Factorisation

Factorisation of Algebraic Expressions Using Method of Common Factors

You know about the prime factorization of numbers. Let us revise the method of prime factorization by taking the example of the number 210.

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

We can write 210 as a product of 2, 3, 5, and 7.

Hence, $210 = 2 \times 3 \times 5 \times 7$

Here, 2, 3, 5, and 7 are the prime factors of 210. In the same way, we can factorize any expression, i.e., we can write any expression as a product of its factors.

For example, $2xyz = 2 \times x \times y \times z$

Here, 2, x, y, and z are the factors of $2xyz$, and we cannot further reduce them. Thus, we say that 2, x, y, and z are the irreducible factors of $2xyz$.

The process of writing a given algebraic expression as a product of two or more expressions is called factorization. Each of the expressions which form the product is called a factor of the given expression.

Let us discuss some more examples based on the above concept.

Example 1:

Find the common factors of the terms $6pq$, $8p^2$, and $4pq^2$.

Solution:

Write the factors of each term.

$6pq = 2 \times 3 \times p \times q$

$8p^2 = 2 \times 2 \times 2 \times p \times p$

$4pq^2 = 2 \times 2 \times p \times q \times q$

Here, 2 and p are the common factors of the given terms $6pq$, $8p^2$, and $4pq^2$.

Example 2:

Factorize $6x^2 - 18x$.

Solution:

Write the factors of each terms.

$6x^2 = 2 \times 3 \times x \times x$

$18x = 2 \times 3 \times 3 \times x$

HCF of $6x^2$ and $18x = 2 \times 3 \times x = 6x$

$\therefore 6x^2 - 18x = 6x\,(x - 3)$

Example 3:

Factorize the following expressions:

(i) $7x^2 + 14x$

(ii) $4a^2bcx^2y^3z^4 - 5ab^2cx^3y^4z^2 + 7abc^2x^4y^2z^3$

(iii) $-4a^2 + 3p^3 - 5b$

Solution:

(i) Write the factors of each term.

$7x^2 = 7 \times x \times x$

$14x = 2 \times 7 \times x$

Here, 7 and x are the common factors.

$7x^2 + 14x = 7 \times x \times x + 2 \times 7 \times x$

$= 7 \times x\,(x + 2)$

$= 7x\,(x + 2)$

(ii) Write the factors of each term.

$$4a^2bcx^2y^3z^4 = 2 \times 2 \times \underline{a} \times a \times \underline{b} \times \underline{c} \times \underline{x} \times \underline{x} \times \underline{y} \times \underline{y} \times y \times \underline{z} \times \underline{z} \times z \times z$$

$$-5ab^2cx^3y^4z^2 = -5 \times \underline{a} \times \underline{b} \times b \times \underline{c} \times \underline{x} \times \underline{x} \times x \times \underline{y} \times \underline{y} \times y \times y \times \underline{z} \times \underline{z}$$

$$7abc^2x^4y^2z^3 = 7 \times \underline{a} \times \underline{b} \times \underline{c} \times c \times \underline{x} \times \underline{x} \times x \times x \times \underline{y} \times \underline{y} \times \underline{z} \times \underline{z} \times z$$

Here, a, b, c, x, x, y, y, z, and z are the common factors of given terms.

Hence, $4a^2bcx^2y^3z^4 - 5ab^2cx^3y^4z^2 + 7abc^2x^4y^2z^3$

$$= a \times b \times c \times x^2 \times y^2 \times z^2 \begin{Bmatrix} 2 \times 2 \times a \times y \times z \times z - 5 \times b \times x \times y \times y \\ + 7 \times c \times x \times x \times z \end{Bmatrix}$$

$$= abcx^2y^2z^2\left(4ayz^2 - 5bxy^2 + 7cx^2z\right)$$

(iii) Write the factors of each term.

$-4a^2 = -2 \times 2 \times a \times a$

$3p^3 = 3 \times p \times p \times p$

$-5b = -5 \times b$

There is no common factor of these terms other than 1.

Factorisation of Algebraic Expressions Using Method of Regrouping Terms

Can you factorize the algebraic expression $4ab + a + 4b + 1$?

All four terms in the expression do not have any common factor except 1. Thus, we cannot factorize the expression by taking the common factors of each term.

Example 1:

Factorize the following expressions:

(i) $2x + ax - 2y - ay$

(ii) $2a + 3b - 2 - 3ab$

Solution:

(i) The given expression is $2x + ax - 2y - ay$.

