3 ALGEBRA

EXERCISE

211. If $\log_2 x + \log_2 y \geq 6$, then the least value of $x + y$ is
(A) 4 (B) 8 (C) 16 (D) 32

212. Suppose $f(x, n) = \sum_{k=1}^{n} \log_x \left(\frac{k}{x}\right)$, then the value of x satisfying the equation f(x, 10) = f(x, 11) is
(A) 9 (B) 10 (C) 11 (D) none

213. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is
(A) 8 (B) 16 (C) 24 (D) 32

214. If the roots of the equation $ax^2 + bx + c = 0$, are of the form $\alpha/(\alpha - 1)$ and $(\alpha + 1)/\alpha$, then the value of $(a + b + c)^2$ is
(A) $2b^2 - ac$ (B) $b^2 - 2ac$ (C) $b^2 - 4ac$ (D) $4b^2 - 2ac$

215. The maximum value of the sum of the AP 50, 48, 46, 44, ... is
(A) 648 (B) 450 (C) 558 (D) 650

216. If $x^a = y^b = z^c$, where a, b, c are unequal positive numbers and x,y,z are in GP, then $a^3 + c^3$ is
(A) $\geq 2b^3$ (B) $> 2b^3$ (C) $\leq 2b^3$ (D) $< 2b^3$

217. If $0 \leq [x] < 2$; $-1 \leq [y] < 1$ and $1 \leq [z] < 3$. where [*] denotes the greatest integer function, then the maximum value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is
(A) 2 (B) 4 (C) 6 (D) 8

218. The equations $\lambda x - y = 2$, $2x - 3y = -\lambda$, $3x - 2y = -1$ are consistent for
(A) $\lambda = -4$ (B) $\lambda = -1, 4$ (C) $\lambda = -1$ (D) $\lambda = 1, -4$

219. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $p^T (Q^{2005})p$ is equal to

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$

220. If $A^5 = O$ such that $A^n \neq I$ for $1 \leq n \leq 4$, then $(I - A)^{-1}$ is equal to

(A) A^4 (B) A^3 (C) $I + A$ (D) none of these

221. The solution of $x - 1 = (x - [x])(x - \{x\})$ (wher $[x]$ and $\{x\}$ are the integral and fractional part of x) is

(A) $x \in R$ (B) $x \in R \sim [1, 2)$ (C) $x \in [1, 2)$ (D) $x \in R \sim [1, 2]$

222. The value of p for which both the roots of the equation $4x^2 - 20px + (25p^2 + 15p - 66) = 0$, are less than 2, lies in

(A) $(4/5, 2)$ (B) $(2, \infty)$ (C) $(-1, -4/5)$ (D) $(-\infty, -1)$

223. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $m = (a + b)(c + d)$ satisfies the relation

(A) $0 < m \leq 1$ (B) $1 \leq m \leq 2$ (C) $2 \leq m \leq 3$ (D) $3 < m \leq 4$

224. Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP. If $a > b > c$ and $a + b + c = 3/2$, then the value of a is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2} + \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} + \frac{1}{\sqrt{2}}$

225. If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & 1 & 2 \end{vmatrix}$, then the value of $\int_{-\pi/2}^{\pi/2} f(x)\, dx$ is equal to

(A) 5 (B) 3 (C) 1 (D) 0

226. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, then

(A) $abc > 1$ (B) $abc > -8$ (C) $abc < -8$ (D) $abc > -2$

227. If $2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ and $2y + x = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$, then

(A) $x + y = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & -2 \end{bmatrix}$ (B) $x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ (C) $x - y = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ (D) $y = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$

228. If $A = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{pmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$, then f(A) equals

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (C) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $(2 + i\sqrt{3})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

229. If the matrices A, B, A + B are non singular, then $[A(A + B)^{-1}B]^{-1}$, is equal to

(A) $A^{-1} + B^{-1}$ (B) $A + B$ (C) $A(A + B)^{-1}$ (D) None of these

230. The number of solution of $|[x] - 2x| = 4$, where $[*]$ denotes the greatest integer $\leq x$, is
(A) infinite (B) 4 (C) 3 (D) 2

231. If $\sum_{i=1}^{21} a_i = 693$, where $a_1, a_2, \ldots, a_{21}$ are in AP, then the value of $\sum_{i=0}^{10} a_{2r+1}$ is
(A) 361 (B) 363 (C) 365 (D) 398

232. If $\log_2 (a + b) + \log_2 (c + d) \geq 4$. Then the minimum value of the expression $a + b + c + d$ is
(A) 2 (B) 4 (C) 8 (D) none of these

233. If $xyz = -2007$ and $\Delta = \begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} = 0$, then value of $ayz + bzx + cxy$ is
(A) -2007 (B) 2007 (C) 0 (D) $(2007)^2$

234. If $\Delta_r = \begin{vmatrix} 1 & r & 2^r \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix}$, then value of $\sum_{r=1}^{n} \Delta_r$ is
(A) n (B) 2n (C) n^2 (D) $-2n$

235. If A is an orthogonal matrix, then A^{-1}, equals
(A) A (B) A′ (C) A^2 (D) none of these

236. If A is a square matrix, then adj $A^T - (\text{adj } A)^T$ is equal to
(A) $2|A|$ (B) $2|A|\ I$ (C) null matrix (D) unit matrix

237. The sum of all values of x, so that $16^{(x^2+3x-1)} = 8^{(x^2+3x+2)}$, is
(A) 0 (B) 3 (C) -3 (D) -5

238. If α, β, γ are the roots of $ax^3 + bx + c = 1$ such that $\alpha + \beta = 0$, then
(A) c = 0 (B) c = 1 (C) b = 0 (D) b = 1

239. If p,q,r are three positive real numbers are in AP, then the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for
(A) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$ (C) all p and r (D) no p and r

240. If the arithmetic progression whose common difference is none zero, the sum of first 3n terms is equal to the sum of the next n terms. Then the ratio of the sum of the first 2n terms to the next 2n terms is
(A) 1/5 (B) 2/3 (C) 3/4 (D) none of these

241. If A is square matrix of order n, a = maximum number of distinct entries. If A is triangular matrix, b = maximum number of distinct entries. If A is a diagonal matrix. c = minimum number of zeros. If A is a triangular matrix if a + 5 = c + 2b
(A) 12 (B) 4 (C) 8 (D) None of these

242. Matrix M_r is defiend as $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r \in N$ value of det (M_1) + det (M_2) + det (M_3) +....+ det (M_{2007}) is

(A) 2007 (B) 2008 (C) 2008^2 (D) 2007^2

243. The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the matrix reflection in the line

(A) $x = 1$ (B) $x + y = 1$ (C) $y = 1$ (D) $x = y$

244. If the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$ is singular, then λ is equal to

(A) 3 (B) 4 (C) 2 (D) 5

245. If the roots of the quadratic equation $ax^2 - 5x + 6 = 0$ are in the ratio 2 : 3, then 'a' is equal to

(A) 3 (B) 1 (C) 2 (D) −1

246. If the sides of a right angled triangle form an AP, then the sines of the acute angles are

(A) $\frac{3}{5}, \frac{4}{5}$ (B) $\sqrt{3}, \frac{1}{3}$ (C) $\sqrt{\left(\frac{\sqrt{5}-1}{2}\right)}, \sqrt{\left(\frac{\sqrt{5}+1}{2}\right)}$ (D) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

247. If $x \in \{1,2,3 ....9\}$ and $f_n(x) = xxx.x$ (n digits), then $f_n^2(3) + f_n(2)$ is equal to

(A) $2f_{2n}(1)$ (B) $f_n^2(1)$ (C) $f_{2n}(1)$ (D) $-f_{2n}(4)$

248. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(A) 128B (B) −128B (C) 4B (D) −64B

249. If $x_1, x_2, ..., x_{20}$ are in H.P. and $x_1, 2, x_{20}$ are in G.P., then $\sum_{r=1}^{19} x_r x_{r+1} =$

(A) 76 (B) 80 (C) 84 (D) none of these

250. If m and x are two real numbers, then $e^{2mi\cot^{-1}x}\left(\frac{xi+1}{xi-1}\right)^m$ (where $i = \sqrt{-1}$) is equal to

(A) $\cos x + i \sin x$ (B) m/2 (C) 1 (D) (m + 1)/2

251. If $|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$, (where $i = \sqrt{-1}$), then z lies on

(A) Re (z) = 2 (B) Im (z) = 2 (C) Re (z) + Im (z) = 2 (D) none of these

252. If z be complex number such that equation $|z - a^2| + |z - 2a| = 3$ always represents an ellipse, then range of a ($\in R^+$) is

(A) $(1, \sqrt{2})$ (B) $[1, \sqrt{3}]$ (C) (−1, 3) (D) (0, 3)

253. The total number of integral solution for x, y, z such that xyz = 24, is

(A) 3 (B) 60 (C) 90 (D) 120

254. If a, b, c are odd positive integers, then number of integral solutions of a + b + c = 13, is
(A) 14 (B) 21 (C) 28 (D) 56

255. Number of point having position vector
$a\hat{i}+b\hat{j}+c\hat{k}$ where a, b, c $\in$ {1, 2, 3, 4, 5} such that $2^a + 3^b + 5^c$ is divisible by 4 is-
(A) 140 (B) 70 (C) 100 (D) None of these

256. Number of terms in the expansion of $\left(\frac{x^3+1+x^6}{x^3}\right)^{\Sigma n}$ (where n $\in$ N) is
(A) $\Sigma n + 1$ (B) ${}^{\Sigma n+2}C_2$ (C) 2n + 1 (D) $n^2 + n + 1$

257. If 5^{40} is divided by 11, then remainder is α and when 2^{2003} is divided by 17, then remainder is β, then the value of $\beta - \alpha$ is
(A) 3 (B) 5 (C) 7 (D) 8

258. The term independent of x in the expansion of $\left(\sqrt{\left(\frac{x}{3}\right)}+\sqrt{\left(\frac{3}{2x^2}\right)}\right)^{10}$ is
(A) 5/12 (B) 1 (C) ${}^{10}C_1$ (D) none of these

259. The probability that the length of a randomly chosen chord of a circle lies between $\frac{2}{3}$ and $\frac{5}{6}$ of its diameter is
(A) 1/4 (B) 5/12 (C) 1/16 (D) 5/16

260. A die is rolled three times, the probability of getting a large number than the previous number is
(A) $\frac{1}{54}$ (B) $\frac{5}{54}$ (C) $\frac{5}{108}$ (D) $\frac{13}{108}$

261. A dice is thrown (2n + 1) times. The probability that faces with even numbers appear odd number is times is
(A) $\frac{2n+1}{2n+3}$ (B) $\frac{n+1}{2n+1}$ (C) $\frac{n}{2n+1}$ (D) none of these

262. If $x_r = \cos(\pi/3^r) - i\sin(\pi/3^r)$, (where $i = \sqrt{-1}$). then value of $x_1 \cdot x_2 \cdot \ldots \infty$, is
(A) 1 (B) −1 (C) −i (D) i

263. Let $z_1 = 6 + i$ and $z_2 = 4 - 3i$ (where $i = \sqrt{-1}$). Let z be a complex number such that
$\arg\left(\frac{z-z_1}{z_2-z}\right) = \frac{\pi}{2}$, then z satisfies
(A) $|z - (5 - i)| = 5$ (B) $|z - (5 - i)| = \sqrt{5}$ (C) $|z - (5 + i)| = 5$ (D) $|z - (5 + i)| = \sqrt{5}$

264. If $z \neq 0$, then $\int_{x=0}^{100} [\arg|z|]\, dx$ is (where [*] denotes the greatest integer function)
(A) 0 (B) 10 (C) 100 (D) not defined

265. The maximum number of points of intersection of 8 circles, is
(A) 16 (B) 24 (C) 28 (D) 56

266. Every one of the 10 available lamps can be switched on to illuminate certain Hall. The total number of ways in which the hall can be illuminated, is
(A) 55 (B) 1023 (C) 2^{10} (D) 10 !

