Chapter 10

Magnetic Effect of Current

LEVEL 1

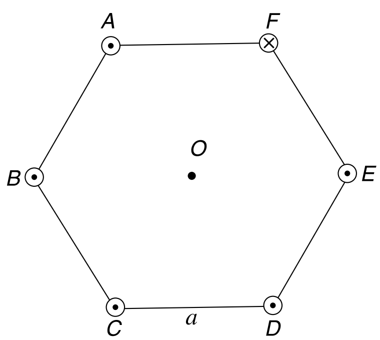
Q. 1: Sketch the magnetic field lines in xy plane for a pair of long parallel wires laid along z direction if-

(a) Both wires carry current in same direction.

(b) Both wire carry current in opposite directions.

Q. 2: A long wire is along $x = 0$, $z = d$ and carries current in positive y direction. Another wire is along $x = y$, $z = 0$ and carries current in direction making acute angle with positive x direction. Both the wires have current I. Find the magnitude of magnetic induction at $(0, 0, 2d)$.

Q. 3: Six long parallel current carrying wires are perpendicular to the plane of the fig. They pass through the vertices of a regular hexagon of side length a. All wires have same current I. Direction of current is out of the plane of the figure in all the wires except the one passing through vertex F; which has current directed into the plane of the figure. Calculate the magnetic induction field at the centre of the hexagon. Also tell the direction of the field.



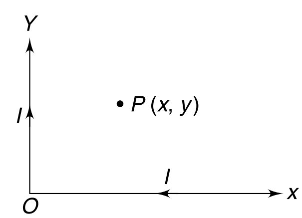
Q. 4: Two infinitely long parallel wires carry current $I_1 = 8$ A and $I_2 = 10$ A in opposite directions. The separation between the wires is $d = 0.12$ m. Find the magnitude of magnetic field at a point P that is at a perpendicular distance $r_1 = 0.16$ m and $r_2 = 0.20$ m respectively from the wires.

Q. 5: A wire frame is in the shape of a regular polygon of 2016 sides. Each side is of length $L = 1$ cm. If a current $I = 5.0$ A is given to the wire frame estimate the magnetic induction field (B) at the centre of the polygon.

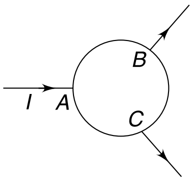
[Take $\pi^2 = 10.08$]

Q. 6: Two coplanar concentric circular wires made of same material have radius R_1 and $R_2 (= 2R_1)$. The wires carry current due to identical source of emf having no internal resistance. Find the ratio of radii of cross section of the two wires if the magnetic induction field at the centre of the circle is zero.

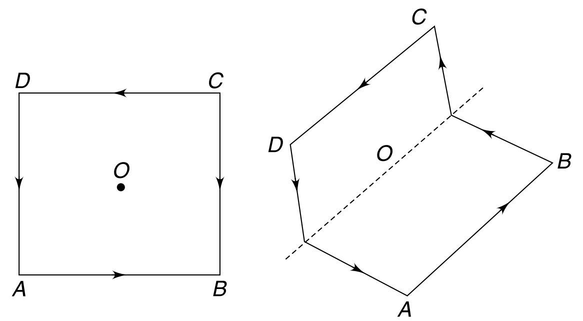
Q. 7: An infinitely long wire carrying current I is bent to from a L shaped wire. Let the bend be the origin and the two arms be along x and y direction (see figure). Calculate the magnitude of magnetic field at point P (in first quadrant) whose co-ordinates are (x, y).



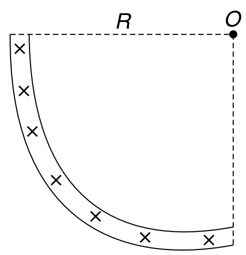
Q. 8: In the figure shown ABC is a circle of radius a. Arc AB and AC each have resistance R. Arc BC has resistance $2R$. A current I enters at point A and leaves the circle at B and C. All straight wires are radial. Calculate the magnetic field at the centre of the circle. Each arc AB, BC and AC subtends 120° at the centre of the circle.



Q. 9: A square loop of side length L carries a current which produces a magnetic field B_0 at the centre (O) of the loop. Now the square loop is folded into two parts with one half being perpendicular to the other (see fig). Calculate the magnitude of magnetic field at the centre O.

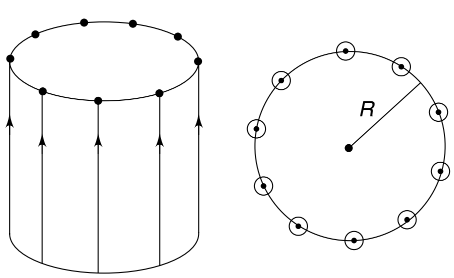


Q. 10: A current I flows in a long straight wire whose cross section is in the shape of a thin quarter ring of radius R. Find the induction of the magnetic field (B) at point O on the axis.

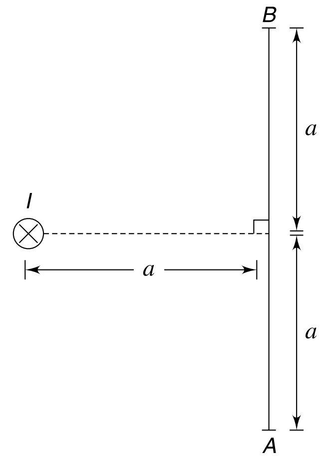


Q. 11: The figure shows a long cylinder and its cross section. There are N (N is a large number) wire on the curved surface of the cylinder at uniform spacing and parallel to its axis. Each wire has current I and cross sectional radius of wires are small compared to radius R of the cylinder. Find magnetic field at a distance x from the axis of the cylinder for

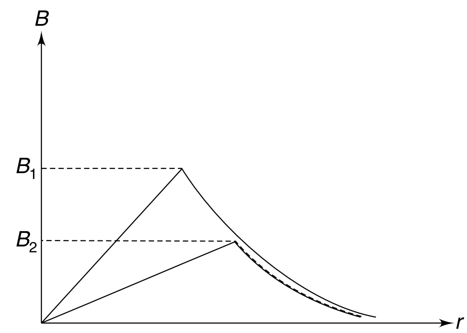
(a) $x < R$ (b) $x > R$



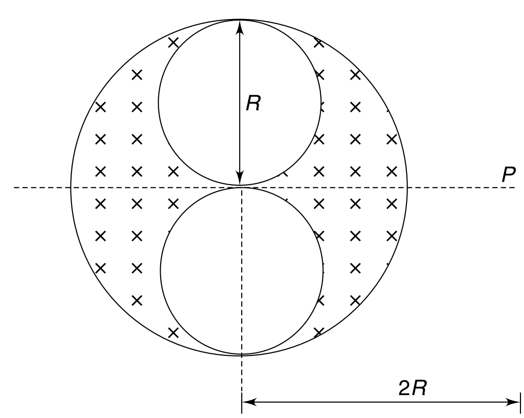
Q. 12: A straight current carrying wire has current I directed into the plane of the fig. There is a line AB of length $2a$ at a distance a from the wire (see fig.). Find the value of line integral $\int_A^B \vec{B} \cdot \vec{dl}$ where $\vec{B}$ represents magnetic field at a point due to current I. Will the value of integral change if a is changed? Length of line AB is always double that of a.



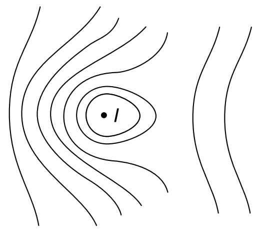
Q. 13: There are two separate long cylindrical wires having uniform current density. The radius of one of the wires is twice that of the other. The fig. shows the plot of magnitude of magnetic field intensity versus radial distance (r) from their axis. The curved parts of the two graphs are overlapping. Find the ratio $B_1 : B_2$.



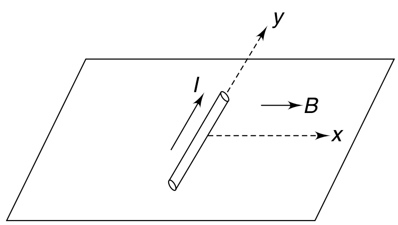
Q. 14: A long cylindrical conductor of radius R has two cylindrical cavities of diameter R through its entire length, as shown in the figure. There is a current I through the conductor distributed uniformly in its entire cross section (apart from the cavity region). Find magnetic field at point P at a distance $r = 2R$ from the axis of the conductor (see figure).



Q. 15: A uniform magnetic field exists in vertical direction in a region of space. A long current carrying wire (having current I) is placed horizontally in the region perpendicular to the figure. The resultant field due to superposition of the uniform field and that due to the current is represented by the field lines shown in the figure. In which direction does the current carrying wire experience the magnetic force?



Q. 16: A conducting wire of length ℓ and mass m is placed on a horizontal surface with its length along y direction. There exists a uniform magnetic field B along positive x direction. With wire carrying a current I in positive y direction, the least value of force required to move it in x and y directions are F_1 and F_2. Now the direction of current in the wire is reversed and the value of two forces becomes F_1' and F_2'. Find the ratio of forces $F_1 : F_2 : F_1' : F_2'$

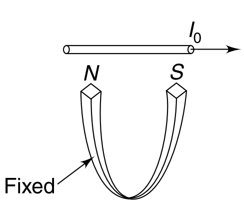


Q. 17: How will the conductor, carrying current I_0 rotate immediately after it is released in following three cases [consider magnetic force only]

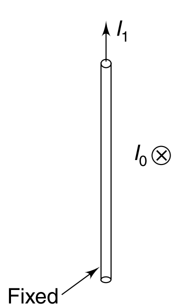
(a) Conductor carrying current I_0 is placed symmetrically above poles of a fixed U shaped magnet (figure a).

(b) Conductor carrying current I_0 is placed symmetrically at a distance from a fixed current (I_1) carrying wire (fig. (b))

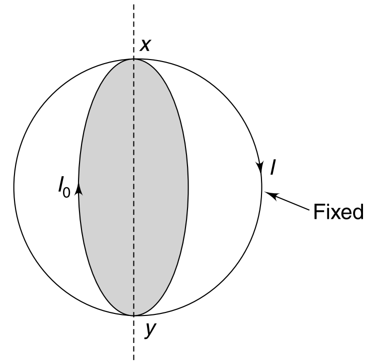
(c) An insulated circular current carrying wire is held fixed in vertical plane. Conductor carrying current I_0 is in the shape of a circle of diameter nearly equal to that of the fixed insulated circle. The planes of the two circles are perpendicular to each other (fig. (c)) with xy as common diameter.



(a)

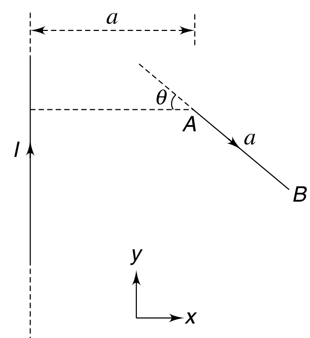


(b)



(c)

Q. 18: A straight wire AB of length a is placed at a distance a from an infinitely long straight wire as shown in the figure. Angle θ is 30°. Find the magnetic force on wire AB if it is also given a current I. Both the wires are in xy plane.

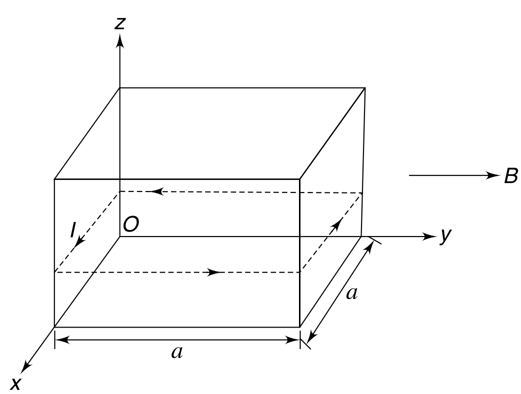


Q.19: A dielectric spherical shell of radius R, having charge Q is rotating with angular speed ω about its diameter. Calculate the magnetic dipole moment (M) of the shell. Write the ratio of M and angular momentum (L) of the rotating shell. This ratio is called gyro-magnetic ratio. Mass of shell is m.

