(2x - \frac{1}{4}\right)\left(4x^2 + \frac{x}{2} - 16\right)$

23. The product of polynomials $3x^3 - 4x^2 + 7$ and $x^2 + 1$ is

(a) $3x^5 - 4x^4 + 3x^3 + 3x^2 + 7$

(b) $x^5 + 4x^2 - 2x + 3$

(c) $3x^5 - 4x^4 - 3x^3 + 4x + 8$

(d) $3x^5 - 5x^4 + 8x^2 + 2x + 1$

24. The LCM and HCF of two monomials is $60x^4y^5a^6b^6$ and $5x^2y^3$ respectively. If one of the two monomials is $15x^4y^3a^6$, then the other monomial is

(a) $12x^2y^3a^6b^6$ (b) $20x^4y^5b^6$

(c) $20x^2y^5b^6$ (d) $15x^2y^5b^6$

25. Which of the following is a factor of the polynomial $f(x) = 2x^3 - 5x^2 + x + 2$?

(a) $x + 1$ (b) $x + 2$

(c) $2x + 1$ (d) $2x - 1$

26. If $3x - 1$ is a factor of the polynomial $81x^3 - 45x^2 + 3a - 6$, then a is ______.

(a) $\frac{8}{3}$ (b) $\frac{-7}{3}$

(c) $\frac{-10}{3}$ (d) $\frac{11}{3}$

27. If $A = 4x^3 - 5x + 7$, $B = 2x^3 - x^2 + 3$ and $C = 5x^3 - 8x^2 + 10$, then $A - 2B - C$ is

(a) $5x^3 - 2x^2 + x + 4$

(b) $-5x^3 + 10x^2 - 5x - 9$

(c) $x^3 + 10x^2 - 5x + 9$

(d) $5x^3 - 8x^2 + x - 1$

28. The square root of $x^{m^2-n^2} \cdot x^{n^2+2mn} \cdot x^{n^2}$ is

(a) x^{m+n} (b) $x^{(m+n)^2}$

(c) $x^{(m+n)/2}$ (d) $x^{\frac{1}{2}(m+n)^2}$

29. $x^{831} + y^{831}$ is always divisible by

(a) $x - y$ (b) $x^2 + y^2$

(c) $x + y$ (d) None of these

30. If $(x + 1)(x + 2)(x + 3)(x + k) + 1$ is a perfect square, then the value of k is

(a) 4 (b) 5

(c) 6 (d) 7

Level 2

31. If $A = 6x^4 + 5x^3 - 14x^2 + 2x + 2$ and $B = 3x^2 - 2x - 1$, then the remainder when $A \div B$ is

(a) x (b) $2x$

(c) $3x$ (d) $4x$

32. The polynomial $x^5 - a^2x^3 - x^2y^3 + a^2y^3$ on factorization gives

(a) $(x - y)(x - a)(x + a)(x^2 + y^2 + xy)$

(b) $(x + a)(x - y)(x - a)(x^2 - y^2 + xy)$

(c) $(x + a)(x + y)(x - a)(x^2 + y^2 + xy)$

(d) None of these

33. The HCF of the polynomials $x^4 + 6x^2 + 25$, $x^3 - 3x^2 + 7x - 5$ and $x^2 + 5 - 2x$ is

(a) $x^2 - 2x - 5$ (b) $x^2 - 2x + 5$

(c) $x - 1$ (d) $3x + 2$

34. The HCF of the polynomials $(2x - 1)(5x^2 - ax + 3)$ and $(x - 3)(2x^2 + x + b)$ is $(2x - 1)(x - 3)$. Then the values of a and b respectively are ______.

(a) 16, −1 (b) −16, 1

(c) −16, −1 (d) 16, 1

35. The remainder when x^{45} is divided by $x^2 - 1$ is

(a) $2x$ (b) $-x$

(c) 0 (d) x

36. Factorize $\sum_{a,b,c} a^2(b^4 - c^4)$.

(a) $(a - b)^2(b - c)^2(c - a)^2$

(b) $(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$

(c) $(a + b)^2(b + c)^2(c + a)^2$

(d) None of these

37. The polynomial $6y^4 - 19y^3 - 23y^2 + 10y + 8$ on factorization gives

(a) $(y + 1)(y - 4)(3y + 2)(2y + 1)$

(b) $(y + 1)(y - 4)(3y - 2)(2y - 1)$

(c) $(y + 1)(y - 4)(3y - 2)(2y + 1)$

(d) $(y + 1)(y - 4)(3y + 2)(2y - 1)$

38. If the LCM of the polynomials $(y - 3)^a(2y + 1)^b(y + 13)^7$ and $(y - 3)^4(2y + 1)^9(y + 13)^c$ is $(y - 3)^6(2y + 1)^{10}(y + 13)^7$, then the least value of $a + b + c$ is