267. The total number of 3 digit even numbers that can be composed from the digitis 1, 2, 3, ..., 9 when the repetition of digits is not allowed, is
(A) 224 (B) 280 (C) 324 (D) 405

268. The number of ways in which a score of 11 can be made from a through by three persons, each throwing a single die once, is
(A) 45 (B) 18 (C) 27 (D) 68

269. The greatest coefficient in the expansion of $(1 + x)^{2n+2}$ is
(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{(2n+2)!}{\{(n+1)!\}^2}$ (C) $\frac{(2n+2)!}{n!(n+1)!}$ (D) $\frac{(2n)!}{n!(n+1)!}$

270. The first integral term in the expansion of $(\sqrt{3} + \sqrt[3]{2})^9$, is its
(A) 2nd term (B) 3rd term (C) 4th term (D) 5th term

271. The number of irrational terms in the expansion of $(2^{1/5} + 3^{1/10})^{55}$ is
(A) 47 (B) 56 (C) 50 (D) 48

272. 10 bulbs out of a sample of 100 bulbs manufactured by a company are defective. The probability that 3 out of 4 bulbs, bought by a customer will not be defective, is
(A) ${}^{4}C_3/{}^{100}C_4$ (B) ${}^{90}C_3/{}^{96}C_4$ (C) ${}^{90}C_3/{}^{100}C_4$ (D) $({}^{90}C_3 \times {}^{10}C_1)/{}^{100}C_4$

273. Two persons each makes a single throw with a pair of dice. The probability that the throws are unequal is given by
(A) $\frac{1}{6^3}$ (B) $\frac{73}{6^3}$ (C) $\frac{51}{6^3}$ (D) none of these

274. Let A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8}. An element (a, b) of their cartesian product $A \times B$ is chosen at random. The probability that $a + b = 9$, is
(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

275. The point of intersection of the curves $\arg(z - 3i) = 3\pi/4$ and $\arg(2z + 1 - 2i) = \pi/4$ (where $i = \sqrt{-1}$) is
(A) 1/4 (3 + 9i) (B) 1/4 (3 – 9i) (C) 1/2 (3 + 2i) (D) no point

276. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is
(A) 0 (B) 2 (C) 7 (D) 17

277. If $|z - i| \le 2$ and $z_1 = 5 + 3i$, (where $i = \sqrt{-1}$) then the maximum value of $|iz + z_1|$ is
(A) $2 + \sqrt{31}$ (B) 7 (C) $\sqrt{31} - 2$ (D) $\sqrt{31} + 2$

278. If $|z - 1| + |z + 3| \le 8$, then the range of values of $|z - 4|$, (where $i = \sqrt{-1}$) is
(A) (0, 7) (B) (1, 8) (C) [1, 9] (D) [2, 5]

279. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no lines passes through both points A and B, and no two are parallel, then the number of intersection points the lines have is equal to
(A) 535 (B) 601 (C) 728 (D) 963

280. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7, so that digits do not repeat and the terminal digits are even is
(A) 144 (B) 72 (C) 288 (D) 720

281. In a polygon, no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70, then number of diagonals of polygon is :
(A) 8 (B) 20 (C) 28 (D) None

282. The coefficient of $a^{10} b^7 c^3$ in the expansion of $(bc + ca + ab)^{10}$ is
(A) 30 (B) 60 (C) 120 (D) 240

283. If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 \ldots\ldots + a_{2n}x^{2n}$. Then value of $a_0 + a_3 + a_6 \ldots\ldots.$ is equal to
(A) 3^n (B) 3^{n+1} (C) 3^{n-1} (D) None of these

284. The value of $\sum_{r=1}^{10} r.\frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to
(A) 5(2n – 9) (B) 10n (C) 9(n – 4) (D) None

285. If two events A and B are such that $P(A) > 0$ and $P(B) \ne 1$, then $p(\overline{A} / \overline{B})$ is equal to
(A) $1 - P(A/B)$ (B) $1 - P(\overline{A}/B)$ (C) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (D) $\frac{P(A)}{P(\overline{B})}$

286. Two numbers x and y are chosen at random from the set {1, 2, 3,....,30}. The probability that $x^2 - y^2$ is divisible by 3 is
(A) 3/29 (B) 4/29 (C) 5/29 (D) none of these

287. Consider $f(x) = x^3 + ax^2 + bx + c$. parameters a, b, c, are chosen, respectively, by throwing a die three times. Then the probability that f(x) is an increasing function is
(A) 5/36 (B) 8/36 (C) 4/9 (D) 1/3

288. Number of solutions of the equation $|z|^2 + 7\overline{z} = 0$ is/are
(A) 1 (B) 2 (C) 4 (D) 6

289. $(1 + i)^6 + (1 - i)^6 =$
(A) $15i$ (B) $-15i$ (C) 15 (D) 0

290. If 1, ω, ω^2,.... ω^{n-1} are n, nth roots of unity, then the value of $(9 - \omega)(9 - \omega^2).....(9 - \omega^{n-1})$ will be
(A) n (B) 0 (C) $\frac{9^n - 1}{8}$ (D) $\frac{9^n + 1}{8}$

291. The number of non-negative integral solutions to the system of equations $x + y + z + u + t = 20$ and $x + y + z = 5$ is-
(A) 336 (B) 346 (C) 246 (D) None of these

292. If 'n' objects are arranged in a row, then number of ways of selecting three of these objects, so that no two of them are next to each other is :
(A) ${}^{n-3}C_3$ (B) ${}^{n-3}C_2$ (C) ${}^{n-2}C_3$ (D) None

293. The sum of 20 terms of a series of which every even term is 2 times the term before it, and every odd term is 3 times the term before it, the first term being unity is

(A) $\left(\frac{2}{7}\right)(6^{10} - 1)$ (B) $\left(\frac{3}{7}\right)(6^{10} - 1)$ (C) $\left(\frac{3}{5}\right)(6^{10} - 1)$ (D) none of these

294. A fair coin is tossed 100 times. The probability of getting tails 1, 3, ... 49 times is
(A) 1/2 (B) 1/4 (C) 1/8 (D) 1/16

295. A six-faced fair dice is thrown until 1 comes. Then the probability that 1 comes in even number of trials is
(A) 5/11 (B) 5/6 (C) 6/11 (D) 1/6

296. Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is
(A) reflexive and transitive (B) reflexive and symmetric
(C) reflexive and antisymmetric (D) None of the above

297. If the heights of 5 persons are 144 cm, 153 cm, 150 cm, 158 cm and 155 cm respectively, then mean height is-
(A) 150 cm (B) 151 cm (C) 152 cm (D) None of these

298. Arithmetic mean of the following frequency distribution :

x :	4	7	10	13	16	19
f :	7	10	15	20	25	30

is -
(A) 13.6 (B) 13.8 (C) 14.0 (D) None of these

299. If p and q are two statements. Then negation of compound statement $(\sim p \vee q)$ is
(A) $\sim p \wedge q$ (B) $p \vee q$ (C) $p \wedge \sim q$ (D) None

300. Negation of statement : if we control population growth we prosper, is
(A) if we do not control population growth, we prosper
(B) if we control population, we do not prosper
(C) we control population and we do not prosper
(D) we do not control population, but we prosper

301. If a set A = {a, b, c} then the number of subsets of the set A is
(A) 3 (B) 6 (C) 8 (D) 9

302. The weighted mean of first n natural number if their weight are the same as the number is-
(A) $\frac{n(n+1)}{2}$ (B) $\frac{n+1}{2}$ (C) $\frac{2n+1}{3}$ (D) None of these

303. The mean income of a group of persons is Rs.400. Another group of persons has mean income Rs.480. If the mean income of all the persons in the two groups together is Rs.430, then ratio of the number of persons in the groups:

(A) $\frac{4}{3}$ (B) $\frac{5}{4}$ (C) $\frac{5}{3}$ (D) None of these

304. If $p \Rightarrow q$ can also be written as

(A) $p \Rightarrow \sim q$ (B) $\sim p \vee q$ (C) $\sim p \Rightarrow \sim q$ (D) none

305. If $p \Rightarrow (\sim p \vee q)$ is false, the truth values of p and q are respectively.

(A) T, F (B) T, T (C) F, T (D) F, F

306. The value of n $\{P[P(\phi)]\}$ is equal to

(A) 0 (B) 2 (C) 3 (D) 4

307. The mean of a set of number is $\bar{x}$ if each number is increased by λ, then mean of the new set is-

(A) $\bar{x}$ (B) $\bar{x} + \lambda$ (C) $\lambda\ \bar{x}$ (D) None of these

308. Mean of 25 observations was found to be 78.4. But later on it was found that 96 was misread 69. The correct mean is

(A) 79.24 (B) 79.48 (C) 80.10 (D) None of these

309. If p, q, r are simple statement. Then $(p \wedge q) \wedge (q \wedge r)$ is true. Then

(A) p, q, r are all false (B) p, q are true and r is false

(C) p, q, r are all true (D) p is true and q and r are false

310. If p, q, r are simple statement with truth values T, F, T then truth values of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is

(A) True (B) False (C) True if r is false (D) None

311. If A = $\{x : x = 2n + 1, n \in Z$ and B = $\{x : x = 2n, n \in Z\}$, then $A \cup B$ is

(A) set of natural numbers (B) set of irrational numbers

(C) set of integers (D) none of these

312. If $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$ then mean of $x_1 + a, x_2 + a, \ldots, x_n + a$ where a is any number positive or negative is-

(A) $\bar{x} + a$ (B) $\bar{x}$ (C) $a\bar{x}$ (D) None of these

313. Mean wage from the following data

Wage (In Rs.)	800	820	860	900	920	980	1000	
No. of workers	7	14	19	25	20	10	5	is

(A) Rs.889 (B) Rs. 890.4 (C) Rs.891.2 (D) None of these

314. Negation of '' 3 is an odd number and 7 is a rational number is -

(A) 3 is not an odd number and 7 is not a rational number

(B) 3 is an odd number or 7 is a rational number

(C) 3 is an odd number or 7 is not a rational number

(D) 3 is not an odd number or 7 is not a rational number.

315. The negation of statement $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$ is -

(A) $(p \vee \sim q) \wedge (p \vee q)$\`
(B) $(p \wedge \sim q) \vee (p \vee q)$
(C) $(\sim p \vee q) \vee (\sim p \wedge \sim q)$
(D) $(p \wedge \sim q) \wedge (p \vee q)$

316. If A = {x : x = 3n, n $\in$ Z} and B = {x : x = 4n, n $\in$ Z} then A $\cap$ B is

(A) {x : x = n, n $\in$ Z}
(B) {x : x = n/2, n $\in$ Z}
(C) {x : x = n–1, n $\in$ Z}(D) {x : x = 12n, n $\in$ Z}

317. The geometric mean of numbers 7, 7^2, 7^3,. ...,7^n is-

$7^{\frac{n+1}{}}$
(D) None of these

318. Harmonic mean of 2, 4, 5 is.....

(A) 4.21
(B) 3.16
(C) 2.98
(D) None of these

319. The negation of statement $(p \wedge q) \vee (q \vee \sim r)$

(A) $(p \wedge q) \vee (\sim q \vee \sim r)$
(B) $(\sim p \wedge \sim q) \wedge (\sim q \wedge r)$
(C) $(\sim p \vee \sim q) \wedge (\sim q \wedge r)$
(D) None of these

320. The statement $(p \wedge \sim q) \vee p$ is logically equivalent to -

(A) p
(B) ~p
(C) q
(D) ~q

321. If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in A $\cup$ B?

(A) 3
(B) 6
(C) 9
(D) 10

322. The number of runs scored by 11 players of a cricket team of school are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27. The median is-

(A) 21
(B) 27
(C) 30
(D) None of these

323. The median for the following frequency distribution :

x :	1	2	3	4	5	6	7	8	9
f :	8	10	11	16	20	25	15	9	6 is :

(A) 4
(B) 5
(C) 6
(D) None of these

324. If $(p \wedge \sim q) \vee (q \wedge r)$ is true and. q and r both true then p is -

(A) True
(B) False
(C) may be true or false
(D) none

325. Negation of the statement If a number is prime then it is odd' is.

(A) A number is not prime but odd.
(B) A number is prime and it is not odd.
(C) A number is neither primes nor odd.
(D) None of these

326. If the number of elements in A is m and number of element in B is n then the number of elements in the power set of A × B is

(A) 2^n
(B) 2^m
(C) 2^{mn}
(D) none of these

327. Median from the following distribution

Class	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
frequency	5	6	15	10	5	4	2	2 is

(A) 19.0
(B) 19.2
(C) 19.3
\`(D) 19.5

328. Mode of the data 3, 2, 5, 2, 3, 5, 6, 6, 5, 3, 5, 2, 5 is-
(A) 6 (B) 4 (C) 5 (D) 3

329. If p, q, r are substatements with truth values. T, T, F then the Statement
$r \to (p \wedge \sim q) \vee (\sim q \wedge \sim r)$ will be
(A) True (B) False
(C) may be true or false (D) None of these

330. The Negation of the statement $(p \wedge q) \to r$ is -
(A) $(\sim p \vee \sim q) \to r$ (B) $(\sim p \wedge \sim q) \wedge \sim r$ (C) $(p \wedge q) \wedge \sim r$ (D) $(\sim p \vee \sim q) \wedge r$

