

Magnetic Effect of Current

- **Hans Christian Oersted** observed that, when a compass is placed near a straight wire carrying a current, the compass needle aligns so that it is tangent to a circle drawn around the wire (neglecting the influence of the Earth's magnetic field on the compass).
- Oersted's discovery provided the first link between electricity and magnetism.
- The space or region around the current carrying conductor within which its influence can be felt by the magnetic needle is called the **magnetic field** of the current carrying conductor in general way.

However, the definition of $\vec{B}$ can be given as the number of lines of magnetic field per unit area or the force acting on unit charge which is moving with unit velocity perpendicular to the magnetic field.

- S.I. unit of magnetic field is Wm^{-2} or T (tesla).
- The strength of magnetic field is called one tesla, if a charge of one coulomb, when moving with a velocity of 1 ms^{-1} along a direction perpendicular to the direction of the magnetic field experiences a force of one newton.

$$1 \text{ tesla (T)} = 1 \text{ weber metre}^{-2} (\text{Wb m}^{-2}) \\ = 1 \text{ newton ampere}^{-1} \text{ metre}^{-1} (\text{N A}^{-1}\text{m}^{-1})$$

- C.G.S. units of magnetic field are called gauss or oersted.
 $1 \text{ gauss} = 10^{-4} \text{ tesla}.$
- **Right hand thumb rule** : If the linear conductor is grasped in the palm of the right hand with thumb pointing along the direction of the current, then the curl fingers will point in the direction of lines of force.

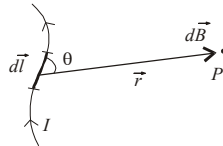


- **Maxwell's cork screw rule** : If a right handed cork screw is rotated so that its tip moves in the direction of flow of current through the conductor, then the rotation of the head of the screw gives the direction of magnetic lines of force.
- The conventional sign for a magnetic field coming out of the plane and normal to it is a dot i.e. $\odot$. The magnetic field perpendicular to the plane in the down ward direction is denoted by $\otimes$.

- **Biot savart's Law:** According to this law, the magnetic field (in magnitude) due to a current element of length dl carrying a current I at a point at distance r from it, is given by

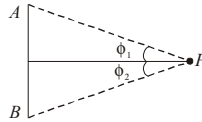
$$|d\vec{B}| = \frac{\mu_0 I}{4\pi} \cdot \frac{|\vec{dl} \times \hat{r}|}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I dl \sin\theta}{r^2}$$

where θ is the angle between the direction of the current and the line joining the current element to the point and μ_0 is absolute permeability of the free space ($\mu_0 = 4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$).



The direction of magnetic field $d\vec{B}$ is that of $I \vec{dl} \times \vec{r}$.

- **Magnetic field due to a current carrying conductor:**



The magnetic field at a point at perpendicular distance a from a straight conductor carrying current I is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \phi_1 + \sin \phi_2),$$

where ϕ_1 and ϕ_2 are angles, which the lines joining the two ends of the conductor to the observation point make with the perpendicular from the observation point to the conductor.

In case, the straight conductor is of infinite length ($\phi_1 \cong \phi_2 \cong \frac{\pi}{2}$), the magnetic field is

given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

- **Magnetic field due to a current carrying circular coil:** For a coil of radius a , consisting of N turns and carrying current I ,

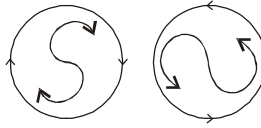
(i) the magnetic field at a point on axis at distance d from its centre is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N I a^2}{(a^2 + d^2)^{3/2}}$$

(ii) the magnetic field at its centre is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N I}{a}$$

- The current carrying loop behaves as a small magnetic dipole placed along the axis one face of the loop behaves as north pole while the other face of loop behaves as south pole.

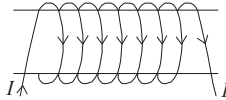


- The face in which the current is flowing in clockwise direction behaves as south pole while the face through which the current is flowing in anticlockwise direction behaves as north pole.
- Ampere's circuital law** states that the line integral of the magnetic field around any closed path in free space is equal to absolute permeability (μ_0) times the net current enclosed by the path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

where $\vec{B}$ is the magnetic field $d\vec{l}$ is small element, μ_0 is the absolute permeability of free space and I is the current.

- Ampere's circuital law holds good for a closed path of any size and shape around a current carrying conductor because the relation is independent of distance from conductor.
- A straight solenoid consists of hollow tube over which a large number of turns of insulated copper wire are uniformly wound.



