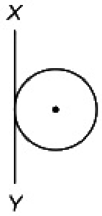
System of Particles and Rotational Motion

Question1

The radius of gyration of a solid sphere of mass 5 kg about X Y is 5m as shown in figure. The radius of the sphere is $\frac{5x}{\sqrt{7}}$m, then the value of x is:



[NEET 2024 Re]

Options:

A.

5

B.

√2

C.

√3

D.

√5

Answer: D

Solution:

$$" I_{XY} = I_{CM} + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2 = \frac{7}{5} \times 5R^2 = 7R^2 \text{ ....(1)}$$

$$I_{XY} = MK^2 = 5 \times 5^2 \text{ ... (2)}$$

$$\therefore 5 \times 5^2 = 7 \times R^2 \quad \text{[From (1) and (2)]}$$

$$\Rightarrow R = \sqrt{\frac{5}{7}} \times 5 = \frac{5x}{\sqrt{7}} \quad \text{(Given)}$$

$$\therefore x = \sqrt{5}$$

Question2

Two bodies A and B of same mass undergo completely inelastic one dimensional collision. The body A moves with velocity v_1 while body B is at rest before collision. The velocity of the system after collision is v_2. The ratio $v_1 : v_2$ is

[NEET 2024]

Options:

A.

1 : 2

B.

2 : 1

C.

4 : 1

D.

1 : 4

Answer: B

Solution:

Before collision $\Rightarrow (A) \longrightarrow v_1 \qquad \underset{\text{rest}}{(B)}$

It undergoes completely inelastic collision

Using conservation of linear momentum

Initial momentum = Final momentum

$\Rightarrow mv_1 = mv_2 + mv_2$

$\Rightarrow mv_1 = 2mv_2$

$\Rightarrow \frac{v_1}{v_2} = \frac{2}{1}$

Question3

A bob is whirled in a horizontal plane by means of a string with an initial speed of ω rpm. The tension in the string is T. If speed becomes 2ω while keeping the same radius, the tension in the string becomes:

[NEET 2024]

Options:

A.

T

B.

4T

C.

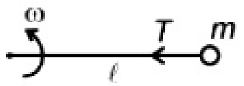
T/4

D.

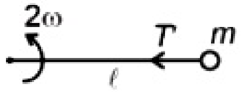
√2T

Answer: B

Solution:



$T = m\ell\omega^2$



$T' = m\ell(2\omega)^2$

$T' = 4T$

Question4

The moment of inertia of a thin rod about an axis passing through its mid point and perpendicular to the rod is 2400gcm2. The length of the 400g rod is nearly:

[NEET 2024]

Options:

A.

8.5cm

B.

17.5cm

C.

20.7cm

D.

72.0cm

Answer: A

Solution:

Moment of inertia of rod $= I = \frac{m\ell^2}{12}$

$\Rightarrow \quad 2400 = 400\,\frac{\ell^2}{12}$

$\Rightarrow \quad 72 = \ell^2$

$\Rightarrow \ell = \sqrt{72} = 8.48\text{ cm} = 8.5\text{ cm}$

Question5

A particle moving with uniform speed in a circular path maintains:

[NEET 2024]

Options:

A.

Constant velocity

B.

Constant acceleration

C.

Constant velocity but varying acceleration

D.

Varying velocity and varying acceleration

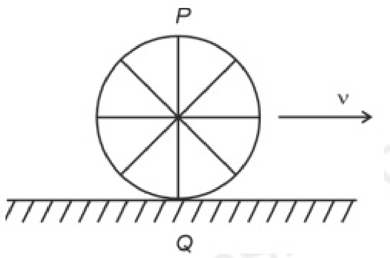
Answer: D

Solution:

A particle moving with uniform speed in a circular path maintains varying velocity and varying acceleration. It is because direction of both velocity as well as acceleration will change continuously.

Question6

A wheel of a bullock cart is rolling on a level road as shown in the figure below. If its linear speed is ν in the direction shown, which one of the following options is correct (P and Q are any highest and lowest points on the wheel, respectively)?



[NEET 2024]

Options:

A.

Point P moves slower than point Q

B.

Point P moves faster than point Q

C.

Both the points P and Q move with equal speed

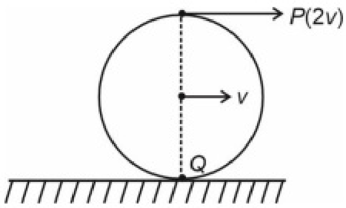
D.

Point P has zero speed

Answer: B

Solution:

In the case of pure rolling,



The topmost point will have velocity 2v while point Q i.e. lowest point will have zero velocity.

Hence point P moves faster than point Q.

Question7

Two particles A and B initially at rest, move towards each other under mutual force of attraction. At an instance when the speed of A is v and speed of B is 3v, the speed of centre of mass is :

[NEET 2023 mpr]

Options:

A.

2v

B.

zero

C.

v

D.

4v

Answer: B

Solution:

Solution:

Final velocity of centre of mass = Initial velocity of centre of mass = 0 because net external force on system is zero.

Question8

A constant torque of 100Nm turns a wheel of moment of inertia 300kgm about an axis passing through its centre. Starting from rest, its angular velocity after 3 s is :-

[NEET 2023 mpr]

Options:

A.

1rad/ s

B.

5rad/ s

C.

10rad/ s

D.

15rad/ s

Answer: A

Solution:

Solution:

$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{100}{300} = \frac{1}{3} \text{rad}/sec^2$

$\omega_i = 0$

$\omega_f = \omega_i + \alpha t$

$= 0 + \frac{1}{3} \times 3$

$\omega_f = 1 \text{ rad}/sec$

Question9

The ratio of radius of gyration of a solid sphere of mass M and radius R about its own axis to the radius of gyration of the thin hollow sphere of same mass and radius about its axis is

[NEET 2023]

Options:

A.

5:3

B.

2:5

C.

5:2

D.

None of Above

Answer: D

Solution:

Solution:

Radius of gyration of solid sphere about its own axis $= \sqrt{\frac{2}{5}}R$

Radius of gyration of hollow sphere about its own axis $= \sqrt{\frac{2}{3}}R$

$\Rightarrow$ Required ratio $= \sqrt{\frac{2}{5}} \times \sqrt{\frac{3}{2}} = \sqrt{\frac{3}{5}}$

* None of the option is correct (correct answer is $\sqrt{\frac{3}{5}}$)

Question10

The angular acceleration of a body, moving along the circumference of a circle, is

[NEET 2023]

Options:

A.

Along the radius towards the centre

B.

Along the tangent to its position

C.

Along the axis of rotation

D.

Along the radius, away from centre

Answer: C

Solution:

Solution:

Angular acceleration of a body, moving along the circumference of a circle is along the axis of rotation.

Question11

A shell of mass m is at rest initially. It explodes into three fragments having mass in the ratio 2 : 2 : 1. If the fragments having equal mass fly off along mutually perpendicular directions with speed v, the speed of the third (lighter) fragment is [NEET-2022]

Options:

A. v

B. $\sqrt{2}$v

C. $2\sqrt{2}$v

D. $3\sqrt{2v}$

Answer: C

Solution:

Momentum of the system would remain conserved.

Initial momentum $= 0$

Final momentum should also be zero.

Let masses be $2m$, $2m$, and m

Momentum along x-direction $= 2mv\hat{i}$

Momentum along y-direction $= 2mv\hat{j}$

Net momentum $= \sqrt{(2mv)^2 + (2mv)^2} = \sqrt{2} \cdot 2mv$

Now, $2\sqrt{2}mv = mv'$

$v' = 2\sqrt{2}v$

Question12

The angular speed of a fly wheel moving with uniform angular acceleration changes from 1200rpm to 3120 rpm in 16 seconds. The angular acceleration in rad $/ s^2$ is [NEET-2022]

Options:

A. 2π

B. 4π

C. 12π

D. 104π

Answer: B

Solution:

Angular acceleration $\alpha = \dfrac{\omega_f - \omega_i}{t}$

$\omega_f = 3120 \times \dfrac{2\pi}{60} rad / s$

$\omega_i = 1200 \times \dfrac{2\pi}{60} rad / s$

$\Rightarrow \alpha = \dfrac{(3120 - 1200)}{16} \times \dfrac{2\pi}{60} = 4\pi$

Question13

Two objects of mass 10kg and 20kg respectively are connected to the two ends of a rigid rod of length 10m with negligible mass. The distance

of the center of mass of the system from the 10kg mass is [NEET-2022]

Options:

A. $\frac{10}{3}$m

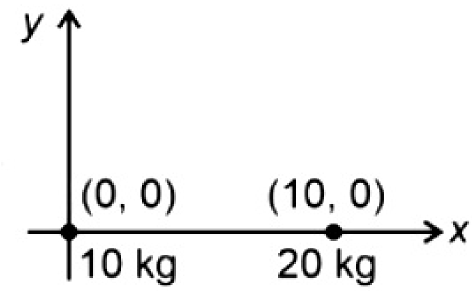
B. $\frac{20}{3}$m

C. 10m

D. 5m

Answer: B

Solution:



$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{10 \times 0 + 20 \times 10}{10 + 20}$$

$$= \frac{200}{30}$$

$$= \frac{20}{3}m$$

Question14

The ratio of the radius of gyration of a thin uniform disc about an axis passing through its centre and normal to its plane to the radius of gyration of the disc about its diameter is [NEET-2022]

Options:

A. 2 : 1

B. $\sqrt{2}$: 1

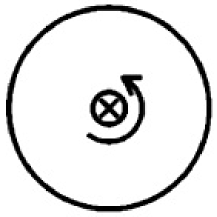
C. 4 : 1

D. 1 : $\sqrt{2}$

Answer: B

Solution:

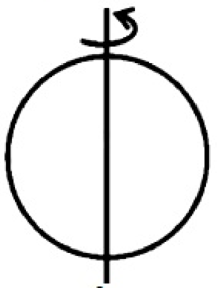
Solution :



$I_1 = \frac{MR^2}{2}$

$k_1 = \sqrt{\frac{I_1}{M}}$

$= \frac{R}{\sqrt{2}}$



$I_2 = \frac{MR^2}{4}$

$k_2 = \sqrt{\frac{I_2}{M}}$

$= \frac{R}{2}$

$\frac{k_1}{k_2} = \frac{\frac{R}{\sqrt{2}}}{\frac{R}{2}}$

$= \sqrt{2} : 1$

Question15

An energy of 484J is spent in increasing the speed of a flywheel from 60 rpm to 360 rpm. The moment of inertia of the flywheel is [NEET Re-2022]

Options:

