18

Electrostatics

In this branch of Physics, we deals with static free charge, its properties and phenomenon exhibited by it. Charge which is static (not flowing in a conductor) and free (unbound unlike an electron of inner shell or proton in nucleus), produces only an electric field.

IN THIS CHAPTER ....

Electric Charge

Charge is the property associated with subatomic matter particles by virtue of which they attract dissimilar particles or repels similar particles.

Charges are of two types namely

(i) Positive charge (ii) Negative charge

It is measured in coulomb (C).

Note Other units of charge are
1 microcoulomb $= 10^{-6}$ coulomb and 1 picocoulomb $= 10^{-12}$ coulomb

Basic Properties of Electric Charge

- **Additivity of Charges** Net electric charge on a body is equal to the algebraic sum of all electric charges distributed on different parts of body.
- **Charge is Conservative** Charge can neither be created nor be destroyed, but can be transferred from one body to another body.
- **Quantisation of Charge** Charge on a body is always an integral multiple of charge of an electron (e).

  i.e. $$q = ne$$

  where, $n = 1, 2, 3, \ldots$ and $e = 1.6 \times 10^{-19}$ C.

  The charge on an electron is taken to be negative.

Example 1. *A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C. Estimate the number of electrons transferred from wool to polythene.*

(a) 1.8×10^{15} (b) 1.8×10^{12} (c) 1.2×10^{11} (d) 1.2×10^{10}

Sol. (b) Here, total charge transferred, $q = -3 \times 10^{-7}$ C
Charge on an electron, $e = -1.6 \times 10^{-19}$ C

From quantisation of charge, $q = ne$

Therefore, number of electrons transferred,

$$n = \frac{q}{e} = \frac{-3 \times 10^{-7}}{-1.6 \times 10^{-19}} = 1.8 \times 10^{12}$$

Methods of Statically Charging a Body

To charge a body statically, we either remove electrons from body to make it positively charged or we add electrons to body to make it negatively charged which can be done by the following methods.

Methods of charging

- Charging by friction
- Charging by conduction
- Charging by induction

Coulomb's Law : Force between Two Point Charges

Coulomb's law is a quantitative statement about the force between two point charges. It states that "the force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them".

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r, then force between two charges is given by

$$F_e = \frac{k|q_1 q_2|}{r^2}$$

where, k is constant of proportionality. Its value depends upon the system of units and on the nature of medium between the charges.

When two charges are located in vacuum or air

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \qquad \text{(In SI unit)}$$

where ε_0 is called *absolute permittivity* of free space and its value is $8.85 \times 10^{-12} C^{-12} N^2 m^{-2}$.

If there is another medium between the point charges except air or vacuum, then ε_0 is replaced by $\varepsilon_0 K$ or $\varepsilon_0 \varepsilon_r$ or ε.

Here, K or ε_r is called *dielectric constant* or *relative permittivity of the medium.*

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where, ε = permittivity of the medium.

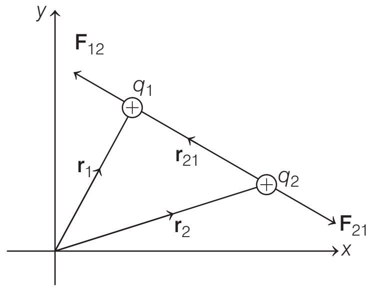
For air or vacuum, $K = 1$

For water, $K = 81$

For metals, $K = \infty$

Coulomb's Law in Vector Form

Consider two point charges q_1 and q_2 separated by distance r in vacuum. Let $\mathbf{F}_{21}$ be the force on q_2 due to q_1 and $\mathbf{F}_{12}$ the force on q_1 due to q_2.



Then, $$\mathbf{F}_{21} = k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}$$

where, $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from q_1 to q_2.

and $$\mathbf{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21}$$

where, $\hat{\mathbf{r}}_{21}$ is a unit vector pointing from q_2 to q_1.

$\therefore$ Force on q_1 due to q_2 = – Force on q_2 due to q_1

or $$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

Note The forces due to two point charges are parallel to the line joining the point charges; such forces are called central forces and so electrostatic forces are conservative forces.

Force between Multiple Charges : Superposition Principle

According to this principle, the net force on a given charge due to number of charges is equal to the vector sum of individual forces exerted on it due to the presence of other charges.

Consider a system of n-point charges $q_1, q_2, q_3 \ldots\ldots q_n$ distributed in space. Let the charges be $q_2, q_3 \ldots q_n$, exert forces $\mathbf{F}_{12}, \mathbf{F}_{13}, \ldots\ldots \mathbf{F}_{1n}$ on charge q_1. The total force on charge q_1 is given by

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_2}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots\ldots + \frac{q_1 q_2}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right)$$

It is important to note here the force of two the charges exert on each other is not changed by the presence of a third charge.

Example 2. *A charge Q is divided into two parts and then they are placed at a fixed distance. The force between the two charges is always maximum when the charges are*

(a) $\frac{Q}{2}, \frac{Q}{2}$ (b) $\frac{Q}{3}, \frac{Q}{3}$

(c) $\frac{Q}{4}, \frac{3Q}{4}$ (d) $\frac{Q}{5}, \frac{4Q}{5}$

Sol. (a) Let the two charges be q and $(Q - q)$.

As, $F = Aq(Q - q)$ (where, A is a constant)

Since, F is maximum when $\frac{dF}{dq} = 0$

$$\Rightarrow \quad \frac{d}{dq}A(qQ - q^2) = 0$$

or $$A(Q - 2q) = 0$$

As $\quad A \neq 0$

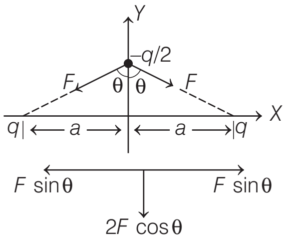
$\therefore \quad Q - 2q = 0$ or $Q = 2q \Rightarrow q = \dfrac{Q}{2}$

Hence, the two charges are $\dfrac{Q}{2}$ and $\left(Q - \dfrac{Q}{2}\right)$ or $\dfrac{Q}{2}$ and $\dfrac{Q}{2}$, when a charge is divided into two equal parts, then force between them is always maximum.

Example 3. *Two charges each equal to q, are kept at x = − a and x = a on the X-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given, a small displacement (y << a) along the Y-axis, the net force acting on the particle is proportional to* **[JEE Main 2013]**

(a) y (b) $-y$ (c) $1/y$ (d) $-1/y$

Sol. (b)



$$F_{\text{net}} = 2F\cos\theta$$

$$\therefore \quad F_{\text{net}} = -\frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2 + a^2})^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

[negative sign indictate the net force is towards the mean position]

$$F_{\text{net}} = -\frac{2kq\left(\frac{q}{2}\right)y}{(y^2 + a^2)^{3/2}} \Rightarrow F \simeq \frac{kq^2 y}{a^3} \propto -y$$

Example 4. *Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d (d << l) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result, charges approach each other with a velocity v. Then, as a function of distance x between them, is* **[AIEEE 2011]**

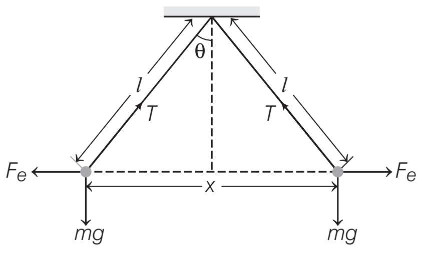
(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$

(c) $v \propto x$ (d) $v \propto x^{-1/2}$

Sol. (d) At any instant,

$$T\cos\theta = mg \quad \text{...(i)}$$

$$T\sin\theta = F_e = \frac{kq^2}{x^2} \quad \text{...(ii)}$$



From Eqs. (i) and (ii), we get

$$\frac{kq^2}{x^2} = mg\tan\theta$$

$$\Rightarrow \quad q^2 = \frac{mg}{k} \cdot \frac{x}{2l} \cdot x^2 \quad \left[\because \tan\theta \approx \frac{x}{2l}\right]$$

$$\Rightarrow \quad q^2 = \frac{mg}{2kl}x^3 \quad \text{...(iii)}$$

$$\Rightarrow \quad 2q\frac{dq}{dt} = \frac{3mg}{2kl}x^2\frac{dx}{dt}$$

$$\Rightarrow \quad 2\left(\frac{mg}{2kl}x^3\right)^{1/2}\frac{dq}{dt} = \frac{3mg}{2kl}x^2 v \quad \left[\because q = \left(\frac{mg}{2kl}x^3\right)^{1/2}\right]$$

$$\Rightarrow \quad vx^{1/2} = \text{constant}$$

$$\Rightarrow \quad v \propto x^{-1/2}$$

Continuous Charge Distribution

There are three types of continuous charge distribution and according to charge distribution, there are three types of charge density as given below

(i) **Linear charge density** (λ) If charge dq is distributed uniformly along a line element dl then the linear charge density, $\lambda = \dfrac{dq}{dl}$.

Its unit is coulomb metre^{-1} (Cm^{-1}).

(ii) **Surface charge density** (σ) If charge dq is distributed over a surface element ds, then surface charge density, $\sigma = dq/ds$.

Its unit is coulomb metre^{-2} (Cm^{-2}).

(iii) **Volume charge density** (ρ) If charge dq is distributed uniformly over the volume element ΔV, then volume charge density, $\rho = \dfrac{dq}{dV}$.

Its unit is coulomb metre^{-3} (Cm^{-3}).

Electric Field

The space surrounding an electric charge q in which another charge q_0 experiences a force of attraction or repulsion, is called the electric field of charge q. The charge q is called the **source charge** and the charge q_0 is called the **test charge**. The test charge must be negligibly small, so that it does not modify the electric field of the source charge.

Intensity of Electric Field (*E*)

The intensity of electric field at a point in an electric field is the ratio of the forces acting on the test charge placed at that point to the magnitude of the test charge.

$\mathbf{E} = \lim\limits_{q_0 \to 0}\left(\dfrac{\mathbf{F}}{q_0}\right)$, where $\mathbf{F}$ is the force acting on q_0.

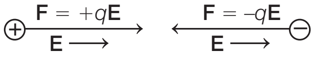
It is a vector quantity.

The direction of electric field is same as that of force acting on the positive test charge. Unit of E is NC^{-1} or Vm^{-1}.

$\therefore$ Dimensions of electric field are $[MLT^{-3}A^{-1}]$.

From $\mathbf{E} = \dfrac{\mathbf{F}}{q}$, we get $\mathbf{F} = q\mathbf{E}$

- If q is positive charge, $\mathbf{F}$ on it is in the direction of $\mathbf{E}$.
- If q is negative charge, $\mathbf{F}$ on it is opposite to the direction of $\mathbf{E}$.



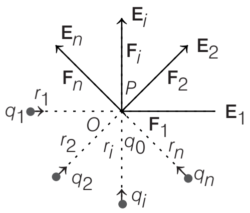
Electric Field due to a Point Charge

Electric field intensity due to a point charge q at a distance r is given by

$$E = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}$$

Electric Field due to System of Charges

Electric field $\mathbf{E}$ at point P due to the systems of charges is given by



$$\mathbf{E}(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$$

Electric Field Due to Continuous Charge Distribution

(i) Electric field due to the line charge distribution at the location of charge q_0 is

$$\mathbf{E}_L = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda}{r^2}\,dL\,\hat{\mathbf{r}}$$

(ii) Electric field due to the surface charge distribution at the location of charge q_0 is

$$\mathbf{E}_S = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma}{r^2}\,dS\,\hat{\mathbf{r}}$$

(iii) Electric field due to the volume charge distribution at the location of charge q_0 is

$$\mathbf{E}_V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r^2}\,dV\hat{\mathbf{r}}$$

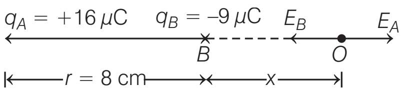
Example 5. *Two point charges of $+16\ \mu C$ and $-9\ \mu C$ are placed 8 cm apart in air. What is the position of the point from $-9\ \mu C$ charge at which the resultant electric field is zero?*

(a) 9 cm (b) 16 cm
(c) 24 cm (d) 35 cm

Sol. (c) Here, $q_A = +16\,\mu C = +16\times10^{-6}$ C; $q_B = -9\,\mu C = -9\times10^{-6}$ C; $r = 8$ cm $= 0.08$ m

Now, the electric field cannot be zero between the two charges. It is because, the charges are of opposite signs. Also, the electric field cannot be zero at a point to the left of charge q_A. It is because the magnitude of charge q_A is greater than that of q_B.

Suppose that the resultant electric field due to the two charges is zero at point O located to the right of charge q_B as shown in figure. If $OB = x$, then $OA = x + 0.08$.



The electric fields E_A and E_B due to the two charges at the point O are in opposite directions. Since, resultant electric field is zero at point O, E_A and E_B are equal in magnitude,

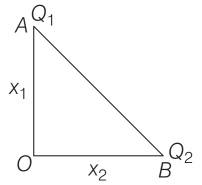
i.e. $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_A}{OA^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_B}{OB^2}$ or $\dfrac{16\times10^{-6}}{(x+0.08)^2} = \dfrac{9\times10^{-6}}{x^2}$

or $\dfrac{(x+0.08)}{4} = \pm\dfrac{x}{3}$ or $x = 0.24$ m or $-\dfrac{0.24}{7}$ m

At, $x = -\dfrac{0.24}{7}$ m

(at point to the left of point B), the magnitudes of E_A and E_B are equal but the two fields will not cancel each other. It is because at this point, both the fields will be in the same direction. Hence, electric field is zero at a point at distance 0.24 m or 24 cm from the charge of $-9\ \mu C$ as shown in figure.

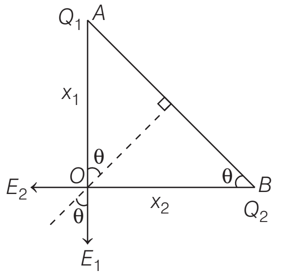
Example 6. *Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to* **[JEE Main 2020]**



(a) $\dfrac{x_1^3}{x_2^3}$ (b) $\dfrac{x_2}{x_1}$
(c) $\dfrac{x_1}{x_2}$ (d) $\dfrac{x_2^2}{x_1^2}$

Sol. (c) Let electric field produced by charges Q_1 and Q_2 at point O be E_1 and E_2, respectively.

The direction of fields are shown in the figure below and a perpendicular is also drawn on side AB, that passes through point O.



If the resultant electric field at point O is perpendicular to hypotenuse, this means resultant of E_1 and E_2 must be along it.

$$\therefore \qquad \tan\theta = \frac{E_2}{E_1} = \frac{\frac{KQ_2}{x_2^2}}{\frac{KQ_1}{x_1^2}} = \frac{Q_2}{Q_1}\frac{x_1^2}{x_2^2} \qquad \ldots \text{(i)}$$

In ΔOAB,
$$\tan\theta = \frac{x_1}{x_2} \qquad \ldots \text{(ii)}$$

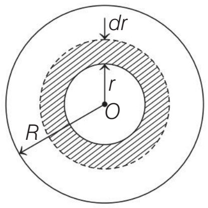
Equating Eqs. (i) and (ii), we get $\frac{Q_2}{Q_1}\cdot\frac{x_1^2}{x_2^2} = \frac{x_1}{x_2}$ or $\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$

Example 7. *Charge is distributed within a sphere of radius R with a volume charge density* $\rho(r) = \frac{A}{r^2} e^{-\frac{2r}{a}}$, *where A and a are constants. If Q is the total charge of this charge distribution, the radius R is* **[JEE Main 2019]**

(a) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ *(b)* $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(c) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ *(d)* $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

Sol. (*d*) Here, volume charge density, $\rho(r) = \frac{A}{r^2}\cdot e^{-\frac{2r}{a}}$



where, a and A are constants.

Let a spherical region of small element of radius r.

If Q is total charge distribution upto radius R, then

$$Q = \int_0^R \rho\cdot dV = \int_0^R \frac{A}{r^2} e^{-2r/a}(4\pi r^2\, dr)$$

(from figure, we observe $dV = A\cdot dr = 4\pi r^2\cdot dr$)

$$= 4\pi A\int_0^R e^{-2r/a}dr$$

$$= 4\pi A\left(\frac{e^{-2r/a}}{-2/a}\right)_0^R$$

$$= 4\pi A\times\left(\frac{-a}{2}\right)(e^{-2R/a} - e^0)$$

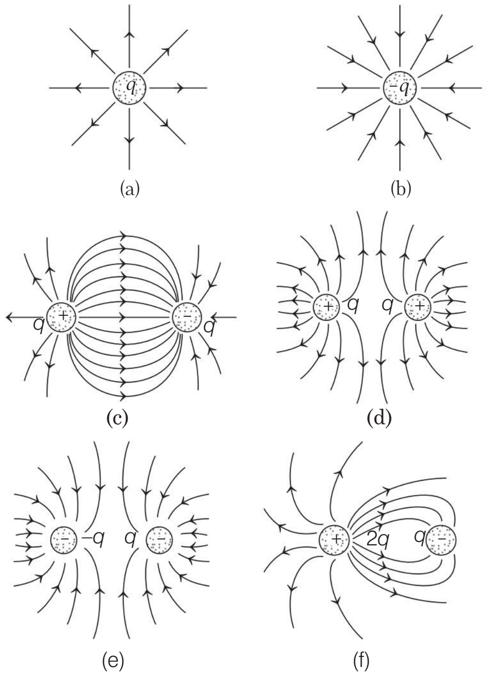
$$= 2\pi A(-a)\,[e^{-2R/a} - 1]$$

or
$$Q = 2\pi a\, A\,(1 - e^{-2R/a})$$

or
$$R = \frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$

Electric Field Lines

"An electric field line is an imaginary line or curve drawn through a region of space, so that its tangent at any point is in the direction of the electric field vector at that point.



Electric field lines associated with a single as well as combination of charges

Following are the properties of electric lines of forces

(i) It is a curved path on which a unit positive charge will move.

(ii) The lines of force do not pass through a conductor.

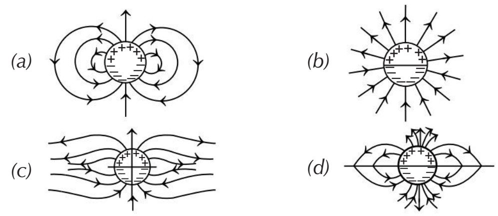
(iii) In the region of strong electric field, lines of forces are closely packed while in the region of weak field they are far apart.

(iv) The electric lines of forces emanate (originate and move outwards) from positive charge and terminate on the nearest negative charge.

(v) Two electric lines of forces never intersect each other.

(vi) The electric lines of forces are always normal, *i.e.* perpendicular to the surface of the charged body, *i.e.* at an angle 90° over the surface of the charged body.

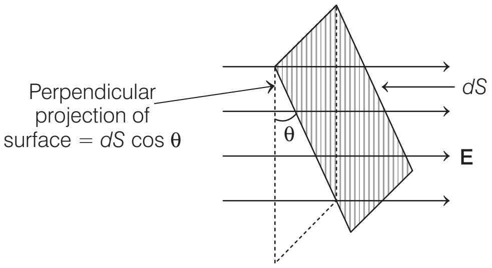
Example 8. *A long cylindrical shell carries positive surface charge* σ *in the upper half and negative surface charge* $-\sigma$ *in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale)* **[JEE Main 2015]**



Sol. (a) Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible. Electric field lines cannot form corners as shown in (d).
Thus, correct option is (a).

Electric Flux

It is the measure of electric field through a surface.

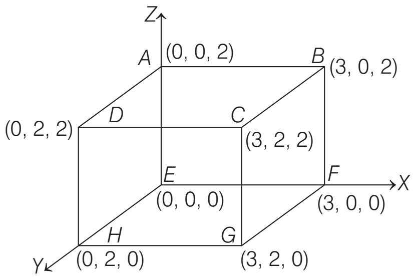


Mathematically, electric flux is the product of an area element $\Delta\mathbf{S}$ and normal component of $\mathbf{E}$ integrated over a surface, $\phi_E = \int \mathbf{E}\cdot\Delta\mathbf{S} = \int \mathbf{E}\cdot\hat{\mathbf{n}}\,\Delta S = \int E\Delta S\cos\theta$.

where, $\hat{\mathbf{n}}$ is the unit vector normal to area element ΔS.

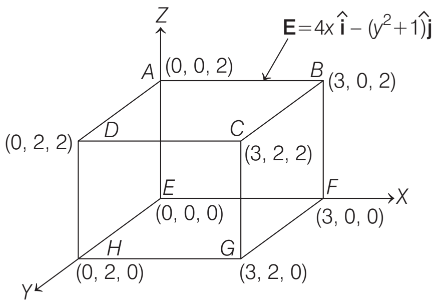
It is a scalar quantity and its SI unit is N-m^2/C m^2 or V-m.

Example 9. *An electric field* $\mathbf{E} = 4x\hat{\mathbf{i}} - (y^2+1)\hat{\mathbf{j}}$ *N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as* ϕ_{I} *and* ϕ_{II}*, respectively. The difference between* $(\phi_{\text{I}} - \phi_{\text{II}})$ *is (in N-m^2/C)* **[JEE Main 2020]**



(a) 45 *(b)* – 45 *(c)* 48 *(d)* – 48

Sol. (d)



Area vector of face *ABCD*, $\mathbf{A}_1 = 2\times 3\hat{\mathbf{k}} = 6\hat{\mathbf{k}}$

Area vector of face *BCGF*, $\mathbf{A}_2 = 2\times 2\hat{\mathbf{i}} = 4\hat{\mathbf{i}}$

So, flux through face *ABCD*,

$$\phi_1 = \mathbf{E}\cdot\mathbf{A}_1 = (4x\hat{\mathbf{i}} - (y^2+1)\hat{\mathbf{j}})\cdot 6\hat{\mathbf{k}} = 0$$

Flux through face *BCGF*,

$$\phi_2 = \mathbf{E}\cdot\mathbf{A}_2 = (4x\hat{\mathbf{i}} - (y^2+1)\hat{\mathbf{j}})\cdot 4\hat{\mathbf{i}} = 16x$$

At face *BCGF*, $x = 3$

So, $\phi_2 = 16\times 3 = 48$ units

$\therefore \quad \phi_{\text{I}} - \phi_2 = 0 - 48 = -48$ N-m^{2}C^{-1}

Gauss's Law

Net flux over any a closed surface is $\dfrac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

i.e. $\phi_E = \oint \mathbf{E}\cdot d\mathbf{S} = \dfrac{1}{\varepsilon_0}\Sigma q$

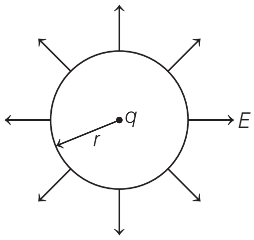
where, Σq = net charge enclosed in that surface.

Note If a closed body, not enclosing any charge, is placed in an electric field (either uniform or non-uniform), total flux linked with it will be zero.

Applications of Gauss's Law

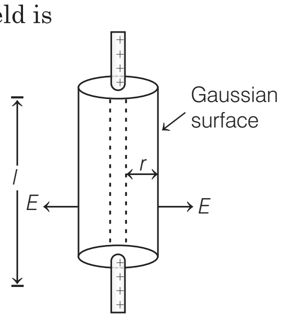
To calculate electric field by Gauss's law, we will draw a Gaussian surface (either sphere or cylinder according to situation) in such a way that electric field is perpendicular at each point of surface and its magnitude is same at every point and then apply Gauss's law.
Following are few application of this law

(i) **Electric field due to a point charge** Electric field due to a point charge at a distance r is



$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

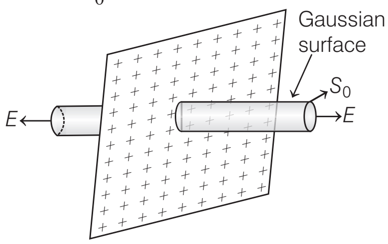
(ii) **Electric field due to infinitely long uniformly charged straight wire** At a distance r from wire electric field is



$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = 2\frac{k\lambda}{r} \quad \text{or} \quad E \propto \frac{1}{r}$$

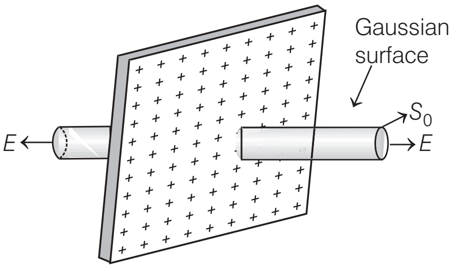
(iii) **Electric field due to an plane sheet of charge** For a thin sheet with charge density σ, electric field

$$E = \frac{\sigma}{2\varepsilon_0}$$



(iv) **Electric field near a charged conducting surface** For a charged conducting surface, field near the surface is

$$E = \frac{\sigma}{\varepsilon_0}$$

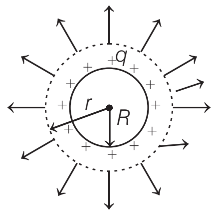


(v) **Electric field due to a uniformly charged spherical shell**

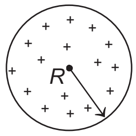
At an internal point ($r < R$), $E_{\text{inside}} = 0$

At the surface of shell ($r = R$), $E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

At an extreme point ($r > R$), $E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$



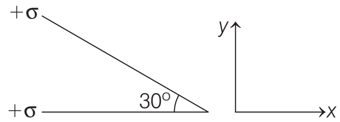
(vi) **Electric field due to a uniformly charged non-conducting solid sphere** Inside the sphere ($r < R$), $E_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r$



On the surface ($r = R$), $E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

At an external point ($r > R$), $E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Example 10. *Two infinite planes each with uniform surface charged density +σ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by* ***[JEE Main 2020]***



(a) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{\mathbf{y}} - \frac{\hat{\mathbf{x}}}{2}\right]$

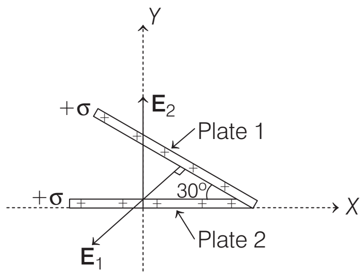
(b) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{\mathbf{y}} - \frac{\hat{\mathbf{x}}}{2}\right]$

(c) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{\mathbf{y}} + \frac{\hat{\mathbf{x}}}{2}\right]$

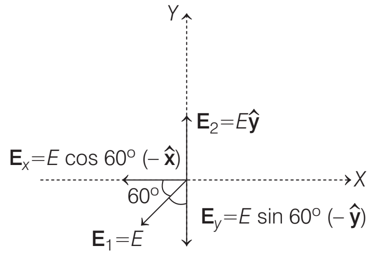
(d) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{\mathbf{y}} + \frac{\hat{\mathbf{x}}}{2}\right]$

Sol. (*b*) Electric field of an infinite plate is perpendicular to the plane of plate and its magnitude is $E = \frac{\sigma}{2\varepsilon_0}$.

We are given with two positively charged plates with a set of coordinate axes as shown in the figure.



From geometry of figure, net electric field in region between plates is resultant of fields of both plates,



Now, field of plate 1 can be resolved along *X* and *Y*-axes as shown in above figure.

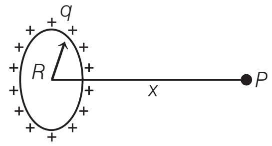
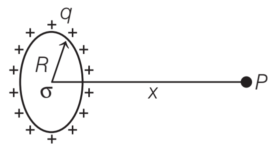
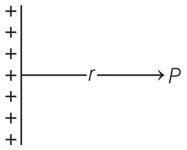
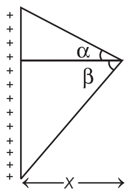
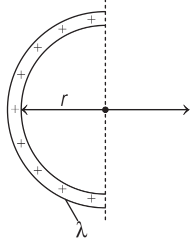
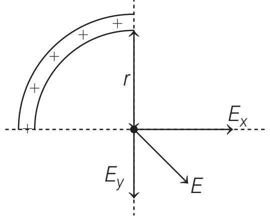
Now, $\mathbf{E}_{\text{net}} = \mathbf{E}_1 + \mathbf{E}_2 = E\cos 60° (-\hat{\mathbf{x}}) + E\sin 60° (-\hat{\mathbf{y}}) + E\hat{\mathbf{y}}$

Here, $E = \frac{\sigma}{2\varepsilon_0}$ and $\hat{\mathbf{x}}$ & $\hat{\mathbf{y}}$ are unit vectors along *X* & *Y*-axes.

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{\hat{\mathbf{x}}}{2} - \frac{\sqrt{3}}{2}\hat{\mathbf{y}} + \hat{\mathbf{y}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{\mathbf{y}} - \frac{\hat{\mathbf{x}}}{2}\right]$$

Electric Field Intensity of Various Systems

System	Electric Field Intensity
Isolated charge 	$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$
A ring of charge 	$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qx}{(R^2 + x^2)^{3/2}}$
A disc of charge 	$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$
Infinitely long line of charge 	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$
Finite line of charge 	$E_\perp = \frac{\lambda}{4\pi\varepsilon_0 x}(\sin\alpha + \sin\beta)$ $E_{\|} = \frac{\lambda}{4\pi\varepsilon_0 x}(\cos\alpha - \cos\beta)$
At centre of semicircular ring 	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$
At centre of quarter circular ring 	$E = \frac{1}{2\sqrt{2}} \cdot \frac{1}{\pi\varepsilon_0 r}$ $E_x = \frac{\lambda}{4\pi\varepsilon_0 r}$ $E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$

Electric Potential (V)

Electric potential at any point is equal to the work done per unit positive charge in carrying it from infinity to that point in electric field.

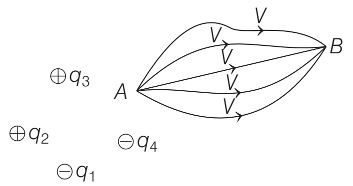
Electric potential, $V = \frac{W}{q}$

Its SI unit is J/C or volt and its dimensions are $[ML^2T^{-3}A^{-1}]$.

It is a scalar quantity.

Electric Potential Difference

The electric potential difference between two points A and B is equal to the work done by the external force in moving a unit positive charge against the electrostatic force from point B to A along any path between these two points.



If V_A and V_B be the electric potentials at points A and B respectively, then $\Delta V = V_A - V_B$

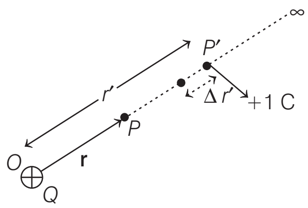
or $$\Delta V = \frac{W_{AB}}{q}$$

The SI unit of potential difference is volt (V).

The dimensional formula for electric potential difference is given by $[ML^2T^{-3}A^{-1}]$.