We can factorize this expression as

$2x + ax - 2y - ay = (2x - 2y) + (ax - ay)$

$= 2\ (x - y) + a\ (x - y)$

$= (x - y)\ (2 + a)$

$= (x - y)\ (2 + a)$

(ii) The given expression is $2a + 3b - 2 - 3ab$.

We can factorize this expression as

$2a + 3b - 2 - 3ab = (2a - 2) + (- 3ab + 3b)$

$= 2\ (a - 1) + (-3b)\ (a - 1)$

$= 2\ (a - 1) - 3b\ (a - 1)$

$= (a - 1)\ (2 - 3b)$

Factorisation of Algebraic Expressions Using Identities (a + b)2, (a - b)2 and a2 - b2

We know the identities

(i). $a^2 + 2ab + b^2 = (a + b)^2$

(ii). $a^2 - 2ab + b^2 = (a - b)^2$

(iii). $a^2 - b^2 = (a + b)\ (a - b)$

We can use these identities to factorise algebraic expressions as well. Let us discuss each identity one by one.

Application of the identity $a^2 + 2ab + b^2 = (a + b)^2$ to factorise an algebraic expression

Let us factorise the expression $x^2 + 6x + 9$.

In this expression, the first term is the square of x, the last term is the square of 3, and the middle term is positive and is twice the product of x and 3.

Thus, $x^2 + 6x + 9$ can be written as

$x^2 + 6x + 9 = (x)^2 + 2 \times x \times 3 + (3)^2$

The right hand side of this expression is in the form of $a^2 + 2ab + b^2$, where $a = x$ and $b = 3$.

We know the identity $a^2 + 2ab + b^2 = (a + b)^2$.

$\therefore (x)^2 + 2 \times x \times 3 + (3)^2 = (x + 3)^2$

Thus, $x^2 + 6x + 9 = (x + 3)^2$.

Application of the identity $a^2 - 2ab + b^2 = (a - b)^2$ to factorise an algebraic expression

Let us factorise the expression $9y^2 - 12y + 4$.

In this expression, the first term is the square of $3y$, the last term is the square of 2, and the middle term is negative and is twice of the product of $3y$ and 2.

Thus, $9y^2 - 12y + 4$ can be written as

$9y^2 - 12y + 4 = (3y)^2 - 2 \times 3y \times 2 + (2)^2$

The right hand side of this expression is in the form of $a^2 - 2ab + b^2$, where $a = 3y$ and

$b = 2$.

We know the identity $a^2 - 2ab + b^2 = (a - b)^2$.

$\therefore (3y)^2 - 2 \times 3y \times 2 + (2)^2 = (3y - 2)^2$

Thus, $9y^2 - 12y + 4 = (3y - 2)^2$.

Application of the identity $a^2 - b^2 = (a + b)(a - b)$ to factorise an algebraic expression

We use this identity when an expression is given as the difference of two squares.

Let us factorise the expression $x^2 - 25$.

We can write it as

$x^2 - 25 = (x)^2 - (5)^2$

The right hand side of this expression is in the form of $a^2 - b^2$, where $a = x$ and $b = 5$

On using the identity $a^2 - b^2 = (a + b)(a - b)$, we obtain

$x^2 - 25 = (x + 5)(x - 5)$

Thus, $(x + 5)$ and $(x - 5)$ are the factors of $x^2 - 25$.

To factorise an algebraic expression, we have to observe the given expression. If it has a form that fits the left hand side of one of the identities mentioned in the beginning, then the expression corresponding to the right hand side of the identity gives the desired factorisation.

Let us discuss some more examples based on what we have discussed so far.

Example 1:

Factorise the given expressions.

1. $\mathbf{25x^2 + 40xy + 16y^2}$
2. $\mathbf{81x^3 + x - 18x^2}$
3. $\mathbf{(p + 1)^2 - (p - 1)^2}$
4. $\mathbf{16a^2 - 25b^2 + 60bc - 36c^2}$
5. $\mathbf{al^2 - bm^2 - am^2 + bl^2}$
6. $\mathbf{81x^4 - 256\,y^4}$
7. $\mathbf{16x^4 - (3a + 5c)^4}$

Solution:

(1) The given expression is $25x^2 + 40xy + 16y^2$.