A. $0.07\,\mathrm{kg}-\mathrm{m}^2$

B. $0.7\,\mathrm{kg}-\mathrm{m}^2$

C. $3.22\,\mathrm{kg}-\mathrm{m}^2$

D. $30.8\,\mathrm{kg}-\mathrm{m}^2$

Answer: B

Solution:

$$\omega_i = 60 \text{ rpm} = 60 \times \frac{2\pi}{60} = 2\pi \text{ ra d/5}$$

$$\omega_f = 360 \text{ rpm} = 360 \times \frac{2\pi}{60} = 12\pi \text{ rad/5}$$

$$\text{Energy spent } = \Delta kE = \frac{1}{2} I (\omega_f^2 - \omega_i^2)$$

$$486 = \frac{1}{2} \times I \times [(12\pi)^2 - (2\pi)^2]$$

$$I = \frac{2 \times 486}{140\pi^2} \approx 0.7 \text{ kg} - \text{m}^2$$

Question16

Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) :
When a fire cracker (rocket) explodes in mid air, its fragments fly in such a way that they continue moving in the same path, which the fire cracker would have followed, had it not exploded.
Reason (R):
Explosion of cracker (rocket) occurs due to internal forces only and no external force acts for this explosion.
In the light of the above statements, choose the most appropriate answer from the options given below
[NEET Re-2022]

Options:

A. (A) is not correct but (R) is correct

B. Both (A) and (R) are correct and (R) is the correct explanation of (A)

C. Both (A) and (R) are correct but (R) is not the correct explanation of (A)

D. (A) is correct but (R) is not correct

Answer: A

Solution:

CoM of rocket follows the same path not the fragments. It is because the explosion takes place due to internal forces.

Question17

From a circular ring of mass ' M and radius ' R ' an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is ' K times ′M R^2 '. Then the value of ' K is
[NEET 2021]

Options:

A. $\frac{3}{4}$

B. $\frac{7}{8}$

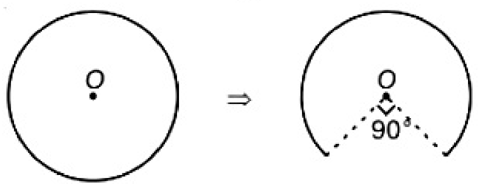
C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: A

Solution:

Solution:
Given that,
Mass of Ring = M; Radius of Ring = R
Now 90° arc is removed from circular ring, then mass removed $= \frac{M}{4}$



Mass of remaining portion $= \frac{3M}{4}$
Moment of inertia of remaining part $= \int d\,mr^2$
$\Rightarrow I = R^2 \int d\,m \quad (\because r = R)$
$\Rightarrow I = \frac{3MR^2}{4}$. So the value of K is $\frac{3}{4}$

Question18

A solid cylinder of mass 2kg and radius 4cm is rotating about its axis at the rate of 3 rpm. The torque required to stop it after 2π revolutions is (NEET 2019)

Options:

A. 2×10^{6}N m

B. 2×10^{-6}N m

C. 2×10^{-3} N m

D. 12×10^{-4} N m

Answer: B

Solution:

Solution:
Given : Mass $M = 2kg$, Radius $R = 4cm$
Initial angular speed
$\omega_0 = 3rpm = 3 \times \frac{2\pi}{60} rad / s = \frac{\pi}{10} rad / s$
We know that, $\omega^2 = \omega_0{}^2 + 2\alpha\theta$
$\Rightarrow 0 = \left(\frac{\pi}{10}\right)^2 + 2 \times \alpha \times 2\pi \times 2\pi$
$\Rightarrow \alpha = \frac{-1}{800} rad / s^2$
Moment of inertia of a solid cylinder,
$I = \frac{MR^2}{2} = \frac{2 \times \left(\frac{4}{100}\right)^2}{2} = \frac{16}{10^4}$
Torque $\tau = I\alpha = \left(\frac{16}{10^4}\right) \times \left(-\frac{1}{800}\right) = -2 \times 10^{-6} N m$

Question19

A disc of radius 2m and mass 100kg rolls on a horizontal floor. Its centre of mass has speed of 20cm / s. How much work is needed to stop it?
(NEET 2019)

Options:

A. 1 J

B. 3J

C. 30kJ

D. 2J

Answer: B

Solution:

Solution:
Required work done $= -(K_f - K_i)$
$= 0 + K_i = K_i$
$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2}\left(\frac{1}{2} mR^2\right)\omega^2 + \frac{1}{2} mv^2$
$= \frac{3}{4} mv^2 = \frac{3}{4} \times 100 \times (20 \times 10^{-2})^2 = 3J$

Question20

A solid cylinder of mass 2kg and radius 50cm rolls up an inclined plane of angle inclination 30°. The centre of mass of cylinder has speed of 4m / s. The distance travelled by the cylinder on the incline surface will be (Take g = 10m / s^2)
(OD NEET 2019)

Options:

A. 2.2m

B. 1.6m

C. 1.2m

D. 2.4m

Answer: D

Solution:

Solution:
Using Law of Conservation of Energy,
$mgh = \frac{1}{2}mv_{cm}{}^2 + \frac{1}{4}mr^2\omega^2$
$\Rightarrow\ mgs\sin 30^{\circ} = \frac{1}{2}mv_{cm}{}^2 + \frac{1}{4}mv_{cm}{}^2$
$\Rightarrow\ s = \frac{3}{4}\frac{v^2}{g\sin 30^{\circ}} \Rightarrow s = \frac{3}{4} \times \frac{4^2}{10 \times \frac{1}{2}}$
$\Rightarrow\ s = \frac{3 \times 4 \times 2}{10} = \frac{12}{5} = 2.4m$

Question21

A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is
(NEET 2018)

Options:

A. 7: 10

B. 5: 7

C. 10: 7

D. 2: 5

Answer: B

Solution:

Solution:
Translational kinetic energy,
$K_t = \frac{1}{2}mv^2$
Rotational kinetic energy,
$K_r = \frac{1}{2}I\omega^2$
$\therefore K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{2}\right)^2$
$\left[\because I = \frac{2}{5}mr^2(\text{ for sphere })\right]$
$\therefore K_t + K_r = \frac{7}{10}mv^2$
So, $\frac{K_t}{K_t + K_r} = \frac{5}{7}$

Question22

A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere? (NEET 2018)

Options:

A. Angular velocity

B. Moment of inertia

C. Rotational kinetic energy

D. Angular momentum

Answer: D

Solution:

Solution:
As there is no external torque acting on a sphere, i.e., $\tau_{ex} = 0$
So, $\frac{dL}{dt} = \tau_{ex} = 0$
i.e., $L =$ constant So angular momentum remains constant.

Question23

Three objects, A: (a solid sphere), B: (a thin circular disk) and C: (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The

amounts of work (W) required to bring them to rest, would satisfy the relation
(NEET 2018)

Options:

A. $W_C > W_B > W_A$

B. $W_A > W_B > W_C$

C. $W_B > W_A > W_C$

D. $W_A > W_C > W_B$

Answer: A

Solution:

Solution:
Work done required to bring a object to rest $\Delta W = \Delta KE$
$\Delta W = \frac{1}{2} I \omega^2$; where $I =$ moment of inertia
For same ω, $\Delta W \propto I$
For a solid sphere, $I_A = \frac{2}{5} MR^2$
For a thin circular disk, $I_B = \frac{1}{2} MR^2$
For a circular ring, $I_C = MR^2$
$\therefore I_C > I_B > I_A \quad \therefore W_C > W_B > W_A$

Question24

The moment of the force, $\vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k}$ at (2, 0, −3), about the point (2, −2, −2), is given by
(NEET 2018)

Options:

A. $-8\hat{i} - 4\hat{j} - 7\hat{k}$

B. $-4\hat{i} - \hat{j} - 8\hat{k}$

C. $-7\hat{i} - 8\hat{j} - 4\hat{k}$

D. $-7\hat{i} - 4\hat{j} - 8\hat{k}$

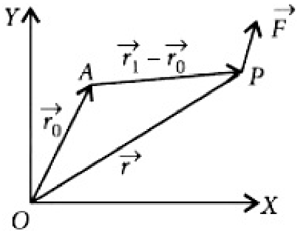
Answer: D

Solution:

Moment of the force is,
$\vec{\tau} = (\vec{r} - \vec{r}_0) \times \vec{F}$

Here, $\vec{r}_0 = 2\hat{i} - 2\hat{j} - 2\hat{k}$

and $\vec{r} = 2\hat{i} + 0\hat{j} - 3\hat{k}$



$\therefore \ \vec{r} - \vec{r}_0 = (2\hat{i} + 0\hat{j} - 3\hat{k}) - (2\hat{i} - 2\hat{j} - 2\hat{k})$

$= 0\hat{i} + 2\hat{j} - \hat{k}$

$$\therefore \ \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 4 & 5 & -6 \end{vmatrix} = -7\hat{i} - 4\hat{j} - 8\hat{k}$$

Question25

A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N?
(2017 NEET)

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Options:

A. 0.25rad s^{-2}

B. 25rad s^{-2}

C. 5ms^{-2}

D. 25ms^{-2}

Answer: B

Solution:

Solution:
$m = 3 \text{kg}, r = 40 \text{cm} = 40 \times 10^{-2} \text{m}, F = 30 \text{N}$

Moment of inertia of hollow cylinder about its axis
$= mr^2 = 3 \text{kg} \times (0.4)^2 \text{m}^2 = 0.48 \text{kgm}^2$
The torque is given by
t = I α
where I = moment of inertia,
α = angular acceleration
In the given case, $\tau = rF$, as the force is acting perpendicularly to the radial vector

$$\therefore \alpha = \frac{\tau}{I} = \frac{F r}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}} = \frac{30 \times 100}{3 \times 40}$$

$\alpha = 25 \text{rad s}^{-2}$

Question26

Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2.Theyare brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is
(2017 NEET)

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Options:

A. $\frac{1}{4}I(\omega_1 - \omega_2)^2$

B. $I(\omega_1 - \omega_2)^2$

C. $\frac{1}{8}I(\omega_1 - \omega_2)^2$

D. $\frac{1}{2}I(\omega_1 + \omega_2)^2$

Answer: A

Solution:

Solution:
Initial angular momentum $= I\omega_1 + \omega_2$
Let w be angular speed of the combined system.
Final angular momentum $= 2I\omega$
∴ According to conservation of angular momentum
$I\omega_1 + I\omega_2 = 2I\omega$ or $\omega = \frac{\omega_1 + \omega_2}{2}$
Initial rotational kinetic energy
$E_1 = \frac{1}{2}I(\omega_1^2 + \omega_2^2)$
Final rotational kinetic energy
$E_f = \frac{1}{2}(2I)\omega^2 = \frac{1}{2}(2I)\left(\frac{\omega_1 + \omega_2}{2}\right)^2 = \frac{1}{4}I(\omega_1 + \omega_2)^2$
∴ Loss of energy $\Delta E = E_i - E_f$
$= \frac{I}{2}(\omega_1^2 + \omega_2^2) - \frac{I}{4}(\omega_1^2 + \omega_2^2 + 2\omega_1 + \omega_2)$
$= \frac{I}{4}[\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2] = \frac{I}{4}(\omega_1 - \omega_2)^2$

Question27

Which of the following statements are correct?
(1) Centre of mass of a body always coincides with the centre of gravity of the body.
(2) Centre of mass of a body is the point at which the total gravitational torque on the body is zero.
(3) A couple on a body produces both transnational and rotational

motion in a body.
(4) Mechanical advantage greater than one means that small effort can be used to lift a large load.
(2017 NEET)

Options:

A. (1) and (2)

B. (2) and (3)

C. (3) and (4)

D. none of the above

Answer: D

Solution:

Solution:
Centre of gravity of a body is the point at which the total gravitational torque on body is zero.
Centre of mass and centre of gravity coincides only for symmetrical bodies.
Hence statements (1) and (2) are incorrect.
A couple of a body produces rotational motion only.
Hence statement (3) is incorrect.
*None of the given options is correct.