331. Let A and B be two non-empty sets having elements in common, then A × B and B × A have elements in common, is
(A) n (B) n – 1 (C) n^2 (D) none of these

332. If the value of mode and mean is 60 and 66 respectively, then the value of median is-
(A) 60 (B) 64 (C) 68 (D) None of these

333. The mean deviation about median from the following data :
340, 150, 210, 240, 300, 310, 320, is
(A) 52.4 (B) 52.5 (C) 52.8 (D) None of these

334. If p is any statement, t is tautology & c is a contradiction, then which of following is not correct-
(A) $p \vee (\sim p) \equiv c$ (B) $p \vee t = t$ (C) $p \wedge t = p$ (D) $p \wedge c = c$

335. $(\sim p \vee q)$ is logically equal to -
(A) $p \to q$ (B) $q \to p$ (C) $\sim (p \to q)$ (D) $\sim (q \to p)$

336. Let R be the relation on the set N of natural numbers defined by
R : {(x, y)} : x + 3y = 12 $x \in N, y \in N$} then domain of R
(A) {1, 2, 3} (B) {2, 3, 5} (C) {9, 6, 3} (D) none of these

337. Mean deviation about mean from the following data :

x_i :	3	9	17	23	27	
f_i :	8	10	12	9	5	is -

(A) 7.15 (B) 7.09 (C) 8.05 (D) None of these

338. Marks of 5 students of a tutorial group are 8, 12, 13, 15, 22 then variance is:
(A) 21 (B) 21.2 (C) 21.4 (D) None of these

339. The statement $p \Leftrightarrow q$ is equal to -
(A) $(\sim p \vee q) \vee (p \vee q)$ (B) $(p \wedge q) \vee (\sim p \wedge \sim q)$
(C) $(\sim p \vee q) \wedge (p \vee \sim q)$ (D) $(p \wedge q) \vee (p \vee q)$

340. The statement $(p \wedge q) \Leftrightarrow \sim p$ is a
(A) Tautology (B) contradiction
(C) Neither tautology nor contradiction (D) None of these

341. Let A be the set of first ten natural numbers and let R be a relation on A defined by $(x, y) \in R \Leftrightarrow x + 2y = 10$ i.e., $R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y=10\}$. Then domains of R^{-1}

(A) {2, 4, 6, 8} (B) {4, 3, 2, 1} (C) {1, 2, 4} (D) none of these

342. Variance of the data given below

size of item	3.5	4.5	5.5	6.5	7.5	8.5	9.5	
frequency	3	7	22	60	85	32	8	is-

(A) 1.29 (B) 2.19 (C) 1.32 (D) None of these

343. If the mean and variance of a variate X having a binomial distribution are 6 and 4 respectively. then the number of values of the variate in the distribution is

(A) 10 (B) 12 (C) 16 (C) 18

344. The statement $p \to p \vee q$ is a -

(A) Tautology (B) Contradiction

(C) Neither tautology nor contradiction (D) None of these

345. The statement $(p \to \sim q) \Leftrightarrow (p \wedge q)$ is a -

(A) Tautology (B) Contradiction

(C) Neither tautology nor contradiction (D) None of these

Questions based on statements (Q. 346 - 360)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement - II is true.

346. **Statement-I :** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4ac = 1$

Statement-II : If a,b,c are odd integer then the roots of the equation $4abc\, x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

347. **Statement-I :** If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$, then x,y,z are in H.P.

Statement-II : If $a_1^2 + a_2^2 + \ldots\ldots + a_n^2 = 0$, then $a_1 = a_2 = a_3 = \ldots\ldots = a_n = 0$

348. **Statement-I :** The number of zeroes at the end of 100! is 24.

Statement-II : The exponent of prime 'p' in n! is $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] \ldots\ldots + \left[\frac{n}{p^r}\right]$

where r is a natural number such that $p^r \le n < p^{r+1}$

349. **Statement-I :** $\sum_{r=0}^{n} \frac{1}{r+1}\, {}^{n}C_r\, x^r = \frac{1}{(n+1)x}\,[(1+x)^{n+1} - 1]$

Statement-II : $\sum_{r=0}^{n} \frac{{}^{n}C_r}{r+1} = \frac{2^{n+1}}{n+1}$

350. **Statement–I :** If all real values of x obtained from the equation $4^x - (a-3)2^x + (a-4) = 0$ are non-positive, then $a \in (4, 5]$.
Statement–II : If $ax^2 + bx + c$ is non-positive for all real values of x, then $b^2 - 4ac$ must be negative or zero and 'a' mst be negative.

351. **Statement–I :** If $\arg(z_1 z_2) = 2\pi$, then both z_1 and z_2 are purely real (z_1 and z_2 have principal arguments).
Statement–II : Principal argument of complex number and between $-\pi$ and π.

352. If $z_1 \neq -z_2$ and $|z_1 + z_2| = |(1/z_1) + (1/z_2)|$ then
Statement–I : $z_1 z_2$ is unimodular.
Statement–II : z_1 and z_2 both are unimodular.

353. **Statement–I :** If an infinite G.P. has 2nd term x and its sum is 4, then x belongs to (–8, 1).
Statement–II : Sum of an infinite G.P. is finite if for its common ratio r, $0 < |r| < 1$.

354. **Statement–I :** $1^{99} + 2^{99} + \ldots\ldots\ldots + 100^{99}$ is divisible by 10100.
Statement–II : $a^n + b^n$ is divisible by a + b if n is odd.

355. **Statement–I :** When number of ways of arranging 21 objects of which r objects are identical of one type and remaining are identical of second type is maximum, then maximum value of ${}^{13}C_r$ is 78.
Statement–II : ${}^{2n+1}C_r$ is maximum when r = n.

356. **Statement–I :** Number of ways of selecting 10 objects from 42 objects of which, 21 objects are identical and remaiing objects are distinct is 2^{20}.
Statement–II : ${}^{42}C_0 + {}^{42}C_1 + {}^{42}C_2 + \ldots\ldots\ldots + {}^{42}C_{21} = 2^{41}$.

357. **Statement–I :** Greatest term in the expansion of $(1 + x)^{12}$, when x = 11/10 is 7th.
Statement–II : 7th term in the expansion of $(1 + x)^{12}$ has the factor ${}^{12}C_6$ which is greatest value of ${}^{12}C_r$.

358. **Statement–I :** If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin\ \ B & \sin C+\sin\ \ C \end{vmatrix} = 0,$$ then triangle may not be equilateral.

Statement–II : If any two rows of a determinant are the same, then the value of that determinant is zero.

359. **Statement–I :** $A = \begin{bmatrix} 4 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Then $(AB)^{-1}$ does not exist.

Statement–II : Since $|A| = 0$, $(AB)^{-1} = B^{-1}A^{-1}$ is meaningless.

360. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.
Statement – 1 :

The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement – 2 :
If the four chosen numbers form an AP, then the set of all possible values of common difference is
$\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

ALGEBRA

211. C	**212.** C	**213.** B	**214.** C	**215.** D	**216.** B
217. B	**218.** B	**219.** A	**220.** D	**221.** C	**222.** D
223. A	**224.** D	**225.** D	**226.** B	**227.** B	**228.** D
229. A	**230.** B	**231.** B	**232.** C	**233.** B	**234.** D

235. B	**236.** C	**237.** C	**238.** B	**239.** A	**240.** A
241. B	**242.** D	**243.** D	**244.** A	**245.** B	**246.** A
247. C	**248.** A	**249.** A	**250.** C	**251.** D	**252.** D
253. D	**254.** B	**255.** B	**256.** D	**257.** C	**258.** D
259. A	**260.** B	**261.** D	**262.** C	**263.** B	**264.** A
265. D	**266.** B	**267.** A	**268.** C	**269.** B	**270.** C
271. C	**272.** D	**273.** D	**274.** A	**275.** D	**276.** B
277. B	**278.** C	**279.** A	2**80.** D	**281.** B	**282.** C
283. C	**284.** A	**285.** C	**286.** D	**287.** C	**288.** B
289. D	**290.** C	**291.** A	**292.** C	**293.** C	**294.** B
295. A	**296.** B	**297.** C	**298.** B	**299.** C	**300.** C
301. C	**302.** C	**303.** C	**304.** B	**305.** A	**306.** D
307. B	**308.** B	**309.** C	**310.** A	**311.** C	**312.** A
313. C	**314.** A	**315.** B	**316.** D	**317.** C	**318.** B
319. C	**320.** A	**321.** B	**322.** B	**323.** B	**324.** C
325. B	**326.** C	**327.** D	**328.** C	**329.** A	**330.** C
331. C	**332.** B	**333.** C	**334.** A	**335.** A	**336.** C
337. B	**338.** B	**339.** C	**340.** C	**341.** B	**342.** C
343. D	**344.** A	**345.** B	**346.** D	**347.** A	**348.** A
349. C	**350.** B	**351.** A	**352.** C	**353.** D	**354.** A
355. D	**356.** C	**357.** B	**358.** A	**359.** A	**360.** C

211. C

$\because \log_2 x + \log_2 y \geq 6 \Rightarrow \log_2 (xy) \geq 6$

$\therefore xy \geq 2^6$ or $\sqrt{xy} \geq 2^3$

$\because \frac{x+y}{2} \geq \sqrt{xy}$ or $x + y \geq 2\sqrt{xy} \geq 16$

$(\because$ AM $\geq$ GM$) \quad \therefore x + y \geq 16.$

212. C

$f(x, 10) = f(x, 11)$

$\Rightarrow \sum_{k=1}^{10} \log_x \left(\frac{k}{x}\right) = \sum_{k=1}^{11} \log_x \left(\frac{k}{x}\right)$

$\Rightarrow 0 = \log_x \left(\frac{11}{x}\right) \Rightarrow \frac{11}{x} = 1 \Rightarrow x = 11$

213. B

We know that only even prime is 2,
then $(2)^2 - \lambda(2) + 12 = 0 \Rightarrow \lambda = 8$ ....(i)
and $x^2 + \lambda x + \mu = 0$ has equal roots
$\therefore \lambda^2 - 4\mu = 0$ or $(8)2 - 4\mu = 0 \Rightarrow \mu = 16$
[from Eq. (i)]

214. C

By hypothesis $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = -\frac{b}{a}$

and $\frac{\alpha}{\alpha-1} \cdot \frac{\alpha+1}{\alpha} = \frac{c}{a}$

$\Rightarrow \frac{2\alpha^2-1}{\alpha^2-\alpha} = -\frac{b}{a}$ and $\alpha = \frac{c+a}{c-a}$

$\Rightarrow (c + a)^2 + 4ac = -2b (c + a)$
$\Rightarrow (c + a)^2 + 2b (c + a) + b^2 = b^2 - 4ac$
$\Rightarrow (a + b + c)^2 = b^2 - 4ac.$

215. D

For maximum value of the given sequence to n terms, when the nth term is either zero or the smallest positive number of the sequence i.e., $50 + (n - 1)(-2) = 0$

$\Rightarrow n = 26 \therefore S_{26} = \frac{26}{2}(50 + 0) = 26 \times 25 = 650$

216. B

$\because x^a = y^b = z^c = \lambda$ (say)
$\therefore x = \lambda^{1/a}, y = \lambda^{1/b}, z = \lambda^{1/c}$
Now, $\because$ x, y, z are in GP
$\therefore y^2 = zx$
$\Rightarrow \lambda^{2/b} = \lambda^{1/c}. \lambda^{1/a} \Rightarrow \lambda^{2/b} = \lambda^{(1/c + 1/a)}$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \therefore$ a,b,c are in HP

Now, GM > HM $\Rightarrow \sqrt{ac} > b$ .....(i)
Now, for three numbers a^3, b^3, c^3
AM > GM

$\Rightarrow \frac{a^3+c^3}{2} > (\sqrt{ac})^3 > b^3$ [from Eq. (i)]

$\therefore a^3 + c^3 > 2b^3$

217. B

$\because 0 \leq [x] < 2 \Rightarrow [x] = 0,1$
$-1 \leq [y] < 1 \Rightarrow [y] = -1, 0$
and $1 \leq [z] < 3 \Rightarrow [z] = 1, 2$
Now, aplying in the given determinant
$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, then

$$\begin{vmatrix} [x]+1 & [y] & [z] \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= ([x] + 1)(1 - 0)-[y] (-1 -0) + [z] (0+1)$
$= [x] + [y] + [z] + 1 = 1 + 0 + 2 + 1 = 4$
$(\therefore$ for maximum value $[x] = 1, [y]=0,[z]= 2)$