Electric Potential due to a Point Charge

Electric potential due to a point charge q at any point P lying at a distance r as shown in the figure below, is given by

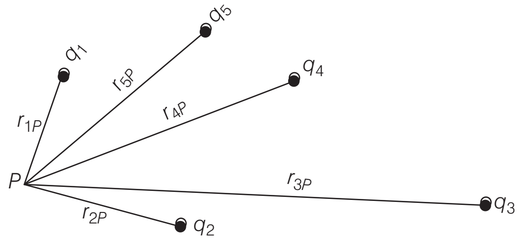


$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Electric Potential due to a System of Charges

Electric potential at any point P due to system of charges is equal to the algebraic sum of potentials due to individual charges.

i.e. $$V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \dots \frac{q_n}{r_{nP}}\right)$$

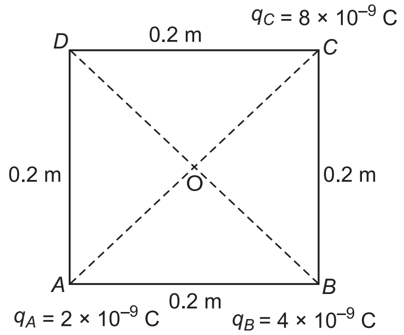


Example 11. *ABCD is a square of side 0.2 m. Charges of 2×10^{-9}, 4×10^{-9} and 8×10^{-9} C are placed at the corners A, B and C respectively. The work required to transfer a charge of 2×10^{-9} C from D to the centre of the square is*

(a) 6.27×10^{-7} J (b) 4.85×10^{-5} J
(c) 4.8×10^{-4} J (d) 3.2×10^{-2} J

Sol. (a) The charges of $q_A = 2\times10^{-9}$ C; $q_B = 4\times10^{-9}$ C and $q_C = 8\times10^{-9}$ C are placed at the corners A, B and C of the square $ABCD$ of each side of length 0.2 m.

Let V_D be potential at point D due to point charges placed at A, B and C, then



V_D = sum of the potentials due to the charges q_A, q_B and q_C

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{AD} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{BD} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C}{CD}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AD} + \frac{q_B}{BD} + \frac{q_C}{CD}\right)$$

Here, $AD = CD = 0.2$ m

and $BD = \sqrt{0.2^2 + 0.2^2} = 0.2\sqrt{2}$ m

$$\therefore \quad V_D = 9\times10^9\left(\frac{2\times10^{-9}}{0.2} + \frac{4\times10^{-9}}{0.2\sqrt{2}} + \frac{8\times10^{-9}}{0.2}\right)$$

$$= \frac{9\times10^9\times2\times10^{-9}}{0.2}(1+\sqrt{2}+4) = 577.26 \text{ V}$$

Now, potential at point O due to charges q_A, q_B and q_C,

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AO} + \frac{q_B}{BO} + \frac{q_C}{CO}\right)$$

Now, $AO = BO = CO = \frac{1}{2}\times$ diagonal of square

$$= \frac{1}{2}\times0.2\sqrt{2} = 0.1\sqrt{2} \text{ m} \qquad (\because BD = 0.2\sqrt{2} \text{ m})$$

$$\therefore \quad V_0 = 9\times10^9\times\left(\frac{2\times10^{-9}}{0.1\sqrt{2}} + \frac{4\times10^{-9}}{0.1\sqrt{2}} + \frac{8\times10^{-9}}{0.1\sqrt{2}}\right)$$

$$= \frac{9\times10^9\times2\times10^{-9}}{0.1\sqrt{2}}(1+2+4) = 890.82 \text{ V}$$

Potential difference between the points O and D,

$V_O - V_D = 890.82 = 577.26 = 313.56$ V

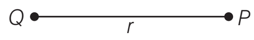
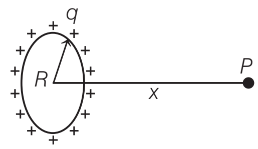
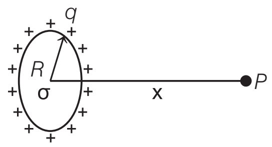
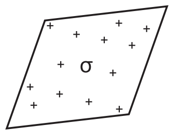
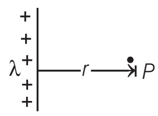
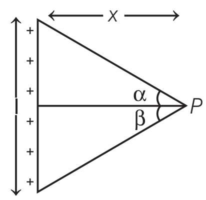
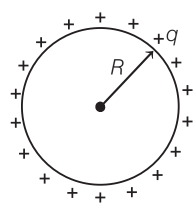
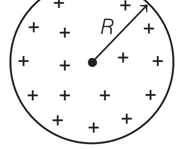
Therefore, work done to transfer a charge,

$q = 2\times10^{-9}$ C from point D to O is given by

$W = q\times(V_O - V_D)$

$= 2\times10^{-9}\times313.56 = 6.27\times10^{-7}$ J

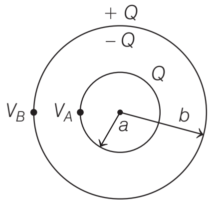
Electric Potential of Various Systems

System	Electrical Potential
Isolated charge 	$V = \frac{q}{4\pi\varepsilon_0 r}$
A ring of charge 	$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{R^2+x^2}}$
A disc of charge 	$V = \frac{\sigma}{2\varepsilon_0}[\sqrt{x^2+R^2} - x]$
Infinite sheet of charge 	$V = \frac{-\sigma r}{2\varepsilon_0}$
Infinitely long line of charge 	$V = \frac{\lambda}{2\pi\varepsilon_0}\log_e r$
Finite line of charge 	$V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left[\frac{\sqrt{x^2+l^2}-l}{\sqrt{x^2+l^2}+1}\right]$
Charged spherical shell 	(a) Inside $0 \le r \le R$, $V = \frac{q}{4\pi\varepsilon_0 R}$ (b) Outside $r \ge R$, $V = \frac{q}{4\pi\varepsilon_0 r}$
Solid sphere of charge 	(a) Inside $0 \le r \le R$, $V = \frac{\rho R^2}{6\varepsilon_0}\left[3 - \frac{r^2}{R^2}\right]$ (b) Outside $r \ge R$, $V = \frac{\rho R^3}{3\varepsilon_0}\left[\frac{1}{r}\right]$

Example 12. *A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of* $-4Q$*, the new potential difference between the same two surfaces is* **[JEE Main 2019]**

(a) $-2V$ (b) $2V$ (c) $4V$ (d) V

Sol. (*d*) Initially when uncharged shell encloses charge Q, charge distribution due to induction will be as shown



The potential on surface of inner shell is

$$V_A = \frac{kQ}{a} + \frac{k(-Q)}{b} + \frac{kQ}{b} \quad \ldots\text{(i)}$$

Potential on surface of outer shell is

$$V_B = \frac{kQ}{b} + \frac{k(-Q)}{b} + \frac{kQ}{b} \quad \ldots\text{(ii)}$$

Then, potential difference is

$$\Delta V_{AB} = V_A - V_B = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

Given, $\Delta V_{AB} = V$

So, $kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \quad \ldots\text{(iii)}$

Finally after giving charge (– 4Q) to outer shell, potential difference will be

$$\Delta V_{AB} = V_A - V_B$$
$$= \left(\frac{kQ}{a} + \frac{k(-4Q)}{b}\right) - \left(\frac{kQ}{b} + \frac{k(-4Q)}{b}\right)$$
$$= kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \quad \text{[from Eq. (iii)]}$$

Hence, we obtain that potential difference does not depend on the charge of outer sphere and so it remains same.

Example 13. *Three concentric spherical metallic spheres A, B and C of radii a, b and c(a < b < c) have surface charge densities* $\sigma, -\sigma$ *and* σ*, respectively. Find the potentials of the three shells A, B and C.*

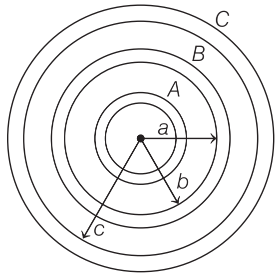
(a) $\frac{\sigma}{\varepsilon_0}(a - b + c), \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right], \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{c} + c\right]$

(b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right], \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{c} + c\right], \frac{\sigma}{\varepsilon_0}(a - b + c)$

(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right], \frac{\sigma}{\varepsilon_0}(a - b + c), \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{c} + c\right]$

(d) *None of the above*

Sol. (*a*) In case of charged sphere,



$$V_{\text{out}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

while $V_{\text{in}} = V_s = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \frac{\sigma R}{\varepsilon_0}$ [as, $q = 4\pi R^2\sigma$]

So, (i) $V_A = (V_A)_s + (V_B)_{\text{in}} + (V_C)_{\text{in}}$

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{\sigma a}{a} + \frac{-\sigma b}{\varepsilon_0} + \frac{\sigma c}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}(a - b + c)$$

where, $q_A = 4\pi a^2\sigma$

(ii) $V_B = (V_A)_{\text{out}} + (V_B)_s + (V_c)_{\text{in}}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_A}{b} + \frac{-\sigma b}{\varepsilon_0} + \frac{\sigma c}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

where, $q_A = 4\pi a^2\sigma$

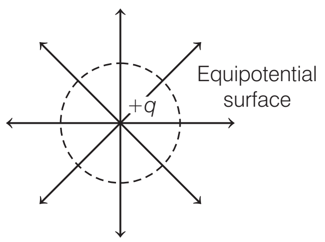
(iii) $V_C = (V_A)_{\text{out}} + (V_B)_{\text{out}} + (V_C)_s$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_A}{c} + \frac{1}{4\pi\varepsilon_0}\frac{q_B}{c} + \frac{\sigma c}{\varepsilon_0}$$
$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2 - b^2}{c} + c\right]$$

where $q_A = 4\pi a^2\sigma$ and $q_B = 4\pi b^2(-\sigma)$.

Equipotential Surfaces

Equipotential surface is an imaginary surface joining the points of same potential in an electric field. So, we can say that the potential difference between any two points on an equipotential surface is zero.



Regarding equipotential surface, following are few important points

(i) Equipotential surface may be planar, etc., but it can never be point size.

(ii) Since, equipotential surface is single valued, so these surfaces never cross each other.

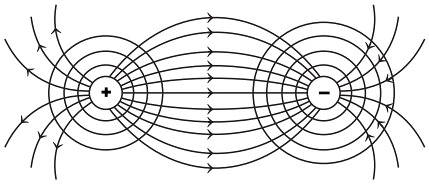
(iii) Electric field is always perpendicular to equipotential surface.

(iv) Work done to move a point charge q between two points on equipotential surface is zero.

(v) Equipotential surface due to an isolated point charge is spherical.

(vi) Equipotential surface are planar in an uniform electric field (arrowed lines show the electric lines of force).

(vii) Equipotential surface due to an electric dipole is shown in the figure.



Relation between Electric Field and Electric Potential

Relation between E and V is given as

$$E = \frac{-dV}{dx}$$

where, dx is the perpendicular distance between two equipotential surfaces and dV is the change in V in the direction of E.

For a three-dimensional potential function,

$$|\mathbf{E}| = \frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial Y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$

where, $\frac{\partial V}{\partial x}$ is partial derivation of potential function which is derivative of V with respect to x treating y and z constants.

Thus, the electric field intensity E is the negative gradient of potential. This means that decrease in potential is along the direction of E. The SI unit of E is volt per metre (Vm^{-1}).

Example 14. *The electric field in a region is given by $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is* **[JEE Main 2019]**

(a) -48 V (b) -520 V (c) 180 V (d) 320 V

Sol. (c) Given, $\text{E} = (Ax + B)\hat{\text{i}}\ \text{NC}^{-1}$

The relation between electric field and potential is given as

$$dV = -\text{E}\cdot dx$$

Integrating both sides within the specified limits, we get

$$\therefore \quad \int_1^2 dV = V_2 - V_1 = -\int_{x_1}^{x_2} \text{E}\cdot dx$$

$$\Rightarrow \quad V_1 - V_2 = \int_{x_1}^{x_2} \text{E}\cdot dx$$

$$= \int_{x_1}^{x_2} (Ax + B)\hat{\text{i}}\cdot(dx\,\hat{\text{i}}) = \int_{x_1}^{x_2} (Ax + B)\cdot dx$$

Here, $A = 20$ SI unit, $B = 10$ SI unit,

$$x_1 = 1 \text{ and } x_2 = -5 \Rightarrow V_1 - V_2 = \int_1^{-5} (20x + 10)\cdot dx$$

$$= \left[\frac{20x^2}{2} + 10x\right]_1^{-5} = 10\ [x^2 + x]_1^{-5}$$

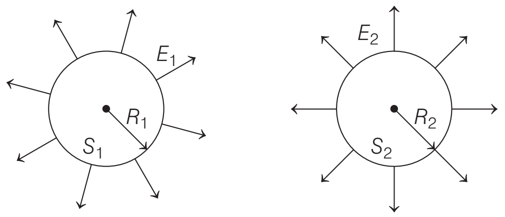
$$= 10\ [(-5)^2 + (-5) - (1)^2 - (1)]$$

$$= 10\ (25 - 5 - 2) = 180 \text{ V}$$

Example 15. *Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1 / E_2 = R_1 / R_2$. Then the ratio V_1(on S_1)/V_2 (on S_2) of the electrostatic potentials on each sphere is* **[JEE Main 2020]**

(a) $\left(\frac{R_1}{R_2}\right)^3$ (b) R_2/R_1 (c) R_1/R_2 (d) $(R_1/R_2)^2$

Sol. (d) Given, $\frac{E_1}{E_2} = \frac{R_1}{R_2}$ ...(i)



For a charged sphere, potential is given by

$$V = \frac{kQ}{R} = \frac{kQ}{R^2}\cdot R \text{ or } V = E\cdot R$$

So, $\frac{V_1}{V_2} = \frac{E_1R_1}{E_2R_2} = \frac{R_1}{R_2}\times\frac{R_1}{R_2}$ [from Eq. (i)]

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{R_1^2}{R_2^2} = \left(\frac{R_1}{R_2}\right)^2$$

Potential Energy of a System of Charges

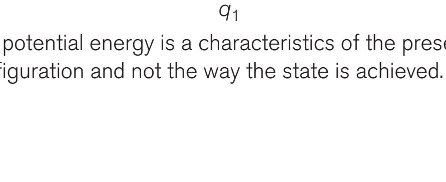
Electrostatic potential energy of a system of two point charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r}$$

where, q_1 and q_2 are charges and r is the separation between the point charges.

The total work done in assembling the charges at the given locations, as shown in the figure below is given as

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$



Note The potential energy is a characteristics of the present state of configuration and not the way the state is achieved.

Potential Energy in an External Field

Potential energy of a single charge q at a point with position vector $\mathbf{r}$, in an external field is $qV(\mathbf{r})$, where $V(\mathbf{r})$ is the potential at the point due to external electric field E.

Potential energy of a system of two charges in an external field can be given as

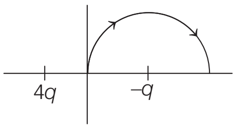
$$U = q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 \mathbf{r}_{12}}$$

where, q_1 and q_2 = two point charges at position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively,

$V(\mathbf{r}_1)$ = potential at $\mathbf{r}_1$ due to the external field

and $V(\mathbf{r}_2)$ = potential at $\mathbf{r}_2$ due to the external field.

Example 16. *Two point charges 4q and – q are fixed on the X-axis at* $x = \frac{-d}{2}$ *and* $x = \frac{d}{2}$, *respectively. If a third point charge q is taken from the origin to x = d along the semi-circle as shown in the figure, the energy of the charge will* **[JEE Main 2020]**



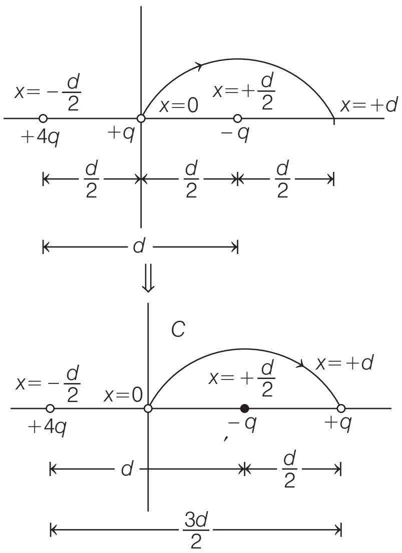
(a) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$

(b) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

(c) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$

(d) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$

Sol. (b)



Change in potential energy, $\Delta U = U_f - U_i$

$$\Delta U = \left[\frac{k(4q)(-q)}{d} + \frac{k(4q)(q)}{\frac{3d}{2}} + \frac{k(-q)(q)}{\frac{d}{2}}\right] - \left[\frac{k(4q)(-q)}{d} + \frac{k(4q)(q)}{\frac{d}{2}} + \frac{k(-q)(q)}{\frac{d}{2}}\right]$$

$$= \frac{k(4q)(-q)}{d} + \frac{k(4q)(q)}{\frac{3d}{2}} + \frac{k(-q)(q)}{\frac{d}{2}} - \frac{k(4q)(-q)}{d} - \frac{k(4q)(q)}{\frac{d}{2}} - \frac{k(-q)(q)}{\frac{d}{2}}$$

$$= \frac{k(4q)(q)}{\frac{3d}{2}} - \frac{k(4q)(q)}{\frac{d}{2}} = \frac{8kq^2}{3d} - \frac{8kq^2}{d}$$

$$= -\frac{16kq^2}{3d} = -\frac{16q^2}{3d \times 4\pi\varepsilon_0}$$

$$= -\frac{4q^2}{3\pi\varepsilon_0 d}$$

So, ΔU will decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$.

Motion of a Charged Particle in an Electric Field

Consider a charged particle having charge q and mass m is initially at rest in an electric field of strength E. The particle will experience an electric force which causes its motion.

The force experienced by the charged particle is F, where

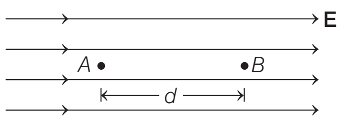
$$F = qE$$

∴ Acceleration produced by this force,

$$a = \frac{F}{m} = \frac{qE}{m}$$

Suppose at point A particle is at rest and after sometime t, it reaches the point B and attains velocity v.

∴ $$v = at$$



If potential difference between A and B be ΔV and the distance between them is d, then

$$v = \frac{qEt}{m} = \sqrt{\frac{2q\Delta V}{m}}$$

Kinetic energy gained by the particle in time t,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} m\left(\frac{qEt}{m}\right)^2 = q\Delta v$$

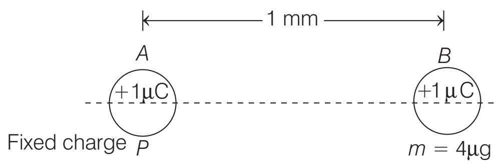
Example 17. *In free space, a particle A of charge $1\mu C$ is held fixed at a point P. Another particle B of the same charge and mass $4\,\mu g$ is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is*

$$\left[\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ N\text{-}m^2C^{-2}\right]$$

[JEE Main 2019]

(a) 1.5×10^2 m/s
(b) 3.0×10^4 m/s
(c) 1.0 m/s
(d) 2.0×10^3 m/s

Sol. (*d*) Given situation is shown in the figure below



When charged particle *B* is released due to mutual repulsion, it moves away from *A*. In this process, potential energy of system of charges reduces and this change of potential energy appears as kinetic energy of *B*.

Now, potential energy of system of charges at separation of 1 mm,

$$U_1 = \frac{Kq_1q_2}{r}$$

Here, $q_1 = q_2 = 1 \times 10^{-6}$ C

$r = 1\text{mm} = 1 \times 10^{-3}$ m

$$\therefore \quad U_1 = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1 \times 10^{-3}} = 9 \text{ J}$$

Potential energy of given system of charges at separation of 9 mm,

$$U_2 = \frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{9 \times 10^{-3}} = 1 \text{ J}$$

By energy conservation,

Change in potential energy of system of *A* and *B*

= Kinetic energy of charged particle *B*

$$\Rightarrow \quad U_1 - U_2 = \frac{1}{2} m_B v_B^2$$

where, m_B = mass of particle $B = 4\,\mu$g

$= 4 \times 10^{-6} \times 10^{-3}$ kg $= 4 \times 10^{-9}$ kg

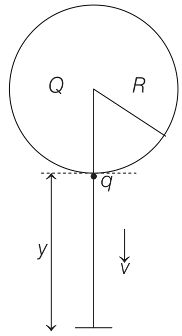
and v_B = velocity of particle *B* at separation of 9 mm

$$\Rightarrow \quad 9 - 1 = \frac{1}{2} \times 4 \times 10^{-9} \times v_B^2$$

$$\Rightarrow \quad v_B^2 = 4 \times 10^9$$

$$\Rightarrow \quad v_B = 2 \times 10^3 \text{ ms}^{-1}$$

Example 18. *A solid sphere of radius R carries a charge Q + q distributed uniformly over its volume. A very small point-like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed v when it has fallen through a vertical height y (see figure), then (Assume the remaining portion to be spherical)* ***[JEE Main 2020]***



(a) $v^2 = y\left[\dfrac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

(b) $v^2 = 2y\left[\dfrac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(c) $v^2 = y\left[\dfrac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(d) $v^2 = 2y\left[\dfrac{qQR}{4\pi\varepsilon_0 (R+y)^3 m} + g\right]$

Sol. (*b*) Using law of conservation of total energy,

$$\frac{1}{2}mv^2 = mgy + (\Delta\text{PE})$$

Here, $\Delta\text{PE} = \text{PE}_i - \text{PE}_f = \dfrac{kQq}{R} - \dfrac{kQq}{R+y}$

$$\Rightarrow \quad \frac{1}{2}mv^2 = mgy + kQq\left[\frac{1}{R} - \frac{1}{(R+y)}\right]$$

$$\Rightarrow \quad v^2 = 2gy + \frac{2kQq}{m}\frac{y}{R(R+y)}$$

$$\Rightarrow \quad v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right] \quad \left[\because k = \frac{1}{4\pi\varepsilon_0}\right]$$

Electric Dipole

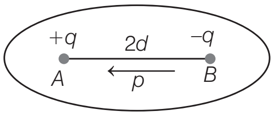
An arrangement of two equal and opposite charges separated by a fixed distance is known as an electric dipole.

Dipole moment is a vector associated with a dipole which is given as

Dipole moment = Magnitude of any one charge
× Distance of separation between two charges

or $\mathbf{p} = q \times 2\mathbf{d}$

SI unit of dipole moment is coulomb-metre (C-m).



Dimensions of $\mathbf{p} = [M^0LTA]$

The dipole moment is always directed from negative charge to the positive charge.

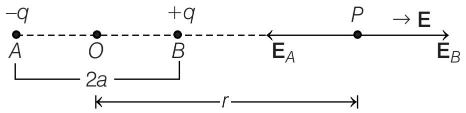
Electric Field Intensity and Potential due to an Electric Dipole

(i) **On Axial Line**

Electric field intensity, $E = \frac{1}{4\pi \varepsilon_0} \frac{2pr}{(r^2 - a^2)^2}$

If $\quad r >> 2a$, then $E = \frac{1}{4\pi \varepsilon_0} \frac{2p}{r^3}$.

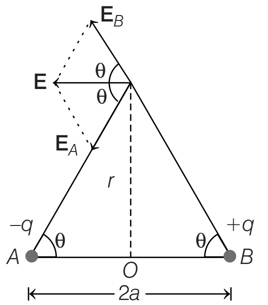
Electric potential, $V = \frac{1}{4\pi \varepsilon_0} \frac{p}{(r^2 - a^2)}$



If $\quad r >> 2a$, then $V = \frac{1}{4\pi \varepsilon_0} \frac{p}{r^2}$.

(ii) **On Equatorial Line**

Electric field intensity, $E = \frac{1}{4\pi \varepsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$



If $r >> 2a$, then

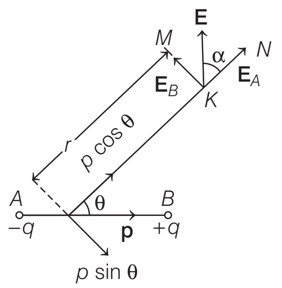
$$E = \frac{1}{4\pi \varepsilon_0} \frac{p}{r^3}$$

Electric potential, $V = 0$.

(iii) **At any Point along a Line Making θ Angle with Dipole Axis**

Electric field intensity,

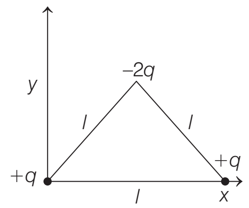
$$E = \frac{1}{4\pi \varepsilon_0} \frac{p\sqrt{(1 + 3\cos^2\theta)}}{r^3}$$



Electric potential, $V = \frac{1}{4\pi \varepsilon_0} \frac{p\cos\theta}{(r^2 - a^2\cos^2\theta)}$

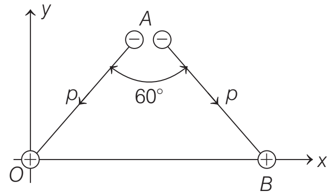
If $r >> 2a$, then $V = \frac{1}{4\pi \varepsilon_0} \frac{p\cos\theta}{r^2}$.

Example 19. *Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure.* **[JEE Main 2019]**



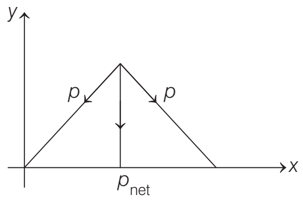
(a) $\sqrt{3}\, ql \frac{\hat{\mathbf{j}} - \hat{\mathbf{i}}}{\sqrt{2}}$ (b) $2ql\, \hat{\mathbf{j}}$ (c) $-\sqrt{3}\, ql\, \hat{\mathbf{j}}$ (d) $(ql) \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$

Sol. (c) Given system is equivalent to two dipoles inclined at 60° to each other as shown in the figure below



Now, magnitude of resultant of these dipole moments is

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2p \cdot p\cos 60^\circ} = \sqrt{3}p = \sqrt{3}ql$$



As, resultant is directed along negative y-direction

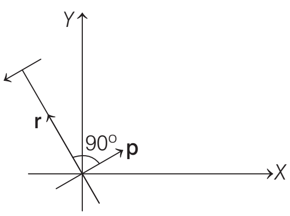
$p_{\text{net}} = -\sqrt{3}p\hat{\mathbf{j}} = -\sqrt{3}ql\,\hat{\mathbf{j}}$

Example 20. *An electric dipole of moment* $\mathbf{p} = (-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \times 10^{-29}$ *C-m is at the origin (0, 0, 0). The electric field due to this dipole at* $\mathbf{r} = +\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ *(note that* $\mathbf{r} \cdot \mathbf{p} = 0$*) is parallel to* **[JEE Main 2020]**

(a) $(+\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ (b) $(-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

(c) $(+\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ (d) $(-\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Sol. (c) Given, $\mathbf{r} \cdot \mathbf{p} = 0$, so $\mathbf{r} \perp \mathbf{p}$, *i.e.* we have following situation



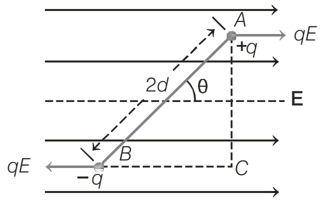
So, we have to find direction of electric field at equatorial line. As $\mathbf{E}$ is directed opposite to $\mathbf{p}$ at all equatorial points, so direction of $\mathbf{E}$ is along $-\mathbf{p}$.

So, $\quad \mathbf{E} = \lambda(-\mathbf{p}) = \lambda[-(-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})] = \lambda(+\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$\therefore$ Electric field due to dipole is parallel to $(+\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Torque on a Dipole in a Uniform Electric Field

When an electric dipole of dipole moment $\mathbf{p} = q \times 2d$ be placed in an uniform electric field intensity it experiences a torque given by



$$\tau = \mathbf{p} \times \mathbf{E}$$

or $$|\tau| = |\mathbf{p}||\mathbf{E}| \sin\theta$$

When $\theta = 90°$, then $\tau_{max} = Ep$.

When electric dipole is parallel to electric field, it is in stable equilibrium and when it is anti-parallel to electric field, it is in unstable equilibrium.

Potential Energy of a Dipole in an External Field

If an external torque τ_{ext} is applied in such a manner that it just neutralises this torque and rotates it in the plane of paper from angle θ_0 to angle θ_1 at an infinitesimal angular speed and without angular acceleration.

The amount of work done by the external torque will be given by

$$W = pE(\cos\theta_0 - \cos\theta_1)$$

If dipole is initially placed parallel to **E** field, then $\theta_1 = 0°$ and when the dipole is rotated by angle θ, then $\theta_2 = \theta$ and work done,

$$W = pE[\cos 0° - \cos\theta] = pE[1 - \cos\theta]$$

Potential energy of a dipole in a uniform electric field,

$$\mathbf{U}(\theta) = -\mathbf{p} \cdot \mathbf{E} = -pE\cos\theta$$

The potential energy $U'(\theta)$ with an inclination θ of the dipole in an external field is given by

$$U'(\theta) = q[V(\mathbf{r}_1) - V(\mathbf{r}_2)] - \frac{q^2}{4\pi\varepsilon_0 \times 2a}$$

where, q is magnitude of either charge of electric dipole and $\mathbf{r}_1$ & $\mathbf{r}_2$ denote the position vectors of $+q$ & $-q$.

Example 21. *An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C-m. What is the potential energy of the electric dipole?* **[JEE Main 2019]**

(a) -9×10^{-20} J (b) -10×10^{-29} J

(c) -20×10^{-18} J (d) -7×10^{-27} J

Sol. (d) Given, $E = 1000$ V/m, $\theta = 45°$

and $p = 10^{-29}$ C-m

We know that, electric potential energy stored in an electric dipole kept in uniform electric field is given by the relation

$$U = -\mathrm{p} \cdot \mathrm{E} = -pE\cos\theta$$
$$= -10^{-29} \times 1000 \times \cos 45°$$

$\Rightarrow$ $$U \approx -7 \times 10^{-27} \text{ J}$$

Example 22. *An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is* **[JEE Main 2019]**

(a) $\sqrt{\frac{2qE}{md}}$ (b) $2\sqrt{\frac{qE}{md}}$

(c) $\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

Sol. (a) When an electric dipole is placed in an electric field **E** at some angle θ, then two forces equal in magnitude but opposite in direction acts on the +ve and –ve charges, respectively. These forces forms a couple which exert a torque, which is given as

$$\tau = \mathbf{p} \times \mathbf{E}$$

where, **p** is dipole moment.

$\therefore$ Torque on the dipole can also be given as

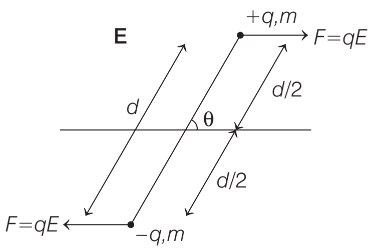
$$\tau = I\alpha = -pE\sin\theta$$

where, I is the moment of inertia and α is angular acceleration.

For small angles, $\sin\theta \approx \theta$

$\therefore$ $$\alpha = -\left(\frac{pE}{I}\right)\theta \quad \text{...(i)}$$

Moment of inertia of the given system is



$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{2md^2}{4} = \frac{md^2}{2}$$

Substituting the value of I in Eq. (i), we get

$\Rightarrow$ $$\alpha = -\left(\frac{2pE}{md^2}\right) \cdot \theta \quad \text{...(ii)}$$

The above equation is similar to the equation for a system executing angular SHM.

Comparing Eq. (ii) with the general equation of angular SHM,

$$\alpha = -\omega^2\theta$$

where, ω is the angular frequency, we get

$$\omega^2 = \frac{2pE}{md^2}$$

or $$\omega = \sqrt{\frac{2pE}{md^2}}$$

As, $$p = qd$$

$\therefore$ $$\omega = \sqrt{\frac{2qdE}{md^2}} = \sqrt{\frac{2qE}{md}}$$

Dielectric

These are insulating (non-conducting) materials that can produce electric effect without conduction.

Dielectrics are of two types

(i) The **non-polar dielectrics** (like N_2, O_2, benzene, methane, etc.) are made up of non-polar atoms/molecules, in which the centre of positive charge coincides with the centre of negative charge of the atom/molecule.

