Chapter 1

Rational Numbers

1.0 Introduction

Salma wants to buy three pens at five rupees each. Her friend Satheesh wants to buy two similar pens. So they go to a wholesale shop. The shopkeeper says that a packet of five pens costs ₹ 22. How much does each pen cost? We can easily calculate the cost of each pen to be ₹ $\frac{22}{5}$. Is there any natural number to represent this cost? Is there any whole number or integer to represent this? This number can not be represented by a whole number. We need a fractional number to represent this quantity.

Consider one more example.

Observe the following various readings of temperature recorded on a particular day in Simla.

Timings	10.00 a.m.	12.00 Noon	3.00 p.m.	7.00 p.m.	10.00 p.m.
Temperature	11 °C	14 °C	17 °C	10 °C	5 °C

In each case what is the change in temperature per hour?

Case I Morning hours : change in temperature per hour = $\frac{14\,^\circ C - 11\,^\circ C}{2} = \frac{3}{2}$ °C/hrs.
(10.00 A.M. - 12.00 Noon)

Case II Afternoon hours: change in temperature per hour = $\frac{17\,^\circ C - 14\,^\circ C}{3}$ = 1 °C/hrs.
(12.00 Noon - 3.00 P.M.)

Case III Evening hours : change in temperature per hour = $\frac{10\,^\circ C - 17\,^\circ C}{4} = \frac{-7}{4}$ °C/hrs.
(3.00 P.M. - 7.00 P.M.)

Case IV Night hours : change in temperature per hour = $\frac{5\,^\circ C - 10\,^\circ C}{3} = \frac{-5}{3}$ °C/hrs.
(7.00 P.M. - 10.00 P.M.)

In the above cases we come across numbers like $\frac{3}{2}$ °C, 1 °C, $\frac{-7}{4}$ °C, $\frac{-5}{3}$ °C.

The numbers used in these temperature are $\frac{3}{2}$, 1 , $\frac{-7}{3}$, $\frac{-5}{3}$. What can be call these numbers? These numbers include both positive and negative fractional numbers and can be written as $\frac{p}{q}$, where $q \neq o$ and p can be positive or negative.

Let us see some more such numbers.

$\frac{3}{4}, \frac{7}{9}, \frac{-10}{17}, \frac{3}{-2}, \frac{2013}{2014}$, ...

The numbers which are expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq o$, are called 'Rational Numbers' and the set of all rational numbers is denoted by 'Q'. These are also called Quotient numbers.

Observe the following

We can express any natural number in $\frac{p}{q}$, form, for ex. 5 as $\frac{5}{1}$ or $\frac{10}{2}$ or $\frac{15}{3}$.........

Similarly we can also express the whole number 0 as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{5}$ ....

We can also express an integer in $\frac{p}{q}$ form, for ex. –3 as $\frac{-3}{1}$ or $\frac{-6}{2}$

That is all the above $\frac{p}{q}$ forms $\frac{15}{3}$, $\frac{0}{5}$ and $\frac{-6}{2}$ are rational numbers. So all integers are also included in rational numbers. From the above observation we can conclude that all natural numbers, all whole numbers and all integers are rational numbers.

Do This

Consider the following collection of numbers 1, $\frac{1}{2}$, –2, 0.5, $4\frac{1}{2}$, $\frac{-33}{7}$, 0, $\frac{4}{7}$, 22, –5, $\frac{2}{19}$, 0.125. Write these numbers under the appropriate category.

[A number can be written in more than one collection]

(i) Natural numbers ____________________

(ii) Whole numbers ____________________

(iii) Integers ____________________

(iv) Rational numbers ____________________

Would you leave out any of the given numbers from rational numbers?

Is every natural number, whole number and integer a rational number ?

Try These

1. Hamid says $\frac{5}{3}$ is a rational number and 5 is only a natural number.

   Sakshi says both are rational numbers. Who do you agree with?

2. Give an example to satisfy the following statements.

   (i) All natural numbers are whole numbers but all whole numbers need not be natural numbers.

   (ii) All whole numbers are integers but all integers are not whole numbers.

   (iii) All integers are rational numbers but all rational numbers need not be integers.

We have already learnt basic operations on rational numbers in earlier classes. Let us explore some properties of operations on rational numbers.

1.1 Operations on Rational numbers

We have already discussed addition and subtraction of Rational numbers in 7th class. Let us recall them by doing the following problems.

Solve

(i) $\frac{9}{10}+\left(\frac{-13}{8}\right)$ (ii) $1\frac{3}{5}+4\frac{2}{7}$

(iii) $\frac{-7}{16}-\left(\frac{-9}{20}\right)$ (iv) $\frac{-11}{14}-\left(\frac{1}{21}\right)$

(v) Find the additive inverse of the following numbers : $\frac{-7}{6}, \frac{1}{10}, \frac{-3}{4}, 8$

1.1.1 Multiplication of Rational Numbers

Now, we learn how to multiply the rational numbers. In class 7 we have learnt how to multiply fractional numbers. We follow a similar process for multiplication of rational numbers also.

Consider the rational numbers $\frac{2}{3}$ and $\frac{5}{7}$. These are also fractional numbers.

We multiply $\frac{2}{3}$ and $\frac{5}{7}$

$$\frac{2}{3}\times\frac{5}{7}=\frac{2\times 5}{3\times 7}=\frac{10}{21} \qquad \left(\frac{\textit{Product of numerator}}{\textit{Product of denominator}}\right)$$

Now consider $\frac{-2}{3} \times \frac{5}{7}$

We get $\frac{-2 \times 5}{3 \times 7} = \frac{-10}{21}$

Let us do one more example $\frac{-10}{21} \times \frac{14}{25}$

$$\frac{-10}{21} \times \frac{14}{25} = \frac{-10 \times 14}{21 \times 25} = \frac{\cancelto{28}{-140}}{\cancelto{105}{525}} = \frac{-28}{105} = \frac{-4}{15}$$

Or we can do like this also.

$$\frac{-\cancelto{2}{10}}{\cancelto{3}{21}} \times \frac{\cancelto{2}{14}}{\cancelto{5}{25}} = \frac{-4}{15}$$

Do this

(i) $\frac{18}{11} \times \frac{-33}{45}$ (ii) $\frac{-7}{17} \times \frac{-1}{10}$

(iii) $\frac{-105}{72} \times \frac{18}{15}$ (iv) $\frac{13}{120} \times \frac{100}{16}$

1.1.2 Division of Rational Numbers

Observe the following

$\frac{2}{5} \times \frac{5}{2} = 1$

$\frac{-9}{11} \times \frac{11}{-9} = 1$

Here we notice that the product is '1'. If the product of any two rational numbers is '1' are called the multiplicative inverse of each other. Here $\frac{2}{5}$ and $\frac{5}{2}$; $\frac{-9}{11}$ and $\frac{11}{-9}$ are multiplicative inverse of each other.

Write multiplicative inverse of $\frac{-3}{7}$, 11, $\frac{9}{5}$, $\frac{1}{-17}$. In class 7 we have learnt to divide fractional numbers. We follow a similar process for division of Rational numbers.

Consider the rational number $\frac{3}{4}$ and $\frac{7}{11}$. These are also fractional numbers.

We devide $\frac{3}{4}$ with $\frac{7}{11}$.

$$\frac{3}{4} \div \frac{7}{11} = \frac{3}{4} \times \frac{11}{7} \qquad \text{(Multiplicative inverse of } \frac{7}{11}\text{)}$$

$$= \frac{3 \times 11}{4 \times 7} = \frac{33}{28} = 1\frac{5}{28}$$

Let's do the following examples :

(i) $\frac{-5}{9} \div \frac{3}{4} = \frac{-5}{9} \times \frac{4}{3}$ (Multiplicative inverse of $\frac{3}{4}$)

$$= \frac{-5 \times 4}{9 \times 3} = \frac{-20}{27}$$

(ii) $\frac{-12}{21} \div \left(\frac{2}{-7}\right) = \frac{-\cancel{12}^{\,6}}{\cancel{21}_{\,3}} \times \left(\frac{-\cancel{7}}{\cancel{2}}\right) = \frac{\cancel{6}}{\cancel{3}} = 2$ (Multiplicative inverse of $\frac{2}{-7}$)

Do this

(i) $\frac{8}{5} \div \frac{2}{3}$ (ii) $\frac{18}{25} \div \left(\frac{-72}{75}\right)$

(iii) $\frac{-125}{64} \div \frac{50}{16}$ (iv) Divide $\frac{-512}{441}$ with $\frac{-1024}{21}$

1.2 Properties of Rational numbers

1.2.1 Closure property

(i) **Whole numbers and Integers**

Let us recall the operations under which the whole numbers and integers are closed.