Q. 20: A wooden cubical block of mass m and side a is resting on a horizontal surface. A wire carrying current I, is wrapped around it in from of a square of side a. A uniform magnetic field $\vec{B} = B_0\hat{j}$ is switched on in the region. Neglect the mass of the wire.

(a) At what distance from the x axis does normal force applied by the horizontal surface on the wooden cube act?

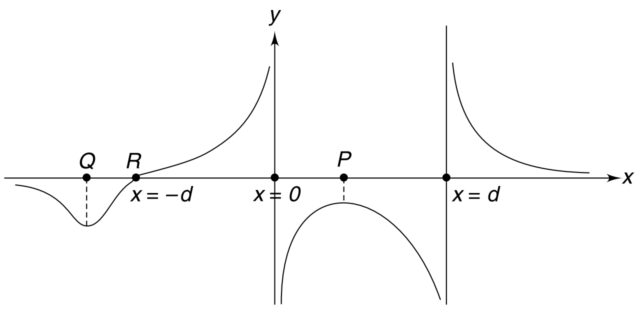
(b) What is the maximum value of current for which the block will not topple?



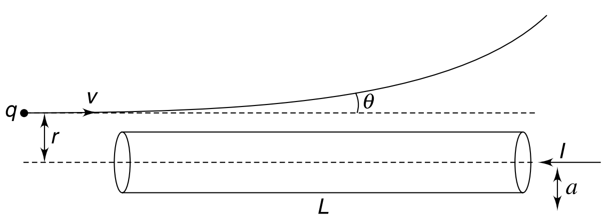
Q.21: A square loop of mass m and side length a lies in xy plane with its centre at origin. It carries a current I. The loop is free to rotate about x axis. A magnetic field $\vec{B} = B_0 \hat{j}$ is switched on in the region. Calculate the angular speed acquired by the loop when it has rotated through 90°. Assume no other force on the loop apart from the magnetic force.

LEVEL 2

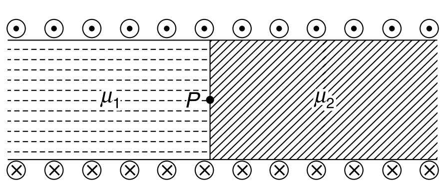
Q. 22: Two long parallel wires are along z direction at $x = 0$ and $x = d$. The magnetic field along x axis has been plotted in the given figure with field (B) positive when it is in positive y direction. The co-ordinate of point R is $x = -d$. Find co-ordinate of points P and Q shown in figure.



Q. 23: A straight wire of length L and radius a has a current I. A particle of mass m and charge q approaches the wire moving at a velocity v in a direction anti parallel to the current. The line of motion of the particle is at a distance r from the axis of the wire. Assume that r is slightly larger than a so that the magnetic field seen by the particle is similar to that caused by a long wire. Neglect end effects and assume that speed of the particle is high so that it crosses the wire quickly and suffers a small deflection θ in its path. Calculate θ.



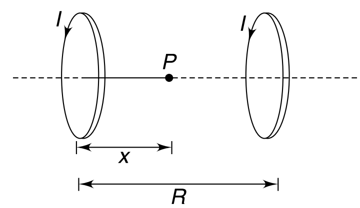
Q. 24: A long narrow solenoid is half filled with material of relative permeability μ_1 and half filled with another material of relative permeability μ_2. The number of turns per meter length of the solenoid is n. Calculate the magnetic field (B) on the axis of the solenoid at boundary of the two material (i.e. at point P). The current in solenoid coil is I.



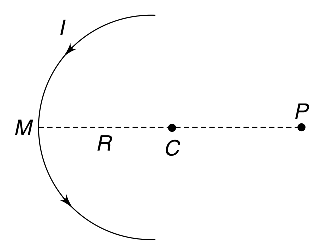
Q. 25: Two identical coils having radius R and number of turns N are placed co-axially with their centres separated by a distance equal to their radius R. The two coils are given same current I in same direction. The configuration is often known as a pair of Helmholtz coil.

(i) Calculate the magnetic field (B) at a point (P) on the axis between the coils at a distance x from the centres of one of the coils.

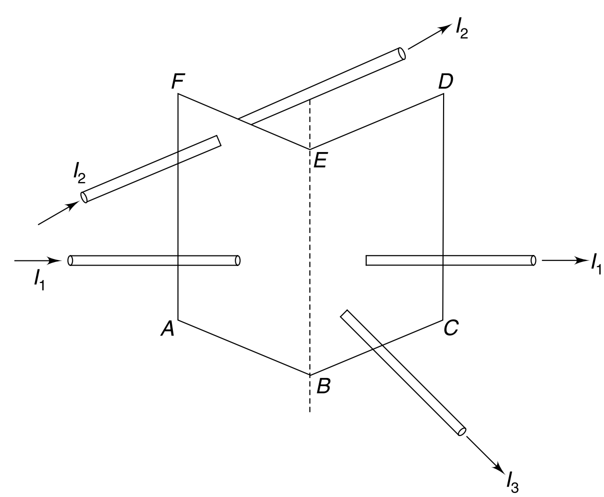
(ii) Prove that $\frac{dB}{dx} = 0$ and $\frac{d^2B}{dx^2} = 0$ $\left[\text{In fact } \frac{d^3B}{dx^3} \text{ is also equal to zero}\right]$ at the point lying midway between the two coils. What conclusion can you draw from these results?



Q. 26: A current carrying wire is in the shape of a semicircle of radius R and has current I. M is midpoint of the arc and point P lies on extension of MC at a distance $2R$ from M. Find the magnetic field due to circular arc at point P.

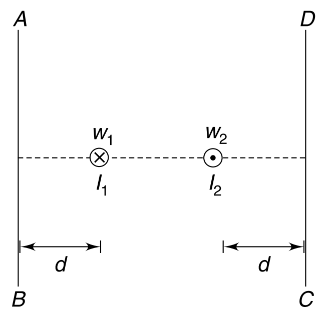


Q. 27: The figure shows three straight current carrying conductors having current I_1, I_2 and I_3 respectively. Calculate line integral of magnetic induction field ($\vec{B}$) along the closed path $ABCDEFA$.

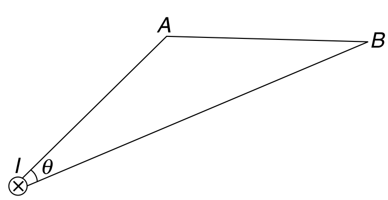


Q.28: A circular coil of N turns carries a current I. Field at a distance x from centre of the loop on its axis is B. Write the value of integral $\int_{-\infty}^{\infty} B \cdot dx$.

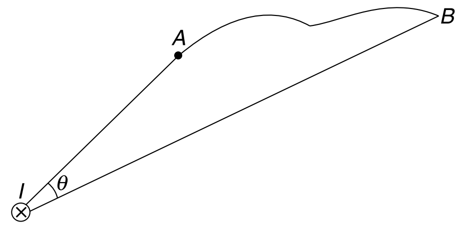
Q. 29: In the figure shown $W1$ represent the cross section of an infinitely long wire carrying current I_1 into the plane of the fig. AB is a line of length L and the wire $W1$ is symmetrically located with respect to the line. The line integral $\int_A^B \vec{B} \cdot \vec{d}l$ along the line from A to B is equal to $-a_0$ where a_0 is a positive number. Another long wire $W2$ is placed symmetrically with respect to AB (see fig) and the value of $\int_A^B \vec{B} \cdot \vec{d}l$ becomes zero. Consider a line DC to the right of $W2$. The line is parallel to AB and has same length. The two wires fall on perpendicular bisector of both lines. If $\int_C^D \vec{B} \cdot \vec{d} = 2a_0$ with both wires $W1$ and $W2$ present, calculate the ratio of current $\frac{I_2}{I_1}$ in the two wires.



Q. 30: (a) A long straight wire carries a current I into the plane of the figure. AB is a straight line in the plane of the figure subtending an angle θ at the point of intersection of the wire with the plane. Find (by integration) the line integral of magnetic field along the line AB.



(b) In the last problem the straight line AB is replaced with a curved line AB as shown in figure. Can you calculate the line integral of magnetic field B along this curved line? If yes, what is its value?



Q. 31: A long straight cylindrical region of radius a curries a current along its length. The current density (J) varies from the axis to the edge of the cylindrical region according to $J = J_0\left(1 - \frac{r}{a}\right)$

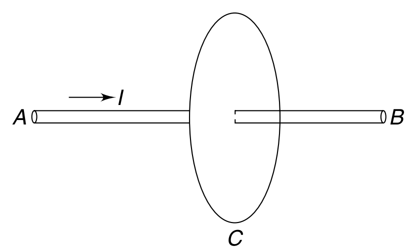
Where r is distance from the axis ($0 \le r \le a$)

(a) Find the mean current density.

(b) Plot the variation of magnetic field (B) with distance r from the axis of the cylinder for $0 \le r \le a$.

Q. 32: A student has studied the use of Ampere's law in calculation of magnetic field (**B**) due to a straight current carrying conductor of infinite length. Now she used similar arguments for calculation of B due to a current carrying conductor (AB) of finite length. She assumes a closed circular path (C) of radius r with the conductor along the axis (see fig.). She argues that because of symmetry the field (B) shall be tangential to C and must have same magnitude at all points on C. Therefore she writes $B = \frac{\mu_0 I}{2\pi r}$

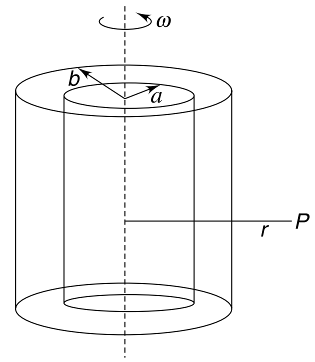
Do you support the answer? Give reasons.



Q. 33: There are two co-axial non conducting cylinders of radii a and $b(>a)$. Length of each cylinder is $L(>>b)$ and their curved surfaces have uniform surface charge densities of $-\sigma$ (on cylinder of radius a) and $+\sigma$ (on cylinder of radius b). The two cylinders are made to rotate with same angular velocity ω as shown in the figure. The charge distribution does not change due to rotation. Find the electric field (E) and magnetic field (B) at a point (P) which is at a distance r from the axis such that

(a) $0 < r < a$ (b) $a < r < b$ (c) $r > b$.

Assume that point P is close to perpendicular bisector of the length of the cylinders



Q. 34: An infinite sheet in xy plane has a uniform surface charge density σ. The thickness of the sheet is infinitesimally small. The sheet begins to move with a velocity $\vec{v} = v\hat{i}$

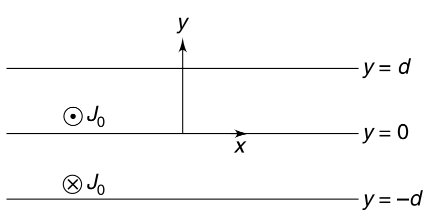
(i) Find the electric field $(\vec{E})$ and magnetic field $(\vec{B})$ above and below the sheet.

(ii) If the velocity of the sheet is changed to $\vec{v} = v\hat{k}$, find the electric and magnetic field above and below the sheet.