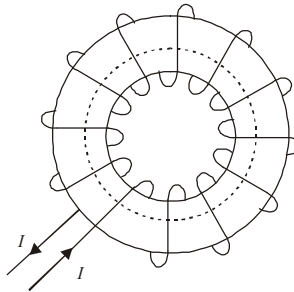
- If a solenoid of n turns per unit length carries a current I , then
 - magnetic field at a point well inside the solenoid is given by

$$B = \mu_0 n I$$

- magnetic field at a point on one end of the solenoid is given by

$$B = \frac{1}{2} \mu_0 n I$$

- A toroidal solenoid is an anchor ring around which is large number of turns of a copper wire are wrapped.



- The magnetic field produced in toroid will be same at all points on the circumference of the circle and at any point it will act along the tangent to the ring

$$B = \frac{\mu_0 NI}{2\pi r}$$

- For any point inside the empty space surrounded by the toroid and outside the toroid, magnetic field is zero because the net current enclosed in these space is zero.
- Magnetic field intensity at a point outside the cylinder ($r > a$)

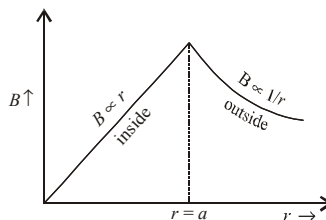
$$B = \frac{\mu_0 I}{2\pi r}$$

- The magnetic field intensity at a point outside the cylinder varies inversely as the distance of the point from the axis of the cylinder.
- Magnetic field intensity on the surface of the cylinder ($r = a$)

$$B = \frac{\mu_0 I}{2\pi a}$$

- Magnetic field intensity at a point inside the cylinder. ($r < a$)

$$B = \frac{\mu_0 I r}{2\pi a^2}$$



- Force on a charge in electric field:** A charge q inside an electric field of strength $\vec{E}$ experiences force $\vec{F}$, which is given by

$$|\vec{F}| = |q\vec{E}| = qE$$

- Motion of a charge inside electric field:** If a potential difference V is applied between two parallel plates, a uniform electric field is set up between the plates. Its strength is given by $E = V/d$

A charge q of mass m experiences force $F = qE$,

which produces an acceleration, $a = \frac{qE}{m}$

The charge entering an electric field into perpendicular direction or at an angle follows a parabolic path.

- Fleming's Left hand rule :** It states that if the forefinger, central finger and thumb are stretched at right angles to each other then, central finger represents the direction of current, fore finger represents field and thumb represents force.
- Lorentz force:** The total force experienced by a charge moving inside the electric and magnetic fields is called Lorentz force. It is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- **Motion of charge inside magnetic field:** A charge q of mass m moving with velocity $\vec{v}$ inside a magnetic field of strength $\vec{B}$ experiences force,

$$|\vec{F}| = q|\vec{v} \times \vec{B}| = Bqv\sin\theta$$

where θ is angle between the direction of motion of charge ($\vec{v}$) and the direction of magnetic field ($\vec{B}$). This force acts perpendicular to both $\vec{v}$ and $\vec{B}$ i.e. the direction of motion of the charge and the direction of applied magnetic field.

- The charge does not experience any force, if it is at rest or if it moves along the direction of magnetic field. The force is maximum, when charge moves perpendicular to the direction of magnetic field.

(i) If $\vec{v}$ and $\vec{B}$ are perpendicular ($\theta = 90^\circ$), the force on the charged particle makes it to move along circular path, whose radius is given by

$$r = \frac{mv}{Bq}$$

(ii) If $\vec{v}$ and $\vec{B}$ act at an angle θ , then due to the component of velocity $v \sin \theta$ (perpendicular to $\vec{B}$, the charge moves along circular path of radius r , which is given by)

$$r = \frac{mv\sin\theta}{Bq}$$

while due to the component of velocity $v\cos\theta$ (along $\vec{B}$), the charge at the same time moves along the direction of magnetic field. As a result, the charge moves along a **helical path** and pitch of the helical path is given by

$$\text{pitch} = \frac{2\pi mv\cos\theta}{Bq}$$

- **Cyclotron** is a particle accelerator and is used to accelerate positive ions. Under the action of magnetic field, the positive ions move along spiral path and gain energy as they cross the alternating electric field again and again.
- Cyclotron is based on the principle that the positive ions can be accelerated to high energies with a comparatively smaller alternating potential difference by making them to cross the electric field again and again, by making use of a strong magnetic field.
- In a cyclotron, the frequency of applied alternating electric field is equal to frequency of oscillation of the positive ion and this frequency is called **cyclotron frequency**. It is given by

$$\nu = \frac{Bq}{2\pi m}$$

where m and q are mass and charge of the positive ion and B is strength of the magnetic field.