Question28

Two rotating bodies A and B of masses m and 2m with moments of inertia I_A and I_B $(I_B > I_A)$ have a equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then
(2016 NEET Phase-II)

Options:

A. $L_A = \frac{L_B}{2}$

B. $L_A = 2L_B$

C. $L_B > L_A$

D. $L_A > L_B$

Answer: C

Solution:

Here, $m_A = m$, $m_B = 2m$

Both bodies A and B have equal kinetic energy of rotation

$k_A = k_B \Rightarrow \frac{1}{2}I_A\omega_A^2 = \frac{1}{2}I_B\omega_B^2$

$\Rightarrow \frac{\omega_A^2}{\omega_B^2} = \frac{I_B}{I_A}$

Ratio of angular momenta,

$\frac{L_A}{L_B} = \frac{I_A\omega_A}{I_B\omega_B} = \frac{I_A}{I_B} \times \sqrt{\frac{I_B}{I_A}}$ [Using eqn.(i)]

$= \sqrt{\frac{I_A}{I_B}} < 1 \quad (\because I_B > I_A) \therefore L_B > L_A$

Question29

A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation $\left(\frac{E_{sphere}}{E_{cylinder}}\right)$ will be

(2016 NEET Phase-II)

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Options:

A. 2 : 3

B. 1 : 5

C. 1 : 4

D. 3 : 1

Answer: B

Solution:

Solution:

$\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2} = \frac{I_s\omega_s^2}{I_c\omega_c^2}$

Here, $I_s = \frac{2}{5}mR^2$, $I_c = \frac{1}{2}mR^2$

$\omega_c = 2\omega_s$

$\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{2}{5}mR^2 \times \omega_s^2}{\frac{1}{2}mR^2 \times (2\omega_s)^2} = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$

Question30

A light rod of length l has two m asses m_1 and m_2 attached to its two ends. The moment of inertia of the system about an axis perpendicular

to the rod and passing through the centre of mass is
(2016 NEET Phase-II)

Options:

A. $\frac{m_1 m_2}{m_1 + m_2} l^2$

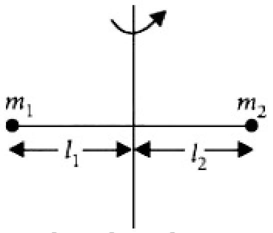
B. $\frac{m_1 + m_2}{m_1 m_2} l^2$

C. $(m_1 + m_2) l^2$

D. $\sqrt{m_1 m_2 l^2}$

Answer: A

Solution:



Here $l_1 + l_2 = l$
Center of mass of the system
$l_1 = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2} = \frac{m_2 l}{m_1 + m_2}$
$l_2 = l - l_1 = \frac{m_1 l}{m_1 + m_2}$
Required moment of inertia of the system, $I = m_1 l_1^2 + m_2 l_2^2$
$= (m_1 m_2^2 + m_2 m_1^2) \frac{l^2}{(m_1 + m_2)^2}$
$= \frac{m_1 m_2 (m_1 + m_2) l^2}{(m_1 + m_2)^2}$
$= \frac{m_1 m_2}{m_1 + m_2} l^2$

Question31

A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first?
(2016 NEET Phase-1)

Options:

A. Both reach at the same time

B. Depends on their masses

C. Disk

D. Sphere

Answer: D

Solution:

Solution:
Time taken by the body to reach the bottom when it rolls down on an inclined plane without slipping is given by

$$\sqrt{\frac{2l\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}}$$

Since g is constant and /, R and sin θ are same for both

$$\therefore \frac{t_d}{t_s} = \frac{\sqrt{1+\frac{{k_d}^2}{R^2}}}{\sqrt{1+\frac{{k_s}^2}{R^2}}} = \sqrt{\frac{1+\frac{R^2}{2R^2}}{1+\frac{2R^2}{5R^2}}}$$

$$\left(\because k_d = \frac{R}{\sqrt{2}}, k_s = \frac{\sqrt{2}}{5}\right)$$

$$= \sqrt{\frac{3}{2}\times\frac{5}{7}} = \sqrt{\frac{15}{14}} \Rightarrow t_d > t_s$$

Hence, the sphere gets to the bottom first.

Question32

From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre? (2016 NEET Phase-1)

Options:

A. $11M R^2/32$

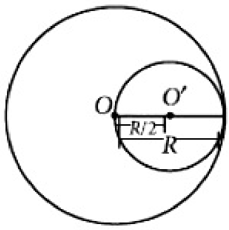
B. $9M R^2/32$

C. $15M R^2/32$

D. $13M R^2/32$

Answer: D

Solution:



Mass per unit area of disc $= \frac{M}{\pi R^2}$
Mass of removed portion of disc,
$M' = \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$
Moment of inertia of removed portion about an axis passing through centre of disc O and perpendicular to the plane of disc,
$I'_O = I_{O'} + M'd^2 = \frac{1}{2} \times \frac{M}{4} \times \left(\frac{R}{2}\right)^2 + \frac{M}{4} \times \left(\frac{R}{2}\right)^2$

$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$
When portion of disc would not have been removed, the moment of inertia of complete disc about centre O is
$I_O = \frac{1}{2}MR^2$
So, moment of inertia of the disc with removed portion is
$I = I_O - I_O' = \frac{1}{2}MR^2 - \frac{3MR^2}{32} = \frac{13MR^2}{32}$

Question33

A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2} Its net acceleration in s^{-2} at the end of 2.0 s is approximately (2016 NEET Phase-1)

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Options:

A. 6.0

B. 3.0

C. 8.0

D. 7.0

Answer: C

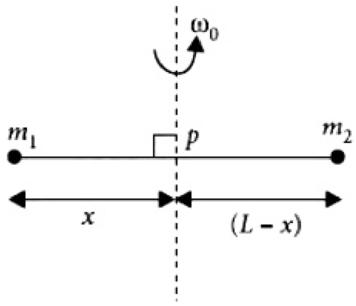
Solution:

Solution:
Given, $r = 50cm = 0.5m$, $\alpha = 2.0 rad\, s^{-2} \omega_0 = 0$
At the end of 2s
Tangential acceleration, $a_t = r\alpha = 0.5 \times 2 = 1, s^{-2}$
Radial accelertion, $a_r = \omega^2 r = (\omega_0 + \alpha t)^2 r$
$= (0 + 2 \times 2)^2 \times 0.5 = 8ms^{-2}$
∴ Net acceleration,

$a = \sqrt{a_t^2 + a_t^2} = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8ms^{-2}$

Question34

Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by



(2015)

Options:

A. $x = \frac{m_2}{m_1}L$

B. $x = \frac{m_2 L}{m_1 + m_2}$

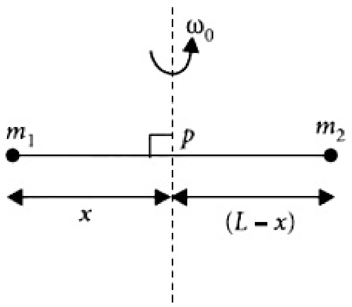
C. $x = \frac{m_1 L}{m_1 + m_2}$

D. $x = \frac{m_1}{m_2}L$

Answer: B

Solution:

Solution:



Moment of inertia of the system about the axis of rotation (through point P) is
$I = m_1x^2 + m_2(L - x)^2$
By work energy theorem, Work done to set the rod rotating with angular velocity ω_0
= Increase in rotational kinetic energy.
$W = \frac{1}{2}I\,\omega_0^2 = \frac{1}{2}[m_1x^2 + m_2(L - x)^2]\omega_0^2$

For W to be minimum, $\frac{dW}{dx} = 0$

i.e. $\frac{1}{2}[2m_1x + 2m_2(L - x)(-1)]\omega_0^2 = 0$

or $m_1x - m_2(L - x) = 0 \quad (\because \omega_0 \neq 0)$

or $(m_1 + m_2)x = m_2L$ or $x = \frac{m_2L}{m_1 + m_2}$

Question35

An automobile moves on a road with a speed of $54kmh^{-1}$.The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is $3kgm^{2}$. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to the wheel is (2015)

Options:

A. $10.86kgm^2s^{-2}$

B. $2.86kgm^2s^{-2}$

C. $6.66kgm^2s^{-2}$

D. $8.58kgm^2s^{-2}$

Answer: C

Solution:

Solution:
Here,
Speed of the automobile,
$v = 54kmh^{-1} = 54 \times \frac{5}{18}ms^{-1} = 15ms^{-1}$
Radius of the wheel of the automobile, $R = 0.45m$
Moment of inertia of the wheel about its axis of rotation,
$I = 3kgm^2$
Time in w hich the veh icle brought to rest, t = 15 s
The initial angular speed of the wheel is
$\omega_i = \frac{v}{R} = \frac{15ms^{-1}}{0.45m} = \frac{1500}{45}rad\,s^{-1} = \frac{100}{3}rad\,s^{-1}$

and its final angular speed is
$\omega_f = 0$ (as the vehicle comes at rest)
∴ The angular retardation of the wheel is
$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - \frac{100}{3}}{15s} = -\frac{100}{45}rad\,s^{-2}$
The magnitude of required torque is
$\tau = I\,|\alpha| = (3kgm^2)\left(\frac{100}{45}rad\,s^{-2}\right)$
$= \frac{20}{3}kgm^2s^{-2} = 6.66kgm^2s^{-2}$

Question36

A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$.The value of α for which angular momentum about origin is conserved is
(2015)

Options:

A. zero

B. 1

C. -1

D. 2

Answer: C

Solution:

Solution:

For the conservation of angular momentum about origin, the torque $\vec{\tau}$ acting on the particle will be zero.

