INDEFINITE INTEGRATION

Integration is the inverse process of differentiation. That is, the process of finding a function, whose differential coefficient is known, is called integration.

If the differential coefficient of F(x) is f (x), i.e. $\frac{d}{dx}[F(x)] = f(x)$, then we say that the **antiderivative** or **integral** of $f(x)$ is F(x), written as $\int f(x)dx = F(x)$,

Here $\int dx$ is the notation of integration $f(x)$ is the integrand, x is the variable of integration and dx denotes the integration with respect to x.

1. INDEFINITE INTEGRAL

1.1 Definition

We know that if $\frac{d}{dx}[F(x)] = f(x)$, then $\int f(x)dx = F(x)$.

Also, for any arbitrary constant C, $\frac{d}{dx}[F(x)+C] = \frac{d}{dx}[F(x)] + 0 = f(x)$.

$\therefore \int f(x)dx = F(x) + C,$

This shows that F(x) and F(x) + C are both integrals of the same function f(x). Thus, for different values of C, we obtain different integrals of f(x). This implies that the integral of f(x) is not definite. By virtue of this property F(x) is called the indefinite integral of f(x).

1.2 Properties of Indefinite Integration

1. $\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$

2. $\int f'(x)dx = \int \frac{d}{dx}[f(x)]dx = f(x) + c$

3. $\int k\, f(x)dx = k\int f(x)dx$, where k is any constant

4. If $f_1(x), f_2(x), f_3(x)$, ... (finite in number) are functions of x, then

$$\int [f_1(x) \pm f_2(x) \pm f_3(x)...]dx$$

$$= \int f_1(x)dx \pm \int f_2(x)dx \pm \int f_3(x)dx \pm ...$$

5. If $\int f(x)dx = F(x) + c$

then $\int f(ax \pm b)dx = \frac{1}{a}F(ax \pm b) + c$

6. Suppose I and J are intervals, g: J → I is differentiable and f : I → R has integral with primitive F. Then (fog).g′: J → R has an integral and

$$\int ((fog).g')(x)\, dx = \int f(g(x))\, g'(x)dx = F(g(x)) + c$$

1.3 Standard Formulae of Integration

The following results are a direct consequence of the definition of an integral.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1.$

2. $\int \frac{1}{x} dx = \log|x| + C$

3. $\int e^x\, dx = e^x + C$

4. $\int a^x\, dx = \frac{a^x}{\log_e a} + C.$

5. $\int \sin x\, dx = -\cos x + C$

6. $\int \cos x\, dx = \sin x + C$

7. $\int \sec^2 x\, dx = \tan x + C$

8. $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$

9. $\int \sec x \tan x\, dx = \sec x + C$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C.$

11. $\int \tan x \, dx = -\log|\cos x| + C = \log|\sec x| + C.$

12. $\int \cot x \, dx = \log|\sin x| + C$

13. $\int \sec x \, dx = \log|\sec x + \tan x| + C$

14. $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$

15. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C \; ; |x| < 1$

16. $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

17. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}|x| + C \; ; |x| > 1$

2. INTEGRATION BY SUBSTITUTION

2.1 Method of Substitution

By suitable substitution, the variable x in $\int f(x)\,dx$ is changed into another variable t so that the integrand $f(x)$ is changed into F(t) which is some standard integral or algebraic sum of standard integrals.

There is no general rule for finding a proper substitution and the best guide in this matter is experience.

However, the following suggestions will prove useful.

(i) If the integrand is of the form $f'(ax+b)$, then we put

$ax + b = t$ and $dx = \frac{1}{a} dt.$

Thus, $\int f'(ax+b)\,dx = \int f'(t) \frac{dt}{a}$

$$= \frac{1}{a}\int f'(t)\,dt = \frac{f(t)}{a} = \frac{f(ax+b)}{a} + c$$

(ii) When the integrand is of the form $x^{n-1} f'(x^n)$, we put $x^n = t$ and $nx^{n-1}\,dx = dt.$

Thus, $\int x^{n-1} f'(x^n)\,dx = \int f'(t)\frac{dt}{n} = \frac{1}{n}\int f'(t)\,dt$

$$= \frac{1}{n} f(t) = \frac{1}{n} f(x^n) + c$$

(iii) When the integrand is of the form $[f(x)]^n . f'(x)$, we put $f(x) = t$ and $f'(x)\,dx = dt.$

Thus, $\int [f(x)]^n f'(x)\,dx = \int t^n\,dt = \frac{t^{n+1}}{n+1} = \frac{[f(x)]^{n+1}}{n+1} + c$

(iv) When the integrand is of the form $\frac{f'(x)}{f(x)}$, we put

$f(x) = t$ and $f'(x)\,dx = dt.$

Thus, $\int \frac{f'(x)}{f(x)} dx = \int \frac{dt}{t} = \log|t| = \log|f(x)| + c$

2.2 Some Special Integrals

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

3. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

4. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

5. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + C$

6. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + C$

7. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

8. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + C$

9. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + C$

2.3 Integrals of the Form

(a) $\int f\left(a^2-x^2\right)dx$,

(b) $\int f\left(a^2+x^2\right)dx$,

(c) $\int f\left(x^2-a^2\right)dx$,

(d) $\int f\left(\frac{a-x}{a+x}\right)dx$,

Working Rule

Integral	Substitution
$\int f\left(a^2-x^2\right)dx$,	$x = a\sin\theta$ or $x = a\cos\theta$
$\int f\left(a^2+x^2\right)dx$,	$x = a\tan\theta$ or $x = a\cot\theta$
$\int f\left(x^2-a^2\right)dx$,	$x = a\sec\theta$ or $x = a\,\mathrm{cosec}\,\theta$
$\int f\left(\frac{a-x}{a+x}\right)dx$ or $\int f\left(\frac{a+x}{a-x}\right)dx$	$x = a\cos 2\theta$

2.4 Integrals of the Form

(a) $\int \frac{dx}{ax^2+bx+c}$,   (b) $\int \frac{dx}{\sqrt{ax^2+bx+c}}$,

(c) $\int \sqrt{ax^2+bx+c}\,dx$

Working Rule

(i) Make the coefficient of x^2 unity by taking the coefficient of x^2 outside the quadratic.

(ii) Complete the square in the terms involving x, i.e. write ax^2+bx+c in the form

$$a\left[\left(x+\frac{b}{2a}\right)^2\right]-\frac{\left(b^2-4ac\right)}{4a}.$$

(iii) The integrand is converted to one of the nine special integrals.

(iv) Integrate the function.

2.5 Integrals of the Form

(a) $\int \frac{px+q}{ax^2+bx+c}dx$,   (b) $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$,

(c) $\int (px+q)\sqrt{ax^2+bx+c}\;dx$

Working Rule

(a) $\int \frac{px+q}{ax^2+bx+c}dx$

Put $px+q=\lambda(2ax+b)+\mu$ or

$px+q=\lambda$ (derivative of quadratic) $+\mu$.

Comparing the coefficient of x and constant term on both sides, we get

$p=2a\lambda$ and $q=b\lambda+\mu \Rightarrow \lambda=\frac{p}{2a}$ and $\mu=\left(q-\frac{bp}{2a}\right)$.Then integral becomes

$$\int \frac{px+q}{ax^2+bx+c}dx$$

$$=\frac{p}{2a}\int \frac{2ax+b}{ax^2+bx+c}dx+\left(q-\frac{bp}{2a}\right)\int \frac{dx}{ax^2+bx+c}$$

$$=\frac{p}{2a}\log|ax^2+bx+c|+\left(q-\frac{bp}{2a}\right)\int \frac{dx}{ax^2+bx+c}$$

(b) $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

In this case the integral becomes

$$\Rightarrow \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$$

$$\Rightarrow \frac{p}{2a}\int \frac{2ax+b}{\sqrt{ax^2+bx+c}}dx+\left(q-\frac{bp}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\Rightarrow \frac{p}{a}\sqrt{ax^2+bx+c}+\left(q-\frac{bp}{2a}\right)\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

(c) $\int (px+q)\sqrt{ax^2+bx+c}\;dx$

The integral in this case is converted to

$$\int (px+q)\sqrt{ax^2+bx+c}\ dx = \frac{p}{2a}\int (2ax+b)\sqrt{ax^2+bx+c}\ dx$$

$$+\left(q-\frac{bp}{2a}\right)\int \sqrt{ax^2+bx+c}\ dx$$

$$=\frac{p}{3a}(ax^2+bx+c)^{3/2}+\left(q-\frac{bp}{2a}\right)\int \sqrt{ax^2+bx+c}\ dx$$

2.6 Integrals of the Form

$\int \frac{P(x)}{\sqrt{ax^2+bx+c}}dx$, where $P(x)$ is a polynomial in x of degree $n \ge 2$.

Working Rule:

Write $\int \frac{P(x)}{\sqrt{ax^2+bx+c}}dx =$

$$=(a_0+a_1x+a_2x^2+...+a_{n-1}x^{n-1})$$

$$\sqrt{ax^2+bx+c}+k\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

where k, a_0, a_1, ... a_{n-1} are constants to be determined by differentiating the above relation and equating the coefficients of various powers of x on both sides.

2.7 Integrals of the Form

$$\int \frac{x^2+1}{x^4+kx^2+1}dx \quad \text{or} \quad \int \frac{x^2-1}{x^4+kx^2+1}dx,$$

where k is a constant positive, negative or zero.

Working Rule

(i) Divide the numerator and denominator by x^2.

(ii) Put $x-\frac{1}{x}=z$ or $x+\frac{1}{x}=z$, whichever subsitution, on differentiation gives, the numerator of the resulting integrand.

(iii) Evaluate the resulting integral in z

(iv) Express the result in terms of x.

2.8 Integrals of the Form

$\int \frac{dx}{P\sqrt{Q}}$, where P, Q are linear or quadratic functions of x.

Integral	Substitution
$\int \frac{1}{(ax+b)\sqrt{cx+d}}dx$	$cx+d=z^2$
$\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$	$px+q=z^2$
$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$	$px+q=\frac{1}{z}$
$\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$	$x=\frac{1}{z}$.

2.9 Integrals of the Form $\int R\left(x, x^{\frac{p_1}{q_1}}, x^{\frac{p_2}{q_2}}, ..., x^{\frac{p_k}{q_k}}\right)dx$

To evaluate $\int R\left(x, x^{\frac{p_1}{q_1}}, x^{\frac{p_2}{q_2}}, ..., x^{\frac{p_k}{q_k}}\right)dx$ where R is a rational function of its variables $x, x^{\frac{p_1}{q_1}}, ..., x^{\frac{p_k}{q_k}}$, put $x = t^n$ where n is the L.C.M of the denomination of the fractions p_1/q_1, p_2/q_2,....., p_k/q_k.

3. INTEGRATION BY PARTIAL FRACTIONS

Integrals of the type $\int \frac{p(x)}{g(x)}$ can be integrated by resolving the integrand into partial fractions. We proceed as follows:

Check degree of $p(x)$ and $g(x)$.

If degree of $p(x) \ge$ degree of $g(x)$, then divide $p(x)$ by $g(x)$ till its degree is less, i.e. put in the form $\frac{p(x)}{g(x)}=r(x)+\frac{f(x)}{g(x)}$ where degree of $f(x)$ < degree of $g(x)$.

CASE 1: When the denominator contains non-repeated linear factors. That is

$$g(x) = (x-\alpha_1)(x-\alpha_2)\ldots(x-\alpha_n).$$

In such a case write $f(x)$ and $g(x)$ as:

$$\frac{f(x)}{g(x)} = \frac{A_1}{(x-\alpha_1)} + \frac{A_2}{(x-\alpha_2)} + \ldots + \frac{A_n}{(x-\alpha_n)}$$

where $A_1, A_2, \ldots A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 2 : When the denominator contains repeated as well as non-repeated linear factor. That is

$$g(x) = (x-\alpha_1)^2(x-\alpha_3)\ldots(x-\alpha_n).$$

In such a case write $f(x)$ and $g(x)$ as:

$$\frac{f(x)}{g(x)} = \frac{A_1}{x-\alpha_1} + \frac{A_2}{(x-\alpha_1)^2} + \frac{A_3}{x-\alpha_3} + \ldots + \frac{A_n}{(x-\alpha_n)}$$

where $A_1, A_2, \ldots A_n$ are constants to determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

Note : Corresponding to repeated linear factor $(x-a)^r$ in the denominator, a sum of r partial fractions of the type $\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_r}{(x-a)^r}$ is taken.

CASE 3 : When the denominator contains a non repeated quadratic factor which cannot be factorised further:

$$g(x) = (ax^2+bx+c)(x-\alpha_3)(x-\alpha_4)\ldots(x-\alpha_n).$$

In such a case express $f(x)$ and $g(x)$ as:

$$\frac{f(x)}{g(x)} = \frac{A_1x+A_2}{ax^2+bx+c} + \frac{A_3}{x-\alpha_3} + \ldots + \frac{A_n}{x-\alpha_n}$$

where $A_1, A_2, \ldots A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 4 : When the denominator contains a repeated quadratic factor which cannot be factorised further: That is

$$g(x) = (ax^2+bx+c)^2(x-\alpha_5)(x-\alpha_6)\ldots(x-\alpha_n)$$

In such a case write $f(x)$ and $g(x)$ as

$$\frac{f(x)}{g(x)} = \frac{A_1x+A_2}{ax^2+bx+c} + \frac{A_3x+A_4}{(ax^2+bx+c)^2} + \frac{A_5}{x-\alpha_5} + \ldots + \frac{A_n}{(x-\alpha_n)}$$

where $A_1, A_2, \ldots A_n$ are constants to be determined by comparing the coefficients of various powers of x on both sides after taking L.C.M.

CASE 5 : If the integrand contains only even powers of x

(i) Put $x^2 = z$ in the integrand.

(ii) Resolve the resulting rational expression in z into partial fractions

(iii) Put $z = x^2$ again in the partial fractions and then integrate both sides.

4. INTEGRATION BY PARTS

The process of integration of the product of two functions is known as integration by parts.

For example, if u and v are two functions of x,

then $\int (uv)\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx.$

In words, integral of the product of two functions = first function × integral of the second – integral of (differential of first × integral of the second function).

Working Hints

(i) Choose the first and second function in such a way that derivative of the first function and the integral of the second function can be easily found.

(ii) In case of integrals of the form $\int f(x).x^n\,dx$, take x^n as the first function and $f(x)$ as the second function.

(iii) In case of integrals of the form $\int (\log x)^n \cdot 1\,dx$, take 1 as the second function and $(\log x)^n$ as the first function.

(iv) Rule of integration by parts may be used repeatedly, if required.

(v) If the two functions are of different type, we can choose the first function as the one whose initial comes first in the word "ILATE", where

I — Inverse Trigonometric function

L — Logarithmic function

A — Algebraic function

T — Trigonometric function

E — Exponential function.

(vi) In case, both the functions are trigonometric, take that function as second function whose integral is simple. If both the functions are algebraic, take that function as first function whose derivative is simpler.

(vii) If the integral consists of an inverse trigonometric function of an algebraic expression in x, first simplify the integrand by a suitable trigonometric substitution and then integrate the new integrand.

4.1 Integrals of the Form:

Where the initial integrand reappears after integrating by parts.

Working Rule

(i) Apply the method of integration by parts twice.

(ii) On integrating by parts second time, we will obtain the given integrand again, put it equal to I.

(iii) Transpose and collect terms involving I on one side and evaluate I.

4.2 Integrals of the Form

$$\int e^x\left[f(x)+f'(x)\right]dx$$

Working Rule

(i) Split the integral into two integrals.