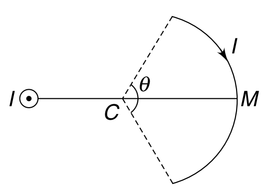
Q. 35: Consider two slabs of current shown in the figure. Both slabs have thickness b in y direction and extend up to infinity in x and z directions. The common face of the two slabs is $y = 0$ plane. The slab in the region $0 < y < d$ has a constant current density $= J_0\hat{k}$ and the other slab in the region $-d < y < 0$ has a constant current density $= J_0(-\hat{k})$.

(a) Find magnetic field at $y = 0$

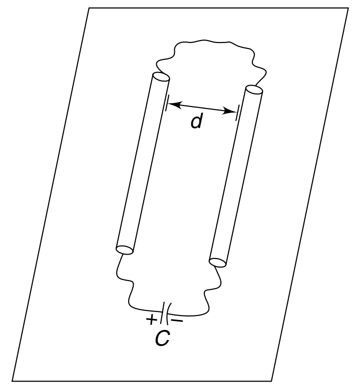
(b) Plot the variation of magnetic field (B) along the y axis.



Q. 36: A current carrying conductor is in the shape of an arc of a circle of radius R subtending an angle θ at the centre (C). A long current carrying wire is perpendicular to the plane of the arc and is at a distance $2R$ from the midpoint (M) of the arc on the line joining the points M and C. Current in the arc as well as straight wire is I. Find the magnetic force on the arc.



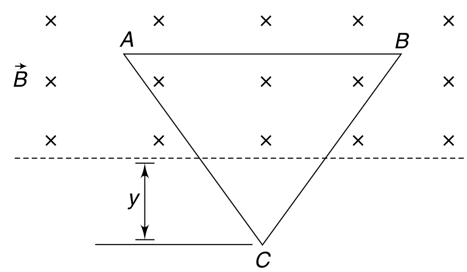
Q. 37: Two long straight conducting wires with linear mass density λ are kept parallel to each other on a smooth horizontal surface. Distance between them is d and one end of each wire is connected to each other using a loose wire as shown in the figure. A capacitor is charged so as to have energy U_0 stored in it. The capacitor is connected to the ends of two wires as shown. The resistance (R) of the entire arrangement is negligible and the capacitor discharges quickly. Assume that the distance between the wires do not change during the discharging process. Calculate the speed acquired by two wires as the capacitor discharges.



Q. 38: A current carrying loop is in the shape of an equilateral triangle of side length a. Its mass is M and it is in vertical plane. There exists a uniform horizontal magnetic field B in the region shown.

(a) The loop is in equilibrium for $y_0 = \frac{\sqrt{3}}{4}a$. Find the current in the loop.

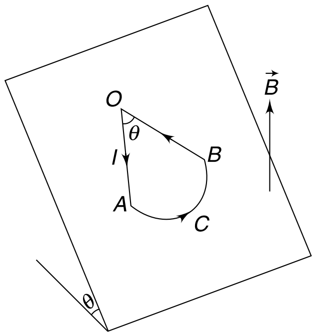
(b) The loop is displaced slightly in its plane perpendicular to its side AB and released. Find time period of its oscillations. Neglect emf induced in the loop. Express your answer in terms of a and g.



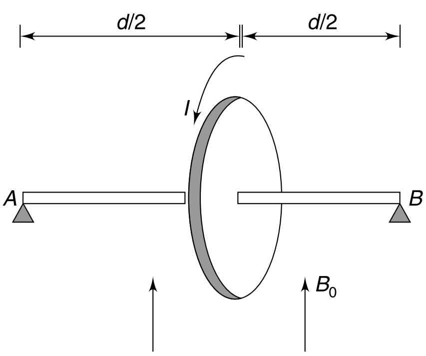
Q. 39: A current loop consist of two straight segments (OA and OB), each of length ℓ, having an angle θ between them and a semicircle (ACB). The loop is placed on an incline plane making an angle θ with horizontal (see figure). The loop carries a current I. A uniform vertical magnetic field B is switched on.

(a) Write the value of magnetic torque on the loop.

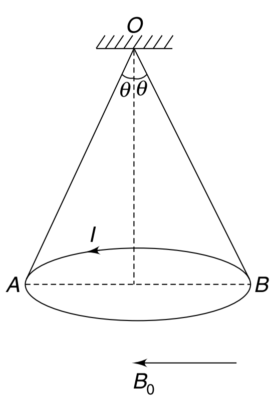
(b) Tell whether the normal contact force between the incline and the loop increases or decreases when magnetic field is switched on. Assume that the loop remains stationary on the incline.



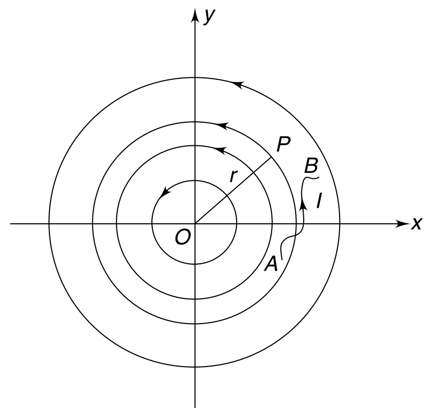
Q. 40: A wooden disc of mass M and radius R has a single loop of wire wound on its circumference. It is mounted on a massless rod of length d. The ends of the rod are supported at its ends so that the rod is horizontal and disc is vertical. A uniform magnetic field B_0 exists in vertically upward direction. When a current I is given to the wire one end of the rod leaves the support. Find least value of I.



Q. 41: A uniform ring of mass M and radius R carries a current I (see figure). The ring is suspended using two identical strings OA and OB. There exists a uniform horizontal magnetic field B_0 parallel to the diameter AB of the ring. Calculate tension in the two strings. [Given $\theta = 60°$]

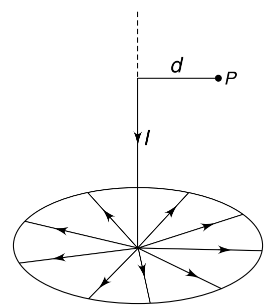


Q. 42: In a two dimensional $x - y$ plane, the magnetic field lines are circular, centred at the origin. The magnitude of the field is inversely proportional to distance from the origin and field at any point P has magnitude given by $B = \frac{k}{r}$; where k is a positive constant. A wire carrying current I is laid in xy plane with its ends at point $A(x_1, y_1)$ and point $B(x_2, y_2)$. Find force on the wire.



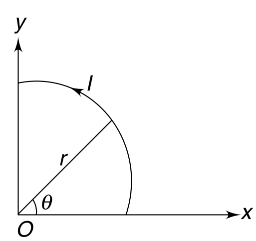
LEVEL 3

Q. 43: A straight current carrying wire has its one end attached to an infinity conducting sheet (shown as a circle in the figure). The other end of the wire goes to infinity and the wire is perpendicular to the sheet. The current spreads uniformly on the surface of the sheet. Calculate the magnitude of magnetic induction field at a point P at a distance d from the straight wire. Current in the wire is I.



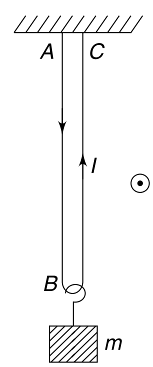
Q. 44: A wire carrying current I is laid in shape of a curve which is represented in plane polar co-ordinate system as

$$r = b + \frac{c}{\pi}\theta \quad \text{for} \quad 0 \le \theta \le \pi/2$$



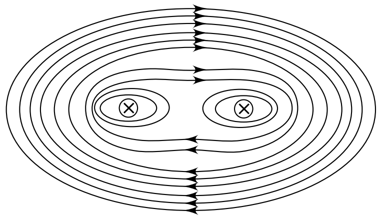
Here b and c are positive constants. θ is the angle measured with respect to positive x direction in anticlockwise sense and r is distance from origin (see figure). Calculate the magnetic field at the origin due to the wire.

Q. 45: A light freely deformable conducting wire with insulation has its two ends (A and C) fixed to the ceiling. The two vertical parts of the wire are close to each other. A load of mass m is attached to the middle of the wire. The entire region has a uniform horizontal magnetic field B directed out of the plane of the figure. Prove that the two parts of the wire take the shape of circular arcs when a current I is passed through the wire. Neglect the magnetic interaction between the two parts of the wire.

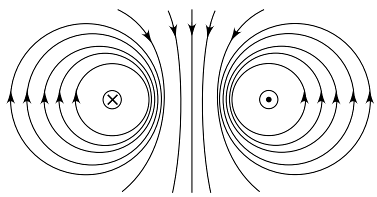


ANSWERS

1. (a)



(b)



2. $\dfrac{\mu_0 I}{2\pi d}\sqrt{\dfrac{5}{4} + \dfrac{1}{\sqrt{2}}}$

3. $\dfrac{\mu_0 I}{\pi a}$ towards midpoint of DE

4. $\sqrt{40}\ \mu$T

5. 1 μT

6. $\dfrac{r_1}{r_2} = \dfrac{1}{2}$

7. $B = \dfrac{\mu_0 I}{4\pi}\left[\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{x}{y\sqrt{x^2 + y^2}} + \dfrac{y}{x\sqrt{x^2 + y^2}}\right]$

8. $B = 0$

9. $\dfrac{B_0}{\sqrt{2}}$

10. $\dfrac{\sqrt{2}\,\mu_0 I}{\pi^2 R}$

11. (a) 0 (b) $\dfrac{\mu_0 NI}{2\pi x}$

12. $-\dfrac{\mu_0 I}{4}$; No

13. 2 : 1

14. $B = \dfrac{9}{34}\dfrac{\mu_0 I}{\pi R}$

15. Right

16. $1 : 1 : \eta : \eta$ where $\eta = \dfrac{mg - BI\ell}{mg + BI\ell}$

17. (a) Conductor will rotate in Horizontal plane

(b) Conductor will rotate so as to get parallel to fixed wire

(c) Circular conductor will rotate so as to set the current (I_0) parallel to (I_1)

18. $\dfrac{\mu_0 I^2}{2\pi}\ell n\left(\dfrac{2 + \sqrt{3}}{2}\right)\left[\dfrac{\hat{i}}{\sqrt{3}} + \hat{j}\right]$

19. $M = \dfrac{Q\omega R^2}{3}$; $\dfrac{M}{L} = \dfrac{Q}{2m}$

20. (a) $\dfrac{Ia^2 B_0}{mg} + \dfrac{a}{2}$ (b) $I_{max} = \dfrac{mg}{2aB_0}$

21. $\omega = \sqrt{\dfrac{12 I B_0}{m}}$

22. $x_P = \dfrac{d}{\sqrt{2} + 1}$; $x_Q = -\left(\dfrac{d}{\sqrt{2} - 1}\right)$

23. $\theta = \dfrac{\mu_0 ILq}{2\pi mvr}$

24. $\dfrac{1}{2}\mu_0(\mu_1 + \mu_2)\, nI$

25. (i) $\dfrac{\mu_0 NIR^2}{2}\left[\dfrac{1}{(R^2 + x^2)^{3/2}} + \dfrac{1}{[R^2 + (R - x^2]^{3/2}}\right]$

(ii) The magnetic field is constant at points close to mid way between the coils.