If the sum of two whole numbers is also a whole number, then, we say that the set of whole numbers satisfy closure property under addition.

Complete the following table with necessary arguements and relevant examples.

Numbers	Operations			
	Addition	**Subtraction**	**Multiplications**	**Division**
Whole numbers	Closed since a + b is a whole number for any two whole numbers *a* and *b* example – –	Not closed since 5–7 = –2 which is not a whole number	Closed since – – – – – –	Not closed since 5 ÷ 8 = $\frac{5}{8}$ which is not a whole number.
Integers	– – –	Closed Since a – b is an integer for any two integers a and b example.	– – – – – –	Not closed since – – –

(ii) Rational numbers - Closure Property

(a) Addition

Consider two rational numbers $\frac{2}{7}, \frac{5}{8}$

$$\frac{2}{7}+\frac{5}{8} = \frac{16+35}{56} = \frac{51}{56}$$

The result $\frac{51}{56}$ is again a rational number

$8+\left(\frac{-19}{2}\right)=$ ______ Is it a rational number?

$\frac{2}{7}+\frac{-2}{7}=$ ______ Do you get a rational number?

Check this for few more in the following pairs.

$3+\frac{5}{7},\quad 0+\frac{1}{2},\quad \frac{7}{2}+\frac{2}{7}$

We observe that the sum of any two rational numbers is again a rational number. Thus we can say that rational numbers are closed under addition. Hence (a + b) is a rational number for any two rational numbers a and b, i.e. $\forall a, b \in$ Q ; $(a + b) \in$ Q.

> $\in$ **belongs to,**
>
> Let A = {1, 2, 3}
>
> The element 3 is in A and it is written as $3 \in A$ and we read it as 3 belongs to A.
>
> $\forall$ **for all**
>
> $\forall$ is the symbol for all or for every.
>
> If we write $\forall\ a, b \in Q$, it means for all *a*, *b* of Q

(b) Subtraction:

Consider two rational numbers $\frac{5}{9}$ and $\frac{3}{4}$

Then $\frac{5}{9}-\frac{3}{4} = \frac{(5\times4)-(3\times9)}{36} = \frac{20-27}{36} = \frac{-7}{36}$

Again we got a rational number $\frac{-7}{36}$ (since –7, 36 are integers and 36 is not a zero, hence $\frac{-7}{36}$ is a rational number).

Check this in the following rational numbers also.

(i) $\frac{2}{3} - \frac{3}{7} = \frac{14-9}{21} =$ _______________ Is it a rational number ?

(ii) $\left(\frac{48}{9}\right) - \frac{11}{18} =$ _______________ Is it a rational number?

We find that the difference is also a rational number for any two rational numbers.

Thus rational numbers are closed under subtraction.

Hence (a – b) is a rational number for any two rational number 'a' and 'b', i.e., $\forall$ a, b $\in$ Q, (a–b) $\in$Q

(c) Multiplication

Observe the following

$3 \times \frac{1}{2} = \frac{3}{2}$

$\frac{6}{5} \times \frac{-11}{2} = \frac{-66}{10} = \frac{-33}{5}$

$\frac{3}{7} \times \frac{5}{2} =$ __________ ; $\frac{2}{1} \times \frac{19}{13} =$ ___________

We can notice that in all the cases the product of two rational numbers is a rational number.

Try for some more pairs of rational numbers and check whether their product is a rational number or not . Can you find any two rational numbers whose product is not a rational number?

We find that rational numbers are closed under multiplication

For any two rational numbers a and b, a×b is also rational number. i.e., $\forall$ a, b $\in$ Q, a × b $\in$Q

(d) Division

Consider two rational numbers.

$\frac{2}{3}, \frac{7}{8}$

Then $\frac{2}{3} \div \frac{7}{8} = \frac{2}{3} \times \frac{8}{7} = \frac{16}{21}$ which is a rational number

Check this for two more example.

$\frac{5}{7} \div 2 = \frac{5}{7} \div \frac{2}{1} = \frac{5}{7} \times \frac{1}{2} = \frac{5}{14}$

$-\frac{2}{3} \div \frac{6}{11} =$ __________ = __________ = __________

$3 \div \frac{17}{13} = \frac{3}{1} \div \frac{17}{13} =$ __________ = __________ = __________

From all the above, we observe that when we divide two rational numbers,we get a rational number. Now can we say that the closure property holds good for rational numbers? Let us check the following: 0, 5 are rational numbers and $\frac{5}{0}$ is not defined. Thus collection of Rational numbers Q is not closed with respect to division.

If we exclude zero from Q then the resulting collection is closed under division.

> Why $\frac{5}{0}$ is not defined ?
> Do the division 5 ÷ 0 = ?
> Or 0) 5 (?
> Can you complete the division? What is the quotient? You may have observed that multiplication of any number with '0', the product will be '0'.
> Thus division is not possible, with '0' as divisor.

Try These

If we exclude zero from the set of integers then is it closed under division?

Do This

Fill the blanks in the table

Numbers	Closure property under			
	Addition	Subtraction	Multiplication	Division
Natural numbers	Yes	—	—	—
Whole numbers	—	—	—	No
Integers	—	Yes	—	—
Rational numbers	—	—	Yes	—

1.2.2. Commutative Property:

The commutative property states that the change in the order of two numbers on binary operation does not change the result.

$a + b = b + a$

$a \times b = b \times a$

here binary operation could be any one of the four fundamental operations i.e., +, –, ×, ÷

Let us recall the commutative property with different operations for both whole numbers and Integers.

Complete the following table.

(i) Whole numbers

Operation	Example	Remark
Addition	2, 3 are whole numbers $2+3 = 5$ and $3 + 2 = 5$ $\therefore 2 + 3 = 3 + 2$	Addition is commutative in W.
Subtraction	Is $3 - 2$ equal to $2 - 3$?	Subtraction is not commutative
Multiplication	------	------
Division	$4 \div 2 = ?$ $2 \div 4 = ?$ Is $4 \div 2 = 2 \div 4$?	-------

(ii) Integers

Operation	**Example**	**Remark**
Addition	---	Addition is commutative in Integers.
Subtraction	2, 3 are integers $2 - (3) = ?; (3) - 2 = ?$ Is $2 - (3) = (3) - 2 = ?$	....
Multiplication	... ...	... ...
Division	... ...	Division is not Commutative in Integers

(iii) Rational Numbers

(a) Addition

Take two rational numbers $\frac{5}{2}, \frac{-3}{4}$ and add them

$$\frac{5}{2} + \frac{(-3)}{4} = \frac{2\times5+1\times(-3)}{4} = \frac{10-3}{4} = \frac{7}{4}$$

and $\frac{(-3)}{4} + \frac{5}{2} = \frac{1\times(-3)+2\times5}{4} = \frac{-3+10}{4} = \frac{7}{4}$

so $\frac{5}{2} + \left(\frac{-3}{4}\right) = \frac{-3}{4} + \frac{5}{2}$

Now check this rule for some more pairs of rational numbers.

Consider $\frac{1}{2} + \frac{5}{7}$ and $\frac{5}{7} + \frac{1}{2}$.

Is $\frac{1}{2} + \frac{5}{7} = \frac{5}{7} + \frac{1}{2}$?

Is $\frac{-2}{3} + \left(\frac{-4}{5}\right) = \frac{(-4)}{5} + \left(\frac{-2}{3}\right)$?

Did you find any pair of rational numbers whose sum changes when we reverse the order of numbers ? So, we can say that a + b = b + a for any two rational numbers a and b.

Thus addition is commutative in the set of rational numbers.