By definition, $\vec{\tau} = \vec{r} \times \vec{F}$

Hence, $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$ and $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$

$$\therefore \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix}$$

$= \hat{i}(-36 + 36) - \hat{j}(12 + 12\alpha) + \hat{k}(6 + 6\alpha)$

$= \hat{j}(12 + 12\alpha) + \hat{k}(6 + 6\alpha)$

But $\vec{\tau} = 0$

$\therefore 12 + 12\alpha = 0$ or $\alpha = -1$

and $6 + 6\alpha = 0$ or $\alpha = -1$

Question37

A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is
(2015)

Options:

A. $\frac{W(d - x)}{x}$

B. $\frac{W(d - x)}{d}$

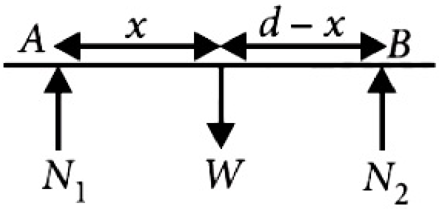
C. $\frac{Wx}{d}$

D. $\frac{W d}{x}$

Answer: B

Solution:

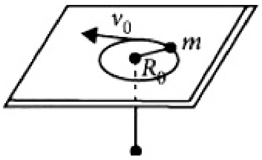
Solution:



Given situation is shown in figure
N_1 = Normal reaction on A
N_2 = Normal reaction on B
W = Weight of the rod
In vertical equilibrium,
$N_1 + N_2 = W$
Torque balance about centre of mass of the rod, $N_1 x = N_2 (d - x)$
Putting value of N_2 from equation (i)
$N_1 x = (W - N_1)(d - x)$
$\Rightarrow N_1 x = W d - W x - N_1 d + N_1 x$
$\Rightarrow N_1 d = W (d - x)$
$\therefore N_1 = \frac{W (d - x)}{d}$

Question38

A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hole in the plane as shown.
The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is



(2015)

Options:

A. $2mv_0^2$

B. $\frac{1}{2}mv_0^2$

C. mv_0^2

D. $\frac{1}{4}mv_0^2$

Answer: A

Solution:

Solution:
According to law of conservation of angular momentum
$mvr = mv'r'$

$v_0R_0 = v\left(\frac{R_0}{2}\right)$; $v = 2v_n$ ... (i)

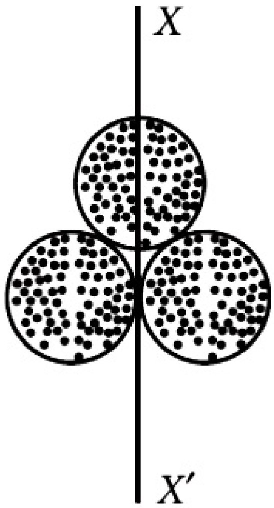
$\therefore \frac{K_0}{K} = \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}mv^2} = \left(\frac{v_0}{v}\right)^2$

or $\frac{K}{K_0} = \left(\frac{v}{v_0}\right)^2 = (2)^2$(Using (i))

$K = 4K_0 = 2mv_0^2$

Question39

Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is (2015)



Options:

A. $\frac{16}{5}mr^2$

B. $4mr^2$

C. $\frac{11}{5}mr^2$

D. $3mr^2$

Answer: B

Solution:

Net moment of inertia of the system,

$I = I_1 + I_2 + I_3$
The moment of inertia of a shell about its diameter,
$I_1 = \frac{2}{3}mr^2$
The moment of inertia of a shell about its tangent is given by
$I_2 = I_3 = I_1 + mr^2 = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2$
$\therefore I = 2 \times \frac{5}{3}mr^2 + \frac{2}{3}mr^2 = \frac{12mr^2}{3} = 4mr^2$

Question40

A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it an other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions s^{-2} is (2014)

Options:

A. 25 N

B. 50 N

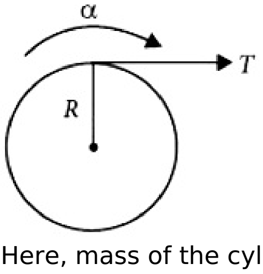
C. 78.5 N

D. 157 N

Answer: D

Solution:

Solution:



Here, mass of the cylinder, M = 50 kg
Radius of the cylinder, R = 0.5 m
Angular acceleration,
$\alpha = 2\text{rev } s^{-2} = 2 \times 2\pi \text{rad } s^{-2} = 4\pi \text{rad } s^{-2}$

Torque, $\tau = TR$
Moment of inertia of the solid cylinder about its axis,
$I = \frac{1}{2}MR^2$
∴ Angular acceleration of the cylinder
$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$
$T = \frac{MR\alpha}{2} = \frac{50 \times 0.5 \times 4\pi}{2} = 157N$

Question41

The ratio of the accelerations for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is
(2014)

Options:

A. 5 : 7

B. 2 : 3

C. 2 : 5

D. 7 : 5

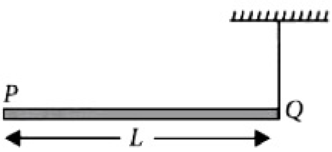
Answer: A

Solution:

Solution:
Acceleration of the solid sphere slipping down the incline without rolling is
$a_{slipping} = g\sin\theta$........(i)
Acceleration of the solid sphere rolling down the incline withoutslipping is
$a_{rolling} = \frac{g\sin\theta}{k^2} = \frac{g\sin\theta}{1 + \frac{2}{5}}$ (∵ For solid sphere, $\frac{k^2}{R^2} = \frac{2}{5}$)
$= \frac{5}{7}g\sin\theta$.......(ii)
Divide eqn.(ii) by eqn. (i), we get
$\frac{a_{rolling}}{a_{slipping}} = \frac{5}{7}$

Question42

A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is



(2013 NEET)

Options:

A. $\frac{2g}{L}$

B. $\frac{2g}{2L}$

C. $\frac{3g}{2L}$

D. $\frac{g}{L}$

Answer: C

Solution:

Solution:
Torque on the rod = moment of weight of the rod about P
$T = mg\frac{L}{2}$
Moment of inertia of rod about,
$P = \frac{ML^2}{3}$ ... (ii)
As^{T} = $|^{\alpha}$
From equations (i) and (ii), we get
$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$
$\therefore\ \alpha = \frac{3g}{2L}$

Question43

A small object of uniform density rolls up a curved surface with an initial velocity V . It reaches up to a maximum height of $3\frac{v^2}{4}$g with respect to the initial position. The object is (2013 NEET)

Options:

A. hollow sphere

B. disc

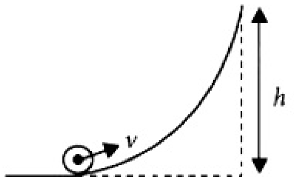
C. ring

D. solid sphere

Answer: B

Solution:

Solution:



The kinetic energy of the rolling object is converted into potential energy at height $h\left(=\frac{3v^2}{4g}\right)$
So by the law of conservation of mechanical energy, we have
$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$
$\frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = Mg\left(\frac{3v^2}{4g}\right)\left(\because \omega = \frac{v}{R}\right)$

$\frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}Mv^2 - \frac{1}{2}Mv^2$

$\frac{1}{2} I \frac{v^2}{R^2} = \frac{1}{4} M v^2$

or $I = \frac{1}{2} M R^2$

Hence, the object is disc.

Question44

Two discs are rotating about their axes, normal to the discs and passing through the centres of the discs. Disc D_1 has 2kg mass and 0.2m radius and initial angular velocity of 50 rad s^{-1}. Disc D_2 has 4kg mass, 0.1m radius and initial angular velocity of 200 rad s^{-1}. The two discs are brought in contact face to face, with their axes of rotation coincident. The final angular velocity (in rad s^{-1}) of the system is (KN NEET 2013)

Options:

A. 60

B. 100

C. 120

D. 40

Answer: B

Solution:

Solution:

(b) : Moment of inertia of disc D_1 about an axis passing through its centre and normal to its plane is

$I_1 = \frac{M R^2}{2} = \frac{(2kg)(0.2m)^2}{2} = 0.04 kgm^2$

Initial angular velocity of disc D_1, $\omega_1 = 50$ rad s^{-1}

Moment of inertia of disc D_2 about an axis passing through its centre and normal to its plane is

$I_2 = \frac{(4kg)(0.1m)^2}{2} = 0.02 kgm^2$

Initial angular velocity of disc D_2, $\omega_2 = 200 rad\, s^{-1}$

Total initial angular momentum of the two discs is

$L_i = I_1\omega_1 + I_2\omega_2$

When two discs are brought in contact face to face (one on the top of the other) and their axes of rotation coincide, the moment of inertia I of the system is equal to the sum of their individual moment of inertia.

$I = I_1 + I_2$

Let ω be the final angular speed of the system. The final angular momentum of the system is

$L_f = I\omega = (I_1 + I_2)\omega$

According to law of conservation of angular momentum, we get

$L_i = L_f$

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

Question45

When a mass is rotating in a plane about a fixed point, its angular momentum is directed along (2012)

Options:

A. a line perpendicular to the plane of rotation

B. the line making an angle of 45° to the plane of rotation

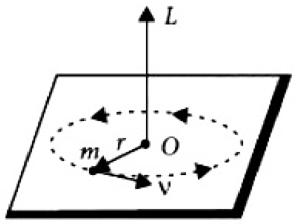
C. the radius

D. the tangent to the orbit

Answer: A

Solution:

Solution:
When a mass is rotating in a plane about a fixed point its angular momentum is directed along a line perpendicular to the plane of rotation.



Question46

Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3.0 m and weighs 100 kg. The 55 kg man walks up to the 65 kg man and sits with him. If the boat is in still water the center of mass of the system shifts by (2012)

Options:

A. 3.0 m

B. 2.3 m

C. zero

D. 0.75 m

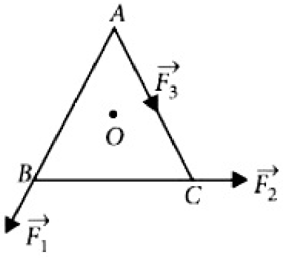
Answer: C

Solution:

As no external force acts on the system, therefore centre of mass will not shift.