(ii) The **polar dielectric** (like H_2O, CO_2, NH_3, etc.) are made up of polar atoms/molecules, in which the centre of positive charge does not coincide with the centre of negative charge of the atom.

Dielectric Constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of electric field on placing the dielectric between the plates of a capacitor is the dielectric constant. It is denoted by K (or ε_r). It is given by $K = \dfrac{E_0}{E}$.

Polarisation (P) and Electric Susceptibility (χ_e)

The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called **polarisation**. It is denoted by P.

$$P = \chi_e E$$

where, χ_e is known as **electric susceptibility of the dielectric medium**.

It is a dimensionless constant.It describes the electrical behaviour of a dielectric. It has different values for different dielectrics.

For vacuum, $\chi = 0$.

Relation between dielectric constant and electric susceptibility can be given as

$$K = 1 + \chi$$

Electrostatics of Conductors

In electrostatics, the conductors have following properties

(i) Inside a conductor, electrostatic field is zero.

(ii) The interior of a conductor can have no excess charge in the static situation.

(iii) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.

(iv) Electric field at the surface of a charged conductor is given as

$$E = \frac{\sigma}{\varepsilon_0}\,\hat{\mathbf{n}}$$

where, σ is the surface charge density and $\hat{\mathbf{n}}$ is a unit vector normal to the surface in the outward direction.

For $\sigma > 0$, electric field is normal to the surface outward and for $\sigma < 0$, the electric field is normal to the surface inward.

(v) For any charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence. The field inside the cavity is always zero, this is known as **electrostatic shielding.**

Capacitor

It is a device that stores electrical energy. It consists of pair of two conductors of any shape and size carrying charges of equal magnitudes and opposite signs and separated by an insulating medium.

Capacity of a capacitor is the amount of charge required to raise the potential of the capacitor by one unit (in SI one volt).

If q = amount of charge given to one of the conductor and V = potential difference between the conductors, then

Capacity of the capacitor, $C = \dfrac{q}{V}$

Its SI unit is farad (F) or coulomb/volt

Its other units are

$$1\mu F = 10^{-6}\ F$$

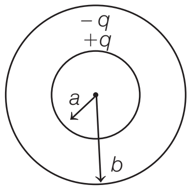
$$1pF = 10^{-12}\ F$$

Its dimensions are $[M^{-1}L^{-2}T^4A^2]$.

Note Capacitance C depends only on the geometrical configuration (shape, size and separation) of the system of conductors. It also depends on the nature dielectric separating two conductors.

Spherical Capacitor

It consists of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential.



Capacitance, $C = \dfrac{Q}{V} = \dfrac{4\pi\varepsilon_0 ab}{b-a}$

For a dielectric (K) between the spheres, $C = \dfrac{4\pi K\varepsilon_0 ab}{b-a}$

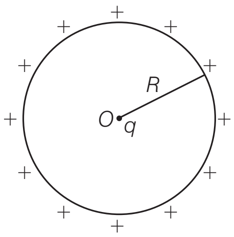
When outer sphere is earthed, $C = \dfrac{4\pi\varepsilon_0 b^2}{(b-a)}$

If both spheres are separated by a distance d, then, the capacitance of the system

$$C = \frac{4\pi\varepsilon_0}{\left(\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{2}{d}\right)}$$

Isolated Spherical Capacitor

For an isolated spherical capacitor of radius R having charge q,



Capacitance in vacuum (or air) is given by

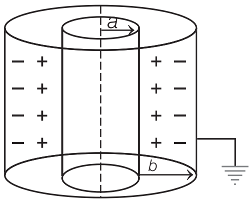
$$C = 4\pi\varepsilon_0 R$$

If sphere is kept in a medium of dielectric constant K or ε_r, then capacity will be

$$C = 4\pi\varepsilon_0\varepsilon_r R = 4\pi\varepsilon_0 KR$$

Cylindrical Capacitor

It consists of two-axial cylinders of radii a and b and length l. The electric field exists in the region between the cylinders. If K is the dielectric constant of the material between the cylinders, then capacitance is given by



$$C = \frac{2\pi K\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)}$$

Example 23. *A cylindrical capacitor has two coaxial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. What will be the potential of the inner cylinder? Neglect and effects (i.e. bending of field lines at the ends).*

(a) 2.8×10^2 V (b) 2.8×10^3 V
(c) 2.8×10^4 V (d) 2.8×10^5 V

Sol. (c) Here, $l = 15$ cm $= 15 \times 10^{-2}$ m; $a = 1.4$ cm $= 1.4 \times 10^{-2}$ m; $b = 1.5$ cm $= 1.5 \times 10^{-2}$ m

$$q = 3.5\,\mu\text{C} = 3.5 \times 10^{-6}\text{ C}$$

Now, $$C = \frac{2\pi\varepsilon_0 l}{2.303\log_{10}\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12} \times 15 \times 10^{-2}}{2.303\log_{10}\frac{1.5 \times 10^{-2}}{1.4 \times 10^{-2}}} = 1.12 \times 10^{-10}\text{ F}$$

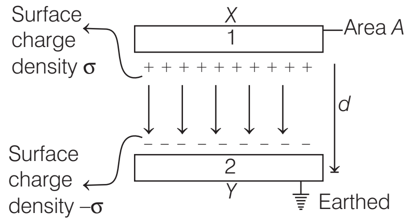
Since, outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them.

Therefore, potential of inner cylinder,

$$V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.21 \times 10^{-10}} = 2.89 \times 10^4\text{ V}$$

Parallel Plate Capacitor

It consists of two metal plates parallel to each other and separated by a distance that is very small as compared to the dimensions of the plates.



Intensity of the electric field between the plates of a capacitor,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

Potential difference between the plates,

$$V = Ed = \frac{qd}{A\varepsilon_0}$$

where, d is the distance between the conductor plates.

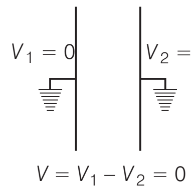
Its capacity, $C = \frac{\varepsilon_0 A}{d}$

Following are few important cases related to parallel plate capacitor

(i) If both plates of parallel plate capacitor are connected by a metallic wire, then

$$C = \frac{q}{V} = \frac{q}{0} = \infty$$

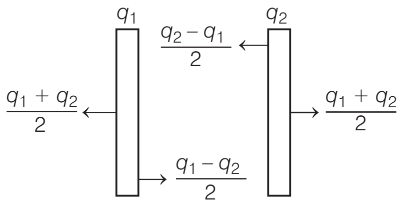
(ii) If both plates are earthed,



$$C = \frac{q}{V} = \frac{q}{0} = \infty$$

(iii) If two plates have different charges, then

$$q = \frac{(q_1 - q_2)}{2} \text{ and } V = \frac{(q_1 - q_2)d}{2A\varepsilon_0}$$



$\therefore$ $$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d}$$

Example 24. *A parallel plate capacitor has 1μF capacitance. One of its two plates is given + 2 μC charge and the other plate + 4μC charge. The potential difference developed across the capacitor is* **[JEE Main 2019]**

(a) 1 V (b) 5 V (c) 2 V (d) 3 V

Sol. *(a)* Net value of charge on plates of capacitor after steady state is reached is

$$q_{\text{net}} = \frac{q_2 - q_1}{2}$$

where, q_2 and q_1 are the charges given to plates.
(Note that this formula is valid for any polarity of charge.)

Here, $q_2 = 4\mu C, q_1 = 2\mu C$

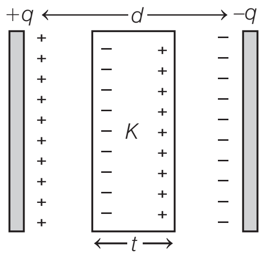
$\therefore$ Charge of capacitor is $q = \Delta q_{\text{net}} = \frac{4-2}{2} = 1\mu C$

Potential difference between capacitor plates is

$$V = \frac{Q}{C} = \frac{1\mu C}{1\mu F} = 1V$$

Effect of Dielectric on Capacitor

When a dielectric slab of dielectric constant K and thickness t is placed between the two plates.



Then, $C = \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{K}} = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{K}\right)}$

(i) If the slab completely fills the space between the plates, then $t = d$, and therefore

$$C = \frac{K\varepsilon_0 A}{d}$$

(ii) If a conducting slab ($K = \infty$) is placed between the plates, then

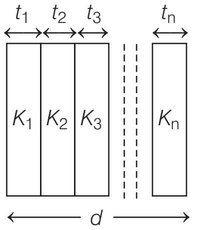
$$C = \frac{\varepsilon_0 A}{d - t}$$

(iii) If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

Then, $C = \infty$

(iv) When more than one dielectric slabs are placed fully between the plates, then

$$C = \frac{A\varepsilon_0}{\left(\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3} + \ldots + \dfrac{t_n}{K_n}\right)}$$



Example 25. *A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be* **[JEE Main 2018]**

(a) 1.2 *nC* *(b)* 0.3 *nC* *(c)* 2.4 *nC* *(d)* 0.9 *nC*

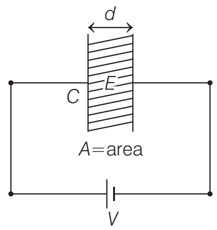
Sol. (a) Magnitude of induced charge is given by

$$Q' = (K-1)\,CV_0$$
$$= \left(\frac{5}{3} - 1\right) 90 \times 10^{-12} \times 20 = 1.2 \times 10^{-9} C$$
$$\Rightarrow \quad Q' = 1.2 \text{ nC}$$

Example 26. *Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is $10^{-4} m^2$. What is the dielectric constant, if the capacitance is 15 pF? (Take, $\varepsilon_0 = 8.86 \times 10^{-12} C^2/N\text{-}m^2$)* **[JEE Main 2019]**

(a) 3.8 *(b)* 8.5 *(c)* 4.5 *(d)* 6.2

Sol. *(b)* As we know, capacitance of a capacitor filled with dielectric medium,



$$C = \frac{\varepsilon_0 KA}{d} \quad \ldots\text{(i)}$$

and electric field between plates is

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E} \quad \ldots\text{(ii)}$$

So, by combining Eqs. (i) and (ii), we get

$$K = \frac{CV}{\varepsilon_0 AE} \quad \ldots\text{(iii)}$$

Given, $C = 15\text{pF} = 15 \times 10^{-12}$ F,

$V = 500$ V, $E = 10^6$ Vm^{-1},

$A = 10^{-4} m^2$ and $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

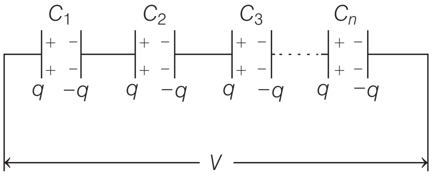
Substituting values in Eq. (iii), we get

$$K = \frac{15 \times 10^{-12} \times 500}{8.85 \times 10^{-12} \times 10^{-4} \times 10^6} = 8.47 \approx 8.5$$

Combination of Capacitors

Series Combination In series combination, capacitors are connected one after another as shown in figure. If a source of emf V volt is connected between points X and Y, then charge on each capacitor is same, *i.e.* $+q$ and $-q$ and the applied potential difference of the source are divided.

If $V_1, V_2, V_3, \ldots, V_n$ are potential differences across capacitors $C_1, C_2, C_3, \ldots, C_n$ respectively, then

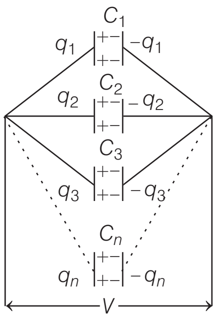


$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

If C_s = equivalent capacity in series combination, then

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

Parallel Combination In parallel combination, capacitors are connected one upon another as shown in figure. If a source of emf V volt is connected between points X and Y, the potential difference across each capacitors is same but charges on different capacitors are different.



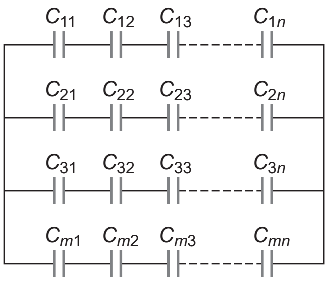
If $q_1, q_2, q_3, \ldots, q_n$ are charges on capacitors $C_1, C_2, C_3, \ldots C_n$ respectively, then charge delivered by source,

$$q = q_1 + q_2 + q_3 + \ldots + q_n$$

If C_p = equivalent capacity in parallel combination, then

$$C_p = C_1 + C_2 + C_3 + \ldots + C_n$$

Mixed Combination If n capacitors each of capacity C are connected in a series to form n row and m such rows are connected in parallel as shown in figure.

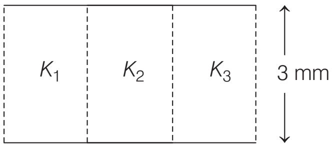


The number of capacitors used, $N = m \times n$

Hence, equivalent capacity of the combination,

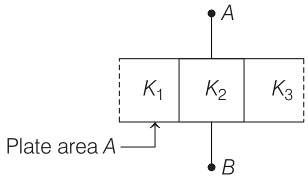
$$C_{eq} = m \times \frac{C}{n}$$

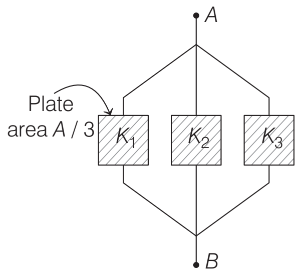
Example 27. *A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which give same capacitance when fully inserted in above capacitor, would be* ***[JEE Main 2019]***



(a) 4 (b) 36
(c) 12 (d) 14

Sol. (c) In the given arrangement, capacitor can be viewed as three-different capacitors connected in parallel as shown below





So, equivalent capacity of the system is

$$C_{eq} = C_1 + C_2 + C_3$$

$$\Rightarrow \quad \frac{K\varepsilon_0 A}{d} = \frac{K_1\varepsilon_0 A/3}{d} + \frac{K_2\varepsilon_0 A/3}{d} + \frac{K_3\varepsilon_0 A/3}{d}$$

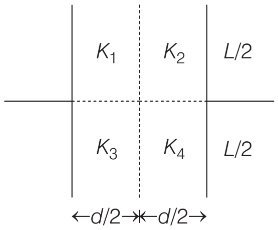
$$\Rightarrow \quad K = \frac{K_1}{3} + \frac{K_2}{3} + \frac{K_3}{3}$$

Here $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$

So, $$K = \frac{10 + 12 + 14}{3}$$

$$\Rightarrow \quad K = 12$$

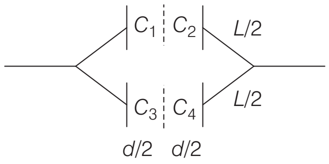
Example 28. *A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be* ***[JEE Main 2019]***



(a) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$ (b) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(c) $K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$ (d) $K = \frac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

Sol. (*) This capacitor system can be converted into two parts as shown in the figure



where C_1, C_2, C_3 and C_4 are capacitance of the capacitor having dielectric constants K_1, K_2, K_3 and K_4, respectively.

Here, $C_1 = \dfrac{K_1\varepsilon_0 A/2}{d/2} = \dfrac{K_1\varepsilon_0 A}{d}$

Similarly, $C_2 = \dfrac{K_2\varepsilon_0 A}{d}$, $C_3 = \dfrac{K_3\varepsilon_0 A}{d}$ and $C_4 = \dfrac{K_4\varepsilon_0 A}{d}$

Since, equivalent capacitance in series combination is

$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Here, C_1, C_2 and C_3, C_4 are in series combination.

$$\therefore \quad (C_{eq})_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$= \frac{\dfrac{K_1\varepsilon_0 A}{d} \cdot \dfrac{K_2\varepsilon_0 A}{d}}{\dfrac{K_1\varepsilon_0 A}{d} + \dfrac{K_2\varepsilon_0 A}{d}}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d}$$

Similarly, $(C_{eq})_{34} = \dfrac{K_3 \cdot K_4}{K_3 + K_4} \cdot \dfrac{\varepsilon_0 A}{d}$

Now, $(C_{eq})_{12}$ and $(C_{eq})_{34}$ are in parallel combination.

$$\therefore \quad C_{net} = (C_{eq})_{12} + (C_{eq})_{34}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d} + \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C_{net} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 A}{d} \quad \text{...(i)}$$

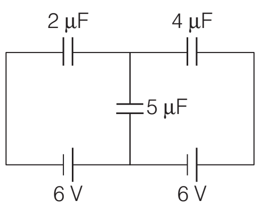
If K is effective dielectric constant, then $C_{net} = \dfrac{K\varepsilon_0 A}{d}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{K\varepsilon_0 A}{d} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 A}{d}$$

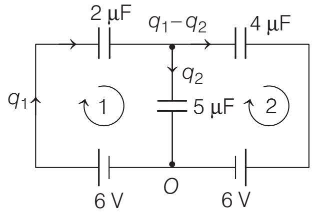
or
$$K = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)$$

Example 29. *In the circuit shown, charge on the* $5\,\mu F$ *capacitor is* ***[JEE Main 2020]***



(a) $18.00\,\mu C$ *(b)* $10.90\,\mu C$
(c) $16.36\,\mu C$ *(d)* $5.45\,\mu C$

Sol. (c) Let q_2 be the required charge on $5\,\mu F$ capacitor. The given circuit is shown below



Applying KVL in loop 1,

$$6 - \frac{q_1}{2} - \frac{q_2}{5} = 0 \quad \text{...(i)}$$

Now, applying KVL in loop 2,

$$-\left(\frac{q_1 - q_2}{4}\right) - 6 + \frac{q_2}{5} = 0 \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$-\left(\frac{q_1 - q_2}{4}\right) - \frac{q_1}{2} = 0$$

$$\Rightarrow \quad -\frac{q_1}{4} + \frac{q_2}{4} - \frac{q_1}{2} = 0$$

$$\frac{q_2}{4} = \frac{q_1}{2} + \frac{q_1}{4}$$

$$\Rightarrow \quad \frac{q_2}{4} = \frac{3q_1}{4}$$

$$\Rightarrow \quad q_1 = \frac{q_2}{3} \quad \text{...(iii)}$$

Substituting the value of q_1 in Eq. (i), we get

$$6 - \frac{q_2}{6} - \frac{q_2}{5} = 0 \Rightarrow 6 = \frac{5q_2 + 6q_2}{30}$$

$$\frac{11}{30} q_2 = 6 \Rightarrow q_2 = \frac{180}{11}\,\mu C = 16.36\,\mu C$$

Example 30. *Effective capacitance of parallel combination of two capacitors* C_1 *and* C_2 *is* $10\,\mu F$*. When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor* C_2 *is 4 times that of* C_1*. If these capacitors are connected in series, then their effective capacitance will be* ***[JEE Main 2020]***

(a) $4.2\,\mu F$ *(b)* $3.2\,\mu F$ *(c)* $1.6\,\mu F$ *(d)* $8.4\,\mu F$

Sol. (c) Let capacitances of capacitors are C_1 and C_2.

Given, energy stored in capacitor 1 = 4 × energy stored in capacitor 2

$$\Rightarrow \quad \frac{1}{2} C_1 \times V^2 = 4 \times \frac{1}{2} C_2 \times V^2$$

$$\Rightarrow \quad C_1 = 4C_2$$

Also, equivalent capacity in parallel combination is $10\,\mu F$.

$$\Rightarrow \quad C_1 + C_2 = 10$$

$$\Rightarrow \quad 5C_2 = 10 \quad (\because C_1 = 4C_2)$$

$$\Rightarrow \quad C_2 = 2\,\mu F$$

So, $C_1 = 4 \times 2 = 8\,\mu F$

Equivalent capacity in series combination will be

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 2}{8 + 2} = 1.6\,\mu F$$

Energy Stored in a Conductor

The work done in charging a conductor is stored as electrostatic potential energy of the charged conductor.

∴ Electrostatic potential energy of the charged conductor or capacitor,

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \times V = \frac{1}{2} CV^2$$

The energy stored per unit volume of space in a capacitor is called **energy density**. Charge on either plate of capacitor is

$$Q = \sigma A = \varepsilon_0 EA$$

Energy stored in the capaitor is

$$U = \frac{Q}{2C} = \frac{(\varepsilon_0 EA)^2}{2 \cdot \varepsilon_0 A/d} = \frac{1}{2} \varepsilon_0 E^2 \cdot Ad$$

Energy density, $u = \frac{\text{Energy stored}}{\text{Volume of capacitor}} = \frac{U}{Ad}$

∴ $$u = \frac{1}{2} \varepsilon_0 E^2$$

When the dielectric is introduced in between parallel plate capacitor

(i) and battery is removed

$$C' = KC, \quad Q' = Q$$

$$V' = \frac{V}{K}, \quad E' = \frac{E}{K}$$

and $$U' = \frac{U}{K}$$

(ii) and battery remains connected

$$C' = KC, \; Q' = KQ, \; V' = V, \; E' = E \text{ and } U' = KU$$

Note Force between the plates of a parallel plate capacitor is

$$|F| = \frac{\sigma A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$$

Example 31. *A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?* **[JEE Main 2019]**

(a) 6.25×10^{-6} J (b) 2.16×10^{-6} J
(c) 2.55×10^{-6} J (d) 3.75×10^{-6} J

Sol. (d) Potential energy stored in a capacitor is

$$U = \frac{1}{2} QV = \frac{1}{2}\frac{Q^2}{C}$$

So, initial energy of the capacitor, $U_i = \frac{1}{2} Q^2 / C_1$

Final energy of the capacitor, $U_f = \frac{1}{2} Q^2 / C_2$

As we know, work done, $W = \Delta U = U_f - U_i = \frac{1}{2} Q^2 \left[\frac{1}{C_2} - \frac{1}{C_1}\right]$

Here, $Q = 5\,\mu\text{C} = 5 \times 10^{-6}$ C,

$C_1 = 5\,\mu\text{F} = 5 \times 10^{-6}$F

and $C_2 = 2\,\mu\text{F} = 2 \times 10^{-6}$ F.

⇒ $$\Delta U = \frac{1}{2} \times (5 \times 10^{-6})^2 \left[\frac{1}{2 \times 10^{-6}} - \frac{1}{5 \times 10^{-6}}\right]$$

$$= \frac{1}{2} \times \frac{5 \times 5 \times 10^{-12}}{10^{-6}} \times \frac{3}{10}$$

$$= \frac{25 \times 3}{20} \times 10^{-6} \text{ J}$$

⇒ $$\Delta U = 3.75 \times 10^{-6} \text{ J}$$

∴ Work done in reducing the capacitance from 5 μF to 2 μF by pulling plates of capacitor apart is 3.75×10^{-6} J.

Redistribution of Charge

When two isolated charged conductors are connected to each other, then charge is redistributed in the ratio of their capacitances.

Common potential, $V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$\text{Energy loss } = \frac{1}{2} \frac{C_1 C_2 (V_1 - V_2)^2}{(C_1 + C_2)}$$

This energy is lost in the form of heat in connecting wires.

When n small drops, each of capacitance C, charged to potential V with charge q, surface charge density σ and potential energy U coalesce to form a single drop.

Then for new drop,

Total charge = nq, total capacitance = $n^{1/3} C$,

total potential = $n^{2/3}\, V$, surface charge density = $n^{1/3}\, \sigma$

and total potential energy = $n^{2/3}\, U$.

Example 32. *A 10 μF capacitor is fully charged to a potential difference of 50 V. After removing the source voltage, it is connected to an uncharged capacitor in parallel. Now, the potential difference across them becomes 20 V. The capacitance of the second capacitor is* **[JEE Main 2020]**

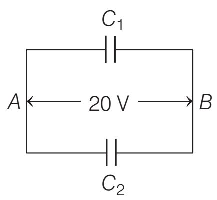
(a) 15 μF (b) 30 μF
(c) 20 μF (d) 10 μF

Sol. (a) Initially we have a capacitor of capacitance, $C_1 = 10$ μF and $V_1 = 50$ V

Charge on capacitor is

$$Q = C_1 V_1 = 10\,\mu\text{F} \times 50\,\text{V} = 500\,\mu\text{C}$$

Now, another uncharged capacitor of capacity C_2 is connected in parallel with C_1 as shown in figure.



Given that, after redistribution, voltage or common potential is 20 V.

Final charge on C_1 is

$$Q_1 = C_1V = 10\ \mu F \times 20V = 200\ \mu C$$

So, another capacitor has a charge of $Q_2 = 500 - 200 = 300\ \mu F$

Potential, $V = 20$ V

$\therefore$ Capacity, $C_2 = \frac{Q_2}{V} = \frac{300\ \mu F}{20\ V} = 15\ \mu F$

Example 33. *Two isolated conducting spheres S_1 and S_2 of radii $\frac{2}{3}R$ and $\frac{1}{3}R$, have charges $12\ \mu C$ and $-3\mu C$ respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done, the charges on S_1 and S_2 respectively, are*

[JEE Main 2020]

(a) $4.5\ \mu C$ on both
(b) $+4.5\ \mu C$ and $-4.5\ \mu C$
(c) $3\ \mu C$ and $6\ \mu C$
(d) $6\ \mu C$ and $3\ \mu C$

Sol. (*d*) When both spheres are connected by a conducting wire, charge from higher potential sphere flows to lower potential sphere till both spheres reach a common potential V_C which is given by

$$V_C = \frac{Q_1 + Q_2}{C_1 + C_2}$$

For a sphere, capacitance, $C = 4\pi\varepsilon_0 R$

so,
$$V_C = \frac{12\times10^{-6} + (-3\times10^{-6})}{4\pi\varepsilon_0(R_1 + R_2)}$$

$$= \frac{9\times10^{-6}}{4\pi\varepsilon_0\left(\frac{2}{3}R + \frac{1}{3}R\right)} = \frac{9\times10^{-6}}{4\pi\varepsilon_0 R}$$

Values of charges on spheres after redistribution are

Q_1 = charge on first sphere

$$Q_1 = C_1V_C = 4\pi\varepsilon_0 R_1 \cdot \frac{(9\times10^{-6})}{4\pi\varepsilon_0 R}s$$

$$= \frac{\left(4\pi\varepsilon_0 \frac{2}{3}R\right)(9\times10^{-6})}{4\pi\varepsilon_0 R}$$

$$= \frac{2}{3}\times9\times10^{-6}\ C$$

$$= \frac{2}{3}\times9\mu C = 6\mu C$$

and Q_2 = charge on second sphere

$$Q_2 = C_2V_C$$

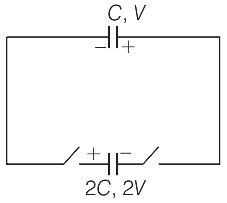
$$= \frac{4\pi\varepsilon_0\left(\frac{1}{3}R\right)\times9\times10^{-6}}{4\pi\varepsilon_0 R} = 3\times10^{-6}\ C = 3\mu C$$

Example 34. *Two capacitors of capacitances C and $2C$ are charged to potential differences V and $2V$, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is*

[JEE Main 2020]

(a) $\frac{9}{2}CV^2$
(b) $\frac{3}{2}CV^2$
(c) $\frac{25}{6}CV^2$
(d) zero

Sol. (*b*) Before connecting the two capacitors in parallel,
Charge on capacitor C, $Q_1 = CV$ and charge on capacitor $2C$, $Q_2 = 2C\times2V = 4CV$

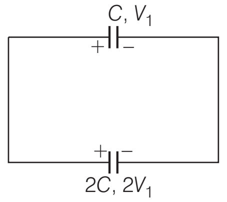


After connecting the two capacitors in parallel,
Using law of conservation of charge,

$$4CV - CV = (C + 2C)V_{common}$$

$$V_{common} = \frac{4CV - CV}{3C} = V$$

$$V_{common} = V$$



Now, final energy of the configuration,

$$U_f = \frac{1}{2}CV^2 + \frac{1}{2}\times2CV^2 = \frac{3}{2}CV^2$$

Practice Exercise

ROUND I Topically Divided Problems

Electric Charge, Coulomb's Force and Electric Field

1. Two identical spheres carrying charges $-9\,\mu C$ and $5\,\mu C$ respectively are kept in contact and then separated from each other. Which amongst the following statement is (are) correct?
(a) 1.25×10^{13} electrons are in excess.
(b) 1.25×10^{13} electrons are in deficit.
(c) 4.15×10^{12} electrons are in excess.
(d) None of the above

2. A glass rod rubbed with silk is used to charge a gold leaf electroscope and the leaves are observed to diverse. The electroscope thin charged leaf is exposed to X-rays for short period, then
(a) the leaves will diverge further
(b) the leaves will melt
(c) the leaves will not be affected
(d) None of the above

3. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and other is reduced by 10%. The new force of repulsion at the same distance would be
(a) 100 N (b) 121 N
(c) 99 N (d) None of these

4. Three charges each of $+1\mu C$ are placed at the corners of an equilateral triangle. If the force between any two charges be F, then the net force on either charge will be
(a) $\sqrt{2}F$ (b) $F\sqrt{3}$
(c) $2F$ (d) $3F$

5. Three charges $+Q$, q and $+Q$ are placed respectively at distance 0, $\frac{d}{2}$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x=0$ is zero, then value of q is **[JEE Main 2019]**
(a) $\frac{+Q}{2}$ (b) $\frac{+Q}{4}$
(c) $\frac{-Q}{2}$ (d) $\frac{-Q}{4}$

6. If two charges are placed at a distance of 5 cm. If a brass sheet is placed between them, the force between two charges will be
(a) decrease to 0 (b) increase to ∞
(c) increase to 0 (d) decrease to ∞

7. The electrostatic force of repulsion between two positively charged ions carrying equal charge is 3.7×10^{-9} N, when they are separated by a distance of $5\,\text{Å}$. What are the number of electrons are missing from each ion?
(a) 2 (b) 4
(c) 0 (d) 10

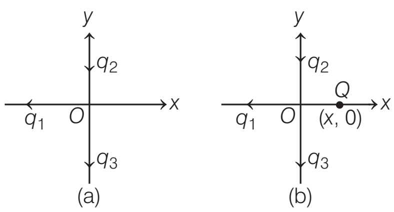
8. Two point charges $+3\mu C$ and $+8\mu C$ repel each other with a force of 40 N. If a charge of $-5\mu C$ is added to each of them, then the force between them will become
(a) -10 N (b) $+10$ N
(c) $+20$ N (d) -20 N

9. A charged spherical conductor of radius R carries a charge q_0. A point test charge q_0 is placed at a distance x from the surface of the conductor. The force experienced by the test charge will be proportional to
(a) $(R+x)^2$ (b) $(R-x)^2$
(c) $\frac{1}{(R-x)^2}$ (d) $\frac{1}{(R+x)^2}$

10. Charge $q_1 = +6.0$ nC is on Y-axis at $y=+3$ cm and charge $q_2 = -6.0$ nC is on Y-axis at $y=-3$ cm. Calculate force on a test charge $q_0 = 2$ nC placed on X-axis at $x=4$ cm.
(a) $-51.8\,\hat{j}\,\mu N$ (b) $+51.8\,\hat{j}\,\mu N$
(c) $-5.18\,\hat{j}\,\mu N$ (d) $5.18\,\hat{j}\,\mu N$

11. Two point charges exert on each other a force F when they are placed r distance apart in air. If they are placed R distance apart in a medium of dielectric constant K, they exert the same force. The distance R equals
(a) $\frac{r}{K}$ (b) rK
(c) $r\sqrt{K}$ (d) $\frac{r}{\sqrt{K}}$

12. In figure, two positive charges q_2 and q_3 fixed along the Y-axis, exert a net electric force in the $+x$-direction on a charge q_1 fixed along the X-axis. If a positive charge Q is added at $(x, 0)$, the force on q_1 **[NCERT Exemplar]**



(a) shall increase along the positive X-axis
(b) shall decrease along the positive X-axis
(c) shall point along the negative X-axis
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

13. A pitch ball A of mass 9×10^{-5} kg carries a charge of $5\,\mu C$. What must be the magnitude and sign of the charge on a pitch ball B held 2 cm directly above the pitch ball A, such that the pitch ball A, remains stationary?