(a) 23 (b) 3

(d) 10 (d) 16

PRACTICE QUESTIONS

39. The LCM of the polynomials $195(x + 3)^2(x - 2)(x + 1)^2$ and $221(x + 1)^3(x + 3)(x + 4)$ is ______.

(a) $221(x + 3)^2 (x + 1)^2(x - 2)(x - 14)$

(b) $13(x + 3)(x + 1)^2$

(c) $3315 (x + 3)^2 (x + 1)^3(x - 2)(x + 4)$

(d) None of these

40. For what value of k the HCF of $x^2 + x + (5k - 1)$ and $x^2 - 6x + (3k + 11)$ is $(x - 2)$?

(a) 2 (b) 2

(c) −2 (d) −1

41. The HCF of the polynomials $9(x + a)^p(x - b)^q(x + c)^r$ and $12(x + a)^{p+3}(x - b)^{q-3}(x + c)^{r+2}$ is $3(x + a)^6(x - b)^6(x + c)^6$, then the value of $p + q - r$ is

(a) 21 (b) 9

(c) 15 (d) 6

42. The remainders obtained when the polynomial $x^3 + x^2 - 9x - 9$ divided by x, $x + 1$ and $x + 2$ respectively are ______.

(a) −9, 0, −15 (b) −9, −16, 5

(c) 0, 0, 5 (d) −9, 0, 5

43. Find the value of

$$\frac{(a+b)^2}{(b-c)(c-a)} + \frac{(b+c)^2}{(a-b)(c-a)} + \frac{(c+a)^2}{(a-b)(b-c)}.$$

(a) −1 (b) 0

(c) 1 (d) 2

44. Find the square root of the expression

$$\frac{1}{xyz}(x^2 + y^2 + z^2) + 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right).$$

(a) $\frac{x + y + z}{xyz}$

(b) $\sqrt{\frac{yz}{x}} + \sqrt{\frac{zx}{y}} + \sqrt{\frac{xy}{z}}$

(c) $\sqrt{x} + \sqrt{y} + \sqrt{z}$

(d) $\sqrt{\frac{x}{yz}} + \sqrt{\frac{y}{xz}} + \sqrt{\frac{z}{xy}}$

45. Factorize the expression $9x^4 + \frac{1}{x^4} + 2$.

(a) $\left(3x^2 - \frac{1}{x^2} + 2\right)\left(3x^2 + \frac{1}{x^2} + 2\right)$

(b) $\left(3x^2 - \frac{1}{x^2} - 2\right)\left(3x^2 + \frac{1}{x^2} + 2\right)$

(c) $\left(3x^2 - \frac{1}{x^2} + 2\right)\left(3x^2 - \frac{1}{x^2} + 2\right)$

(d) $\left(3x^2 + \frac{1}{x^2} + 2\right)\left(3x^2 + \frac{1}{x^2} - 2\right)$

46. The following are the steps involved in factorizing $64x^6 - y^6$. Arrange them in sequential order.

(A) $\{(2x)^3 + y^3\}\ \{(2x)^3 - y^3\}$

(B) $(8x^3)^2 - (y^3)^2$

(C) $(8x^3 + y^3)(8x^3 - y^3)$

(D) $(2x + y)(4x^2 - 2xy + y^2)(2x - y)(4x^2 + 2xy + y^2)$

(a) BADC (b) BDAC

(c) BCAD (d) BACD

47. If $a + b + c = 0$, show that $a^3 + b^3 + c^3 = 3abc$. The following are the steps involved in showing the above result. Arrange them in sequential order.