$\therefore\ \forall$ a, b $\in$ Q, a + b = b + a

(b) Subtraction: Take two rational numbers $\frac{2}{3}$ and $\frac{7}{8}$

$\frac{2}{3} - \frac{7}{8} = \frac{16-21}{24} = \frac{-5}{24}$ and $\frac{7}{8} - \frac{2}{3} = \frac{21-16}{24} = \frac{5}{24}$

So $\frac{2}{3} - \frac{7}{8} \neq \frac{7}{8} - \frac{2}{3}$

Check the following.

Is $2 - \frac{5}{4} = \frac{5}{4} - 2$?

Is $\frac{1}{2} - \frac{3}{5} = \frac{3}{5} - \frac{1}{2}$?

Thus we can say that subtraction is not commutative in the set of rational numbers .

a – b $\neq$ b – a for any two rational numbers a and b.

(c) Multiplication: Take two rational numbers 2, $-\frac{5}{7}$

$$2\times\frac{-5}{7}=\frac{-10}{7}\ ;\quad \frac{-5}{7}\times 2=\frac{-10}{7}\quad \text{therefore}\quad 2\times\frac{-5}{7}=\frac{-5}{7}\times 2$$

Is $\frac{-1}{2}\times\left(\frac{-3}{4}\right)=\left(\frac{-3}{4}\right)\times\left(\frac{-1}{2}\right)$?

Check for some more rational numbers .

We can conclude that multiplication is commutative in the set of rational numbers.

It means a × b = b × a for any two rational numbers a and b.

i.e. $\forall$ a, b $\in$ Q, a × b = b × a

(d) Division

Is $\frac{7}{3}\div\frac{14}{9}=\frac{14}{9}\div\frac{7}{3}$?

$$\frac{7}{3}\div\frac{14}{9}=\frac{\cancel{7}^{1}}{\cancel{3}_{1}}\times\frac{\cancel{9}^{3}}{\cancel{14}_{2}}=\frac{3}{2}\quad\text{and}\quad\frac{14}{9}\div\frac{7}{3}=\frac{\cancel{14}^{2}}{\cancel{9}_{3}}\times\frac{\cancel{3}^{1}}{\cancel{7}_{1}}=\frac{2}{3}$$

$$\frac{7}{3}\div\frac{14}{9}\neq\frac{14}{9}\div\frac{7}{3}$$

Thus we can say that division of rational numbers is not commutative in the set of rational numbers .

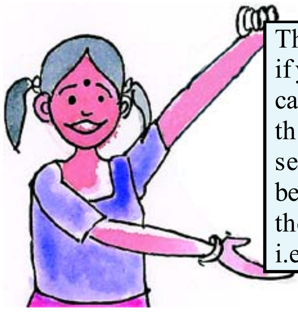
Do This

Complete the following table.

Numbers	commutative with respect to			
	Addition	Subtraction	Multiplication	Division
Natural numbers	Yes	No	Yes	— — —
Whole numbers	— — —	— — —	— — —	No
Integers	— — —	— — —	— — —	— — —
Rational numbers	— — —	— — —	— — —	No

1.2.3 Associative Property

Recall the associative property of whole numbers with respect to four operations, i.e. addition, subtraction, multiplication & division.



The associative property states that if you have to add three numbers, you can add the first two numbers and then add the third or you add the second and third number in the beginning and then add the first to the sum. The result will be the same i.e. $(3+2)+5$ or $3+(2+5)$.

(i) Whole numbers

Complete the table with necessary illustrations and remarks.

Operation	Examples with whole numbers	Remark
Addition	Is $2 + (3 + 0) = (2 + 3) + 0$? $2 + (3 + 0) = 2 + 3 = 5$ $(2 + 3) + 0 = 5 + 0 = 5$ $\Rightarrow 2 + (3 + 0) = (2 + 3) + 0$ $a + (b + c) = (a + b) + c$ for any three whole numbers a, b, c	— — — — — —
Subtraction	$(2-3) - 2 = ?$; $2-(3-2) = ?$ Is $(2-3) -2 = 2-(3-2)$?	Subtraction is not associative
Multiplication	— — — — — — — — — — — —	Multiplication is associative
Division	Is $2 \div (3 \div 5) = (2 \div 3) \div 5$? $2 \div (3 \div 5) = 2 \div \frac{3}{5} = 2 \times \frac{5}{3} = \frac{10}{3}$ $(2 \div 3) \div 5 = \frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$ $2 \div (3 \div 5) \neq (2 \div 3) \div 5$	Division is not associative

(ii) Integers

Recall associativity for integers under four operations.

Complete the following table with necessary remarks.

Operation	Integers with example	Remark
Addition	Is 2 + [(–3) + 5)] = [(2 + (–3)] + 5 ? $2+\left[(-3)+5\right]=2+\left[-3+5\right]=2+2=4$ $\left[2+(-3)\right]+5=\left[2-3\right]+5=-1+5=4$ For any three integers a, b and c a + (b + c) = (a + b) + c	— — —— —
Subtraction	Is 6 – (9 – 5) = (6 – 9) – 5 ?	— — —— —
Multiplication	Is $2\times[7\times(-3)]=(2\times7)\times(-3)$?	— — —— —
Division	$10\div\left[2\div(-5)\right]=[10\div2]\div(-5)$? $10\div\left[2\div(-5)\right]=10\div\frac{-2}{5}=10\times\frac{-5}{2}=-25$ Now $(10\div2)\div(-5)=\frac{10}{2}\div(-5)=5\div(-5)=\frac{5}{-5}=-1$ Thus $10\div\left[2\div(-5)\right]\neq[10\div2]\div(-5)$	— — —— —

(iii) Rational numbers

(a) Addition

Let us consider three rational numbers $\frac{2}{7}, 5, \frac{1}{2}$ and verify whether

$$\frac{2}{7}+\left[5+\left(\frac{1}{2}\right)\right]=\left[\left(\frac{2}{7}+5\right)\right]+\left(\frac{1}{2}\right)$$

$$\text{L.H.S.}=\frac{2}{7}+\left[5+\left(\frac{1}{2}\right)\right]=\frac{2}{7}+\left[5+\frac{1}{2}\right]=\frac{2}{7}+\left[\frac{10+1}{2}\right]=\frac{4+77}{14}=\frac{81}{14}$$

$$\text{R.H.S.}=\left[\left(\frac{2}{7}+5\right)\right]+\left(\frac{1}{2}\right)=\left[\left(\frac{2+35}{7}\right)\right]+\frac{1}{2}=\frac{37}{7}+\frac{1}{2}=\frac{74+7}{14}=\frac{81}{14}$$

L.H.S. = R.H.S.

Find $\frac{1}{2}+\left[\frac{3}{7}+\frac{4}{3}\right]$ and $\left[\frac{1}{2}+\frac{3}{7}\right]+\left(\frac{4}{3}\right)$

Are the two sums equal?

Take some more rational numbers and verify the associativity.

We find rational numbers satisfy associative property under addition.

a + (b + c) = (a + b) + c for any three rational numbers a, b and c.

i.e., $\forall$ a, b, c $\in$ Q, a + (b + c) = (a + b) + c

(b) Subtraction

Let us take any three rational numbers $\frac{1}{2}, \frac{3}{4}$ and $\frac{-5}{4}$

Verify whether $\frac{1}{2} - \left[\frac{3}{4} - \left(\frac{-5}{4}\right)\right] = \left[\frac{1}{2} - \frac{3}{4}\right] - \left(\frac{-5}{4}\right)$

$$\text{L.H.S.} = \frac{1}{2} - \left[\frac{3}{4} - \left(\frac{-5}{4}\right)\right] = \frac{1}{2} - \left[\frac{3}{4} + \frac{5}{4}\right] = \frac{1}{2} - \left[\frac{8}{4}\right]$$

$$= \frac{1}{2} - 2 = \frac{1-4}{2} = \frac{-3}{2}$$

$$\text{R.H.S.} = \left(\frac{1}{2} - \frac{3}{4}\right) - \left(\frac{-5}{4}\right) = \left(\frac{1\times 2 - 3}{4}\right) + \frac{5}{4} = \left(\frac{-1}{4}\right) + \frac{5}{4}$$

$$= \frac{-1+5}{4} = \frac{4}{4} = 1$$

$$\therefore \frac{1}{2} - \left[\frac{3}{4} - \left(\frac{-5}{4}\right)\right] \neq \left(\frac{1}{2} - \frac{3}{4}\right) - \left(\frac{-5}{4}\right)$$

L.H.S. $\neq$ R.H.S.