218. B

$$\begin{vmatrix} \lambda & -1 & -2 \\ 2 & -3 & \lambda \\ 3 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda (-3 + 2\lambda) + 1 (2 - 3\lambda) - 2(-4 + 9) + 0$
$\Rightarrow 2\lambda^2 - 6\lambda - 8 = 0 \Rightarrow \lambda^2 - 3\lambda - 4 = 0$
$\Rightarrow (\lambda - 4)(\lambda + 1) = 0 \quad \therefore \lambda = -1, 4$

219. A

$\because Q = PAP^T \Rightarrow P^T Q = AP^T \quad (\because PP^T = I)$
$\therefore P^T Q^{2005} P = AP^T Q^{2004} P$
$= A^2 P^T Q^{2003} P = A^3 P^T Q^{2002} P = ...$
$= A^{2004} P^T (QP) = A^{2004} P^T (PA)$
$(\because Q = PAP^T \Rightarrow QP = PA)$

$= A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

220. D

$\because A^4 (I - A) = A^4 I - A^5 = A^4 - O = A^4 \neq I$
$A^3 (I - A) = A^3 I - A^4 = A^3 - A^4 \neq I$
and $(I + A)(I - A) = I^2 - A^2 = I - A^2 \neq I.$

221. C

$\because (x - 1) = (x - [x])(x - \{x\})$
$\Rightarrow x = 1 + \{x\}[x] \Rightarrow [x]+\{x\} = 1 + \{x\}[x]$
$\Rightarrow (\{x\} - 1)([x] - 1) = 0$

$\Rightarrow \{x\} - 1 \neq 0, \therefore [x] - 1 = 0 \Rightarrow [x] = 1$
$\Rightarrow x \in [1, 2)$

222. **D**

Discriminant ≥ 0 $\therefore \pi < 22/5$
Roots less than 2 $\therefore f(2) > 0$
$\therefore p^2 - p - 2 > 0 \Rightarrow p > 2$ or $p < -1$
combine both casses, we get

$p \in (-\infty, -1)\ e\left(2, \frac{22}{5}\right]$

223. **A**

$\because$ a, b, c, d are positive real numbers.
$\therefore m > 0$ ....(i) Now, AM $\geq$ GM

$\Rightarrow \frac{(a+b)+(c+d)}{2} > \sqrt{(a+b)(c+d)}$

$\Rightarrow \frac{2}{2} \geq \sqrt{m}$ or $m \leq 1$ ....(ii)

From Eqs. (1) and (ii), we get, $0 < m \leq 1$

224. **D**

Let $b = a + d$, $c = a + 2d$ ...(i)
$\because a^2, b^2, c^2$ are in GP $\therefore (b^2)^2 = a^2c^2$
or $\pm b^2 = ac$ ....(ii)
$\because$ a, b, c are in AP $\therefore 2b = a + c$
Given, $a + b + c = 3/2 \Rightarrow 3b = 3/2 \Rightarrow b = 1/2$

From Eq. (i), $a = \frac{1}{2} - d$, $c = \frac{1}{2} + d$

$\therefore$ From Eq. (ii),

$\pm \frac{1}{4} = \left(\frac{1}{2} - d\right)\left(\frac{1}{2} + d\right) \Rightarrow \pm \frac{1}{4} = \frac{1}{4} - d^2$

Taking (–ve) sign, $\therefore d = \pm \frac{1}{\sqrt{2}}$

$\therefore a = \frac{1}{2} - d = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$

$\Rightarrow a = \frac{1}{2} + \frac{1}{\sqrt{2}}$ ($\because$ a > b)

225. **D**

$$\therefore f(-x) = \begin{vmatrix} -x & \cos x & e^{x^2} \\ -\sin x & x^2 & \sec x \\ -\tan x & 1 & 2 \end{vmatrix} = -f(x)$$

$\therefore \int_{-\pi/2}^{\pi/2} f(x)\,dx = 0$ [$\because$ f(x) is an odd function]

226. **B**

$\Delta > 0 \Rightarrow abc + 2 > 3(abc)^{1/3}$
Let $(abc)^{1/3} = x$
$x^3 + 2 > 3x \Rightarrow (x - 1)^2 (x + 2) > 0$
$\therefore x + 2 > 0 \Rightarrow x > -2$
$(abc)^{1/3} > -2 \Rightarrow abc > -8$

227. **B**

$\because 2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ ...(i)

$\Rightarrow 4x - 2y = \begin{bmatrix} 6 & -6 & 0 \\ 6 & 6 & 4 \end{bmatrix}$ ....(ii)

and $x + 2y = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$ .....(iii)
Adding Eqs. (ii) and (iii), then

$5x = \begin{bmatrix} 10 & -5 & 5 \\ 5 & 10 & 0 \end{bmatrix} \therefore x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

From Eq. (iii), $2x+4y = \begin{bmatrix} 8 & 2 & 10 \\ -2 & 8 & -8 \end{bmatrix}$ ....(iv)

Substracting Eq. (i) from (iv), then

$5y = \begin{bmatrix} 5 & 5 & 10 \\ -5 & 5 & -10 \end{bmatrix} \therefore y = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$

228. **D**

$\because \omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$
Also, $\omega^3 = 1$ and $\omega + \omega^2 = -1$

Then, $A = \begin{pmatrix} \frac{\omega}{i} & \frac{\omega^2}{i} \\ -\frac{\omega^2}{i} & -\frac{\omega}{i} \end{pmatrix} = \frac{\omega}{i}\begin{pmatrix} 1 & \omega \\ -\omega & -1 \end{pmatrix}$

$\therefore A^2 = \frac{\omega^2}{i^2}\begin{pmatrix} 1 & \omega \\ -\omega & -1 \end{pmatrix}\begin{pmatrix} 1 & \omega \\ -\omega & -1 \end{pmatrix}$

$= -\omega^2\begin{pmatrix} 1-\omega^2 & 0 \\ 0 & 1-\omega^2 \end{pmatrix} = \begin{pmatrix} -\omega^2+\omega^4 & 0 \\ 0 & -\omega^2+\omega^4 \end{pmatrix}$

$= \begin{pmatrix} -\omega^2+\omega & 0 \\ 0 & -\omega^2+\omega \end{pmatrix}$

$\because f(x) = x^2 + 2 \therefore f(A) = A^2 + 2I$

$= \begin{pmatrix} -\omega^2+\omega & 0 \\ 0 & -\omega^2+\omega \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$= \begin{pmatrix} -\omega^2+\omega+2 & 0 \\ 0 & -\omega^2+\omega+2 \end{pmatrix}$

$= (-\omega^2 + \omega + 2)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$= (3 + 2\omega)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \left(3+2\left(\frac{-1+i\sqrt{3}}{2}\right)\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$= (2 + i\sqrt{3})\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$.

229. A

$[A(A + B)^{-1} B]^{-1}$
$= B^{-1}(A + B) A^{-1}$
$= (B^{-1}A + I) A^{-1} = B^{-1} + A^{-1}$

230. B

$|[x] - 2x| = 4 \Rightarrow |[x] - 2([x] + \{x\})| = 4$
$\Rightarrow |[x] + 2\{x\}| = 4$
Which is possible only when $2\{x\} = 0,1$.
If $\{x\} = 0$, then $[x]=\pm4$ and then $x = -4, 4$

and if $\{x\} = \frac{1}{2}$, then $[x] + 1 = \pm4$

$\Rightarrow [x] = 3, -5 \therefore x = 3 + \frac{1}{2}$ and $-5 + \frac{1}{2}$

$\Rightarrow x = 7/2, -9/2$ Hence, $x=-4, -9/2, 7/2, 4$

231. B

$\because a_1, a_2, \ldots a_{21}$, are in AP

$\therefore a_1 + a_2 + \ldots + a_{21} = \frac{21}{2}(a_1 + a_{21})$

$\Rightarrow 693 = \frac{21}{2}(a_1 + a_{21})$ (given)

$\therefore a_1 + a_{21} = 66$ ....(i)

$\therefore \sum_{r=0}^{10} a_{2r+1} = a_1 + a_3 + a_5 + a_7 + a_9 + \ldots + a_{21}$
$= (a_1 + a_{21}) + (a_3 + a_{19}) + (a_5 + a_{17}) + (a_7 + a_{15}) + (a_9 + a_{13}) + a_{11}$
$= 5\times(a_1 + a_{21}) + a_{11}$ $(\therefore T_n + T_n' = a + \ell)$
$= 5 \times 66 + a_{11} = 330 + a_{11}$

$= 330 + \left(\frac{a_1 + a_{21}}{2}\right)$ ($\because a_{11}$ is middle term)

$= 330 + 33 = 363$

232. C

$\log_2 (a + b) + \log_2 (c + d) \geq 4$
$\Rightarrow \log_2 \{(a + b)(c + d)\} \geq 4$
$\Rightarrow (a + b)(c + d) \geq 2^4$
But AM $\geq$ GM

$\therefore \frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)} = 2^2$

$\therefore a + b + c + d \geq 8$

233. B

$R_2 \to R_2 - R_3, R_1 \to R_1 - R_2$

$$\begin{vmatrix} x & -y & 0\\ 0 & y & -z\\ a & b & c+z\end{vmatrix} = 0$$

$\Rightarrow x(cy + yz + bz) + y(az) = 0$
$cxy + xyz + bzx + ayz = 0$
$cxy + bzx + ayz = 2007$

234. D

$$\sum_{r=1}^{n}\Delta_r = \begin{vmatrix} \Sigma(1) & \Sigma r & \Sigma 2^r\\ 2 & n & n^2\\ n & \frac{(n)(n+1)}{2} & 2^{n+1}\end{vmatrix}$$

$$= \begin{vmatrix} n & \frac{(n)(n+1)}{2} & 2^{n+1}-2\\ 2 & n & n^2\\ n & \frac{(n)(n+1)}{2} & 2^{n+1}\end{vmatrix} = \begin{vmatrix} 0 & 0 & -2\\ 2 & n & n^2\\ n & \frac{n(n+1)}{2} & 2^{n+1}\end{vmatrix} = -2n$$

235. B

$\because$ A is orthogonal, $\therefore AA' = I \Rightarrow A^{-1} = A'$

236. C

By property, adj $A^T - (\text{adj } A)^T = O$ (null matrix)

237. C

Given that $16^{x^2+3x-1} = 8^{x^2+3x+2}$

$\Rightarrow 2^{4(x^2+3x-1)} = 2^{3(x^2+3x+2)}$
$\Rightarrow 4(x^2 + 3x - 1) = 3(x^2 + 3x + 2)$
$\Rightarrow x^2 + 3x - 10 = 0$
$\Rightarrow (x + 5)(x - 2) = 0 \Rightarrow x = -5, 2$
Sum of all values $= -5 + 2 = -3$

238. B

Sum of the roots $\alpha + \beta + \gamma = 0 \Rightarrow \gamma = 0$
$\therefore$ 0 is a root of the equation $\Rightarrow c - 1 = 0$
$\Rightarrow c = 1$

239. A

$\because$ p, q, r are in AP $\therefore$ 2q = p + r ...(i)

$\because$ Roots of $px^2 + qx + r = 0$ are all real, then $q^2 - 4pr \geq 0$

$$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow (p+r)^2 - 16\,pr \geq 0 \Rightarrow p^2 + r^2 - 14pr \geq 0$$

$$\Rightarrow \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \geq 0$$

$$\Rightarrow \left(\frac{r}{p} - 7\right)^2 \geq 48 \Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$$

240. A

Let $S_n = Pn^2 + Qn$ = Sum of first n terms according to question, Sum of first 3n terms = sum of the next n terms

$\Rightarrow S_{3n} = S_{4n} - S_{3n}$ or $2S_{3n} = S_{4n}$

or $2[P(3n)^2 + Q(3n)] = P(4n)^2 + Q(4n)$

$\Rightarrow 2Pn^2 + 2Qn = 0$ or $Q = -nP$ ....(i)

$$\text{Then } \frac{\text{Sum of first 2n terms}}{\text{Sum of next 2n terms}} = \frac{S_{2n}}{S_{4n} - S_{2n}}$$

$$= \frac{P(2n)^2 + Q(2n)}{[P(4n)^2 + Q(4n)] - [P(2n)^2 + Q(2n)]}$$

$$= \frac{2nP + Q}{6Pn + Q} = \frac{nP}{5nP} = \frac{1}{5} \quad \text{[from Eq. (i)]}$$

