MISCELLANEOUS EXERCISE

QNo 1 Prove that $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Sol: LHS $= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$$=\cos\left(\frac{9\pi}{13}+\frac{\pi}{13}\right)+\cos\left(\frac{9\pi}{13}-\frac{\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$\{$using $2\cos A\cos B = \cos(A+B)+\cos(A-B)\}$

$$=\cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=-\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$ $[\because \cos(\pi-\theta) = -\cos\theta]$

$= 0$

QNo. 2: $(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x = 0$

Sol LHS $=\left(2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}\right)\sin x+\left(-2\sin\frac{3x+x}{2}\sin\frac{3x-x}{2}\right)\cos x$

$$=(2\sin 2x\cos x)\sin x+(-2\sin 2x\sin x)\cdot\cos x$$

$$=2\sin x\cos x\sin 2x-2\sin x\cos x\sin 2x$$

$= 0 = $ RHS.

QNo 3: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Sol LHS $(\cos x+\cos y)^2+(\sin x-\sin y)^2$

$$=\left(2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right)^2+\left(2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2$$

$$=4\cos^2\frac{x+y}{2}\cos^2\frac{x-y}{2}+4\cos^2\frac{x+y}{2}\sin^2\frac{x-y}{2}$$

$$=4\cos^2\frac{x+y}{2}\left[\cos^2\frac{x-y}{2}+\sin^2\frac{x-y}{2}\right]=4\cos^2\frac{x+y}{2}(1)=\text{RHS}$$

Q No 4: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$

Sol. $LHS = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$$= \left[-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}\right]^2 + \left[2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\right]^2$$

$$= 4\sin^2\frac{x+y}{2}\sin^2\frac{x-y}{2} + 4\cos^2\frac{x+y}{2}\sin^2\frac{x-y}{2}$$

$$= 4\sin^2\left(\frac{x-y}{2}\right)\left[\sin^2\left(\frac{x+y}{2}\right) + \cos^2\left(\frac{x+y}{2}\right)\right]$$

$$= 4\sin^2\left(\frac{x-y}{2}\right)[1] \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= RHS$

Q No 5. $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Sol $LHS = \sin x + \sin 3x + \sin 5x + \sin 7x$

$$= (\sin 7x + \sin x) + (\sin 5x + \sin 3x)$$

$$= 2\sin\frac{7x+x}{2}\cos\frac{7x-x}{2} + 2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}.$$

$$= 2\sin 4x \cos 3x + 2\sin 4x \cos x$$

$$= 2\sin 4x\left[\cos 3x + \cos x\right]$$

$$= 2\sin 4x\left[2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}\right]$$

$$= 4\sin x\left(\cos 2x \cdot \cos x\right) = 4\cos x \cos 2x \sin 4x = RHS$$

Q No 6. $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x.$

LHS $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$$= \frac{2\sin 6x \cos x + 2\sin 6x \cos 3x}{2\cos 6x \cos x + 2\cos 6x \cos 3x} = \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]} = \tan 6x$$

Q No. 7 : $\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$

Sol. LHS $= \sin 3x + \sin 2x - \sin x$

$$= 2 \sin \frac{3x+2x}{2} \cos \frac{3x-2x}{2} - \sin x$$

$$= 2 \sin \frac{5x}{2} \cos \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

$$= 2 \cos \frac{x}{2} \left[\sin \frac{5x}{2} - \sin \frac{x}{2} \right]$$

$$= 2 \cos \frac{x}{2} \left[2 \cos \frac{\frac{5x}{2} + \frac{x}{2}}{2} \sin \frac{\frac{5x}{2} - \frac{x}{2}}{2} \right]$$

$$= 2 \cos \frac{x}{2} \left[2 \cos \frac{3x}{2} \sin x \right] = 4 \sin x . \cos \frac{x}{2} \cos \frac{3x}{2} = \text{RHS}$$

Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, $\tan \frac{x}{2}$ in each of the following :

Q No 8 : $\tan x = -\frac{4}{3}$; x in quadrant II

Sol : Here $\tan x = -\frac{4}{3}$; x in Second quadrant.

Now $\sec^2 x = 1 + \tan^2 x = 1 + \left(-\frac{4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\therefore$ $\sec x = \pm \frac{5}{3} \Rightarrow \cos x = \pm \frac{3}{5}$

$\because$ x is in Second quadrant

$\therefore$ $\cos x$ is $-$ve $\therefore$ $\cos x = -\frac{3}{5}$

Now If x lies in Second quadrant

$\Rightarrow$ $\frac{x}{2}$ lies in First quadrant

$\therefore$ All t-ratios of $\frac{x}{2}$ are +ve.

$$\therefore \quad \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

$$\tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = 2.$$

QNo.9 : $\cos x = -\frac{1}{3}$; x in third quadrant. (III)

Sol Here $\cos x = -\frac{1}{3}$

Now as x lies in third quadrant

$\Rightarrow \frac{x}{2}$ lies in 2nd quadrant. $\left[\begin{array}{l} 180 < x < 270 \\ \Rightarrow 90 < \frac{x}{2} < 135 \end{array}\right]$

$\therefore$ $\sin\frac{x}{2}$ is +ve and $\cos\frac{x}{2}$, $\tan\frac{x}{2}$ are −ve.

$$\therefore \quad \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+1/3}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-\frac{1}{3}}{2}} = -\sqrt{\frac{1}{3}} = -\frac{1}{\sqrt{3}}$$

$$\tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \frac{\frac{\sqrt{2}}{\sqrt{3}}}{-\frac{1}{\sqrt{3}}} = -\sqrt{2}.$$

QNo 10: $\sin x = \frac{1}{4}$; x in Quadrant II

Sol: Here $\sin x = \frac{1}{4}$; x is Second quadrant

$\Rightarrow \cos x = -\sqrt{1-\sin^2 x} = -\sqrt{1-\frac{1}{16}} = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$ [Cos x is −ve in Second quad]

Now Since x lies in Second quadrant

$\Rightarrow \frac{x}{2}$ lies in First quadrant

$\Rightarrow \sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$ are all +ve.

$$\therefore \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4+\sqrt{15}}{8}} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{8}}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4-\sqrt{15}}{8}} = \frac{\sqrt{4-\sqrt{15}}}{\sqrt{8}}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{4-\sqrt{15}}} = \frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}}$$

$\#$.