(ii) Integrate only the first integral by parts, i.e.

$$\int e^x\left[f(x)+f'(x)\right]dx$$

$$=\int e^x f(x)\,dx+\int e^x f'(x)\,dx$$

$$=\left[f(x).e^x-\int f'(x).e^x dx\right]+\int e^x f'(x)\,dx$$

$$=e^x f(x)+C.$$

4.3 Integrals of the Form

$$\int\left(f(x)+xf'(x)\right)dx$$

$$=\int f(x)\,dx+\int x\,f'(x)\,dx$$

$$=\int f(x)\,dx+\left[x\,f(x)-\int 1.f(x)\,dx\right]=xf(x)+c$$

5. INTEGRATION OF VARIOUS TRIGONOMETRIC FUNCTIONS

5.1 Integral of the Form

(a) $\int\frac{dx}{a+b\cos x}$ (b) $\int\frac{dx}{a+b\sin x}$

(c) $\int\frac{dx}{a+b\cos x+c\sin x}$

Working Rule

(i) Put $\cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\sin x=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ so that the given integrand becomes a function of $\tan\frac{x}{2}$.

(ii) Put $\tan\frac{x}{2}=z\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dz$

(iii) Integrate the resulting rational algebraic function of z

(iv) In the answer, put $z=\tan\frac{x}{2}$.

5.2 Integrals of the Form

(a) $\int\frac{dx}{a+b\cos^2 x}$ (b) $\int\frac{dx}{a+b\sin^2 x}$

(c) $\int\frac{dx}{a\cos^2 x+b\sin x\cos x+c\sin^2 x}$

Working Rule

(i) Divide the numerator and denominator by $\cos^2 x$.

(ii) In the denominator, replace $\sec^2 x$, if any, by $1+\tan^2 x$.

(iii) Put $\tan x=z\Rightarrow\sec^2 x\,dx=dz$.

(iv) Integrate the resulting rational algebraic function of z.

(v) In the answer, put $z=\tan x$.

5.3 Integrals of the Form

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx$$

Working Role

(i) Put Numerator = λ (denominator) + μ (derivative of denominator)

$a\cos x + b\sin x = \lambda(c\cos x + d\sin x) + \mu(-c\sin x + d\cos x)$.

(ii) Equate coefficients of sin x and cos x on both sides and find the values of λ and μ.

(iii) Split the given integral into two integrals and evaluate each integral separately, i.e.

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx =$$

$$\lambda\int 1dx + \mu\int \frac{-c\sin x + d\cos x}{c\cos x + d\sin x}\,dx = \lambda x + \mu \log|c\cos x + d\sin x|.$$

(iv) Substitute the values of λ and μ found in step 2.

5.4 Integrals of the Form

$$\int \frac{a + b\cos x + c\sin x}{e + f\cos x + g\sin x}\,dx$$

Working Rule

(i) Put Numerator = l (denominator) + m (derivative of denominator) + n

$a + b\cos x + c\sin x = l(e + f\cos x + g\sin x) + m(-f\sin x + g\cos x) + n$

(ii) Equate coefficients of sin x, cos x and constant term on both sides and find the values of l, m, n.

(iii) Split the given integral into three integrals and evaluate each integral separately, i.e.

$$\int \frac{a + b\cos x + c\sin x}{e + f\cos x + g\sin x}\,dx$$

$$= l\int 1dx + m\int \frac{-f\sin x + g\cos x}{e + f\cos x + g\sin x}\,dx + n\int \frac{dx}{e + f\cos x + g\sin x}$$

$$= lx + m\log|e + f\cos x + g\sin x| + n\int \frac{dx}{e + f\cos x + g\sin x}\,dx$$

(iv) Substitute the values of l, m, n found in Step (ii).

5.5 Integrals of the Form

$$\int \sin^m x \cos^n x\,dx$$

Working Rule

(i) If the power of sin x is an odd positive integer, put cos x = t.

(ii) If the power of cos x is an odd positive integer, put sin x = t.

(iii) If the power of sin x and cos x are both odd positive integers, put sin x = t or cos x = t.

(iv) If the power of sin x and cos x are both even positive integers, then express it as sines or cosines of multiple angles. Further integrate term by term.

(v) If the sum of powers of sin x and cos x is an even negative integer, put tan x = z.

5.6 Integrating $\int \tan^m x\sec^n x dx$

1. When m is odd and any n, rewrite the integrand in terms of sin x and cos x:

$$\tan^m x\sec^n x dx = \left(\frac{\sin x}{\cos x}\right)^m \left(\frac{1}{\cos x}\right)^n dx$$

$$= \frac{\sin^{m-1} x}{\cos^{n+m} x}\sin x\,dx$$

and then substitute u = cosx., du = - sin x dx

$\sin^2 x = 1 - \cos^2 x = 1 - u^2$.

2. Alternatively, if m is odd and $n \geq 1$ move one factor of sec x tan x to the side so that you can see secx tanx dx in the integral, and substitute u = sec x. du = sec x tan x dx and $\tan^2 x = \sec^2 x - 1 = u^2 - 1$.

3. If n is even with $n \geq 2$, move one factor of $\sec^2 x$ to the side so that you can see $\sec^2 x dx$ in the integral, and substitute u = tan x, du = $\sec^2 x$ dx and $\sec^2 x = 1 + \tan^2 x = 1 + u^2$.

4. When m is even and n = 0 – that is the integrand is just an even power of tangent - we can still use the u = tan x substitution, after using $\tan^2 x = \sec^2 x - 1$ (possibly more than once) to create a $\sec^2 x$.

6. REDUCTION FORMULA

Reduction formulae makes it possible to reduce an integral depending on the index n > 0, called the order of the integral, to an integral of the same type with smaller index. (i.e. To reduce the integral into similar integral of order less than or greater than given integral). Application of reduction formula is given with the help of some examples.

6.1 Reduction Formula for $\int \sin^n x \, dx$

Let $I_n = \int \sin^n x \, dx = \int \sin^{n-1} x \sin x \, dx$

I II

$$= -\sin^{n-1} x \cos x + \int (n-1) \sin^{n-2} x \cos^2 x \, dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int \sin^{n-2} x(1-\sin^2 x) \, dx$$

$$= -\sin^{n-1} x \cos x + (n-1)\int (\sin^{n-2} x - \sin^n x) \, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) I_{n-2} - (n-1) I_n$$

$$\therefore nI_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2}$$

$$\Rightarrow I_n = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

Thus, $\int \sin^n x \, dx = \dfrac{-\sin^{n-1} x \cos x}{n} + \dfrac{n-1}{n}\int \sin^{n-2} x \, dx$

6.2 Reduction Formula for $\int \cos^n x \, dx$

Let $I_n = \int \cos^n x \, dx = \int \cos^{n-1} x \cos x \, dx$

$$nI_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}$$

or $\int \cos^n x \, dx = \dfrac{\cos^{n-1} x \sin x}{n} + \dfrac{n-1}{n}\int \cos^{n-2} x \, dx$

6.3 Reduction Formula for $\int \tan^n x \, dx$

$$I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$$

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$$

SOLVED EXAMPLES

Example – 1

Evaluate : $\int \left(x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}} \right) dx$

Sol. $\int \left(x^3 + 5x^2 - 4 + \frac{7}{x} + \frac{2}{\sqrt{x}} \right) dx$

$= \int x^3\, dx + \int 5x^2\, dx - \int 4dx + \int \frac{7}{x} dx + \int \frac{2}{\sqrt{x}} dx$

$= \int x^3\, dx + 5 \,.\, \int x^2\, dx - 4 \,.\, \int 1 \,.\, dx + 7 \,.\, \int \frac{1}{x} dx + 2 \,.\, \int x^{-1/2} dx$

$= \frac{x^4}{4} + 5.\frac{x^3}{3} - 4x + 7\log|x| + 2\left(\frac{x^{1/2}}{1/2}\right) + C$

$= \frac{x^4}{4} + \frac{5}{3}x^3 - 4x + 7\log|x| + 4\sqrt{x} + C$

Example – 2

Evaluate : $\int e^{x \log a} + e^{a \log x} + e^{a \log a}\, dx$

Sol. We have,

$\int e^{x \log a} + e^{a \log x} + e^{a \log a}\, dx$

$= \int e^{\log a^x} + e^{\log x^a} + e^{\log a^a}\, dx$

$= \int (a^x + x^a + a^a)\, dx$

$= \int a^x\, dx + \int x^a\, dx + \int a^a\, dx$

$= \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a . x + C.$

Example – 3

Evaluate : $\int \frac{x^4}{x^2+1} dx$

Sol. $\int \frac{x^4}{x^2+1} dx$

$= \int \frac{x^4 - 1 + 1}{x^2+1} dx = \int \frac{x^4 - 1}{x^2+1} + \frac{1}{x^2+1} dx$

$= \int (x^2 - 1)\, dx + \int \frac{1}{x^2+1} dx = \frac{x^3}{3} - x + \tan^{-1} x + C$

Example – 4

Evaluate : $\int \frac{2^x + 3^x}{5^x} dx$

Sol. $\int \frac{2^x + 3^x}{5^x} dx$

$= \int \left(\frac{2^x}{5^x} + \frac{3^x}{5^x} \right) dx$

$= \int \left[\left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x \right] dx = \frac{(2/5)^x}{\log_e 2/5} + \frac{(3/5)^x}{\log_e 3/5} + C$

Example – 5

Evaluate : $\int x^3 \sin x^4\, dx$

Sol. We have

$I = \int x^3 \sin x^4\, dx$

Let $x^4 = t \quad \Rightarrow \quad d(x^4) = dt$

$\Rightarrow \quad 4x^3\, dx = dt \Rightarrow \quad dx = \frac{1}{4x^3} dt$

$I = \frac{1}{4}\int \sin t\, dt = -\frac{\cos t}{4} + C = -\frac{\cos (x^4)}{4} + C$

Example – 6

Evaluate : $\int \frac{x}{x^4 + x^2 + 1}\,dx$

Sol. We have,

$$I = \int \frac{x}{x^4 + x^2 + 1}\,dx = \int \frac{x}{(x^2)^2 + x^2 + 1}\,dx$$

Let $x^2 = t$, then, $d(x^2) = dt$

$\Rightarrow$ $2x\,dx = dt$

$\Rightarrow$ $dx = \frac{dt}{2x}$

$$I = \int \frac{x}{t^2 + t + 1} \cdot \frac{dt}{2x}$$

$$= \frac{1}{2}\int \frac{1}{t^2 + t + 1}\,dt$$

$$= \frac{1}{2}\int \frac{1}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dt$$

$$= \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2t + 1}{\sqrt{3}}\right) + C = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + C.$$

Example – 7

Evaluate : $\int \sqrt{x^2 + 2x + 5}\,dx$

Sol. We have,

$$\int \sqrt{x^2 + 2x + 5}$$

$$= \int \sqrt{x^2 + 2x + 1 + 4}\,dx = \int \sqrt{(x+1)^2 + 2^2}\,dx$$

$$= \frac{1}{2}(x+1)\sqrt{(x+1)^2 + 2^2} + \frac{1}{2} \cdot (2)^2 \log|(x+1)$$

$$+ \sqrt{(x+1)^2 + 2^2}\,| + C$$

$$= \frac{1}{2}(x+1)\sqrt{x^2 + 2x + 5} + 2\log|(x+1) + \sqrt{x^2 + 2x + 5}\,| + C$$

Example – 8

Evaluate : $\int \frac{1}{x^2 - x + 1}\,dx$

Sol. $\int \frac{1}{x^2 - x + 1}\,dx$

$$= \int \frac{1}{x^2 - x + \frac{1}{4} - \frac{1}{4} + 1}\,dx$$

$$= \int \frac{1}{(x - 1/2)^2 + 3/4}\,dx$$

$$= \int \frac{1}{(x - 1/2)^2 + \left(\sqrt{3}/2\right)^2}\,dx$$

$$= \frac{1}{\sqrt{3}/2} \tan^{-1}\left(\frac{x - 1/2}{\sqrt{3}/2}\right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x - 1}{\sqrt{3}}\right) + C.$$

Example – 9

Evaluate : $\int \frac{1}{\sqrt{9 + 8x - x^2}}\,dx$

Sol. $\int \frac{1}{\sqrt{9 + 8x - x^2}}\,dx$

$$= \int \frac{1}{\sqrt{-\{x^2 - 8x - 9\}}}\,dx$$

$$= \int \frac{1}{\sqrt{-\{x^2 - 8x + 16 - 25\}}}\,dx$$

$$= \int \frac{1}{\sqrt{-\{(x-4)^2 - 5^2\}}}\,dx$$

$$= \int \frac{1}{\sqrt{5^2 - (x-4)^2}}\,dx = \sin^{-1}\left(\frac{x - 4}{5}\right) + C$$

Example – 10

Evaluate : $\int \frac{2x+3}{\sqrt{x^2+4x+1}}\,dx$

Sol. $\int \frac{2x+3}{\sqrt{x^2+4x+1}}\,dx$

$= \int \frac{(2x+4)-1}{\sqrt{x^2+4x+1}}\,dx$

$= \int \frac{2x+4}{\sqrt{x^2+4x+1}}\,dx - \int \frac{1}{\sqrt{x^2+4x+1}}\,dx$

$= \int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}}\,dx$, where $t = x^2+4x+1$

$= 2\sqrt{t} - \log|(x+2)+\sqrt{x^2+4x+1}|+C$

$= 2\sqrt{x^2+4x+1} - \log|x+2+\sqrt{x^2+4x+1}|+C$

Example – 11

Evaluate : $\int (x-5)\sqrt{x^2+x}\,dx$

Sol Let $(x-5) = \lambda.\frac{d}{dx}(x^2+x)+\mu$. Then,

$x-5 = \lambda(2x+1)+\mu$.

Comparing coefficients of like powers of x, we get

$1 = 2\lambda$ and $\lambda+\mu = -5 \Rightarrow \lambda = \frac{1}{2}$ and $\mu = -\frac{11}{2}$

$\int (x-5)\sqrt{x^2+x}\,dx$

$= \int \left(\frac{1}{2}(2x+1)-\frac{11}{2}\right)\sqrt{x^2+x}\,dx$

$= \int \frac{1}{2}(2x+1)\sqrt{x^2+x}\,dx - \frac{11}{2}\int \sqrt{x^2+x}\,dx$

$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x}\,dx - \frac{11}{2}\int \sqrt{x^2+x}\,dx$

$= \frac{1}{2}\int \sqrt{t}\,dt - \frac{11}{2}\int \sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\,dx$ where

$t = x^2+x$

$= \frac{1}{2}.\frac{t^{3/2}}{3/2} - \frac{11}{2}\left[\left\{\frac{1}{2}\left(x+\frac{1}{2}\right)\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right.\right.$

$\left.\left. -\frac{1}{2}.\left(\frac{1}{2}\right)^2 \log\left[\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right\}\right]\right]+C$

$= \frac{1}{3}t^{3/2} - \frac{11}{2}\left[\frac{2x+1}{4}\sqrt{x^2+x}-\frac{1}{8}\ell n\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|\right]+C$

$= \frac{1}{3}(x^2+x)^{3/2}$

$-\frac{11}{2}\left[\frac{2x+1}{4}\sqrt{x^2+x}-\frac{1}{8}\ell n\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|\right]+C$

Example – 12

$\int \frac{1-x^2}{1+x^2+x^4}\,dx$

Sol. $\int \frac{-\left(1-\frac{1}{x^2}\right)dx}{x^2+\frac{1}{x^2}+1}$ (Dividing numerator and denominator by x^2)

Put $x+\frac{1}{x} = t$

$\Rightarrow -\int \frac{dt}{t^2-1}$

$= -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|+C$

$= -\frac{1}{2}\ln\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right|+C$

Example – 13

Evaluate : $\int \frac{1}{x^4+1}\,dx$

Sol. We have,

$$I = \int \frac{1}{x^4+1}\,dx$$

$$\Rightarrow \quad I = \int \frac{\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int \frac{\frac{2}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} - \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx - \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}\,dx$$

$$\Rightarrow \quad I = -\frac{1}{2}\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-2}\,dx$$

Putting $x - \frac{1}{x} = u$ in 1st integral and $x + \frac{1}{x} = v$ in 2nd integral, we get

$$I = \frac{1}{2}\int \frac{du}{u^2+\left(\sqrt{2}\right)^2} - \frac{1}{2}\int \frac{dv}{v^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) - \frac{1}{2}\,\frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x-1/x}{\sqrt{2}}\right) - \frac{1}{4\sqrt{2}}\log\left|\frac{x+1/x-\sqrt{2}}{x+1/x+\sqrt{2}}\right| + C$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right) - \frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}\,x+1}{x^2+x\sqrt{2}+1}\right| + C$$

Example – 14

Evaluate : $\int x \log(1+x)\,dx$

Sol. $\int \underset{II}{x} \underset{I}{\log(1+x)}\,dx$

$$= \log(x+1)\,.\,\frac{x^2}{2} - \int \frac{1}{x+1}\,.\,\frac{x^2}{2}\,dx$$

$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx$$

$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int \frac{x^2-1+1}{x+1}\,dx$$

$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int \frac{x^2-1}{x+1} + \frac{1}{x+1}\,dx$$

$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\left[\int\left((x-1)+\frac{1}{x+1}\right)dx\right]$$

$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\left[\frac{x^2}{2} - x + \log|x+1|\right] + C$$

Example – 15

Evaluate $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\,dx$

Sol. $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\,dx$

$$= \int \frac{\sin^{-1}\sqrt{x}-(\pi/2-\sin^{-1}\sqrt{x})}{\pi/2}\,dx$$

$\{\because \sin^{-1}\theta + \cos^{-1}\theta = \pi/2\}$.