$25x^2 + 40xy + 16y^2 = (5x)^2 + 2 \times 5x \times 4y + (4y)^2$

$= (5x + 4y)^2$ $[a^2 + 2ab + b^2 = (a + b)^2]$

$\therefore 25x^2 + 40xy + 16y^2 = (5x + 4y)^2$

(2) The given expression is $81x^3 + x - 18x^2$.

Here, x is a factor common to all terms in the expression.

$\therefore 81x^3 + x - 18x^2 = x\,(81x^2 + 1 - 18x)$

$= x\,[(9x)^2 + (1)^2 - 2 \times 9x \times 1]$

$= x\,[9x - 1]^2$ $[a^2 - 2ab + b^2 = (a - b)^2)$

$\therefore 81x^3 + x - 18x^2 = x\,(9x - 1)^2$

(3) The given expression is $(p + 1)^2 - (p - 1)^2$.

On using the identity $a^2 - b^2 = (a + b)\,(a - b)$, we obtain

$(p + 1)^2 - (p - 1)^2 = \{(p + 1) + (p - 1)\}\,\{(p + 1) - (p - 1)\}$

$= (p + 1 + p - 1)\,(p + 1 - p + 1)$

$= (2p)\,(2) = 4p$

$\therefore (p + 1)^2 - (p - 1)^2 = 4p$

(4) The given expression is $16a^2 - 25b^2 + 60bc - 36c^2$.

$16a^2 - 25b^2 + 60bc - 36c^2$

$= 16a^2 - (25b^2 - 60bc + 36c^2)$

$= (4a)^2 - \{(5b)^2 - 2\,(5b)\,(6c) + (6c)^2\}$

= $(4a)^2 - (5b - 6c)^2$ [Using the identity $a^2 - 2ab + b^2 = (a - b)^2$]

= $\{(4a) + (5b - 6c)\}\{(4a) - (5b - 6c)\}$[Using the identity $a^2 - b^2 = (a + b)\,(a - b)$]

= $(4a + 5b - 6c)\,(4a - 5b + 6c)$

(5) The given expression is $al^2 - bm^2 - am^2 + bl^2$.

$al^2 - bm^2 - am^2 + bl^2$

$= al^2 - am^2 + bl^2 - bm^2$ {Regrouping the terms}

$= a\,(l^2 - m^2) + b\,(l^2 - m^2)$

$= (l^2 - m^2)\,(a + b)$

$= (l + m)\,(l - m)\,(a + b)$ [Using the identity $a^2 - b^2 = (a + b)\,(a - b)$]

(6) The given expression is $81x^4 - 256y^4$.

$81x^4 - 256y^4$

$$= (9x^2)^2 - (16y^2)^2$$
$$= (9x^2 + 16y^2)(9x^2 - 16y^2) \qquad \left[\text{Using the identity } a^2 - b^2 = (a+b)(a-b)\right]$$
$$= (9x^2 + 16y^2)\left\{(3x)^2 - (4y)^2\right\}$$
$$= (9x^2 + 16y^2)(3x + 4y)(3x - 4y) \qquad \left[\text{Using the identity } a^2 - b^2 = (a+b)(a-b)\right]$$

(7) The given expression is $16x^4 - (3a + 5c)^4$.

$16x^4 - (3a + 5c)^4$

$$= (4x^2)^2 - \left[(3a + 5c)^2\right]^2$$
$$= \left\{4x^2 + (3a + 5c)^2\right\}\left\{4x^2 - (3a + 5c)^2\right\}$$
$$\left[a^2 - b^2 = (a+b)(a-b)\right]$$
$$= \left\{4x^2 + (3a)^2 + 2(3a)(5c) + (5c)^2\right\}\left\{(2x)^2 - (3a + 5c)^2\right\}$$
$$\left[(a+b)^2 = a^2 + 2ab + b^2\right]$$
$$= (4x^2 + 9a^2 + 30ac + 25c^2)\left\{(2x) + (3a + 5c)\right\}\left\{(2x) - (3a + 5c)\right\}$$
$$\left[a^2 - b^2 = (a+b)(a-b)\right]$$
$$= (4x^2 + 9a^2 + 30ac + 25c^2)(2x + 3a + 5c)(2x - 3a - 5c)$$

Factorisation of Algebraic Expressions Using the Identity (x + a) (x + b) = x^2 + (a + b)x + ab

How can we factorise the algebraic expression $x^2 + 8x + 15$?