(a) 5×10^{-6} C (b) 5×10^{-12} C
(c) 8×10^{-6} C (d) 7.84×10^{-12} C

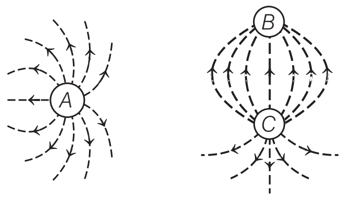
14. There are two charged identical metal spheres A and B repel each other with a force 3×10^{-5} N. Another identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Net force on C is

(a) 1×10^{-5} N (b) 2×10^{-5} N
(c) 1.5×10^{-5} N (d) 3×10^{-5} N

15. Two small conducting sphere of equal radii have charges $+10\,\mu C$ and $-20\,\mu C$ respectively and placed at a distance R from each other experience force F_1. If they are brought in contact and separated to the same distance,they experience force F_2. The ratio of F_1 to F_2 is

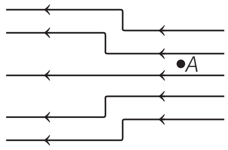
(a) 1 : 2 (b) –8 : 1
(c) 1 : 8 (d) –2 : 1

16. Figure shows the electric field lines around three point charges A, B and C. Which charge has the largest magnitude? **[NCERT Exemplar]**



(a) Charge A
(b) Charge B
(c) Charges A and B
(d) Charge C

17. In the electric field shown in figure, the electric lines in the left have twice the separation as that between those on right. If the magnitude of the field at point A is $40\ NC^{-1}$, the force experienced by a proton placed at point A is

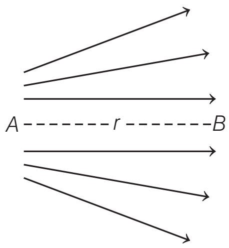


(a) 6.4×10^{-18} N (b) 3.2×10^{-15} N
(c) 5.0×10^{-12} N (d) 1.2×10^{-18} N

18. Two conducting sphere of radii r_1 and r_2 are charged to the same surface charge density. The ratio of electric field near their surface is

(a) r_1^2 / r_2^2 (b) r_2^2 / r_1^2
(c) r_1 / r_2 (d) 1 : 1

19. Figure shows the electric lines of force energy from a charged body. If the electric field at A and B are E_A and E_B, respectively and the displacement between A and B is r, then



(a) $E_A < E_B$ (b) $E_A > E_B$
(c) $E_A = E_B$ (d) $E_A = 2E_B$

20. Two point charges $-q$ and $+q/2$ are situated at the origin and at the point $(a, 0, 0)$, respectively. The point along the X-axis, whereas the electric field vanished, is

(a) $x = \dfrac{\sqrt{2}a}{\sqrt{2}-1}$ (b) $X = \sqrt{2}a - \sqrt{2} - 1$
(c) $x = (\sqrt{2}-1)\sqrt{2}a$ (d) None of these

21. The maximum field intensity on the axis of a uniformly charged ring of charge q and radius R will be

(a) $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{3\sqrt{3}R^2}$

(b) $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2q}{3R^2}$

(c) $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2q}{3\sqrt{3}R^2}$

(d) $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{3q}{2\sqrt{3}R^2}$

22. An insulated sphere of radius R has charge density ρ. The electric field at a distance r from the centre of the sphere ($r < R$)

(a) $\frac{\rho r}{3\varepsilon_0}$ (b) $\frac{\rho R}{3\varepsilon_0}$

(c) $\frac{\rho r}{\varepsilon_0}$ (d) $\frac{\rho R}{\varepsilon_0}$

23. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then, value of h is **[JEE Main 2019]**

(a) $\frac{R}{\sqrt{2}}$ (b) $R\sqrt{2}$

(c) R (d) $\frac{R}{\sqrt{5}}$

24. Two point charges q_1 ($\sqrt{10}\,\mu$C) and q_2 ($-25\,\mu$C) are placed on the X-axis at $x = 1$ m and $x = 4$ m, respectively. The electric field (in V/m) at a point $y = 3$ m on Y-axis is

$\left(\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N-m}^2\text{C}^{-2}\right)$ **[JEE Main 2019]**

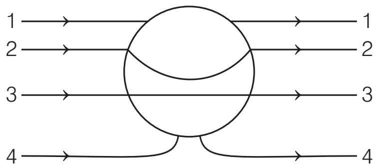
(a) $(63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2$

(b) $(81\,\hat{\mathbf{i}} - 81\hat{\mathbf{j}}) \times 10^2$

(c) $(-81\,\hat{\mathbf{i}} + 81\,\hat{\mathbf{j}}) \times 10^2$

(d) $(-63\,\hat{\mathbf{i}} + 27\hat{\mathbf{j}}) \times 10^2$

25. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the paths shown in figure



(a) 1 (b) 2

(c) 3 (d) 4

26. A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is **[NCERT Exemplar]**

(a) directed perpendicular to the plane and away from the plane

(b) directed perpendicular to the plane but towards the plane

(c) directed radially away from the point charge

(d) directed radially towards the point charge

27. A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed **[NCERT Exemplar]**

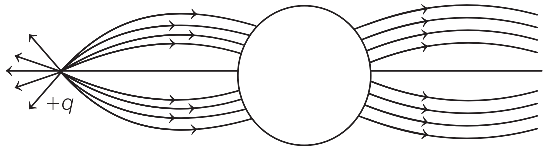
(a) perpendicular to the diameter

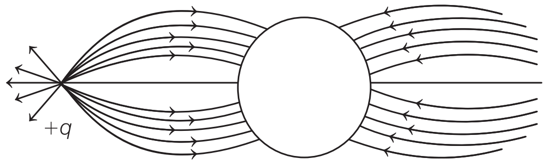
(b) parallel to the diameter

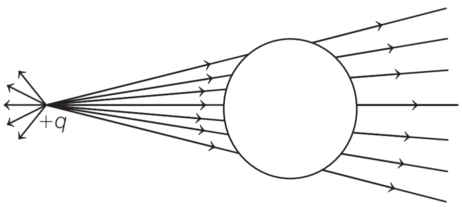
(c) at an angle tilted towards the diameter

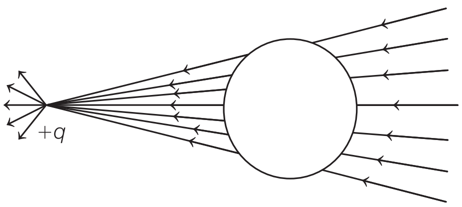
(d) at an angle tilted away from the diameter

28. A point positive charge brought near an isolated conducting sphere (figure). The electric field is best given by **[NCERT Exemplar]**

(a) 

(b) 

(c) 

(d) 

29. In a region of space, the electric field is given by $\mathbf{E} = 8\hat{\mathbf{i}} +`4\hat{\mathbf{j}} + 3\mathbf{k}$. The electric flux through a surface of area of 100 units XY-plane is

(a) 800 units

(b) 300 units

(c) 400 units

(d) 1500 units

Electric Flux and Gauss's Theorem

30. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by

(a) zero

(b) $\pi R^2 / E$

(c) $2\,\pi R^2 E$

(c) None of the above

31. Suppose an imaginary cube is with a charge situated at the centre of it. The total electric flux passing through each of the faces of the cube will be

(a) $\frac{q}{6\varepsilon_0}$

(b) $\frac{q}{2\varepsilon_0}$

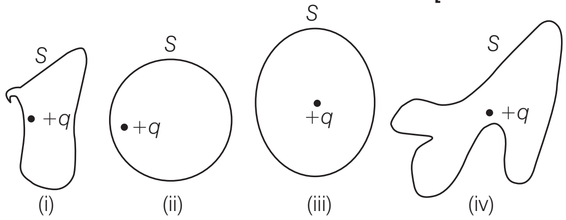
(c) $\frac{q}{12\varepsilon_0}$

(c) None of the above

32. In finding the electric field using Gauss's law, the formula $|\mathbf{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula, ε_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation? **[JEE Main 2020]**

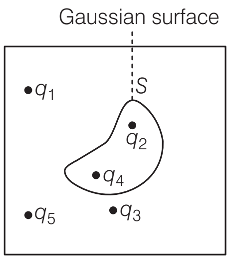
(a) Only when the Gaussian surface is an equipotential surface and $|\mathbf{E}|$ is constant on the surface.
(b) Only when the Gaussian surface is an equipotential surface.
(c) For any choice of Gaussian surface.
(d) Only when $|\mathbf{E}|$ = constant on the surface.

33. The electric flux through the surface **[NCERT Exemplar]**



(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig .(ii) is same as fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

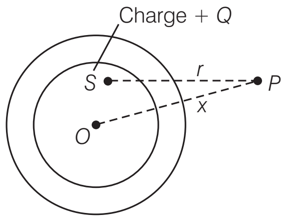
34. Five charges q_1, q_2, q_3, q_4 and q_5 are fixed at their positions as shown in figure, S is a Gaussian surface. The Gauss's law is given by $\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$.



Which of the following statements is correct? **[NCERT Exemplar]**

(a) $\mathbf{E}$ on the LHS of the above equation will have a contribution from q_1, q_5 and q_3 on the RHS will have a contribution from q_2 and q_4 only.
(b) $\mathbf{E}$ on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.
(c) $\mathbf{E}$ on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only.
(d) Both $\mathbf{E}$ on the LHS and q on the RHS will have contributions from q_2 and q_4 only.

35. The adjacent diagram shows a charge $+Q$ held on an insulating support S and enclosed by a hollow spherical conductor, O represents the centre of the spherical conductor and P is a point such that $OP = x$ and $SP = r$. The electric field at point, P will be



(a) zero
(b) $\frac{Q}{4\pi\varepsilon_0 x^2}$
(c) $\frac{Q}{\varepsilon_0 x^2}$
(d) None of these

36. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre, a and b are constants. Then, the charge density inside the ball is

(a) $-24\pi\, a\, \varepsilon_0 r$
(b) $-6a\varepsilon_0$
(c) $-24\,\pi\,\varepsilon_0$
(d) $-6\,a\varepsilon_0\, r$

37. In infinite parallel plane sheet of a metal is charged to charge density σ coulomb per square metre in a medium of dielectric constant K. Intensity of electric field near the metallic surface will be

(a) $E = \frac{\sigma}{\varepsilon_0 K}$
(b) $E = \frac{K}{3\,\varepsilon_0}$
(c) $E = \frac{\sigma}{2\,\varepsilon_0 K}$
(d) $E = \frac{K}{2\,\varepsilon_0}$

38. A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders? **[NCERT]**

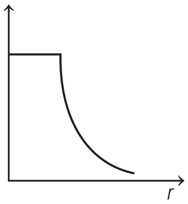
(a) $\frac{\lambda}{2\pi\varepsilon_0 r}$
(b) $\frac{\lambda r}{\sqrt{2}\,\pi\varepsilon_0}$
(c) $\frac{\lambda}{\sqrt{2}\,\pi\varepsilon_0 r}$
(d) None of these

39. A spherical charged conductor has σ as the surface density of charge. The electric field on its surface is E. If the radius of the sphere is doubled, keeping the surface density of the charge unchanged, what will be the electric field on the surface of the new sphere?

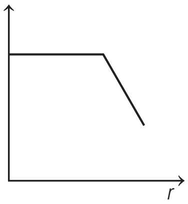
(a) $\frac{E}{4}$
(b) $\frac{E}{2}$
(c) E
(d) $2E$

40. Which one of the following graphs shows the variation of electric field strength E with distance d from the centre of the hollow conducting sphere?

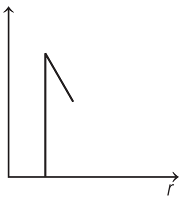
(a)



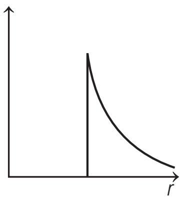
(b)



(c)



(d)



41. A positive point charge q is carried from a point B to a point A in the electric field of a point charge $+Q$ at O. If the permittivity of free space is ε_0, the work done in the process is given by (where, $a = OA$ and $b = OR$)

(a) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{a}+\frac{1}{b}\right)$ (b) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

(c) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$ (d) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$

42. The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard.

S_1 : At any point inside the sphere, electric intensity is zero.

S_2 : At any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statements. **[NCERT Exemplar]**

(a) S_1 is true but S_2 is false.

(b) Both S_1 and S_2 are false.

(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2.

(d) S_1 is true, S_2 is also true but the statements are independent.

43. A hollow conducting sphere of radius R has a charge $(+Q)$ on its surface. What is the electric potential within the sphere at a distance $r = R/3$ from its centre?

(a) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$ (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$ (d) Zero

44. n small drops of same size are charged to V volt each. If they coalesce to form a single large drop, then its potential will be

(a) Vn (b) Vn^{-1}

(c) $Vn^{1/3}$ (d) $Vn^{2/3}$

45. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0 \ldots\infty$, on the X-axis and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0 \ldots \infty$. Here, x_0 is the constant. Take, the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\varepsilon_0 r$. Then, the potential at the origin due to the above system of charges is

(a) $\frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$ (b) $\frac{q}{8\pi\varepsilon_0 x_0}\log_e 2$

(c) 0 (d) ∞

46. The charge of $+\frac{10}{3}\times 10^{-9}$ C are placed at each of the four corners of a square of side 8 cm. The potential at the point of intersection of the diagonals, is

(a) $1500\sqrt{2}$ V (b) $1800\sqrt{2}$ V

(c) $600\sqrt{2}$ V (d) $900\sqrt{2}$ V

47. A cube of side b has a charge q at each of its vertices. Determine the potential due to this charge array at the centre of the cube.

(a) $\frac{4q}{\sqrt{3}\,\pi\varepsilon_0 b}$ (b) $\frac{3q}{\sqrt{2}\,\pi\,\varepsilon_0 b}$

(c) $\frac{3q}{\sqrt{2}\,\pi\varepsilon_0 b^2}$ (d) $\frac{2q}{\sqrt{3}\,\pi\varepsilon_0 b}$

48. The tangential component of electrostatic field is continuous from one side of a charged surface to another is **[NCERT]**

(a) $\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_A}+\frac{1}{r_B}-\frac{1}{r_C}\right)$ (b) zero

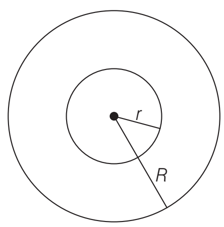
(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_A}-\frac{1}{r_B}+\frac{1}{r_C}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_A}+\frac{1}{r_B}+\frac{1}{r_C}\right)$

49. Four equal point charges Q each are placed in the XY-plane at (0, 2), (4, 2), (4, −2) and (0, − 2). The work required to put a fifth charge Q at the origin of the coordinate system (in joule) will be **[JEE Main 2019]**

(a) $\frac{Q^2}{4\pi\varepsilon_0}$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

50. A charge Q is distributed over two concentric conducting thin spherical shells of radii r and R $(R > r)$. If the surface charge densities on the two shells are equal, the electric potential at the common centre is **[JEE Main 2020]**



(a) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

51. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge $+q$ each, while 2, 4, 6, 8, 10 have charge $-q$ each. The potential V and the electric field E at the centre of the circle respectively, are (Take, $V=0$ at infinity) **[JEE Main 2020]**

(a) $V=\frac{10q}{4\pi\varepsilon_0 R}$; $E=0$ (b) $V=0$; $E=\frac{10q}{4\pi\varepsilon_0 R^2}$

(c) $V=0$; $E=0$ (d) $V=\frac{10q}{4\pi\varepsilon_0 R}$; $E=\frac{10q}{4\pi\varepsilon_0 R^2}$

52. A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge **[NCERT Exemplar]**

(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

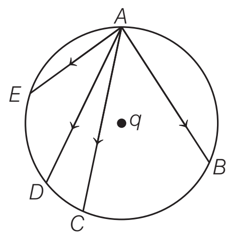
53. The work done by electric field done during the displacement of a negatively charged particle towards a fixed positively charged particle is 9 J. As a result, the distance between the charges has been decreased by half. What work is done by the electric field over the first half of this distance?

(a) 3 J (b) 6 J
(c) 1.5 J (d) 9 J

54. A charge 5 μC is placed at a point. What is the work required to carry 1C of charge once round it in a circle of 12 cm radius?

(a) 100 (b) 0 (c) 1 (d) ∞

55. In the electric field of a point charge q at a certain point charge is carried from point A to B, C, D and E as shown in figure. The work done is



(a) least along the path AE
(b) least along the path AC
(c) zero along any of the paths
(d) least along AB

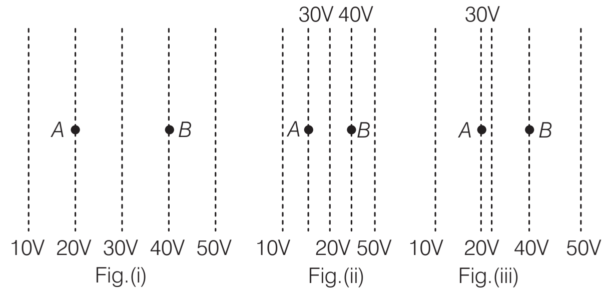
56. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately **[NCERT Exemplar]**

(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

57. An electric field is given by $\mathbf{E}=(y\hat{\mathbf{i}}+x\hat{\mathbf{j}})\ \text{NC}^{-1}$. The work done (in joule) in moving a 1 C charge from $\mathbf{r}_A=(2\hat{\mathbf{i}}+2\hat{\mathbf{j}})$ m to $\mathbf{r}_B=$ (in joule $4\hat{\mathbf{i}}+2\hat{\mathbf{j}}$) m is

(a) 2 y (b) 3 y
(c) zero (d) infinity

58. Figures shows some equipotential lined distributed in space. A charged object is moved from point A to point B. **[NCERT Exemplar]**

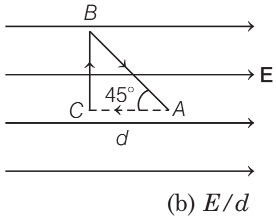


(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done in Fig. (i), Fig. (ii) and Fig. (iii) is equal
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in Fig. (i)

59. The potential at a point x (measured in μm) due to some charges situated on the X-axis is given by $V(x)=20/(x^2-4)$ volt.
The electric field E at $x=4\ \mu$m is given by **[AIEEE 2007]**

(a) $\frac{5}{3}$ V/μm and in the –ve x-direction
(b) $\frac{5}{3}$ V/μm and in the +ve x-direction
(c) $\frac{10}{9}$ V/μm and in the –ve x-direction
(d) $\frac{10}{9}$ V/μm and in the +ve x-direction

60. A test charge q_0 is moved without acceleration from A to C and covers the path ABC as shown in figure. The potential difference between A and C is



(a) Ed (b) E/d
(c) 2 Ed (d) $Ed/2$

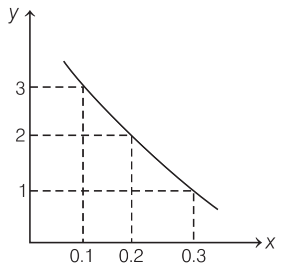
Electric Potential and Electric Potential Energy

61. A non-conducting ring of radius 0.5 m carries total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field everywhere in space.

The value of the line integral $\oint_{l=\infty}^{l=0} -E \cdot dl$ ($l = 0$, being centre of ring) in volt is

(a) + 2 (b) –1
(c) – 2 (d) zero

62. The variation of potential V with distance x from a fixed point charge is shown in figure. The electric field strength between $x = 0.1$ m and 0.3 m is



(a) $+ 0.4 \text{ Vm}^{-1}$ (b) $- 0.4 \text{ Vm}^{-1}$
(c) $+ 10 \text{ Vm}^{-1}$ (d) $- 10 \text{ Vm}^{-1}$

63. The electric potential V at any point (x, y, z) in space is given by $V = 4x^2$. The electric field at (1, 0, 2) m in Vm^{-1} is

(a) 8, along negative X-axis
(b) 8, along positive X-axis
(c) 16, along negative X-axis
(d) 16, along positive Z-axis

64. A ball of mass 1 kg carrying a charge 10^{-8} C moves from a point A at potential 600 V to a point B at zero potential. The change in its kinetic energy is

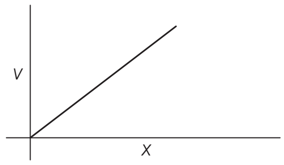
(a) -6×10^{-6} erg
(b) -6×10^{-6} J
(c) 6×10^{-6} J
(d) 6×10^{-6} erg

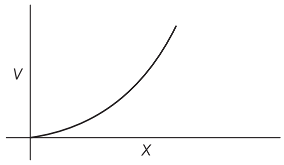
65. A charge $-q$ and another charge $+Q$ are kept at two points A and B, respectively. Keeping the charge $+Q$ fixed at B, the charge $-q$ at A is moved to another point C such that ABC forms an equilateral triangle of side l. The net work done in moving the charge $-q$ is

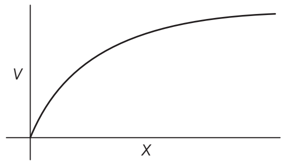
(a) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l^2}$

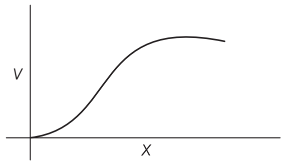
(c) $\frac{1}{4\pi\varepsilon_0}Qql$ (d) zero

66. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale) **[JEE Main 2020]**

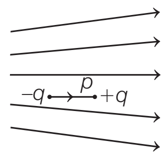
(a)


(b)


(c)


(d)


67. Figure shows electric field lines in which an electric dipole **p** is placed as shown. Which of the following statements is correct? **[NCERT Exemplar]**



(a) The dipole will not experience any force.
(b) The dipole will experience a force towards left.
(c) The dipole will experience a force towards right.
(d) The dipole will experience a force upwards.

68. An electric dipole is placed at an angle of 60° with an electric field of intensity 10^5 NC^{-1}. It experiences a torque equal to $8\sqrt{3}$ N-m. Calculate the charge on the dipole, if the dipole length is 2 cm.

(a) -8×10^3 C (b) 8.54×10^{-4} C
(c) 8×10^{-3} C (d) 0.85×10^{-6} C

69. A point Q lies on the perpendicular bisector of an electrical dipole of dipole moment p. If the distance of Q from the dipole is r (much larger than the size of the dipole), then the electric field intensity E at Q is proportional to

(a) r^{-2} (b) r^{-4} (c) r^{-1} (d) r^{-3}

70. A given charge situated at a certain distance from an electric dipole in the end on position, experiences a force F. If the distance of charge is doubled, the force acting on the charge will be

(a) $2F$ (b) $F/2$
(c) $F/4$ (d) $F/8$

71. Two point charges of $1\ \mu C$ and $-1\ \mu C$ are separated by a distance of 100 Å. A point P is at a distance of 10 cm from the mid-point and on the perpendicular bisector of the line joining the two charges. The electric field at P will be

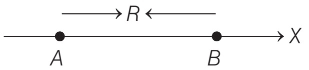
(a) $9\ NC^{-1}$ (b) $0.9\ Vm^{-1}$
(c) $90\ Vm^{-1}$ (d) $0.09\ NC^{-1}$

72. A point dipole $\mathbf{p} = -p_0\hat{\mathbf{x}}$ is kept at the origin. The potential and electric field due to this dipole on the Y-axis at a distance d are respectively (Take, $V = 0$ at infinity) **[JEE Main 2019]**

(a) $\frac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (b) $0, \frac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (d) $\frac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

73. Two electric dipoles A, B with respective dipole moments $\mathbf{d}_A = -4qa\,\hat{\mathbf{i}}$ and $\mathbf{d}_B = -2qa\,\hat{\mathbf{i}}$ are placed on the X-axis with a separation R, as shown in the figure



The distance from A at which both of them produce the same potential is **[JEE Main 2019]**

(a) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

(c) $\frac{R}{\sqrt{2}+1}$ (d) $\frac{R}{\sqrt{2}-1}$

74. An electric dipole has a fixed dipole moment $\mathbf{p}$, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is **[JEE Main 2017]**

(a) 45° (b) 60°
(c) 90° (d) 30°

75. Two electric dipoles of moment p and $64\,p$ are placed in opposite direction on a line at a distance of 25 cm. The electric field will be zero at point between the dipoles whose distance from dipole of moment p is

(a) 10 cm (b) 5 cm
(c) 8 cm (d) 20 cm

76. Two identical electric point dipoles have dipole moments $\mathbf{p}_1 = p\hat{\mathbf{i}}$ and $\mathbf{p}_2 = -p\hat{\mathbf{i}}$ are held on the X-axis at distance a from each other. When released, they move along the X-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is m, their speed when they are infinitely far apart is **[JEE Main 2020]**

(a) $\frac{p}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (b) $\frac{p}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$

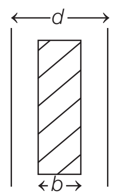
(c) $\frac{p}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$ (d) $\frac{p}{a}\sqrt{\frac{3}{2\pi\varepsilon_0 ma}}$

Capacitors and Capacitance

77. A capacitor connected to a 10 V battery collects a charge of $40\ \mu C$ with air as dielectric and $100\ \mu C$ with a given oil as dielectric. The dielectric constant of the oil is

(a) 1.5 (b) 2.0
(c) 2.5 (d) 3.0

78. A slab of copper of thickness b is inserted in between the plates of parallel plate capacitor as shown in figure. The separation between the plates is d, if $b = d/2$, then the ratio of capacities of capacitors after and before inserting the slab will be



(a) $\sqrt{2}:1$ (b) $2:1$
(c) $1:1$ (d) $1:\sqrt{2}$

79. The capacitance of a spherical condensers is $1\ \mu F$. If the spacing between two spheres is 1 mm, the radius of the outer sphere is

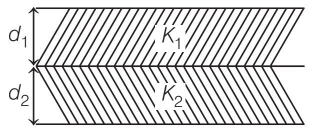
(a) 3 m (b) 7 m
(c) 8 m (d) 9 m

80. Two identical metal plates are given positive charges Q_1 and Q_2 ($< Q_1$), respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is

(a) $\frac{Q_1 + Q_2}{2C}$ (b) $\frac{Q_1 + Q_2}{C}$

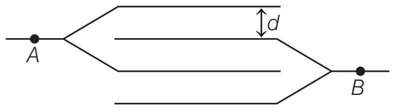
(c) $\frac{Q_1 - Q_2}{C}$ (d) $\frac{Q_1 - Q_2}{2C}$

81. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d(= d_1 + d_2)$ and effective dielectric constant K. The K is **[NCERT Exemplar]**



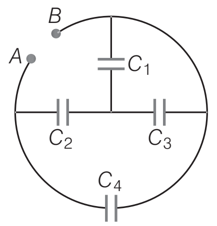
(a) $\dfrac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (b) $\dfrac{K_1d_1 + K_2d_2}{K_1 + K_2}$
(c) $\dfrac{K_1K_2(d_1 + d_2)}{(K_2d_1 + K_1d_2)}$ (d) $\dfrac{2K_1K_2}{K_1 + K_2}$

82. The equivalent capacity between points A and B in figure will be, while capacitance of each capacitor is $3\,\mu F$.



(a) $2\,\mu F$ (b) $4\,\mu F$
(c) $7\,\mu F$ (d) $9\,\mu F$

83. In the arrangement of capacitors shown in figure, each capacitor is of $9\,\mu F$, then the equivalent capacitance between the points A and B is

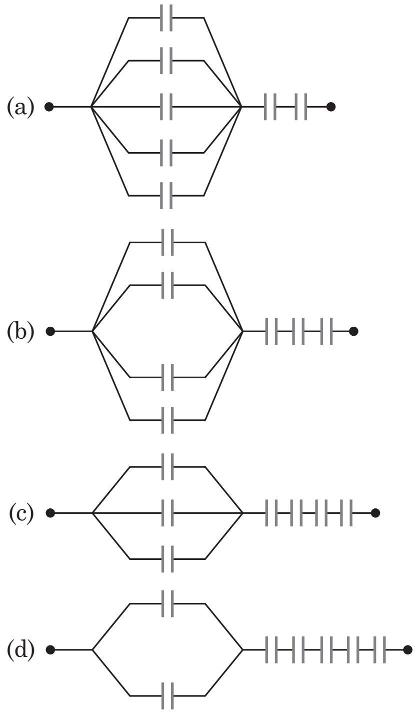


(a) $9\,\mu F$
(b) $18\,\mu F$
(c) $4.5\,\mu F$
(d) $15\,\mu F$

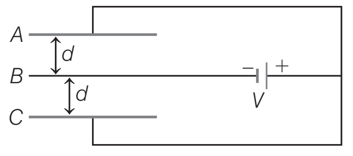
84. An electrical technician requires a capacitance of $2\,\mu F$ in a circuit across a potential difference of 1 kV. A large number of $1\,\mu F$ capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors. **[NCERT]**

(a) Six rows having 3 capacitors in each row
(b) Three rows having 6 capacitors in each row
(c) Nine rows having 2 capacitors in each row
(d) Two rows having 9 capacitors in each row

85. Seven capacitors each of the capacitance $2\,\mu F$ are connected in a configuration to obtain an effective capacitance of $\frac{10}{11}\,\mu F$. Which of the combination(s) shown in figure will achieve the desired result?