$\Rightarrow \quad I = \frac{2}{\pi}\int (2\sin^{-1}\sqrt{x} - \pi/2)\,dx$

$I = \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - \int 1.dx$

$I = \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - x + c \qquad \text{... (i)}$

Let $x = \sin^2\theta$, then $dx = 2\sin\theta\cos\theta\,d\theta = \sin 2\theta\,d\theta$

$\therefore \quad \int \sin^{-1}\sqrt{x}\,dx = \int \underset{I}{\theta}.\sin \underset{II}{2\theta}\,d\theta$

applying integration by parts

$\int \sin^{-1}\sqrt{x}\,dx = -\theta.\frac{\cos 2\theta}{2} + \int \frac{1}{2}\cos 2\theta\,d\theta$

$= \frac{-\theta}{2}.\cos 2\theta + \frac{1}{4}\sin 2\theta$

$= \frac{-1.\theta}{2}.(1 - 2\sin^2\theta) + \frac{1}{2}.\sin\theta.\sqrt{1-\sin^2\theta}$

$= \frac{-1}{2}\sin^{-1}\sqrt{x}\,(1-2x) + \frac{1}{2}.\sqrt{x}\,\sqrt{1-x} \qquad \text{... (ii)}$

from (i) and (ii)

$I = \frac{4}{\pi}\left\{\frac{-1}{2}(\sin^{-1}\sqrt{x})(1-2x) + \frac{1}{2}\sqrt{x}\,\sqrt{1-x}\right\} - x + c$

$= \frac{2}{\pi}\{\sqrt{x-x^2} - (1-2x)\sin^{-1}\sqrt{x}\} - x + c$

Example – 16

If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(a) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

(b) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(c) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

Ans. (c)

Sol. Given, $\int f(x)dx = \psi(x)$

Let $I = \int x^5 f(x^3)dx$

put $x^3 = t$

$\Rightarrow x^2 dx = \frac{dt}{3} \qquad \text{....(i)}$

$\therefore I = \frac{1}{3}\int t\,f(t)\,dt$

$= \frac{1}{3}\left[t\int f(t)dt - \int\left\{\frac{d}{dt}(t)\int f(t)dt\right\}dt\right]$

[Integration by parts]

$= \frac{1}{3}\left[t\psi(t) - \int \psi(t)dt\right]$

$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3)dx\right] + c$ from ....(i)

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

Example – 17

Evaluate $\int\left(e^{\log x} + \sin x\right)\cos x\,dx$.

Sol. $\int\left(e^{\log x} + \sin x\right)\cos x\,dx$

$I = \int (x + \sin x)\cos x\,dx$

$I = \int x\cos x\,dx + \frac{1}{2}\int \sin 2x\,dx$

$= x\sin x - \int \sin x + x - \frac{1}{4}\cos 2x + c$

$= x\sin x + \cos x - \frac{1}{4}\cos 2x + c$

Example – 18

Evaluate

(i) $\int e^x\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)dx$ (ii) $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

Sol. (i) $I=\int e^x\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)dx$

$$I=\int e^x\left\{\frac{1}{\cos^2 x}+\frac{\sin x\cos x}{\cos^2 x}\right\}dx$$

$$I=\int e^x\{\tan x+\sec^2 x\}dx$$

M–I : $I=\int \underset{II}{e^x}.\underset{I}{\tan x}\,dx+\int e^x(\sec^2 x)\,dx$

$$I=\tan x.e^x-\int \sec^2 x.e^x dx+\int e^x.\sec^2 x\,dx+c$$

$I=e^x\tan x+c.$

M–II : $\int e^x(f(x)+f'(x))\,dx=e^x f(x)+c$

$$=e^x\tan x+c$$

(ii) $I=\int e^{2x}\left\{\frac{1+\sin 2x}{1+\cos 2x}\right\}dx$

$$=\int e^{2x}\left\{\frac{1+2\sin x\cos x}{2\cos^2 x}\right\}dx$$

$$=\int e^{2x}\left\{\frac{1}{2\cos^2 x}+\frac{2\sin x\cos x}{2\cos^2 x}\right\}dx$$

$$=\int e^{2x}\left\{\frac{1}{2}\sec^2 x+\tan x\right\}dx$$

M–I : $I=\int \underset{II}{e^{2x}}.\underset{I}{\tan x}\,dx+\frac{1}{2}\int e^{2x}.\sec^2 x\,dx$

$$=\tan x.\frac{e^{2x}}{2}-\int \sec^2 x.\frac{e^{2x}}{2}dx+\frac{1}{2}\int e^{2x}.\sec^2 x\,dx$$

$$I=\frac{1}{2}e^{2x}.\tan x+c.$$

M–II : $I=\int \frac{1}{2}e^{2x}\left(2\tan x+\sec^2 x\right)dx$

$\because \int e^{g(x)}\left(f(x).g'(x)+f'(x)\right)dx=e^{g(x)}f(x)+c$

$\therefore I=\frac{1}{2}e^{2x}\tan x+c$

Example – 19

Evaluate $\int \frac{(x-1)e^x}{(x+1)^3}dx.$

Ans. $\frac{e^x}{(x+1)^2}+c$

Sol. $I=\int \frac{(x-1)e^x}{(x+1)^3}dx=\int \frac{(x+1-2)e^x}{(x+1)^3}dx$

$$=\int e^x\left[\frac{1}{(x+1)^2}+\frac{-2}{(x+1)^3}\right]dx$$

$\because \int e^x[f(x)+f'(x)]dx=e^x f(x)+c$

$$=\frac{e^x}{(x+1)^2}+c$$

Example – 20

Evaluate $\int \frac{3x-5}{(3x-2)(x+1)^2}dx$

Sol. Let $\frac{3x-5}{(3x-2)(x+1)^2}=\frac{A}{3x-2}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$

$\therefore$ $3x-5=A(x+1)^2+B(3x-2)(x+1)+C(3x-2)$

Putting $x=\frac{2}{3}$, we get $A=-\frac{27}{25}$

Putting $x=-1$, we get $C=\frac{8}{5}$;

Putting $x=0$, we get $B=\frac{9}{25}$

$\therefore$ $I=\int\left[-\frac{27}{25}.\frac{1}{3x-2}+\frac{9}{25}.\frac{1}{(x+1)}+\frac{8}{5}.\frac{1}{(x+1)^2}\right]dx$

$$=-\frac{9}{25}\log|3x-2|+\frac{9}{25}\log|x+1|-\frac{8}{5}.\frac{1}{(x+1)}+c.$$

Example – 21

Evaluate $\int \frac{dx}{(x-1)(x^2+1)}$

Sol. Let $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$= \frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$

or $1 = A(x^2+1)+(Bx+C)(x-1)$

Putting $x = 1$, we get $A = \frac{1}{2}$;

Putting $x = 0$, we get $A - C = 1 \therefore C = A - 1 = -\frac{1}{2}$

Putting $x = -1$, we get $2A - 2(-B + C) = 1$

or $1 + 2B - 2\left(-\frac{1}{2}\right) = 1 \therefore B = -\frac{1}{2}$

Now $\frac{1}{(x-1)(x^2+1)} = \frac{1}{2(x-1)} - \frac{1}{2}\cdot\frac{x+1}{x^2+1}$

$\therefore \int \frac{dx}{(x-1)(x^2+1)} = \int \frac{1}{2(x-1)}dx - \frac{1}{2}\int \frac{x}{x^2+1}dx - \frac{1}{2}\int \frac{dx}{x^2+1}$

$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + c$

Example – 22

Find $\int \frac{x^4}{(x-1)(x^2+1)}dx$

Sol.

$$\begin{array}{r|l} x^3 - x^2 + x - 1 & x^4 \quad (x+1 \\ & x^4 - x^3 + x^2 - x \\ & \underline{\;\;\;\; + \;\;\; - \;\;\; +} \\ & x^3 - x^2 + x \\ & x^3 - x^2 + x - 1 \\ & \underline{\;\;\;\; + \;\;\; - \;\;\; +} \\ & 1 \end{array}$$

$\therefore \quad \frac{x^4}{(x-1)(x^2+1)} = x + 1 + \frac{1}{(x-1)(x^2+1)}$

$\therefore \quad \int \frac{x^4}{(x-1)(x^2+1)}dx = \int (x+1)dx + \int \frac{dx}{(x-1)(x^2+1)}$

$= \frac{x^2}{2} + x + \left(\int \frac{1}{2(x-1)} - \frac{x}{2(x^2+1)} - \frac{1}{2}\frac{1}{x^2+1}\right)dx$

(By Partial Fraction)

$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + c$

Example – 23

Evaluate $\int \frac{x^3+3x+2}{(x^2+1)^2(x+1)}dx$

Sol. $I = \int \frac{x(x^2+1)+2(x+1)}{(x^2+1)^2(x+1)}dx$

$= \int \frac{x}{(x^2+1)(x+1)}dx + 2\int \frac{dx}{(1+x^2)^2}$ ... (1)

Let $\frac{x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$

$\therefore \quad x = A(1+x^2) + (Bx+C)(1+x)$

Putting $x = -1$, we get $A = -\frac{1}{2}$

Putting $x = 0$, we get, $0 = A + C \Rightarrow C = -A = \frac{1}{2}$

Putting $x = 1$, we get $1 = 2A + 2(B + C)$

$= 2A + 2B + 2C = -1 + 2B + 1 \therefore B = \frac{1}{2}$

$\therefore \quad \int \frac{x}{(1+x)(1+x^2)}dx = \int\left(-\frac{1}{2(1+x)} + \frac{\frac{1}{2}x + \frac{1}{2}}{1+x^2}\right)dx$

$= -\frac{1}{2}\log|1+x| + \frac{1}{2}\int \frac{x}{1+x^2}dx + \frac{1}{2}\int \frac{dx}{1+x^2}$

$$= -\frac{1}{2}\log|1+x| + \frac{1}{4}\log(1+x^2) + \frac{1}{2}\tan^{-1}x \qquad ...(2)$$

To evaluate : $\int \frac{dx}{(1+x^2)^2}$, put $x = \tan\theta$

Then, $\int \frac{dx}{(1+x^2)^2} = \int \frac{\sec^2\theta}{(1+\tan^2\theta)^2}d\theta$

$$= \int \cos^2\theta d\theta = \int \frac{1+\cos 2\theta}{2}d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] = \frac{1}{2}[\theta + \sin\theta\cos\theta]$$

$$= \frac{1}{2}\left[\tan^{-1}x + \frac{x}{\sqrt{1+x^2}}.\frac{1}{\sqrt{1+x^2}}\right]$$

$$= \frac{1}{2}\tan^{-1}x + \frac{1}{2}\frac{x}{1+x^2}$$

Now from (1),

$$I = \frac{3}{2}\tan^{-1}x - \frac{1}{2}\log|1+x| + \frac{1}{4}\log(1+x^2) + \frac{x}{1+x^2} + c$$

Example – 24

Evaluate $\int \frac{dx}{\sin x + \sin 2x}$

Sol. Let the given integral be I, then

$$I = \int \frac{dx}{\sin x + 2\sin x\cos x} = \int \frac{dx}{\sin x(1+2\cos x)}$$

$$= \int \frac{\sin x\, dx}{\sin^2 x(1+2\cos x)} = \int \frac{\sin x dx}{(1-\cos^2 x)(1+2\cos x)}$$

Put cos x = y so that –sin x dx = dy.

$$\therefore \quad I = \int \frac{-dy}{(1-y^2)(1+2y)}$$

$$= -\int \frac{dy}{(1-y)(1+y)(1+2y)}$$

Now we break $\frac{1}{(1-y)(1+y)(1+2y)}$ into partial fractions

Let $\frac{1}{(1-y)(1+y)(1+2y)} = \frac{A}{1-y} + \frac{B}{1+y} + \frac{C}{1+2y}$

$$= \frac{A(1+y)(1+2y) + B(1-y)(1+2y) + C(1-y)(1+y)}{(1-y)(1+y)(1+2y)}$$

$$\therefore \quad 1 = A(1+y)(1+2y) + B(1-y)(1+2y) + C(1-y)(1+y)$$

Putting y = 1, we get $A = \frac{1}{6}$; putting $y = -\frac{1}{2}$, we get $C = \frac{4}{3}$

Putting y = –1, we get $B = -\frac{1}{2}$

Now $I = -\int \left[\frac{1}{6(1-y)} - \frac{1}{2(1+y)} + \frac{4}{3(1+2y)}\right]dy$

$$= -\frac{1}{6}\log|1-y| + \frac{1}{2}\log|1+y| - \frac{2}{3}\log|1+2y| + c$$

$$= -\frac{1}{6}\log(1-\cos x) + \frac{1}{2}\log(1+\cos x)$$

$$-\frac{2}{3}\log|1+2\cos x| + c$$

Example – 25

Evaluate $\int \sin^{-11/3}x \cos^{-1/3}x\, dx$

Sol. Here, $\int \sin^{-11/3}x, \cos^{-1/3}x\, dx$

i.e., $\left(-\frac{11}{3} - \frac{1}{3}\right) = -4$

$$\therefore \quad I = \int \frac{\cos^{-1/3}}{\sin^{-1/3}x.\sin^4 x}dx = \int (\cot^{-1/3}x)(\operatorname{cosec}^2 x)^2.dx$$

$$I = \int (\cot^{-1/3}x)(1+\cot^2 x)\operatorname{cosec}^2 x\, dx.$$

{let $\cot x = t, -\operatorname{cosec}^2 x\, dx = dt$}

$$= -\int t^{-1/3}(1+t^2)\, dt \quad = -\int (t^{-1/3} + t^{5/3})\, dt$$

$$= -\left\{\frac{3}{2}t^{2/3} + \frac{3}{8}t^{8/3}\right\} + c$$

$$= -\left\{\frac{3}{2}(\cot^{2/3}x) + \frac{3}{8}(\cot^{8/3}x)\right\} + c.$$

Example – 26

Evaluate : $\int \frac{1}{1+\sin x+\cos x}\,dx$

Sol. $I=\int \frac{1}{1+\sin x+\cos x}\,dx$

$$=\int \frac{1}{1+\frac{2\tan x/2}{1+\tan^2 x/2}+\frac{1-\tan^2 x/2}{1+\tan^2 x/2}}\,dx$$

$$=\int \frac{1+\tan^2 x/2}{1+\tan^2 x/2+2\tan x/2+1-\tan^2 x/2}\,dx$$

$$=\int \frac{\sec^2 x/2}{2+2\tan x/2}\,dx$$

Putting $\tan\frac{x}{2}=t$ and $\frac{1}{2}\sec^2\frac{x}{2}\,dx=dt$, we get

$$I=\int \frac{1}{t+1}\,dt=\log|t+1|+C=\log\left|\tan\frac{x}{2}+1\right|+C$$

Example – 27

Evaluate : $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}\,dx$

Sol. $I=\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}\,dx$

Let $3\sin x+2\cos x=\lambda.\frac{d}{dx}(3\cos x+2\sin x)+$

$\mu(3\cos x+2\sin x)$

$\Rightarrow$ $3\sin x+2\cos x=\lambda(-3\sin x+2\cos x)+$

$\mu(3\cos x+2\sin x)$

Comparing the coefficients of sin x and cos x on both sides, we get

$$-3\lambda+2\mu=3 \text{ and } 2\lambda+3\mu=2$$

$\Rightarrow$ $\mu=\frac{12}{13}$ and $\lambda=-\frac{5}{13}$

$\therefore$ $I=\int \frac{\lambda(-3\sin x+2\cos x)+\mu(3\cos x+2\sin x)}{3\cos x+2\sin x}\,dx$

$$=\mu\int 1.\,dx+\lambda\int \frac{-3\sin x+2\cos x}{3\cos x+2\sin x}\,dx$$

$=\mu x+\lambda\int \frac{dt}{t}$, where $t=3\cos x+2\sin x$

$=\mu x+\lambda\, ln|t|+C$

$$=\frac{12}{13}x+\frac{-5}{13}\,ln|3\cos x+2\sin x|+C$$

Example – 28

Evaluate : $\int \frac{3\cos x+2}{\sin x+2\cos x+3}\,dx$

Sol. We have,

$$I=\int \frac{3\cos x+2}{\sin x+2\cos x+3}\,dx$$

Let $3\cos x+2=\lambda(\sin x+2\cos x+3)+$

$\mu(\cos x-2\sin x)+\nu$

Comparing the coefficients of sin x, cos x and constant term on both sides, we get

$$\lambda-2\mu=0,\ 2\lambda+\mu=3,\ 3\lambda+\nu=2$$

$\Rightarrow$ $\lambda=\frac{6}{5},\ \mu=\frac{3}{5}$ and $\nu=-\frac{8}{5}$

$\therefore$ $I=\int \frac{\lambda(\sin x+2\cos x+3)+\mu(\cos x-2\sin x)+\nu}{\sin x+2\cos x+3}\,dx$