We find subtraction is not associative in the set of rational numbers. That is a - (b - c) $\neq$ (a - b) - c for any three rational numbers a, b, c.

(c) Multiplication

Take three rational numbers $\frac{2}{3}, \frac{4}{7}, \frac{-5}{7}$

Is $\frac{2}{3} \times \left[\frac{4}{7} \times \left(\frac{-5}{7}\right)\right] = \left(\frac{2}{3} \times \frac{4}{7}\right) \times \left(\frac{-5}{7}\right)$?

$$\text{L.H.S} = \frac{2}{3} \times \left[\frac{4}{7} \times \left(\frac{-5}{7}\right)\right] = \frac{2}{3}\left[\frac{-20}{49}\right] = \frac{-40}{147}$$

$$\text{R.H.S} = \left(\frac{2}{3} \times \frac{4}{7}\right) \times \left(\frac{-5}{7}\right) = \left(\frac{8}{21}\right) \times \left(\frac{-5}{7}\right) = \frac{-40}{147}$$

L.H.S = R.H.S

Check the following.

Find $2 \times \left(\frac{1}{2} \times 3\right)$ and $\left(2 \times \frac{1}{2}\right) \times 3$

Is $2 \times \left(\frac{1}{2} \times 3\right) = \left(2 \times \frac{1}{2}\right) \times 3$?

Find $\frac{5}{3} \times \left(\frac{3}{7} \times \frac{7}{5}\right)$ and $\left(\frac{5}{3} \times \frac{3}{7}\right) \times \frac{7}{5}$

Is $\frac{5}{3} \times \left(\frac{3}{7} \times \frac{7}{5}\right) = \left(\frac{5}{3} \times \frac{3}{7}\right) \times \frac{7}{5}$?

We can find in all the above cases L.H.S = R.H.S

Thus multiplication is associative in rational numbers

a × (b × c) = (a × b) × c for any three rational numbers a, b, c.

i.e., $\forall$ a, b, c $\in$ Q, a × (b × c) = (a × b) × c

(d) Division

Take any three rational numbers $\frac{2}{3}, \frac{3}{4}$ and $\frac{1}{7}$

Is $\frac{2}{3} \div \left(\frac{3}{4} \div \frac{1}{7}\right) = \left(\frac{2}{3} \div \frac{3}{4}\right) \div \frac{1}{7}$?

L.H.S. $= \frac{2}{3} \div \left(\frac{3}{4} \div \frac{1}{7}\right) = \frac{2}{3} \div \left(\frac{3}{4} \times \frac{7}{1}\right) = \frac{2}{3} \div \frac{21}{4} = \frac{2}{3} \times \frac{4}{21} = \frac{8}{63}$

R.H.S. $= \left(\frac{2}{3} \div \frac{3}{4}\right) \div \frac{1}{7} = \left(\frac{2}{3} \times \frac{4}{3}\right) \div \frac{1}{7} = \left(\frac{8}{9}\right) \div \frac{1}{7} = \frac{8}{9} \times \frac{7}{1} = \frac{56}{9}$

$$\frac{2}{3} \div \left(\frac{3}{4} \div \frac{1}{7}\right) \neq \left(\frac{2}{3} \div \frac{3}{4}\right) \div \frac{1}{7}$$

L.H.S. ≠ R.H.S.

Thus $a \div (b \div c) \neq (a \div b) \div c$ for any three rational numbers a, b, c.

So, division is not associative in rational numbers.

Do This

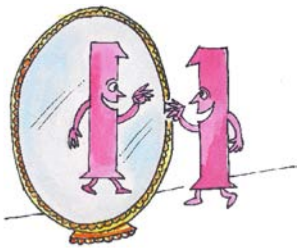
Complete the following table

Numbers	Associative under			
	Addition	**Subtraction**	**Multiplication**	**Division**
Natural numbers	Yes	No	. . . . .	. . . . .
Whole numbers	. . . . .	. . . . .	. . . . .	No
Integers	. . . . .	No	Yes	. . . . .
Rational numbers	. . . . .	. . . . .	. . . . .	. . . . .

1.2.4 The Role of Zero

Can you find a number, when it is added to $\frac{1}{2}$ gives the same number $\frac{1}{2}$?

When the number '**0**' is added to any rational number, the rational number remains the same. For example



$1 + 0 = 1$ and $0 + 1 = 1$

$-2 + 0 = -2$ and $0 + (-2) = -2$

$\frac{1}{2} + 0 = \frac{1}{2}$ and $0 + \frac{1}{2} = \frac{1}{2}$

For this reason we call '0' as an identity element of addition or "additive identity".

If **a** represents any rational number then $a + 0 = a$ and $0 + a = a$

Does the set of natural numbers have additive identity ?

1.2.5 The Role of 1

Fill in the following blanks :

$3 \times \square = 3$ and $\square \times 3 = 3$

$-2 \times \square = -2$ and $\square \times -2 = -2$

$\frac{7}{8} \times \square = \frac{7}{8}$ and $\square \times \frac{7}{8} = \frac{7}{8}$

What observations have you made in the above multiplications?

You will find that when you multiply any rational number with '1', you will get the same rational number as the product.

We say that '1' is the multiplicative identity for rational numbers

What is the multiplicative identity for integers and whole numbers?

We often use the identity properties without realizing that we are using them.

For example when we write $\frac{15}{50}$ in the simplest form we may do the following.

$$\frac{15}{50} = \frac{3 \times 5}{10 \times 5} = \frac{3}{10} \times \frac{5}{5} = \frac{3}{10} \times 1 = \frac{3}{10}$$

When we write that $\frac{3}{10} \times 1 = \frac{3}{10}$. Here we used the identity property of multiplication.

1.2.6 Existence of Inverse

(i) Additive inverse:

$3 + (-3) = 0$ and $-3 + 3 = 0$

$-5 + 5 = 0$ and $5 + (-5) =$ ______

$\frac{2}{3} + ? = 0$ and ____ $+ \frac{2}{3} = 0$

$\left(-\frac{1}{2}\right) + ? = 0$ and $? + \left(-\frac{1}{2}\right) = 0$

Here -3 and 3 are called the additive inverses of each other because on adding them we get the sum '0' i.e. the additive identity. Any two numbers whose sum is '0' are called the additive inverses of each other. In general if **a** represents any rational number then **a + (–a) = 0 and (–a) + a = 0**.

Then 'a', '–a' are additive inverse of each other.

The additive inverse of 0 is only 0 as $0 + 0 = 0$.

(ii) Multiplicative inverse:

By which rational number $\frac{2}{7}$ is multiplied to get the product 1 ?

We can see $\frac{2}{7} \times \frac{7}{2} = 1$ and $\frac{7}{2} \times \frac{2}{7} = 1$

Fill the boxes below-

$2 \times \square = 1$ and $\square \times 2 = 1$

$-5 \times \square = 1$ and $\square \times 5 = 1$

$\frac{-17}{19} \times \square = 1$ and $\square \times \frac{-17}{19} = 1$

$1 \times ? = 1$

$-1 \times ? = 1$

Any two numbers whose product is '1' are called the multiplicative inverses of each other.

For example, $4 \times \frac{1}{4} = 1$ and $\frac{1}{4} \times 4 = 1$, therefore the numbers 4 and $\frac{1}{4}$ are the multiplicative inverses (or the reciprocals) of each other.

We say that a rational number $\frac{c}{d}$ is called the reciprocal or the multiplicative inverse of another rational number $\frac{a}{b}$ if $\frac{a}{b} \times \frac{c}{d} = 1$

Think, discuss and write

1. If a property holds good with respect to addition for rational numbers, whether it holds good for integers? And for whole numbers? Which one holds good and which doesn't hold good?
2. Write the numbers whose multiplicative inverses are the numbers themselves
3. Can you find the reciprocal of '0' (zero)? Is there any rational number such that when it is multiplied by '0' gives '1'?