(A) $a^3 + b^3 + 3ab(-c) = -c^3$

(B) $(a + b)^3 = (-c)^3$

(C) $a + b + c = 0 \Rightarrow a + b = -c$

(D) $a^3 + b^3 + 3ab(a + b) = -c^3$

(E) $a^3 + b^3 + c^3 = 3abc$

(a) ABDCE (b) BCDAE

(c) CBDAE (d) CADBE

48. If the HCF of $8x^3y^a$ and $12x^by^2$ is $4x^ay^b$, then find the maximum value of $a + b$.

(a) 2 (b) 4

(c) 6 (d) Cannot be determined

49. The polynomial $5x^5 - 3x^3 + 2x^2 - k$ gives a remainder 1, when divided by $x + 1$. Find the value of k.

(a) 5 (b) −1

(c) 2 (d) 1

50. Factorize: $a^3 + b^3 + 3ab - 1$.

(a) $(a + b - 1)(a^2 + b^2 + a + b + 1 - ab)$

(b) $(a + b - 1)(a^2 + b^2 + a + b - 1 + ab)$

(c) $(a + b - 1)(a^2 + b^2 - a - b + 1 + ab)$

(d) None of these

PRACTICE QUESTIONS

51. If f and g are two polynomials of degrees 3 and 4 respectively, then what is the degree of $f - g$?
(a) 1
(b) 3
(c) 4
(d) Cannot be determined

52. Find the square root of $\frac{x^2}{9} + \frac{9}{4x^2} - \frac{x}{3} - \frac{3}{2x} + \frac{5}{4}$.
(a) $\frac{2x}{3} + \frac{3}{2x} - \frac{1}{2}$ (b) $\frac{x}{3} - \frac{3}{2x} + 1$
(c) $\frac{3}{x} + \frac{2}{3x} - \frac{1}{2}$ (d) $\frac{x}{3} + \frac{3}{2x} - \frac{1}{2}$

53. The square root of $(xy + xz - yz)^2 - 4xyz(x - y)$ is ______.
(a) $xy + yz - 2xyz$
(b) $(x + y - 2xy)$
(c) $(xy + 3 - y)$
(d) $(xy + yz - zx)$

54. $\left(\sum_{x,y,z} (x+1)^2\right) - \left(\sum_{x,y,z} (x)\right)^2 - 3 =$ ______.
(a) $2\left[\sum_{x,y,z} x - \sum_{x,y,z} xy\right]$
(b) $3\left[\sum_{x,y,z} x^2 - \sum_{x,y,z} x\right]$
(c) $2\left[\sum_{x,y,z} xy - \sum_{x,y,z} x^2\right]$
(d) $3\left[\sum_{x,y,z} x^2 - \sum_{x,y,z} x\right]$

55. $\left(\sum_{x,y,z} x\right)^2 - \left(\sum_{x,y,z} x^2\right) =$ ______.
(a) $\sum_{x,y,z} x$ (b) $2\left(\sum_{x,y,z} xy\right)$
(c) $\pi_{x,y,z}\ xy$ (d) $2\left(\sum_{x,y,z} x + y\right)$

Level 3

56. If $\sqrt{4x^4 + 12x^3 + 25x^2 + 24x + 16} = ax^2 + bx + c$, then which of the following is true?
(a) $2b = a - c$
(b) $2a = b + c$
(c) $2b = a + c$
(d) $2b = c - a$

57. Find the square root of the algebraic expression which is the average of the following expressions
$x^2 + \frac{1}{x^2}$, $-2\left(x - \frac{1}{x}\right)$ and -1.
(a) $\frac{x}{\sqrt{3}} - \frac{1}{\sqrt{3}} + \frac{1}{x}$
(b) $\frac{x}{\sqrt{3}} + 1 + x$
(c) $\frac{1}{\sqrt{3}}\left(x - 1 - \frac{1}{x}\right)$
(d) None of these

58. If the each of algebraic expressions $lx^2 + mx + n$, $mx^2 + nx + l$ and $nx^2 + lx + m$ are prefect squares, then $\frac{l+m}{n} =$ ______.
(a) -4 (b) 6
(c) -8 (d) None of these

59. Which of the following is to be added to make $x^6 - 6x^4 + 4x^3 + 8x^2 - 10x + 3$ a perfect square?
(a) $(x - 1)^2$ (b) $(x - 2)^2$
(c) $(2x - 3)^2$ (d) $(2x + 1)^2$

60. Resolve into factors: $\left(\sum_{x,y,z} x\right)^3 - \sum_{x,y,z} x^3$.
(a) $(x + y)(y + z)(z + x)$
(b) $-(x + y)(y + z)(z + x)$
(c) $3(x + y)(y + z)(z + x)$
(d) $-3(x + y)(y + z)(z + x)$