$\Rightarrow$ $I=\lambda\int dx+\mu\int \frac{\cos x-2\sin x}{\sin x+2\cos x+3}\,dx+$

$\nu\int \frac{1}{\sin x+2\cos x+3}\,dx$

$\Rightarrow$ $I=\lambda x+\mu\log|\sin x+2\cos x+3|+\nu I_1$, where

$$I_1=\int \frac{1}{\sin x+2\cos x+3}\,dx$$

Putting, $\sin x=\frac{2\tan x/2}{1+\tan^2 x/2}$, $\cos x=\frac{1-\tan^2 x/2}{1+\tan^2 x/2}$ we get

$$I_1=\int \frac{1}{\frac{2\tan x/2}{1+\tan^2 x/2}+\frac{2(1-\tan^2 x/2)}{1+\tan^2 x/2}+3}\,dx$$

$$= \int \frac{1+\tan^2 x/2}{2\tan x/2 + 2 - 2\tan^2 x/2 + 3(1+\tan^2 x/2)}\, dx$$

$$= \int \frac{\sec^2 x/2}{\tan^2 x/2 + 2\tan x/2 + 5}\, dx$$

Put $\tan x/2 = t$

$$\sec^2 x/2 . \frac{1}{2} dx = dt$$

$$\sec^2 x/2 dx = 2dt = 2\int \frac{dt}{t^2+2t+5} = 2\int \frac{dt}{(t+1)^2+(2)^2}$$

$$= \tan^{-1}\left(\frac{t+1}{2}\right) + C$$

$$= 2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+1}{2}\right) + C$$

Example – 29

Integrate $\frac{1}{1-\cot x}$ or $\frac{\sin x}{\sin x - \cos x}$.

Sol. Let $I = \int \frac{\sin x}{\sin x - \cos x} dx$

Again, let $\sin x = A(\cos x + \sin x) + B(\sin x - \cos x)$ then

$A + B = 1$ and $A - B = 0$

$$\Rightarrow A = \frac{1}{2}, B = \frac{1}{2}$$

$$\therefore I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx$$

$$= \frac{1}{2}\int \frac{\cos x + \sin x}{\sin x - \cos x} dx + \frac{1}{2}\int 1 dx + c$$

$$= \frac{1}{2}\log(\sin x - \cos x) + \frac{1}{2}x + c$$

Example – 30

Evaluate $\int \sin^3 x . \cos^5 x \, dx$

Sol. $I = \int \sin^3 x . \cos^5 x \, dx$

Let $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$I = -\int (1-t^2) . t^5 . dt$$

$$I = \int t^7 dt - \int t^5 dt = \frac{t^8}{8} - \frac{t^6}{6} + c$$

$$I = \frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + c$$

Method II: $I = \int R^3 (1-R^2)^2 dR$, if $\sin x = R$, $\cos x \, dx = dR$.

$$I = \int R^3 \, dR - \int 2R^5 dR + \int R^7 dR$$

$$I = \frac{\sin^4 x}{4} - \frac{2\sin^6 x}{6} + \frac{\sin^8 x}{8} + c$$

Example – 31

Evaluate

(i) $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$

(ii) $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$

Sol. (i) $I = \int \frac{1}{\sin(x-a)\cos(x-b)} dx$

$$I = \frac{\cos(a-b)}{\cos(a-b)} . \int \frac{dx}{\sin(x-a)\cos(x-b)}$$

$$= \frac{1}{\cos(a-b)} . \int \frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\cos(a-b)}$$

$$. \int \left\{ \frac{\cos(x-b) . \cos(x-a)}{\sin(x-a)\cos(x-b)} + \frac{\sin(x-b) . \sin(x-a)}{\sin(x-a)\cos(x-b)} \right\} dx$$

$$I=\frac{1}{\cos(a-b)}\int\{\cot(x-a)+\tan(x-b)\}\,dx$$

$$I=\frac{1}{\cos(a-b)}\{\log|\sin(x-a)|-\log|\cos(x-b)|\}+c$$

$$I=\frac{1}{\cos(a-b)}\log_e\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+c$$

(ii) $$I=\int\frac{1}{\cos(x-a)\cos(x-b)}dx$$

$$=\frac{1}{\sin(a-b)}\int\frac{\sin(a-b)\,dx}{\cos(x-a)\cos(x-b)}$$

$$=\frac{1}{\sin(a-b)}\int\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)}dx$$

$$=\frac{1}{\sin(a-b)}\int\left\{\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)}-\frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right\}dx$$

$$=\frac{1}{\sin(a-b)}\int\{\tan(x-b)-\tan(x-a)\}\,dx$$

$$=\frac{1}{\sin(a-b)}[-\log|\cos(x-b)|+\log|\cos(x-a)|]+c$$

$$=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+c$$

Example – 32

If $\int\frac{\sin x}{\sin(x-\alpha)}dx=Ax+B\log\sin(x-\alpha)+C$, then the value of (A, B) is

(a) (–sin α, cos α) (b) (cos α, sin α)

(c) (sin α, cos α) (d) (–cos α, sin α)

Ans. (b)

So $$\int\frac{\sin x}{\sin(x-\alpha)}dx$$

Let $x-\alpha=t\Rightarrow x=t+\alpha\Rightarrow dx=dt$

$$I=\int\frac{\sin(t+\alpha)}{\sin t}dt$$

$$I=\int\frac{\sin t\cos\alpha+\cos t\sin\alpha}{\sin t}dt$$

$$=\int\cos\alpha\,dt+\int\sin\alpha\cot t\,dt$$

$$=t\cos\alpha+\sin\alpha\ln\sin t+c$$

$$=(x-\alpha)\cos\alpha+\sin\alpha\ln\sin(x-\alpha)+c$$

$$\Rightarrow A=\cos\alpha,\ B=\sin\alpha$$

Example – 33

$\int\frac{dx}{\cos x-\sin x}$ is equal to

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

Ans. (d)

Sol. $$\int\frac{dx}{\cos x-\sin x}=\frac{1}{\sqrt{2}}\int\frac{dx}{\cos\left(x+\frac{\pi}{4}\right)}$$

$$\int\frac{dx}{\cos x-\sin x}=\frac{1}{\sqrt{2}}$$

$$\int\sec\left(x+\frac{\pi}{4}\right)dx=\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)+c$$

Example – 34

$\int\frac{\sin^8x-\cos^8x}{(1-2\sin^2x\cos^2x)}dx$ is equal to:

(a) $\frac{1}{2}\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $-\frac{1}{2}\sin x+c$ (d) $-\sin^2x+c$

Ans. (b)

Sol. $$=\int\frac{\sin^8x-\cos^8x}{1-2\sin^2x\cos^2x}dx$$

$$=\int \frac{\left(\sin^4 x-\cos^4 x\right)\left(\sin^4 x+\cos^4 x\right)}{1-2\sin^2 x\cos^2 x}dx$$

$$=\int \frac{\left(\sin^2 x-\cos^2 x\right)\left(1-2\sin^2 x\cos^2 x\right)}{1-2\sin^2 x\cos^2 x}dx$$

$$=\int -\cos 2x\,dx$$

$$=\frac{-1}{2}\sin 2x+c$$

Example – 35

Evaluate : $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\,dx$

Sol. Let $I=\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}\,dx$

Putting $x + 1 = t^2$, and $dx = 2t\,dt$, we get

$$I=\int \frac{(t^2+1)\,2t\,dt}{\{(t^2-1)^2+3(t^2-1)+3\}\sqrt{t^2}}$$

$$\Rightarrow \quad I=2\int \frac{(t^2+1)}{t^4+t^2+1}\,dt=2\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}+1}\,dt$$

$$\Rightarrow \quad I=2\int \frac{du}{u^2+\left(\sqrt{3}\right)^2} \text{ where } t-\frac{1}{t}=u.$$

$$\Rightarrow \quad I=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)+C=\frac{2}{\sqrt{3}}\tan^{-1}\left\{\frac{t-\frac{1}{t}}{\sqrt{3}}\right\}+C$$

$$\Rightarrow \quad I=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t^2-1}{t\sqrt{3}}\right)+C=\frac{2}{\sqrt{3}}\tan^{-1}\left\{\frac{x}{\sqrt{3(x+1)}}\right\}+C$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Integral as an anti-derivative

1. $\int\left(x-\frac{1}{x}\right)^3 dx, (x>0)$ *equals*

(a) $\frac{x^3}{3}-\frac{3}{2}x^2+3\log x+\frac{1}{2x^2}+C$

(b) $\frac{x^4}{4}-\frac{3}{2}x^2+3\log x+\frac{1}{2x^2}+C$

(c) $\frac{x^4}{4}+3\log x+\frac{1}{2x^2}+C$

(d) none of these

2. The value of $\int\left(\frac{6}{1+x^2}+10^x\right)dx$ is

(a) $6\tan^{-1}x+10^x\log_e 10+C$

(b) $6\tan^{-1}x+\frac{10^x}{\log_e 10}+C$

(c) $3\tan^{-1}x+\frac{10^x}{\log_e 10}+C$

(d) none of these

3. $\int(\tan x+\cot x)^2 dx$ is equal to

(a) $\tan x-\cot x+c$ (b) $\tan x+\cot x+c$

(c) $\cot x-\tan x+c$ (d) none of these

4. $\int\frac{x^2+\cos^2 x}{(1+x^2)\sin^2 x}dx$ is equals to

(a) $\tan^{-1}x+\cot x+c$ (b) $\tan^{-1}x-\cot x+c$

(c) $\cot^{-1}x-\tan x+c$ (d) $-\tan^{-1}x-\cot x+c$

5. $\int\left(\frac{2^x-5^x}{10^x}\right)dx$ is equal to

(a) $\frac{2^{-x}}{\log_e 2}-\frac{5^{-x}}{\log_e 5}+c$ (b) $\frac{2^x}{\log_e 2}-\frac{5^x}{\log_e 5}+c$

(c) $\frac{2^x}{\log_e 2}+\frac{5^x}{\log_e 5}+c$ (d) $\frac{5^{-x}}{\log_e 5}-\frac{2^{-x}}{\log_e 2}+c$

6. $\int\sec^2(ax+b)\,dx$ equals

(a) $\tan(ax+b)+C$ (b) $\frac{1}{2}\tan x+C$

(c) $\frac{1}{a}\tan(ax+b)+C$ (d) none of these

7. $\int\frac{dx}{\sin^2 x\cos^2 x}$ is equal to

(a) $\tan x+\cot x+C$ (b) $(\tan x+\cot x)^2+C$

(c) $\tan x-\cot x+C$ (d) $(\tan x-\cot x)^2+C$

8. $\int\frac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}dx$ is equal to

(a) $\tan x+\cot x+3x+c$ (b) $\tan x+\cot x-3x+c$

(c) $\tan x-\cot x-3x+c$ (d) $\tan x-\cot x+3x+c$

Integrations by substitution

9. $\int\sqrt{\frac{1+x}{1-x}}\,dx$ equals

(a) $\sin^{-1}x+\sqrt{1-x^2}+c$ (b) $\sin^{-1}x+\sqrt{x^2-1}+c$

(c) $\sin^{-1}x-\sqrt{1-x^2}+c$ (d) $\sin^{-1}x-\sqrt{x^2-1}+c$

10. $\int \frac{\left(\sin^{-1}x\right)^3}{\sqrt{1-x^2}}dx$ is equal to

(a) $\frac{\left(\sin^{-1}x\right)^2}{2}+c$ (b) $\frac{\left(\sin^{-1}x\right)^3}{3}+c$

(c) $\frac{\sin^{-1}x}{x}+c$ (d) $\frac{\left(\sin^{-1}x\right)^4}{4}+c$

11. $\int \sec^n x\tan x\, dx$ is equal to

(a) $\frac{\sec^n x}{n}+c$ (b) $\frac{\sec^2 x}{2}+c$

(c) $\frac{\tan x}{n}+c$ (d) $\frac{\left(\sec^n x\right)\tan x}{n}+c$

12. $\int \frac{\cos^3 x}{\sin^2 x+\sin x}dx$ is equal to

(a) log |cos x| – sin x + c (b) log |sin x| – sin x + c

(c) log |sin x| + cos x + c (d) log |cos x| – cos x + c

13. $\int \frac{\log_e x}{x\sqrt{1+\log_e x}}dx=$

(a) $(1+\log_e x)^{3/2}+c$

(b) $\frac{2}{3}(1+\log_e x)(\log_e x-2)+c$

(c) $\frac{2}{3}(1+\log_e x)^{1/2}(\log_e x-5)+c$

(d) $\frac{2}{3}(1+\log_e x)^{1/2}(\log_e x-2)+c$

14. $\int \frac{1}{x\log x}dx$ is equal to

(a) log |x log x| + C (b) log |log x + x| + C

(c) log x + C (d) log |log x| + C

15. $\int \sec^2 x\cos(\tan x)dx$ equals

(a) sin (cos x) + C (b) sin (tan x) + C

(c) cosec (tan x) + C (d) none of these

16. $\int \tan^n x\sec^2 x\,dx$ equals

(a) $\frac{\tan^{n-1}x}{n-1}+C$ (b) $\frac{\tan^{n+1}x}{n+1}+C$

(c) $\tan^{n+1}x+C$ (d) none of these

17. $\int \frac{\sin 2x}{1+\cos^4 x}dx$ is equal to

(a) $\cos^{-1}(\cos^2 x)+c$ (b) $\sin^{-1}(\cos^2 x)+c$

(c) $\cot^{-1}(\cos^2 x)+c$ (d) none of these

18. $\int \frac{dx}{x+\sqrt{x}}$ equals

(a) $2\log\left(\sqrt{x}-1\right)+c$ (b) $2\log\left(\sqrt{x}+1\right)+c$

(c) $\tan^{-1}x+c$ (d) none of these

19. $\int \frac{x^5}{\sqrt{1+x^3}}dx$ equals

(a) $\frac{2}{9}\left(x^3-2\right)\sqrt{1+x^3}+c$ (b) $\frac{2}{9}\left(x^3+2\right)\sqrt{1+x^3}+c$