26. $\dfrac{\mu_0 I}{4\pi R}\ell n(\sqrt{2} + 1)$

27. $\mu_0(I_3 - I_2)$

28. $\mu_0 NI$

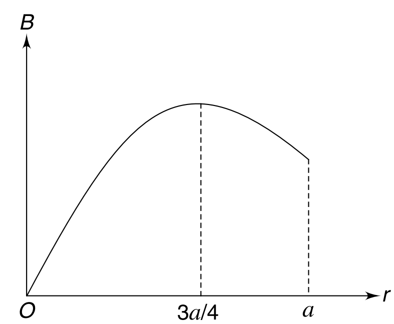
29. $\dfrac{I_2}{I_1} = 1 + \sqrt{2}$

30. (a) $\dfrac{\theta}{2\pi}\mu_0 I$

(b) Yes, line integral is $\dfrac{\theta}{2\pi}\mu_0 I$

31. (a) $\dfrac{J_0}{3}$

(b)



32. No

33. $E = 0$ for $r < a$

$= \dfrac{a\sigma}{\epsilon_0 r}$ radially inward for $a < r < b$

$= \dfrac{(b - a)\sigma}{\epsilon_0 r}$ radially outward for $r > b$

$B = \sigma\omega(b - a)$ up along the axis for $r < a$

$= b\sigma\omega$ up along the axis for $a < r < b$

$= 0$ for $r > b$

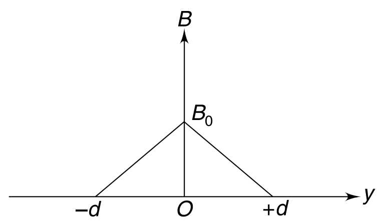
34. (i) $E = \dfrac{\sigma}{2\epsilon_0}$ perpendicularly away from the sheet

$B = \dfrac{\mu_0 \sigma v}{2}$ parallel to the sheet in -ve and +ve y direction

(ii) $E =$ same as in (i); $B = 0$

35. (a) $B_0 = \mu_0 J_0 d$

(b)



36. Zero

37. $\dfrac{\mu_0 U_0}{2\pi\lambda R d}$

38. (a) $I = \dfrac{2Mg}{aB}$ (b) $T = \pi\sqrt{\dfrac{\sqrt{3}a}{g}}$

39. (a) $\tau = IB\ell^2 \sin\theta \cdot \sin\dfrac{\theta}{2}\left[\cos\dfrac{\theta}{2} + \pi\sin\dfrac{\theta}{2}\right]$

(d) No change

40. $I = \dfrac{Mgd}{2\pi R^2 B_0}$

41. $T_{AO} = Mg + \pi IRB_0$

$T_{BO} = Mg - \pi IRB_0$

42. $\vec{F} = \dfrac{kI}{2}\ell n\left(\dfrac{x_2^2 + y_2^2}{x_1^2 + y_1^2}\right)\hat{k}$

43. $\dfrac{\mu_0 I}{2\pi d}$

44. $\dfrac{\mu_0 I}{4c}\,\ell n\left(1 + \dfrac{c}{2b}\right) \odot$

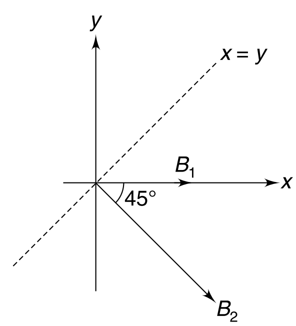
OLUTIONS

2. Field due to first wire is

$$B_1 = \frac{\mu_0 I}{2\pi d} \text{ along positive } x$$

Field due to second wire is

$$B_2 = \frac{\mu_0 I}{2\pi(2d)} \text{ in the direction shown.}$$

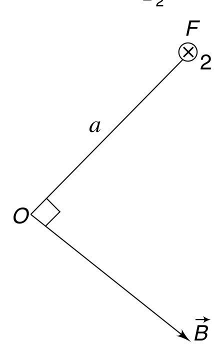


$\therefore$
$$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos 45^\circ}$$

$$= \frac{\mu_0 I}{2\pi d}\sqrt{1 + \frac{1}{4} + 2\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2}}} = \frac{\mu_0 I}{2\pi d}\sqrt{\frac{5}{4} + \frac{1}{\sqrt{2}}}$$

3. Imagine a current I at F in the same direction as that of other 5 currents. Such 6 currents will produce zero field at the centre.

Hence field at the centre will be due to a current $2I$ at F directed into the plane of the figure.

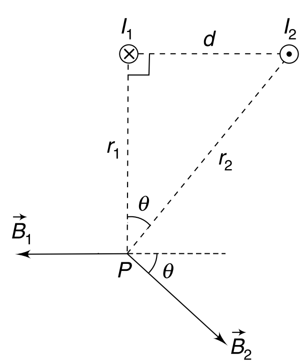


$$B = \frac{\mu_0(2I)}{2\pi a} = \frac{\mu_0 I}{\pi a}.$$

The direction is perpendicular to the line FO. This is a direction towards midpoint of side DE.

4. The situation is as shown in the figure.

Note that $d^2 + r_1^2 = r_2^2$



$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{4\pi \times 10^{-7} \times 8}{2\pi \times 0.16} = 10\ \mu\text{T}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.20} = 10\ \mu\text{T}$$

Angle between $\vec{B}_1$ and $\vec{B}_2$ is $180 - \theta$ where $\cos\theta = \dfrac{r_1}{r_2} = \dfrac{0.16}{0.20} = \dfrac{4}{5}$

$\therefore$ Resultant field at P is

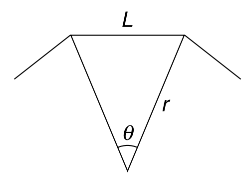
$$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos(180 - \theta)}$$

$$= \sqrt{10^2 + 10^2 - 2 \times 10 \times 10 \times \frac{4}{5}} = \sqrt{40}\ \mu\text{T}$$

5. Since number of sides in the polygon is large we can approximately treat it as a circular loop of radius r given by

$$r = \frac{L}{\theta} = \frac{1\text{cm}}{\frac{2\pi}{2016}}$$

$$= \frac{1008}{\pi}\text{cm} = \frac{10.08}{\pi}\text{m}$$



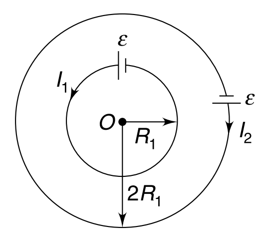
$$\therefore \quad B = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 5 \times \pi}{2 \times 10.08}$$

$$= 10^{-6} T$$

6. The wires must have current in opposite sense and

$$\frac{\mu_0 I_1}{2R_1} = \frac{\mu_0 I_2}{2\cdot(2R_1)}$$

$$\therefore \quad 2I_1 = I_2 \qquad \text{...(i)}$$



And $\quad \varepsilon = I_1 \dfrac{\rho(2\pi R_1)}{\pi r_1^2} = I_2 \dfrac{\rho(2\pi 2R_1)}{\pi r_2^2}$

[r_1 and r_2 are radii of cross section]

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{1}{2}\frac{I_1}{I_2}$$

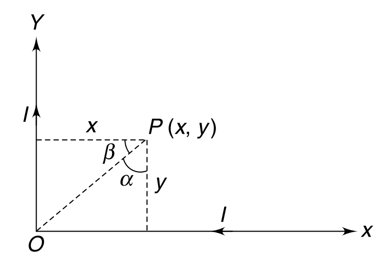
$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{1}{4} \qquad \text{[using (i)]}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{2}$$

7. Field at P due to wire along x axis is

$$B_1 = \frac{\mu_0 I}{4\pi y}[\sin\alpha + 1] \otimes$$

$$= \frac{\mu_0 I}{4\pi y}\left[\frac{x}{\sqrt{x^2 + y^2}} + 1\right] \otimes$$



Field due to wire along y axis is

$$B_2 = \frac{\mu_0 I}{4\pi x}[\sin\beta + 1]$$

$$= \frac{\mu_0 I}{4\pi x}\left[\frac{y}{\sqrt{x^2 + y^2}} + 1\right] \otimes$$

Resultant field at P

$$B = B_1 + B_2$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{1}{x} + \frac{1}{y} + \frac{x}{y\sqrt{x^2 + y^2}} + \frac{y}{x\sqrt{x^2 + y^2}}\right] \otimes$$

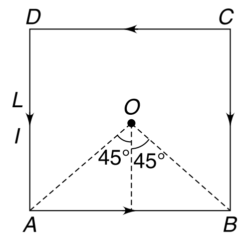
8. Point B and C will have equal potential and there will be no current in arc BC. Current I gets divided into two equal parts at A.

$$\therefore \quad B_{AB} = \frac{\mu_0 (I/2)}{2a} \otimes$$

$$B_{AC} = \frac{\mu_0 (I/2)}{2a} \odot$$

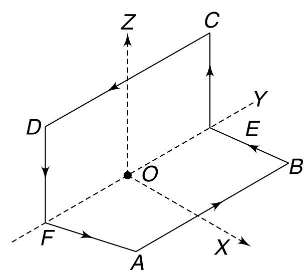
$$\therefore \quad B_{\text{centre}} = 0$$

9.



$$B_0 = \frac{4\mu_0 I}{4\pi \frac{L}{2}} (\sin 45° + \sin 45°) = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$

After folding



$$\vec{B}_{AB} = \frac{\mu_0 I}{4\pi \frac{L}{2}} (\sin 45° + \sin 45°)\, \hat{k} = \frac{2\sqrt{2}}{4\pi} \frac{\mu_0 I}{L} \hat{k}$$

$$\vec{B}_{BE} = \frac{\mu_0 I}{4\pi \frac{L}{2}} (\sin 0° + \sin 45°)\, \hat{k} = \frac{\sqrt{2}\mu_0 I}{4\pi L} \hat{k}$$

Similarly we have:

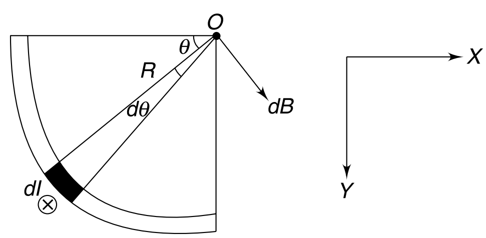
$$\vec{B}_{EC} = \frac{\sqrt{2}\mu_0 I}{4\pi L} \hat{i}$$

$$\vec{B}_{CD} = \frac{2\sqrt{2}}{4\pi} \frac{\mu_0 I}{L} \hat{i}; \; \vec{B}_{DF} = \frac{\sqrt{2}\mu_0 I}{4\pi L} \hat{i}; \; \vec{B}_{FA} = \frac{\sqrt{2}\mu_0 I}{4\pi L} \hat{k}$$

$$\therefore \quad \vec{B} = \frac{\sqrt{2}\mu_0 I}{\pi L} \hat{i} + \frac{\sqrt{2}\mu_0 I}{\pi L} \hat{k}$$

$$\therefore \quad |\vec{B}| = \frac{2\mu_0 I}{\pi L} = \frac{B_0}{\sqrt{2}}$$

10. Consider a long wire element as shown. The figure shows the cross section of the wire.



Current through the wire element is $dI = \frac{I}{\frac{\pi}{2}} d\theta = \frac{2I}{\pi} d\theta$

Magnetic field due to the current dI at O is

$$dB = \frac{\mu_0 dI}{2\pi R}$$

$$= \frac{\mu_0 \frac{2I}{\pi} d\theta}{2\pi R} = \frac{\mu_0 I}{\pi^2 R} d\theta$$

x component of the field is

$$dB_x = \frac{\mu_0 I}{\pi^2 R} \sin\theta d\theta$$

$$B_x = \int dB_x = \frac{\mu_0 I}{\pi^2 R} \int_0^{\pi/2} \sin\theta d\theta = \frac{\mu_0 I}{\pi^2 R}$$

From symmetry $B_y = B_x$

$\therefore$ Resultant B at O is

$$B = \sqrt{B_x^2 + B_y^2} = \frac{\sqrt{2}\mu_0 I}{\pi^2 R}$$

11. (a) Consider a circle of radius $x < R$ with its axis along the axis of the cylinder. Assuming this circle to be Amperian loop we can write–