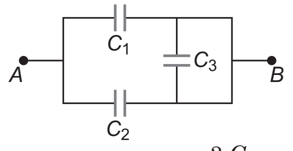


86. Consider the arrangement of three metal plates A, B and C of equal surface area and separation d as shown in figure. The energy stored in the arrangement, when the plates are fully charged, is



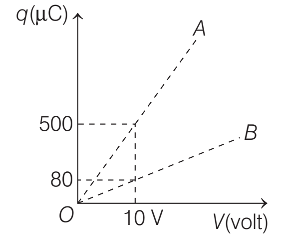
(a) $\dfrac{\varepsilon_0AV^2}{2d}$ (b) $\dfrac{\varepsilon_0AV^2}{d}$
(c) $\dfrac{2\varepsilon_0AV^2}{d}$ (d) $\dfrac{3\varepsilon_0AV^2}{2d}$

87. The equivalent capacitance of the combination of three capacitors, each of capacitance C as shown in figure between points A and B is



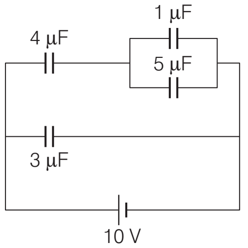
(a) $\dfrac{C}{2}$ (b) $\dfrac{3C}{2}$
(c) $\dfrac{1}{3C}$ (d) $2C$

88. Figure shows charge (q) *versus* voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are **[JEE Main 2019]**



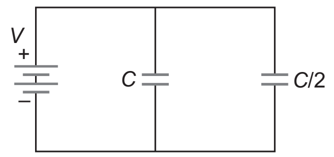
(a) 60 μF and 40 μF
(b) 50 μF and 30 μF
(c) 20 μF and 30 μF
(d) 40 μF and 10μF

89. In the given circuit, the charge on 4 μF capacitor will be **[JEE Main 2019]**



(a) 5.4 μC
(b) 9.6 μC
(c) 13.4 μC
(d) 24 μC

90. Two condenser one of capacity C and other of capacity $C/2$ are connected to 9V battery as shown in figure. The work done in charging fully both condensers is



(a) $(1/4)CV^2$
(b) $2CV^2$
(c) $(3/4)CV^2$
(d) $(1/2)CV^2$

91. Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the system is

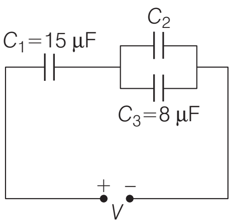
(a) $\frac{1}{4}C(V_1^2 - V_2^2)$ (b) $\frac{1}{4}C(V_1^2 + V_2^2)$

(c) $\frac{1}{4}C(V_1 - V_2)^2$ (d) $\frac{1}{4}C(V_1 + V_2)^2$

92. The force on each plate of parallel plate capacitor has a magnitude equal to $\frac{1}{2}QE$, where Q is the charge on the capacitor and E is the magnitude of electric field between the plates. Then, **[NCERT]**

(a) $\frac{E}{2}$ contributes to the force against which the plates are moved
(b) $\frac{E}{3}$ contributes to the force against which the plates are moved
(c) E contributes force against which the plates are moved
(d) None of the above

93. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then, the charge on capacitor C_2 is **[JEE Main 2020]**



(a) 450 μC
(b) 590 μC
(c) 160 μC
(d) 650 μC

94. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is **[JEE Main 2020]**

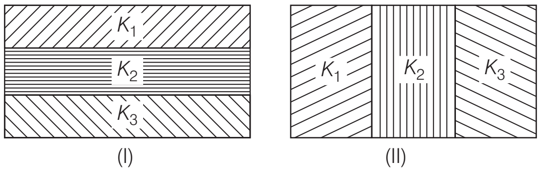
(a) $\frac{1}{3}CV_0^2$

(b) $\frac{1}{6}CV_0^2$

(c) $\frac{1}{2}CV_0^2$

(d) $\frac{1}{4}CV_0^2$

95. Two identical parallel plate capacitors of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in Fig. I, and the second one is filled as shown in Fig. II. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) : **[JEE Main 2019]**



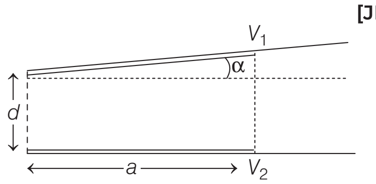
(a) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(b) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{K_1K_2K_3}$

(c) $\frac{E_1}{E_2} = \frac{9\,K_1K_2K_3}{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}$

(d) $\frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2K_3 + K_3K_1 + K_1K_2)}{9\,K_1K_2K_3}$

96. A capacitor is made of two square plates each of side a making a very small angle α between them as shown in figure. The capacitance will be close to **[JEE Main 2020]**



(a) $\frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{4d}\right)$

(b) $\frac{\varepsilon_0 a^2}{d}\left(1 + \frac{\alpha a}{d}\right)$

(c) $\frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$

(d) $\frac{\varepsilon_0 a^2}{d}\left(1 - \frac{3\alpha a}{2d}\right)$

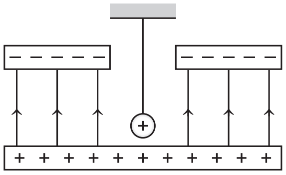
ROUND II Mixed Bag

Only One Correct Option

1. Charges $2q$, $-q$ and $-q$ lie at the vertices of an equilateral triangle. The value of E and V at the centroid of the triangle will be

(a) $E \neq 0$ and $V \neq 0$ (b) $E = 0$ and $V = 0$
(c) $E \neq 0$ and $V = 0$ (d) $E = 0$ and $V \neq 0$

2. If a positively charged pendulum is oscillating in a uniform electric field as shown in figure. Its time period as compared to that when it was uncharged will



(a) increase
(b) decrease
(c) not change
(d) first increase and then decrease

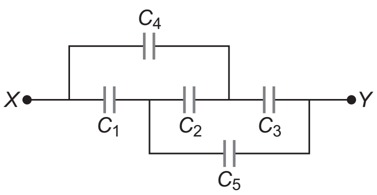
3. Two particles of equal mass m and charge q are placed at a distance of 16 cm. They do not experience any force. The value of $\frac{q}{m}$ is

(a) $\sqrt{\frac{\pi\varepsilon_0}{G}}$ (b) $\sqrt{\frac{G}{\pi\varepsilon_0}}$
(c) $\sqrt{4\pi\varepsilon_0}G$ (d) l

4. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab $(3/4)d$, where d is the separation between the plates of parallel plate capacitor. The new capacitance C in terms of original capacitance C_0 is given by the following relation **[JEE Main 2021]**

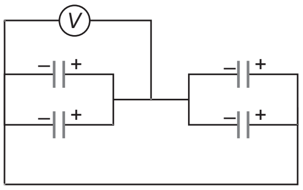
(a) $C' = \frac{3 + K}{4K} C_0$ (b) $C' = \frac{4 + K}{3} C_0$
(c) $C' = \frac{4K}{K + 3} C_0$ (d) $C' = \frac{4}{3 + K} C_0$

5. The effective capacitance between points X and Y shown in figure is (Assuming, $C_2 = 10\,\mu F$ and that outer capacitors are all $4\,\mu F$)



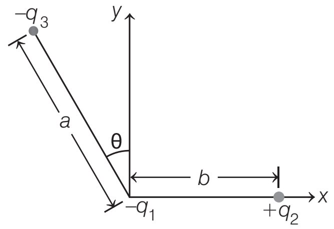
(a) $1\,\mu F$ (b) $3\,\mu F$ (c) $4\,\mu F$ (d) $5\,\mu F$

6. The four capacitors, each of $25\,\mu F$ are connected as shown in figure. The DC voltmeter reads 200 V. The charge on each plate of capacitor is



(a) $\pm 2 \times 10^{-3}$ C (b) $\pm 5 \times 10^{-3}$ C
(c) $\pm 2 \times 10^{-2}$ C (d) $\pm 5 \times 10^{-2}$ C

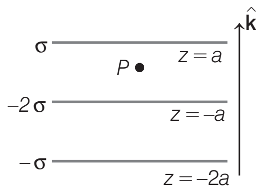
7. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in figure. The x-component of the force on $-q_1$ is proportional to



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$ (b) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$ (d) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$

8. Three large parallel plates have uniform surface charge densities as shown in the figure. Find the electric field at point P.



(a) $\frac{-4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (b) $\frac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$

(c) $\frac{-2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (d) $\frac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$

9. A 100 eV electron is fired directly towards a large metal plate having surface charge density $2\times10^{-6}\,\text{cm}^{-2}$. The distance from where the electrons be projected so that it just fails to strike the plate is

(a) 0.22 mm (b) 0.44 mm

(c) 0.66 mm (d) 0.88 mm

10. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the X-axis at $x = +1$ cm and C be the point on the Y-axis at $y = +1$ cm. Then, the potentials at the points A, B and C satisfy the condition

(a) $V_A < V_B$ (b) $V_A > V_B$

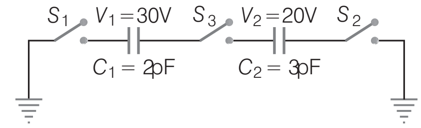
(c) $V_A < V_C$ (d) $V_A > V_C$

11. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge $-3Q$, the new potential difference between the same two surfaces is

(a) V (b) $2V$

(c) $4V$ (d) $-2V$

12. For the circuit shown in figure, which of the following statements is true?



(a) With S_1 closed, $V_1 = 15\text{V}$, $V_2 = 20\text{V}$.

(b) With S_3 closed, $V_1 = V_2$, $V_2 = 20\text{V}$.

(c) With S_1 and S_3 closed, $V_1 = V_2 = 0$.

(d) With S_1 and S_3 closed, $V_1 = 30\text{V}$, $V_2 = 20\text{V}$.

13. Consider the force F on a charge q due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following option is correct for F, if q is placed at distance r from the centre of the shell? **[JEE Main 2020]**

(a) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ (for $r < R$)

(b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} > F > 0$ (for $r < R$)

(c) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ (for $r > R$)

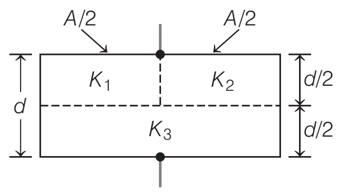
(d) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ (for all r)

14. Point charges 4×10^{-6} C and 2×10^{-6} C are placed at the vertices A and B of a right angled triangle ABC, respectively. B is the right angle, $AC = 2\times10^{-2}$ m and $BC = 10^{-2}$ m. Find the magnitude and direction of the resultant electric intensity at C.

(a) $1.73\times10^4\ \text{NC}^{-1}$; 34 .7° (b) $2.38\times10^8\ \text{NC}^{-1}$; 40.9°

(c) $4.28\times10^9\ \text{NC}^{-1}$; 45° (d) $4.9\times10^{10}\ \text{NC}^{-1}$; 34.7°

15. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown in figure. If a single dielectric material is to be used to have the same capacitance C is this capacitors, then its dielectric constant K is given by



(a) $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{2K_3}$ (b) $\frac{1}{K} = \frac{1}{K_1+K_2} + \frac{1}{2K_3}$

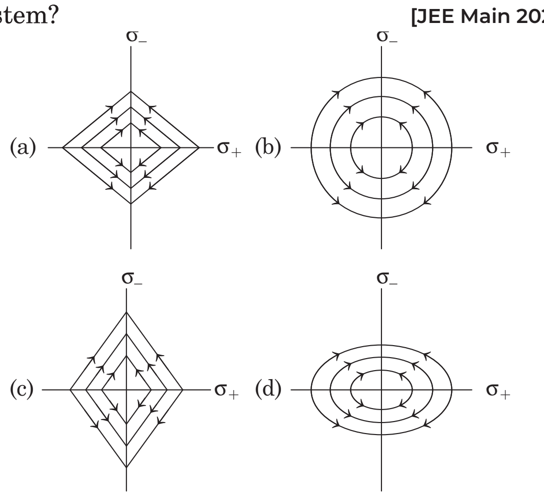
(c) $K = \frac{K_1K_2}{K_1+K_2} + 2K_3$ (d) $K = K_1 + K_2 + 2K_3$

16. A large insulated sphere of radius r charged with Q units of electricity is placed in contact with a small insulated uncharged sphere of radius r' and in then separated. The charge on smaller sphere will now be

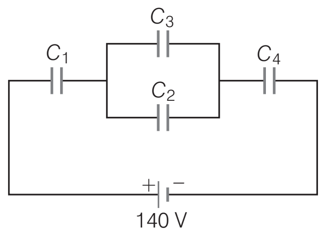
(a) $Q(r + r')$ (b) $\frac{Qr'}{r' + r}$

(c) $Q(r + r')$ (d) $\frac{Q}{r' + r}$

17. Two charged thin infinite plane sheets of uniform surface charge densities σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system? **[JEE Main 2020]**

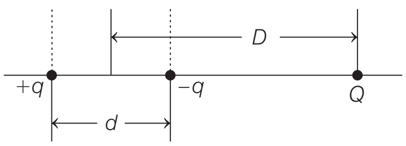


18. In the circuit arrangement shown in figure, the value of $C_1 = C_2 = C_3 = 30$ pF and $C_4 = 120$ pF. If the combination of capacitors is charged with 140V DC supply, the potential differences across the four capacitors will be respectively



(a) 80 V, 40V, 40V and 20 V
(b) 20V, 40V, 40V and 80 V
(c) 35V, 35V, 35V and 35 V
(d) 80V, 20V, 20V and 20 V

19. A system of three charges are placed as shown in the figure



If $D >> d$, the potential energy of the system is best given by **[JEE Main 2019]**

(a) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{2qQd}{D^2}\right]$

(b) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$

(d) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

20. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is

(a) 4 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

21. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is **[JEE Main 2013]**

(a) $\frac{Q}{8\pi\,\varepsilon_0 L}$ (b) $\frac{3\,Q}{4\pi\,\varepsilon_0 L}$

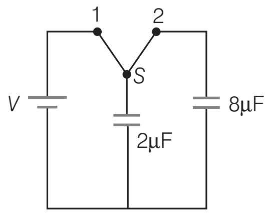
(c) $\frac{Q}{4\pi\,\varepsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\,\varepsilon_0 L}$

22. Concentric metallic hollow spheres of radii R and $4R$ hold charges Q_1 and Q_2, respectively. Given that, surface charge densities of the concentric spheres are equal. The potential difference $V(R) - V(4R)$ is **[JEE Main 2020]**

(a) $\frac{3Q_2}{4\pi\varepsilon_0 R}$ (b) $\frac{3Q_1}{16\pi\varepsilon_0 R}$

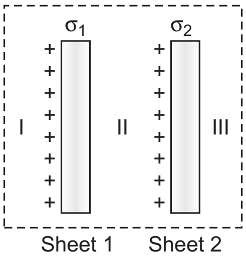
(c) $\frac{3Q_1}{4\pi\varepsilon_0 R}$ (d) $\frac{Q_2}{4\pi\varepsilon_0 R}$

23. A 2μF capacitor is charged as shown in figure. The percentage of the stored energy dissipated after the switch S is turned to position 2 is



(a) 20% (b) 80%
(c) 10% (d) 100%

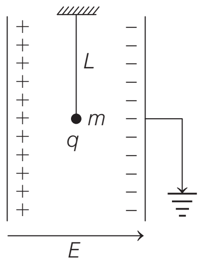
24. Two parallel plane sheets 1 and 2 carry uniform charge densities σ_1 and σ_2, as shown in figure. The magnitude of the resultant electric field in the region marked III is $(\sigma_1 > \sigma_2)$



(a) $\frac{\sigma_1 + \sigma_2}{2\varepsilon_0}$ (b) $\frac{\sigma_1 \sigma_2}{\varepsilon_0}$

(c) $\frac{\sigma_1}{2\sigma_2 \varepsilon_0}$ (d) $\frac{\sigma_2}{2\varepsilon_0}$

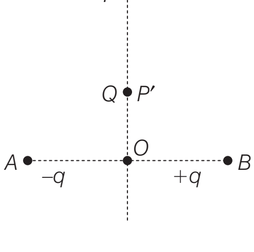
25. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by **[JEE Main 2019]**



(a) $2\pi \sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$ (b) $2\pi \sqrt{\frac{L}{\sqrt{g^2 - \frac{q^2E^2}{m^2}}}}$

(c) $2\pi \sqrt{\frac{L}{\left(g + \frac{qE}{m}\right)}}$ (d) $2\pi \sqrt{\frac{L}{\left(g - \frac{qE}{m}\right)}}$

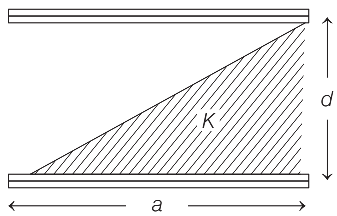
26. Charges $-q$ and $+q$ located at A and B respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid-point of the dipole and OP is perpendicular to AB. A charge Q is placed at P, where $OP = y$ and $y >> 2a$. The charge Q experiences an electrostatic force F.



If Q is now moved along the equatorial line to P' such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} >> 2a\right)$ **[JEE Main 2019]**

(a) $\frac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

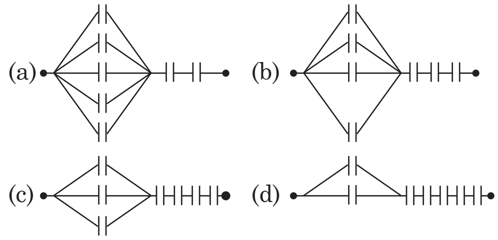
27. A parallel plate capacitor is made of two square plates of side a separated by a distance d $(d << a)$. The lower triangular portions is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is **[JEE Main 2019]**



(a) $\frac{K\varepsilon_0 a^2}{d} \ln K$ (b) $\frac{K\varepsilon_0 a^2}{d(K-1)} \ln K$

(c) $\frac{K\varepsilon_0 a^2}{2d(K+1)}$ (d) $\frac{1}{2} \cdot \frac{K\varepsilon_0 a^2}{d}$

28. Seven capacitors, each of capacitance $2\,\mu F$ are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations shown in figures below will achieve the desired value? **[JEE Main 2019]**



29. A parallel plate capacitor has plate of length l, width w and separation of plates is d. It is connected to a battery of emf V. A dielectric slab of the same thickness d and of dielectric constant $K = 4$ is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored? **[JEE Main 2020]**

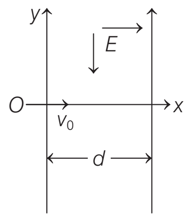
(a) $\frac{2l}{3}$ (b) $\frac{l}{3}$ (c) $\frac{l}{4}$ (d) $\frac{l}{2}$

30. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is **[JEE Main 2019]**

(a) $\frac{(n+1)V}{(K+n)}$ (b) $\frac{nV}{K+n}$

(c) V (d) $\frac{V}{K+n}$

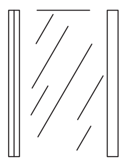
31. A charged particle (mass m and charge q) moves along X-axis with velocity v_0. When it passes through the origin, it enters a region having uniform electric field $\mathbf{E} = -E\,\hat{\mathbf{j}}$ which extends upto $x = d$. Equation of path of electron in the region $(x > d)$ is **[JEE Main 2020]**



(a) $y = \frac{qEd}{mv_0^2}x$ (b) $y = \frac{qEd}{mv_0^2}(x-d)$

(c) $y = \frac{qEd}{mv_0^2}\left(\frac{d}{2}-x\right)$ (d) $y = \frac{qEd^2}{mv_0^2}x$

32. A parallel plate capacitor has plates of area A separated by distance d between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1+\alpha x)$, where x is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression **[JEE Main 2020]**



(a) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$ (b) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(c) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$ (d) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

33. If on the concentric hollow sphere of radii r and $R(> r)$ the charge Q is distributed such that their surface densities are same, then the potential at their common centre is

(a) $\frac{Q(R^2+r^2)}{4\pi\varepsilon_0(R+r)}$ (b) $\frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$

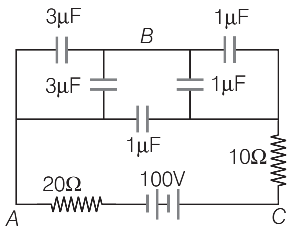
(c) zero (d) $\frac{QR}{R+r}$

34. Two infinitely long parallel wires having linear charge densities λ_1 and λ_2 respectively are placed at a distance of R metres. The force per unit length on either wire will be $\left(K = \frac{1}{4\pi\varepsilon_0}\right)$

(a) $K\frac{2\lambda_1\lambda_2}{R^2}$ (b) $K\frac{2\lambda_1\lambda_2}{R}$

(c) $K\frac{\lambda_1\lambda_2}{R^2}$ (d) $K\frac{\lambda_1\lambda_2}{R}$

35. In the figure below, what is the potential difference between the points A and B and between B and C respectively, in steady state?

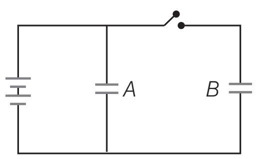


(a) $V_{AB} = V_{BC} = 50\text{V}$
(b) $V_{AB} = 25\text{V}, V_{BC} = 75\text{V}$
(c) $V_{AB} = 75\text{V}, V_{BC} = 25\text{V}$
(d) $V_{BC} = V_{AB} = 100\text{V}$

36. Two equal negative charges $-q$ are fixed at the points $(0, a)$ and $(0, -a)$ on the Y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the X-axis. The charge Q will

(a) execute simple harmonic motion about the origin
(b) move to the origin and remain at rest
(c) move to infinity
(d) execute oscillatory but not simple harmonic motion

37. Figure given below shows two identical parallel plate capacitors connected to a battery with switch S closed. The switch is now opened and the free space between the plate of capacitors is filled with a dielectric constant 3. What will be the ratio of total electrostatic energy stored in both capacitors before and after the introduction of the dielectric?

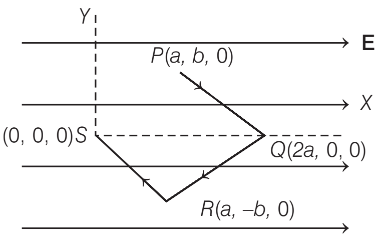


(a) 1 : 2 (b) 1 : 5
(c) 3 : 5 (d) 5 : 2

38. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is connected to another battery and is charged to potential difference $2V$. The charging batteries are now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) infinite
(b) zero
(c) $\frac{3CV^2}{2}$
(d) $\frac{6CV^2}{2}$

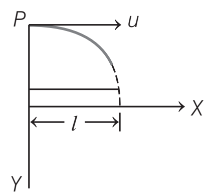
39. A point charge q moves from point P to point S along the path $PQRS$ in a uniform electric field $\mathbf{E}$ pointing parallel to the positive direction of the X-axis as shown in figure.



The coordinates of the points P, Q, R and S are $(a, b, 0)$ $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$, respectively. The work done by the field in the above process is given by the expression

(a) qE
(b) $-qaE$
(c) $q(\sqrt{a^2+b^2})+E$
(d) $3qE(\sqrt{a^2+b^2})$

40. The electron is projected from a distance d and with initial velocity u parallel to a uniformly charged flat conducting plate as shown in figure. It strikes the plate after travelling a distance l along the direction. The surface charge density of conducting plate is equal to

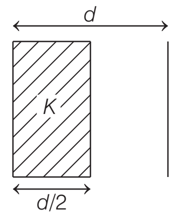


(a) $\frac{2d\varepsilon_0 m u^2}{el^2}$
(b) $\frac{2d\varepsilon_0 m u}{el}$
(c) $\frac{d\varepsilon_0 m u^2}{el}$
(d) $\frac{d\varepsilon_0 m u}{el}$

Numerical Value Questions

41. 27 identical drops of mercury are charged simultaneously to the same potential of 10 V each. Assuming drops to be spherical, if all the charged drops are made to combine to form one larger drop, then the potential (in volt) of larger drop would be ....

42. In a parallel plate capacitor set up, the plate area of capacitor is 2m^2 and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2m^2 (see figure) the capacitance of the set up will be .... ε_0. (Dielectric constant of the material = 3.2)
(Round off to the nearest integer)



[JEE Main 2021]

43. A 10 μF capacitor and a 20 μF capacitor are connected in series across 200 V supply line. The charged capacitors are then disconnected from the line and reconnected with their positive plates together and negative plates together and no external voltage is applied. What is the potential difference (in volt) across each capacitor?

44. A 5 μF capacitor is charged fully by a 220V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J, then value of X to the nearest integer is .... **[JEE Main 2020]**

45. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time, the charge is redistributed between them is(in nJ) .... **[JEE Main 2020]**

46. A parallel plate capacitor whose capacitance C is 14pF is charged by a battery to a potential difference $V = 12\text{V}$ between its plates. The charging battery is now disconnected and a porcelain plate with $K = 7$ is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of .... pJ. (Assume no friction) **[JEE Main 2021]**

47. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to 6×10^{-x} C/m^2, where the value of x is ....

Answers

Round I

1. *(a)*	***2.*** *(a)*	***3.*** *(c)*	***4.*** *(b)*	***5.*** *(d)*	***6.*** *(a)*	***7.*** *(a)*	***8.*** *(a)*	***9.*** *(d)*	***10.*** *(a)*
11. *(d)*	***12.*** *(a)*	***13.*** *(d)*	***14.*** *(d)*	***15.*** *(b)*	***16.*** *(d)*	***17.*** *(a)*	***18.*** *(d)*	***19.*** *(b)*	***20.*** *(a)*
21. *(c)*	***22.*** *(a)*	***23.*** *(a)*	***24.*** *(a)*	***25.*** *(d)*	***26.*** *(a)*	***27.*** *(a)*	***28.*** *(a)*	***29.*** *(b)*	***30.*** *(a)*
31. *(a)*	***32.*** *(a)*	***33.*** *(d)*	***34.*** *(b)*	***35.*** *(b)*	***36.*** *(b)*	***37.*** *(c)*	***38.*** *(a)*	***39.*** *(c)*	***40.*** *(d)*
41. *(b)*	***42.*** *(c)*	***43.*** *(c)*	***44.*** *(d)*	***45.*** *(a)*	***46.*** *(a)*	***47.*** *(a)*	***48.*** *(b)*	***49.*** *(d)*	***50.*** *(d)*
51. *(c)*	***52.*** *(c)*	***53.*** *(a)*	***54.*** *(b)*	***55.*** *(c)*	***56.*** *(a)*	***57.*** *(a)*	***58.*** *(c)*	***59.*** *(d)*	***60.*** *(a)*
61. *(a)*	***62.*** *(c)*	***63.*** *(a)*	***64.*** *(c)*	***65.*** *(d)*	***66.*** *(c)*	***67.*** *(b)*	***68.*** *(c)*	***69.*** *(d)*	***70.*** *(d)*
71. *(d)*	***72.*** *(b)*	***73.*** *(a)*	***74.*** *(b)*	***75.*** *(b)*	***76.*** *(b)*	***77.*** *(c)*	***78.*** *(b)*	***79.*** *(a)*	***80.*** *(d)*
81. *(c)*	***82.*** *(d)*	***83.*** *(d)*	***84.*** *(a)*	***85.*** *(a)*	***86.*** *(b)*	***87.*** *(d)*	***88.*** *(d)*	***89.*** *(d)*	***90.*** *(c)*
91. *(c)*	***92.*** *(a)*	***93.*** *(b)*	***94.*** *(b)*	***95.*** *(d)*	***96.*** *(c)*				

Round II

1. *(c)*	***2.*** *(a)*	***3.*** *(c)*	***4.*** *(c)*	***5.*** *(c)*	***6.*** *(b)*	***7.*** *(c)*	***8.*** *(c)*	***9.*** *(b)*	***10.*** *(b)*
11. *(a)*	***12.*** *(d)*	***13.*** *(c)*	***14.*** *(b)*	***15.*** *(b)*	***16.*** *(b)*	***17.*** *(c)*	***18.*** *(a)*	***19.*** *(d)*	***20.*** *(c)*
21. *(d)*	***22.*** *(b)*	***23.*** *(b)*	***24.*** *(a)*	***25.*** *(a)*	***26.*** *(d)*	***27.*** *(b)*	***28.*** *(c)*	***29.*** *(b)*	***30.*** *(a)*
31. *(c)*	***32.*** *(c)*	***33.*** *(b)*	***34.*** *(b)*	***35.*** *(b)*	***36.*** *(d)*	***37.*** *(c)*	***38.*** *(c)*	***39.*** *(b)*	***40.*** *(a)*
41. *90*	***42.*** *3*	***43.*** *88.89*	***44.*** *4*	***45.*** *6*	***46.*** *864*	***47.*** *7*			

Solutions

Round I

1. Each sphere having $-2\,\mu$C.

$$\therefore \qquad n = \frac{q}{e} = -\frac{2\times10^{-6}}{-1.6\times10^{-19}} = 1.25\times10^{13}$$

Thus, 1.25×10^{13} electrons are in excess.

2. Charge on the glass rod is positive, so charge on the gold leaves will also be positive. Due to X-rays, more electrons from leaf will be emitted, so leaf becomes more positive and diverse further.

3. $\therefore\ 100 = k\dfrac{q_1q_2}{r^2}$

Now, $\qquad q_1 = 0.9q_1 \qquad$ (decrease by 10%)

and $\qquad q_2 = 1.1q_2 \qquad$ (increase by 10%)

$$\because \qquad F' = \frac{k(0.9\,q_1)(1.1\,q_2)}{r^2}$$

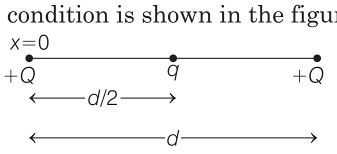
$$\Rightarrow \qquad \frac{F'}{100} = \frac{0.9\times1.1}{1}$$

$$\therefore \qquad F' = 99\text{ N}$$

4. Angle between two forces due to individual charges is equal to 60°.

$$\therefore \qquad R = \sqrt{F^2+F^2+2\,F\,F\cos60^\circ} = F\sqrt{3}$$

5. The given condition is shown in the figure given below



Then, according to the Coulomb's law, the electrostatic force between two charges q_1 and q_2 such that the distance between them is (r) given as

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$

$\therefore$ Net force on charge Q placed at origin, *i.e.* at $x=0$ in accordance with the principle of superposition can be given as

$$F_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times Q}{(d)^2}$$

Since, it has been given that, $F_{\text{net}} = 0$.

$$\Rightarrow \qquad \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times Q}{(d)^2} = 0$$

$$\Rightarrow \qquad \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\times Q}{(d)^2}$$

$$\text{or} \qquad q = -\frac{Q}{4}$$

6. From $F = \dfrac{1}{4\pi\varepsilon_0K}\dfrac{q_1\,q_2}{r^2}$

For a brass sheet, $K = \infty$

$$\therefore \qquad F = \frac{1}{4\pi\varepsilon_0(\infty)}\frac{q_1\,q_2}{r^2}$$

$$\Rightarrow \qquad F = 0$$

7. From Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$\Rightarrow \quad 3.7 \times 10^{-9} = 9 \times 10^9 \times \frac{q^2}{(5 \times 10^{-10})^2} \quad (\because q_1 = q_2 = q)$

$\Rightarrow \quad q = 3.2 \times 10^{-19}$ C

As, $\quad q = ne$

$\Rightarrow \quad n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2$

8. Here, $F = 40 = \frac{k(3)(8)}{r^2} = \frac{24k}{r^2}$

and $\quad F' = \frac{k(3-5)(8-5)}{r^2} = \frac{k(-2)(3)}{r^2} = \frac{-6k}{r^2}$

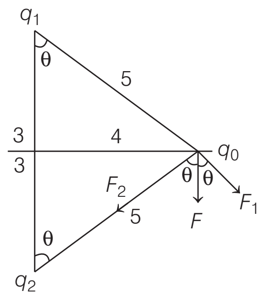
$\because \quad \frac{F'}{F} = \frac{-6k}{r^2} \times \frac{r^2}{24k} = -\frac{1}{4}$

$\Rightarrow \quad F' = -\frac{F}{4} = -\frac{40}{4} = -10$ N

9. In case of spherical conductor, the whole charge is concentrated at the centre. Now, the distance between two charges will be $(R + x)$. Thus, the force will be proportional to $1/(R + x)^2$.

10. Here, $q = \pm\ 6.0$ nC $= \pm\ 6.0 \times 10^{-9}$ C,

$2a = 6$ cm $= 6 \times 10^{-2}$ m



$r = 4$ cm (on equatorial line)

$= 4 \times 10^{-2}$ m

and $\quad q_0 = 2$ nC $= 2 \times 10^{-9}$ C, $F = ?$

$\therefore \quad F = F_1 \cos\theta + F_2 \cos\theta$

$= 2 \times \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cos\theta$

$= 2 \times 9 \times 10^9 \times \frac{6 \times 10^{-9} \times 2 \times 10^{-9}}{(5 \times 10^{-2})^2} \times \frac{3}{5}$

or $\quad F = 5.18 \times 10^{-5}$ N

Clearly, this force is along $\hat{\mathbf{j}}$.