(c) $\left(x^3+2\right)\sqrt{1+x^3}+c$ (d) none of these

20. $\int \frac{dx}{3x^2+2x+1}$ equals

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$

(b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x-1}{\sqrt{2}}\right)+C$

(c) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x-1}{\sqrt{2}}\right)+C$

(d) none of these

21. $\int \frac{dx}{\sqrt{3-5x-x^2}}$ equals

(a) $\sin^{-1}\left(\frac{2x+5}{\sqrt{37}}\right)+c$ (b) $\cos^{-1}\left(\frac{2x+5}{\sqrt{37}}\right)+c$

(c) $\sin^{-1}(2x+5)+c$ (d) none of these

22. $\int \frac{e^x}{e^{2x}+5e^x+6}dx$ equals

(a) $\log\left(\frac{e^x+3}{e^x+2}\right)+c$ (b) $\log\left(\frac{e^x+2}{e^x+3}\right)+c$

(c) $\frac{1}{2}\log\left(\frac{e^x+2}{e^x+3}\right)+c$ (d) none of these

23. $\int \frac{x^2}{x^2-1}dx$ equals

(a) $x+\log\sqrt{\left|\frac{x-1}{x+1}\right|}+c$ (b) $x+\log\sqrt{\left|\frac{x+1}{x-1}\right|}+c$

(c) $x+\log\left|\frac{x-1}{x+1}\right|+c$ (d) $x+\log\left|\frac{x+1}{x-1}\right|+c$

24. If m is a non-zero number and

$\int \frac{x^{5m-1}+2x^{4m-1}}{\left(x^{2m}+x^m+1\right)^3}dx = f(x)+c$, then f(x) is

(a) $\frac{x^{5m}}{2m\left(x^{2m}+x^m+1\right)^2}$ (b) $\frac{x^{4m}}{2m\left(x^{2m}+x^m+1\right)^2}$

(c) $\frac{2m\left(x^{5m}+x^{4m}\right)}{\left(x^{2m}+x^m+1\right)^2}$ (d) $\frac{\left(x^{5m}-x^{4m}\right)}{2m\left(x^{2m}+x^m+1\right)^2}$

25. $\int \frac{dx}{1+e^x}=$

(a) $\log_e\left(\frac{e^x+1}{e^x}\right)+c$ (b) $\log_e\left(\frac{e^x}{e^x+1}\right)+c$

(c) $x+\log_e(e^x+1)+c$ (d) e^x+x+c

26. $\int \sin^9 x.\cos^3 x\,dx$ is equal to

(a) $\frac{\sin^{10}x}{10}-\frac{\sin^{12}x}{12}+c$ (b) $\frac{\cos^6 x}{6}-\frac{\cos^8 x}{8}+c$

(c) $\frac{\cos^6 x}{6}-\frac{\sin^8 x}{8}+c$ (d) none of these

27. If $\int \frac{\sin^4 x}{\cos x}dx=\frac{1}{2}\log_e\left(\frac{1+\sin x}{1-\sin x}\right)-g(x)+c$ where g(x) equals.

(a) $\frac{1}{3}\sin^3 x+\sin x$ (b) $\frac{1}{3}\cos^3 x+\cos x$

(c) $\frac{1}{3}\sin^3 x-\sin x$ (d) $\frac{1}{3}\cos^3 x-\cos x$

28. $\int \frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$ is equal to

(a) $\tan^{-1}\left(\tan x+\frac{1}{2}\right)+c$ (b) $\frac{1}{4}\tan^{-1}\left(\tan x+\frac{1}{2}\right)+c$

(c) $4\tan^{-1}\left(\tan x+\frac{1}{2}\right)+c$ (d) none of these

29. The integral $\int \frac{\sin^2 x\cos^2 x}{\left(\sin^3 x+\cos^3 x\right)^2}dx$ equal to:

(a) $\frac{1}{\left(1+\cot^3 x\right)}+c$ (b) $-\frac{1}{3\left(1+\tan^3 x\right)}+c$

(c) $\frac{\sin^3 x}{\left(1+\cos^3 x\right)}+c$ (d) $-\frac{\cos^3 x}{3\left(1+\sin^3 x\right)}+c$

30. If $\int \tan^4 x\,dx=\lambda\tan^3 x+\mu\tan x+x+C$, then

(a) $\lambda=\frac{1}{3}$ (b) $\mu=1$

(c) $\lambda=-\frac{1}{3}$ (d) none of these

Standard algebraic formats

31. $\int \frac{2x-3}{x^2+3x-18}dx$ is equal to

(a) $\log|x^2+3x-18|-\frac{3}{2}\log\left|\frac{x-3}{x+6}\right|+C$

(b) $\log|x^2+3x-18|-\frac{2}{3}\log\left|\frac{x-3}{x+6}\right|+C$

(c) $\log|x^2+3x-18|+\frac{2}{3}\log\left|\frac{x-3}{x+6}\right|+C$

(d) None of these

32. Evaluate $\int \frac{x^2+x+5}{x^2-x-1}dx$

(a) $x+\log|x^2-x-1|+\frac{7}{\sqrt{5}}\log\left|\frac{2x-1-\sqrt{5}}{2x-1+\sqrt{5}}\right|+C$

(b) $x+\log|x^2-x-1|+\frac{7}{2\sqrt{5}}\log\left|\frac{2x-1-\sqrt{5}}{2x-1+\sqrt{5}}\right|+C$

(c) $x+\log|x^2-x-1|+\frac{14}{\sqrt{5}}\log\left|\frac{2x-1-\sqrt{5}}{2x-1+\sqrt{5}}\right|+C$

(d) None of these

33. Evaluate $\int \frac{x^3+x+1}{x^2-1}dx$

(a) $\frac{x^2}{2}+\log|x^2-1|+\log\left|\frac{x-1}{x+1}\right|+C$

(b) $\frac{x^2}{2}+\log|x^2-1|+\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$

(c) $x^2+\log|x^2-1|+\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$

(d) None of these

34. Evaluate $\int \frac{x^2+2}{x^4+4}dx$

(a) $\tan^{-1}\left(\frac{x-\frac{2}{x}}{2}\right)+C$ (b) $\frac{1}{4}\tan^{-1}\left(\frac{x-\frac{2}{x}}{2}\right)+C$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{x-\frac{2}{x}}{2}\right)+C$ (d) None of these

35. Evaluate $\int \frac{dx}{(x^2+2x+6)}$

(a) $\tan^{-1}\left(\frac{x+1}{\sqrt{5}}\right)+C$

(b) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x+1}{\sqrt{5}}\right)+C$

(c) $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{x+1}{\sqrt{5}}\right)+C$

(d) None of these

36. Evaluate the following $\int \frac{dx}{\sqrt{2ax-x^2}}$

(a) $\sin^{-1}\left(\frac{a-x}{a}\right)+c$

(b) $\frac{1}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+c$

(c) $\sin^{-1}\left(\frac{x-a}{a}\right)+c$

(d) $2\sin^{-1}\left(\frac{x-a}{a}\right)+c$

37. Evaluate $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}dx$

(a) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)+c$

(b) $\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)+c$

(c) $\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)+c$

(d) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{3(x+1)}\right)+c$

38. Evaluate $\int \frac{dx}{(x-2)\sqrt{x^2-4}}$

(a) $\frac{1}{2}\sqrt{\frac{x+2}{x-2}}+c$ (b) $-\frac{1}{2}\sqrt{\frac{x+2}{x-2}}+c$

(c) $\frac{1}{2}\sqrt{\frac{x-2}{x+2}}+c$ (d) $-\frac{1}{2}\sqrt{\frac{x-2}{x+2}}+c$

39. Evaluate $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

(a) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)+c$

(b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)+c$

(c) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}x}\right)+c$

(d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}x}\right)+c$

40. $\int \sqrt{1+x-2x^2}\,dx$ equals

(a) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2}+\frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right)+C$

(b) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2}-\frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right)+C$

(c) $\frac{1}{8}(4x-1)\sqrt{1+x-2x^2}+\frac{\sqrt{2}}{32}\sin^{-1}\left(\frac{4x+1}{3}\right)+C$

(d) None of these

Integration by partial fractions

41. $\int \frac{x^3+3}{(x+1)(x^2+1)}dx$ equals.

(a) $x+\log_e|x+1|-\log_e(x^2+1)+\cot^{-1}x+c$

(b) $x-\log_e|x+1|+\log_e(x^2+1)+\tan^{-1}x+c$

(c) $x+\log_e|x+1|-\log_e(x^2+1)+\tan^{-1}x+c$

(d) $x-\log_e|x+1|-\log_e(x^2+1)-\tan^{-1}x+c$

42. $\int \frac{x^2+1}{(x-1)(x-2)}dx$ equals

(a) $\log\left|\frac{(x-2)^5}{(x-1)^2}\right|+C$ (b) $x+\log\left|\frac{(x-2)^5}{(x-1)^2}\right|+C$

(c) $x+\log\left|\frac{(x-1)^5}{(x-2)^5}\right|+C$ (d) none of these

43. The value of $\int \frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$ *is*

(a) $\frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b}-a\tan^{-1}\frac{x}{a}\right]+C$

(b) $\frac{1}{b^2-a^2}\left[a\tan^{-1}\frac{x}{b}-b\tan^{-1}\frac{x}{a}\right]+C$

(c) $\frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b}+a\tan^{-1}\frac{x}{a}\right]+C$

(d) none of these

Integrations by parts

44. $\int \log x \, dx$ is equal to

(a) $x (\log x) - 1 + c$ (b) $x (\log x) - x + c$

(c) $x (\log x) - 1/x + c$ (d) $x (\log x) + c$

45. $\int \frac{\sin^{-1}\sqrt{x}}{\sqrt{1-x}} dx$ equals

(a) $2\left[\sqrt{x} - \sqrt{1-x}\sin^{-1}\sqrt{x}\right] + c$

(b) $2\left[\sqrt{x} + \sqrt{1-x}\sin^{-1}\sqrt{x}\right] + c$

(c) $\left[\sqrt{x} - \sqrt{1-x}\sin^{-1}\sqrt{x}\right] + c$

(d) none of these

46. $\int x^2 e^x \, dx$ is equal to

(a) $x^2 e^x - 2\left[e^{2x} - xe^x\right] + c$

(b) $x^2 e^x - 2\left[e^x - xe^x\right] + c$

(c) $x^2 e^x - 2\left[xe^{2x} - e^x\right] + c$

(d) $x^2 e^x - 2\left[xe^x - e^x\right] + c$

47. $\int (\log x)^2 \, dx$ is equal to

(a) $x (\log x)^2 - 2 [x \log x - x] + c$

(b) $x (\log x)^2 - 2 [\log x - x] + c$

(c) $x (\log x)^2 - 2 [\log x^2 - x] + c$

(d) $x (\log x)^2 - 2 [\log x - 2x] + c$

48. $\int \frac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}} dx$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}\sin^{-1} x + \frac{1}{2}\log\left|\sqrt{1-x^2}\right| + c$

(b) $\frac{x}{\sqrt{1-x^2}}\sin^{-1} x + \frac{1}{2}\log\left|1-x^2\right| + c$

(c) $\frac{x}{\sqrt{1-x^2}}\sin^{-1} x + \frac{1}{4}\log\left|\sqrt{1-x^2}\right| + c$

(d) none of these

49. The value of $\int x \sec x \tan x \, dx$ is

(a) $x \sec x + \log|\sec x + \tan x| + c$

(b) $x \sec x - \log|\sec x + \tan x| + c$

(c) $x \sec x + \log|\sec x - \tan x| + c$

(d) none of these

50. $\int \frac{x - \sin x}{1 - \cos x} dx$ is equal to

(a) $-x\cot\frac{x}{2} + c$ (b) $\cot\frac{x}{2} + c$

(c) $-\cot\frac{x}{2} + c$ (d) none of these

51. $\int e^x \frac{(1-x)^2}{\left(1+x^2\right)^2} dx$ is equal to

(a) $\frac{1}{x^2+1} + c$ (b) $\frac{e^x}{x^2+1} + c$

(c) $\frac{e^x - 1}{x^2+1} + c$ (d) $\frac{1-e^x}{x^2+1} + c$

52. $\int \frac{xe^x}{(x+1)^2}$ dx is equal to

(a) $\frac{e^x}{(x+1)^2} + c$ (b) $\frac{e^x}{x+1} + c$

(c) $\frac{e^x}{(x+1)^3} + c$ (d) none of these

53. $\int e^x \frac{x-1}{(x+1)^3} dx$ equals

(a) $-\frac{e^x}{x+1} + C$ (b) $\frac{e^x}{x+1} + C$

(c) $\frac{e^x}{(x+1)^2} + C$ (d) $-\frac{e^x}{(x+1)^2} + C$

54. $\int \left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{x}{x^2+1}+C$ (b) $\frac{\log x}{(\log x)^2+1}+C$

(c) $\frac{x}{(\log x)^2+1}+C$ (d) $\frac{xe^x}{1+x^2}+C$

55. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to :

(a) $-xe^{x+\frac{1}{x}}+c$ (b) $(x-1)\,e^{x+\frac{1}{x}}+c$

(c) $xe^{x+\frac{1}{x}}+c$ (d) $(x+1)\,e^{x+\frac{1}{x}}+c$

Standard trigonometric Formats

56. $\int \sin^2(x/2)\,dx$ equals

(a) $\frac{1}{2}(x+\sin x)+c$ (b) $\frac{1}{2}(x+\cos x)+c$

(c) $\frac{1}{2}(x-\sin x)+c$ (d) none of these

57. $\int \frac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}dx$ is equal to

(a) $2(\sin x+x\cos\theta)+C$

(b) $2(\sin x-x\cos\theta)+C$

(c) $2(\sin x+2x\cos\theta)+C$

(d) $2(\sin x-2x\cos\theta)+C$

58. $\int\frac{1-\cos 2x}{1+\cos 2x}dx=$

(a) $\tan x-x+C$ (b) $x+\tan x+C$

(c) $x-\tan x+C$ (d) $-x-\cot x+C$

59. If the integral $\int\frac{5\tan x}{\tan x-2}dx = x + a\ \ln|\sin x-2\cos x|+k$, then a is equal to

(a) –1 (b) –2

(c) 1 (d) 2

60. $\int\sqrt[3]{\frac{\sin^n x}{\cos^{n+6}x}}\,dx,\ n\in N$ is equal to

(a) $\frac{3}{n}(\tan x)^{\frac{n}{3}+1}+c$ (b) $\frac{3}{3+n}(\tan x)^{\frac{n}{3}+1}+c$

(c) $\frac{3}{n}(\cos x)^{n+1}+c$ (d) none of these

61. $\int\frac{\sin^4 x}{\cos^8 x}dx$ is equal to

(a) $\frac{(1+\tan^5 x)}{5}-\frac{\tan^7 x}{7}+c$

(b) $\frac{\tan^5 x}{5}+\frac{\tan^7 x}{7}+c$

(c) $\frac{\tan^5 x}{7}+\frac{\tan^7 x}{5}+c$

(d) none of these

62. $\int\frac{\sin^{10}x}{\cos^{12}x}dx=$

(a) $10\tan^9 x+C$ (b) $\frac{\tan^{11}x}{11}+C$

(c) $\frac{\tan 11x}{11}+C$ (d) none of these

63. $\int\sqrt{\frac{1+\cos x}{1-\cos x}}\,dx$ equals

(a) $\log\cos\left(\frac{x}{2}\right)+C$ (b) $2\log\sin\left(\frac{x}{2}\right)+C$

(c) $2\log\sec\left(\frac{x}{2}\right)+C$ (d) none of these

64. $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$ is equal to

(a) $\sin(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

(b) $\text{cosec}(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

(c) $\text{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

(d) $\sin(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

65. The value of $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(a) $x-\log\left|\cos x\left(x-\frac{\pi}{4}\right)\right|+C$

(b) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+C$

(c) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+C$

(d) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+C$

66. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$ (d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

Numerical Value Type Questions

67. $\int \frac{e^{5\log_e x}-e^{4\log_e x}}{e^{3\log_e x}-e^{2\log_e x}} dx = \frac{x^k}{k}+c$. Then k is equal to

68. $\int \left(\frac{1-x}{1+x}\right)^2 dx = x-a\log|x+1|-\frac{b}{x+1}+c$. Then a + b is equal to

69. If $\int \frac{dx}{1+\tan x} = px+q\log_e|\cos x+\sin x|+c$

then p + q equals.