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{\text{enclosed}} = 0$$

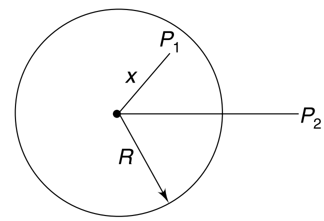
$$\Rightarrow \quad B = 0$$

(b)
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 NI$$

$$\Rightarrow \quad B 2\pi x = \mu_0 NI$$

$$\Rightarrow \quad B = \frac{\mu_0 NI}{2\pi x}$$

13. Field at a point P_1 ($x < R$) is



$$B = \frac{\mu_0 j}{2} x$$

Slope of 1st graph is higher

$$\therefore \quad j_1 > j_2.$$

Also $\quad R_2 = 2R_1$

Field at a point P_2 ($x > R$) is

$$B = \frac{\mu_0 I}{2\pi x} \qquad [I = \text{current in the wire}]$$

Since the two graphs are overlapping

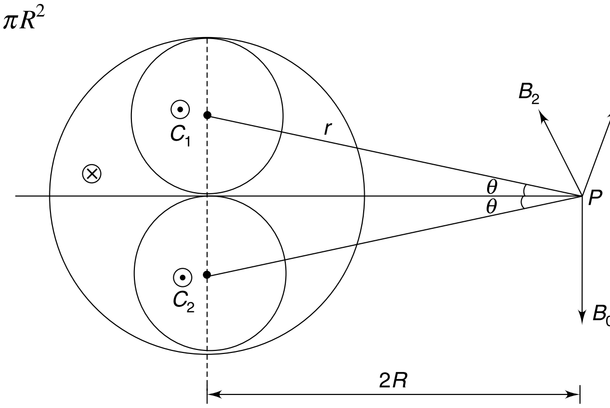
$$\therefore \quad I_1 = I_2$$

Field on the surface of two wires are–

$$B_1 = \frac{\mu_0 I_1}{2\pi R_1} \quad \text{and} \quad B_2 = \frac{\mu_0 I_2}{2\pi R_2}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{R_2}{R_1} = \frac{2}{1}$$

14. Current density
$$J = \frac{I}{A} = \frac{I}{\pi\left[R^2 - \frac{R^2}{4} - \frac{R^2}{4}\right]} = \frac{2I}{\pi R^2}$$



Let B_0 = field at P, had there been no cavity and B_1 and B_2 be field at P due to conductors of radius $\frac{R}{2}$ occupying the cavity space and carrying current in direction opposite to the given current.

Field at P will be vector sum of B_0, B_1 and B_2

$$B_0 = \frac{\mu_0(\pi R^2 J)}{2\pi(2R)} = \frac{\mu_0}{4} JR \text{ In the direction shown.}$$

$$B_1 = \frac{\mu_0\left(\pi \frac{R^2}{4} J\right)}{2\pi r} = \frac{\mu_0 R^2 J}{8r} \text{ perpendicular to } C_1P$$

$$B_2 = \frac{\mu_0 R^2 J}{8r} \text{ perpendicular to } C_2P$$

Resultant field at P is

$$B = B_0 - (B_1 + B_2)\cos\theta$$

$$= \frac{\mu_0 JR}{4} - \frac{\mu_0 R^2 J}{4r} \cdot \frac{2R}{r}$$

Put $\quad r = \sqrt{4R^2 + \frac{R^2}{4}} = \frac{\sqrt{17}}{2} R$

$\therefore \quad B = \frac{\mu_0 JR}{4} - \frac{\mu_0 JR^3}{2 \cdot \frac{17}{4} R^2}$

$\Rightarrow \quad B = \frac{\mu_0 JR}{4} - \frac{2}{17} \mu_0 JR = \frac{9}{68} \mu_0 JR$

$\therefore \quad B = \frac{9}{68} \mu_0 R \frac{2I}{\pi R^2} = \frac{9}{34} \frac{\mu_0 I}{\pi R}$

15. Let the direction of uniform B be vertically down. The current in the horizontal wire must be towards you. This current will create a field in upward direction to the right of the wire and resultant field will get reduced to the right of the wire. In fact it will be zero at same point. This is what is depicted in the figure.

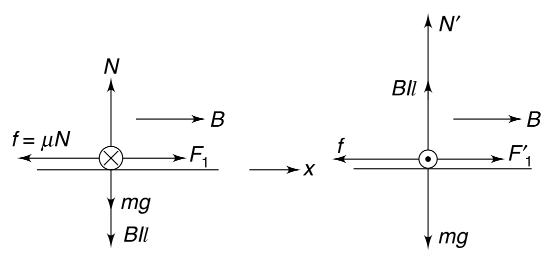
Force on the wire is along $I\vec{L} \times \vec{B}$ which is to right.

16. When current is along positive y, the direction of magnetic force is vertically down.

Hence normal reaction on the wire is

$$N = mg + BI\ell$$

$$F_1 = \mu N = \mu(mg + BI\ell)$$



when we pull the wire in y direction, the friction force will be in negative y direction but its maximum value will remain same as there is no change in N.

$\therefore \quad F_1 = F_2$

With direction of current reversed the magnetic force becomes vertically up

$\therefore \quad N' = mg - BI\ell$

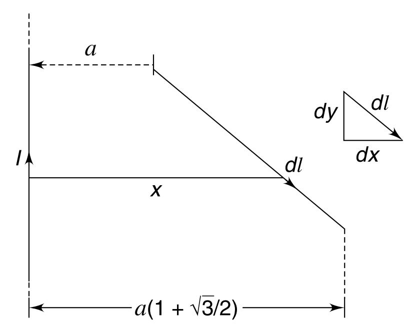
$\therefore \quad F_1' = F_2' = \mu N' = \mu(mg - BI\ell)$

$\therefore \quad \frac{F_1'}{F_1} = \frac{mg - BI\ell}{mg + BI\ell} = \eta$ (say)

$\therefore \quad F_1 : F_2 : F_1' : F_2' = 1 : 1 : \eta : \eta$

18. An infinitesimal length $d\ell$ on wire AB can be expressed as $\overrightarrow{d\ell} = \hat{i}\,dx - \hat{j}dy$

Where $\quad dy = (\tan 30°)\ dx = \frac{dx}{\sqrt{3}}$



Field due to long wire at a distance x is

$$\vec{B} = \frac{\mu_0 I}{2\pi x}(-\hat{k})$$

Force on length $d\ell$ of wire AB is

$$\overrightarrow{dF} = I\overrightarrow{d\ell} \times \vec{B} = I\left(\hat{i}\,dx - \hat{j}\frac{dx}{\sqrt{3}}\right) \times \frac{\mu_0 \mathrm{I}}{2\pi x}(-\hat{k})$$

$$= \frac{\mu_0 I^2}{2\pi}\left[\hat{i}\frac{dx}{\sqrt{3}\cdot x} + \hat{j}\frac{dx}{x}\right]$$

$\therefore$ Force on AB is

$$\vec{F} = \frac{\mu_0 I^2}{2\pi}\left[\frac{\hat{i}}{\sqrt{3}} \int_a^{a\left(1 + \frac{\sqrt{3}}{2}\right)} \frac{dx}{x} + \hat{j} \int_a^{a\left(1 + \frac{\sqrt{3}}{2}\right)} \frac{dx}{x}\right]$$

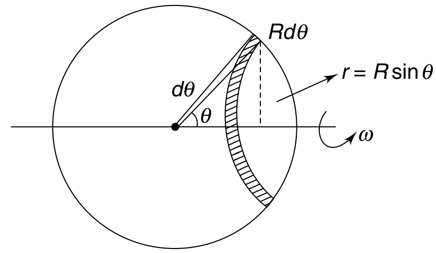
$$= \frac{\mu_0 I^2}{2\pi} \ell n\left(\frac{2 + \sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\hat{i} + \hat{j}\right)$$

19. The shell can be throught to be made up of a number of co-axial rings. We will consider one such ring as shown in figure.

Surface charge density $\sigma = \frac{Q}{4\pi R^2}$

Charge on ring element $dq = \sigma(2\pi r)(Rd\theta)$

$$= \frac{Q}{2} \sin\theta d\theta$$



This charge rotates with frequency $f = \frac{\omega}{2\pi}$

$\therefore$ Current associated with this charge is

$$dI = (dq)\,f = \frac{Q\,\omega}{4\pi} \sin\theta d\theta$$

Magnetic dipole moment of the elemental ring is

$$dM = (dI)(\pi r^2) = \left(\frac{Q\,\omega}{4\pi} \sin\theta d\theta\right)(\pi R^2 \sin^2\theta)$$

$$\therefore \quad M = \frac{Q\omega R^2}{4} \int_0^{\pi} \sin^3\theta d\theta$$

$$= \frac{Q\,\omega R^2}{4 \times 4}\left[\int_0^{\pi} 3\sin\theta d\theta - \int_0^{\pi} \sin 3\theta d\theta\right]$$

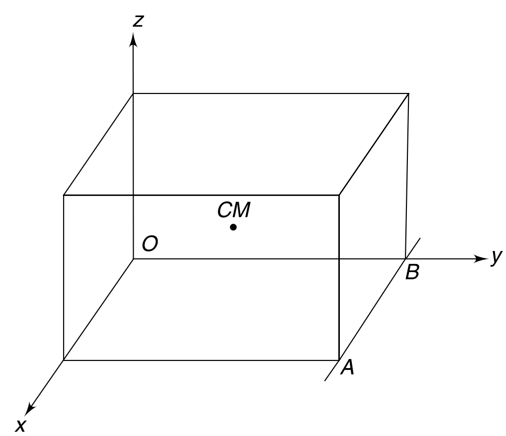
$$= \frac{Q\omega R^2}{16}\left[3(2) + \frac{1}{3}(-2)\right]$$

$$M = \frac{Q\omega R^2}{3} \quad \text{and} \quad L = \frac{2}{3} mR^2\omega$$

$$\therefore \quad \frac{M}{L} = \frac{Q}{2m}$$

20. Magnetic dipole moment of the wire loop is

$$\vec{\mu} = a^2 I\hat{k}$$



Magnetic torque on the loop

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (a^2 I\hat{k}) \times (B_0\hat{j})$$

$$= -a^2 I B_0 \hat{i}$$

For rotational equilibrium, taking torque about COM we get–

$a^2 I B_0 = Nx$ [x = distance of normal force from centre]

$$\Rightarrow \quad a^2 I B_0 = mgx \quad \Rightarrow \quad x = \frac{a^2 I B_0}{mg}$$

(a) $\therefore$ Distance of N from x axis $= \frac{a^2 I B_0}{mg} + \frac{a}{2}$

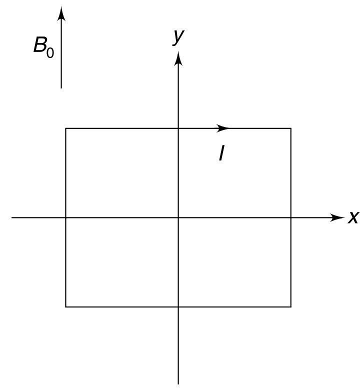
(b) Block will not topple if $x < \frac{a}{2}$

$$\Rightarrow \quad \frac{a^2 I B_0}{mg} < \frac{a}{2} \quad \Rightarrow \quad I < \frac{mg}{2aB_0}$$

21. $\vec{\mu}_1 = a^2 I(-\hat{k})$

Potential energy of the loop

$$U_1 = -\vec{\mu}_1 \cdot \vec{B} = -a^2 I(-\hat{k}) \cdot B_0(\hat{j}) = 0$$



After the loop rotates through 90°

$$\vec{\mu}_2 = a^2 I(\hat{j})$$

$$U_2 = -\vec{\mu}_2 \cdot \vec{B} = -a^2 I B_0$$

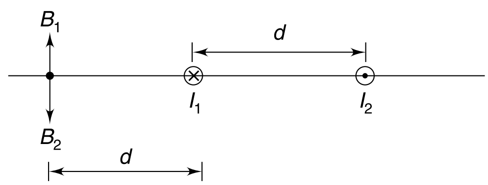
$\therefore$ Loss in $PE = a^2 I B_0$

$\therefore$ Gain in $KE = a^2 I B_0$

$$\frac{1}{2} I_x \omega^2 = a^2 I B_0$$

$$\Rightarrow \quad \frac{1}{2} \frac{ma^2}{6} \omega^2 = a^2 I B_0 \quad \Rightarrow \quad \omega = \sqrt{\frac{12 I B_0}{m}}$$

