Chapter - 12 LINEAR PROGRAMMING PROBLEMS
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Solve the L.P.P graphically.

आलेख द्वारा निम्न रैखिक प्रोग्रामन समस्या को हल कीजिए:

1. Maximize $z = 4x + y$ s.t. $x+y \le 50$,

$3x+y \le 90$

$x \ge 0,\ y \ge 0$

निम्न अवरोधो के अंतर्गत $z = 4x + y$ का अधिकतम मान ज्ञात कीजिए : $x + y \le 50, 3x + y \le 90$

$x \ge 0,\ y \ge 0$

2. Minimize $Z = -3x + 4y$ subject to $x + 2y \le 8, 3x + 2y \le 12,$

$x \ge 0, y \ge 0$

निम्न अवरोधो के अंतर्गत $Z = -3x + 4y$ का न्यूनतमीकरण कीजिए :

$x + 2y \le 8, 3x + 2y \le 12,$

$x \ge 0, y \ge 0$

3. Maximize $Z = 3x + 2y$ subject to

$x + 2y \le 10, 3x + y \le 15 \quad x \ge 0, y \ge 0$

निम्न अवरोधो के अंतर्गतः $Z = 3x + 2y$ का

अधिकतम मान ज्ञात कीजिए –

$x + 2y \le 10, 3x + y \le 15 \quad x \ge 0, y \ge 0$

4. Find Maximum and Minimum value of $Z = 5x + 10y$ subject to $x + 2y \le 120$,

$x + y \ge 60, x - 2y \ge 0 \quad x, y \ge 0$

निम्न अवरोधो के अंतर्गत $Z = 5x + 10y$ का न्यूनतमीकरण तथा अधिकतमीकरण कीजिए :

$x + 2y \le 120$,

$x + y \ge 60, x - 2y \ge 0 \quad x, y \ge 0$

5. Maximize $Z = y - 2x$ subject to $x \le 2, x + y \le 3, -2x + y \le 1 \quad x, y \ge 0$

निम्न अवरोधो के अंतर्गत $Z = y - 2x$

का अधिकतमीकरण कीजिए –

$x \le 2, x + y \le 3, -2x + y \le 1 \quad x, y \ge 0$.

6. Minimize and Maximize $Z = x + 2y$ subject to $x + 2y \ge 100, 2x - y \le 0, 2x + y \le 200$.

$x, y \ge 0$

निम्न अवरोधों के अंतर्गत $Z = x + 2y$ का न्यूनतमीकरण तथा अधिकतमीकरण कीजिए :

$x + 2y \ge 100, 2x - y \le 0, 2x + y \le 200$

$x, y \ge 0$

5 MARKS SOLUTION

1.

Sol -

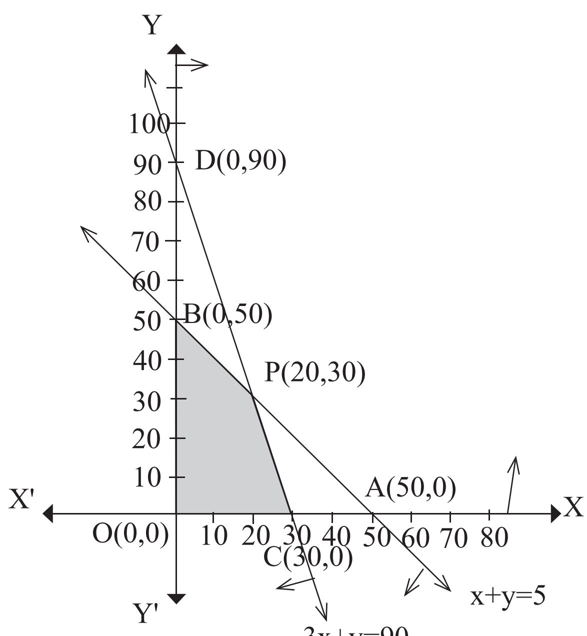
Convert all the constraints into equation, we get,

$x + y = 50,\ 3x + y = 90, x = 0, y = 0$

$\Rightarrow \frac{x}{50} + \frac{y}{50} = 1, \frac{x}{30} + \frac{y}{90} = 1$

Hence Eq^n $x + y = 50$ cuts the $x - axis$ at $A(50,0)$ and $y - axis$ at $B(0,50)$

Similarly eq^n $3x + y = 90$ cuts the $x - axis$ at $C(30,0)$ and $y - axis$ at $D(0,90)$.