Note that we cannot express this expression as $(a + b)^2$, since 15 is not the square of any natural number. What do we do in such a case?

In this case, we can use the identity $x^2 + (a + b)\ x + ab = (x + a)\ (x + b)$.

If we compare $x^2 + 8x + 15$ with $x^2 + (a + b)\ x + ab$, then we obtain $a + b = 8$ and $ab = 15$.

Hence, we need to find two numbers, a and b, such that their sum is 8 and their product is 15.

The only numbers that fulfil these two conditions are 3 and 5.

Hence, we can write $x^2 + 8x + 15$ as

$x^2 + (5 + 3)\ x + 5 \times 3$

$= (x^2 + 5x) + (3 \times x + 5 \times 3)$

$= x(x + 5) + 3(x + 5)$

$= (x + 5)\ (x + 3)$ {Taking a common factor from each group}

Let us practise some more questions based on this concept.

Example 1:

Factorise the expression $x^2 - x - 42$.

Solution:

On comparing $x^2 - x - 42$ with $x^2 + (a + b)\ x + ab$, we obtain $a + b = -1$ and $ab = -42$.

Here, we have to find two numbers, a and b, such that their sum is -1 and their product is -42.

Since the product (-1) of the numbers a and b is negative and their sum (-42) is also negative, we have to choose two numbers such that the bigger number is negative and the smaller number is positive.

The numbers that fulfil these conditions are -7 and 6.

Thus, obtain $a + b = -1$ and $ab = -\ 42$.
Hence,
$x^2 - x - 42 = x^2 - 7x + 6x - 42$
$= x\ (x - 7) + 6\ (x - 7)$
$= (x - 7)\ (x + 6)$

Example 2:

Factorise the expression $y^2 - 13y + 36$.

Solution:

On comparing the expression $y^2 - 13y + 36$ with $y^2 + (a + b)\, y + ab$, we obtain

$a + b = -13$ and $ab = 36$.

Since the product of the numbers a and b is positive and their sum is negative, we have to choose two negative numbers.

The numbers that fulfil these conditions are -9 and -4.

$\therefore y^2 - 13y + 36 = y^2 - 4y - 9y + 36$
$= y\,(y - 4) - 9\,(y - 4) = (y - 4)\,(y - 9)$

Example 3:
Factorise the expression $7(x^2 - x)(2x^2 - 2x - 1) - 42$.

Solution:
We have
$7(x^2 - x)(2x^2 - 2x - 1) - 42$
$= 7(x^2 - x)\{2(x^2 - x) - 1)\} - 42$
Let $x^2 - x = m$
Thus, $7(x^2 - x)\{2(x^2 - x) - 1)\} - 42 = 7m(2m - 1) - 42$
$= 14m^2 - 7m - 42$
$= 7(2m^2 - m - 6)$
$= 7(2m^2 - 4m + 3m - 6)$
$= 7\{2m\,(m - 2) + 3(m - 2)\}$
$= 7(2m + 3)(m - 2)$
On re-substituting the value of m, we obtain
$7(x^2 - x)\{2(x^2 - x) - 1)\} - 42 = 7\{2(x^2 - x) + 3)\}(x^2 - x - 2)$
$= 7(2x^2 - 2x + 3)(x^2 - x - 2)$

Division of Polynomials by Monomials Using Factorization Method

Division is exactly the opposite of multiplication. For example, if $4 \times 5 = 20$, then it is also correct to say that $20 \div 5 = 4$ and $20 \div 4 = 5$.

We can use the same concept to divide algebraic expressions.

Let us try to divide the expression $3x$ by 3 and x.

We can factorize $3x$ as $3 \times x$.

This means that $3x$ is a product of 3 and x.

$\therefore 3x \div 3 = x$ and $3x \div x = 3$

Each of the expressions i.e., 3, x, and $3x$ is a monomial. Hence, these were examples of division of monomials by monomials.

When we divide a monomial by another monomial, we first need to factorise each monomial. Next, we divide the monomial by cancelling the common factors.