Question47

ABC is an equilateral triangle with O as its centre.$\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ represent three forces acting along the sides AB, BC and AC respectively. If the total torque about O is zero then the magnitude of $\vec{F}_3$ is



(2012)

Options:

A. $F_1 + F_2$

B. $F_1 - F_2$

C. $\frac{F_1 + F_2}{2}$

D. $2(F_1 + F_2)$

Answer: A

Solution:

Solution:
Let x be the distance of centre O of equilateral triangle from each side.Total torque about O = 0
$\Rightarrow F_1 x + F_2 x - F_3 = 0$ or $F_3 = F_1 + F_3$

Question48

A circular platform is mounted on a frictionless vertical axle. Its radius R = 2 m and its moment of inertia about the axle is $200 kgm^2$.It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of $1 ms^{-1}$ relative to the ground.Time taken by the man to complete one revolution is
(2012 Mains)

Options:

A. πs

B. $\frac{3\pi}{2}$s

C. 2πs

D. $\frac{\pi}{2}$s

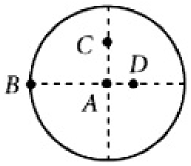
Answer: C

Solution:

Solution:
As the system is initially at rest, therefore, initial angular momentum $L_i = 0$.
According to the principle of conservation of angular momentum final angular momentum, $L_f = 0$
∴ Angular momentum = Angular momentum of man of platform in opposite direction
i.e., $mvR = I\omega$
or $\omega = \frac{mvR}{I} = \frac{50 \times 1 \times 2}{200} = \frac{1}{2} \text{rad s}^{-1}$
Angular velocity of man relative to platform is
$\omega_r = \omega + \frac{v}{R} = \frac{1}{2} + \frac{1}{2} = 1 \text{rad s}^{-1}$
Time taken by the man to complete one revolution is
$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{1} = 2\pi s$

Question49

The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through



(2012 Mains)

Options:

A. B

B. C

C. D

D. A

Answer: A

Solution:

Solution:
According to the theorem of parallel axes,
$I = I_{CM} + Ma^2$
As a is maximum for point B.
Therefore I is maximum about B.

Question50

Three masses are placed on the x-axis : 300 g at origin, 500 g at x = 40 cm and 400 g at x = 70 cm. The distance of the centre of mass from the origin is
(2012 Mains)

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Options:

A. 40 cm

B. 45 cm

C. 50 cm

D. 30 cm

Answer: A

Solution:

Solution:
The distance of the centre of mass of the system of three masses from the origin O is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_2x_3}{m_1 + m_2 + m_3} = \frac{300 \times 0 + 500 \times 40 + 400 \times 70}{300 + 500 + 400}$$

$$= \frac{500 \times 40 + 400 \times 70}{1200} = \frac{400[50 + 70]}{1200}$$

$$= \frac{50 + 70}{3} = \frac{120}{3} = 40\text{cm}$$

Question51

The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$.The torque on the wheel becomes zero at
(2011)

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Options:

A. t = 1s

B. t = 0.5s

C. t = 0.25s

D. t = 2s

Answer: A

Solution:

Given: $\theta(t) = 2t^3 - 6t^2$
$\therefore \frac{d\theta}{dt} = 6t^2 - 12t$
$\frac{d^2\theta}{dt^2} = 12t - 12$
Angular acceleration, $\alpha = \frac{d^2\theta}{dt^2} = 12t - 12$
When angular acceleration (α) is zero, then the torque on the wheel becomes zero ($\because \tau = I\alpha$)
$\Rightarrow 12t - 12 = 0$ or $t = 1s$

Question52

The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0 Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is (2011)

©

Options:

A. $I_0 + \frac{ML^2}{2}$

B. $I_0 + \frac{ML^2}{4}$

C. $I_0 + 2ML^2$

D. $I_0 + ML^2$

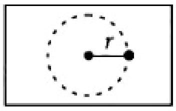
Answer: B

Solution:

Solution:
According to the theorem of parallel axes, the moment of inertia of the thin rod of mass M and length L about an axis passing through one of the ends is
$I = I_{CM} + Md^2$
Where I_{CM} is the moment of inertia of the given rod about an axis passing through its centre of mass and perpendicular to its length and d is the distance between two parallel axes.
Here. $I_{CM} = I_0$, $d = \frac{L}{2}$
$\therefore I = I_0 + M\left(\frac{L}{2}\right)^2 = I_0 + \frac{ML^2}{4}$

Question53

A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will



(2011 Mains)

Options:

A. decrease by a factor of 2

B. remain constant

C. increase by a factor of 2

D. increase by a factor of 4

Answer: D

Solution:

According to law of conservation of angular momentum
$mvr = mv'r'$
$vr = v'\left(\frac{r}{2}\right)$
$v' = 2v$..........(i)
$\therefore \frac{K}{K'} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv'^2}\left(\frac{v}{v'}\right)^2$
or $\frac{K'}{K} = \left(\frac{v'}{v}\right)^2 = (2)^2$ (Using (i))
$K' = 4K$

Question54

A circular disk of moment of inertia I_t is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed ω_i.Another disk of moment of inertia I_b is dropped co-axially I_b is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed ω_f.The lost by the initially rotating disc to friction is
(2010)

Options:

A. $\frac{1}{2}\frac{I_b^2}{(I_t + I_b)}\omega_i^2$

B. $\frac{1}{2}\frac{I_t^2}{(I_t + I_b)}\omega_i^2$

C. $\frac{I_b - I_t}{(I_t + I_b)}\omega_i^2$

D. $\frac{1}{2}\frac{I_b I_t}{(I_t + I_b)}\omega_i^2$

Answer: D

Question55

Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and 2v at any instant, then the speed of centre of mass of the system will be (2010)

©

Options:

A. 2v

B. zero

C. 1.5 v

D. v

Answer: B

Solution:

Solution:
As no external force is acting on the system, the centre of mass must be at rest i.e. $v_{CM} = 0$

Question56

A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if (2010)

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Options:

A. $r = mg\omega^2$

B. $r < \frac{\omega^2}{\mu g}$

C. $r \le \frac{\mu g}{\omega^2}$

D. $r \ge \frac{\mu g}{\omega^2}$

Answer: C

Solution:

Solution:
(c) : The coin will revolve with the record,
if Force of friction ≥ centripetal force
$\mu mg \ge mr\omega^2$
or $r \le \frac{\mu g}{\omega^2}$

Question57

From a circular disc of radius R and mass 9M, a small disc of mass M and radius $\frac{R}{3}$ is removed concentrically. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is (2010 Mains)

Options:

A. $\frac{40}{9}MR^2$

B. MR^2

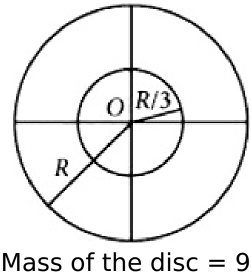
C. $4MR^2$

D. $\frac{4}{9}MR^2$

Answer: A

Solution:

Solution:



Mass of the disc = 9M
Mass of removed portion of disc = M
The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is
$I_1 = \frac{9}{2}MR^2$
Now, the moment of inertia of the disc with removed portion
$I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{1}{18}MR^2$

Therefore, moment of inertia of the remaining portion of disc about O is
$I = I_1 - I_2$
$= \frac{9MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$

Question58

A solid cylinder and a hollow cylinder, both of the same mass and same external diameter are released from the same height at the same time on an inclined plane. Both roll down without slipping. Which one will reach the bottom first?
(2010 Mains)

Options:

A. Both together only when angle of inclination of plane is 45°

B. Both together

C. Hollow cylinder

D. Solid cylinder

Answer: D

Solution:

Solution:
Time taken to reach the bottom of inclined plane is
$$\sqrt{\frac{2l\left(1 + \frac{K^2}{R^2}\right)}{g\sin\theta}}$$
Here, l is length of incline plane.
For solid cylinder $K^2 = \frac{R^2}{2}$
For hollow cylinder $K^2 = R^2$
Hence, solid cylinder will reach the bottom first

Question59

A thin circular ring of mass M and radius r is rotating about its axis with constant angular velocity ω. Two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity given by
(2010 Mains)

Options:

A. $\frac{(M + 2m)\omega}{2m}$

B. $\frac{2M\omega}{M + 2m}$

C. $\frac{(M + 2m)\omega}{M}$

D. $\frac{M\omega}{M + 2m}$

Answer: D

Solution:

Solution:
As no external torque is acting about the axis, angular momentum of system remains conserved.
$I_1\omega_1 = I_2\omega_2$
$\Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mr^2\omega}{(M + 2m)r^3} = \frac{M\omega}{(M + 2m)}$

Question60

A thin circular ring of mass M and radius R is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity ω. If two objects each of mass m be attached gently to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity (2009)

Options:

A. $\frac{\omega M}{M + 2m}$

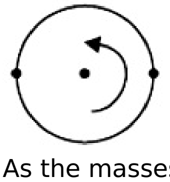
B. $\frac{\omega(M + 2m)}{M}$

C. $\frac{\omega M}{M + m}$

D. $\frac{\omega(M - 2m)}{M + 2m}$

Answer: A

Solution:



As the masses are added to the ring gently, there is no external torque and angular momentum is conserved. $I\omega = I'\omega'$
$\Rightarrow MR^2\omega = (MR^2 + 2mR^2)\omega'$
$\Rightarrow \omega' = \frac{MR^2\omega}{(M + 2m)R^2} \Rightarrow \omega' = \frac{M\omega}{M + 2m}$

Question61

If $\vec{F}$ is the force on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin, then (2009)

Options:

A. $\vec{r} \cdot \vec{\tau} > 0$ and $\vec{F} \cdot \vec{\tau} < 0$

B. $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} = 0$

C. $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$

D. $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} = 0$

Answer: B

Solution:

Solution:

Torque is always perpendicular to $\vec{F}$ as well as $\vec{r}$.

$\therefore \vec{r} \cdot \vec{\tau} = 0$ as well as $\vec{F} \cdot \vec{\tau} = 0$.