$\square \times 0 = 1$ and $0 \times \square = 1$

1.3 Distributivity of multiplication over addition

Take three rational numbers $\frac{2}{5}, \frac{1}{2}, \frac{3}{4}$

Let us verify whether $\frac{2}{5} \times \left(\frac{1}{2} + \frac{3}{4}\right) = \left(\frac{2}{5}\right) \times \left(\frac{1}{2}\right) + \left(\frac{2}{5}\right) \times \left(\frac{3}{4}\right)$

$$\text{L.H.S} = \frac{2}{5} \times \left(\frac{1}{2} + \frac{3}{4}\right) = \frac{2}{5} \times \left(\frac{2+3}{4}\right) = \frac{2}{5} \times \frac{5}{4} = \frac{10}{20} = \frac{1}{2}$$

$$\text{R.H.S} = \frac{2}{5} \times \left(\frac{1}{2}\right) + \frac{2}{5} \times \left(\frac{3}{4}\right) = \frac{2}{10} + \frac{6}{20} = \frac{4+6}{20} = \frac{10}{20} = \frac{1}{2}$$

$$\text{L.H.S} = \text{R.H.S}$$

Thus $$\frac{2}{5} \times \left(\frac{1}{2} + \frac{3}{4}\right) = \left(\frac{2}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{5}\right)\left(\frac{3}{4}\right)$$

This property is called the distributive law of multiplication over addition.

Now verify the following

Is $\frac{2}{5} \times \left(\frac{1}{2} - \frac{3}{4}\right) = \frac{2}{5} \times \left(\frac{1}{2}\right) - \frac{2}{5} \times \left(\frac{3}{4}\right)$

What do you observe? Is L.H.S = R.H.S?

This property is called the distributive law over subtraction.

Take some more rational numbers and verify the distributive property

For all rational numbers a, b and c

We can say-

$a\,(b + c) = ab + ac$

$a\,(b - c) = ab - ac$

Try these: find using distributivity

(1) $\left\{\frac{7}{5} \times \left(\frac{-3}{10}\right)\right\} + \left\{\frac{7}{5} \times \frac{9}{10}\right\}$

(2) $\left\{\frac{9}{16} \times 3\right\} + \left\{\frac{9}{16} \times (-19)\right\}$

Do These

Complete the following table.

Numbers	Additive properties				
	Closure	Commutative	Associative	Existence of Identity	Existence of Inverse
Rational Numbers	Yes	——	——	——	——
Integers	Yes	——	——	——	——
Whole Numbers	——	——	——	Yes	No
Natural Numbers	Yes	——	——	——	——

Complete the following table

Numbers	**Multiplicative properties**				
	Closure	Commutative	Associative	Existence of Identity	Existence of Inverse
Rational Numbers	Yes	— —	— —	— —	— —
Integers	— —	Yes	— —	— —	— —
Whole Numbers	— —	— —	Yes	— —	— —
Natural Numbers	— —	— —	— —	Yes	— —

Example 1. Simplify $\frac{2}{5}+\frac{3}{7}+\left(\frac{-6}{5}\right)+\left(\frac{-13}{7}\right)$

Solution: Rearrange the given fractions keeping like fractions together.

$$\frac{2}{5}+\frac{3}{7}+\left(\frac{-6}{5}\right)+\left(\frac{-13}{7}\right)=\frac{2}{5}+\frac{3}{7}-\frac{6}{5}-\frac{13}{7}$$

$$=\left(\frac{2}{5}-\frac{6}{5}\right)+\left(\frac{3}{7}-\frac{13}{7}\right) \quad \text{(by commutative law of addition)}$$

$$=\frac{2-6}{5}+\frac{3-13}{7}$$

$$=\frac{-4}{5}+\frac{-10}{7}=\frac{-4}{5}-\frac{10}{7}$$

$$=\frac{(-4\times7)-(10\times5)}{35}=\frac{-28-50}{35}=\frac{-78}{35}$$

Example 2: Write the additive inverses of each of the following rational numbers.

(i) $\frac{2}{7}$ (ii) $\frac{-11}{5}$ (iii) $\frac{7}{-13}$ (iv) $\frac{-2}{-3}$

Solution : (i) The additive inverse of $\frac{2}{7}$ is $\frac{-2}{7}$

because $\frac{2}{7}+\left(\frac{-2}{7}\right)=\frac{2-2}{7}=0$

(ii) The additive inverse of $\frac{-11}{5}$ is $-\left(\frac{-11}{5}\right) = \frac{11}{5}$

(iii) The additive inverse of $\frac{7}{-13}$ is $-\left(\frac{7}{-13}\right) = \frac{-7}{-13} = \frac{7}{13}$

(iv) The additive inverse of $\frac{-2}{-3}$ is $-\left(\frac{\cancel{-}2}{\cancel{-}3}\right) = -\frac{2}{3}$

Example 3 : Find $\frac{2}{5} \times \frac{-1}{9} + \frac{23}{180} - \frac{1}{9} \times \frac{3}{4}$

Solution : $\frac{2}{5} \times \frac{-1}{9} + \frac{23}{180} - \frac{1}{9} \times \frac{3}{4} = \frac{2}{5} \times \frac{-1}{9} - \frac{1}{9} \times \frac{3}{4} + \frac{23}{180}$

(by the commutative law of addition)

$= \frac{2}{5} \times \left(\frac{-1}{9}\right) + \left(\frac{-1}{9}\right) \times \frac{3}{4} + \frac{23}{180}$

$= \frac{-1}{9}\left(\frac{2}{5} + \frac{3}{4}\right) + \frac{23}{180}$

$= -\frac{1}{9}\left(\frac{8+15}{20}\right) + \frac{23}{180}$ (by the distributive law)

$= -\frac{1}{9}\left(\frac{23}{20}\right) + \frac{23}{180} = \frac{-23}{180} + \frac{23}{180} = 0$ (by the additive inverse law)

Example 4: Find the product of the reciprocals of $\frac{-9}{2}, \frac{5}{18}$ and add the additive inverse of $\left(\frac{-4}{5}\right)$ to the product. What is the result?

Solution : The reciprocal of $\frac{-9}{2}$ is $\frac{-2}{9}$

The reciprocal of $\frac{5}{18}$ is $\frac{18}{5}$

Product of reciprocals $= \frac{-2}{\cancel{9}_{1}} \times \frac{\cancel{18}^{2}}{5} = \frac{-4}{5}$

The additive inverse of $\left(\frac{-4}{5}\right)$ is $\frac{4}{5}$

Thus the product + the additive inverse $= \frac{-4}{5} + \frac{4}{5} = 0$ (the additive Inverse property)

Exercise - 1.1

1. Name the property involved in the following examples

(i) $\frac{8}{5} + 0 = \frac{8}{5} = 0 + \frac{8}{5}$

(ii) $2\left(\frac{3}{5} + \frac{1}{2}\right) = 2\left(\frac{3}{5}\right) + 2\left(\frac{1}{2}\right)$

(iii) $\frac{3}{7} \times 1 = \frac{3}{7} = 1 \times \frac{3}{7}$

(iv) $\left(\frac{-2}{5}\right) \times 1 = \frac{-2}{5} = 1 \times \left(\frac{-2}{5}\right)$

(v) $\frac{2}{5} + \frac{1}{3} = \frac{1}{3} + \frac{2}{5}$

(vi) $\frac{5}{2} \times \frac{3}{7} = \frac{15}{14}$

(vii) $7a + (-7a) = 0$

(viii) $x \times \frac{1}{x} = 1 \ (x \neq 0)$

(ix) $(2 \times x) + (2 \times 6) = 2 \times (x + 6)$

2. Write the additive and the multiplicative inverses of the following.

(i) $\frac{-3}{5}$ (ii) 1 (iii) 0 (iv) $\frac{7}{9}$ (v) -1

3. Fill in the blanks

(i) $\left(\frac{-1}{17}\right) + (\_\_\_\_) = \left(\frac{-12}{5}\right) + \left(\frac{-1}{17}\right)$