22. The two wires have current in opposite directions as shown in the figure. Let the currents be I_1 and I_2 Field at R is zero.



$$B_1 = B_2 \text{ at } R$$

$$\Rightarrow \quad \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I_2}{2\pi(2d)} \quad \Rightarrow \quad I_2 = 2I_1$$

At point P field is minimum. Field at a point having coordinate $0 < x < d$ is

$$B = \frac{\mu_0 I_1}{2\pi x} + \frac{\mu_0 I_2}{2\pi(d - x)} \text{ in negative } y \text{ direction}$$

$$= \frac{\mu_0 I_1}{2\pi}\left[\frac{1}{x} + \frac{2}{d - x}\right]$$

B is minimum if $\frac{dB}{dx} = 0$

$$\Rightarrow \quad -\frac{1}{x^2} + \frac{2}{(d - x)^2} = 0$$

$$\Rightarrow \quad \frac{x}{d - x} = \pm \frac{1}{\sqrt{2}}$$

Taking positive sign (as ratio of two positive numbers cannot be negative)

$$\sqrt{2}x = d - x \quad \Rightarrow \quad x = \frac{d}{\sqrt{2} + 1}$$

At point Q, the field is locally maximum. Let distance of point Q from origin be x. Field at such a point is

$$B = \frac{\mu_0 I_2}{2\pi(d + x)} - \frac{\mu_0 I_1}{2\pi x} \text{ in negative } y \text{ direction}$$

$$= \frac{\mu_0 I_1}{2\pi}\left[\frac{2}{d + x} - \frac{1}{x}\right]$$

This is maximum if

$$\frac{dB}{dx} = 0$$

$$\Rightarrow \quad -\frac{2}{(d + x)^2} + \frac{1}{x^2} = 0$$

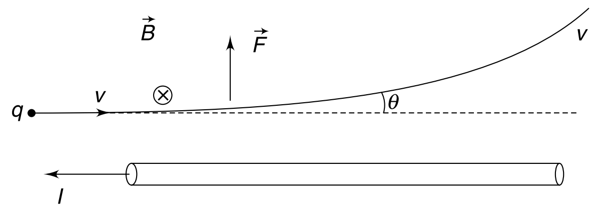
$$\Rightarrow \quad \frac{d + x}{x} = \sqrt{2}$$

$$\Rightarrow \quad d + x = \sqrt{2}x$$

$$\Rightarrow \qquad x = \frac{d}{\sqrt{2}-1}$$

$\therefore$ Co-ordinate of point Q is $-\left(\frac{d}{\sqrt{2}-1}\right)$

23. $$B = \frac{\mu_0 I}{2\pi r} \otimes$$

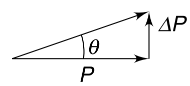


Force on the particle is $$F = qvB = \frac{\mu_0 Iqv}{2\pi r} (\uparrow)$$

This force is always perpendicular to the velocity. Since deflection is small, the force is nearly in ($\uparrow$) direction always.

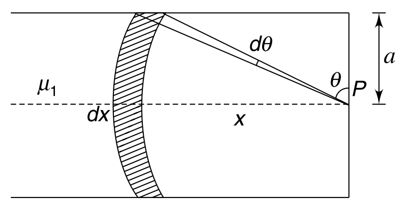
Impulse is $$F\Delta t = \frac{\mu_0 Iqv\Delta t}{2\pi r} = \frac{\mu_0 IqL}{2\pi r} \qquad [\because \; v\Delta t = L]$$

$$\therefore \qquad \Delta P = \frac{\mu_0 IqL}{2\pi r}(\uparrow)$$



$$\theta = \frac{\Delta P}{P} = \frac{\mu_0 IqL}{2\pi rmV}$$

24. Let's calculate the field due to one half by way of integration. (This is just to demonstrate ; otherwise the result is obviously half the field due to a solenoid extending to large length on both sides.)



Consider a rig element of angular width $d\theta$ as shown in the figure.

$$x = a\tan\theta$$

$$\therefore \qquad dx = a\sec^2\theta d\theta$$

Number of turns in width dx is ndx

Field due to ring at P is along the axis given by

$$dB = \frac{\mu_0\mu_1 ndx Ia^2}{2(a^2+x^2)^{3/2}} = \frac{\mu_0\mu_1 nIa^2}{2}\frac{(a\sec^2\theta d\theta)}{(a\sec\theta)^3} \qquad [\because \; \sqrt{a^2+x^2} = a\sec\theta]$$

$$\therefore \qquad dB = \frac{\mu_0\mu_1 nI}{2}\cos\theta d\theta$$

$$\therefore \qquad B_1 = \frac{\mu_0\mu_1 nI}{2}\int_0^{\pi/2}\cos\theta d\theta = \frac{1}{2}\mu_0\mu_1 nI$$

Similarly, field due to second half will be

$$B_2 = \frac{1}{2}\mu_0\mu_2 nI$$

Resultant field at P is

$$B = B_1 + B_2 = \frac{1}{2}(\mu_1+\mu_2)\mu_0 nI$$

25. (i) $B = \frac{\mu_0 NIR^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+(R-x^2)^{3/2}}\right]$

[Note that fields due to both the coils are in same direction]

(ii) $\frac{dB}{dx} = \frac{\mu_0 NIR^2}{2}\left[-\frac{3}{2}\frac{2x}{(R^2+x^2)^{5/2}} - \frac{3}{2}\frac{2(R-x)(-1)}{[R^2+\ (R-x^2)^{5/2}}\right]$

$$= \frac{\mu_0 NIR^2}{2}\left[-\frac{3x}{(R^2 + x^2)^{5/2}} + \frac{3(R - x)}{[R^2 + (R - x)^2]^{5/2}}\right]$$

At $\quad x = \frac{R}{2};\ \frac{dB}{dx} = 0$

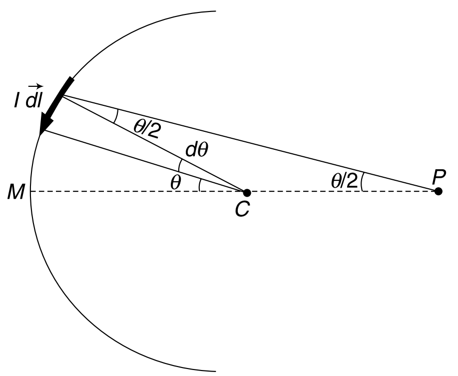
$$\frac{d^2B}{dx^2} = \frac{3\mu_0 NIR^2}{2}$$

$$\left[-\frac{(R^2 + x^2)^{5/2} - \frac{5x}{2}(R^2 + x^2)^{3/2}(2x)}{(R^2 + x^2)^5} + \frac{[R^2 + (R - x)^2]^{5/2}(-3)}{[R^2 + (R - x)^2]^5} - \frac{3(R - x)\,\frac{5}{2}[R^2 + (R - x)^2]^{3/2}\,2(R - x)(-1)}{[R^2 + (R - x)^2]^5}\right]$$

By putting $x = \frac{R}{2}$, one can show that $\frac{d^2B}{dx^2} = 0$

$\frac{dB}{dx} = \frac{d^2B}{dx^2} = 0$ at $x = \frac{R}{2}$ implies that B is constant for small variations in x

26. Point P lies on the circumference of the circle. Consider an element subtending angle $d\theta$ at the centre of the circle. Field at P due to this element is given by Biot – Savart law.



$$dB = \frac{\mu_0 Id\ell}{4\pi}\frac{\sin\left(90 + \frac{\theta}{2}\right)}{r^2}$$

Where $\quad d\ell = Rd\theta \quad$ and $\quad r = 2R\cos\left(\frac{\theta}{2}\right)$

$$\therefore \quad dB = \frac{\mu_0 IRd\theta}{4\pi}\frac{\cos\left(\frac{\theta}{2}\right)}{\left[2R\cos\left(\frac{\theta}{2}\right)\right]^2}$$

$$= \frac{\mu_0 I}{16\pi R}\sec\left(\frac{\theta}{2}\right)d\theta$$

Direction of dB is $\odot$

Note: If you consider an identical element in the other quadrant, field at P due to the element is -

$$dB = \frac{\mu_0 IRd\theta}{4\pi}\frac{\sin\left(90 - \frac{\theta}{2}\right)}{\left[2R\cos\left(\frac{\theta}{2}\right)\right]^2} = \frac{\mu_0 I}{16\pi R}\sec\left(\frac{\theta}{2}\right)d\theta \odot$$

All elements contribute in same direction and resultant field is

$$B = 2 \times \frac{\mu_0 I}{16\pi R}\int_0^{\pi/2}\sec\left(\frac{\theta}{2}\right)d\theta$$

$$= \frac{\mu_0 I}{4\pi R}\left[\ell n\left(\sec\frac{\theta}{2} + \tan\frac{\theta}{2}\right)\right]_0^{\pi/2}$$

$$= \frac{\mu_0 I}{4\pi R}\left[\ell n(\sqrt{2} + 1) - \ell n(1 + 0))\right]$$

$$= \frac{\mu_0 I}{4\pi R}\ell n(\sqrt{2} + 1)$$

27. $$\oint_{ABEFA} \vec{B}\cdot\vec{dl} = -\mu_0(I_1 + I_2)$$

And $$\oint_{BCDEB} \vec{B}\cdot\vec{dl} = \mu_0(I_1 + I_3)$$

$\therefore$ $$\oint_{ABCDEFA} \vec{B}\cdot\vec{dl} = \oint_{ABEFA} \vec{B}\cdot\vec{dl} + \oint_{BCDEB} \vec{B}\cdot\vec{dl}$$

$$= \mu_0(I_3 - I_2)$$

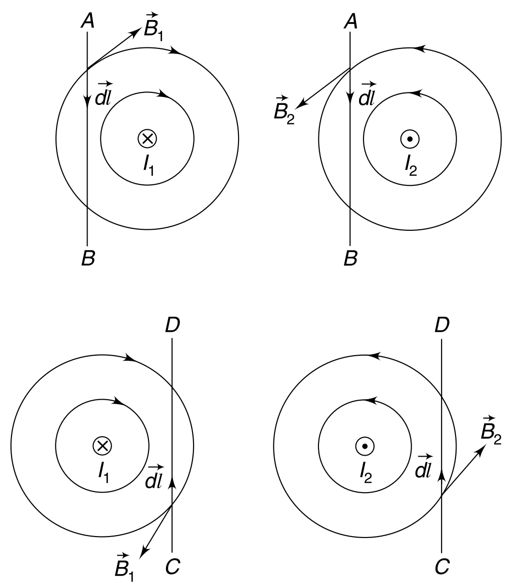
28. Think of a closed rectangular path with its one side of infinite length along the axis of the loop and other parallel side at ∞ distance from the axis. Apart from the side along the axis, the integral $\int \vec{B}\cdot\vec{dl}$ along all three sides will be zero since $B = 0$

Using Ampere's law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 NI$$

$\Rightarrow$ $$\int_{-\infty}^{\infty} B\cdot dx = \mu_0 NI$$

29. Current in $W2$ must be outward.



In the field of I_1 we have $\int_A^B \vec{B}_1\vec{dl} = -a_0$

In the field of I_2 we have $\int_A^B \vec{B}_2\vec{dl} = +a_0$

For CD

$$\int_C^D \vec{B}_1\vec{dl} = -a\left(\frac{I_1}{I_2}\right)$$

$$\int_C^D \vec{B}_2\vec{dl} = a_0\left(\frac{I_2}{I_1}\right)$$

$\therefore$ $$\int_C^D \vec{B}\cdot\vec{dl} = a_0\left(\frac{I_2}{I_1} - \frac{I_1}{I_2}\right)$$

$\Rightarrow$ $$2a_0 = a_0\left(\eta - \frac{1}{\eta}\right) \quad \left[\text{Let } \left(\frac{I_2}{I_1} = \eta\right)\right]$$

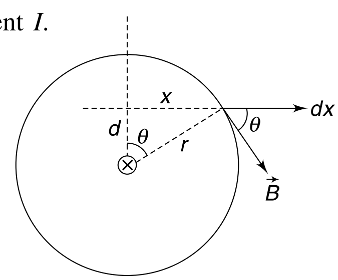
$\Rightarrow$ $$\eta^2 - 2\eta - 1 = 0$$

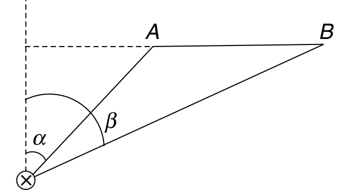
$\therefore$ $$\eta = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$\therefore$ $$\eta = 1 + \sqrt{2} \quad [\eta \text{ cannot be negative}]$$

$\therefore$ $$\frac{I_2}{I_1} = 1 + \sqrt{2}$$

30. **(a)** The figure shows a circular field line due to current I.