Question62

Four identical thin rods each of mass M and length l, form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is (2009)

Options:

A. $\frac{2}{3} M l^2$

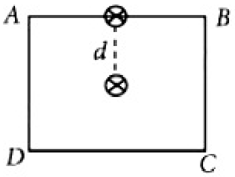
B. $\frac{13}{3} M l^2$

C. $\frac{1}{3} M l^2$

D. $/3 M l^2$

Answer: D

Solution:



Moment of inertia for the rod AB rotating about an axis through the midpoint of AB perpendicular to the plane of the paper is $\frac{Ml^2}{12}$

∴ M.I. about the axis passing through the center of the square and parallel to this axis,

$I = I_0 + Md^2 = M\left(\frac{l^2}{12} + \frac{l^2}{4}\right) = \frac{Ml^2}{3}$

For all the four rods, $I = \frac{4}{3}Ml^2$

Question63

Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$, respectively. The centre of mass of this system has a position vector (2009)

Options:

A. $-2\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{i} - \hat{j} - 2\hat{k}$

C. $-\hat{i} + \hat{j} + \hat{k}$

D. $-2\hat{i} + 2\hat{k}$

Answer: A

Solution:

$\vec{r}_1$ for $M_1 = \hat{i} + 2\hat{j} + \hat{k}$ for $M_1 = 1kg$

$\vec{r}_2$ For $M_2 = -3\hat{i} - 2\hat{j} + \hat{k}$ for $M_2 = 3kg$

$\vec{r}_{CM} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$

$\Rightarrow \vec{r}_{CM} = \frac{(\hat{i} + 2\hat{j} + 1\hat{k}) \times 1 + (-3\hat{i} - 2\hat{j} + \hat{k}) \times 3}{4}$

$\Rightarrow \vec{r}_{CM} = \frac{(1\hat{i} + 2\hat{j} + 1\hat{k}) \times 1 + (-9\hat{i} - 6\hat{j} + 3\hat{k})}{4}$

$\Rightarrow \vec{r}_{CM} = \frac{-8\hat{i} - 4\hat{j} + 4\hat{k}}{4} = -2\hat{i} - \hat{j} + \hat{k}$

Question64

A thin rod of length L and mass M is bent at its midpoint into two halves so that the angle between them is 90°. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is (2008)

Options:

A. $\frac{ML^2}{6}$

B. $\frac{\sqrt{2}ML^2}{24}$

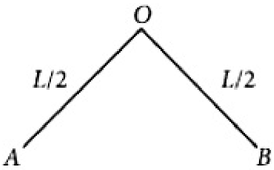
C. $\frac{ML^2}{24}$

D. $\frac{ML^2}{12}$

Answer: D

Solution:

Solution:



Total Mass=M, total length = L
Moment of inertia of OA = OB about O
$\Rightarrow$ Total M.L $= 2 \times \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2\frac{1}{3} = \frac{ML^2}{12}$

Question65

The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is (2008)

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Options:

A. $\sqrt{2} : 1$

B. $\sqrt{2} : \sqrt{3}$

C. $\sqrt{3} : \sqrt{2}$

D. $1 : \sqrt{2}$

Answer: D

Solution:

M.I. of a circular disc, $Mk^2 = \frac{MR^2}{2}$
M.I. of a circular ring $= MR^2$
∴ Ratio of their of gyration
$= \frac{1}{\sqrt{2}} : 1$ or $1 : \sqrt{2}$

Question66

A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then (2007)

Options:

A. $L_A = L_B$

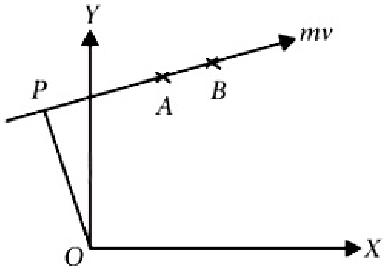
B. the relationship between L_A and L_B depends upon the slope of the line A_B

C. $L_A < L_B$

D. $L_A > L_B$

Answer: A

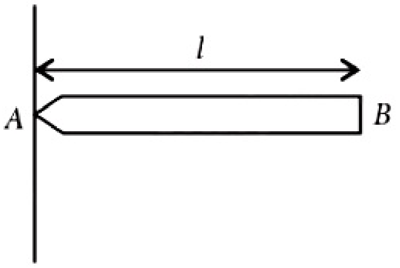
Solution:



Moment of momentum is angular momentum OP is the same whether the mass is at A or B.
$\therefore L_A = L_B$

Question67

A uniform rod AB of length l and mass m is free to rotate about point A. The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about A is $ml^2/3$, the initial angular acceleration of the rod will be



(2007, 2006)

Options:

A. $\frac{mgl}{2}$

B. $\frac{3}{2}gl$

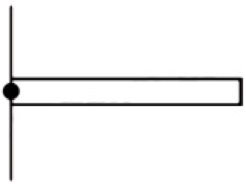
C. $\frac{3g}{2l}$

D. $\frac{2g}{3l}$

Answer: C

Solution:

Solution:



Torque about A ,

$\tau = mg \times \frac{l}{2} = \frac{mgl}{2}$

Also $\tau = I\,\alpha$

∴ Angular acceleration,

$\alpha = \frac{\tau}{I} = \frac{mgl\ /\ 2}{ml^2\ /\ 3} = \frac{3}{2}\frac{g}{l}$

Question68

A wheel has angular acceleration of 3.0 rad / sec^2 and an initial angular speed of 2.00 rad / sec. In a time of 2 sec it has rotated through an angle (in radian) of
(2007)

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Options:

A. 10

B. 12

C. 4

D. 6

Answer: A

Solution:

Solution:
Given: Angular acceleration, $\alpha = 3\,\mathrm{rad}/\mathrm{s}^2$
Initial angular velocity $\omega_i = 2\,\mathrm{rad}/\mathrm{s}$
Time $t = 2\mathrm{s}$
Using, $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$\therefore \theta = 2 \times 2 + \frac{1}{2} \times 3 \times 4 = 4 + 6 = 10\,\mathrm{radian}$

Question69

The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at its diameter and normal to the disc (2006)

Options:

A. $\frac{1}{2}MR^2$

B. MR^2

C. $\frac{2}{5}MR^2$

D. $\frac{3}{2}MR^2$

Answer: D

Solution:

Solution:
Moment of inertia of a uniform circular disc about an axis through its centre and perpendicular to its plane is $I_C = \frac{1}{2}MR^2$
$\therefore$ Moment of inertia of a uniform circular disc about an axis touching the disc at its diameter and normal to the disc is I.
By the theorem of parallel axes,
$I = I_C + Mh^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Question70

A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(2006)

Options:

A. $\frac{ML^2\omega^2}{2}$

B. $\frac{ML\omega^2}{2}$

C. $\frac{ML^2\omega}{2}$

D. $ML\omega^2$

Answer: B

Solution:

Solution:

The centre of the tube will be at length $\frac{L}{2}$.

So radius $r = \frac{L}{2}$

The force exerted by the liquid at the other end = centrifugal force

Centrifugal force $= Mr\omega^2 = M\left(\frac{L}{2}\right)\omega^2 = \frac{ML\omega^2}{2}$

Question71

The moment of inertia of a uniform circular disc of radius R and mass M about an axis passing from the edge of the disc and normal to the disc is (2005)

Options:

A. MR^2

B. $\frac{1}{2}MR^2$

C. $\frac{3}{2}MR^2$

D. $\frac{7}{2}MR^2$

Answer: C

Solution:

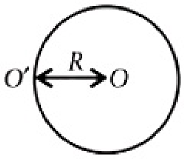
Solution:

M.I. of disc about its normal $= \frac{1}{2}MR^2$

M.I. about its one edge $= MR^2 + \frac{MR^2}{2}$

(Perpendicular to the plane)

Moment of inertia $= \frac{3}{2}MR^2$



Question72

A drum of radius R and mass M , rolls down without slipping along an inclined plane of angle θ. The frictional force (2005)

Options:

A. dissipates energy as heat

B. decreases the rotational motion

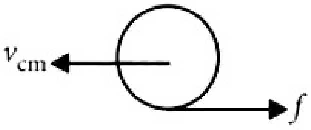
C. decreases the rotational and translational motion

D. converts translational energy to rotational energy.

Answer: D

Solution:

Solution:



Required frictional force convert some part of translational energy into rotational energy.

Question73

Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular velocity will be in the ratio (2005)

Options:

A. 2 : 1

B. 1 : 2

C. $\sqrt{2}$: 1

D. 1 : $\sqrt{2}$

Answer: C

Solution:

Solution:

$K.E. = \frac{1}{2}I\omega^2$

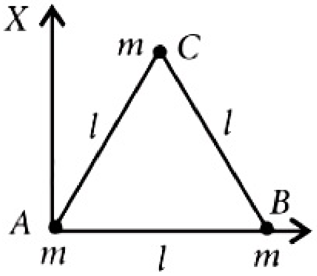
$\therefore \frac{1}{2}I_1\omega_1^{\ 2} = \frac{1}{2} \times 2I_1\omega_2^{\ 2}$

$\frac{\omega_1^{\ 2}}{\omega_2^{\ 2}} = \frac{2}{1}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{\sqrt{2}}{1}$

Question74

Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side l cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram $-cm^2$ units will be



(2004)

Options:

A. $\frac{3}{4}ml^2$

B. $2ml^2$

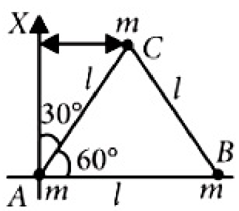
C. $\frac{5}{4}ml^2$

D. $\frac{3}{2}ml^2$

Answer: C

Solution:

Solution:



The moment of inertia of the system

$= m_A r_A{}^2 + m_B r_B{}^2 + m_C r_C{}^2$
$= m_A(0)^2 + m(l)^2 + m(l \sin 30°)^2$
$= m_1{}^2 + ml^2 \times \left(\frac{1}{4}\right) = \left(\frac{5}{4}\right) ml^2$

Question75

Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass of the particles through a distance d , by what distance would be particle of mass m_2 move so as to keep the centre of mass of the particles at the original position?
(2004)

Options:

A. $\frac{m_1}{m_1 + m_2}d$

B. $\frac{m_1}{m_2}d$

C. d

D. $\frac{m_2}{m_1}d$

Answer: B

Solution:

Solution:

C.M. $= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$...(i)

After changing position of m_1 and to keep the position of C.M. same

C.M. $= \frac{m_1(x_1 - d) + m_2(x_2 + d_2)}{m_1 + m_2}$

$0 = \frac{m_1 d + m_2 d_2}{m_1 + m_2}$ [Substituting value of C.M. from (i)]

$\Rightarrow d_2 = \frac{m_1}{m_2}d$

Question76

A wheel having moment of inertia $2\,kg\,m^2$ about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be
(2004)

Options:

A. $\frac{2\pi}{15}$ Nm

B. $\frac{\pi}{12}$ Nm

C. $\frac{\pi}{15}$ Nm

D. $\frac{\pi}{18}$ Nm

Answer: C

Solution:

Solution:

$\omega_f = \omega_i - \alpha t \Rightarrow 0 = \omega_i - \omega_t$

$\therefore \alpha = \frac{\omega_i}{t}$, where α is retardation.