- The positive ions of charge q and mass m in cyclotron attain maximum energy which is given by

(i) $E_{\max} = \frac{1}{2} \cdot \frac{B^2 q^2 R^2}{m}$ where R is radius of the dees of the cyclotron.

(ii) $E_{\max} = 2n (V q) .$

where n is number of revolutions completed by the positive ions before leaving the dees.

- **Limitations of the cyclotron**

- (i) Cyclotron can not accelerate uncharged particle like neutron.
- (ii) The positively charged particles having large mass i.e. ions cannot move at limitless speed in a cyclotron.

- **Force on a current carrying conductor placed inside a magnetic field:** A conductor of length l carrying current I and placed inside a magnetic field of strength $\vec{B}$ is given by

$$|\vec{F}| = I |\vec{l} \times \vec{B}| = BIl \sin\theta$$

- The conductor experiences maximum force, when the magnetic field acts at right angle to the length of the conductor; and the force is zero, when the length of the conductor is parallel to the direction of the magnetic field.
- **Force between two infinitely long parallel current carrying conductors.** When two infinitely long parallel conductors carrying currents I_1 and I_2 are placed a distance r apart, then force on the unit length of a conductor due to the other conductor is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r}$$

The force is attractive, if currents in two conductors is in same direction; and repulsive, if currents are in opposite directions.

- If the currents in both parallel wires are equal and in same direction then magnetic field at a point exactly half way between the wire is zero.
- **Torque on a current carrying coil placed inside a magnetic field:** When a coil of area A having N turns and carrying current I is suspended inside a magnetic field of strength B , then torque on the coil is given by

$$\tau = NBIA \sin\theta,$$

when θ is angle between the direction of magnetic field and normal to the plane of the coil.

- If the direction of magnetic field makes an angle α with the plane of the coil, then
- $\tau = NBIA \cos\alpha.$
- The torque on the coil is maximum, when the plane of the coil is parallel to the magnetic field i.e. $\theta = 90^\circ$ or $\alpha = 0^\circ$
- **Moving coil galvanometer** is a device used to detect or measure small electric current in the electric circuit.
- When a current carrying loop or coil is placed in the uniform magnetic field, it experience a torque. This basic principle is used in moving coil galvanometer.
- The deflection of the coil in moving coil galvanometer is directly proportional to current flowing through it

$$I \propto \theta$$

$$I = G \theta$$

where $G = \frac{k}{NAB}$ = galvanometer constant

k = The restoring torque per unit twist

N = Number of turns in the coil

A = Area of coil

B = Intensity of magnetic field

- **Sensitivity of a galvanometer.** A galvanometer is said to be sensitive, if it gives a large deflection, even when a small current is passed through it or when a small voltage is applied across its coil.
- **Current sensitivity.** It is defined as the deflection produced in the galvanometer on passing unit current through its coil.

$$\text{Current sensitivity, } \frac{\theta}{I} = \frac{NBA}{k} (\text{rad A}^{-1})$$

- **Voltage sensitivity.** It is defined as the deflection produced in the galvanometer, when a unit voltage is applied across its coil.

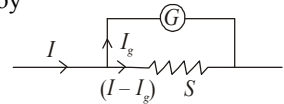
$$\text{Voltage sensitivity, } \frac{\theta}{V} = \frac{NBA}{kR} (\text{rad V}^{-1})$$

Here, R is resistance of the coil of the galvanometer.

- A small resistance usually put in parallel to the coil of a galvanometer is called **shunt**. The most part of the current in the circuit passes through the shunt and thus shunt allows only a very small part of current to pass through the galvanometer.
- **Ammeter** is an instrument used to measure current in an electrical circuit.

A galvanometer of resistance G can be converted into an ammeter of range I by putting a small resistance S in parallel to its coil, which is given by

$$S = \frac{I_g \times G}{I - I_g}$$



Here, I_g is maximum current that can pass through the galvanometer.

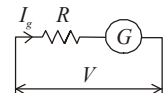
The resistance of the ammeter so obtained is given by $R_A = \frac{GS}{G + S}$

Ammeter is a low resistance instrument and it is always connected in series to the circuit.

- **Voltmeter.** It is an instrument used to measure potential difference across a conductor in an electrical circuit.

A galvanometer of resistance G can be converted into a voltmeter to read upto V by connecting a large resistance R in series to its coil, which is given by

$$R = \frac{V}{I_g} - G$$



The resistance of the voltmeter so obtained is given by

$$R_V = G + R$$

Voltmeter is a high resistance instrument and it is always connected in parallel to the conductor, across which potential difference is to be measured.