JEE 2023

Definite Integration

DPP-05

1. The value of

$$\lim_{n\to\infty}\left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right).....\left(1+\frac{n^2}{n^2}\right)\right\}^{\frac{1}{n}}$$ will be:

(1) $e^{\frac{\pi-4}{2}}$ (2) $\frac{1}{2}e^{\frac{\pi-2}{2}}$

(3) $2e^{\frac{\pi-2}{2}}$ (4) $2e^{\frac{\pi-4}{2}}$

2. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+.....+\frac{1}{n}\sec^2 1\right]$ equals

(1) $\tan 1$ (2) $\frac{1}{2}\tan 1$

(3) $\frac{1}{2}\sec 1$ (4) $\frac{1}{2}\operatorname{cosec} 1$

3. The value of x satisfying

$\int_0^{2[x+14]}\left\{\frac{x}{2}\right\}dx=\int_0^{\{x\}}[x+14]dx$ is equal to (where. [.] and {.} denotes the greatest integer and fraction part of x)

(1) $[-14,-13)$ (2) $(0, 1)$

(3) $(-15,-14]$ (4) None of these

4. Consider $I_1=\int_0^1 e^{-x}\cos^2 x\,dx$,

$I_2=\int_0^1 e^{-x^2}\cos^2 x dx$, $I_3=\int_0^1 e^{\frac{-x^2}{2}}\cos^2 x dx$,

$I_4=\int_0^1 e^{\frac{-x^2}{2}}dx$, then correct sequence is:

(1) $I_2>I_4>I_1>I_3$ (2) $I_2<I_4<I_1<I_3$

(3) $I_1<I_2<I_3<I_4$ (4) $I_1>I_2>I_3>I_4$

5. Maximum value of integral $\int_0^1\sqrt{(1+x)(1+x^3)}\,dx$ is

(1) $\sqrt{\frac{15}{8}}$ (2) $\sqrt{\frac{8}{15}}$

(3) $\sqrt{\frac{1}{8}}$ (4) $\sqrt{\frac{13}{8}}$

6. Prove that $0<\int_0^1\frac{x^7dx}{\sqrt[3]{1+x^8}}<\frac{1}{8}$.

7. If $\int_0^x\frac{bt\cos 4t-a\sin 4t}{t^2}dt=\frac{a\sin 4x}{x}$ for all $x\neq 0$, then a and b are given by

(1) $a=\frac{1}{4}, b=1$ (2) $a = 2, b = 2$

(3) $a = -1, b = 4$ (4) $a = 2, b = 4$

8. Let $I=\int_0^1\frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx$. Then, which one of the following is true?

(1) $I>\frac{2}{3}$ and $J<2$ (2) $I>\frac{2}{3}$ and $J>2$

(3) $I<\frac{2}{3}$ and $J<2$ (4) $I<\frac{2}{3}$ and $J>2$

9. If $f(x)=\int_{x^2}^{x^2+1}e^{-t^2}dt$, then $f(x)$ increases in

(1) $(2, 2)$ (2) No value of x

(3) $(0, \infty)$ (4) $(-\infty, 0)$

10. If $\int_{\frac{\pi}{2}}^{x}\sqrt{(3-x\sin^2 t)}dt+\int_0^y\cos t\,dt=0$ then, $\left(\frac{dy}{dx}\right)_{(\pi,\pi)}$ is equal to

(1) -3 (2) 0

(3) $\sqrt{3}$ (4) None of these

Answer Key

1. **(4)**
2. **(2)**
3. **(1)**
4. **(3)**
5. **(1)**
6. **(Proof)**
7. **(1)**
8. **(3)**
9. **(4)**
10. **(3)**