The torque on the wheel is given by

$\tau = I\,\alpha = \frac{I\,\omega}{t} = \frac{I\ .\ 2\pi v}{t} = \frac{2 \times 2 \times \pi \times 60}{60 \times 60}$

$\tau = \frac{\pi}{15}$ Nm

This is the torque required to stop the wheel in 1 min. (or 60s)

Question77

A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is (2004)

Options:

A. $\frac{I_2\omega}{I_1 + I_2}$

B. ω

C. $\frac{I_1\omega}{I_1 + I_2}$

D. $\frac{I_1 + I_2)\omega}{I_1}$

Answer: C

Solution:

Applying conservation of angular momentum.

$I_1\omega = (I_1 + I_2)\omega_1$ or $\omega_1 = \frac{I_1}{(I_1 + I_2)}\omega$

Question78

The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is (2004)

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Options:

A. 2 : 3

B. 2 : 1

C. $\sqrt{5} : \sqrt{6}$

D. $1 : \sqrt{2}$

Answer: C

Solution:

Solution:

Radius of gyration of disc about a tangential axis in the plane of disc is $\frac{\sqrt{5}}{2}R = K_1$ radius of gyration of circular ring of same radius about a tangential axis in the plane of circular ring is

$K_2 = \sqrt{\frac{3}{2}}R$

$\therefore \frac{K_1}{K_2} = \frac{\sqrt{5}}{\sqrt{6}}$

Question79

A stone is tied to a string of length l and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is (2003)

Options:

A. $\sqrt{2(u^2 - gl)}$

B. $\sqrt{u^2 - gl}$

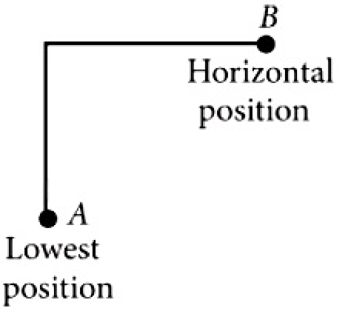
C. $u - \sqrt{u^2 - 2gl}$

D. $\sqrt{2gl}$

Answer: A

Solution:

Solution:



The total energy at A = the total energy at B

$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl$

$\Rightarrow v = \sqrt{u^2 - 2gl}$

The change in magnitude of velocity $= \sqrt{u^2 + v^2} = \sqrt{2(u^2 - gl)}$

Question80

A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K . If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be
(2003)

Options:

A. $\frac{K^2 + R^2}{R^2}$

B. $\frac{K^2}{R^2}$

C. $\frac{K^2}{K^2 + R^2}$

D. $\frac{R^2}{K^2 + R^2}$

Answer: C

Solution:

Total energy $= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

Required fraction $= \frac{K^2/R^2}{1+K^2/R^2} = \frac{K^2}{R^2+K^2}$

Question81

A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom?
(2003, 1989)

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Options:

A. $\sqrt{2gh}$

B. $\sqrt{\frac{3}{4}gh}$

C. $\sqrt{\frac{4}{3}gh}$

D. $\sqrt{4gh}$

Answer: C

Solution:

Potential energy of the solid cylinder at height $h = Mgh$
K.E. of centre of mass when it reaches the bottom

$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\frac{v^2}{R^2}$

$= \frac{1}{2}Mv^2\left(1 + \frac{k^2}{R^2}\right)$

For a solid cylinder $\frac{k^2}{R^2} = \frac{1}{2}$

$\therefore$ K.E. $= \frac{3}{4}Mv^2$

$\therefore Mgh = \frac{3}{4}Mv^2,\ v = \sqrt{\frac{4}{3}gh}$

Question82

A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be
(2003)

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Options:

A. $\frac{M\omega}{4m}$

B. $\frac{M\omega}{M+4m}$

C. $\frac{(M+4m)\omega}{M}$

D. $\frac{(M-4m)\omega}{M+4m}$

Answer: B

Solution:

Solution:
According to conservation of angular momentum, $L = I\omega =$ constant.
Therefore, $I_2\omega_2 = I_1\omega_1$
or $\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mk^2\omega}{(M+4m)k^2}$
$= \frac{M\omega}{M+4m}$

Question83

A rod of length 3m and its mass per unit length is directly proportional to distance x from one of its end then its centre of gravity from that end will be at
(2002)

Options:

A. 1.5m

B. 2m

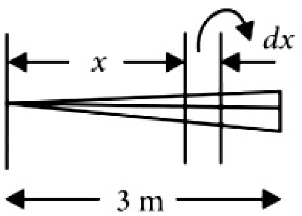
C. 2.5m

D. 3.0m

Answer: B

Solution:

Solution:



Let us consider an elementary length dx at a distance x from one end.
It's mass $= k.x.dx$
[$k =$ proportionality constant]
Then centre of gravity of the rod x_c is given by

$$x_c = \frac{\int_0^3 kx\,dx \cdot x}{\int_0^3 kx\,dx} = \frac{\int_0^3 x^2\,dx}{\int_0^3 x\,dx} = \frac{\left.\frac{x^3}{3}\right|_0^3}{\left.\frac{x^3}{3}\right|_0^3}$$

or $x_c = \frac{27/3}{9/2} = 2$

∴ Centre of gravity of the rod will be at distance of 2m from one end.

Question84

Consider a contact point P of a wheel on ground which rolls on ground without slipping. Then value of displacement of point P when wheel completes half of rotation (If radius of wheel is 1m) (2002)

Options:

A. 2m

B. $\sqrt{\pi^2 + 4}\text{m}$

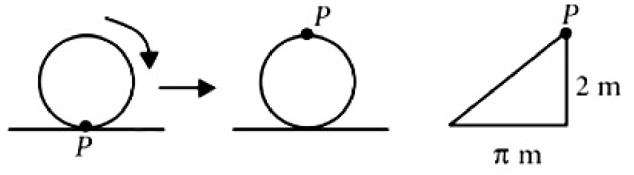
C. πm

D. $\sqrt{\pi^2 + 2}\text{m}$

Answer: B

Solution:

Solution:



In half rotation point P has moved horizontally.

$\frac{\pi d}{2} = \pi r = \pi \times 1\text{m} = \pi\text{m}$ [∵ radius = 1 m]

In the same time, it has moved vertically a distance which is equal to its diameter $= 2\text{m}$.

∴ Displacement of $\text{P} = \sqrt{\pi^2 + 2^2} = \sqrt{\pi^2 + 4}\text{m}$

Question85

A solid sphere of radius R is placed on a smooth horizontal surface. A horizontal force F is applied at height h from the lowest point. For the maximum acceleration of centre of mass, which is correct? (2002)

©

Options:

A. $h = R$

B. h = 2R

C. h = 0

D. no relation between h and R

Answer: D

Solution:

Solution:
Since there is no friction at the contact surface (smooth horizontal surface) there will be no rolling. Hence, the acceleration of the centre of mass of the sphere will be independent of the position of the applied force F. Therefore, there is no relation between h and R.

Question86

A disc is rotating with angular speed ω. If a child sits on it, what is conserved?
(2002)

Options:

A. linear momentum

B. angular momentum

C. kinetic energy

D. potential energy.

Answer: B

Solution:

Solution:
When a child sits on a rotating disc, no external torque is introduced. Hence the angular momentum of the system is conserved. But the moment of inertia of the system will increase and as a result, the angular speed of the disc will decrease to maintain constant angular momentum.
[∵ angular momentum = moment of inertia × angular velocity]

Question87

A circular disc is to be made by using iron and aluminium so that it acquires maximum moment of inertia about geometrical axis. It is possible with
(2002)

Options:

A. aluminium at interior and iron surrounding it

B. iron at interior and aluminium surrounding it

C. using iron and aluminium layers in alternate order

D. sheet of iron is used at both external surface and aluminium sheet as internal layers.

Answer: A

Solution:

Solution:
A circular disc may be divided into a large number of circular rings. Moment of inertia of the disc will be the summation of the moments of inertia of these rings about the geometrical axis. Now, moment of inertia of a circular ring about its geometrical axis is MR^2, where M is the mass and R is the radius of the ring.

since the density (mass per unit volume) for iron is more than that of aluminium, the proposed rings made of iron should be placed at a higher radius to get more value of MR^2. Hence to get maximum moment of inertia for the circular disc, aluminium should be placed at interior and iron at the exterior position.

Question88

A disc is rolling, the velocity of its centre of mass is v_{cm}. Which one will be correct?
(2001)

Options:

A. the velocity of highest point is $2v_{cm}$ and at point of contact is zero

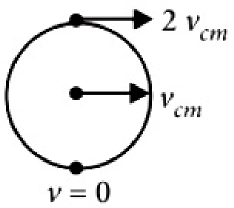
B. the velocity of highest point is v_{cm} and at point of contact is v_{cm}

C. the velocity of highest point is $2v_{cm}$ and point of contact is v_{cm}

D. the velocity of highest point is $2v_{cm}$ and point of contact is $2v_{cm}$

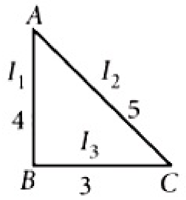
Answer: A

Solution:



Question89

For the adjoining diagram, the correct relation between I_1, I_2, and I_3 is, (I − moment of inertia)



(2000)

Options:

A. $I_1 > I_2$

B. $I_2 > I_1$

C. $I_3 > I_1$

D. $I_3 > I_2$

Answer: B

Solution:

Solution:
As effective distance of mass from BC is greater than the effective distance of mass from AB, therefore $I_2 > I_1$

Question90

For a hollow cylinder and a solid cylinder rolling without slipping on an inclined plane, then which of these reaches earlier?
(2000)

Options:

A. solid cylinder

B. hollow cylinder

C. both simultaneously

D. can't say anything.

Answer: A

Solution:

Solution:
Solid sphere reaches the bottom first because for solid cylinder $\frac{K^2}{R^2} = \frac{1}{2}$,

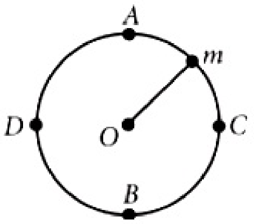
and for hollow cylinder $\frac{K^2}{R^2} = 1$

Acceleration down the inclined plane $\propto \frac{1}{K^2/R^2}$

Solid cylinder has greater acceleration, so it reaches the bottom first.

Question91

As shown in the figure at point O, a mass is performing vertical circular motion. The average velocity of the particle is increased, then at which point will the string break?
(2000)



Options:

A. A

B. B

C. C

D. D

Answer: B

Solution:

Solution:

When a sphere is rotating in a vertical circle, it exerts the maximum outward pull when it is at the lowest point B.

Therefore, tension at B is maximum $= \text{Weight} + \frac{mv^2}{R}$

So, the string breaks at point B.