70. $\int \frac{x+1}{x^2+x+3} dx$

$=\frac{1}{a}\ln|x^2+x+3|+\frac{1}{\sqrt{b}}\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)+c$

then (b – a) equals

71. If $\int \frac{2x+3}{(x-1)(x^2+1)} dx$

$=\log\left[|x-1|^{5/2}(x^2+1)^{-\frac{5}{a}}\right]-\frac{1}{2}\tan^{-1}x+C$

where C is any arbitrary constant, then a is equal to

72. If $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = a\sin\sqrt{x}+c$, then a equals

73. $\int \frac{x^2}{1+x^6} dx = \frac{1}{k}\tan^{-1}x^3+c$. Then k is equal to

74. $\int \frac{x+1}{\sqrt{x+2}} dx =$

$\frac{a}{b}(x+2)^{\frac{3}{2}}-a\sqrt{x+2}+c$ then ab equals

75. $\int \sqrt{1+x-2x^2}\, dx$

$=\frac{1}{2a}(4x-1)\sqrt{1+x-x^2}+\frac{9\sqrt{2}}{b}\sin^{-1}\left(\frac{4x-1}{3}\right)+c$.

Then b/a is equal to

76. $\int \frac{(x-x^3)^{1/3}}{x^4} dx = -\frac{3}{k}\left(\frac{1}{x^2}-1\right)^{4/3} + c$. Then k is equal to

77. $\int \frac{\log x}{x^2} dx = -\frac{1}{x}(a+\log bx)+c$. (a,b $\in$ Integers). Then a + b equal to

78. $\int x^3 (\log x)^2 \, dx = \frac{1}{p} x^4 \left[q(\log x)^2 + r \log x + 1 \right] + c$.

Then p + q + r is equal to

79. $\int \frac{x+\sin x}{1+\cos x} dx = \frac{x}{a} \tan\left(\frac{x}{b}\right) + C$. Then b – a is equal to

80. If $\int \sin 2x \sin 3x \, dx$ equals $\frac{1}{k}$ $(5\sin x - \sin 5x) + c$. Then k is equal to

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equal **(2015)**

(a) $-(x^4+1)^{\frac{1}{4}}+c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (d) $(x^4+1)^{\frac{1}{4}}+c$

2. The integral $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$ is equal to

(2015/Online Set–1)

(a) $4\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$ (b) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$

(c) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}}+C$ (d) $4\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}}+C$

3. If $\int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}dt = \frac{1}{2}(g(t))^2 + C$, where C is constant, then g(2) is equal to :

(2015/Online Set–2)

(a) $\frac{1}{\sqrt{5}}\log(2+\sqrt{5})$ (b) $\frac{1}{2}\log(2+\sqrt{5})$

(c) $2\log(2+\sqrt{5})$ (d) $\log(2+\sqrt{5})$

4. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to : **(2016)**

(a) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$ (b) $\frac{x^5}{2(x^5+x^3+1)^2}+C$

(c) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$ (d) $\frac{-x^5}{(x^5+x^3+1)^2}+C$

Where C is an arbitrary constant.

5. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then A+B+C equals :

(2016/Online Set–1)

(a) $\frac{21}{5}$ (b) $\frac{16}{5}$

(c) $\frac{10}{7}$ (d) $\frac{27}{10}$

6. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration.)

(2016/Online Set–2)

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$ (b) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(c) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

7. Let $I_n = \int \tan^n x\,dx, (n>1)$. If $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to: **(2017)**

(a) $\left(-\frac{1}{5},1\right)$ (b) $\left(\frac{1}{5},0\right)$

(c) $\left(\frac{1}{5},-1\right)$ (d) $\left(-\frac{1}{5},0\right)$

8. The integral

$\int\sqrt{1+2\cot x(\operatorname{cosec} x+\cot x)}\,dx \left(0<x<\frac{\pi}{2}\right)$ is equal to (where C is a constant of integration)

(2017/Online Set–1)

(a) $4\log\left(\sin\frac{x}{2}\right)+C$ (b) $2\log\left(\sin\frac{x}{2}\right)+C$

(c) $2\log\left(\cos\frac{x}{2}\right)+C$ (d) $4\log\left(\cos\frac{x}{2}\right)+C$

9. If $f\left(\frac{3x-4}{3x+4}\right)=x+2,\ x\neq-\frac{4}{3}$, and

$\int f(x)\,dx = A\log|1-x|+Bx+C$, then the ordered pair (A, B) is equal to :

(where C is a constant of integration)

(2017/Online Set–2)

(a) $\left(\frac{8}{3},\frac{2}{3}\right)$ (b) $\left(-\frac{8}{3},\frac{2}{3}\right)$

(c) $\left(-\frac{8}{3},-\frac{2}{3}\right)$ (d) $\left(\frac{8}{3},-\frac{2}{3}\right)$

10. The integral

$$\int\frac{\sin^2x\cos^2x}{\left(\sin^5x+\cos^3x\sin^2x+\sin^3x\cos^2x+\cos^5x\right)^2}dx \text{ is}$$

(2018)

(a) $\frac{-1}{1+\cot^3x}+C$ (b) $\frac{1}{3\left(1+\tan^3x\right)}+C$

(c) $\frac{-1}{3\left(1+\tan^3x\right)}+C$ (d) $\frac{1}{1+\cot^3x}+C$

(Where C is a constant of integration)

11. If $f\left(\frac{x-4}{x+2}\right)=2x+1,\ \left(x\in R-\{1,-2\}\right)$, then $\int f(x)dx$ is equal to

(where C is a constant of integration)

(2018/Online Set–1)

(a) $12\log_e|1-x|+3x+C$

(b) $-12\log_e|1-x|-3x+C$

(c) $12\log_e|1-x|-3x+C$

(d) $-12\log_e|1-x|+3x+C$

12. If $\int\frac{2x+5}{\sqrt{7-6x-x^2}}dx=A\sqrt{7-6x-x^2}+B\sin^{-1}\left(\frac{x+3}{4}\right)+c$

(where C is a constant of integration), then the ordered pair (A, B) is equal to : **(2018/Online Set–2)**

(a) (2, 1) (b) (-2, -1)

(c) (-2, 1) (d) (2,-1)

13. If $\int\frac{\tan x}{1+\tan x+\tan^2x}dx=x-\frac{k}{\sqrt{A}}\tan^{-1}\left(\frac{k\tan x+1}{\sqrt{A}}\right)+c$,

(C is a constant of integration), then the ordered pair (K, A) is equal to : **(2018/Online Set–3)**

(a) (2, 1) (b) (-2, 3)

(c) (2, 3) (d) (-2, 1)

14. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration.)

(8-04-2019/Shift-1)

(a) $2x+\sin x+2\sin 2x+c$

(b) $x+2\sin x+2\sin 2x+c$

(c) $x+2\sin x+\sin 2x+c$

(d) $2x+\sin x+\sin 2x+c$

15. If $\int\frac{dx}{x^3\left(1+x^6\right)^{2/3}}=xf(x)\left(1+x^6\right)^{\frac{1}{3}}+C$, where C is a constant of integration, then the function $f(x)$ is equal to:

(08-04-2019/Shift-2)

(a) $\frac{3}{x^2}$ (b) $-\frac{1}{6x^3}$

(c) $-\frac{1}{2x^2}$ (d) $-\frac{1}{2x^3}$

16. The integral $\int\sec^{\frac{2}{3}}x\,\text{cosec}^{\frac{4}{3}}x\,dx$ is equal to:

(9-04-2019/Shift-1)

(a) $-3\tan^{\frac{-1}{3}}x+C$ (b) $-\frac{3}{4}\tan^{\frac{-4}{3}}x+C$

(c) $-3\cot^{\frac{-1}{3}}x+C$ (d) $3\tan^{\frac{-1}{3}}x+C$

(Here C is a constant of integration)

17. If $\int e^{\sec x}\left(\sec x \tan x\, f(x)+\left(\sec x \tan x+\sec^2 x\right)\right)dx$ $=e^{\sec x} f(x)+C$, then a possible choice of $f(x)$ is:

(9-04-2019/Shift-2)

(a) $\sec x+\tan x+C$ (b) $\sec x-\tan x-C$

(c) $\sec x+2\tan x-C$ (d) $x\sec x+\tan x+C$

18. If $\int x^5 e^{-x^2}dx = g(x)\, e^{-x^2}+c$, where c is a constant of integration, then g (–1) is equal to:

(10-4-2019/Shift-2)

(a) –1 (b) 1

(c) $-\frac{5}{2}$ (d) $-\frac{1}{2}$

19. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to : (Here C is a constant of integration) **(12-04-2019/Shift-1)**

(a) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$ (b) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$

(c) $\log_e\left|\frac{x^3+1}{x}\right|+C$ (d) $\log_e\frac{|x^3+1|}{x^2}+C$

20. Let $\alpha\in\left(0,\frac{\pi}{2}\right)$ be fixed. If the integral

$\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$,

where C is a constant of integration, then the functions A(x) and B(x) are respectivelty:

(12-04-2019/Shift-2)

(a) $x+\alpha$ and $\log_e|\sin(x+\alpha)|$

(b) $x-\alpha$ and $\log_e|\sin(x-\alpha)|$

(c) $x-\alpha$ and $\log_e|\cos(x-\alpha)|$

(d) $x+\alpha$ and $\log_e|\sin(x-\alpha)|$

21. For $x^2\neq n\pi, n\in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$ is equal to :

(where c is a constant of integration)

(9-01-2019/Shift-1)

(a) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

(b) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

(c) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(d) $2\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|-c$

22. If $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx,\ (x\geq 0)$, and $f(0)=0$, then the value of $f(1)$ is **(9-01-2019/Shift-2)**

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

23. Let $n\geq 2$ be a natural number and $0<\theta<\frac{\pi}{2}$ then

$\int\frac{(\sin^n\theta-\sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to:

(10-1-2019/Shift-1)

(a) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(b) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(c) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(d) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(where C is a constant of integration)

24. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to:

(10-01-2019/Shift-2)

(a) $-2x^3 - 1$ (b) $-4x^3 - 1$

(c) $-2x^3 + 1$ (d) $4x^3 + 1$

25. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals :

(11-01-2019/Shift-1)

(a) $\frac{-1}{27x^9}$ (b) $\frac{-1}{3x^3}$

(c) $\frac{1}{27x^6}$ (d) $\frac{1}{9x^4}$

26. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to :

(11-01-2019/Shift-2)

(a) $\frac{1}{3}(x+1)$ (b) $\frac{2}{3}(x+2)$

(c) $\frac{2}{3}(x-4)$ (d) $\frac{1}{3}(x+4)$

27. The integral $\int \cos(\log_e x)\, dx$ is equal to (Where C is a constant of integration) **(12-01-2019/Shift-1)**

(a) $\frac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right] + C$

(b) $x\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

(c) $\frac{x}{2}\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

(d) $x\left[\cos(\log_e x) - \sin(\log_e x)\right] + C$

28. The integral $\int \frac{3x^{13} + 2x^{11}}{\left(2x^4 + 3x^2 + 1\right)^4} dx$ is equal to (where C is a constant to integration) **(12-01-2019/Shift-2)**

(a) $\frac{x^4}{6\left(2x^4 + 3x^2 + 1\right)^3} + C$ (b) $\frac{x^{12}}{6\left(2x^4 + 3x^2 + 1\right)^3} + C$

(c) $\frac{x^4}{\left(2x^4 + 3x^2 + 1\right)^3} + C$ (d) $\frac{x^{12}}{\left(2x^4 + 3x^2 + 1\right)^3} + C$

29. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair (A (x), B (x)) can be : **(3-09-2020/Shift-2)**

(a) $(x+1, -\sqrt{x})$ (b) $(x-1, -\sqrt{x})$

(c) $(x+1, \sqrt{x})$ (d) $(x-1, \sqrt{x})$

30. The integral $\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to (where C is a constant of integration) : **(4-09-2020/Shift-1)**

(a) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

(b) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(c) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

(d) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

31. If $\int (e^{2x} + 2e^x - e^{-x} - 1)\, e^{(e^x + e^{-x})} dx$

$= g(x)e^{(e^x + e^{-x})} + c$ where c is a constant of integration, then g(0) is equal to: **(5-09-2020/Shift-1)**

(a) 2 (b) e

(c) 1 (d) e^2

32. If $\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)|+C$ where c is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

(5-09-2020/Shift-2)

(a) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$ (b) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$

(c) $\frac{2\sin\theta+1}{\sin\theta+3}$ (d) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

33. If $\int \frac{\cos x}{\sin^3 x(1+\sin^6 x)^{\frac{2}{3}}}dx = f(x)(1+\sin^6 x)^{\frac{1}{\lambda}}+c$,

where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to: **(8-01-2020/Shift-1)**

(a) $-\frac{9}{8}$ (b) $\frac{9}{8}$

(c) 2 (d) –2

34. If $f'(x)=\tan^{-1}(\sec x+\tan x), -\frac{\pi}{2}<x<\frac{\pi}{2}$

and $f(0)=0$, then $f(1)$ is equal to:

(9-01-2020/Shift-1)

(a) $\frac{\pi+1}{4}$ (b) $\frac{\pi+2}{4}$

(c) $\frac{1}{4}$ (d) $\frac{\pi-1}{4}$

35. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to: (where C is a constant of integration) **(9-01-2020/Shift-1)**

(a) $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}}+C$ (b) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}}+C$

(c) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$ (d) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}}+C$

36. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)} = \lambda\tan\theta+2\log_e|f(\theta)|+C$

where C is constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to: **(9-1-2020/Shift-2)**

(a) $(-1,1-\tan\theta)$ (b) $(-1,1+\tan\theta)$

(c) $(1,1+\tan\theta)$ (d) $(1,1-\tan\theta)$

37. If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+b\left(\frac{2x+1}{x^2+x+1}\right)+C$,

$x>0$ where C is the constant of integration, then the value of $9(\sqrt{3}a+b)$ is equal to _______.