Example 1:

Divide the following expressions:

(i). $27x^2y^2z \div 27xyz$

(ii). $144pq^2r \div (-48qr)$

Solution:

(i). $27x^2y^2z \div 27xyz$

Dividend = $27x^2y^2z$

$= 3 \times 3 \times 3 \times x \times x \times y \times y \times z$

Divisor = $27xyz$

$= 3 \times 3 \times 3 \times x \times y \times z$

$$\Rightarrow \frac{27x^2y^2z^2}{27xyz} = \frac{3 \times 3 \times 3 \times x \times x \times y \times y \times z}{3 \times 3 \times 3 \times x \times y \times z}$$

$= xy$

$\therefore 27x^2y^2z \div 27xyz = xy$

(ii). $144pq^2r \div (-48qr)$

Dividend = $144\,pq^2r$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times p \times q \times q \times r$

Divisor = $-48\,qr$

$= -2 \times 2 \times 2 \times 2 \times 3 \times q \times r$

$$\Rightarrow \frac{144pq^2r}{-48qr} = \frac{2\times2\times2\times2\times3\times3\times p\times q\times q\times r}{-2\times2\times2\times2\times3\times q\times r}$$
$$= -3\times p\times q$$
$$= -3pq$$

$\therefore 144pq^2r \div (-48qr) = -3pq$

Example 2:

Carry out the following divisions:

(i). $(x^3y^6 - x^6y^3) \div x^3y^3$

(ii). $26xy\,(x + 5) \div 13xy$

(iii). $27\,(-a^2bc + ab^2c - abc^2) \div (-3abc)$

Solution:

(i). $(x^3y^6 - x^6y^3) \div x^3y^3$

Dividend = $x^3y^6 - x^6y^3$

$= x^3y^3\,(y^3 - x^3)$

Divisor = x^3y^3

$$\Rightarrow \frac{x^3y^6 - x^6y^3}{x^3y^3} = \frac{x^3y^3\left(y^3 - x^3\right)}{x^3y^3}$$

$= y^3 - x^3$

Another method of simplifying this expression is

$$\frac{x^3y^6 - x^6y^3}{x^3y^3} = \frac{x^3y^6}{x^3y^3} - \frac{x^6y^3}{x^3y^3}$$
$$= y^3 - x^3$$

$\therefore (x^3y^6 - x^6y^3) \div x^3y^3 = y^3 - x^3$

(ii). Dividend = $26xy\ (x + 5)$

$= 2 \times 13 \times x \times y \times (x + 5)$

Divisor = $13xy$

$= 13 \times x \times y$

$$\therefore \frac{26xy(x+5)}{13xy} = \frac{2\times13\times x\times y\times(x+5)}{13\times x\times y}$$

$= 2\ (x + 5)$

$= 2x + 10$

$\therefore 26xy\ (x + 5) \div 13xy = 2x + 10$

(iii). $27\left(-a^2bc + ab^2c - abc^2\right) \div \left(-3abc\right)$

$$= \frac{27\left(-a^2bc + ab^2c - abc^2\right)}{\left(-3abc\right)}$$

$$= \frac{-27a^2bc + 27ab^2c - 27abc^2}{-3abc}$$

$$= \left(\frac{-27a^2bc}{-3abc}\right) + \left(\frac{27ab^2c}{-3abc}\right) + \left(\frac{-27abc^2}{-3abc}\right)$$

$= 9a - 9b + 9c$

Division of Polynomials by Polynomials

We know how to divide a given polynomial by a monomial. But how do we go about dividing a polynomial by another polynomial.

Example 1:

Factorise the following expressions and divide as directed.

(i) $6ab\ (9a^2 - 16b^2) \div 2ab\ (3a + 4b)$

(ii) $(x^2 - 14x - 32) \div (x + 2)$

(iii) $36abc\ (5a - 25)\ (2b - 14) \div 24(a - 5)\ (b - 7)$

Solution:

(i) We can factorise the given expressions as

$6ab\ (9a^2 - 16b^2) = 2 \times 3 \times a \times b\ [(3a)^2 - (4b)^2]$

$= 2 \times 3 \times a \times b\ [(3a + 4b)\ (3a - 4b)]\ [a^2 - b^2 = (a + b)(a - b)]$

And, $2ab\ (3a + 4b) = 2 \times a \times b \times (3a + 4b)$

$$\Rightarrow \frac{6ab\left(9a^2 - 16b^2\right)}{2ab\left(3a + 4b\right)} = \frac{2 \times 3 \times a \times b \times \left(3a + 4b\right) \times \left(3a - 4b\right)}{2 \times a \times b \times \left(3a + 4b\right)}$$