So, $\quad \mathbf{F} = -51.8\ \hat{\mathbf{j}}\ \mu$N

11. As, $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

In a medium of dielectric constant, $K = \frac{\varepsilon}{\varepsilon_0}$

$$F' = \frac{q_1 q_2}{4\pi\varepsilon R^2}$$

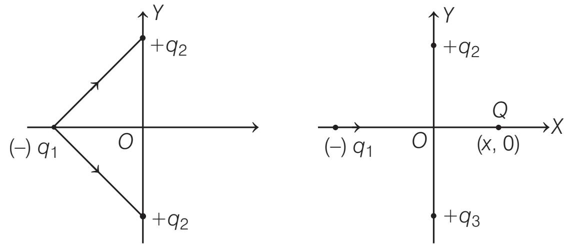
As, $\quad F' = F$

$\Rightarrow \quad \frac{q_1 q_2}{4\pi\varepsilon R^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

or $\quad R^2 = \frac{\varepsilon_0}{\varepsilon} r^2 = \frac{r^2}{K}$

$\therefore \quad R = \frac{r}{\sqrt{K}}$

12. As, q_2 and q_3 are positive charges and net force on q_1 is along + x-direction, therefore q_1 must be negative as shown in figure



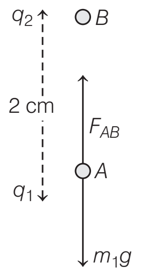
(a) (b)

When a positive charge Q is added at $(x, 0)$, it will attract $(-q_1)$ along + x-direction, Fig. (b). Therefore, force on q_1 will increase along the positive X-axis.

13. Here, charge on pitch ball A, $\quad q_1 = 5\,\mu\text{C} = 5 \times 10^{-6}$ C

Mass of pitch ball A, $\quad m_1 = 9 \times 10^{-5}$ kg

The weight $m_1 g$ of the pitch ball A acts vertically downwards.



Let q_2 be charge on the pitch ball B held 2 cm above the pitch ball A, so that the pitch ball A remains stationary. It can be possible only, if the charges on two pitch balls are of opposite signs, *i.e.* if charge on pitch ball A is positive, charge on B must be negative. Then, the force on pitch ball A due to B, *i.e.* F_{AB} will act vertically upwards (figure). For charge q_1 to remain stationary,

$$F_{AB} = m_1 g \quad \text{or} \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{AB^2} = m_1 g$$

Here, $\quad AB = 2$ cm $= 0.02$ m

$\therefore \quad 9 \times 10^9 \times \frac{5 \times 10^{-6} \times q_2}{(0.02)^2} = 9 \times 10^{-5} \times 9.8$

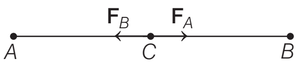
or $\quad q_2 = 7.84 \times 10^{-12}$ C

14. If same charges on spheres A and B are q, then

force, $\quad F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 3 \times 10^{-5}$ N

Charge on A and C after touching,

$$q'_A = q_C = \frac{q}{2}$$



$\therefore$ Net force on C,

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B$$

$$\therefore \quad F = \left| \frac{1}{4\pi\varepsilon_0} \frac{(q/2)\,(q/2)}{(r/2)^2} - \frac{1}{4\,\pi\varepsilon_0} \frac{(q/2)\times q}{(r/2)^2} \right|$$

$$= \left| \frac{1}{4\pi\varepsilon_0 r} \frac{q^2}{r^2} - 2\frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2} \right|$$

$$= \left| -\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \right) \right| = 3\times10^{-5}\ \text{N}$$

15. Here, $F_1 = \dfrac{k\,(+10)\,(-20)}{R^2} = \dfrac{-k\times200}{R^2}$

As spheres are of equal radii, so on touching, the net charge $= (+10-20)\,\mu\text{C} = -10\,\mu\text{C}$, is shared equally between them, *i.e.* each sphere carries $-5\,\mu\text{C}$ charge.

$$F_2 = \frac{k\,(-5)\,(-5)}{R^2} = \frac{k\times25}{R^2}$$

$$\because \quad \frac{F_1}{F_2} = \frac{-8}{1} \Rightarrow F_1 : F_2 = -8:1$$

16. Charge (C) has the largest magnitude, since maximum number of field lines are associated with it.

17. Force on proton at point A,

$$F_A = qE_A = 1.6\times10^{-19}\times40 = 6.4\times10^{-18}\ \text{N}$$

18. As, $\sigma_1 = \sigma_2 \Rightarrow \dfrac{Q_1}{4\pi r_1^2} = \dfrac{Q_2}{4\pi r_2^2}$

$$\Rightarrow \quad \frac{Q_1}{4\pi\varepsilon_0 r_1^2} = \frac{Q_2}{4\pi\varepsilon_0 r_2^2}$$

$$\Rightarrow \quad E_1 = E_2 \qquad \left[\because E = \frac{Q}{4\pi\varepsilon_0 r^2}\right]$$

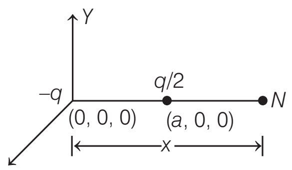
or $\quad E_1/E_2 = 1$

19. The density of lines of force $\propto E$.

Here, the density of lines of force at A is greater than at B.

Thus, $\quad E_A > E_B$

20. According to the question,



$$\frac{kq}{x^2} = \frac{kq/2}{(x-a)^2}$$

or $\quad 2\,(x-a)^2 = x^2$

$$\sqrt{2}\,(x-a) = x$$

or $\quad (\sqrt{2}-1)\,x = \sqrt{2}\ a$

$$\Rightarrow \quad x = \frac{\sqrt{2}\ a}{\sqrt{2}-1}$$

21. For a ring, $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{qx}{(x^2+R^2)}$

and E is maximum when $x = \dfrac{R}{\sqrt{2}}$

$$\Rightarrow \quad E_{\max} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{3\sqrt{3}R^2}$$

22. Charge enclosed in the sphere of radius r,

$$q = \frac{4}{3}\pi r^3 \rho$$

Electric field, $E = \dfrac{q}{4\pi\varepsilon_0 r^2} = \dfrac{\frac{4}{3}\pi r^3 \rho}{4\pi\varepsilon_0 r^2} = \dfrac{r\rho}{3\,\varepsilon_0}$

23. Electric field at a distance h from the centre of uniformly charged ring of total charge q (say) on its axis is given as

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qh}{(h^2+R^2)^{3/2}}$$

For the magnitude to be maximum, then $\dfrac{dE}{dh} = 0$

$$\Rightarrow \quad \frac{dE}{dh} = \frac{q}{4\,\pi\,\varepsilon_0}$$

$$\left[\frac{(h^2+R^2)^{3/2} - h[3/2(h^2+R^2)^{1/2}\,2h]}{(h^2+R^2)^3} \right] = 0$$

$$\Rightarrow \quad 0 = \frac{(h^2+R^2)^{3/2} - 3h^2(h^2+R^2)^{1/2}}{(h^2+\ R^2)^3}$$

$$\Rightarrow \quad (h^2+R^2)^{3/2} = 3h^2(h\ \ +R^2)^{1/2}$$

$$\Rightarrow \quad 3h^2 = (h^2+R^2)$$

$$\Rightarrow \quad 3h^2 - h^2 = R^2$$

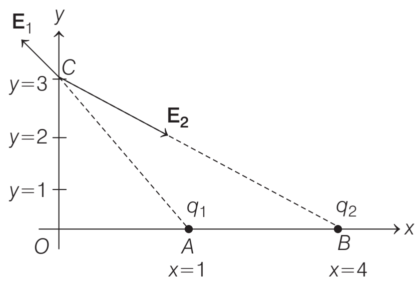
$$\Rightarrow \quad 2h^2 = R^2$$

$$\Rightarrow \quad h = \pm\frac{R}{\sqrt{2}}$$

$\therefore$ At $\dfrac{R}{\sqrt{2}}$, the value of electric field associated with a charged ring on its axis has the maximum value.

24. Here, $\quad q_1 = \sqrt{10}\ \mu\text{C} = \sqrt{10}\times10^{-6}\ \text{C}$,

$$q_2 = -25\ \mu\text{C} = -25\times10^{-6}\ \text{C}$$



Let $\mathbf{E}_1$ and $\mathbf{E}_2$ are the values of electric field due to q_1 and q_2, respectively.

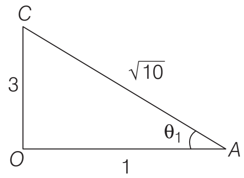
Here, $E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{AC^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{\sqrt{10} \times 10^{-6}}{(1^2 + 3^2)}$

$= 9 \times 10^9 \times \sqrt{10} \times 10^{-7}$

$= 9\sqrt{10} \times 10^2$

$\therefore \quad \mathsf{E}_1 = 9\sqrt{10} \times 10^2 [\cos\theta_1 (-\hat{i}) + \sin\theta_1 \hat{j}]$

From ΔOAC,



$\sin\theta_1 = \frac{3}{\sqrt{10}}$

and $\cos\theta_1 = \frac{1}{\sqrt{10}}$

$\therefore \quad E_1 = 9\sqrt{10} \times 10^2 \left[\frac{1}{\sqrt{10}} (-\hat{\mathbf{i}}) + \frac{3}{\sqrt{10}} \hat{\mathbf{j}}\right]$

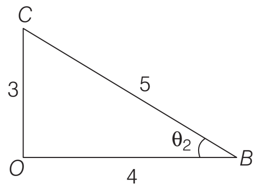
$= 9 \times 10^2 [-\hat{\mathbf{i}} + 3\hat{\mathbf{j}}]$

$= (-9\hat{\mathbf{i}} + 27\hat{\mathbf{j}}) \times 10^2$ V/m

and $E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-25 \times 10^{-6}}{(4^2 + 3^2)}$

$= 9 \times 10^3$ V/m

From ΔOBC,



$\sin\theta_2 = \frac{3}{5}$

and $\cos\theta_2 = \frac{4}{5}$

$\therefore \quad \mathbf{E}_2 = 9 \times 10^3 [\cos\theta_2 \hat{\mathbf{i}} - \sin\theta_2 \hat{\mathbf{j}}]$

$\mathbf{E}_2 = 9 \times 10^3 \left[\frac{4}{5}\hat{\mathbf{i}} - \frac{3}{5}\hat{\mathbf{j}}\right]$

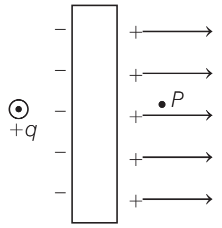
$= (72\hat{\mathbf{i}} - 54\hat{\mathbf{j}}) \times 10^2$

$\therefore \quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$= (63\hat{\mathbf{i}} - 27\hat{\mathbf{j}}) \times 10^2$ V/m

25. In a uniform electric field, field lines should be straight but line of force cannot pass through the body of metal sphere and must end/start from the sphere normally. All these conditions are fulfilled only in option (d).

26. When a point charge $+q$ is placed at a distance (d) from an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge develops on opposite side of the plane.



Hence, the field at a point P on the other side of the plane is directed perpendicular to the plane and away from the plane as shown in figure.

27. When the point is situated at a point on diameter away from the centre of hemisphere charged uniformly positively, the electric field is perpendicular to the diameter. The components of electric intensity parallel to the diameter cancel out.

28. Electric field lines always move from positive to negative charge and is always normal to the surface of conductor.

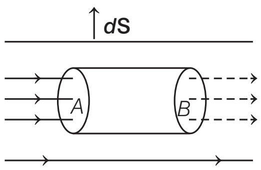
29. Here, $\mathbf{E} = 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\mathbf{S} = 100\,\hat{\mathbf{k}}$

(direction of area is perpendicular to XY-plane)

$\therefore \quad \phi = \mathbf{E} \cdot \mathbf{S} = (8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot 100\,\hat{\mathbf{k}} = 300$ units

30. Flux through surface A,



$\phi_A = E \times \pi R^2$

and $\phi_B = -E \times \pi R^2$

Electric flux through the curved surface,

$$\phi = \int \mathbf{E} \cdot d\mathbf{S} = \int E\, dS \cos 90^\circ = 0$$

$\therefore$ Total flux $= E\pi R^2 - E\pi R^2 + 0 = 0$

32. Equation $|\mathbf{E}| = \frac{q_{\text{enc}}}{\varepsilon_0 |A|}$ gives $\int \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$

Now, in finding the electric field by above equation, the integral is easy to evaluate, if $|\mathbf{E}| =$ constant. Also, if $|\mathbf{E}| =$ constant for the surface, then surface is equipotential.

33. The electric flux through a surface depends only on amount of charge enclosed by that surface. It does not depend on size and shape of the surface, as per Gauss's theorem in electrostatic. Therefore, electric flux through the given surfaces is the same for all the figures.

34. According to Gauss's theorem in electrostatics, $\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$. Here, $\mathbf{E}$ is due to all the charges q_1, q_2, q_3, q_4 and q_5. As, q is charge enclosed by the Gaussian surface, therefore $q = q_2 + q_4$.

35. According to Gauss's theorem,

$$\oint \mathbf{E}\cdot d\mathbf{S} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

$$\Rightarrow \quad E\cdot 4\pi x^2 = \frac{Q}{\varepsilon_0}$$

or $$E = \frac{Q}{4\pi\varepsilon_0 x^2}$$

36. Here, $\phi = ar^2 + b$

As, $$E = -\frac{d\phi}{dr} = -2ar$$

$$\therefore \quad \oint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$-2ar\cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

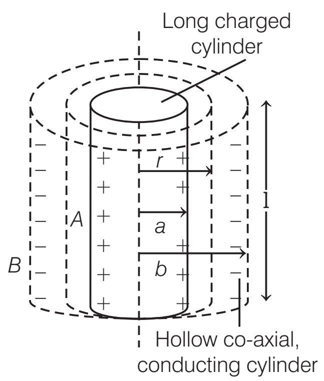
$$\Rightarrow \quad q = -8\,\varepsilon_0 a\pi r^3 \quad \ldots\text{(i)}$$

$$\therefore \quad \rho = \frac{q}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \rho = -6\,a\varepsilon_0 \quad \text{[from Eq. (i)]}$$

37. We have, $E = \dfrac{\sigma}{2\,\varepsilon} = \dfrac{\sigma}{2\,\varepsilon_0 K}$ $\left(\because \dfrac{\varepsilon}{\varepsilon_0} = K\right)$

38. The charge on cylinder A, $q = \lambda l$



Total charge = Linear charge density × Length

This charge spreads uniformly on A and a charge $-q$ is induced on B. Let $\mathbf{E}$ be the electric field produced in the space between the two cylinders. Consider a Gaussian cylindrical surface of radius r between the two given cylinders.

Electric flux linked with the Gaussian surface,

$$\phi_E = \int \mathbf{E}\cdot d\mathbf{S} = \int E\cdot dS\cos 0^\circ = E\int dS = E\times 2\pi rl$$

(As angle between the direction of electric field and area vector is zero)

According to Gauss's theorem, $\phi_E = E\times 2\pi rl = \dfrac{q}{\varepsilon_0}$

$$\Rightarrow \quad E\times 2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

39. As, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2} = \dfrac{1}{4\pi\varepsilon_0}\times\dfrac{4\pi R^2\sigma}{R^2} = \dfrac{\sigma}{\varepsilon_0}$

This is independent of radius and depends on σ. Hence, the electric field on the surface of new sphere will be E.

40. Inside hollow sphere, $E = 0$. On the surface of hollow sphere, $E =$ maximum and outside the sphere, $E \propto 1/r^2$. So the variations are correctly shown in option (d).

41. As, work done, $W_{BA} = q\,(V_A - V_B)$

$$= q\left(\frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}\right) = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

42. As we know, at any point inside a charged conducting sphere, $E = 0$ and $V = \dfrac{kq}{R}$ = potential on the surface = 100 V

Also, $$E = -\frac{dV}{dr} = -\frac{d}{dr}\text{(constant)} = \text{Zero}$$

43. Electric potential inside the hollow conducting sphere is constant and equal to potential at the surface of the sphere, *i.e.* $\dfrac{Q}{4\pi\varepsilon_0 R}$.

44. As, $$\frac{4}{3}\pi R^3 = n\times\frac{4}{3}\pi r^3$$

$$\therefore \quad R = n^{1/3} r$$

New potential, $V' = \dfrac{nq}{4\pi\varepsilon_0 r} = n^{2/3} V$

45. As we have,

$$V = \frac{q}{4\pi\varepsilon_0 x_0}\left(1 + \frac{1}{3} + \frac{1}{5} + \ldots\right) - \frac{q}{4\pi\varepsilon_0 x_0}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\ldots\right)$$

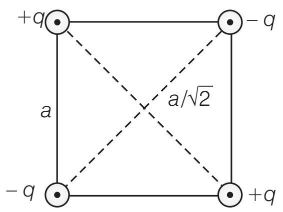
$$= \frac{q}{4\pi\varepsilon_0 x_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right) = \frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$$

46. Potential at the centre O,

$$V = 4\times\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a\sqrt{2}}$$

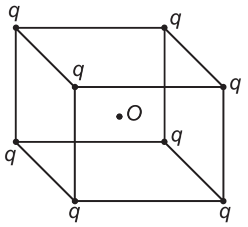
where, $q = \dfrac{10}{3}\times 10^{-9}$ C, in magnitude

and $a = 8\text{ cm} = 8\times 10^{-2}\text{ m}$



Then, $$V = 4\times 9\times 10^9\times\frac{\frac{10}{3}\times 10^{-9}}{\frac{8\times 10^{-2}}{\sqrt{2}}} = 1500\sqrt{2}\text{ V}$$

47. Let there is a cube of side b and its centre is O. The charge q is placed at each of the corners. Side of the cube = b



Length of the main diagonal of the cube

$$= \sqrt{b^2 + b^2 + b^2} = \sqrt{3}b$$

Distance of centre O from each of the vertices,

$$r = \frac{b\sqrt{3}}{2} \quad \text{...(i)}$$

Potential at point O due to one charge,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Potential at point O due to all charges placed at the vertices of the cube,

$$V' = 8V = \frac{8 \times 1 \times q}{4\pi\varepsilon_0 \cdot r}$$

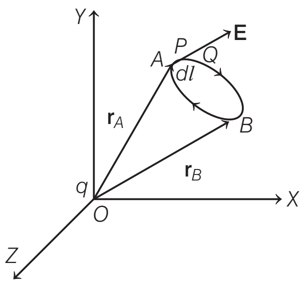
$$= \frac{8q \times 2}{4\pi\varepsilon_0 \cdot b\sqrt{3}} \quad \text{[from Eq. (i)]}$$

$$= \frac{4\,q}{\sqrt{3}\pi\varepsilon_0 b}$$

48. The tangential component of electrostatic field is continuous from one side of a charged surface to another, we use that the work done by electrostatic field on a closed loop is zero.

Let ABA be a charged surface in the field of a point charge q lying at origin.

Let $\mathbf{r}_A$ and $\mathbf{r}_B$ be its positive vectors at points A and B, respectively.



Let $\mathbf{E}$ be the electric field at point P, thus $E \cos\theta$ is the tangential component of electric field $\mathbf{E}$.

To prove that $E\cos\theta$ is continuous from one to another side of the charge surface, we have to find the value of $\oint_{ABA} \mathbf{E} \cdot d\mathbf{l}$. If it comes to be zero, then we can say that tangential component of $\mathbf{E}$ is continuous.

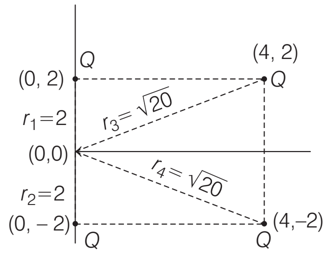
$$\int_A^B \mathbf{E} \cdot d\mathbf{l} = \frac{1}{4\pi\varepsilon_0} q \cdot \left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

and $$\int_B^A \mathbf{E} \cdot d\mathbf{l} = \frac{1}{4\pi\varepsilon_0} q \cdot \left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\therefore \quad \oint_{ABA} \mathbf{E} \cdot d\mathbf{l} = \int_A^B \mathbf{E} \cdot d\mathbf{l} + \int_B^A \mathbf{E} \cdot d\mathbf{l}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q \cdot \left(\frac{1}{r_A} - \frac{1}{r_B} + \frac{1}{r_B} - \frac{1}{r_A}\right) = 0$$

49. The four charges are shown in the figure below



Electric potential at origin (0, 0) due to these charges can be found by scalar addition of electric potentials due to each charge.

$$\therefore \quad V = \frac{kQ}{r_1} + \frac{kQ}{r_2} + \frac{kQ}{r_3} + \frac{kQ}{r_4} \quad \text{... (i)}$$

$$\Rightarrow \quad V = kQ\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{\sqrt{20}} + \frac{1}{\sqrt{20}}\right] = kQ\left[1 + \frac{1}{\sqrt{5}}\right]$$

$$\Rightarrow \quad V = kQ\frac{(\sqrt{5}+1)}{\sqrt{5}} \text{ volt} \quad \text{...(ii)}$$

Now, if another charge Q is placed at origin, then work done to get the charge at origin,

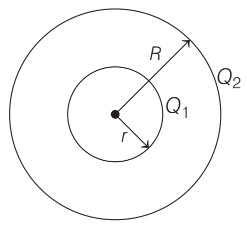
$$W = QV \quad \text{...(iii)}$$

By putting the value of V from Eq. (ii) in Eq. (iii), we get

$$W = kQ^2\frac{(\sqrt{5}+1)}{\sqrt{5}} \text{ J}$$

or $$W = \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right) \text{ J}$$

50. Given, total charge Q is uniformly distributed over concentric shells of radii r and R.



Let Q_1 and Q_2 be the charges over inner and outer shells, such that charge densities are equal on both shells.

Charge density of inner shell, $\sigma = \dfrac{Q_1}{4\pi r^2}$

Charge density of outer shell, $\sigma = \dfrac{Q_2}{4\pi R^2}$

$\because \quad \dfrac{Q_1}{4\pi r^2} = \dfrac{Q_2}{4\pi R^2}$

$\Rightarrow \quad \dfrac{Q_1}{r^2} = \dfrac{Q_2}{R^2}$

$\Rightarrow \quad Q_2 = \dfrac{Q_1 R^2}{r^2}$

Also, $\quad Q_1 + Q_2 = Q$

$\Rightarrow \quad Q_1 + \dfrac{Q_1 R^2}{r^2} = Q$

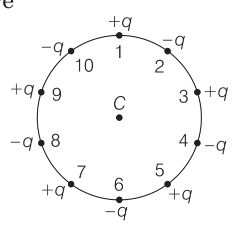
or $\quad Q_1 = \dfrac{Qr^2}{(r^2 + R^2)}$

and $\quad Q_2 = \dfrac{QR^2}{(r^2 + R^2)}$

Potential due to Q_1 and Q_2 at common centre of shells is

$$V = V_{Q_1} + V_{Q_2} = \frac{kQ_1}{r} + \frac{kQ_2}{R}$$
$$= \frac{kQr}{r^2 + R^2} + \frac{kQR}{r^2 + R^2}$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q(r + R)}{r^2 + R^2}$$

51. The arrangement of all the charges is shown in the following figure



Potential at the centre of the circle C,

$$V = \frac{kq}{R} + \frac{k(-q)}{R} + \frac{kq}{R} + \frac{k(-q)}{R} + \dots$$
$$= \Sigma \frac{kq}{R} = k\frac{\Sigma(q)}{R} = \frac{k(0)}{R} = 0$$

Electric field at centre of the circle C,

$\mathbf{E}$ = ($\mathbf{E}$ due to 5 + ve charges) + ($\mathbf{E}$ due to 5 – ve charges)

As, all the charges are equidistant from centre. So, the electric field of one charge will get cancelled due to another symmetrical charge in front of it.

$\therefore \quad E = 0$

52. In a uniform electric field, when a positively charged particle is released from rest, it moves along the electric field (*i.e.* from higher potential to lower potential). Therefore, electric potential energy of charge decreases.

53. Here, $U_1 = \dfrac{Q(-q)}{4\pi\varepsilon_0 r}$; $U_2 = \dfrac{Q(-q)}{4\pi\varepsilon_0 (r/2)}$

$$\therefore \quad U_1 - U_2 = \frac{Q(-q)}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{2}{r}\right]$$
$$= \frac{Qq}{4\pi\varepsilon_0 r} = 9\text{ J} \quad \dots\text{(i)}$$

When negative charge travels first half of distance, *i.e.* $r/4$, potential energy of the system,

$$U_3 = \frac{Q(-q)}{4\pi\varepsilon_0 (3r/4)} = -\frac{Qq}{4\pi\varepsilon_0 r} \times \frac{4}{3}$$

$\therefore$ Work done $= U_1 - U_3$

$$= -\frac{Q(q)}{4\pi\varepsilon_0 r} + \frac{Qq}{4\pi\varepsilon_0 r} \times \frac{4}{3}$$
$$= \frac{Qq}{4\pi\varepsilon_0 r} \times \frac{1}{3} = \frac{9}{3} = 3\text{ J}$$

54. Because, all the points on the circular path are at same potential. So, work done is zero.

55. For charge q placed at the centre of circle, the circular path is an equipotential surface and hence, work done along all paths AB or AC or AD or AE is zero.

56. From a collection of charges, whose total sum is not zero, equipotentials at large distances must be spheres only.

57. Here, displacement would be

$$\Delta\mathbf{r} = (4 - 2)\,\hat{\mathbf{i}} + (2 - 2)\,\hat{\mathbf{j}} = 2\hat{\mathbf{i}}$$

Now, $\quad \Delta\mathbf{V} = \mathbf{E} \cdot \Delta\mathbf{r} = (y\hat{\mathbf{i}} + x\hat{\mathbf{j}}) \cdot 2\hat{\mathbf{i}} = 2y$

$\therefore$ Work done, $W = q(\Delta\mathbf{V}) = q \cdot 2y = 1 \times 2y = 2y$ J

58. We observe that in all the three parts, $V_A = 20$ V and $V_B = 40$ V. Work done in carrying a charge q from A to B is $W = q(V_B - V_A)$ = same in all the three figures.

59. $E = -\dfrac{\partial V}{\partial x}\hat{\mathbf{i}} - \dfrac{\partial V}{\partial y}\hat{\mathbf{j}} - \dfrac{\partial V}{\partial z}\hat{\mathbf{k}}$

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx}\left[\frac{20}{x^2 - 4}\right] = \frac{40x}{(x^2 - 4)^2}$$

$$\Rightarrow \quad E_x \text{ at } x = 4\,\mu\text{m} = \frac{10}{9}\text{ V}/\mu\text{m}$$

and is along positive x-direction.

60. As, it is clear from figure

$$\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$$

As, $\quad E = -\dfrac{dV}{dr} = -\dfrac{(V_C - V_A)}{d} = \dfrac{V_A - V_C}{d}$

$\therefore \quad V_A - V_C = Ed$

61. As, $\int_{l=\infty}^{l=0} -E \cdot dl = V$, the potential at the centre of ring,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{9 \times 10^9 \times 1.11 \times 10^{-10}}{0.5} = 2\text{V}$$

62. As, $E = -\dfrac{dV}{dr} = -\dfrac{(1 - 3)}{0.3 - 0.1}\text{ Vm}^{-1} = 10\text{ Vm}^{-1}$

63. As, $E = -\dfrac{dV}{dr} = -\dfrac{d}{dx}(4x^2) = -8x = -8\,(1) = -8\text{Vm}^{-1}$

Negative sign indicates $\mathbf{E}$ is along negative direction of X-axis.

64. As the work has been done by field, so it should be positive.

The work done, $W = q(V_A - V_B)$

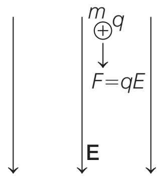
We know that, change in kinetic energy = Work done

$$= q(V_A - V_B)$$
$$= 10^{-8}(600 - 0) = 6 \times 10^{-6}\,\text{J}$$

65. As, net work done = final PE – initial PE

$$= \frac{Qq}{4\pi\varepsilon_0 l} - \frac{Qq}{4\pi\varepsilon_0 l} = \text{zero}$$

66.



Acceleration of the particle, $a = \frac{F}{m} = \frac{q}{m} \cdot E$

Velocity v and distance x can be related using $v^2 - u^2 = 2ax$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{q}{m}E\right)x} \qquad (\because u = 0)$$

or $$v^2 = 2\left(\frac{q}{m}E\right)x$$

This equation resembles a parabola $y^2 = 4ax$. So, the graph between v and x will be as shown in option (c).

67. In figure, spacing between electric lines of force increases from left to right. Therefore, E on left is greater that E on right. Force on + q charge of dipole is smaller and to the right. Force on – q charge of dipole is bigger and to the left. Hence, the dipole will experience a net force towards the left.

68. Here, $\theta = 60^\circ$, $E = 10^5\ \text{NC}^{-1}$,

$$\tau = 8\sqrt{3}\ \text{N-m}, q = ?, 2a = 2\ \text{cm} = 2 \times 10^{-2}\,\text{m}$$

From, $\tau = pE\sin\theta = q(2a)E\sin\theta$

or $$q = \frac{\tau}{2aE\sin\theta}$$
$$= \frac{8\sqrt{3}}{2 \times 10^{-2} \times 10^5 \times \sin 60^\circ}$$
$$= \frac{8\sqrt{3}}{2 \times 10^3 \times \sqrt{3}/2}$$

or $q = 8 \times 10^{-3}\ \text{C}$

69. On equatorial line of electric dipole, $E \propto \frac{1}{r^3}$

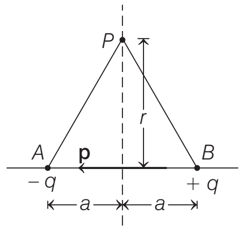
70. In case of an electric dipole, $F \propto \frac{1}{r^3}$

$\therefore$ New force = $F/2^3 = F/8$

71. The point lies on equatorial line of a short dipole.

$$\therefore \quad E = \frac{p}{4\pi\varepsilon_0 r^3}$$
$$= \frac{9 \times 10^9 (10^{-6} \times 10^{-8})}{(10^{-1})^3}$$
$$= 9 \times 10^{-2}\ \text{NC}^{-1} = 0.09\ \text{NC}^{-1}$$

72. The given problem can be shown as clearly potential difference at point P due to dipole is



$$V = V_{AP} + V_{BP} \qquad \text{(scalar addition)}$$

$$\Rightarrow \quad V = \frac{k(-q)}{AP} + \frac{k(q)}{BP} \qquad \ldots\text{(i)}$$

Here, $AP = BP = \sqrt{a^2 + r^2}$

$$\therefore \quad V = -\frac{kq}{\sqrt{a^2+r^2}} + \frac{kq}{\sqrt{a^2+r^2}} = 0 \qquad \ldots\text{(ii)}$$

Now, electric field at any point on Y-axis, *i.e.* equatorial line of the dipole can be given by

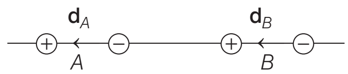
$$\mathbf{E} = -\frac{k\,\mathbf{p}}{r^3} \qquad \text{(standard expression)}$$

$$\Rightarrow \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{r^3}$$

Given, $r = d$

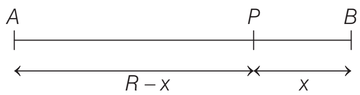
$$\therefore \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{d^3} \qquad \ldots\text{(iii)}$$

73. As, dipole moments point in same direction



So, potential of both dipoles can be same at some point between A and B.