(27-08-2021/Shift-1)

38. If $\int \frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx = \frac{1}{14}\left(ux+v\log_e\left(4e^x+7e^{-x}\right)\right)+C$,

where C is a constant of integration, then $u+v$ is equal to ________. **(27-08-2021/Shift-2)**

39. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to

(where C is a constant of integration)

(31-08-2021/Shift-1)

(a) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+c$ (b) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+c$

(c) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+c$ (d) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+c$

40. $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx = \alpha\log_e|1+\tan x|+\beta\log_e$

$|1-\tan x+\tan^2 x|+\gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$,

When C is constant of integration, then the value of $18(\alpha+\beta+\gamma^2)$ is **(31-08-2021/Shift-2)**

41. If $\int \frac{\cos x - \sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right)+c$ where c is a constant of integration, then the ordered pair (a, b) is equal to **(24-02-2021/Shift-1)**

(a) $(3,1)$ (b) $(1,-3)$

(c) $(1,3)$ (d) $(-1,3)$

42. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$$

is:

(where c is a constant of integration)

(25-02-2021/Shift-1)

(a) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$

(b) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

(c) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$

(d) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

43. The integral $\int \frac{e^{3\log_e 2x}+5e^{2\log_e 2x}}{e^{4\log_e x}+5e^{3\log_e x}-7e^{2\log_e x}}dx, x>0$ is equal to:

(where c is a constant of integration)

(25-02-2021/Shift-2)

(a) $4\log_e\left|x^2+5x-7\right|+c$

(b) $\log_e\sqrt{x^2+5x-7}+c$

(c) $\frac{1}{4}\log_e\left|x^2+5x-7\right|+c$

(d) $\log_e\left|x^2+5x-7\right|+c$

44. For the real numbers α, β, γ and δ if

$$\int \frac{\left(x^2-1\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma\left(x^2-1\right)}{x}\right)$$

$$+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

where C is an arbitrary constant, then the value of $10(\alpha+\beta\gamma+\delta)$ is equal to ___ **(16-03-2021/Shift-2)**

45. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to :

(where c is a constant of integration)

(18-03-2021/Shift-1)

(a) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

(b) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

(c) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(d) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

1. Evaluate : $\int \frac{\sin x\, dx}{\sqrt{3\sin^2 x + 4\cos^2 x}}$

(a) $-l n(\cos x + \sqrt{3+\cos^2 x}) + c$

(b) $l n(\cos x + \sqrt{3+\cos^2 x}) + c$

(c) $-l n(\sin x + \sqrt{3+\sin^2 x}) + c$

(d) None of these

2. If $\int \log(x^2 + x)\, dx = x \log|x| + (x+1)\log|x+1| + k$, then k equals

(a) $2x + \log|x+1| + c$ (b) $2x - \log|x+1| + c$

(c) constant (d) None

3. If $I = \int \cos\theta \log\left(\tan\frac{\theta}{2}\right) d\theta$, then I equals

(a) $\sin\theta \log(\tan\theta/2) + \theta + c$

(b) $\cos\theta \log(\tan\theta/2) + \theta + c$

(c) $\sin\theta \log(\tan\theta/2) - \theta + c$

(d) None of these

4. If $\int g(x)\,dx = g(x)$ then $\int g(x)(f(x) + f'(x))\,dx$ is equal to

(a) $g(x) f(x) - g(x) f'(x) + c$ (b) $g(x) f'(x) + c$

(c) $g(x) f(x) + c$ (d) $g(x) f^2(x) + c$

5. If $\int (x^3 - 2x^2 + 5) e^{3x}\, dx = e^{3x}(Ax^3 + Bx^2 + Cx + 13/9)$ then which of the following statement is incorrect :

(a) $3C = 2$ (b) $A + B + \frac{2}{3} = 0$

(c) $C + 2B = 0$ (d) $A + B + C = 0$

6. $\int \frac{e^{2\tan^{-1}x}(1+x)^2}{(1+x^2)} dx$ is equal to

(a) $xe^{\tan^{-1}x} + c$ (b) $xe^{2\tan^{-1}x} + c$

(c) $2xe^{2\tan^{-1}x} + c$ (d) none of these

7. If $I = \int \sin^7 x\, dx$, then I equals

(a) $-\cos x + \cos^3 x - \cos^5 x + \frac{1}{7}\cos^7 x + c$

(b) $-\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

(c) $\tan x - \tan^3 x + \sin x - \frac{1}{3}\cos^3 x + c$

(d) $-\cos x + \cos^5 x - \frac{1}{3}\cos^3 x + \frac{1}{7}\cos^7 x + c$

8. If $\int \frac{dx}{(x^2-4)\sqrt{x+1}} = \frac{1}{k}\log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right|$

$-\frac{1}{2}\tan^{-1}\sqrt{x+1} + c$ then k equals

(a) $2\sqrt{3}$ (b) $4\sqrt{3}$

(c) $\frac{1}{4\sqrt{3}}$ (d) none of these

9. Evaluate $\int \frac{(x+2)\,dx}{(x^2+3x+3)\sqrt{x+1}}$

(a) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{-x}\right) + c$

(b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{x}\right) + c$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{-x}\right) + c$

(d) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{x}\right) + c$

10. Evaluate : $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}}dx$

(a) $l\text{n}\,(x+\sqrt{x^2+1})+\sec^{-1}x+c$

(b) $l\text{n}\,(x-\sqrt{x^2+1})-\sec^{-1}x+c$

(c) $l\text{n}\,(x+\sqrt{x^2-1})-\sec^{-1}x+c$

(d) None of these

11. Let g(x) be an antiderivative of f(x). Then ln $(1+(g(x))^2)$ is an antiderivative for :

(a) $\frac{2f(x).g(x)}{1+(f(x))^2}$ (b) $\frac{2f(x).g(x)}{1+(g(x))^2}$

(c) $\frac{2f(x)}{1+(f(x))^2}$ (d) None

12. If $I_n = \int (\ln x)^n\,dx$ then $I_n + n\,I_{n-1} =$

(a) $(ln\,x)^n$ (b) $x\,(ln\,x)^n$

(c) $x^n\,ln\,x$ (d) $x\,(ln\,x)^{n-1}$

13. If $I_n = \int \cos^n x\,dx$ then $I_n - \left(\frac{n-1}{n}\right)I_{n-2} =$

(a) $\cos^{n-1}x\,.\,\sin x + c$ (b) $\frac{1}{n}(\cos^{n-1}x\sin x)+c$

(c) $\frac{1}{n}(\cos^n x\sin x)+c$ (d) None of these

14. If $I = \int \frac{dx}{(a^2-b^2x^2)^{3/2}}$

(a) $\frac{x}{\sqrt{a^2-b^2x^2}}+c$ (b) $\frac{x}{a^2\sqrt{a^2-b^2x^2}}+c$

(c) $\frac{ax}{\sqrt{a^2-b^2x^2}}+c$ (d) none of these

15. If $I = \int \frac{x^3+x}{x^4-9}dx$ then I equals :

(a) $\frac{1}{4}\log|x^4-9|+\frac{1}{12}\log\left|\frac{x^2+3}{x^2-3}\right|+c$

(b) $\frac{1}{4}\log|x^4-9|+\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+c$

(c) $\frac{1}{4}\log|x^4-9|-\frac{1}{12}\log\left|\frac{x-3}{x+3}\right|+c$

(d) None

16. Evaluate $I = \int\left(\sqrt{\frac{a+x}{a-x}}+\sqrt{\frac{a-x}{a+x}}\right)dx$

(a) $2\sin^{-1}\left(\frac{x}{a}\right)+c$ (b) $2a\sin^{-1}\left(\frac{x}{a}\right)+c$

(c) $2\cos^{-1}\left(\frac{x}{a}\right)+c$ (d) $2a\cos^{-1}\left(\frac{x}{a}\right)+c$

17. $\int\sqrt{\frac{e^x+a}{e^x-a}}\,dx$

(a) $l\text{n}\,(e^x+\sqrt{e^{2x}-a^2})+\sin^{-1}(ae^{-x})+c$

(b) $l\text{n}\,(e^x-\sqrt{e^{2x}-a^2})+\cos^{-1}(ae^{-x})+c$

(c) $l\text{n}\,(e^x+\sqrt{e^{2x}-a^2})+\cos^{-1}(ae^{-x})+c$

(d) None

18. $\int \frac{x^2-1}{\left(x^4+3x^2+1\right)tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$ is equal to

(a) $tan^{-1}\left(x+\frac{1}{x}\right)+c$

(b) $\log_e\left|tan^{-1}\left(x+\frac{1}{x}\right)\right|+c$

(c) $\log_e\left|tan\left(\frac{x^2+1}{x}\right)\right|+c$

(d) $\left(x+\frac{1}{x}\right)tan^{-1}\left(x+\frac{1}{x}\right)+c$

19. $\int \sqrt{e^{2x}-1}\,dx$ is equal to

(a) $\sqrt{e^{2x}-1}+\sec^{-1}e^{2x}+c$

(b) $\sqrt{e^{2x}-1}-\sec^{-1}e^{2x}+c$

(c) $\sqrt{e^{2x}-1}-\sec^{-1}e^{x}+c$

(d) none of these

20. Evaluate : $\int \frac{(1+x)}{x\,(1+xe^x)^2}dx$

(a) $l\,n\left(\frac{xe^x}{1+xe^x}\right)+\frac{1}{1+xe^x}+c$

(b) $l\,n\left(\frac{1+xe^x}{xe^x}\right)+\frac{1}{xe^x+1}+c$

(c) $l\,n\left(\frac{xe^x}{xe^x+1}\right)-\frac{1}{xe^x+1}+c$

(d) None of these

21. If $f(x)$ is a polynomial function of the n^{th} degree, then $\int e^x f(x)\,dx$ is equal to

(a) $e^x\{f(x)-f'(x)-f''(x)-f'''(x)-...-(-1)^n f^n(x)\}$

(b) $e^x\{f(x)-f'(x)+f''(x)-f'''(x)+...+(-1)^n f^n(x)\}$

(c) $e^x\{f(x)-f'(x)+f''(x)-f'''(x)+...+(1)^n f^n(x)\}$

(d) none of these

22. If $I=\int e^x(x\cos x+\sin x)\,dx$ then I equals :

(a) $\frac{1}{2}e^x(x\sin x-\cos x)+c$

(b) $\frac{1}{2}e^x(x\sin x+\cos x)+c$

(c) $\frac{1}{2}e^x(x\cos x-\sin x)+c$

(d) None

23. $\int \sqrt[3]{x}\,(\log_e x)^2\,dx=$

(a) $\frac{3}{4}(x)^{4/3}\left[(\log_e x)^2-\frac{3}{2}\log_e x-\frac{9}{8}\right]+c$

(b) $\frac{3}{4}(x)^{4/3}\left[(\log_e x)^2+\frac{3}{2}\log_e x+\frac{9}{8}\right]+c$

(c) $\frac{3}{4}(x)^{4/3}\left[(\log_e x)^2-\frac{3}{2}\log_e x+\frac{9}{8}\right]+c$

(d) $\frac{3}{4}(x)^{1/3}\left[(\log_e x)^2+\frac{3}{2}\log_e x-\frac{9}{8}\right]+c$

24. If $I=\int \frac{\sqrt{\sin^3 2x}}{\sin^5 x}dx$, and $f(x)=(\cot x)^{3/2}$, $g(x)=(\cot x)^{5/2}$, then I equals

(a) $\frac{2\sqrt{3}}{3}f(x)-\frac{1}{5}g(x)+c$ (b) $-\frac{4\sqrt{2}}{5}g(x)+c$

(c) $\frac{1}{2\sqrt{3}}f(x)+c$ (d) $\frac{2\sqrt{2}}{3}f(x)+\frac{1}{5}g(x)+c$

25. $\int \frac{dx}{\tan x+\cot x+\sec x+\operatorname{cosec} x}$ is equal to

(a) $\frac{1}{2}(\sin x-\cos x+x)+c$

(b) $\frac{1}{2}(\sin x-\cos x-\tan x+\cot x)+c$

(c) $\frac{1}{2}(\sin x-\cos x-x)+c$

(d) $\frac{1}{2}(\sin x+\cos x-\tan x-\cot x+x)+c$

26. $\int \frac{\sin x-\cos x}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}}dx=$

(a) $-\sin(\sin 2x+1)+c$

(b) $\operatorname{cosec}(\sin 2x+1)$

(c) $-\sec^{-1}(\sin 2x+1)+c$

(d) $\tan^{-1}(\sin 2x+1)+c$

27. If $y=\sqrt{x^2-x+1}$ and for n ≥ 1, $I_n=\int x^n/y\,dx$ and $aI_3+bI_2+cI_1=x^2y$, then (a, b, c) is equal to

(a) $\left(\frac{3}{2},\frac{1}{2},-1\right)$ (b) $(1,-1,1)$

(c) $\left(3,-\frac{5}{2},2\right)$ (d) $\left(\frac{1}{2},-\frac{1}{2},1\right)$

28. If $\lim\limits_{x\to 0}\frac{f(x)}{x^2}$ exists finitely and

$\lim\limits_{x\to 0}\left(1+x+\frac{f(x)}{x}\right)^{1/x}=e^3$, where $f(x)=ax^2+bx+c$

then $\int f(x)\log_e x\,dx$ is equal to

(a) $\frac{2}{3}x^3\left(\log_e x-\frac{1}{3}\right)+c$ (b) $\frac{x^3}{3}\left(\log_e x-\frac{1}{3}\right)+c$

(c) $\frac{2}{3}x^3(\log_e x+1)+c$ (d) $\frac{2}{3}x^3(\log_e x-1)+c$

29. For $0<x<1$, let

$f(x)=\lim\limits_{n\to\infty}(1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^n})$

then $\int\frac{f(x)}{1-x}\log_e x\,dx$ equals

(a) $\log_e\left(\frac{x}{1-x}\right)+c$

(b) $-\log_e\left(\frac{x}{1-x}\right)+\frac{\log_e x}{1-x}+c$

(c) $\frac{\log_e x}{1-x}+\log_e(1-x)+c$

(d) $x\log_e x+\log_e(1-x)+c$

30. If $\int\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)^{1/2}\frac{dx}{x}=2\cos^{-1}\sqrt{x}-f(x)+c$, then f (x) equals.

(a) $\log_e\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right)$ (b) $\frac{1}{2}\log_e\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right)$

(c) $2\log_e\left(\frac{1-\sqrt{1-x}}{\sqrt{x}}\right)$ (d) $2\log_e\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right)$

31. $\int\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x+a}}\,dx$

(a) $\sqrt{ax+x^2}-2\sqrt{a^2+ax}-a\,ln(\sqrt{x}+\sqrt{a+x})+c$

(b) $\sqrt{ax+x^2}-2\sqrt{a+x}-a\,ln(\sqrt{x}+\sqrt{a+x})+c$

(c) $\sqrt{ax+x^2}-2\sqrt{a^2+ax}+a\,ln(\sqrt{x}+\sqrt{a-x})+c$

(d) None of these

32. If $I=\int\frac{x^2+a^2}{x^4-a^2x^2+a^4}dx$, then I =

(a) $\frac{1}{a}\tan^{-1}\left(\frac{ax}{x^2-a^2}\right)+c$ (b) $\frac{1}{a}\tan^{-1}\left(\frac{x^2-a^2}{ax}\right)+c$

(c) $\log|x+\sqrt{x^2-a^2}|+c$ (d) None of these

33. Evaluate : $\int\sqrt{x+\sqrt{x^2+2}}\,dx$

(a) $\frac{2}{3}(x+\sqrt{x^2+2})^{3/2}-\frac{1}{\sqrt{x+\sqrt{x^2+2}}}+c$

(b) $\frac{1}{3}(x+\sqrt{x^2+2})^{3/2}-\frac{2}{\sqrt{x+\sqrt{x^2+2}}}+c$

(c) $\frac{1}{3}(x+\sqrt{x^2+2})^{1/2}-\frac{2}{\sqrt{x+\sqrt{x^2+2}}}+c$

(d) None of these

34. $\int \frac{x-1}{(x+1)\sqrt{x(x^2+x+1)}}dx$ is

(a) $\tan^{-1}\left(\frac{x^2+x+1}{x}\right)+c$

(b) $2\tan^{-1}\left(\frac{x^2+x+1}{x}\right)+c$

(c) $\tan^{-1}\left(\frac{\sqrt{x^2+x+1}}{x}\right)+c$

(d) $2\tan^{-1}\sqrt{x+\frac{1}{x}+1}+c$

35. $\int \frac{(1+x)\sin x}{(x^2+2x)\cos^2 x-(1+x)\sin 2x}dx = \frac{1}{2}\log_e\left|\frac{t+1}{t-1}\right|+c$

where t is

(a) $(x+1)\cos x-\sin x$ (b) $(x+1)\sin x-\cos x$

(c) $(x+1)\sin x+\cos x$ (d) $(x+1)\cos x+\sin x$

36. $\int \frac{(x^2-1)dx}{2x\sqrt{x^4+4x^3-6x^2+4x+1}}$ is

(a) $\frac{1}{2}\ln\left|x+\frac{1}{x}+2+\sqrt{\left(x+\frac{1}{x}+2\right)^2-12}\right|+c$

(b) $\frac{1}{2}\ln\left|x-\frac{1}{x}+2+\sqrt{\left(x-\frac{1}{x}+2\right)^2-12}\right|+c$