(ii) $\frac{-2}{3} + \_\_\_\_ = \frac{-2}{3}$

(iii) $1 \times \_\_\_\_ = \frac{9}{11}$

(iv) $-12 + \left(\frac{5}{6} + \frac{6}{7}\right) = \left(-12 + \frac{5}{6}\right) + (\_\_\_\_)$

(v) $(\_\_\_\_) \times \left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{3}{4} \times \frac{1}{2}\right) + \left(\frac{3}{4} \times \_\_\_\_\right)$

(vi) $\frac{-16}{7} + \_\_\_\_ = \frac{-16}{7}$

4. Multiply $\frac{2}{11}$ by the reciprocal of $\frac{-5}{14}$

5. Which properties can be used in computing $\frac{2}{5} \times \left(5 \times \frac{7}{6}\right) + \frac{1}{3} \times \left(3 \times \frac{4}{11}\right)$

6. Verify the following and write the property used

$$\left(\frac{5}{4} + \frac{-1}{2}\right) + \frac{-3}{2} = \frac{5}{4} + \left(\frac{-1}{2} + \frac{-3}{2}\right)$$

7. Evaluate $\frac{3}{5} + \frac{7}{3} + \left(\frac{-2}{5}\right) + \left(\frac{-2}{3}\right)$ after rearrangement.

8. Subtract

(i) $\frac{3}{4}$ from $\frac{1}{3}$ (ii) $\frac{-32}{13}$ from 2 (iii) -7 from $\frac{-4}{7}$

9. What number should be added to $\frac{-5}{8}$ so as to get $\frac{-3}{2}$?

10. The sum of two rational numbers is 8 If one of the numbers is $\frac{-5}{6}$ find the other.

11. Is subtraction associative in rational numbers? Explain with an example.

12. Verify that $-(-x) = x$ for

(i) $x = \frac{2}{15}$ (ii) $x = \frac{-13}{17}$

13. Write-

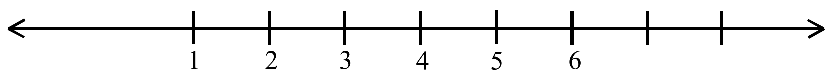
(i) The set of numbers which do not have an additive identity

(ii) The rational number that does not have any reciprocal

(iii) The reciprocal of a negative rational number.

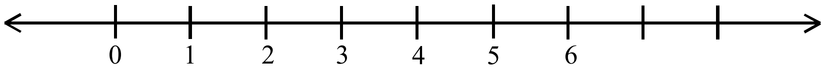
1.4 Representation of Rational numbers on Number line.

Gayathri drew a number line and labelled some numbers on it.



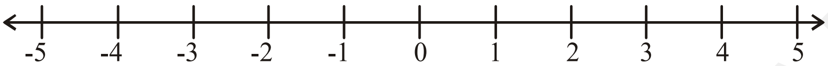
What set of numbers are marked on the line?

Sujatha said "They are Natural numbers". Paramesh said "They are rational numbers" Whom do you agree with?



Which set of numbers are marked on the above line?

Are they whole numbers or rational numbers?



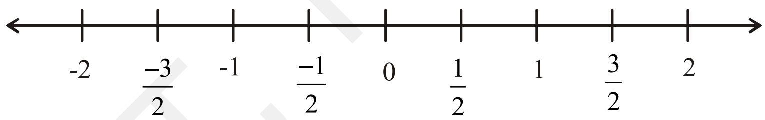
Which set of numbers are marked on the above line?

Can you find any number between –5 and 3 on the above line?

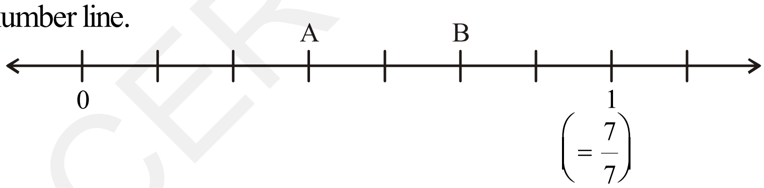
Can you see any integers between 0 and 1 or –1 and 0 in the above line?

Numbers in the middle of 0 and 1 is $\frac{1}{2}$; in the middle of 1 and 2 is $1\frac{1}{2} = \frac{3}{2}$, in the middle of 0 and –1 is $-\frac{1}{2}$ and in the middle of – 1 and –2 is $-1\frac{1}{2} = -\frac{3}{2}$.

These above rational numbers can be represented on the number line as follows:



Example 5: Identify the rational numbers shown as A and B that are marked on the following number line.

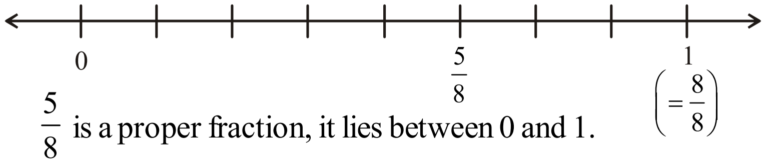


Solution: Here a unit, 0 to 1 is divided into 7 equal parts. A is representing 3 out of 7 parts. So, A represents $\frac{3}{7}$ and B represents $\frac{5}{7}$.

Any rational number can be represented on the number line. Notice that in a rational number the denominator tells the number of equal parts in which the each unit has been divided. The numerator tells 'how many' of these parts are considered.

Example 6: Represent $\frac{5}{8}$ on the number line.

Solution:



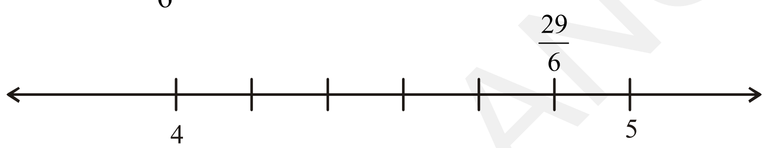
$\frac{5}{8}$ is a proper fraction, it lies between 0 and 1.

So divide the number line between 0 and 1 into 8 equal parts.

Then mark 5th part (numerator) $\frac{5}{8}$ counting from 0 is the required rational number $\frac{5}{8}$.

Example 7: Represent $\frac{29}{6}$ on the number line.

Solution:



$\frac{29}{6} = 4\frac{5}{6} = 4+\frac{5}{6}$. This lies between 4 and 5 on the number line

Divide the number line between 4 and 5 into 6 (denominator) equal parts.

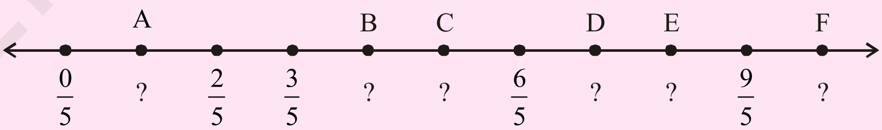
Mark 5th part (numerator of rational part) counting from 4.

This is the place of the required rational number $4+\frac{5}{6} = 4\frac{5}{6} = \frac{29}{6}$.

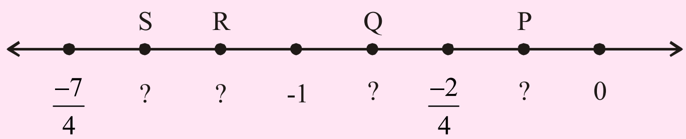
Try These

Write the rational number for the points labelled with letters, on the number line

(i)



(ii)



Do This

(i) Represent $-\frac{13}{5}$ on the number line.

1.5 Rational Number between Two Rational Numbers

Observe the following

The natural numbers between 5 and 1 are 4, 3, 2.

Are there any natural numbers between 1 and 2?

The integers between –4 and 3 are –3, –2, –1, 0, 1, 2.

Write the integers between –2 and –1.Did you find any?

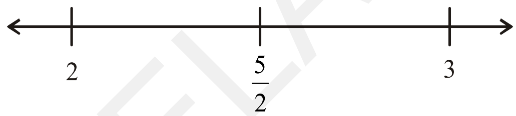
We cannot find integers between any two successive integers.

But we can write rational numbers between any two successive integers.

Let us write the rational numbers between 2 and 3.