Question92

Three identical metal balls, each of radius r are placed touching each other on a horizontal surface such that an equilateral triangle is formed when centres of three balls are joined. The centre of the mass of the system is located at
(1999)

Options:

A. line joining centres of any two balls

B. centre of one of the balls

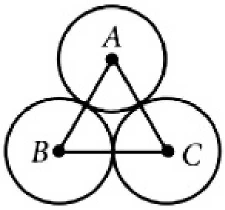
C. horizontal surface

D. point of intersection of the medians

Answer: D

Solution:

Solution:
Centre of mass of each ball lies on the centre.
⇒Centre of mass of combined body will be at the centroid of equilateral triangle.



Question93

The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter is
(1999)

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Options:

A. $\frac{5}{4} M R^2$

B. $\frac{2}{3} M R^2$

C. $\frac{3}{2} M R^2$

D. $\frac{4}{5} M R^2$

Answer: A

Solution:

Solution:
Moment of inertia of a disc about its diameter $= \frac{1}{4} M R^2$
Using theorem of parallel axes,
$I = \frac{1}{4} M R^2 + M R^2 = \frac{5}{4} M R^2$

Question94

Find the torque of a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point

$\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$
(1997)

Options:

A. $-21\hat{i} + 4\hat{j} + 4\hat{k}$

B. $-14\hat{i} + 34\hat{j} - 16\hat{k}$

C. $14\hat{i} - 38\hat{j} + 16\hat{k}$

D. $4\hat{i} + 4\hat{j} + 6\hat{k}$

Answer: C

Solution:

Solution:
$\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ and distance of the point $\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

Torque $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\hat{i} - 38\hat{j} + 16\hat{k}$

Question95

The centre of mass of system of particles does not depend on (1997)

Options:

A. position of the particles

B. relative distances between the particles

C. masses of the particles

D. forces acting on the particle.

Answer: D

Solution:

Solution:
The resultant of all forces, on any system of particles, is zero. Therefore their centre of mass does not depend upon the forces acting on the particles.

Question96

A couple produces
(1997)

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Options:

A. linear and rotational motion

B. no motion

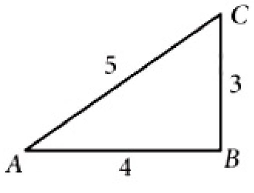
C. purely linear motion

D. purely rotational motion.

Answer: D

Question97

The ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC and CA respectively. Which one of the following relations is correct?
(1995)



Options:

A. $I_{AB} + I_{BC} = I_{CA}$

B. I_{CA} is maximum

C. $I_{AB} > I_{BC}$

D. $I_{BC} > I_{AB}$

Answer: D

Solution:

Solution:
The intersection of medians is the centre of mass of the triangle. since the distances of centre of mass from the sides is

related as $x_{BC} < x_{AB} < x_{AC}$
Therefore $I_{BC} > I_{AB} > I_{AC}$ or $I_{BC} > I_{AB}$

Question98

What is the torque of the force $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$N acting at the point $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$m about origin? (1995)

©

Options:

A. $-6\hat{i} + 6\hat{j} - 12\hat{k}$

B. $-17\hat{i} + 6\hat{j} + 13\hat{k}$

C. $6\hat{i} - 6\hat{j} + 12\hat{k}$

D. $17\hat{i} - 6\hat{j} - 13\hat{k}$

Answer: D

Solution:

Solution:
$\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ N and distance of the point from origin $(r) = 3\hat{i} + 2\hat{j} + 3\hat{k}$ m
Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$$= (3\hat{i} + 2\hat{j} + 3\hat{k}) \times (2\hat{i} - 3\hat{j} + 4\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= 17\hat{i} - 6\hat{j} - 13\hat{k}$$

Question99

A solid spherical ball rolls on a table. Ratio of its rotational kinetic energy to total kinetic energy is (1994)

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Options:

A. $\frac{1}{2}$

B. $\frac{1}{6}$

C. $\frac{7}{10}$

D. $\frac{2}{7}$

Answer: D

Solution:

Solution:

Linear K.E. of ball $= \frac{1}{2}mv^2$ and rotational K.E. of ball

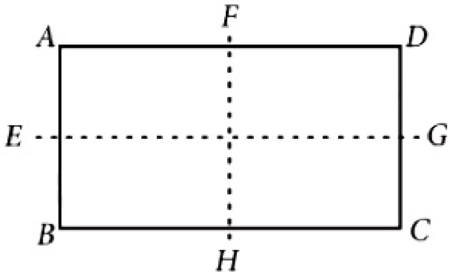
$= \frac{1}{2}I\,\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = \frac{1}{5}mv^2$

Therefore total K.E. $= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$

And ratio of rotational K.E. and total K.E. $= \dfrac{\left(\frac{1}{5}\right)mv^2}{\left(\frac{7}{10}\right)mv^2} = \frac{2}{7}$

Question100

In a rectangle ABCD(BC = 2AB). The moment of inertia is minimum along axis through



(1993)

Options:

A. BC

B. BD

C. H F

D. E G

Answer: D

Solution:

Solution:

The moment of inertia is minimum about E G because mass distribution is at minimum distance from E G.

Question101

A solid sphere, disc and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on the inclined

plane, then
(1993)

Options:

A. solid sphere reaches the bottom first

B. solid sphere reaches the bottom last

C. disc will reach the bottom first

D. all reach the bottom at the same time

Answer: A

Solution:

Solution:

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

For disc and solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

As for solid sphere is smallest, it takes minimum time to reach the bottom of the incline, disc and cylinder reach together later.

Question102

The speed of a homogenous solid sphere after rolling down an inclined plane of vertical height h from rest without sliding is (1992)

Options:

A. $\sqrt{\frac{10}{7}gh}$

B. $\sqrt{gh}$

C. $\sqrt{\frac{6}{5}gh}$

D. $\sqrt{\frac{4}{3}gh}$

Answer: A

Solution:

Solution:

P.E. = total K.E.

$mgh = \frac{7}{10}mv^2, v = \sqrt{\frac{10gh}{7}}$

Question103

If a sphere is rolling, the ratio of the translational energy to total kinetic energy is given by (1991)

Options:

A. 7: 10

B. 2: 5

C. 10: 7

D. 5: 7

Answer: D

Solution:

(d) : Total kinetic energy $= E_{trans} + E_{rot}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \times \left(\frac{2}{5}mr^2\right)\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$\therefore \frac{E_{trans}}{E_{total}} = \frac{\frac{1}{2}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7}$$

Question104

A particle of mass m = 5 is moving with a uniform speed $v = 3\sqrt{2}$ in the XOY plane along the line $Y = X + 4$. The magnitude of the angular momentum of the particle about the origin is (1991)

Options:

A. 60 units

B. $40\sqrt{2}$ units

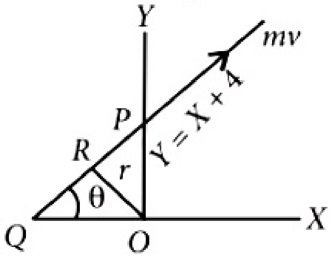
C. zero

D. 7.5 units

Answer: A

Solution:

$\vec{L} = \vec{r} \times \vec{p}$
$Y = X + 4$ line has been shown in the figure.



When $X = 0$,
$Y = 4$, So $OP = 4$.
The slope of the line can be obtained by comparing with the equation of line
$y = mx + c$
$m = \tan\theta = 1 \Rightarrow \theta = 45°$
$\angle OQP = \angle OPQ = 45°$
If we draw a line perpendicular to this line.
Length of the perpendicular $= OR$
$\Rightarrow OR = OP \sin 45° = 4\frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$
Angular momentum of particle going along this line $= r \times mv = 2\sqrt{2} \times 5 \times 3\sqrt{2} = 60\,\text{units}$

Question105

A fly wheel rotating about fixed axis has a kinetic energy of 360 joule when its angular speed is 30 radian/sec. The moment of inertia of the wheel about the axis of rotation is (1990)

Options:

A. $0.6\,\text{kg m}^2$

B. $0.15\,\text{kg m}^2$

C. $0.8\,\text{kg m}^2$

D. $0.75\,\text{kg m}^2$

Answer: C

Solution:

K.E. $= \frac{1}{2} I \omega^2$
$I = \frac{2K.E.}{\omega^2} = \frac{2 \times 360}{30 \times 30} = 0.8\,\text{kg m}^2$

Question106

The moment of inertia of a body about a given axis is $1.2\,\mathrm{kg\,m^2}$. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of 25 radian/sec ^{2} must be applied about that axis for a duration of
(1990)

Options:

A. 4s

B. 2s

C. 8s

D. 10s

Answer: B

Solution:

Solution:
$I = 1.2\,\mathrm{kg\,m^2}$, $E_r = 1500\mathrm{J}$
$\alpha = 25\,\mathrm{rad/s^2}$, $\omega_1 = 0$, $t = ?$
As $E_r = \frac{1}{2} I \omega^2$, $\omega = \sqrt{\frac{2E_r}{I}}$
$\omega = \sqrt{\frac{2 \times 1500}{1.2}} = 50\,\mathrm{rad/s}$
From $\omega_2 = \omega_1 + \alpha t$
$50 = 0 + 25t$, or $t = 2\mathrm{s}$

Question107

Moment of inertia of a uniform circular disc about a diameter is I . Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be
(1990)

Options:

A. 5I

B. 3I

C. 6I

D. 4I

Answer: C

Solution:

Solution:
Moment of inertia of uniform circular disc about diameter $= I$
According to theorem of perpendicular axes.
Moment of inertia of disc about axis $= 2I = \frac{1}{2}mr^2$
Applying theorem of parallel axes
Moment of inertia of disc about the given axis = $2I + mr^2 = 2I + 4I = 6I$

Question108

A solid homogenous sphere of mass M and radius is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere (1988)

Options:

A. total kinetic energy is conserved

B. the angular momentum of the sphere about the point of contact with the plane is conserved

C. only the rotational kinetic energy about the centre of mass is conserved

D. angular momentum about the centre of mass is conserved

Answer: B

Solution:

Solution:
Angular momentum about the point of contact with the surface includes the angular momentum about the centre.
Because of friction, linear momentum will not beconserved.

Question109

A ring of mass m and radius r rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is (1988)

Options:

A. $\frac{1}{2}mr^2\omega^2$

B. $mr\omega^2$

C. $mr^2\omega^2$

D. $\frac{1}{2}mr\omega^2$

Answer: A

Solution:

Kinetic energy $= \frac{1}{2}I\,\omega^2$, and for ring $I = mr^2$

Hence $K\,E = \frac{1}{2}mr^2\omega^2$