Let potentials are same at P, distant x from B as shown below



Then, $$\frac{4qa}{(R-x)^2} = \frac{2qa}{(x)^2}$$

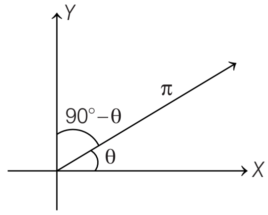
$$\Rightarrow \quad 2x^2 = (R-x)^2 \Rightarrow \sqrt{2}\,x = R - x$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}+1}$$

Distance from A is

$$R - x = R - \frac{R}{\sqrt{2}+1} = \frac{\sqrt{2}R}{\sqrt{2}+1}$$

74.



Torque applied on a dipole, $\tau = pE \sin\theta$

where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\,\hat{\mathbf{i}}$

It means field is directed along positive x-direction, so angle between dipole and field will remain θ, therefore torque in this direction,

$$E_1 = pE_1 \sin\theta$$

For electric field $E_2 = \sqrt{3}\,E\,\hat{\mathbf{j}}$,

It means field is directed along positive Y-axis, so angle between dipole and field will be $90° - \theta$.

Torque in this direction, $\tau_2 = pE \sin(90° - \theta)$

$$= p\sqrt{3}\,E_1 \cos\theta$$

According to question, $\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$

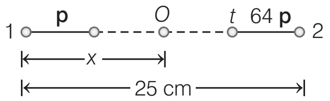
$\therefore \quad pE_1 \sin\theta = p\sqrt{3}\,E_1\cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan 60°$

$\therefore \quad \theta = 60°$

75. Let us consider a neutral point O lies at a distance x from the dipole of moment p or at a distance $(25 - x)$ from dipole of 64 p.



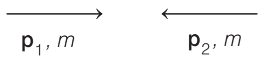
At O, electric field due to dipole (1) = electric field due to dipole (2)

$\Rightarrow \quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{x^3} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2(64\,p)}{(25-x)^3}$

$\Rightarrow \quad \dfrac{1}{x^3} = \dfrac{64}{(25-x)^3}$

$\Rightarrow \quad x = 5$ cm

76. When two electric dipoles of opposite dipole moments are placed on a line, they experience force of attraction along the same line as shown below



Considering both dipoles as a system, we find that net external force on system is zero, *i.e.* $\mathbf{F}_{\text{ext}} = 0$.

So, total mechanical energy = constant

$$(\text{ME})_i = (\text{ME})_f$$

or $\quad (\text{KE})_i + (\text{PE})_i = (\text{KE})_f + (\text{PE})_f$

As, initially they are released from rest, so initial KE is zero and finally they are infinite apart, so final PE is zero.

$$0 + \left(-\frac{2kp_1p_2}{r^3}\cos 180°\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + 0$$

Here, $\quad m_1 = m_2 = m,\ p_1 = p_2 = p,\ r = a$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{2kp^2}{a^3} \quad \text{...(i)}$$

Using conservation of momentum,

$$\mathbf{p}_i = \mathbf{p}_f$$

$\Rightarrow \quad 0 = m\mathbf{v}_1 + m\mathbf{v}_2$

or $\quad \mathbf{v}_1 = -\mathbf{v}_2$

$\Rightarrow \quad v_1 = v_2 = v \quad$ ...(ii)

Putting this value in Eq. (i), we get

$$mv^2 = \frac{2kp^2}{a^3}$$

or $\quad v = \dfrac{p}{a}\sqrt{\dfrac{2k}{ma}}$

or $\quad v = \dfrac{p}{a}\sqrt{\dfrac{1}{2\pi\varepsilon_0 ma}} \quad \left(\because k = \dfrac{1}{4\pi\varepsilon_0}\right)$

77. As, charge on capacitor, $Q = CV$

Since, V is constant, therefore $Q \propto C$

Hence, C becomes $\dfrac{100}{40} = 2.5$ times

$\therefore \quad K = 2.5$

78. As, $C = \dfrac{A\varepsilon_0}{d}$ ...(i)

After inserting the slab,

$$C' = \frac{A\varepsilon_0}{(d-b)} = \frac{A\varepsilon_0}{d - \dfrac{d}{2}}$$

or $\quad C' = \dfrac{2A\varepsilon_0}{d} \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{C'}{C} = \frac{2}{1} = 2:1$$

79. As, $r_b - r_a = 1 \text{ mm} = 10^{-3}$ m

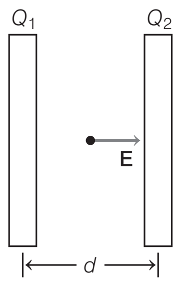
From $\quad C = \dfrac{4\pi\varepsilon_0 r_a r_b}{r_b - r_a}$

$\Rightarrow \quad 10^{-6} = \dfrac{1\,(r_b - 10^{-3})\,r_b}{9\times10^9\,(10^{-3})}$

$\Rightarrow \quad r_b^2 = 9$

or $\quad r_b = 3$ m

80. On bringing the charged metal plates closer, electric field $\mathbf{E}$ in the intervening space is



$\therefore \quad E = \dfrac{\sigma_1}{2\varepsilon_0} - \dfrac{\sigma_2}{2\varepsilon_0} = \dfrac{Q_1}{2A\varepsilon_0} - \dfrac{Q_2}{2A\varepsilon_0}$

or $$E = \frac{(Q_1 - Q_2)}{2A\varepsilon_0} = \frac{V}{d}$$

or $$V = \frac{(Q_1 - Q_2)d}{2A\varepsilon_0}$$

$$\Rightarrow \quad V = \frac{Q_1 - Q_2}{2C} \qquad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

81. The capacities of two individual condensers are

$$C_1 = \frac{K_1\varepsilon_0 A}{d_1}$$

and $$C_2 = \frac{K_2\varepsilon_0 A}{d_2}$$

The arrangement is equivalent to two capacitors joined in series. Therefore, the combined capacity (C_s) is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{K_1\varepsilon_0 A} + \frac{d_2}{K_2\varepsilon_0 A}$$

$$\Rightarrow \quad \frac{1}{C_s} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right] = \frac{d_1K_2 + d_2K_1}{\varepsilon_0 AK_1K_2}$$

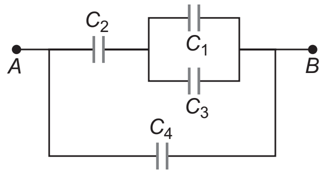
$$\Rightarrow \quad C_s = \frac{\varepsilon_0 AK_1K_2}{d_1K_2 + d_2K_1} \text{ but } C_s = \frac{K\varepsilon_0 A}{(d_1 + d_2)}$$

$$\therefore \quad K = \frac{K_1K_2(d_1 + d_2)}{(d_1K_2 + d_2K_1)}$$

82. Positive plate of all the three condensers is connected to one point A and negative plate of all the three condensers is connected to point B, *i.e.* they are joined in parallel.

$$\therefore \quad C_p = 3 + 3 + 3 = 9\,\mu\text{F}$$

83. The arrangement can be redrawn as shown in the adjoining figure



$$\therefore \quad C_{13} = C_1 + C_3 = 9 + 9 = 18\,\mu\text{F}$$

and $$C_{2-13} = \frac{C_2 \times C_{13}}{C_2 + C_{13}} = \frac{9\,\mu\text{F} \times 18\,\mu\text{F}}{(9 + 18)\,\mu\text{F}} = 6\,\mu\text{F}$$

$$\therefore \quad C = C_{2-13} + C_4 = 6\,\mu\text{F} + 9\,\mu\text{F} = 15\,\mu\text{F}$$

84. The required capacitance, $C = 2\,\mu\text{F}$

Potential difference,

$$V = 1\text{ kV} = 1000\text{ V}$$

Capacitance of each capacitor $C_1 = 1\,\mu\text{F}$ and it can withstand a potential difference of $V_1 = 400\text{ V}$

Let the n capacitors are connected in series and there are m rows of such capacitors.

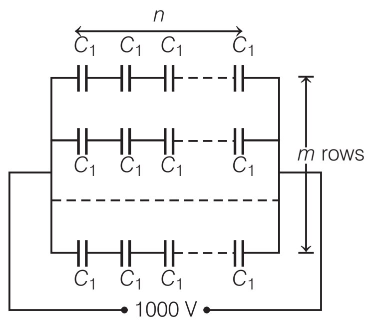
As the potential difference across each row is 1000 V.

So, the potential difference across each capacitor $= \frac{1000}{n}$

Minimum number of capacitors that must be connected in series in a row are

$$\frac{1000}{n} = 400$$

$$\Rightarrow \quad n = 2.5$$



Here, n is the number of capacitors, so it should be a whole number. If we take $n = 2$, then potential difference across each capacitor is 500 V.

Here, according to question, a capacitor can bear only 400 V, so they burst. We take the value of $n = 3$.

So, the capacitance of each row (in series),

$$\frac{1}{C'} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{3}{1}$$

$$\Rightarrow \quad C' = \frac{1}{3}$$

The total capacitance of m rows is $m \times \frac{1}{3} = \frac{m}{3}$

According to question, the total capacitance required is $2\,\mu\text{F}$.

So, $$\frac{m}{3} = 2 \Rightarrow m = 6$$

Thus, the total number of capacitor $= m \times n = 3 \times 6 = 18$
So, $1\,\mu\text{F}$ capacitors are connected in 6 rows having 3 capacitors in each row.

85. Net capacity of 5 capacitors joined in parallel $= 5 \times 2 = 10\,\mu\text{F}$

Now, it is connected with two capacitors of $2\,\mu\text{F}$ each in series, thus equivalent capacitance is $\frac{10}{11}\,\mu\text{F}$.

86. The arrangement behaves as a combination of 2 capacitors each of capacitance, $C = \frac{\varepsilon_0 A}{d}$.

Thus, equivalent capacity $= 2C$

$\therefore$ Total energy stored, $U = \frac{1}{2} \times (2C)V^2 = CV^2 = \frac{\varepsilon_0 A}{d}V^2$

87. In the arrangement shown both plates of capacitor C_3 are joined to point B. Hence, it does not act as a capacitor and is superfluous. Now, C_1 and C_2 are in parallel, hence

$$C_{AB} = C_1 + C_2 = C + C = 2C$$

88. In the given figure,

slope of OA > slope of OB

Since, we know that,

net capacitance of parallel combination > net capacitance of series combination

$\therefore$ Parallel combination's capacitance,

$$C_p = C_1 + C_2 = \frac{500\,\mu\text{C}}{10\text{ V}} = 50\,\mu\text{F} \quad \text{... (i)}$$

Series combination's capacitance,

$$C_s = \frac{C_1 C_2}{C_1 + C_2} = \frac{80\,\mu\text{C}}{10\text{ V}} = 8\,\mu\text{F} \quad \text{... (ii)}$$

or $\quad C_1 C_2 = 8 \times (C_1 + C_2) = 8 \times 50\,\mu\text{F}$

$= 400\,\mu\text{F} \quad$ [using Eq. (i)] $\quad$ ...(iii)

From Eqs. (i) and (iii), we get

$$C_1 = 50 - C_2 \text{ and } C_1 C_2 = 400$$

$\Rightarrow \quad C_2 (50 - C_2) = 400$

$\Rightarrow \quad 50\,C_2 - C_2^2 = 400$

or $\quad C_2^2 - 50\,C_2 + 400 = 0$

$\Rightarrow \quad C_2 = \dfrac{+50 \pm \sqrt{2500 - 1600}}{2} = \dfrac{+50 \pm 30}{2}$

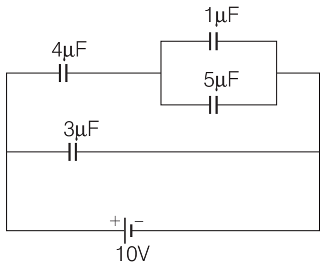
$\Rightarrow \quad C_2 = +40\,\mu\text{F}$ or $+10\,\mu\text{F}$

Also, $\quad C_1 = 50 - C_2$

$\Rightarrow \quad C_1 = +10\,\mu\text{F}$ or $+40\,\mu\text{F}$

Hence, capacitance of two given capacitors is $10\,\mu\text{F}$ and $40\,\mu\text{F}$.

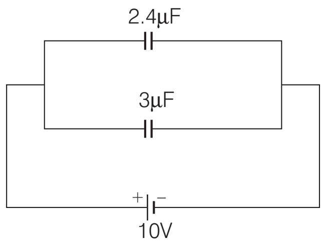
89. Given circuit is



In parallel, $\quad C_{eq} = 5 + 1 = 6\,\mu\text{F}$

and in series, $C'_{eq} = \dfrac{6 \times 4}{6 + 4} = 2.4\,\mu\text{F}$

This is equivalent to



So, potential difference across upper branch = 10 V

Using $Q = C \times V$, charge delivered to upper branch is

$$Q = C'_{eq} \cdot V$$
$$= 2.4\,\mu\text{F} \times 10\text{V} = 24\,\mu\text{C}$$

As we know, in series connection, same charge is shared by capacitors, so charge on $4\,\mu$F capacitor and $6\,\mu$F capacitor would be same,

i.e. $\quad Q'_{4\mu\text{F}} = 24\,\mu\text{C}$

90. The two condensers are connected in parallel, we have

$$C_p = C + \frac{C}{2} = \frac{3C}{2}$$

$\therefore$ Total work done in charging both the condensers,

$$W = \frac{1}{2} C_p V^2 = \frac{1}{2} \times \frac{3C}{2} V^2 = \frac{3}{4} CV^2$$

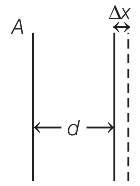
91. On sharing of charges loss in electrical energy,

$$\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$

In present case, $C_1 = C_2 = C$

$$\therefore \quad \Delta U = \frac{C^2}{2(2C)} (V_1 - V_2)^2 = \frac{1}{4} C (V_1 - V_2)^2$$

92. Let the distance between the plates be increased by a very small distance Δx. The force on each plate is F.



The amount of work done in increasing the separation by Δx

$= \text{Force} \times \text{Increased distance}$

$= F \cdot \Delta x \quad$ ...(i)

Increase in volume of capacitor

$= \text{Area of plates} \times \text{Increased distance}$

$= A \cdot \Delta x$

$$u = \text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$\text{Energy} = u \times \text{Volume} = u \cdot A \cdot \Delta x \quad$ ...(ii)

As, $\quad$ energy = work done

$u \cdot A \cdot \Delta x = F \cdot \Delta x \quad$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad F = u \cdot A$

$$= \frac{1}{2} \varepsilon_0 E^2 \cdot A \quad \left(\because \; u = \frac{1}{2} \varepsilon_0 E^2 \text{ and } E = \frac{V}{d}\right)$$

$$= \frac{1}{2} \varepsilon_0 \cdot \frac{V^2}{d^2} \cdot A$$

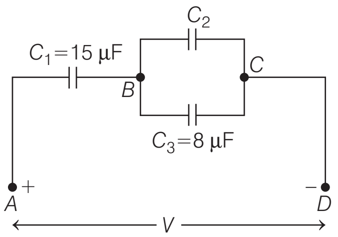
$$= \left(\frac{\varepsilon_0 A}{d} \cdot V\right) \frac{V}{d} \times \frac{1}{2} \quad \left(\because \; C = \frac{\varepsilon_0 A}{d}, CV = q\right)$$

$$= \frac{1}{2} \cdot E \cdot C \cdot V$$

$$= \frac{1}{2} QE$$

The factor of $\frac{1}{2}$ in the force can be explained by the fact that the field is zero inside the conductor and outside the conductor, field is E. So, the average value of the field, *i.e.* $\frac{E}{2}$ contributes to the force against which the plates are moved.

93.



In given arrangement,

$$V_{BC} = \text{potential drop across } C_2 = 20\text{V}$$

As combination of capacitors C_2 and C_3 is in parallel, so potential drop across C_3 is also 20 V.

So, $\quad Q_3 = \text{charge on } C_3 = C_3 \times V$
$= 8 \times 10^{-6} \times 20 = 160\,\mu\text{C}$

Also, total charge given by cell, $Q = 750\ \mu\text{C}$

As combination of capacitors C_2 and C_3 is in series with capacitor C_1, so charge on

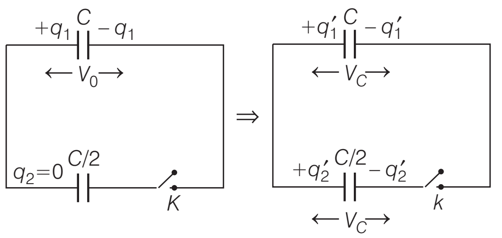
C_1 = charge on combination of capacitors C_2 and C_3 = total charge given by cell.

Hence, $Q_2 + Q_3 = 750\ \mu\text{C}$

$\Rightarrow \quad Q_2 + 160\,\mu\text{C} = 750\,\mu\text{C}$

$\Rightarrow \quad Q_2 = 750 - 160 = 590\,\mu\text{C}$

94.



Before charge distribution   After charge distribution

Using law of conservation of charge, we have

$$q_1 + q_2 = q_1' + q_2'$$

$$CV_0 + \frac{C}{2}(0) = CV_C + \frac{C}{2}(V_C)$$

$$CV_0 + 0 = \frac{3}{2}CV_C$$

$$\Rightarrow \quad CV_0 = \frac{3}{2}CV_C$$

$$\Rightarrow \quad V_C = \frac{2}{3}V_0 \qquad \ldots\text{(i)}$$

Here, V_C = common potential.

Now, initial electrical energy,

$$U_i = \frac{1}{2}CV_0^2 + \frac{1}{2}\left(\frac{C}{2}\right)(0)^2$$

$$= \frac{1}{2}CV_0^2 + 0$$

$$= \frac{1}{2}CV_0^2$$

and final electrical energy,

$$U_f = \frac{1}{2}\left(C + \frac{C}{2}\right)V_C^2$$

$$= \frac{1}{2}\left(\frac{3C}{2}\right)\left(\frac{2}{3}V_0\right)^2$$

$$= \frac{1}{2} \times \frac{3C}{2} \times \frac{4}{9}V_0^2$$

$$= \frac{1}{3}CV_0^2$$

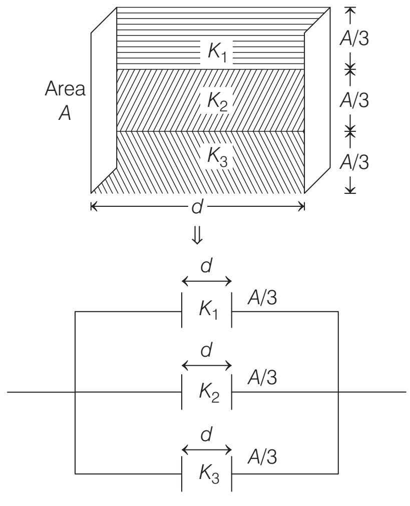
Loss in electrical energy,

$$\Delta U_{\text{loss}} = U_i - U_f$$

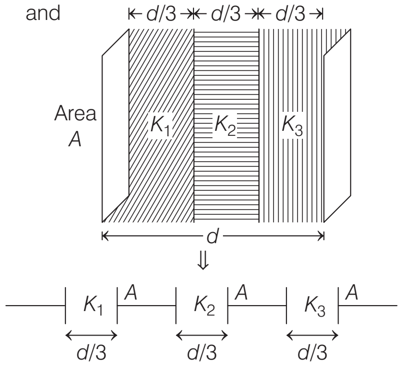
$$= \frac{1}{2}CV_0^2 - \frac{1}{3}CV_0^2$$

$$= \frac{1}{6}CV_0^2$$

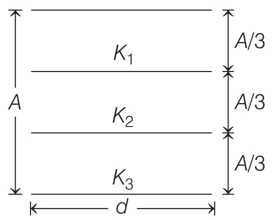
95. A capacitor filled with dielectrics can be treated/compared as series/parallel combinations of capacitor having individual dielectric. *e.g.*

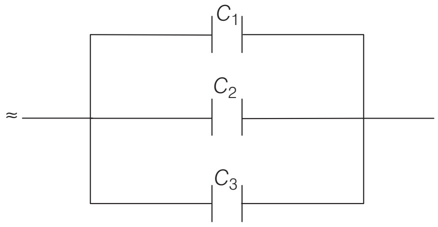


and



***Case* I**





Capacitance in the equivalent circuit are

$$C_1 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_1 = \frac{\varepsilon_0 A}{3d} K_1$$

$$C_2 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_2 = \frac{\varepsilon_0 A}{3d} K_2$$

and $$C_3 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_3 = \frac{\varepsilon_0 A}{3d} K_3$$

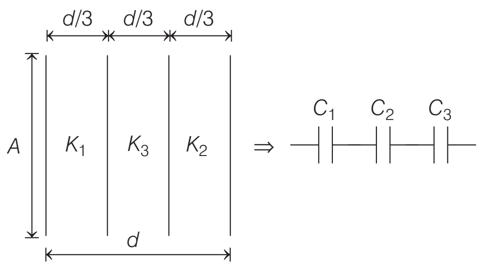
So, equivalent capacitance,

$$C_{\text{I}} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d} K_1 + \frac{\varepsilon_0 A}{3d} K_2 + \frac{\varepsilon_0 A}{3d} K_3$$

$$C_{\text{I}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \quad \ldots\text{(i)}$$

***Case* II**



Capacitance of equivalent circuit are

$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} \cdot K_1 = \frac{3\varepsilon_0 A}{d} K_1$$

$$C_2 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_2 = \frac{3\varepsilon_0 A}{d} K_2$$

and $$C_3 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_3 = \frac{3\varepsilon_0 A}{d} K_3$$

So, equivalent capacitance,

$$\frac{1}{C_{\text{II}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{d}{3\varepsilon_0 AK_1} + \frac{d}{3\varepsilon_0 AK_2} + \frac{d}{3\varepsilon_0 AK_3}$$

$$\Rightarrow \quad \frac{1}{C_{\text{II}}} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right]$$

$$= \frac{d}{3\varepsilon_0 A}\left[\frac{K_2K_3 + K_1K_3 + K_1K_2}{K_1K_2K_3}\right]$$

$$C_{\text{II}} = \frac{3\varepsilon_0 A}{d}\left[\frac{K_1 K_2 K_3}{K_1 K_2 + K_2 K_3 + K_3 K_1}\right] \quad \ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{C_{\text{I}}}{C_{\text{II}}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \times \frac{d(K_1K_2 + K_2K_3 + K_3K_1)}{3\varepsilon_0 A(K_1K_2K_3)}$$

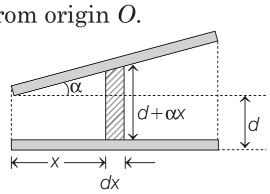
$$= \frac{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}{9K_1K_2K_3}$$

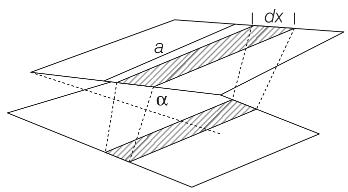
Now, energy stored in capacitor, $E = \frac{1}{2}CV^2$

$$\Rightarrow \quad E \propto C$$

$$\therefore \quad \frac{E_{\text{I}}}{E_{\text{II}}} = \frac{C_{\text{I}}}{C_{\text{II}}}$$

96. For given capacitor, consider a small width dx at a distance x from origin O.





For this elemental capacitor, we have

Area of each plate $= a \times dx$

Distance between plates $= d + \alpha x$

Capacity of this elemental capacitor, $dC = \frac{\varepsilon_0 a dx}{d + \alpha x}$

Total capacity of arrangement is sum of capacitances of all such elemental capacitors because they all are in parallel.

Hence, $C_{\text{system}} = \int_0^a dC = \int_0^a \frac{\varepsilon_0 a}{d + \alpha x} \cdot dx = \frac{\varepsilon_0 a}{\alpha} \ln\left(1 + \frac{\alpha a}{d}\right)$

As, for small values of $\frac{a\alpha}{d}$,

$$\ln\left(1 + \frac{a\alpha}{d}\right) \approx \frac{a\alpha}{d} - \frac{1}{2}\left(\frac{a\alpha}{d}\right)^2$$

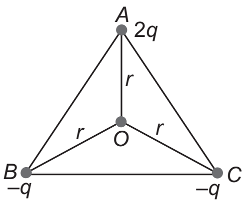
So, $$C_{\text{system}} = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{a\alpha}{2d}\right)$$

Round II

1. In an equilateral triangle, distance of centroid from all the vertices is same (say r).

$\therefore \quad V = V_1 + V_2 + V_3$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{r} - \frac{q}{r} - \frac{q}{r}\right] = 0$$



But $\quad \mathbf{E}_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{r^2}$ along AO,

$\mathbf{E}_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ along OB

and $\quad \mathbf{E}_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ along OC

Obviously, $\mathbf{E}_B + \mathbf{E}_C$ will also be in the direction of AO (extended) and hence, $\mathbf{E}_A$ and $(\mathbf{E}_B + \mathbf{E}_C)$ being in same direction will not give zero resultant.

2. Bob will experience an additional force, $F = qE$ in vertically upward direction and hence, effective acceleration due to gravity is reduced from g to $(g - a)$. Consequently, time period of oscillation will become

$$T = 2\pi\sqrt{\frac{l}{(g-a)}}$$

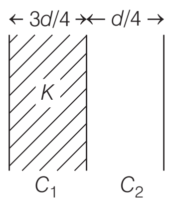
i.e. time period will increase.

3. They will not experience any force, if $|\mathbf{F}_G| = |\mathbf{F}_e|$

$$\Rightarrow \quad G\frac{m^2}{(16\times10^{-2})^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(16\times10^{-2})^2}$$

$$\Rightarrow \quad \frac{q}{m} = \sqrt{4\pi\varepsilon_0 G}$$

4. The given situation can be shown as a combination of two capacitors



The equivalent capacitance is

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{3d/4}{\varepsilon KA} + \frac{d/4}{\varepsilon_0 A}$$

$$= \frac{d}{4\varepsilon_0 A}\left(\frac{3+K}{K}\right)$$

$$C' = \frac{4K}{3+K}\,\frac{\varepsilon_0 A}{d}$$

$$= \frac{4K}{K+3}C_0 \qquad \left(\because C_0 = \frac{\varepsilon_0 A}{d}\right)$$

5. The arrangement shows a Wheatstone bridge.

As $\frac{C_1}{C_3} = \frac{C_4}{C_5} = 1$, therefore the bridge is balanced.

$$\frac{1}{C_{s_1}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}, C_{s_1} = 2\,\mu\text{F}$$

Similarly, $C_{s_2} = 2\,\mu\text{F}$

$\therefore$ Effective capacitance

$$= C_p = C_{s_1} + C_{s_2}$$

$$= 2 + 2 = 4\,\mu\text{F}$$

6. Charge on each plate of capacitor will be

$$Q = \pm CV$$

$$= \pm 25\times10^{-6}\times200$$

$$= \pm 5\times10^{-3}\text{ C}$$

7. Force on $-q_1$ due to q_2,

$$F_{12} = \frac{kq_1 q_2}{b^2}, \text{ along } X\text{-axis}$$

Force on $-q_1$ due to $-q_3$, $F_{13} = \frac{kq_1 q_3}{a^2}$,

at $\angle\theta$ with negative direction of Y-axis.

$\therefore$ x-component of force on $-q_1$ is

$$F_x = F_{12} + F_{13}\sin\theta$$

$$= kq_1\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

i.e. $\quad F_x \propto \left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$

8. Here, $\quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$

$$= \left(+\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathbf{k}}) + \left(\frac{2\sigma}{2\varepsilon_0}\right)(-\hat{\mathbf{k}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathbf{k}})$$

$$= -\left(\frac{2\sigma}{\varepsilon_0}\right)\hat{\mathbf{k}}$$

9. Here, KE = 100 eV = $100\times1.6\times10^{-19}$ J = 1.6×10^{-17} J.

This is lost when electron moves through a distance (d) towards the negative plate.

$\therefore \quad$ KE = Work done

$$= F\times s = qE\times s = e\left(\frac{\sigma}{\varepsilon_0}\right)d$$

$$d = \frac{(\text{KE})\varepsilon_0}{e\sigma}$$

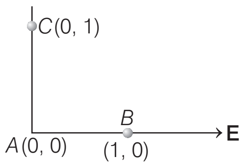
$$\Rightarrow \quad d = \frac{1.6\times10^{-17}\times8.86\times10^{-12}}{1.6\times10^{-19}\times2\times10^{-6}}$$

$$= 4.43\times10^{-4}\text{m}$$

$$= 0.44\text{ mm}$$

10. As, electric field is along positive x-axis and $E = -\frac{dV}{dx}$.

Hence, potential at A must be greater than that at B, *i.e.* $V_A > V_B$.



11. Due to additional charge of $-3Q$, given to external spherical shell, the potential difference between conducting sphere and the outer shell will not change because by presence of charge on outer shell, potential everywhere inside and on the surface of the shell, will change by same amount. Therefore, the potential difference between sphere and shell will remain unchanged.

12. With S_1 and S_3 closed, the capacitors C_1 and C_2 are in series arrangement. In series arrangement, potential difference developed across capacitors are in the inverse ratio of their capacities. Hence,

$$\frac{V_1'}{V_2'} = \frac{C_2}{C_1} = \frac{3 \text{ pF}}{2 \text{ pF}} = \frac{3}{2} \text{ and}$$

$$V_1' + V_2' = V_1 + V_2 = 30 + 20 = 50\text{V}$$

On simplification, we get

$$V_1' = V_1 = 30\text{V}$$

and $$V_2' = V_2 = 20\text{V}$$

13. To calculate force on a point charge q, we need to find electric field due to uniformly charged spherical shell at various points.