$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi d \sec\theta}$$

And $$x = d\tan\theta$$

$\therefore$ $$dx = \sec^2\theta d\theta$$

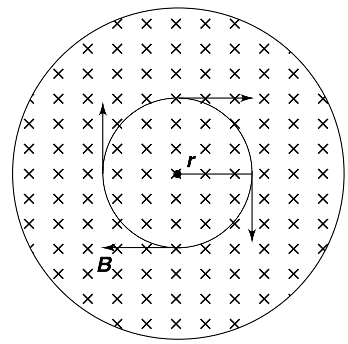
$$\int_A^B \vec{B}\cdot\vec{dx} = \int_A^B B\cdot dx \cos\theta$$

$$= \int_A^B \left(\frac{\mu_0 I}{2\pi r d \sec\theta}\right)(d\sec^2\theta d\theta)\cos\theta$$

$$= \frac{\mu_0 I}{2\pi}\int_\alpha^\beta d\theta = \frac{\mu_0 I}{2\pi}(\beta - \alpha)$$

$$= \frac{\mu_0 I}{2\pi}\theta$$

31. (a) Current $\quad I = \int_0^a (2\pi r dr)\, J = 2\pi J_0 \int_0^a \left(r - \frac{r^2}{a}\right) dr$



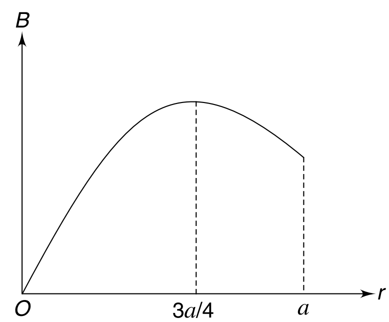
$\Rightarrow \quad I = \frac{1}{3}\pi a^2 J_0$

Mean current density is $\quad \bar{J} = \frac{I}{\pi a^2} = \frac{J_0}{3}$

(b) Current enclosed within distance r from the axis is

$$I_r = \int_0^r (2\pi r dr) J = 2\pi J_0 \int_0^r \left(r - \frac{r^2}{a}\right) dr$$

$$= \pi J_0 r^2 \left(1 - \frac{2r}{3a}\right)$$



Using ampere's law on a circle of radius r gives

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$B\cdot 2\pi r = \mu_0 I_r$$

$$B = \frac{\mu_0 J_0}{2} r \left(1 - \frac{2r}{3a}\right)$$

At $r = 0$; $B = 0$

At $r = a$; $B = \frac{\mu_0 J_0 a}{6}$

The graph is parabolic with maxima at distance given by

$$\frac{dB}{dr} = 0 \quad \Rightarrow \quad 1 - \frac{4r}{3a} = 0$$

$\Rightarrow \quad r = \frac{3a}{4}$

32. If the wire is of finite length there must be a return path for the current which will cause the value and direction of $\vec{B}$ to change from point to point on C. In the expression $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$; $\vec{B}$ is net field at a point produced by all sources.

33. Electric field will be calculated using Gauss' law. We need to consider a co – axial cylindrical Gaussian surface.

For $r < a$; $E = 0$

For $a < r < b$; $\quad E = \frac{-\lambda}{2\pi\epsilon_0 r} = \frac{-2\pi a\sigma}{2\pi\epsilon_0 r}$

$$= \frac{a\sigma}{\epsilon_0 r} \text{ (radially inward)}$$

For $r > b$; $\quad E = \frac{-2\pi a\sigma + 2\pi b\sigma}{2\pi\epsilon_0 r}$

$$E = \frac{(b - a)\sigma}{\epsilon_0 r} \text{ radially outward}$$

For magnetic field, we can consider the two cylinders just like ideal solenoids.

Field inside an ideal solenoid = $\mu_0 ni$ (parallel to the axis)

For inner cylinder $ni = (2\pi a)(1)(\sigma)\left(\frac{\omega}{2\pi}\right) = a\sigma\omega$

$\therefore$ For $r < a$

$$B = b\sigma\omega - a\sigma\omega = \sigma\omega(b - a)\ (\uparrow)$$ [sense of current is opposite in two solenoids]

For $a < r < b$

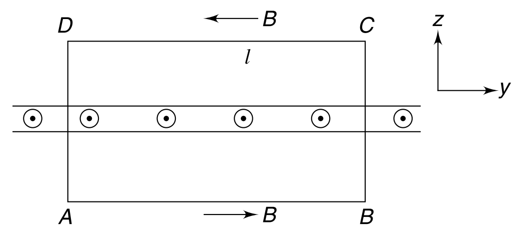
$$B = b\sigma\omega\ (\uparrow)$$ [There is no field due to a solenoid at an outside point]

For $r > b$

$$B = 0$$

34. (i) The $\overline{E}$ has got nothing to do with the motion of the sheet.

$\therefore$ $E = \frac{\sigma}{2\epsilon_0}$ perpendicular to the sheet away from it.



The motion of the sheet in x direction creates a current in x direction.

The symmetry (with right hand rule) shows that $\vec{B}$ will be in negative y direction above the sheet and in positive y direction below the sheet. Consider rectangular Amperian loop $ABCD$ as shown in the figure. AB and CD are equidistant from the surface.

$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = 2B\ell$$

Current through the loop $I = \sigma\ell v$

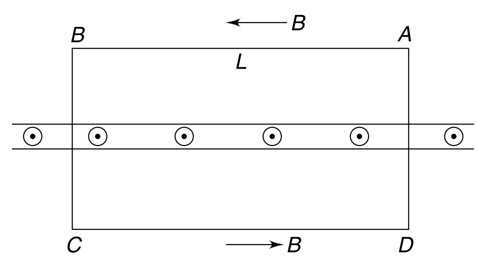
$\therefore$ Ampere's law gives

$$2B\ell = \mu_0 I$$

$$2B\ell = \mu_0\sigma\ell v \qquad \therefore \quad B = \frac{\mu_0\sigma v}{2}$$

(ii) E will not change. There will be no B.

35. (a) Due to symmetry the field at all point will be along $\pm x$ direction only.



Let's first find field due to one slab only. Consider an Amperian loop in shape of a rectangle as shown in the figure. Line AB and CD are located symmetrically wrt the slab. Field at AB and CD are of equal strength but in opposite directions.

$$\oint \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\cdot\vec{dl} + \int_B^C \vec{B}\cdot\vec{dl} + \int_C^D \vec{B}\cdot\vec{dl} + \int_D^A \vec{B}\cdot\vec{dl}$$

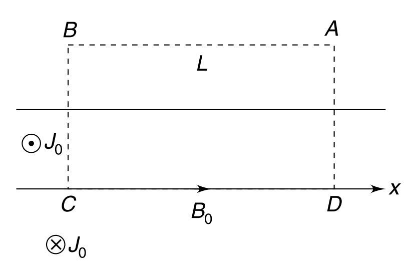
$$= BL + 0 + BL + + 0 = 2BL$$

Ampere's law gives

$$2BL = \mu_0 J_0 dL$$

$$\Rightarrow \qquad B = \frac{\mu_0 J_0 d}{2}$$

This is independent of distance of line AB from the slab.



It is obvious that $B = 0$ due to both the slabs at all points outside the slabs. Now consider a rectangular loop $ABCD$ with its side CD along x axis as shown.

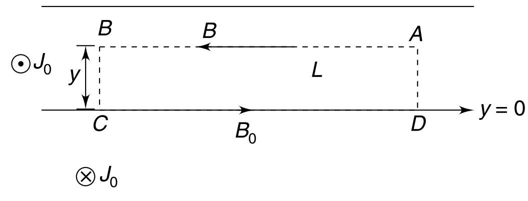
$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\cdot\vec{dl} + \int_B^C \vec{B}\cdot\vec{dl} + \int_C^D \vec{B}\cdot\vec{dl} + \int_D^A \vec{B}\cdot\vec{dl}$$

$$= 0 + 0 + B_0L + 0 = B_0L$$

Where B_0 is field at $y = 0$

Using Ampere's law gives

$$B_0L = \mu_0(J_0 dL) \Rightarrow B_0 = \mu_0 J_0 d$$



(b) Once again consider an Amperian loop as shown in figure. Let the field be B in negative x direction at line AB.

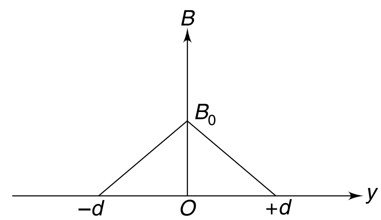
$$\oint_{ABCD} \vec{B}\cdot\vec{dl} = BL + 0 + B_0L + 0 = BL + B_0L$$

Using Ampere's law

$$BL + B_0L = \mu_0(J_0 yL)$$

$$\Rightarrow \quad B + \mu_0 J_0 d = \mu_0 J_0 y$$

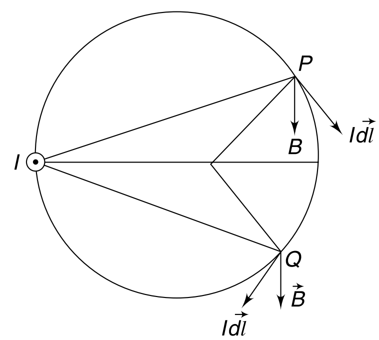
$$\Rightarrow \quad B = -\mu_0 J_0 (d - y)$$



Negative sign indicates that field is in positive x direction. Similarly, field at points

$$-d < y < 0 \text{ will be}$$

$$B = \mu_0 J_0(d + y) \text{ in positive } x \text{ direction}$$



36. Consider two symmetrically located elements $(I\vec{dl})$ on the arc at points P and Q. The direction of $I\vec{dl} \times \vec{B}$ at these two locations is opposite Hence, the force on the arc will be zero.

37. We will assume that the capacitor discharges quickly and there is no appreciable displacement of the wires in that interval.