241. B

a + 5 = c + 2b

$$\frac{n^2 - n}{2} + n + 1 + 5 = \frac{n^2 - n}{2} + 2n + 2 \Rightarrow n = 4$$

242. D

$$\text{Det}(M_r) = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix} = 2r - 1$$

$$\sum_{r=1}^{2007} \det(M_r) = 2\sum_{r=1}^{2007} r - 2007$$

$$= 2\,\frac{(2007)(2008)}{2} - 2007 = (2007)^2$$

243. D

$$\because \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Then, X = y and Y = x ie, y = x

244. A

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$

$\Rightarrow \lambda = 3$

245. B

Let the roots of $ax^2 - 5x + 6 = 0$ be $2\alpha, 3\alpha$,

$$5\alpha = \frac{5}{a} \text{ and } 6\alpha^2 = \frac{6}{a} \Rightarrow \frac{6}{a^2} = \frac{6}{a} \Rightarrow a = 1$$

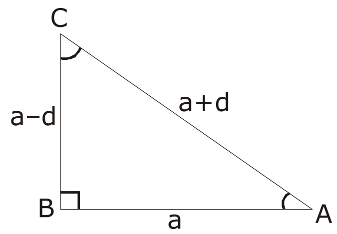
246. A

Let sides be a − d, a, a + d

$\therefore (a+d)^2 = (a-d)^2 + a^2$

$\Rightarrow 4ad = a^2 \quad \therefore a = 4d$

Then sides are 3d, 4d and 5d



$$\sin A = \frac{a-d}{a+d} = \frac{3d}{5d} = \frac{3}{5}$$

$$\text{and } \sin C = \frac{a}{a+d} = \frac{4d}{5d} = \frac{4}{5}$$

247. C

$$f_n(x) = x\,x\,x \ldots x(\text{n digits}) = x\left\{\frac{(10^n - 1)}{9}\right\}$$

$$\therefore f_n^2(3) = 3^2\left\{\frac{(10^n - 1)}{9}\right\}^2 = \frac{(10^n - 1)^2}{9}$$

$$\text{and } f_n(2) = \frac{2(10^n - 1)}{9}$$

$$\therefore f_n^2(3) + f_n(2) = \left(\frac{10^n - 1}{9}\right)(10^n - 1 + 2)$$

$$= \frac{(10^n - 1)(10^n + 1)}{9} = \frac{10^{2n} - 1}{9} = f_{2n}(1).$$

248. A

$$A^2 = -B^2 \Rightarrow A^2 = \begin{vmatrix} 2 & -2 \\ -2 & 2 \end{vmatrix} = -2B$$

$A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$A^8 = 64B^2 = 128B$

249. **A**

$x_1, x_2, x_3 \ldots\ldots\ldots x_{20}$ are in HP

then $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3} \ldots\ldots\ldots \frac{1}{x_{20}}$ are in A.P.

Let $\left(\frac{1}{x_i} = a_i\right)$ then $a_1, a_2, a_3 \ldots a_{20}$ are in

A.P.

$x_1x_2 + x_2x_3 + x_3x_4 + \ldots\ldots\ldots + x_{19}x_{20}$

$\frac{1}{a_1.a_2} + \frac{1}{a_2.a_3} + \ldots\ldots + \frac{1}{a_{19}.a_{20}}$

$$= \frac{1}{d}\left[\left(\frac{1}{a_1} - \frac{1}{a_2}\right) + \left(\frac{1}{a_2} - \frac{1}{a_3}\right) + \ldots\ldots + \left(\frac{1}{a_{19}} - \frac{1}{a_{20}}\right)\right]$$

$$= \frac{1}{d}\left[\frac{a_{20} - a_1}{a_1a_{20}}\right] = \frac{1}{d}\left[\frac{a_1 + 19d - a_1}{a_1a_{20}}\right]$$

$$= \frac{19}{a_1a_{20}} = 19.\, x_1x_{20} = 19 \times 4 = 76$$

250. **C**

Let $\cot^{-1} x = \theta$ $\quad\therefore \cot\theta = x$

$$\Rightarrow \left(\frac{xi + 1}{xi - 1}\right) = \frac{i\cot\theta + 1}{i\cot\theta - 1} = \frac{\cot\theta - i}{\cot\theta + i}$$

$$= \frac{\cos\theta - i\sin\theta}{\cos\theta + i\sin\theta} = \frac{e^{-i\theta}}{e^{i\theta}} = \frac{1}{e^{2i\theta}}.$$

$$\Rightarrow e^{2i\theta}\left(\frac{xi + 1}{xi - 1}\right) = 1 \Rightarrow e^{2mi\theta}\left(\frac{xi + 1}{xi - 1}\right)^m = 1$$

$$\therefore e^{2mi\cot^{-1}x}\left(\frac{xi + 1}{xi - 1}\right)^m = 1.$$

251. **D**

$|z - i\,\text{Re}(z)| = |z - \text{Im}(z)|$ If $z = x + iy$ then $|x + iy - ix| = |x + iy - y|$

$$\Rightarrow \sqrt{x^2 + (y - x)^2} = \sqrt{(x - y)^2 + y^2}$$

or $x^2 = y^2$ $\therefore x = \pm y \Rightarrow \text{Re}(z) = \pm \text{Im}(z)$

$\Rightarrow \text{Re}(z) + \text{Im}(z) = 0$ and $\text{Re}(z) - \text{Im}(z) = 0$

252. **D**

$|a^2 - 2a| < 3 \Rightarrow -3 < a^2 - 2a < 3$

$\Rightarrow -3 + 1 < a^2 - 2a + 1 < 3 + 1$

$\Rightarrow -2 < (a - 1)^2 < 4$

$\therefore 0 \le (a - 1)^2 < 4 \Rightarrow -2 < a - 1 < 2$

or $-1 < a < 3$ But $a \in R^+$

$\therefore 0 < a < 3 \Rightarrow a \in (0, 3)$.

253. **D**

$\because xyz = 2^3 \times 3^1$

Let $\alpha + \beta + \gamma = 3, \alpha + \beta + \gamma = 1$

Number of integral positive solutions

$= {}^{3+3-1}C_{3-1} \times {}^{1+3-1}C_{3-1}$

$= {}^5C_2 \times {}^3C_2 = 30$

Since, negative values of x, y, z is also allowed but since product is positive and hence any two of them may be negative.

$\therefore$ Number of negative integral solutions $= {}^3C_2 \times 30 = 90$

Hence, total number of integral solutions of $xyz = 24$ is $30 + 90 = 120$

254. **B**

Let $a = 2x + 1, b = 2y + 1, c = 2z + 1$

where $x, y, z \in$ whole number

$\therefore a + b + c = 13$

$\Rightarrow 2x + 1 + 2y + 1 + 2z + 1 = 13$

or $x + y + z = 5$

The number of integrals solutions

$$= {}^{5+3-1}C_{3-1} = {}^7C_2 = \frac{7.6}{1.2} = 21$$

255. **B**

$4m = 2^a + 3^b + 5^c = 2^a + (4 - 1)^b + (4 + 1)^c$

$= 4k + 2^a + (-1)^b + 1^c$

$\because a = 1, b =$ even, $c =$ any number

OR

$a \ne 1, b =$ odd, c any number.

Required number $1 \times 2 \times 5 + 4 \times 3 \times 5 = 70$

256. **D**

Let $\Sigma n = \lambda$ $\quad\ldots$(i)

$$\therefore \left(\frac{x^3 + 1 + x^6}{x^3}\right)^{\Sigma n} = \left[1 + \left(x^3 + \frac{1}{x^3}\right)\right]^{\lambda}$$

$$= 1 + {}^{\lambda}C_1\left(x^3 + \frac{1}{x^3}\right) + {}^{\lambda}C_2\left(x^3 + \frac{1}{x^3}\right)^2 + \ldots.$$

$$\ldots\ldots + {}^{\lambda}C_{\lambda}\left(x^3 + \frac{1}{x^3}\right)^{\lambda}$$

On expanding each term, two dissimilar terms are added in the expansion

$$\because \left(x^3 + \frac{1}{x^3}\right)^3 = x^9 + \frac{1}{x^9} + 3\left(x^3 + \frac{1}{x^3}\right)$$

Only x^9 and $\frac{1}{x^9}$ are new terms. Coefficient of x^3 and $\frac{1}{x^3}$

have occured earlier in ${}^{\lambda}C_1\left(x^3+\frac{1}{x^3}\right)$

Hence, number of terms

$= 1 + 2 + 2 + 2 + \ldots$ upto λ

$= 1 + 2\lambda = 1 + 2\,\Sigma n = 1 + 2.\frac{n(n+1)}{2}$

$= 1 + n + n^2$

257. C

$\because\ 5^{40} = (5^2)^{20} = (22+3)^{20} = 22\lambda + 3^{20},\ \lambda \in N$

Also, $3^{20} = (3^2)^{10} = (11-2)^{10} = 11\mu + 2^{10},\ \mu \in N$

Now, $2^{10} = 1024 = 11 \times 93 + 1$

$\therefore$ Remainder = 1 ie, $\alpha = 1$

Also, $2^{2003} = 2^3.\ 2^{2000} = 8\ (2^4)^{500} = 8\ (16)^{500}$

$= 8(17-1)^{500} = 8(17v + 1),\ v \in N$

$= 8 \times 17v + 8$

$\therefore$ Remainder = 8 ie, $\beta = 8$

$\Rightarrow \beta - \alpha = 8 - 1 = 7$

258. D

$$(r+1)^{th} \text{ term} = T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^r$$

$$= {}^{10}C_r\left(\frac{x}{3}\right)^{5-\frac{r}{2}}\left(\frac{3}{2x^2}\right)^{\frac{r}{2}} = {}^{10}C_r\ \frac{x^{5-\frac{r}{2}-r}}{3^{5-\frac{r}{2}-\frac{r}{2}}.2^{\frac{r}{2}}}$$

For independent of x, Put $5 - \frac{r}{2} - r = 0$

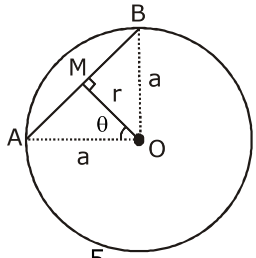
$\therefore 5 = \frac{3r}{2} \Rightarrow r = \frac{10}{2}$ impossible $\because r \neq$ whole number

259. A

Let a be the radius of the circle, ℓ be the length of the chord and r be the distance of the mid point of the chord from the centre of the circle.

Let $\angle AOM = \theta$

$\therefore \sin\theta = \frac{AM}{a}$



and $\cos\theta = \frac{r}{a}$

$\Rightarrow r = a\cos\theta$,

$\ell = 2\,AM = 2a\sin\theta$

Given, $(2a)\frac{2}{3} < AB < (2a)\ \frac{5}{6}$

$\Rightarrow \frac{4a}{3} < \ell < \frac{10a}{6} \Rightarrow \frac{4a}{3} < 2a\sin\theta < \frac{10a}{6}$

$\Rightarrow \frac{2}{3} < \sin\theta < \frac{5}{6} \Rightarrow \frac{4}{9} < 1 - \cos^2\theta < \frac{25}{36}$

or $\frac{4}{9} - 1 < -\cos^2\theta < \frac{25}{36} - 1$

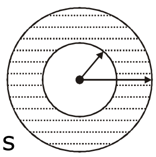
$\Rightarrow \frac{5}{9} > \cos^2\theta > \frac{11}{36}$

$\Rightarrow \frac{\sqrt{11}}{6}\ a < a\cos\theta < \frac{\sqrt{5}}{3}\ a$

or $\frac{\sqrt{11}a}{6} < r < \frac{\sqrt{5}a}{6}$

$\therefore$ The given condition is satisfied, if the mid point of the chord lies within the region between the concentric circles of radius

$\frac{\sqrt{11}a}{6}$ and $\frac{\sqrt{5}}{3}$ a



Hence, the required probability

$= \frac{\text{the area of the circular annulus}}{\text{area of the given circle}}$

$$= \frac{\pi\left(\frac{5}{9}a^2 - \frac{11}{36}a^2\right)}{\pi a^2} = \frac{5}{9} - \frac{11}{36} = \frac{20-11}{36} = \frac{9}{36} = \frac{1}{4}$$

260. B

Let S be the sample space and E be the event of getting a large number than the previous number. $\therefore$ n(S) = 6 × 6 × 6 = 216

Now, we count the number of favourable ways. Obviously, the second number has to be greater than 1. If the second number is i (i > 1), then the number of favourable ways, = (i – 1) × (6 – i)