Check the region at (0,0) we get

$0+0 \leq 50 \;\&\; 0+0 \leq 90$

$\therefore$ all inequation satisfies (0,0)

Corner point	objective function Z_{max}=4x+y
O(0,0)	Z=0
C(30,0)	Z=4.30+0=120 (Max^m)
P(20,30)	Z=4.20+30=110
B(0,50)	Z=0+50=50

$\therefore$ Z is maximum at C(30,0)

$\therefore$ Z_{max}=120

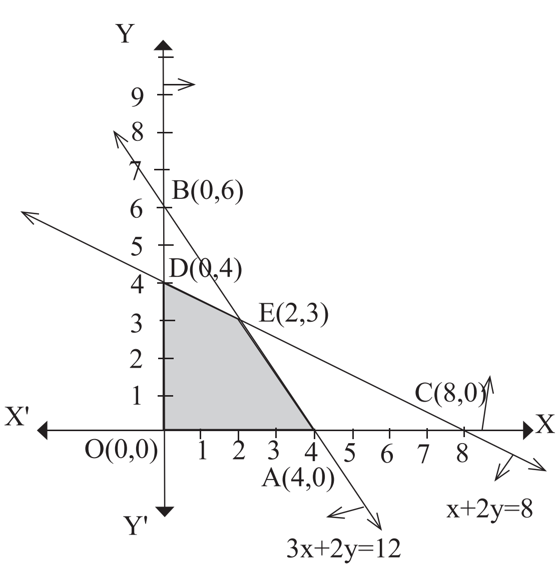
2. Convert all the constraints into eq^n-

$x+2y=8\,,\,3x+2y=12\,,\,x=0,y=0$

Draw the graph-

x	8	0
y	0	4

x	4	0
y	0	6



put (0,0) into equation

we get, $0 \leq 8$, $0 \leq 12$

All the euation satisfies (0,0)

The corner points of the feasible region are

O(0,0), A(4,0) , E(2,3) and D(0,4)

The values of Z at there corner points are-

Corner points	Z=-3x+4y
O(0,0)	Z=0
A(4,0)	Z=-12+0=-12 (Min^m)
E(2,3)	Z=-6+12=6
D(0,4)	Z=0+16=16

The minimum value of Z is -12 at (4,0).

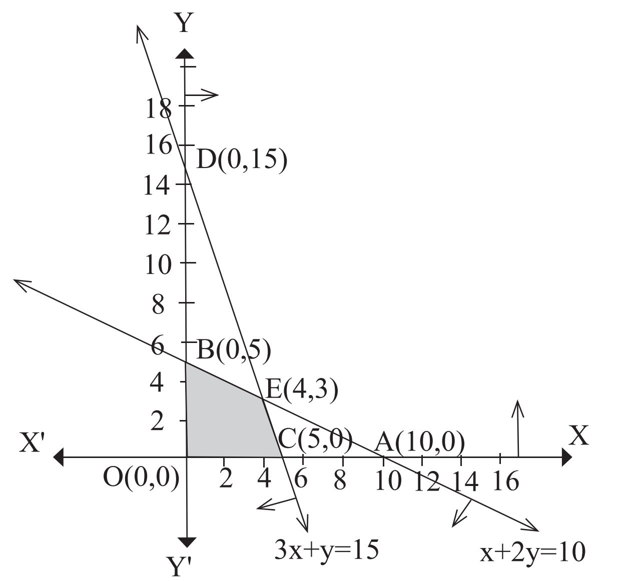
3. Convert all the constraints into eq^n.

$x+2y=10\,,\,3x+y=15\,,\,x=0,y=0$

Draw the graph-

x	10	0
y	0	5

x	5	0
y	0	15



The corner points of the feasible region are

O(0,0), C(5,0) , E(4,3) and B(0,5)

The values of Z at these corner points are-

Corner points	Z=3x+2y
O(0,0)	Z=0+0=0
C(5,0)	Z=3.5+0=15
E(4,3)	Z=3.4+2.3=18 (Max^m)
B(0,5)	Z=0+2.5=10

$\therefore$The maximum value of Z is 18 at the point (4,3)

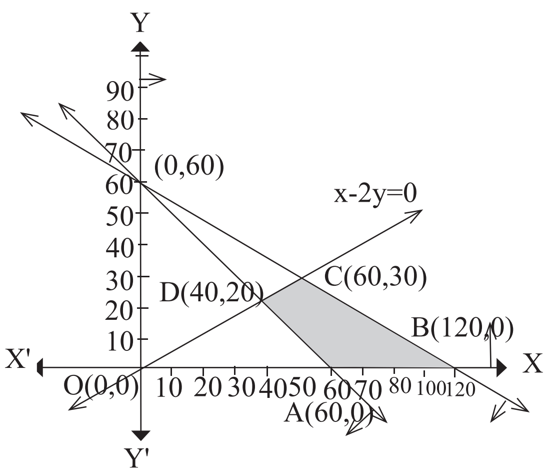
4. Convert all the constraints into eq^n-

We get,

$x + 2y = 120$, $x + y = 60$, $x - 2y = 0$,
$x = y = 0$

$$\Rightarrow \frac{x}{120} + \frac{y}{60} = 1, \; \frac{x}{60} + \frac{y}{60} = 1,$$
$x = 2y$

Hence the equation x+2y=120 cut at (120,0) and (0, 60) and x+y=60 cuts x-axis at (60, 0) and y-axis at (0,60) and x=2y passes through the origin and all equation satisfies (0.0)



The corner points of the feasible region are A(60,0) , B(120,0) , C(60,30) and D(40,20)

The value of z at these corner points are-

Corner points	Z=5x+10y
A(60,0)	Z=5.60+0=300 (Min^{m})
B(120,0)	Z=5.120+0=600 (Max^{m})
C(60,30)	Z=5.60+10.30=600(Max^{m})
D(40,20)	Z=5.40+10.20=400

The minimum value of Z is 300 at (60,0) and

the maximum value of Z is 600 at all the points on the line segment joining (120,0) and (60,30).