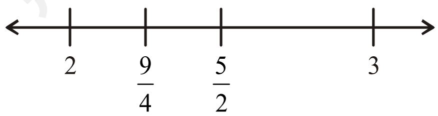
We know if a and b are any two rational numbers then $\frac{a+b}{2}$ (and it is also called the mean of a and b) is a rational number between them. So $\frac{2+3}{2} = \frac{5}{2}$ is a rational number which lies exactly between 2 and 3.

Thus $2 < \frac{5}{2} < 3$.

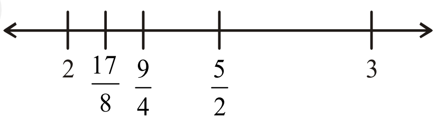


Now the rational number between 2 and $\frac{5}{2}$ is $\frac{2+\frac{5}{2}}{2} = \frac{\frac{9}{2}}{2} = \frac{9}{2} \times \frac{1}{2} = \frac{9}{4}$.

Thus $2 < \frac{9}{4} < \frac{5}{2} < 3$



Again the mean of 2, $\frac{9}{4}$ is $\frac{2+\frac{9}{4}}{2} = \frac{\frac{17}{4}}{2} = \frac{17}{8}$



So $2 < \frac{17}{8} < \frac{9}{4} < \frac{5}{2} < 3$

In this way we can go on inserting rational numbers between any two numbers. Infact, there are infinite number of rational numbers between any two given rational numbers.

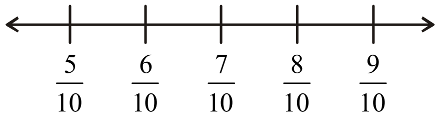
Another Method :

Can you write hundred rational numbers between $\frac{5}{10}$ and $\frac{9}{10}$ in mean method?

You may feel difficult because of the lengthy process.

Here is another method for you.

We know that $\frac{5}{10} < \frac{6}{10} < \frac{7}{10} < \frac{8}{10} < \frac{9}{10}$



Here we wrote only three rational numbers between $\frac{5}{10}$ and $\frac{9}{10}$.

But if we consider $\frac{5}{10} = \frac{50}{100}$ and $\frac{9}{10} = \frac{90}{100}$

Now the rational numbers between $\frac{50}{100}$ and $\frac{90}{100}$ are

$$\frac{5}{10} = \frac{50}{100} < \frac{51}{100} < \frac{52}{100} < \frac{53}{100} < \ldots\ldots. < \frac{89}{100} < \frac{90}{100} = \frac{9}{10}$$

Similarly, when we consider

$$\frac{5}{10} = \frac{500}{1000} \text{ and } \frac{9}{10} = \frac{900}{1000}$$

So $\frac{5}{10} = \frac{500}{1000} < \frac{501}{1000} < \frac{502}{1000} < \frac{503}{1000} < \ldots\ldots\ldots\ldots < \frac{899}{1000} < \frac{900}{1000} = \frac{9}{10}$

In this way we can go on inserting required number of rational numbers.

Example 8: Write any five rational numbers between -3 and 0.

Solution: $-3 = -\frac{30}{10}$ and $0 = \frac{0}{10}$ so

$-\frac{29}{10}, -\frac{28}{10}, -\frac{27}{10}, \ldots\ldots\ldots, -\frac{2}{10}, -\frac{1}{10}$ lies between -3 and 0.

We can take any five of these.

Exercise - 1.2

1. Represent these numbers on the number line.

   (i) $\frac{9}{7}$ (ii) $-\frac{7}{5}$

2. Represent $-\frac{2}{13}, \frac{5}{13}, \frac{-9}{13}$ on the number line.

3. Write five rational numbers which are smaller than $\frac{5}{6}$.

4. Find 12 rational numbers between -1 and 2.

5. Find a rational number between $\frac{2}{3}$ and $\frac{3}{4}$.

   [Hint : First write the rational numbers with equal denominators.]

6. Find ten rational numbers between $-\frac{3}{4}$ and $\frac{5}{6}$

1.6 Decimal representation of Rational numbers

We know every rational number is in the form of $\frac{p}{q}$ where $q \neq 0$ and p, q are integers. Let us see how to express a rational number in decimal form.

To convert a rational number into decimal by division method.

Consider a rational number $\frac{25}{16}$.

Step1: Divide the numerator by the denominator

Step2: Continue the division till the remainder left is less than the divisor.

$$\begin{array}{r} 16\,\overline{)\,25\,(1} \\ \underline{16} \\ 9 \end{array}$$

Step3: Put a decimal point in the dividend and at the end of the quotient.

Step4: Put a zero on the right of decimal point in the dividend as well as right of the remainder.

Divide again just as whole numbers.

$$\begin{array}{r} 16\,\overline{)\,25.0\,(1.} \\ \underline{16} \\ 90 \end{array}$$

Step 5: Repeat step 4 till either the remainder is zero or requisite number of decimal places have been obtained

$$\begin{array}{r} 16\,\overline{)\,25.0000\,(1.5625} \\ \underline{16} \\ 90 \\ \underline{80} \\ 100 \\ \underline{96} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ \underline{0} \end{array}$$

Therefore $\frac{25}{16} = 1.5625$

Consider $\frac{17}{5}$

$$\begin{array}{r} 5\,\overline{)\,17.0\,(3.4} \\ \underline{15} \\ 20 \\ \underline{20} \\ \underline{0} \end{array}$$

Therefore $\frac{17}{5} = 3.4$

Try to express $\frac{1}{2}, \frac{13}{25}, \frac{8}{125}, \frac{1974}{10}$ in decimal form and write the values.

We observe that there are only finite number of digits in the decimal part of these decimal numbers. Such decimals are known as terminating decimals.

Non terminating recurring decimals:

Consider the rational number $\frac{5}{3}$

By long division method we have →

3) 5.000 (1.666
3
20
18
20
18
20
18
2

Therefore $\frac{5}{3} = 1.666...$

We write this as $\frac{5}{3} = 1.\bar{6}$ the bar on '6' in the decimal part indicates it is recurring.

We observe that in the above division the same remainder is repeating itself and the digit 6 in the quotient is repeated.

Consider the rational number $\frac{1}{7}$ →

By long division method

7) 10.00000000 (0.14285714
7
30
28
20
14
60
56
40
35
50
49
10
7
30
28
2

$\frac{1}{7} = 0.142857142857....$

$\frac{1}{7} = 0.\overline{142857}$. The bar on decimal part 142857 indicates that these digits are repeating in the same order.

The above examples are illustrating the representation of rational numbers in the form of non-terminating recurring decimals or we call them as non-terminating repeating decimals.

Try to express $\frac{1}{3}$, $\frac{17}{6}$, $\frac{11}{9}$ and $\frac{20}{19}$ in decimal form

$\frac{1}{3} =$ ☐ $\frac{17}{6} =$ ☐ $\frac{11}{9} =$ ☐ $\frac{20}{19} =$ ☐

When we try to express some rational numbers in decimal form by division method, we find that the division never comes to an end. This is due to the reason that in the division process the remainder starts repeating after a certain number of steps. In these cases in the quotient a digit or set of digits repeats in the same order.

For example 0.33333------.= $0.\overline{3}$

0.12757575---------= $0.12\overline{75}$

123.121121121121----------= $123.\overline{121}$

5.678888-----------= $5.67\overline{8}$ etc.

Such decimals are called non-terminating and repeating decimal or non-terminating recurring decimals.

The set of digits which repeats in non-terminating recurring decimal is called period.

For example

In 0.3333 ...... = $0.\overline{3}$ the period is 3

In 0.12757575 .... = $0.12\overline{75}$ the period is 75

The number of digits in a period of non-terminating recurring decimal is called periodicity.

For example

In 0.3333 ...... = $0.\overline{3}$ the periodicity is 1

In 0.12757575 .... = $0.12\overline{75}$ the periodicity is 2

The period of 0.23143143143....... = _______ periodicity = _______

The period of 125.6788989 .... = _______ periodicity = _______

Think and Discuss:

1. Express the following in decimal form.

(i) $\frac{7}{5}, \frac{3}{4}, \frac{23}{10}, \frac{5}{3}, \frac{17}{6}, \frac{22}{7}$

(ii) Which of the above are terminating and which are non-terminating decimals ?

(iii) Write the denominators of above rational numbers as the product of primes.