$\therefore$ n(E) = Total number of favourable ways

$$= \sum_{i=1}^{6}(i-1)\times(6-i)$$

$= 0 + 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1 + 0 = 20$

Therefore, the required probability = $\frac{n(E)}{n(S)}$

$= \frac{20}{216} = \frac{5}{54}$

261. D

Even numbers are 2,4,6

$\therefore$ The probability that an even number appear $= \frac{3}{6} = \frac{1}{2}$ $\therefore$ The required probability

= P(that an even number occurs once or thrice or five times....or (2n + 1) times)

$$= {}^{2n+1}C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{2n} + {}^{2n+1}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{2n-2} + {}^{2n+1}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{2n-4} + \ldots + {}^{2n+1}C_{2n+1}\left(\frac{1}{2}\right)^{2n+1}\left(\frac{1}{2}\right)^0$$

$$= \frac{1}{2^{2n+1}}\{{}^{2n+1}C_1 + {}^{2n+1}C_3 + {}^{2n+1}C_5 + \ldots + {}^{2n+1}C_{2n+1}\}$$

$$= \frac{2^{2n}}{2^{2n+1}} = \frac{1}{2}$$

262. C

Since $x_r = \cos\left(\frac{\pi}{3^r}\right) - i\sin\left(\frac{\pi}{3^r}\right)$

$$\therefore x_1 \cdot x_2 \cdot x_3 \ldots\infty = \cos\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3} + .\, 3\infty\right) - i\sin\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3} + .\, 3\infty\right)$$

$$= \cos\left(\frac{\pi/3}{1-1/3}\right) - i\sin\left(\frac{\pi/3}{1-1/3}\right)$$

$= \cos(\pi/2) - i\sin(\pi/2) = -i$

263. B

$\arg\left(\frac{z-z_1}{z_2-z}\right) = \frac{\pi}{2}$ $\therefore z_1, z_2, z_3$ lie on a circle

$\Rightarrow z_1$ and z_2 are the end points of diameter

$\therefore$ center $(z_0) = \frac{z_1+z_2}{2}$

$\therefore z_0 = 5-i$ and radius $= |z_1 - z_0| = |1 + 2i|$ $r = \sqrt{5}$

$\therefore$ Equation circle is $|z - z_0| = r$ or $|z-(5-i)| = \sqrt{5}$

264. A

$\because |z|$ = Real and positive, imaginary part is zero

$\therefore \arg|z| = 0 \Rightarrow [\arg|z|] = 0$

$$\therefore \int_{x=0}^{100} [\arg|z|]\,dx = \int_{x=0}^{100} 0\,dx = 0$$

265. D

Maximum number of points $= {}^8P_2 = 56$

266. B

$2^{10} - 1 = 1023$

267. A

8 type 7 type 2 or 4 or 6 or 8

Total number $= {}^8P_2 \times 4 = 224$

268. C

Coefficient of x^{11} in

$(x^1 + x^2 + x^3 + x^4 + x^5 + x^6)^3$

Coefficient of x^{11} in

$x^3(1 + x + x^2 + x^3 + x^4 + x^5)^3$

or Coefficient of x^{11} in

$x^3(1 - x^6)^3(1 - x)^{-3}$

or Coefficient of x^8 in

$(1 - 3x^6)(1 + {}^3C_1 x + {}^4C_2 x^2 + \ldots.)$

$= {}^{10}C_8 - 3.{}^4C_2$

$= {}^{10}C_2 - 3.{}^4C_2$

$= 45 - 18 = 27$

269. B

Here, 2n + 2 is even $\therefore$ Greatest coefficient

$${}^{2n+2}C_{\frac{2n+2}{2}} = {}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!} = \frac{(2n+2)!}{\{(n+1)!\}^2}$$

270. C

$(\sqrt{3} + \sqrt[3]{2})^9 = (3^{1/2} + 2^{1/3})^9$

$\therefore T_{r+1} = {}^9C_r(3^{1/2})^{9-r}(2^{1/3})^r = {}^9C_r\, 3^{(9-r)/2}\, 2^{r/3}$

For first integral term for r = 3

$T_{3+1} = {}^9C_3\, 3^2\, 2^1$ ie, $T_{3+1} = T_4$ (4th term)

271. C

$(2^{1/5} + 3^{1/10})^{55}$

Total terms = 55 + 1 = 56

$T_{r+1} = {}^{55}C_r 2^{(55-r)/5} 3^{r/10}$

Here, r = 0 , 10, 20, 30, 40, 50

Number of rational terms = 6

$\therefore$ Number of irrational terms = 56 – 6 = 50

272. D

100 bulbs = 10 defective + 90 non defective

Probability that 3 out of 4 bulbs, bought by a customer will not to be defective.

$$= \frac{{}^{90}C_3 \times {}^{10}C_1}{{}^{100}C_4}$$

273. D

Let A denotes the event that the throws of the two persons are unequal. Then A′ denote the event that the throws of the two persons are equal. The total number of cases for A′ is $(36)^2$. we now proceed to find out the number of favourable cases for A′. suppose

$(x + x^2 + x^3 + ...x^6)^2 = a_2x^2 + a_3x^3 + ...+a_{12}x^{12}$

The number of favourable ways for

$A' = a_2^2 + a_3^2 + ... + a_{12}^2$

= coefficient of constant term in

$$(a_2x^2+a_3x^3 + ...+a_{12}x^{12}) \times \left(\frac{a_2}{x^2} + \frac{a_3}{x^3} + ... + \frac{a_{12}}{x^{12}}\right)$$

= coefficient of constant term in

$$\frac{(1-x^6)^2(1-1/x^6)^2}{(1-x)^2(1-1/x)^2}$$

= coefficient of x^{10} in $(1 - x^6)^4 (1 - x)^{-4}$

= coefficient of x^{10} in

$(1 - 4x^6 + 6x^{12}) (1 + {}^4C_1x + {}^5C_2x^2 + {}^6C_3x^3+...)$

$= {}^{13}C_{10} - 4.\,{}^7C_4 = 146$

$$\therefore P(A')=\frac{146}{36^2} = \frac{73}{648} \Rightarrow P(A) = 1 - P(A') = \frac{575}{648}$$

274. A

$A \times B = (1, 3, 5, 7, 9) \times (2, 4, 6, 8)$

= (1, 2), (1, 4), (1, 6), (1, 8), (3, 2), (3, 4), (3, 6), (3, 8), (5, 2), (5, 4), (5, 6), (5, 8), (7, 2), (7, 4), (7, 6), (7, 8), (9, 2), (9, 4), (9, 6), (9, 8) Total ways = 5 × 4 = 20

Favourable case:(1, 8), (3, 6), (5, 4), (7, 2) $(\because a + b = 9)$

$\therefore$ Number of favourable cases = 4

$\therefore$ Required probability $= \frac{4}{20} = \frac{1}{5}$

275. D

$\because \arg(z - 3i) = \arg(x + iy - 3i) = 3\pi/4$

$\Rightarrow x < 0, y - 3 > 0 \left(\because \frac{3\pi}{4} \text{ is in II quadrant}\right)$

$\frac{y-3}{x} = \tan\frac{3\pi}{4} = -1 \Rightarrow y = -x + 3$ .....(i)

$\forall\ x < 0$ and $y > 3$

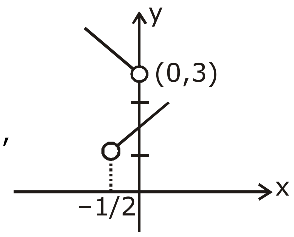
& $\arg(2z + 1 - 2i) = \arg[(2x+1) + i(2y-2)] = \pi/4$

$\Rightarrow 2x + 1 > 0, 2y - 2 > 0$ ($\because \pi/4$ is in I quadrant)

$\therefore \frac{2y-2}{2x+1} = \tan \pi/4 = 1 \Rightarrow 2y - 2 = 2x + 1$

$\Rightarrow y = x + 3/2\ \forall\ x > -1/2, y > 1$ ....(ii)

From Eqs. (i) and (ii),



it is clear from the graph No point of intersection

276. B

$\because |z_1 - z_2| = |z_1 - (z_2 - 3 - 4i) - (3 + 4i)|$

$\geq |z_1| - |z_2 - 3 - 4i| - |3 + 4i|$

$= 12 - 5 - 5 = 2 \Rightarrow |z_1 - z_2| \geq 2.$

277. B

$|iz + z_1| = |i(z - i) + z_1 - 1|$

$\leq |i(z - i)| + |z_1 - 1|$

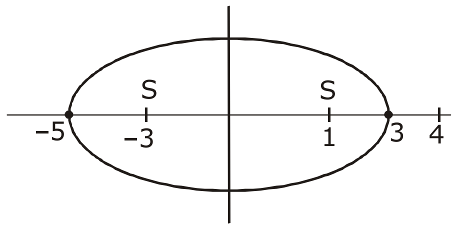
$= |z - i| + |z_1 - 1|$

$\leq 2 + |4 + 3i| = 2 + 5 \leq 7.$

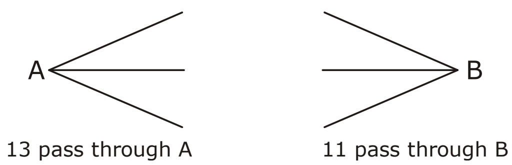
278. C

$\because |z - 1| + |z + 3| \leq 8$

$\because$ z lies inside or on the ellipse whose foci are (1, 0) and (−3, 0) and vertices are (−5, 0) and (3, 0) Now minimum and maximum value of $|z - 4|$ are 1 and 9 respectively $\therefore |z - 4| \in [1, 9]$



279. A



$\therefore$ Number of intersection points

$= {}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2$

($\because$ two points A and B) = 535

280. D

Terminal digits are the first and last digits.

$\because$ Terminal digits are even

$\therefore$ Ist place can be filled in 3 ways and last place can be filled in 2 ways and remaining places can be filled in $^5P_4 = 120$ ways. Hence, the number of six digit numbers, the terminal digits are even, is $= 3 \times 120 \times 2 = 720$

281. B

$^nC_4 = 70$

$n(n-1)(n-2)(n-3) = 1680 \Rightarrow n = 8$

Diagonals $= {}^nC_2 - n$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2} = \frac{8 \times 5}{2} = 20$$

282. C

$(bc + ca + ab)^{10}$

$$\text{General term} = \frac{10!}{p!q!r!}(bc)^p(ca)^q(ab)^r$$

$$= \frac{10!}{p!q!r!} a^{q+r}.\ b^{r+p}.\ c^{p+q}$$

Let $q + r = 10$, $r + p = 7$, $p + q = 3$

$\therefore p + q + r = 10 \quad \therefore p = 0, q = 3, r = 7$

$\therefore$ Coefficient of $a^{10}\ b^7\ c^3$ is

$$\frac{10!}{0!3!7!} = \frac{10\cdot 9\cdot 8\cdot 7!}{1\cdot 1\cdot 2\cdot 3\cdot 7!} = 120$$

283. C

Put $x = 1, \omega, \omega^2$ & add them

$3(a_0 + a_3 + \ldots) = 3^n \Rightarrow a_0 + a_3 + a_6 \ldots = 3^{n-1}$

284. A

$$\Sigma\, r.\ \frac{n-r+1}{r} = \Sigma\,(n+1) - r$$

$$= (n+1)\,\Sigma 1 - \Sigma r = (n+1).\ 10 - \frac{10.11}{2}$$

$$= 10n + 10 - 55 = 10n - 45 = 5(2n-9)$$

285. C

$$P(\overline{A}/\overline{B}) = \frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})} = \frac{P(\overline{A\cup B})}{P(\overline{B})} = \frac{1-P(A\cup B)}{P(\overline{B})}$$

286. D

The total number of ways of choosing two numbers out of 1, 2, 3,...,30 is $^{30}C_2 = 435$

Since, $x^2 - y^2$ is divisible by 3 iff either a and b are divisible by 3 or none of a and b divisible by 3. Thus, the favourable number of cases $^{10}C_2 + {}^{20}C_2 = 235$

$$\therefore \text{Required probability} = \frac{235}{435} = \frac{47}{87}$$

287. C

Total ways $= 6 \times 6 \times 6 = 216$

for increasing function $f'(x) \geq 0$

$\Rightarrow 3x^2 + 2ax + b \geq 0$

$\Rightarrow D \leq 0 \Rightarrow 4(a^2 - 3b) \leq 0 \Rightarrow a^2 \leq 3b$

favourable

a	b
1	1, 2, 3, 4, 5, 6
2	2, 3, 4, 5, 6
3	3, 4, 5, 6
4	6
5	–
6	–

Total favourable for (a, b) = 16

c can be any out of 1, 2, 3, 4, 5, 6

$$\therefore P = \frac{16 \times 6}{6 \times 6 \times 6} = \frac{4}{9}$$

288. B

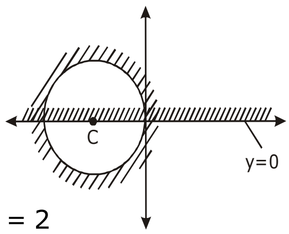
$|z|^2 + 7(\bar{z}) = 0 \Rightarrow x^2 + y^2 + 7(x - iy) = 0 + i0$