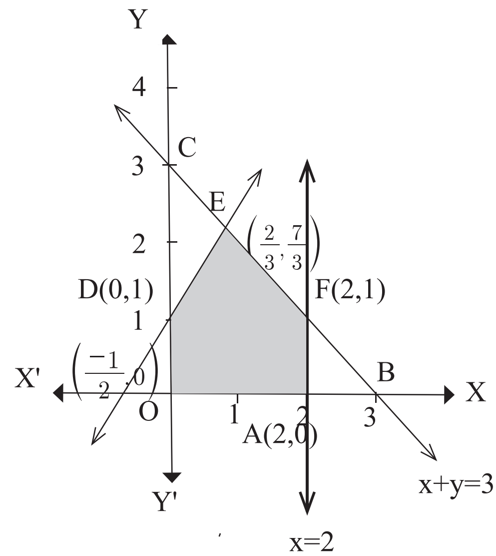
5. Sol- Convert all the constraints into eq^{n}-

We, get

$x = 2, x + y = 3$, $-2x + y = 1$, $x = 0, y = 0$

$$\Rightarrow \frac{x}{3} + \frac{y}{3} = 1, \; \frac{x}{\left(-\frac{1}{2}\right)} + \frac{y}{1} = 1$$

Hence, the equation x=2 cut x-axis at (2,0) and parallel to y-axis, eq^{n} x+y=3 cuts x-axis at (3,0) and y-axis at (0,3) and -2x+y=1 cuts x-axis at $\left(-\frac{1}{2},0\right)$ and y-axis at (0,1) and all inequation satisfies at (0,0).



The corner points of the feasible region are

O(0,0), A(2,0), F(2,1), E$\left(\frac{2}{3},\frac{7}{3}\right)$ and D(0,1)

The value of Z at these corner points are-

Corner points	Z=y-2x
O(0,0)	0+0=0
A(2,0)	0-4=-4
F(2,1)	1-4=-3
E(2/3,7/3)	$\frac{7}{3} - \frac{4}{3} = 1$ (Max^{m})
D(0,1)	1-0=1 (Max^{m})

The maximum value of Z is 1 at all points on the line segment joining D(0,1) and E($\frac{2}{3}$ - $\frac{7}{3}$)

6. Convert all the constraints into eq^{n}-

We get,

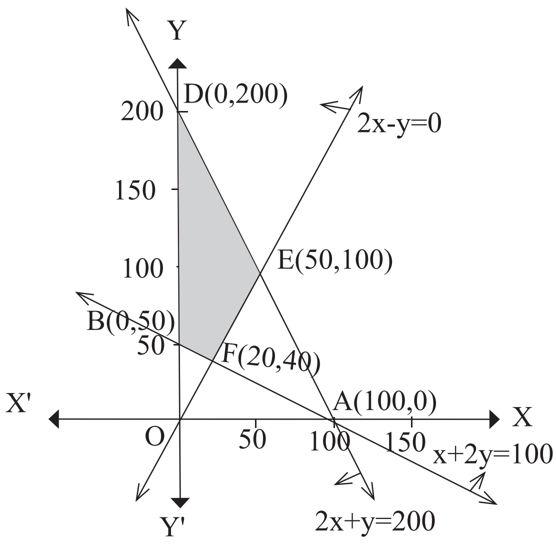
$x + 2y = 100$, $2x - y = 0$, $2x + y = 200$,
$x = y = 0$

$$\Rightarrow \frac{x}{100} + \frac{y}{50} = 1, \; 2x = y \quad \frac{x}{100} + \frac{y}{200} = 1$$

Equation x+2y=100 cuts x-axis at points

A(100,0) and y axis at B(0,50).

Equation 2x=y passes through origin and equation 2x+y=200 cuts x-axis at point C(100,0) and y-axis at D(0,200) and all the equation satisfies (0,0).



The Corner points of the feasible region are B(0,50), E(50,100), F(20,40) and D(0,200).

The value of Z at these corner points are -

Corner points	Z=x+2y
B(0,50)	Z= $0+2.50=100$ (Min^{m})
F(20,40)	Z=20+80=100 (Min^{m})
E(50,100)	Z=50+200=250
D(0,200)	Z=0+400=400 (Max^{m})

The Maximum value of Z is 400 at (0,200) and the minimum value of Z is 100 at all points on the line segment joining the points B(0,50) and F(20,40).