(c) $\frac{1}{2}\ln\left|x+\frac{1}{x}-2+\sqrt{\left(x+\frac{1}{x}-2\right)^2-12}\right|+c$

(d) None of these

37. Evaluate : $\int \sqrt{\frac{1+x^{2n}}{x^{2n}}}\cdot\frac{\ln(1+x^{2n})-2n\ln x}{x^{2n+1}}dx$

(a) $\frac{2P^3}{9n}(1-3\ln P)+C$ (b) $\frac{P^3}{3n}(3\ln P-1)+C$

(c) $\frac{2P^3}{3n}(3\ln P-1)+C$ (d) None of these

where $P=\left(1+\frac{1}{x^{2n}}\right)^{1/2}$

38. Evaluate $I=\int \frac{dx}{\sin^3 x+\cos^3 x}$

(a) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+\tan^{-1}t+c$

(b) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}-t}{\sqrt{2}+t}\right|+\frac{1}{3}\tan^{-1}t+c$

(c) $\frac{1}{3\sqrt{2}}\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+\frac{2}{3}\tan^{-1}t+c$

(d) None of these

where $t=\sin x-\cos x$

39. Evaluate : $\int \frac{\sec x\,dx}{\sqrt{\sin(2x+A)+\sin A}}$

(a) $2\cos A\sqrt{2\cos A+\sin A\tan x}+c$

(b) $\sqrt{2}\sec A\sqrt{2\cos A\tan x+2\sin A}+c$

(c) $\sqrt{2}\sec A\sqrt{\cos A\tan x+\sin A}+c$

(d) None of these

40. $\int\sqrt{\sec x-1}\,dx$

(a) $2\ln(\sqrt{\cos x}+\sqrt{1+\cos x})+c$

(b) $2\ln(\sqrt{\sin x}+\sqrt{1+\sin x})+c$

(c) $-2\ln(\sqrt{\cos x}+\sqrt{1+\cos x})+c$

(d) None of these

41. $\int (\sin 4x) e^{\tan^2 x} dx =$

(a) $-2e^{\tan^2 x}\cos^4 x + c$ (b) $2e^{\tan^2 x}\sec^4 x + c$

(c) $-2e^{\tan^2 x}\sec^2 x + c$ (d) $2e^{\tan^2 x}\cos^2 x + c$

42. If $I = \int \frac{(2x+3)dx}{(x^2+2x+3)\sqrt{x^2+2x+4}}$, then I equals

(a) $\log\left|\frac{\sqrt{x^2+2x+4}-1}{\sqrt{x^2+2x+4}+1}\right| + c$

(b) $\log\left|\frac{\sqrt{x^2+2x+4}-1}{\sqrt{x^2+2x+4}+1}\right| + \tan^{-1}\left(\frac{x+2}{3}\right) + c$

(c) $\log\tan^{-1}\left(\frac{\sqrt{x+3}}{2}\right) + c$

(d) None

43. If $f: R \to R$ is a function satisfying the following :

(i) $f(-x) = -f(x)$

(ii) $f(x+1) = f(x) + 1$

(iii) $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2} \ \forall\ x \neq 0$

then $\int e^x f(x) dx$ is equal to

(a) $e^x(x-1) + c$ (b) $e^x \log x + c$

(c) $\frac{e^x}{x} + c$ (d) $\frac{e^x}{x+1} + c$

Objective Questions II [One or more than one correct option]

44. If primitive of sin (log x) is $f(x)$ (sin (g(x)) – cos (h(x)) + c then

(a) $\lim_{x\to 2} f(x) = 1$ (b) $\lim_{x\to 1} \frac{g(x)}{h(x)} = 1$

(c) $g(e^3) = 3$ (d) $h(e^5) = 5$

45. $\int \frac{dx}{5+4\cos x} = \lambda \tan^{-1}\left(m \tan\frac{x}{2}\right) + c$ then

(a) $\lambda = \frac{2}{3}$ (b) $m = \frac{1}{3}$

(c) $\lambda = \frac{1}{3}$ (d) $m = \frac{2}{3}$

46. $\int \frac{1}{x^2-1} ln\left(\frac{x-1}{x+1}\right) dx$ equals

(a) $\frac{1}{2} ln\left(\frac{x-1}{x+1}\right) + c$ (b) $\frac{1}{4} ln^2\left(\frac{x-1}{x+1}\right) + c$

(c) $\frac{1}{2} ln^2\left(\frac{x+1}{x-1}\right) + c$ (d) $\frac{1}{4} ln^2\left(\frac{x+1}{x-1}\right) + c$

47. If $f(x) = \lim_{n\to\infty} e^{x\tan(1/n)\log(1/n)}$ and $\int \frac{f(x)dx}{\sqrt[3]{\sin^{11} x \cos x}} = g(x) + c$

then

(a) $g\left(\frac{\pi}{4}\right) = \frac{3}{2}$

(b) g (x) is continuous for all x

(c) $g\left(\frac{\pi}{4}\right) = \frac{-15}{8}$

(d) g (x) is non-differentiable at infinitely many points

Numerical Value Type Questions

48. Let f be a function satisfying $f''(x) = x^{-3/2}$, $f'(4) = 2$ and $f(0) = 0$ then $f(784)$ is equal to

49. If the graph of the antiderivative F (x) of f (x) = log (log x) + (log x)$^{-2}$ passes through (e, 1998–e), then the term independent of x in F (x) is

50. Let F (x) be the antiderivative of

$f(x) = \frac{1}{(3+5\sin x+3\cos x)}$ whose graph passes through the point (0, 0) then the value of $F\left(\frac{\pi}{2}\right) - \frac{1}{5}\log\frac{8}{3} + 1982$ is equal to

51. f (x) is the integral of $\frac{2\sin x - \sin 2x}{x^3}$, $x \neq 0$ find $\lim_{x\to 0} f'(x)$.

52. If $\int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}} dx$ is equal to

$\frac{\sqrt{2}}{4} A \tan^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{x^2+\frac{1}{x^2}}\right) + c$ then A is equal to

Assertion & Reason

(A) **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

(B) **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

(C) **If ASSERTION is true, REASON is false.**

(D) **If ASSERTION is false, REASON is true.**

53. **Assertion :** If $\int \frac{1}{f(x)} dx = 2\log|f(x)| + c$, then $f(x) = \frac{x}{2}$

Reason : When $f(x) = \frac{x}{2}$, then

$$\int \frac{1}{f(x)} dx = \int \frac{2}{x} dx = 2\log|x| + c$$

(a) A (b) B

(c) C (d) D

54. **Assertion :** $\int \frac{x^2-1}{x^2} e^{\frac{x^2+1}{x}} dx = e^{\frac{x^2+1}{x}} + c$

Reason : $\int f(x) e^{f(x)} dx = e^{f(x)} + c$

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

55. $\int f(x)\, dx$ when

(A)	$f(x) = \frac{1}{e^x + 1}$	**(P)**	$2\log(e^{x/2} + e^{-x/2}) + c$
(B)	$f(x) = e^{e^x + x}$	**(Q)**	$\log \frac{e^x}{e^x + 1} + c$
(C)	$f(x) = \frac{e^x - 1}{e^x + 1}$	**(R)**	$\frac{1}{4} e^{2x^2} + c$
(D)	$f(x) = e^{2x^2 + \log x}$	**(S)**	$e^{e^x} + c$

The correct matching is :

(a) (A-S, B-Q, C-P, D-R)

(b) (A-Q, B-S, C-R, D-P)

(c) (A-Q, B-S, C-P, D-R)

(d) (A-R, B-S, C-P, D-Q)

56. $\int f(x)\, dx$, when

(A) $f(x) = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$	**(P)** $c - \frac{2}{\sqrt{15}} \log\left|\frac{x+6+\sqrt{60x-15x^2}}{2x-3}\right|$
(B) $f(x) = \sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{x}$	**(Q)** $\log\left|\frac{\sqrt{x+1}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right| + 2\tan^{-1}\sqrt{\frac{1-x}{1+x}} + c$
(C) $f(x) = \frac{1}{\left(x-\frac{3}{2}\right)\sqrt{4x-x^2}}$	**(R)** $\left(\sqrt{x}-2\right)\sqrt{1-x} - \sin^{-1}\sqrt{x} + c$
(D) $f(x) = \sqrt{\frac{1+\sqrt{x}}{x}}$	**(S)** $\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2} + c$

The correct matching is :

(a) (A-Q, B-R, C-P, D-S)

(b) (A-R, B-Q, C-P, D-S)

(c) (A-R, B-P, C-Q, D-S)

(d) (A-R, B-Q, C-S, D-P)

Using the following passage, solve Q.57 to Q.60

Passage – 1

We can derive reduction formula for the integration of the form $\int \sin^n x\, dx, \int \cos^n x\, dx, \int \tan^n x\, dx$ and other integrals of these form using integration by parts. In turn these reduction formulas can be used to compute integrals of higher power of sin x and cos x.

57. If $I = \int \sec^6 x\, dx = \frac{1}{5}\tan^5 x + A\tan^3 x + \tan x + c$ then A is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

58. If $\int \sin^5 x\,dx = -\frac{1}{5}\sin^4 x\cos x + A\sin^2 x\cos x - \frac{8}{15}\cos x + c$ then A is equal to

(a) $-\frac{2}{15}$ (b) $-\frac{3}{5}$

(c) $-\frac{4}{15}$ (d) $-\frac{1}{15}$

59. If $\int \tan^6 x\,dx = \frac{1}{5}\tan^5 x + A\tan^3 x + \tan x - x + c$ then A is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

60. If $\int \operatorname{cosec}^n x\,dx = -\frac{\operatorname{cosec}^{n-2} x\cot x}{n-1} + A\int \operatorname{cosec}^{n-2} x\,dx$

then A is equal to

(a) $\frac{1}{n-2}$ (b) $\frac{n}{n-2}$

(c) $\frac{n-1}{n-2}$ (d) $\frac{n-2}{n-1}$

Using the following passage, solve Q.61 and Q.62

Passage – 2

If the integrand is a rational function of x and fractional powers of a linear fractional function of the form $\frac{ax+b}{cx+d}$. Then rationalization of the integral is affected by the substitution $\frac{ax+b}{cx+d} = t^m$, where m is the L.C.M. of fractional powers of $\frac{ax+b}{cx+d}$.

61. If $I = \int \frac{dx}{\sqrt[4]{(x-1)^3 (x+2)^5}} = A\sqrt[4]{\frac{x-1}{x+2}} + c$ then A is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

62. If $I = \int \frac{(2x-3)^{1/2}}{(2x-3)^{1/3}+1}dx = 3\left[\frac{1}{7}(2x-3)^{7/6} - \frac{1}{5}(2x-3)^{5/6} + \frac{1}{3}(2x-3)^{1/2} - (2x-3)^{1/6} + g(x)\right] - 1$ then g (x) is equal to

(a) $\tan^{-1}(2x-3)^{1/6}$ (b) $(2x-3)^{1/2}$

(c) $3\tan^{-1}(2x-3)^{1/6}$ (d) $4(2x-3)^{1/6}$

Text

63. Integrate $\int \frac{x^3+3x+2}{(x^2+1)^2(x+1)}dx$

64. Evaluate $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$

65. Evaluate $\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. Let $f(x)=\dfrac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and

$g(x)=\underbrace{(f \text{ o } f \text{ o...o } f)}_{\text{f occurs n times}}(x)$

Then $\int x^{n-2} g(x)\,dx$ equals **(2007)**

(a) $\dfrac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}}+c$

(b) $\dfrac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+c$

(c) $\dfrac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+c$

(d) $\dfrac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+c$

2. Let $I=\int \dfrac{e^x}{e^{4x}+e^{2x}+1}dx,\ J=\int \dfrac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$

Then, for an arbitrary constant c, the value of J–I equals **(2008)**

(a) $\dfrac{1}{2}\log\left(\dfrac{e^{4x}-e^{2x}+1}{e^{4x}+e^x+1}\right)+c$

(b) $\dfrac{1}{2}\log\left(\dfrac{e^{2x}+e^x+1}{e^{2x}-e^x+1}\right)+c$

(c) $\dfrac{1}{2}\log\left(\dfrac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right)+c$

(d) $\dfrac{1}{2}\log\left(\dfrac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+c$

3. The integral $\int \dfrac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals to (for some arbitrary constant C) **(2012)**

(a) $\dfrac{-1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+C$

(b) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+C$

(c) $\dfrac{-1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+C$

(d) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+C$

Objective Questions II [One or more than one correct option]

4. Let b be a nonzero real number. Suppose $f: R \to R$ is a differentiable function such that $f(0) = 1$. If the derivative f' of f satisfies the equation $f'(x)=\dfrac{f(x)}{b^2+x^2}$ for all $x \in R$, then which of the following statements is/are TRUE? **(2020)**

(a) If $b > 0$, then f is an increasing function

(b) If $b < 0$, then f is a decreasing function

(c) $f(x).f(-x)=1$ for all $x \in R$

(d) $f(x)-f(-x)=0$ for all $x \in R$

Assertion & Reason

(A) **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

(B) **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

(C) **If ASSERTION is true, REASON is false.**

(D) **If ASSERTION is false, REASON is true.**

5. Let F(x) be an indefinite integral of $\sin^2 x$.

Assertion : The function F(x) satisfies $F(x + \pi) = F(x)$ for all real x.

Reason : $\sin^2(x + \pi) = \sin^2 x$ for all real x. **(2007)**

(a) A (b) B

(c) C (d) D

Answer Key

CHAPTER -5 | INDEFINITE INTEGRATION

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)
6. (c)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (d)
11. (a)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (b)
16. (b)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (a)
21. (a)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (b)
26. (a)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (a)
31. (b)	**32.** (a)	**33.** (b)	**34.** (c)	**35.** (b)
36. (c)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (a)
41. (c)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (a)
46. (d)	**47.** (a)	**48.** (b)	**49.** (b)	**50.** (a)
51. (b)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (c)
56. (c)	**57.** (a)	**58.** (a)	**59.** (d)	**60.** (b)
61. (b)	**62.** (b)	**63.** (b)	**64.** (c)	**65.** (d)
66. (c)	**67.** (3)	**68.** (8)	**69.** (1)	**70.** (9)
71. (4)	**72.** (2)	**73.** (3)	**74.** (6)	**75.** (8)
76. (8)	**77.** (2)	**78.** (36)	**79.** (1)	**80.** (10)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)
6. (b)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (d)
16. (a)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (b)
21. (c)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (a)
26. (d)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (a)
31. (a)	**32.** (a)	**33.** (d)	**34.** (a)	**35.** (c)
36. (b)	**37.** (15)	**38.** (7)	**39.** (c)	**40.** (3)
41. (c)	**42.** (b)	**43.** (a)	**44.** (6)	**45.** (d)

CHAPTER - 5 | INDEFINITE INTEGRATION

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (a)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (c)
6. (b)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (c)
11. (b)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)
16. (b)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (a)
21. (b)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (c)
26. (c)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (d)
31. (a)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (a)
36. (a)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (c)
41. (a)	**42.** (d)	**43.** (a)	**44.** (a,b,c,d)	
45. (a,b)	**46.** (b,d)	**47.** (c,d)	**48.** (2240)	**49.** (1998)
50. (1982)	**51.** (1)	**52.** (2)	**53.** (d)	**54.** (c)
55. (c)	**56.** (b)	**57.** (b)	**58.** (c)	**59.** (d)
60. (d)	**61.** (d)	**62.** (a)		

63. $-\frac{1}{2}\log|x+1|+\frac{1}{4}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}x+\frac{x}{x^2+1}+c$

64. $(x+1)\tan^{-1}\left(\frac{2x+2}{3}\right)-\frac{3}{4}\log\left(4x^2+8x+13\right)+c$

65. $-\ln\left(\frac{1}{x+1}-\frac{1}{2}+\frac{x+1}{\sqrt{x^2+x+1}}\right)+C$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (a)	**2.** (c)	**3.** (c)	**4.** (a,c)	**5.** (d)