$\Rightarrow (x^2 + y^2 + 7x) - i(7y) = 0 + i0$

$\Rightarrow x^2 + y^2 + 7x = 0$ and $-7y = 0$

$$\text{centre} = \left(\frac{-7}{2}, 0\right)$$

$$\text{radius} = \frac{7}{2}$$



Circle and x-axis cut at 2 points

$\therefore$ no. of solution = 2

289. D

$(1+i)^6 + (1-i)^6$

$= 2\,[^6C_0\,(i)^0 + {}^6C_2\,(i)^2 + {}^6C_4\,(i)^4\ {}^6C_4\,(i)^6]$

$= 2\,[1 + (-15) + 15 - 1] = 0$

290. C

Let $x = (1)^{1/n} \Rightarrow x^n - 1 = 0$

or $x^n - 1 = (x-1)(x-\omega)(x-\omega^2)\ldots(x-\omega^{n-1})$

$$\Rightarrow \frac{x^n - 1}{x-1} = (x-\omega)(x-\omega^2)\ldots\ldots(x-\omega^{n-1})$$

Putting x = 9 in both sides, we have

$$(9-\omega)(9-\omega^2)(9-\omega^3)\ldots(9-\omega^{n-1}) = \frac{9^n-1}{8}$$

291. A

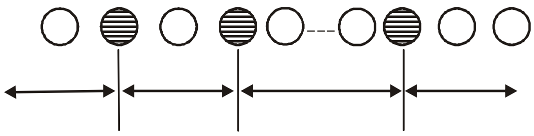
Given x + y + z + u + t = 20

x + y + z = 5

x + y + z = 5 and u + t = 15

Required number = ${}^{7}C_5 \times {}^{16}C_{15} = 336$

292. C



$\therefore x + y + z + u = n - 3;\ y \ge 1$

$z \ge 1$

$x \ge 0$

$u \ge 0$

$\therefore x + y + z + u = n - 1 \qquad \therefore n = {}^{n-2}C_3$

293. C

Series is 1 + 2 + 6 + 12 + 36 + 72 +...20 term

(1+6+36+...10 terms) + (2 + 12 + 72...10 terms)

$$= 3\left(\frac{6^{10}-1}{6-1}\right) = \frac{3}{5}\left(6^{10}-1\right)$$

294. B

$$P = \frac{{}^{100}C_0 + {}^{100}C_1 + {}^{100}C_5 + \ldots + {}^{100}C_{49}}{100}$$

$$P = \frac{s}{(2)^{100}}$$

where $s = {}^{100}C_1 + {}^{100}C_3 + \ldots\ldots\ldots + {}^{100}C_{49}$

But $({}^{100}C_1 + {}^{100}C_3 + \ldots + {}^{100}C_{49}) + ({}^{100}C_{51} + {}^{100}C_{53} + \ldots + {}^{100}C_{99}) = \frac{2^{100}}{2} = 2^{99}$

$s + s = 2^{99}$

$$s = 2^{98} \Rightarrow P = \frac{2^{98}}{2^{100}} = \frac{1}{4}$$

295. A

1^{st}	2^{nd}	3^{ed}	4^{th}	5^{th}	6^{th}	∞
$\bar{}$	1					
$\bar{}$	$\bar{1}$	$\bar{1}$	1			
$\bar{}$	$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{1}$	1	

....

....

....

∞

$$P = \left(\frac{5}{6}\cdot\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^5\cdot\frac{1}{6} + \ldots\infty$$

$$= \frac{\frac{5}{36}}{1-\frac{25}{36}} = \frac{5}{11}$$

296. B

reflexive and symmetric

297. C

$$\text{Mean Height} = \frac{144+153+150+158+155}{5}$$

$$= \frac{760}{5} = 152 \text{ cm.}$$

298. B

The given frequency distribution is-

x_i	f_i	$\Sigma f_i x_i$
4	7	28
7	10	70
10	15	150
13	20	260
16	25	400
19	30	570
$\Sigma f_i =$ 107		$\Sigma f_i x_i$ = 1478

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1478}{107} = 13.81$$

299. C

$\sim(\sim p \vee q) = \sim(\sim p) \wedge \sim q = p \wedge \sim q$

300. C

p : we control population growth

q : we prosper

So, negative of $(p \to q)$ is $\sim(p \to q) \equiv p \wedge \sim q$

301. C

Since n(A) = 3

$\therefore$ number of subsets of A is $2^3 = 8$

302. C

Here the numbers are 1, 2, 3,.., n and their weights also are respectively 1, 2, 3.....n so

$$\text{weighted mean} = \frac{\sum wx}{\sum w}$$

$$= \frac{1.1+2.2+3.3+\ldots\ldots+n.n}{1+2+3+\ldots..+n}$$

$$= \frac{1^2 + 2^2 + 3^2 + ...n^2}{1 + 2 + 3 + ... + n}$$

$$= \frac{n(n+1)(2n+1)}{6} \times \frac{2}{n(n+1)} = \frac{2n+1}{3}$$

303. C

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \because \bar{x}_1 = 400, \bar{x}_2 = 480, \bar{x} = 430$$

$$\therefore \quad 430 = \frac{n_1(400) + n_2(480)}{n_1 + n_2}$$

$$\Rightarrow 30n_1 = 50n_2 \Rightarrow \frac{n_1}{n_2} = \frac{5}{3}$$

304. B

Because $(p \Rightarrow q) \equiv (\sim p \vee q)$

305. A

p	~p	q	$(\sim p \vee q)$	$p \Rightarrow (\sim p \vee q)$
T	F	F	F	F

306. D

We have $P(\phi) = \{\phi\} \therefore P(P(\phi)) = \{\phi, \{\phi\}\}$

$\Rightarrow P[P(P(\phi))] = \{ \phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$.

Hence, $n\{P[P(\phi)]\} = 4$

307. B

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum x_i}{n} \quad \therefore \sum x_i = n\bar{x}$$

$$\text{New mean} = \frac{\sum(x_i + \lambda)}{n} = \frac{\sum x_i + n\lambda}{n} = \bar{x} + \lambda$$

308. B

Mean $\bar{x} = \frac{\sum x}{n}$ or $\sum x = n\bar{x}$

$\sum x = 25 \times 78.4 = 1960$ But this $\sum x$ is incorrect as 96 was misread as 69.

$\therefore$ correct $\sum x = 1960 + (96 - 69) = 1987$

$$\therefore \text{correct mean} = \frac{1987}{25} = 79.48$$

309. C

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge (q \wedge r)$
T	T	T	T	T	T

310. A

p	q	r	~p	~r	$(\sim p \vee q)$	$(\sim p \vee q) \wedge \sim r$	$(\sim p \vee q) \wedge \sim r \Rightarrow p$
T	F	T	F	F	F	F	T

311. C

$A \cup B = \{x : x \text{ is an odd integer}\}$
$\cup \{x : x \text{ is an even integer}\}$
$= \{x : x \text{ is an integer}\} = Z$

312. A

We have $\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n}$ Let $\bar{x}'$ be the mean of $x_1 + a$, $x_2 + a$,...., $x_n + a$ then

$$\bar{x}' = \frac{(x_1 + a) + (x_2 + a) + ... + (x_n + c)}{n}$$

$$= \frac{(x_1 + x_2 + ... + x_n)....+ na}{n}$$

$$= \frac{x_1 + x_2 + ... + x_n}{n} + a = \bar{x} + a$$

313. C

Let the assumed mean be, A = 900.The given data can be written as under :

Wage (in Rs.) x_i	**No. of workers f_i**	**$d_i = x_i - A = x_i - 900$**	**$u_i = \frac{x_i - 900}{20}$**	**$f_i u_i$**
800	7	– 100	- 5	– 35
820	14	– 80	- 4	– 56
860	19	– 40	- 2	– 38
900	25	0	0	0
920	20	20	1	20
980	10	80	4	40
1000	5	100	5	25

N $= \sum f_i = 100$ $\qquad \sum f_i u_i = -44$

Here A = 900, h = 20

$$\therefore \text{Mean} = \bar{X} = A + h\left(\frac{1}{N}\sum f_i u_i\right)$$

$= 900 + 20\left(-\frac{44}{100}\right) = 891.2$

Hence, mean wage = Rs. 891.2.

314. A

use the property~(p and q)=~p or ~q

3 is not an odd number or 7 is not a rational number.

315. B

Use the property $\sim(a \wedge b) = \sim a \vee \sim b$

$(p \wedge \sim q) \vee (p \vee q)$

316. D

We have, $x \in A \cap B \Leftrightarrow x = 3n, n \in Z$ and $x = 4n, n \in Z$

$\Leftrightarrow$ x is a multiple of 3 and x is a multiple of 4

$\Leftrightarrow$ x is a multiple of 3 and 4 both

$\Leftrightarrow$ x is a multiple of 12 $\Leftrightarrow x = 12n, n \in Z$

Hence $A \cap B = \{x : x = 12n, n \in Z\}$

317. C

Geometric mean of number

$7, 7^2, 7^3, \ldots, 7^n = (7.7^2.7^3.\ldots\ldots 7^n)^{1/n}$

$= (7^{1+2+3+\ldots\ldots+n})^{1/n}$

$$= \left[7^{\frac{n(n+1)}{2}}\right]^{1/n} = 7^{\frac{(n+1)}{2}}$$

318. B

The harmonic mean of 2, 4 and 5 is

$$\frac{3}{\frac{1}{2}+\frac{1}{4}+\frac{1}{5}} = \frac{60}{19} = 3.16$$

319. C

use the property $\sim(a \vee b) = \sim a \wedge \sim b$

$(\sim p \vee \sim q) \wedge (\sim q \wedge r)$

320. A

p	q	~q	$p \wedge \sim q$	$(p \wedge \sim q) \vee p$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	F
F	F	T	F	F

Hence statement $(p \wedge \sim q) \vee p$ is logically equal to statement $p \Rightarrow p$

321. B

We have, $n(A \cup B)=n(A)+n(B) - n(A \cap B)$

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

Case-I: When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$ This is possible only when $A \cap B = \phi$. In this case, $n(A \cup B)$

$= n(A)+n(B) - 0 = n(A) + n(B) = 3 + 6=9.$

So, maximum number of elements in $A \cup B$ is 9

Case-II: When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$.

In this case, $n(A \cap B) = 3$

$\therefore n(A \cup B)=n(A)+n(B)-n(A \cap B)=(3 + 6 - 3)=6$

So, minimum number of elements in $A \cup B$ is 6

322. B

Let us arrange the value in ascending order 0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

$\therefore$ Median M $= \left(\frac{n+1}{2}\right)^{th}$ value

$= \left(\frac{11+1}{2}\right)^{th}$ value $= 6^{th}$ value

Now 6^{th} value in data is 27

$\therefore$ Median = 27 runs.

323. B

x	f	c.f.
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120

$N = 120 = \sum f_i$

$\therefore \frac{N}{2} = 60$ We find that the c.f. just greater than $\frac{N}{2}$ is 65 and the value of x corresponding to 65 is 5, therefore median is 5.

324. C

Let p be true then statement $(p \wedge \sim q) \vee (q \wedge r) = (T \wedge F) \vee (T \wedge T) = F \vee T = T$. also let p be false then statement $(p \wedge \sim q) \vee (q \wedge r) = (F \wedge F) \vee (T \wedge T) = F \vee T = T$. $\therefore$ p may be true or false.

325. B

use the property $\sim (p \rightarrow q) = p \wedge \sim q$ Hence (B) is correct option.

326. C

Since $n(A)=m$; $n(B) = n$ then $n(A \times B) = mn$

So number of subsets of $A \times B = 2^{mn}$

$\Rightarrow n(P(A \times B)) = 2^{mn}$

327. D

Class	Frequency	Cumulative frequency
5 - 10	5	5
10 - 15	6	11
15 - 20	15	26
20 - 25	10	36
25 - 30	5	41
30 - 35	4	45
35 - 40	2	47
40 - 45	2	49
	N = 49	

We have N = 49. $\therefore \frac{N}{2} = \frac{49}{2} = 24.5$

The cumulative frequency just greater than N/2, is 26 and the corresponding class is 15-20. Thus 15-20 is the median class such that $\ell=15$, $f=15$, $F=11$, $h=5$.