61. Find the square root of $\frac{a^2}{4}+\frac{1}{a^2}-\frac{1}{a}+\frac{a}{2}-\frac{3}{4}$.

(a) $\frac{a}{2}-\frac{1}{a}+\frac{1}{2}$ (b) $\frac{a}{2}+\frac{2}{a}-1$

(c) $\frac{a}{2}+\frac{1}{a}-\frac{1}{2}$ (d) $\frac{a}{2}-\frac{2}{a}-\frac{1}{2}$

62. $\frac{(x+y)^3+(x-y)^3}{2}-y(3x^2+y^2)=$ ______.

(a) x^3-y^3 (b) $(x-y)^3$

(c) $2x^3-3x^2y$ (d) x^3-6xy^2

63. Find the square root of $(4a+5b+5c)^2-(5a+4b+4c)^2+9a^2$.

(a) $\sqrt{3}\,(b+c)$

(b) $3(b+c-a)$

(c) $3(b+c)$

(d) $3(b+c-a)$

64. $\frac{(a-b)^3-(a+b)^3}{2}+a(a^2+3b^2)=$ ________.

(a) a^3-b^3 (b) $(a+b)^3$

(c) a^3+b^3 (d) $(a-b)^3$

65. The square root of $(3a+2b+3c)^2-(2a+3b+2c)^2+5b^2$ is

(a) $\sqrt{5}(a+b+c)$ (b) $\sqrt{5}(a+b)$

(c) $\sqrt{5}(a+c)$ (d) $\sqrt{5}(a+c-b)$

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. biquadratic
2. 15
3. False
4. $x^2 + 10x + 1$
5. $a^3 + b^3 + c^3 = 3abc$
6. $(x - y)(x + y)(x^2 + y^2 - xy)(x^2 + y^2 + xy)$
7. $\sqrt{8}x^7y^2$
8. any real number
9. $x + \dfrac{1}{x}$
10. $4x^2 - \dfrac{7}{2}x + \dfrac{5}{2}$
11. −288.
12. $(2x - 1)\ (3x + 2)$
13. The LCM of given polynomials $= 3 \times 5 \times x^3 \times y^3 \times z^3 = 15x^3y^3z^3$

    The HCF of given polynomials $= 3x^2yz^2$
14. $f(2) = 2^{15}$
15. $f\left(\dfrac{1}{2}\right) = \dfrac{101}{32}$
16. $\sqrt{a} - \sqrt{b}$
17. symmetric
18. $a^{\frac{m^2}{2}} \cdot b^{\frac{n^2}{2}}$
19. 20
20. $ab(a^2 + b^2)(a + b)(a - b)$
21. 49
22. $4x^5 + x^4 - x^3$
23. $m^4(m + 1)(m^2 - m + 1)$
24. $\dfrac{1}{6}(a - 2)(a - 4)$
25. $a = \pm\dfrac{3}{4}$
26. $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
27. $(a^2 + 1)(a + 11)$
28. −461700
29. $-8x^2 + 6x - 14$
30. 8

Short Answer Type Questions

31. 1, 1
32. $a^2(b + c)b^2(c + a)c^2(a + b)$
33. $3(a - b)(b - c)(c - a)$
34. $c^2(a^2 - b^2) + a^2(b^2 - c^2) + b^2(c^2 - a^2)$
35. $-x^3 - 16x^2 + 19x - 53$
36. $x^6 - 4x^5 - 2x^4 + 4x^3 + 5x^2 - 1$
37. $x^2 + x - 30$
38. $\left(a + \dfrac{x}{4} - \dfrac{1}{2}\right)\left(a^2 + \dfrac{x^2}{16} + \dfrac{1}{4} - \dfrac{ax}{4} + \dfrac{x}{8} + \dfrac{a}{2}\right)$
39. HCF $= 9(x + 2)^2(x - 1)^2(x + 3)^4$,

    LCM $= 1260\ (x + 2)^5(x - 1)^5(x + 3)^5$
40. $a = 6,\ b = 2$
41. $(2^{23} - 1)x + (2 - 2^{23})$
43. $2c$
44. $25abc$
45. $(x + 2)(x + 3)(x + 4)$