If $r < R$, *i.e.* inside the shell, then $E = 0$

$\Rightarrow$ $$F = qE = 0$$

If $r > R$, *i.e.* outside the shell, then $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

$\Rightarrow$ $$F = qE$$

$\Rightarrow$ $$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

If $r = R$, *i.e.* at surface of shell, then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$\Rightarrow$ $$F = \frac{Qq}{4\pi\varepsilon_0 R^2}$$

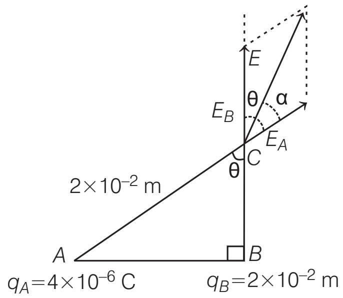
If $E = R$, $E = \frac{kQ}{R^2}$ and $F = \frac{kQq}{R^2}$

14. Figure shows the right angled triangle ABC, such that $AC = 2 \times 10^{-2}$ m and $BC = 10^{-2}$ m. The charges of $q_A = 4 \times 10^{-6}$ C and $q_B = 2 \times 10^{-6}$ C are placed at the vertices A and B respectively. Let E_A and E_B be electric field intensity at point C due to charges q_A and q_B respectively, then

$$E_A = \frac{1}{4\pi\varepsilon_0} \times \frac{q_A}{(AC)^2}$$

$$= 9 \times 10^9 \times \frac{4 \times 10^{-6}}{(2 \times 10^{-2})^2}$$

$$= 9 \times 10^7 \text{ NC}^{-1} \quad \text{(along } AC\text{)}$$



$$E_B = \frac{1}{4\pi\varepsilon_0} \times \frac{q_B}{BC^2} = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{(10^{-2})^2}$$

$$= 18 \times 10^7 \text{ NC}^{-1} \quad \text{(along } BC\text{)}$$

If θ is angle between the directions of E_A and E_B, then

$$E = \sqrt{E_A^2 + E_B^2 + 2E_A E_B \cos\theta}$$

In right angled ΔABC, $\angle ACB = \theta$

$\therefore$ $$\cos\theta = \frac{BC}{AC} = \frac{10^{-2}}{2 \times 10^{-2}} = 0.5 \text{ or } \theta = 60^\circ$$

Hence,

$$E = \sqrt{(9 \times 10^7)^2 + (18 \times 10^7)^2 + 2 \times 9 \times 10^7 \times 18 \times 10^7 \times 0.5}$$

$$= 9 \times 10^7 \sqrt{1 + 4 + 2}$$

$$= 2.38 \times 10^8 \text{ NC}^{-1}$$

Suppose that the resultant electric intensity E makes an angle α with line AC, then

$$\tan\alpha = \frac{E_B \sin\theta}{E_A + E_B \cos\theta}$$

$$= \frac{18 \times 10^7 \sin 60^\circ}{9 \times 10^7 + 18 \times 10^7 \times \cos 60^\circ}$$

$$= \frac{18 \times 10^7 \times 0.866}{9 \times 10^7 + 18 \times 10^7 \times 0.5} = 0.866$$

or $$\alpha = 40.9^\circ$$

15. Capacitance of two capacitors each of area $\frac{A}{2}$, plate separation d but dielectric constants K_1 and K_2, respectively joined in parallel

$$C_1 = \frac{K_1\varepsilon_0\left(\frac{A}{2}\right)}{d/2} + \frac{K_2\varepsilon_0\left(\frac{A}{2}\right)}{d/2} = \frac{(K_1 + K_2)\varepsilon_0 A}{d}$$

It is in series with a capacitor of plate area A, plate separation $d/2$ and dielectric constant K_3, *i.e.* $C_2 = \frac{K_3\varepsilon_0 A}{d/2}$.

If resultant capacitance be taken as $C = \frac{K\varepsilon_0 A}{d}$, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad \frac{d}{K\varepsilon_0 A} = \frac{d}{(K_1 + K_2)\varepsilon_0 A} + \frac{d/2}{K_3 \varepsilon_0 A}$$

$$\Rightarrow \quad \frac{1}{K} = \frac{1}{(K_1 + K_2)} + \frac{1}{2K_3}$$

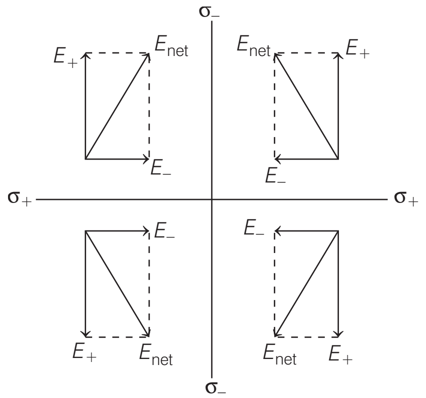
16. Common potential,

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q + 0}{4\pi\varepsilon_0 (r + r')}$$

$\therefore$ Charge on smaller sphere

$$= 4\pi\varepsilon_0 r' \times V = \frac{Qr'}{r + r'}$$

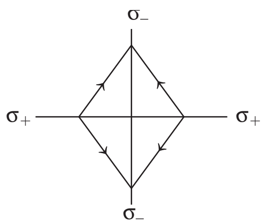
17. Electric field in each quadrant will look like this



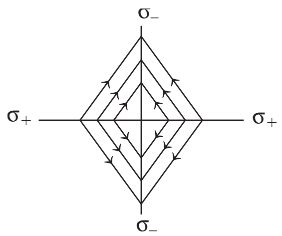
$$\because \quad |\sigma_+| > |\sigma_-|$$

$$\therefore \quad |E_+| > |E_-|$$

So, the final electric field will become as shown below



So, the nearest matching option is



Hence, correct option is (c).

18. Here, $C_{23} = 30 + 30 = 60$ pF.

Total equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{30} + \frac{1}{60} + \frac{1}{120} = \frac{7}{120}$$

$$\Rightarrow \quad C = \frac{120}{7} \text{ pF}$$

$\therefore$ Total charge, $Q = CV = \frac{120}{7} \times 140 \text{ pC} = 2400 \text{ pC}$

$$\therefore \quad V_1 = \frac{Q}{C_1} = \frac{2400 \text{ pC}}{30 \text{ pF}} = 80 \text{ V}$$

$$V_2 = V_3 = V_{23} = \frac{Q}{C_{23}} = \frac{2400 \text{ pC}}{60 \text{ pF}} = 40 \text{ V}$$

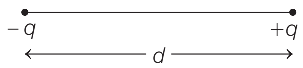
and $$V_4 = \frac{Q}{C_4} = \frac{2400 \text{ pC}}{120 \text{ pF}} = 20 \text{ V}$$

19. The system of two charges, *i.e.* $+q$ and $-q$ that are separated by distance d can be considered as a dipole. Thus, the charge Q would be at D distance from the centre of an electric dipole on its axial line.

So, the total potential energy of the system will be due to two components.

(1) Potential energy of dipole's own system,

$$(\text{PE})_1 = \frac{Kq_1 q_2}{d} = -\frac{Kq^2}{d} \qquad \text{...(i)}$$



(2) Potential energy of charge Q and dipole system,

$$(\text{PE})_2 = -\frac{KQq}{D^2} \cdot d \qquad \text{...(ii)}$$

Hence, total potential energy of the system,

$$(\text{PE})_{\text{total}} = (\text{PE})_1 + (\text{PE})_2 = -\frac{Kq^2}{d} - \frac{KQq}{D^2} \cdot d$$

$$\Rightarrow \quad (\text{PE})_{\text{total}} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{d} + \frac{Qqd}{D^2}\right]$$

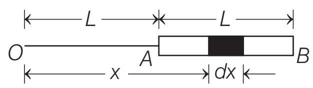
20. Here, $\frac{r_1}{r_2} = \frac{1 \text{ mm}}{2 \text{ mm}} = \frac{1}{2}$. When the spheres are connected by a conducting wire, then $V_1 = V_2$

or $$\frac{q_1}{4\pi\varepsilon_0 r_1} = \frac{q_2}{4\pi\varepsilon_0 r_2}$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{r_1}{r_2} = \frac{1}{2}$$

Now, $$\frac{E_1}{E_2} = \frac{q_1}{q_2} \cdot \left(\frac{r_2}{r_1}\right)^2 = \frac{1}{2} \times \left(\frac{2}{1}\right)^2 = \frac{4}{2} = 2:1$$

21.



Let an element of length dx, charge dq, at distance x from point O.

$$dV = k\frac{dq}{x}$$

where, $$dq = \frac{Q}{L} dx$$

$$\therefore \quad V = \int_L^{2L} \frac{k\,dq}{x} = k\int_L^{2L} \frac{\left(\frac{Q}{L}\right) dx}{x}$$

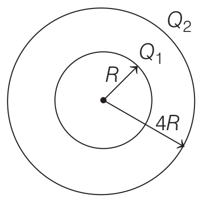
$$= \frac{Q}{4\pi\varepsilon_0 L} \int_L^{2L} \left(\frac{1}{x}\right) dx$$
$$= \frac{Q}{4\pi\varepsilon_0 L} [\log_e x]_L^{2L}$$
$$= \frac{Q}{4\pi\varepsilon_0 L} [\log_e 2L - \log_e L]$$
$$= \frac{Q}{4\pi\varepsilon_0 L} \left[\log_e^2 \frac{2L}{L}\right]$$
$$= \frac{Q}{4\pi\varepsilon_0 L} \ln (2)$$

22. Potential due to a hollow spherical charge distribution is

$$V = \begin{cases} \dfrac{kq}{r}, & r > R \text{ (outside points)} \\ \dfrac{kq}{R}, & r = R \text{ (at surface)} \\ \dfrac{kq}{R}, & r < R \text{ (inside points)} \end{cases}$$

Given arrangement is

Potential on the surface of inner sphere,



$V(R)$ = potential due to charge Q_1 + potential due to charge Q_2

$$= \frac{kQ_1}{R} + \frac{kQ_2}{4R}$$

Potential on the surface of outer sphere,

$V(4R)$ = potential due to charge Q_1 + potential due to charge Q_2

$$= \frac{kQ_1}{4R} + \frac{kQ_2}{4R}$$

The potential difference $V(R) - V(4R)$

$$= \left(\frac{kQ_1}{R} + \frac{kQ_2}{4R}\right) - \left(\frac{kQ_1}{4R} + \frac{kQ_2}{4R}\right)$$
$$= \frac{kQ_1}{R}\left(\frac{3}{4}\right) = \frac{3}{16\pi\varepsilon_0} \cdot \frac{Q_1}{R}$$

23. We have, $U_i = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times V^2$

$\Rightarrow \quad U_i = V^2 \quad$ ...(i)

Final voltage, $V_f = \dfrac{C_1 V_1}{C_1 + C_2} = \dfrac{2V}{10} = \dfrac{V}{5}$

and $\quad U_f = \frac{1}{2}(C_1 + C_2) V_f^2$

$$= \frac{1}{2}(2 + 8)\left(\frac{V}{5}\right)^2$$
$$= 5\left(\frac{V}{5}\right)^2 = \frac{5V^2}{25} = \frac{V^2}{5} \quad \text{...(ii)}$$

Now, $\dfrac{U_i - U_f}{U_i} \times 100 = \dfrac{V^2 - \dfrac{V^2}{5}}{V^2} \times 100$

$$= \frac{4V^2/5}{V^2} \times 100 = 80\%$$

24. The resultant electric field is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

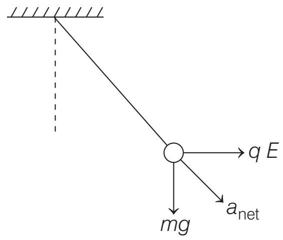
or $\quad E = \dfrac{\sigma_1}{2\varepsilon_0} + \dfrac{\sigma_2}{2\varepsilon_0} = \dfrac{\sigma_1 + \sigma_2}{2\varepsilon_0}$

25. When pendulum is oscillating between capacitor plates, it is subjected to two forces

(i) Weight downwards $(ma_1) = mg$

(ii) Electrostatic force acting horizontally $(ma_2) = qE$

So, net acceleration of pendulum bob is resultant of accelerations produced by these two perpendicular forces.



Net acceleration, $a_{net} = \sqrt{a_1^2 + a_2^2}$

$$= \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

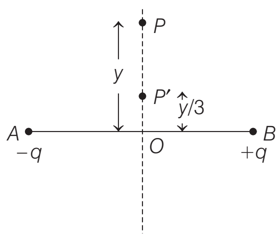
So, time period of oscillations of pendulum,

$$T = 2\pi\sqrt{\frac{l}{a_{net}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

26. Electric field on the equatorial line of a dipole at any point, which is at distance r from the centre is given by

$$E = \frac{2kp}{(r^2 + a^2)^{3/2}} \quad \text{... (i)}$$

where, p is the dipole moment of the charges.



In first case $\quad r = y$

$\Rightarrow \quad E_1 = \dfrac{2kp}{(y^2 + a^2)^{3/2}}$

Here, $\quad y^2 >> a^2$

$\Rightarrow \quad y^2 + a^2 \approx y^2 \quad \text{or} \quad E_1 = \dfrac{2kp}{y^3}$ ... (ii)

So, force on the charge in its position at P will be

$$F = QE_1 = \frac{2kpQ}{y^3} \quad \text{... (iii)}$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kp}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}}$$

Again, $\dfrac{y}{3} >> a$

$$\Rightarrow \quad \left(\frac{y}{3}\right)^2 + a^2 \approx \left(\frac{y}{3}\right)^2$$

$$\Rightarrow \quad E_2 = \frac{2kp}{(y/3)^3} \Rightarrow E_2 = 27 \times \frac{2kp}{y^3}$$

Force on charge in this position,

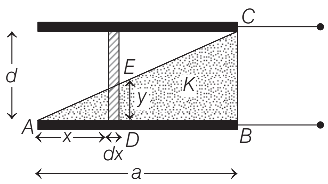
$$F' = QE_2 = 27 \times \frac{2kpQ}{y^3} \quad \text{... (iv)}$$

From Eqs. (iii) and (iv), we get

$$F' = 27\,F$$

27. Let's consider a strip of thickness dx at a distance of x from the left end as shown in figure. From the figure, ΔABC and ΔADE are similar triangles,

$$\Rightarrow \quad \frac{y}{x} = \frac{d}{a} \Rightarrow y = \left(\frac{d}{a}\right)x \quad \text{...(i)}$$



We know that, the capacitance of parallel plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$

$$C_1 = \frac{\varepsilon_0 (adx)}{(d-y)} \quad \text{and} \quad C_2 = \frac{K\varepsilon_0 (adx)}{y}$$

Here, two capacitors are placed in series with variable thickness, therefore

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow \quad C_{eq} = \frac{K\varepsilon_0 adx}{Kd + (1-K)y} \quad \text{...(ii)}$$

Now, integrate it from 0 to a

$$C = \int_0^a \frac{K\varepsilon_0 adx}{Kd + (1-K)y}$$

Using Eq. (i), we get

$$C = \varepsilon_0 a \int_0^a \frac{dx}{d + \left(\frac{1}{K} - 1\right)\frac{d}{a}x}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a}{\left(\frac{1-K}{K}\right)\frac{d}{a}} \ln\left[\frac{1}{K}\right]$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 a^2 K \ln K}{(K-1)\,d}$$

28. Here, effective capacitance $= \left(\dfrac{6}{13}\right)\mu\text{F}$

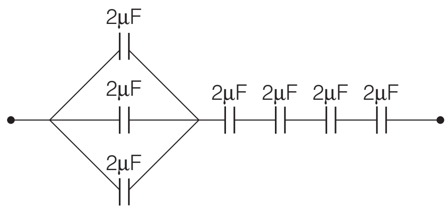
In the given options, some capacitors are joined in parallel and some are in series.

For parallel combination, $C_P = C_1 + C_2 + C_3 + \ldots$

For series combination, $\dfrac{1}{C_S} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} \ldots$

Here, capacity of each capacitor is 2μF.

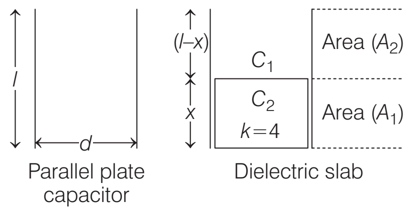
$\therefore$ Three capacitors must be in parallel to get 6 μF. Now, we consider the combinations with three capacitors in parallel (as shown in the figure below)



$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= \frac{1}{6} + 2 = \frac{13}{6} \text{ or } C_{eq} = \frac{6}{13}\mu\text{F}$$

29. Let x be the required length of the slab.



The capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

Capacitance of parallel plate capacitor before inserting the dielectric slab,

$$C_{\text{initial}} = \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C_{\text{initial}} = \frac{\varepsilon_0 lw}{d}$$

Now, capacitance of parallel plate capacitor after inserting the dielectric slab,

$$C_{\text{final}} = C_1 + C_2$$

$$= \frac{k\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$$

$$= \frac{k\varepsilon_0 wx}{d} + \frac{\varepsilon_0 w(l-x)}{d}$$

$$= \frac{\varepsilon_0 w}{d}[kx + (l - x)]$$

$$= \frac{\varepsilon_0 w}{d}[l + (k-1)x]$$

According to question,

Energy stored in capacitor $= 2 \times$ Initial energy stored

$$\Rightarrow \quad \frac{1}{2} C_{\text{final}} V^2 = 2\left(\frac{1}{2} C_{\text{initial}} V^2\right)$$

$$\Rightarrow \quad C_{\text{final}} = 2\, C_{\text{initial}}$$

$$\Rightarrow \quad \frac{\varepsilon_0 w}{d}[l + (4-1)x] = \frac{2\varepsilon_0 w l}{d}$$

$$\Rightarrow \quad \frac{\varepsilon_0 w}{d}[l + 3x] = \frac{2\varepsilon_0 w l}{d}$$

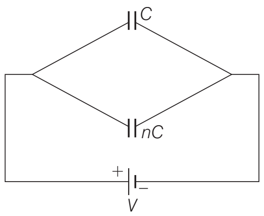
$$\Rightarrow \quad l + 3x = 2l$$

$$\Rightarrow \quad 3x = l$$

$$\Rightarrow \quad x = \frac{l}{3}$$

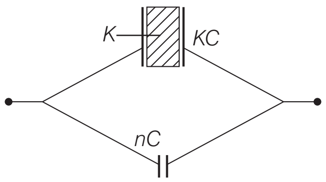
30. When a dielectric slab is inserted to fill the space between the plates, then the charge on the capacitor plates remains same. However, the capacitance increase, *i.e.* $C = KC_0$, where C_0 is the capacitance of the capacitor without slab and K is the dielectric constant of slab.

When parallel combination is fully charged, charge on the combination is



$$Q = C_{\text{eq}} V = C(1 + n)\, V$$

When battery is removed and a dielectric slab is placed between two plates of first capacitor, then charge on the system remains same. Now, equivalent capacitance after insertion of dielectric is



$$C_{\text{eq}} = KC + nC = (n + K)\, C$$

If potential value after insertion of dielectric is V', then charge on system is

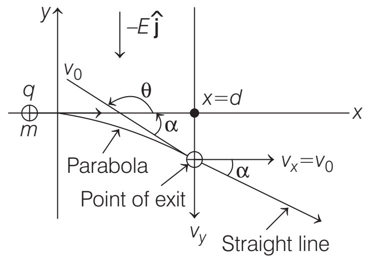
$$Q' = C_{\text{eq}} V' = (n + K)\; CV'$$

As $Q = Q'$, we have

$$C(1 + n)V = (n + K)\, CV'$$

$$\therefore \quad V' = \frac{(1 + n)\, V}{(n + K)}$$

31. The path followed by charged particle is shown in figure below



When particle is in the region of electric field $(0 < x \le d)$, it has two velocity components.

Along X-axis, $\mathbf{v}_x = v_0 \hat{\mathbf{i}}$ ...(i)

Along Y-axis, $\mathbf{v}_y = a_y \cdot t\, \hat{\mathbf{j}}$

where, $t =$ time in which particle crosses region of field.

$$\Rightarrow \quad \mathbf{v}_y = \frac{F}{m} \cdot t\, \hat{\mathbf{j}} \Rightarrow \mathbf{v}_y = \frac{-qEt}{m}\, \hat{\mathbf{j}} \qquad \text{...(ii)}$$

Now, if particle crosses region of field in time t, then

$$d = v_x t \quad \Rightarrow \quad t = \frac{d}{v_0} \qquad \text{...(iii)}$$

So, from Eqs. (ii) and (iii), we get

$$\mathbf{v}_y = \frac{-qEd}{mv_0} \cdot \hat{\mathbf{j}}$$

Hence, angle α is given by $\tan \alpha = \frac{|\mathbf{v}_y|}{|\mathbf{v}_x|} = \frac{qEd}{mv_0^2}$

So, slope of path (straight line) of particle when it comes out of region of field is

$$m' = \tan \theta = \tan (180° - \alpha)$$

$$= -\tan \alpha = \frac{-qEd}{mv_0^2} \qquad \text{...(iv)}$$

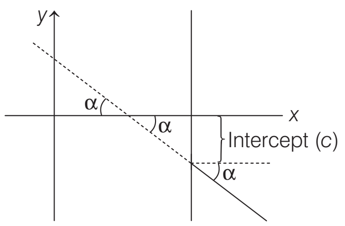
Now, y-coordinate at point of exit is

$$y = \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad y = \frac{-1}{2}\left(\frac{qE}{m}\right) \cdot t^2 = -\frac{1}{2} \frac{qEd^2}{mv_0^2}$$

So, the intercept length c is $-y$ as shown in figure.

$$c = \left(\frac{1}{2} \frac{qEd^2}{mv_0^2}\right) \qquad \text{...(v)}$$

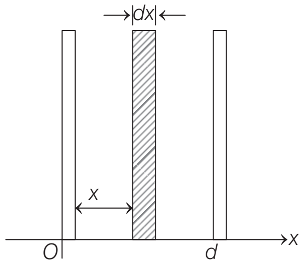


$\therefore$ Equation of path will be

$$y = m'x + c \qquad \text{[from Eqs. (iv) and (v)]}$$

$$\Rightarrow \quad y = \frac{-qEdx}{mv_0^2} + \frac{1}{2} \frac{qEd^2}{mv_0^2} = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)$$

32. Consider a dielectric slab of differentiable thickness dx at a distance x from left hand side plate.



Capacity of this differential thickness capacitor,

$$dC = \frac{\varepsilon_0 K(x) A}{dx} = \frac{\varepsilon_0 K(1 + \alpha x) A}{dx}$$

Total capacity of system is equivalent capacitance of series combination of these differential capacitors.

As in series, $\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$

So, $$\frac{1}{C_{eq}} = \int_0^d \frac{1}{dC} = \int_0^d \frac{dx}{\varepsilon_0 K(1 + \alpha x) A}$$

or $$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 AK} \int_0^d \frac{dx}{1 + \alpha x}$$

$$= \frac{1}{\varepsilon_0 A \alpha K} [\ln(1 + \alpha x)]_0^d$$

$$= \frac{1}{\varepsilon_0 A \alpha K} [\ln(1 + d\alpha) - \ln(1)]$$

$$= \frac{1}{\varepsilon_0 A \alpha K} \ln(1 + d\alpha)$$

Now, we use expansion,

$$\ln(1 + y) = y - \frac{y^2}{2} + \frac{y^3}{3} \text{ ...., we get}$$

$$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 A \alpha K} \left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} \text{ ....} \right)$$

$$= \frac{\alpha d}{\varepsilon_0 A \alpha K} \left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} \ldots \right)$$

or $$C_{eq} = \frac{\varepsilon_0 AK}{d} \left(1 - \frac{\alpha d}{2} \right)^{-1}$$

From binomial approximation $(1 + x)^n = 1 + nx$, we have

$$C_{eq} = \frac{\varepsilon_0 AK}{d} \left(1 + \frac{\alpha d}{2} \right)$$

33. As, $q_1 + q_2 = Q$

Here, $$\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$\therefore$ $$q_1 = \frac{Qr^2}{R^2 + r^2}$$

and $$q_2 = \frac{QR^2}{R^2 + r^2}$$

$\therefore$ Potential at component centre

$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{Qr^2}{(R^2 + r^2) r} + \frac{QR^2}{(R^2 + r^2) R} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q(R + r)}{(R^2 + r^2)}$$

34. The force on l length of the wire 2,

$$f_2 = QE_1 = (\lambda_2 l) \frac{2K\lambda_1}{R}$$

$\Rightarrow$ $$\frac{f_2}{l} = \frac{2K\lambda_1\lambda_2}{R}$$

Also, $$\frac{f_1}{l} = \frac{f_2}{l} = \frac{f_1}{l} = \frac{2K\lambda_1\lambda_2}{R}$$

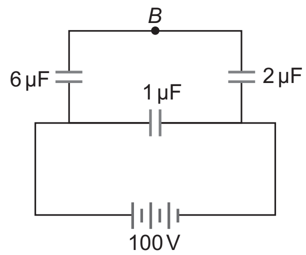
35. We have,

$$C_{eq} = \frac{(3+3)\times(1+1)}{(3+3)\times(1+1)} + 1 = \frac{6\times 2}{6+2} + 1 = \frac{12}{8} + 1$$

$$= \frac{12+8}{8} = \frac{20}{8} = \frac{5}{2}\,\mu\text{F}$$

$\because$ $Q = CV$

$\therefore$ $Q = \frac{5}{2} \times 100 = 250\,\mu\text{C}$



Charge in 6 μF branch

$$= CV = \left(\frac{6\times 2}{6+2} \right) \times 100 = 150\,\mu\text{C}$$

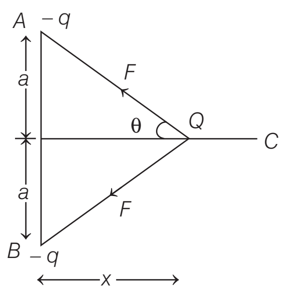
$$V_{AB} = \frac{150}{6} = 25\text{ V}$$

and $V_{BC} = 100 - V_{AB} = 75\text{ V}$ ...(i)

36. By symmetry of problem, the components of force on Q due to charges at A and B along y will cancel each other while along X-axis it will be added up. If at any time, charge Q at a distance x from O, then net force on charge Q,

$$F_{net} = 2F\cos\theta$$

Here, $$F = \frac{-qQ}{4\pi\varepsilon_0 (a^2 + x^2)}$$



and $\cos\theta = \frac{x}{(a^2 + x^2)^{1/2}}$

$\Rightarrow$ $F_{\text{net}} = -\frac{1}{4\pi\varepsilon_0} \frac{2qQx}{(a^2 + x^2)^{3/2}}$

As the resulting force F_{net} is not linear. So, the motion will be oscillating, but not simple harmonic.

37. The energy of the system when the both capacitors are same.

$$U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 \quad \ldots\text{(i)}$$

In 2nd case, when K is opened and dielectric medium is filled between the plates, capacitance of both the capacitors becomes $3C$, while potential difference across A is V and potential difference across B is $V/3$.

Then, $U_2 = \frac{1}{2}(3C)V^2 + \frac{1}{3}(3C)\frac{V^2}{3}$

$= \frac{10}{6}CV^2 = \frac{5}{3}CV^2 \quad \ldots\text{(ii)}$

Dividing Eq. (i) by Eq. (ii), we get

$\therefore$ $\frac{U_1}{U_2} = 3:5$

38. The total charge $= (2C) \times (2V) + (C)(-V)$

$= 4CV - CV = 3CV$

Common potential $= \frac{3CV}{C} = 3V$

$\therefore$ Energy $= \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$

39. As the electrostatic forces are conservative, so work done is independent of path

$\therefore$ $W = \mathbf{F} \cdot d\mathbf{s} = qE\hat{\mathbf{i}} \cdot [(0-a)\hat{\mathbf{i}} + (0-b)\hat{\mathbf{j}}]$

$= -qEa$

40. Here, $E = \frac{\sigma}{\varepsilon_0}$ and $t = \frac{l}{u}$

Along Y-axis, $u = 0, a = \frac{F}{m} = \frac{eE}{m}$

$\therefore$ $s = d = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}t^2 = \frac{1}{2}\frac{e\sigma}{m\varepsilon_0}\frac{l^2}{u^2}$

or $\sigma = \frac{2d\varepsilon_0 mu^2}{el^2}$

41. Radius of bigger drop,

$R = 3r \qquad \left[\because \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3\right]$

$\therefore$ $V = \frac{27q}{4\pi\varepsilon_0 R} = \frac{27q}{4\pi\varepsilon_0 (3r)}$

$= 9\left(\frac{q}{4\pi\varepsilon_0 R}\right)$

$= 9 \times 10 = 90$ V

42. The capacitance with dielectric,

$$C = \frac{\varepsilon_0 A}{\frac{d}{2K} + \frac{d}{2}} = \frac{2\varepsilon_0 A}{\frac{d}{K} + d}$$

Here, $A = 2\text{m}^2$, $d = 1$ m and $K = 3.2$

$$C = \frac{2 \times \varepsilon_0 \times 2}{\frac{1}{3.2} + 1} = 3.04\varepsilon_0 \simeq 3\varepsilon_0$$

43. For equivalent capacitance,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$$

$$C_s = \frac{20}{3}\,\mu\text{F}$$

$\therefore$ Charge on each capacitor

$= C_s V = \frac{20}{3} \times 200$

$= \frac{4000}{3}\,\mu\text{C}$

Common potential $= \frac{\text{Total charge}}{\text{Total capacity}}$

$= \frac{2 \times 4000/3}{10 + 20} = \frac{800}{9}$ V

$= 88.89$ V

44. Initial energy of charged capacitor,

$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (220)^2$

$= 121 \times 10^{-3}$ J

Common potential after redistribution of charges,

$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$\simeq \frac{C_1V_1}{C_1 + C_2}$

$\simeq \frac{5 \times 10^{-6} \times 220}{5 \times 10^{-6} + 2.5 \times 10^{-6}}$

$= 220 \times \frac{2}{3}$ V

Final stored energy,

$U_2 = \frac{1}{2}(C_1 + C_2)V^2$

$= \frac{1}{2} \times 7.5 \times 10^{-6} \times \left(220 \times \frac{2}{3}\right)^2$

$\approx 80 \times 10^{-3}$ J

Loss of energy

$\Delta U = (121 - 80) \times 10^{-3}$ J

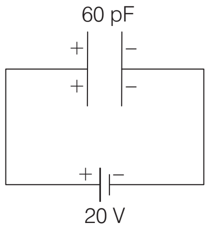
$= 41 \times 10^{-3}$ J

$= \frac{4.1}{100}$ J $\approx \frac{4}{100}$ J

Given, $\Delta U = \frac{X}{100}$ J

$\therefore$ $X = 4$

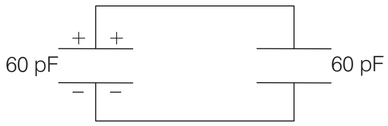
45. Initially a 60 pF capacitor is fully charged by a 20 V supply as shown in the figure.



Its energy, $U_1 = \frac{1}{2}CV^2$

$$\Rightarrow \quad U_1 = \frac{1}{2} \times 60 \times 10^{-12} \times (20)^2$$
$$= 12 \times 10^{-9} \text{ J}$$

Then, it is connected to another uncharged 60 pF capacitor as shown in the figure.



Transfer of charge from charged to uncharged capacitor occurs till both reaches a common potential.

If V = common potential, then

$$V = \frac{\text{total charge}}{\text{total capacitance}}$$

$$\Rightarrow \quad V = \frac{Q_1}{C_1 + C_2} = \frac{C_1 V_1}{C_1 + C_2}$$
$$= \frac{60 \times 10^{-12} \times 20}{60 \times 10^{-12} + 60 \times 10^{-12}} = 10 \text{ V}$$

Final energy of system,

$$U_2 = \frac{1}{2}(C_1 + C_2) \cdot V^2$$
$$= \frac{1}{2} \times 2 \times 60 \times 10^{-12} \times (10^2)$$
$$= 60 \times 10^{-10}$$
$$= 6 \times 10^{-9} \text{ J}$$

Loss of energy,

$$\Delta U = U_1 - U_2$$
$$= 12 \times 10^{-9} - 6 \times 10^{-9}$$
$$= 6 \times 10^{-9} \text{ J}$$
$$= 6 \text{ nJ}$$

46. Initial energy of capacitor,

$$U_1 = \frac{1}{2}CV^2$$
$$= \frac{1}{2} \times 14 \times 12 \times 12$$
$$= 1008 \text{ pJ}$$

Final energy, $U_f = \frac{U_i}{K} = \frac{1008}{7} = 144$ pJ $\quad (\because C_m = KC_0)$

Mechanical energy,

$$\Delta U = U_i - U_f = 1008 - 144 = 864 \text{ pJ}$$

47. Electric field inside dielectric, $\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

$$\Rightarrow \quad \sigma = 2.2 \times 8.58 \times 10^{-12} \times 3 \times 10^4$$
$$= 6.6 \times 8.85 \times 10^{-8}$$
$$= 5.841 \times 10^{-7}$$
$$= 6 \times 10^{-7} \text{ C/m}^2$$