Let initial charge on the capacitor be Q_0. Current at time t is

$$I = \frac{Q_0}{RC} e^{-t/RC}$$

Force between two parallel wires per unit length is

$$F = \frac{\mu_0 I^2}{2\pi d} = \frac{\mu_0 Q_0^2}{2\pi d R^2 C^2} e^{-2t/RC}$$

$$\Rightarrow \quad \lambda dv = \frac{\mu_0 Q_0^2}{2\pi d R^2 C^2} e^{-2t/RC}\, dt$$

$$\Rightarrow \quad \int_0^v dv = \frac{\mu_0 Q_0^2}{2\pi \lambda d R^2 C^2} \int_0^\infty e^{-2t/RC}\, dt$$

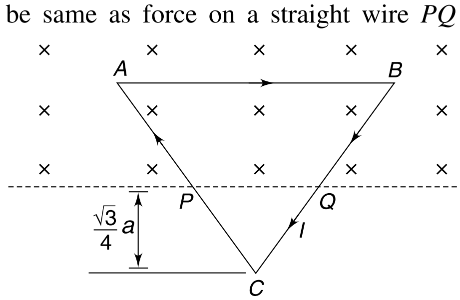
$$\Rightarrow \quad v = \frac{\mu_0 Q_0^2}{4\pi\lambda RCd} = \frac{\mu_0 U_0}{2\pi\lambda Rd} \qquad \left[\because \; U_0 = \frac{Q_0^2}{2C}\right]$$

38. (a) The current in the loop is clockwise. Magnetic force on the loop will be same as force on a straight wire PQ (current from P to Q)

Length $\quad PQ = 2\cdot\frac{\sqrt{3}}{4} a \tan 30° = \frac{a}{2}$

For equilibrium $\quad I\left(\frac{a}{2}\right) B = Mg$

$$\Rightarrow \quad I = \frac{2Mg}{aB} \qquad ...(i)$$



(b) Assume that the loop is displaced downward by a small distance y.

$$M\frac{d^2y}{dt^2} = Mg - I\left[2\left(\frac{\sqrt{3}}{4}a + y\right)\tan 30°\right]B$$

$$\Rightarrow \quad M\frac{d^2y}{dt^2} = Mg - I\frac{a}{2}B - \frac{2IB}{\sqrt{3}}y$$

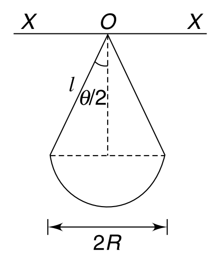
$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{2IB}{\sqrt{3}M}y \qquad \text{[using (i)]}$$

This is equation of SHM

$$\omega = \sqrt{\frac{2IB}{\sqrt{3}M}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\sqrt{3}M}{2IB}} = \pi\sqrt{\frac{\sqrt{3}a}{g}} \qquad \text{[using (i)]}$$

39. Area of the loop



$$A = \frac{1}{2}(2R)\left(\ell\cos\frac{\theta}{2}\right) + \pi R^2$$

Where $R = \ell\sin\frac{\theta}{2}$

$$\therefore \quad A = \ell^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \pi\ell^2\sin^2\frac{\theta}{2}$$

$$= \ell^2\sin\frac{\theta}{2}\left[\cos\frac{\theta}{2} + \pi\sin\frac{\theta}{2}\right]$$

Area vector is normal to the incline plane. Angle between $\vec{A}$ and $\vec{B}$ is θ.

$$\mu = IA$$

$$\tau = IAB\sin\theta = IB\ell^2\sin\theta\cdot\sin\frac{\theta}{2}\left[\cos\frac{\theta}{2} + \pi\sin\frac{\theta}{2}\right]$$

This torque is along XX shown in the figure. It will try to rotate the loop (about XX) so as to lift it up from the plane with support at O. But net force on the loop is zero.

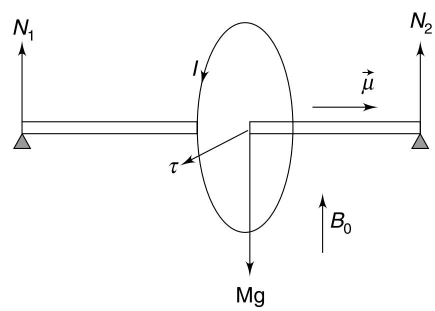
$\therefore$ Normal contact force will not change

40. Magnetic dipole moment of loop is $\mu = I\pi R^2$ (direction as shown)

Magnetic torque on the loop is

$$\tau = \mu B_0\sin 90° = I\pi R^2 B_0 \text{ (in the direction shown)}$$

Balancing torque on the disc gives



$$\tau + N_2\frac{d}{2} = N_1\frac{d}{2}$$

$$\Rightarrow \quad N_1 - N_2 = \frac{2I\pi R^2 B_0}{d} \qquad \text{...(i)}$$

Also $\quad N_1 + N_2 = Mg \qquad$ ...(ii)

Solving (i) and (ii) gives

$$N_1 = \frac{Mg}{2} + \frac{I\pi R^2 B_0}{d}$$

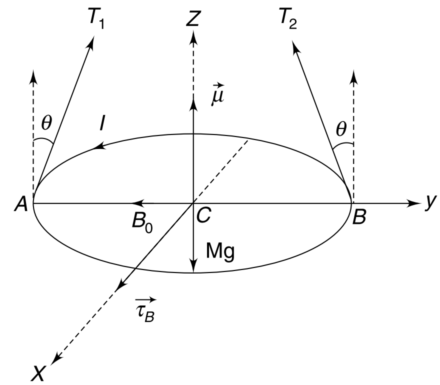
$$N_2 = \frac{Mg}{2} - \frac{I\pi R^2 B_0}{d}$$

$$N_2 < N_1$$

N_2 will become zero if

$$\frac{I\pi R^2 B_0}{d} = \frac{Mg}{2}$$

$$\Rightarrow \qquad I = \frac{Mgd}{2\pi R^2 B_0}$$



41. Magnetic dipole moment $\qquad \vec{\mu} = I \cdot \pi \; R^2 \hat{k}$

Magnetic torque $\qquad \vec{\tau}_B = \vec{\mu} \times \vec{B}$

$$= I\pi R^2 B_0 \,(\hat{k} \times (-\hat{j})) = I\pi R^2 B_0 \;(\hat{i})$$

To counterbalance this torque we must have $T_1 > T_2$

Torque about centre C

$$-(T_1 \cos\theta \cdot R - T_2 \cos\theta R)\,\hat{i} + I\pi R^2 B_0 \hat{i} = 0$$

$$\therefore \qquad T_1 - T_2 = \frac{I\pi R B_0}{\cos\theta} \qquad ...(i)$$

And $\qquad (T_1 + T_2)\cos\theta = Mg$

$$\Rightarrow \qquad T_1 + T_2 = \frac{Mg}{\cos\theta} \qquad ...(ii)$$

Solving (i) and (ii)

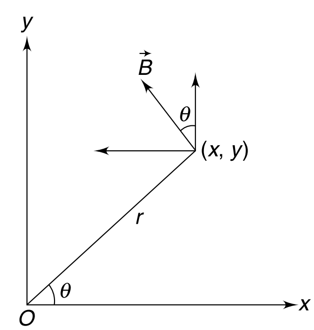
$$T_1 = \frac{1}{2\cos\theta}(Mg + \pi IRB_0) = Mg + \pi IRB_0 \;\text{ and }\; T_2 = Mg - \pi IRB_0$$

42. Field at any point (x, y) can be written as

$$B = -B\sin\theta\hat{i} + B\cos\theta\hat{j}$$

$$= -\frac{k}{\sqrt{x^2+y^2}} \cdot \frac{y}{\sqrt{x^2+y^2}}\hat{i} + \frac{k}{\sqrt{x^2+y^2}} \cdot \frac{x}{\sqrt{x^2+y^2}}\hat{j}$$

$$= -\frac{ky}{(x^2+y^2)}\hat{i} + \frac{kx}{(x^2+y^2)}\hat{j}$$



A small segment of the wire at point (x, y) can be taken as

$$\vec{dl} = dx\hat{i} + dy\hat{j}$$

Force on segment $\vec{dl}$ is

$$d\vec{F} = I\vec{dl} \times \vec{B}$$

$$= I\frac{kx\,dx}{x^2+y^2}\hat{k} + I\frac{ky\,dy}{x^2+y^2}\hat{k}$$

$\therefore$ Force on wire AB is

$$\vec{F} = \int d\vec{F} = kI \int_{A(x_1,\, y_1)}^{B(x_2,\, y_2)} \frac{xdx + ydy}{x^2+y^2}\hat{k}$$

If $\qquad x^2 + y^2 = t$

then $\qquad 2x\,dx + 2y\,dy = dt$

$$x\,dx + y\,dy = \frac{1}{2}dt$$

$$\therefore \qquad \vec{F} = \frac{kI}{2}\int_A^B \frac{dt}{t} \cdot \hat{k}$$

$$= \frac{kI}{2}[\ell nt]_{x_1, y_1}^{x_2, y_2}\hat{k}$$

$$= \frac{kI}{2}\ell n\left(\frac{x_2^2 + y_2^2}{x_1^2 + y_1^2}\right)\hat{k}$$

43. Consider an Amperian loop in shape of a circle passing through P. Due to symmetry field at all points on the circle will have same magnitude and tangential direction.

$$\therefore \quad \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi d}$$

44. Unit vectors along $\vec{r}$ and perpendicular to $\vec{r}$ (known as azimuthal direction) are $\hat{r}$ and $\hat{\theta}$ respectively. A small element of length $d\ell$ along the curve can be represented as

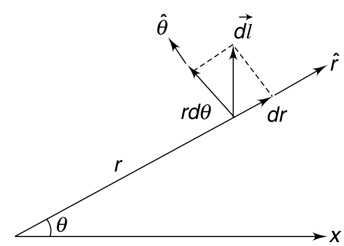
$$\vec{dl} = (dr)\hat{r} + (rd\theta)\ \hat{\theta}$$

Field due to such an element at O is

$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times (-\hat{r})}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{1}{r^2}[(dr)\hat{r} + (rd\,\theta)\,\hat{\theta}) \times (-\hat{r})]$$

$$= \frac{\mu_0 I}{4\pi}\frac{I d\theta}{r}\hat{k}$$



$$\therefore \quad B = \frac{\mu_0 I}{4\pi}\int_0^{\pi/2}\frac{d\theta}{b + \frac{c}{\pi}\theta} \odot$$

$$= \frac{\mu_0 I}{4\pi}\frac{\pi}{c}\left[\ell n\left(b + \frac{c}{\pi}\theta\right)\right]_0^{\pi/2}$$

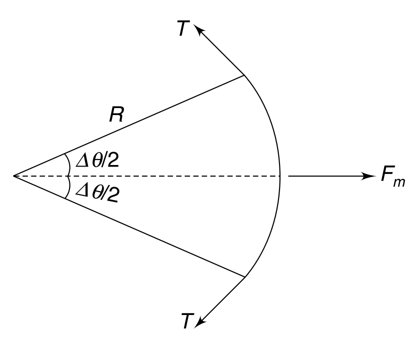
$$= \frac{\mu_0 I}{4c}\left[\ell n\left(b + \frac{c}{2}\right) - \ell nb\right]$$

$$B = \frac{\mu_0 I}{4c}\left[\ell n\left(1 + \frac{c}{2b}\right)\right] \odot$$

45. Magnetic force on any small element of the wire is perpendicular to its length. It means that the tension will be constant along the wire.

Consider a small segment of the wire subtending a small angle $\Delta\theta$ at the centre of curvature. Magnetic force is

$$F_m = I(R\Delta\,\theta)B$$



This is balanced by components of tension

$$= 2\cdot T\sin\left(\frac{\Delta\theta}{2}\right) = T\Delta\theta$$

$$\therefore \quad T\Delta\theta = I(R\Delta\,\theta)B \quad \Rightarrow \quad T = IRB$$

Or, $$R = \frac{T}{IB} = \text{constant}$$

Radius of curvature remains constant means that the two segments will take the shape of circular arcs.