Essay Type Questions

46. $(x + 2)(x - 3)(2x + 1)(3x - 1)$
48. $p = -154,\ q = 121$
49. $3x^2 - 2x + 1$
50. $(a + b),\ (b + c),\ (c + a)$

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (d)
11. (c)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (b)
21. (b)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (c)	**30.** (a)

Level 2

31. (a)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (b)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (d)
41. (b)	**42.** (d)	**43.** (a)	**44.** (d)	**45.** (d)	**46.** (c)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (a)
51. (c)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (b)					

Level 3

56. (c)	**57.** (c)	**58.** (a)	**59.** (a)	**60.** (c)	**61.** (a)	**62.** (b)	**63.** (c)	**64.** (d)	**65.** (c)

CONCEPT APPLICATION

Level 1

1. Degree of AB = degree of A + degree of B.
2. Factorize the given expression.
3. Use the formula, HCF × LCM = product of polynomials.
4. Use remainder theorem.
5. Degree of every term should be same.
6. Use factorization concept.
7. Use remainder theorem.
8. Find the common factors with the least exponents.
9. Use factor theorem.
10. Take the terms in common and factorize.
11. An expression is symmetric if all the coefficients are equal.
12. Use $a^2 + b^2 + 2ab = (a + b)^2$ identity.
13. Use the concept of multiplication of polynomials.
14. Factorize the given polynomials.
15. Use the addition and subtraction concept of polynomials.
16. The degree of the homogeneous expression is 3.
17. (i) Factorize the given polynomials.
    (ii) Factorize the middle term such that product obtained is 22 and sum obtained is $-12\sqrt{2}$.
18. (i) Use factor theorem.
    (ii) Put $x = -1$.
    (iii) Check for what values of n, $(-1)^n + 1$ is divisible by $x + 1$.
19. Use division method to find p and q.
20. Apply the division method to find the square root.
21. Recall the concept of finding the square root of monomial.
22. Use algebraic identities.
23. Use the concept of polynomials multiplication.
24. Use the formula (LCM) (HCF) = $f(x) \cdot g(x)$
25. Use factor theorem.
26. Use factor theorem.
27. Use addition and subtraction concept of polynomials.
28. Find the square root.
29. (i) $x^n + y^n$ is always disable by $x + y$ if n is odd.
    (ii) $x^n - y^n$ is divisible by $x - y$, if n is odd number.
30. (i) The product of four consecutive numbers added to 1 is a perfect square.
    (ii) The continued product of 4 consecutive integers added by 1 is always a perfect square.

Level 2

31. (i) Divide A by B.
    (ii) Divide the polynomial A by B and then write the remainder.
32. (i) Take common terms and factorize.
    (ii) From the first two terms take x^3 common and from last two terms take y^3 common.
    (iii) Again take $x^2 - a^2$ common in the product.
    (iv) Now write the factors of $a^2 - b^2$ and $a^3 + b^3$.
33. (i) Factorize the polynomials.
    (ii) Find the HCF of the first two polynomials.
    (iii) Now find the HCF of the third polynomial and HCF of the first two polynomials.
34. (i) Factorize the polynomials and use factor theorem.
    (ii) $(x - 3)$ is a factor of $5x^2 - ax + 3$ and $2x - 1$ is a factor of $2x^2 + x + b$.
    (iii) Substitute $x = 1$ in $5x^2 - ax + 3$ and $x = \frac{1}{2}$ in $2x^2 + x + b$, then obtain the values of a and b.
35. (i) Use division algorithm.
    (ii) $f(x) = g(x)Q(x) + [ax + b]$ where $Q(x)$ is quotient and $(ax + b)$ is the remainder.
    (iii) Put $x = 1$ and $x = -1$, then find the values of a and b.
36. (i) Factorize the cyclic expression.
    (ii) Put $b = c$; the expression become zero, i.e., $b - c$ is a factor of the expression.

(iii) Similarly $(a - b)$ and $(c - a)$ are also the factors of the expression.

(iv) Since the degree of the expression is 4, the fourth factor is $k(a + b + c)$.

37. (i) Find a factor by hit and trial method and remaining factors by division method.

(ii) By observation, sum of the coefficients of odd terms = Sum of the coefficients of even terms.

(iii) Now divide the expression by $y + 1$.

(iv) Find the roots of quotient by trial and error method.

38. (i) LCM is the product of the all the factors with highest powers.