$\therefore$ Median $= \ell + \frac{N/2 - F}{f} \times h$

$= 15 + \frac{24.5 - 11}{15} \times 5 = 15 + \frac{13.5}{3} = 19.5$

328. C

Since 5 is repeated maximum number of times, therefore mode of the given data is 5.

329. A

$F \rightarrow (T \wedge F) \vee (F \wedge T)$ $F \rightarrow (F \vee F)$

$F \rightarrow F = T$ $\therefore$ True

330. C

use the property $\sim (a \rightarrow b) = a \wedge \sim b$. Hence (C) is correct option

331. C

We have $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

On replacing C by B and D by A, we get

$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

It is given that AB has n elements so

$(A \cap B) \times (B \cap A)$ has n^2 elements

But $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

$\therefore$ $(A \times B) \cap (B \times A)$ has n^2 elements

Hence $A \times B$ and $B \times A$ have n^2 elements in common.

332. B

Mode = 3 Median − 2 mean

$\therefore$ Median $= \frac{1}{3}$(mode + 2 mean)

$= \frac{1}{3}(60 + 2 \times 66) = 64$

333. C

Arranging the observations in ascending order of magnitude, we have 150, 210, 240, 300, 310, 320, 340. Clearly, the middle observation is 300. So, median = 300 Calculation of Mean deviation

x_i	$\lvert d_i \rvert = \lvert x_i - 300 \rvert$
340	40
150	150
210	90
240	60
300	0
310	10
320	20
Total	$\sum \lvert d_i \rvert = \sum \lvert x_i - 300 \rvert$ = **370**

Mean deviation $= \frac{1}{n} \sum |d_i| = \frac{1}{7} \sum |x_i - 300|$

$= \frac{370}{7} = 52.8$

334. A

$p \vee (\sim p) \equiv t$

So (1) is incorrect.

335. A

~ (p → q) = p ∧ ~q, p → q = ~p ∨ q. Hence (A) is correct option

336. C

We have, x + 3y = 12 ⇒ x = 12 – 3y

Putting y = 1, 2, 3, we get x = 9, 6, 3 respectively For y = 4, we get x = 0 ∉ N.

Also for y > 4, x ∉ N

∴ R = {(9, 1), (6, 2), (3, 3)}

Domain of R = {9, 6, 3}

337. B

Calculation of mean deviation about mean.

x_i	f_i	$f_i x_i$	$\lvert x_i - 15 \rvert$	$f_i \lvert x_i - 15 \rvert$
3	8	24	12	96
9	10	90	6	60
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60

$N = \sum f_i = 44$ $\sum f_i x_i = 660$ $\sum f_i \lvert x_i - 15 \rvert = 312$

Mean = $\overline{X} = \frac{1}{N}(\sum f_i x_i) = \frac{660}{44} = 15$

Mean deviation = M.D. = $\frac{1}{N}\sum f_i \lvert x_i - 15 \rvert$

$= \frac{312}{44} = 7.09.$

338. B

$\overline{x} = \frac{8+12+13+15+22}{5} = 14$

CALCULATION OF VARIANCE

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
8	– 6	36
12	– 2	4
13	– 1	1
15	1	1
22	8	64
		$\sum(x_i - \overline{x})^2 = 106$

∵ $n = 5, \ \sum(x_i - \overline{x})^2 = 106$

$\therefore \ \text{var}(x) = \frac{1}{n}\sum(x_i - \overline{x})^2 = \frac{106}{5} = 21.2$

339. C

We know that ~ (p ⇔ q)

= (p ∧ ~q) ∨ (~p ∧ q) (p ⇔ q)

= (~p ∨ q) ∧ (p ∨ ~q)

Hence (C) is correct option

340. C

p	q	p ∧ q	~p	(p ∧ q) ⇔ ~p
T	T	T	F	F
T	F	F	F	T
F	T	F	T	F
F	F	F	T	F

Hence the statement neither tautology nor contradiction.

341. B

We have (x, y) ∈ R ⇔ x + 2y = 10

⇔ y = , x, y ∈ A where A

= {1,2,3,4,5,6,7,8,9,10}

Now, x = 1 ⇒ y = ∉ A. This shows that 1 is not related to any element in A. Similarly we can observe that 3, 5, 7, 9 and 10 are not related to any element of a under the defined relation. Further we find that

for x = 2, y = = 4 ∈ A ∴ (2, 4) ∈ R

for x = 4, y = = 3 ∈ A ∴ (4, 3) ∈ R

for x = 6, y = = 2 ∈ A ∴ (6, 2) ∈ R

for x = 8, y = = 1 ∈ A ∴ (8, 1) ∈ R

Thus R = {(2, 4), (4, 3), (6, 2), (8, 1)}

⇒ R^{-1} = {(4, 2), (3, 4), (2, 6), (1, 8)}

Clearly, Dom (R)={2, 4, 6, 8}=Range (R^{-1})

and Range (R) = {4, 3, 2, 1} = Dom (R^{-1})

342. C

Let the assumed mean be A = 6.5

Calculation of variance

size of item x_i	f_i	$d_i = x_i - 6.5$	d_i^2	$f_i d_i$	$f_i d_i^2$
3.5	3	–3	9	–9	27
4.5	7	–2	4	–14	28
5.5	22	–1	1	–22	22
6.5	60	0	0	0	0
7.5	85	1	1	85	85
8.5	32	2	4	64	128
9.5	8	3	9	24	72

$N = \sum f_i = 217$ $\sum f_i d_i = 128$ $\sum f_i d_i^2 = 362$

Here, $N = 217$, $\sum f_i d_i = 128$ and $\sum f_i d_i^2 = 362$

$$\therefore \text{Var}(X) = \left(\frac{1}{N}\sum f_i d_i^2\right) - \left(\frac{1}{N}\sum f_i d_i\right)^2$$

$$= \frac{362}{217} - \left(\frac{128}{217}\right)^2 = 1.668 - 0.347 = 1.321$$

343. D

Here $np = 6$, $npq = 4$

$\Rightarrow q = \frac{2}{3}$, $p = 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore np = 6 \Rightarrow n = 18$

344. A

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Hence the statement is a tautology

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345. B

p	q	$\sim q$	$p \to \sim q$	$p \wedge q$	$(p \to \sim q) \Leftrightarrow p \wedge q$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

Hence the statement is a contradiction

346. D

Given equation $x^2 - bx + c = 0$

Let α, β, be two root's such that $|\alpha - \beta| = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \Rightarrow b^2 - 4c = 1$

Statement-II given equation abc

$x^2 + (b^2 - 4ac)x - b = 0$

$D = (b^2 - 4ac)^2 + 16b^2ac$

$D = (b^2 + 4ac)^2 > 0$

Hence root's are real and unequal.

347. A

$x^2 + 9y^2 + 25z^2 - 15yz - 5xz - 3xy = 0$

$\Rightarrow 2x^2 + 18y^2 + 50z^2 - 30yz - 10xz - 6xy = 0$

$\Rightarrow (x - 3y)^2 + (3y - 5z)^2 + (5z - x)^2 = 0$

$\therefore x = 3y = 5z = k$ (say)

$\Rightarrow x = k,\ y = \frac{k}{3},\ z = \frac{k}{5} \Rightarrow k, \frac{k}{3}, \frac{k}{5}$ are in H.P.

$\Rightarrow x, y, z$ are in H.P.

348. A

Exponent of 5

$$= \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right] = 20 + 4 = 24$$

Both correct & correct explanation

349. C

$$\Sigma\, {}^nC_r x^r = (1 + x)^n \Rightarrow \Sigma\, {}^nC_r \frac{x^{r+1}}{r+1} = \frac{(1+x)^{n+1} - 1}{(n+1)}$$

(I) is true & (II) is false

350. B

The equation can be written as

$(2^x)^2 - (a - 3)2^x + (a - 4) = 0$

$\Rightarrow 2^x = 1$ and $2^x = a - 4$

We have,

$x \le 0$ and $2^x = a - 4$ [$\because$ x is non-positive]

$\therefore 0 < a - 4 \le 1 \Rightarrow 4 < a \le 5 \therefore a \in (4, 5]$

351. A

$\arg(z_1 z_2) = 2\pi \Rightarrow \arg(z_1) + \arg(z_2) = 2\pi$

$\Rightarrow \arg(z_1) = \arg(z_2) = \pi$, as principal arguments are from $-\pi$ to π.

Hence both the complex numbers are purely real. Hence both the statements are true and statement 2 is correct explanation of statement 1.

352. C

$$|z_1 + z_2| = \left|\frac{z_1 + z_2}{z_1 z_2}\right|$$

$$\Rightarrow |z_1 + z_2|\left(1 - \frac{1}{|z_1 z_2|}\right) = 0 \Rightarrow |z_1 z_2| = 1$$

Hence, statement 1 is unimodular.

However, it is not necessary that

$|z_1| = |z_2| = 1$. Hence, statement 2 is false.

353. D

Sum $= \frac{x/r}{1-r} = 4$ (where r is common ratio)

$x = 4r(1 - r) = 4(r - r^2)$

For $r \in (-1, 1) - \{0\}$

$r - r^2 \in \left(-2, \frac{1}{4}\right) - \{0\} \Rightarrow x$ d $(-8, 1) - \{0\}$

354. A

Statement 2 is ture as

$$\frac{a^n + b^n}{a+b} = \frac{a^n - (-b)^n}{a - (-b)}$$

$= a^{n-1} - a^{n-2}b + a^{n-3}b^2 - .....(-1)^{n-1} b^{n-1}$

Now,

$1^{99} + 2^{99} + ... + 100^{99} = (1^{99} + 100^{99}) + (2^{99} + 99^{99}) + ... + (50^{99} + 51^{99})$

Each bracket is divisible by 101 ; hence the sum is divisible by 101. Also,

$1^{99} + 2^{99} +$ L ...... $100^{99} = (1^{99} + 100^{99}) + (2^{99} + 98^{99}) + ... + (49^{99} + 51^{99}) + 50^{99} + 100^{99}$

Here, each bracket and 50^{99} and 100^{99} are divisible by $101 \times 100 = 10100$.

355. D

Number of ways of arranging 21 identical objects when r is identical of one type and remaining are identical of second type is

$\frac{21!}{r!(21-r)!} = {}^{21}C_r$ which is maximum when

r=10 or 11. Therefore, ${}^{13}C_r = {}^{13}C_{10}$ or ${}^{13}C_{11}$, hence maximum value of ${}^{13}C_r$ is ${}^{13}C_{10} = 286$. Hence, statement 1 is false, Obviously statement 2 is true.

356. C

Number of objects from 21 different objects	Number of objects from 21 identical objects	Number of ways of selections
10	0	${}^{21}C_{10} \times 1$
9	1	${}^{21}C_9 \times 1$
⋮	⋮	⋮
0	10	${}^{21}C_0 \times 1$

Thus, total number of ways of selection is ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10} = 2^{20}$. Statement 2 is false, as gien series is not exact half series. (for details, see the theory in binomial theorem.)

357. B

$$\frac{T_{r+1}}{T_r} = \frac{12 - r + 1}{r} \; \frac{11}{10}$$

Let, $T_{r+1} \geq T_r \Rightarrow 13 - r \geq 1.1x$

$\Rightarrow 13 \geq 2.1r \quad \Rightarrow r \leq 6.19$

358. A

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0 \quad (i)$$

Then A = B or B = C or C = A, for which any two rows are same. For (1) to hold it is not necessary that all the three rows are same or A = B = C.

359. A

Statement 1 is true as $|A| = 0$.

Since $|B| \neq 0$, statement 2 is also true and correct explanation of statement 1.

360. C

S : 1 required no of groups

(1,2,3,4) ....(17,18,19,20) = 17 ways

(1,3,5,7)..........(14,16,18,20) = 14 ways

(1,4,7,10). ....(11,14,17,20) = 11 ways

(1,5,9,13). ....(8,12,16,20) = 8 ways

(1,6,11,16). ....(15,10,15,20) = 5 ways

(1,7,13,19). ....(2,8,14,20) = 2 ways

required arability

$$= \frac{(17 + 14 + 11 + 8 + 5 + 2)4!}{{}^{20}C_4 4!}$$

$$= \frac{57\ 4!}{20.19.18.17} = \frac{3.4.3.2.1}{20.18.17.}$$

$$= \frac{1}{85}$$

S : 1 is true.

S : 2

possible cases of common difference are [±1, +2, ±3, ±4, ±5, ±6]

S : 2 is false