(iv) If the denominators of the above simplest rational numbers has no prime divisors other than 2 and 5 what do you observe?

1.7 Conversion of decimal form into rational form

1.7.1 Converting terminating decimal into rational form

Consider a decimal number 15.75

Find the number of places after the decimal point in the given number. In 15.75 there are 2 decimals places.

Therefore 15.75 can be written as $\frac{1575}{100}$. Now

$$\frac{1575}{100} = \frac{1575 \div 5}{100 \div 5} = \frac{315 \div 5}{20 \div 5} = \frac{63}{4}$$

$\frac{63}{4}$ is the rational form of 15.75.

Example 9: Express each of the following decimals in the $\frac{p}{q}$ form

(i) 0.35 (ii) −8.005 (iii) 2.104

Solution: (i) Since 0.35 has two places after decimal point. It is written as $0.35 = \frac{35}{100}$.

Now, $0.35 = \frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$

(ii) $-8.005 = \frac{-8005}{1000} = \frac{-8005 \div 5}{1000 \div 5} = \frac{-1601}{200}$

(iii) $2.104 = \frac{2104}{1000} = \frac{2104 \div 4}{1000 \div 4} = \frac{526 \div 2}{250 \div 2} = \frac{263}{125}$

1.7.2 Converting a non-terminating recurring decimal into rational form

Let us discuss the method of conversion by following example.

Example 10: Express each of the following decimals numbers in the rational form.

(i) $0.\overline{4}$ (ii) $0.\overline{54}$ (iii) $4.\overline{7}$

Solution (i): $0.\overline{4}$

let $x = 0.\overline{4}$

$\Rightarrow$ $x = 0.444 \ldots$ ----------------(i)

here the periodicity of the decimal is one.

So we multiply both sides of (i) by 10 and we get

$10x = 4.44$ .... ----------------(ii)

Subtracting (i) from (ii)

$$\begin{aligned} 10x &= 4.444...... \\ x &= 0.444...... \\ \hline 9x &= 4.000...... \\ \hline x &= \frac{4}{9} \end{aligned}$$

Hence $0.\overline{4} = \frac{4}{9}$

Observe

$0.\overline{4} = \frac{4}{9}$

$0.\overline{5} = \frac{5}{9}$

$0.\overline{54} = \frac{54}{99}$

$0.\overline{745} = \frac{745}{999}$

Solution (ii): $0.\overline{54}$

let $x = 0.\overline{54}$

$\Rightarrow x = 0.545454....$ ------------------ (i)

here the periodicity of the decimal is two.

So we multiply both sides of (i) by 100, we get

$100x = 54.5454....$ ------------------- (ii)

On subtracting (i) from (ii)

$$\begin{aligned} 100x &= 54.5454 ...... \\ x &= 0.5454 ..... \\ \hline 99x &= 54.0000...... \\ \hline \end{aligned}$$

$x = \frac{54}{99}$. Hence $0.\overline{54} = \frac{54}{99}$.

Solution (iii) : $4.\overline{7}$

let $x = 4.\overline{7}$

$x = 4.777....$ ---------------- (i)

here the periodicity of the decimal is one.

So multiply both sides of (i) by 10, we get

$10x = 47.777....$ ---------------- (ii)

Subtracting (i) from (ii) we get

$$\begin{aligned} 10x &= 47.777 ..... \\ x &= 4.777 ..... \\ \hline 9x &= 43.000.... \\ \hline \end{aligned}$$

$$x = \frac{43}{9}$$

Hence $4.\overline{7} = \frac{43}{9}$.

Alternative Method :

$$4.\overline{7} = 4 + 0.\overline{7}$$

$$= 4 + \frac{7}{9}$$

$$= \frac{9 \times 4 + 7}{9}$$

$$\therefore 4.\overline{7} = \frac{43}{9}$$

Example 11: Express the mixed recurring decimal $15.7\overline{32}$ in $\frac{p}{q}$ form.

Solution : Let $x = 15.7\overline{32}$

x =15.7323232.... ---------------(i)

Since two digits 32 are repeating therefore

the periodicity of the above decimal is two.

So multiply (i) both sides by 100, we get

100 x = 1573.2323.... -------------(ii)

Subtracting (i) from (ii), we get

$$100x = 1573.232323....$$

$$x = 15.732323....$$

$$99x = 1557.50000$$

$$x = \frac{1557.5}{99} = \frac{15575}{990}$$

$$= 15.7\overline{32} = \frac{15575}{990}$$

Think Discuss and Write

Convert the decimals $0.\overline{9}$, $14.\overline{5}$ and $1.\overline{24}$ to rational form. Can you find any easy method other than formal method?

Exercise - 1.3

1. Express each of the following decimal in the $\frac{p}{q}$ form.

   (i) 0.57 (ii) 0.176 (iii) 1.00001 (iv) 25.125

2. Express each of the following decimals in the rational form ($\frac{p}{q}$).

   (i) $0.\overline{9}$ (ii) $0.\overline{57}$ (iii) $0.7\overline{29}$ (iv) $12.2\overline{8}$

3. Find $(x + y) \div (x - y)$ if

   (i) $x = \frac{5}{2}, y = -\frac{3}{4}$ (ii) $x = \frac{1}{4}, y = \frac{3}{2}$

4. Divide the sum of $-\frac{13}{5}$ and $\frac{12}{7}$ by the product of $-\frac{13}{7}$ and $-\frac{1}{2}$.

5. If $\frac{2}{5}$ of a number exceeds $\frac{1}{7}$ of the same number by 36. Find the number.

6. Two pieces of lengths $2\frac{3}{5}$ m and $3\frac{3}{10}$ m are cut off from a rope 11 m long. What is the length of the remaining rope?

7. The cost of $7\frac{2}{3}$ meters of cloth is ₹ $12\frac{3}{4}$. Find the cost per metre.

8. Find the area of a rectangular park which is $18\frac{3}{5}$ m long and $8\frac{2}{3}$ m broad.

9. What number should $-\frac{33}{16}$ be divided by to get $-\frac{11}{4}$?

10. If 36 trousers of equal sizes can be stitched with 64 meters of cloth. What is the length of the cloth required for each trouser?

11. When the repeating decimal 10.363636 .... is written in simplest fractional form $\frac{p}{q}$, find the value of $p + q$.

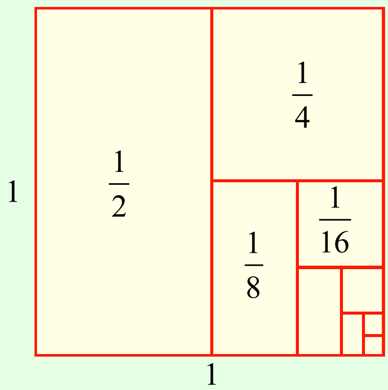
What we have discussed

1. Rational numbers are closed under addition, subtraction and multiplication.
2. The addition and multiplications are
   (i) Commutative for rational numbers
   (ii) Associative for rational numbers
3. '0' is the additive identity for rational numbers.
4. '1' is the multiplicative identity for rational numbers.
5. A rational number and its additive inverse are opposite in their sign.
6. The multiplicative inverse of a rational number is its reciprocal.
7. Distributivity of rational numbers a,b and c,
   a (b + c) = ab + ac and a (b – c) = ab – ac
8. Rational numbers can be represented on a number line
9. There are infinite rational numbers between any two given rational numbers. The concept of mean help us to find rational numbers between any two rational numbers.
10. The decimal representation of rational numbers is either in the form of terminating decimal or non-terminating recurring decimals.

N7E1D4

Can you find?

Guess a formula for a_n. Use the subdivided unit square below to give a visual justification of your conjecture.



Hint : $a_1 = \frac{1}{2}, \; a_2 = \frac{1}{2} + \frac{1}{4}, \; a_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots\ldots\ldots \quad a_n = \frac{1}{2} + \frac{1}{4} + \ldots. + \frac{1}{2^n}$

$a_1 = 1 - \frac{1}{2}, \; a_2 = 1 - \frac{1}{4}, \; a_3 = 1 - \frac{1}{8} \ldots\ldots\ldots$ then $a_n = ?$