(ii) Use the LCM concept and find the values of a and b.

(iii) $a + b + c$ is least when $c = 0$.

39. (i) Find the common and uncommon factors with highest powers.

(ii) Find LCM of 195 and 221.

(iii) Find the LCM of the expressions given.

40. (i) Use factor theorem.

(ii) If $x - a$ is HCF of $f(x)$, then $f(a) = 0$.

(iii) Substitute $x = 2$ in $f(x)$ or $g(x)$ and obtained the value of k.

41. (i) HCF is the product of all common factors with least exponents.

(ii) Use the HCF concept and find the values of a, b and c.

42. (i) Use remainder theorem.

(ii) Put $x = 0$, $x = -1$, and $x = -2$ in the given expression, and obtain the corresponding reminders.

43. (i) Find the LCM of denominations and add the polynomials.

(ii) Take LCM.

(iii) Find the factors and simplify.

44. (i) Use algebraic identities to factorize the given polynomial.

(ii) Multiply the first three terms with xyz.

(iii) Now express the expression in $(a + b + c)^2$ form.

45. (i) Use algebraic identities to factorize the given polynomial.

(ii) Add and subtract 4 to the expression.

(iii) Express the expression of the form $a^2 - b^2$.

46. BCAD is the required sequential order of steps in solving the given problem.

47. CBDAE is the required sequential order.

48. Given polynomials are $8x^3y^a$ and $12x^by^2$ HCF = $4x^ay^b$

$\Rightarrow \quad a \le 3$ or $a \le b$ and $b \le a$ or $b \le 2 \Rightarrow a = b$

$\therefore$ Each of a and b is less than or equal to 2.

$\therefore$ The maximum value of $a + b$ is $2 + 2$, i.e., 4.

49. Let $f(x) = 5x^5 - 3x^3 + 2x^2 - k$

$f(-1) = 1$ (given)

$\Rightarrow 5(-1)^5 - 3(-1)^3 + 2(-1)^2 - k = 1$

$-5 + 3 + 2 - k = 1$

$\therefore k = -1$.

50. $a^3 + b^3 + 3ab - 1$

$= a^3 + b^3 + (-1)^3 - 3 \cdot a \cdot b(-1)$

$= (a + b - 1)(a^2 + b^2 + 1 - ab + b + a)$.

51. Degree of $f = 3$

Degree of $g = 4$

$\Rightarrow$ Degree of $(f - g) = 4$, since the term of degree 4 cannot be vanished.

52. $\dfrac{x^2}{9} + \dfrac{9}{4x^2} - \dfrac{x}{3} + \dfrac{3}{2x} - \dfrac{5}{4}$

$\dfrac{x}{3}$	$\dfrac{x^2}{9} + \dfrac{9}{4x^2} - \dfrac{x}{3} - \dfrac{3}{2x} + \dfrac{5}{4}$	$\dfrac{x}{3} - \dfrac{1}{2} + \dfrac{3}{2x}$
$\dfrac{2x}{3} - \dfrac{1}{2}$	$\dfrac{9}{4x^2} - \dfrac{x}{3} - \dfrac{3}{2x} + \dfrac{5}{4}$ $-\dfrac{x}{3} \qquad +\dfrac{1}{4}$ $+ \qquad -$	
$\dfrac{2x}{3} - 1 + \dfrac{3}{2x}$	$\dfrac{9}{4x^2} - \dfrac{3}{2x} + 1$ $\dfrac{9}{4x^2} - \dfrac{3}{2x} + 1$	
	0	

$$\therefore \sqrt{\frac{x^2}{9} + \frac{9}{4x^2} - \frac{x}{3} - \frac{3}{2x} + \frac{5}{4}} = \frac{x}{3} - \frac{1}{2} + \frac{3}{2x}.$$

53. $(xy + xz - yz)^2 - 4xyz(x - y)$

$= (xy + z(x - y))^2 - 4(xy)\ [z(x - y)]$

$= [xy - z(x - y)]^2$

$[\because (a + b)^2 - 4ab = (a - b)^2]$

$= (xy - zx + yz)^2$

$= (xy + yz - zx)^2$

$\therefore$ The square root of the given expression is $(xy + yz - zx)$.

54. $\sum_{x,y,z} (x+1)^2 - \left(\sum_{x,y,z} x\right)^2 - 3$

$= (x + 1)^2 + (y + 1)^2 + (z + 1)^2 - (x + y + z)^2 - 3$

$= x^2 + 2x + 1 + y^2 + 2y + 1 + z^2 + 2z + 1 - x^2 - y^2 - z^2 - 2xy - 2yz - 2zx - 3$

$= 2x + 2y + 2z - 2xy - 2yz - 2zx$

$= 2\sum_{x,y,z} x - 2\sum_{x,y,z} xy = 2\left(\sum_{x,y,z} x - \sum_{x,y,z} xy\right)$

55. $\left(\sum_{x,y,z} x\right)^2 - \left(\sum_{x,y,z} x^2\right)$

$= (x + y + z)^2 - (x^2 + y^2 + z^2)$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx - x^2 - y^2 - z^2$

$= 2xy + 2yz + 2zx$

$= 2\left(\sum_{x,y,z} xy\right)$.

Level 3

56. (i) Find the square root and equate it to $ax^2 + bx + c$.

(ii) Square on both sides and obtain the values of a, b and c.

(iii) Verify the relation between a, b and c.

57. (i) Find average of the given polynomials and then apply the division method.

(ii) Average of a, b and c is $\frac{a+b+c}{3}$.

(iii) Express the numerator in the form of $(a + b + c)^2$.

61.

$\frac{a}{2}$	$\frac{a^2}{4} + \frac{1}{a^2} - \frac{1}{a} + \frac{a}{2} - \frac{3}{4} - \frac{a^2}{4}$	$\frac{a}{2} + \frac{1}{2} - \frac{1}{a}$
$\frac{2a}{2} + \frac{1}{2}$	$\frac{a^2}{4} + \frac{1}{a^2} - \frac{1}{a} + \frac{a}{2} - \frac{3}{4} - \frac{a^2}{4}$	
$a + 1 - \frac{1}{a}$	$\frac{1}{a^2} - \frac{1}{a} - 1$ $\frac{1}{a^2} - \frac{1}{a} - 1$	
	0	

$\therefore$ The square root of the given expression is $\frac{a}{2} - \frac{1}{a} + \frac{1}{2}$.

62. $\frac{(x+y)^3 + (x-y)^3}{2} - (y^3 + 3x^2y)$

$= \frac{x^3 + y^3 + 3x^2y + 3xy^2 + x^3 - y^3 - 3x^2y + 3xy^2}{2} - y^3 - 3x^2y$

$= \frac{2x^3 + 6xy^2}{2} - y^3 - 3x^2y$

$= x^3 - 3x^2y + 3xy^2 - y^3$

$= (x - y)^3$.

63. $(4a + 5b + 5c)^2 - (5a + 4b + 4c)^2 + 9a^2$

$= (4a + 5b + 5c + 5a + 4b + 4c)\ (4a + 5b + 5c - 5a - 4b - 4c) + 9a^2$

$= (9a + 9b + 9c)(-a + b + c) + 9a^2$

$= 9(a + b + c)(-a + b + c) + 9a^2$

$= 9(b + c)^2 - 9a^2 + 9a^2$

$= 9(b + c)^2$.

$\therefore$ The square root of the given expression is $\sqrt{9(b+c)^2} = 3(b + c)$.

64. $\frac{(a-b)^3 - (a+b)^3}{2} + a(a^2 + 3b^2)$

$$= \frac{a^3 - 3a^2b + 3ab^2 - b^3 - a^3 - 3a^2b - 3ab^2 - b^3}{2}$$

$$+ a^3 + 3ab^2$$

$$= \frac{-6a^2b - 2b^3}{2} + a^3 + 3ab^2$$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

$$= (a - b)^3.$$

65. $(3a + 2b + 3c)^2 - (2a + 3b + 2c)^2 + 5b^2$

$$= (3a + 2b + 3c + 2a + 3b + 2c)\ (3a + 2b + 3c - 2a - 3b - 2c) + 5b^2$$

$$= (5a + 5b + 5c)(a - b + c) + 5b^2$$

$$= 5(a + c + b)(a + c - b) + 5b^2$$

$$= 5[(a + c)^2 - b^2] + 5b^2$$

$$= 5(a + c)^2$$

$\therefore$ Square root of the given expression is

$$\sqrt{5(a+c)^2} = \sqrt{5}(a+c).$$

HINTS AND EXPLANATION