$= 3 \times (3a - 4b)$

$= 9a - 12b$

$\therefore 6ab\ (9a^2 - 16b^2) \div 2ab\ (3a + 4b) = 9a - 12b$

(ii) We can factorise the given expression as

$x^2 - 14x - 32 = x^2 - (16 - 2)\ x - 32$

$= x^2 - 16x + 2x - 32$

$= x\ (x - 16) + 2\ (x - 16)$

$= (x - 16)\ (x + 2)$

$= (x + 2) = (x + 2)$

$$\Rightarrow \frac{x^2 - 14x - 32}{x + 2} = \frac{(x - 16)(x + 2)}{(x + 2)}$$

$= x - 16$

$\therefore (x^2 - 14x - 32) \div (x + 2) = (x - 16)$

(iii) We can factorise the given expression as

$$36abc(5a-25)\ (2b-14)\ =\ 2\times2\times3\times3\times a\times b\times c(5\times a-5\times5)\ (2\times b-2\times7)$$
$$=\ 2\times2\times3\times3\times5\times a\times b\times c\ (a-5)\times2(b-7)$$
$$=\ 2\times2\times2\times3\times3\times5\times a\times b\times c\ (a-5)\ (b-7)$$

$$24(a-5)(b-7) = 24\ (a-5)\ (b-7)\ =\ 2\times2\times2\times3(a-5)\ (b-7)$$

$$\therefore \frac{36abc(5a-25)(2a-14)}{24(a-5)(b-7)}$$

$$= \frac{2\times2\times2\times3\times3\times5\times a\times b\times c(a-5)(b-7)}{2\times2\times2\times3(a-5)(b-7)}$$

$$= 3\times5\times a\times b\times c$$

$$= 15abc$$

Hence, $36abc\ (5a - 25)\ (2b - 14) \div 24(a - 5)\ (b - 7) = 15abc$

Correcting Errors in Relations and Solutions of Equations

Rajan and Sneha were asked to solve the equation $2(x - 3) = 3x + 4$.

However, both of them solved the equation using different methods.

Rajan solved the equation as

$2(x - 3) = 3x + 4$

$2x - 3 = 3x + 4$

$2x - 3x = 4 + 3$

$-x = 7$

$x = -7$

On the other hand, Sneha solved the same equation as

$2(x - 3) = 3x + 4$

$2x - 6 = 3x + 4$

$2x - 3x = 4 + 6$

$-x = 10$

$x = -10$

Who solved the equation in the correct manner? To find the answer, we need to check if both Rajan and Sneha attempted all the steps correctly or not.

Actually, Sneha solved the equation correctly. Rajan made a mistake in the second step, where he multiplied the expression ($x - 3$) with 2. He solved the second step as

$2(x - 3) = 2x - 3$.

We know that **when an expression is multiplied with a constant or a variable, each term of the expression has to be multiplied with the constant or the variable.**

However, Rajan only multiplied the term x with 2 and did not multiply the other term i.e., 3 with 2. This is the reason why his solution was incorrect.

Now, we will study some common errors that we make in Algebra.

Let us first look at the following table. In the two given columns, the same expression has been solved using two different methods. Out of the two columns containing the two methods, can you find out the correct method of solving each of the given expressions?

S. No.	**Column A**	**Column B**
(i)	When x = −1, 4x = 4 − 1 = 3	When x = −1, 4x = 4(−1) = − 4
(ii)	(5x)2 = 25x2	(5x)2 = 5x2
(iii)	(x − 2)2 = x2 − 4	(x − 2)2 = x2 − 4x + 4
(iv)	(x + 6)2 = x2 + 36	(x + 6)2 = x2 + 12x + 36
(v)	(3a − 4) (a + 2) = 3a2 − 8	(3a − 4) (a + 2) = 3a2 + 2a − 8
(vi)	$\frac{4x+7}{4} = x + 7$	$\frac{4x+7}{4} = x + \frac{7}{4}$

Let us now look at the solution of each expression one by one.

In **(i)**, we were required to find the value of the expression $4x$, when $x = -1$.

$4x$ is the product of 4 and x. **Thus, we need to perform the multiplication operation on 4 and the value of x. This should not be confused with the addition or subtraction operations.**

Thus, as shown in column B, when $x = -1$, $4x = 4(-1) = -4$.

In **(ii)**, we were required to square the monomial $5x$.

We know that **we need to ensure that we square each factor of the monomial while squaring a monomial**.

Thus, as shown in column A, $(5x)^2 = (5)^2 \times (x)^2 = 25x^2$.

In **(iii) and (iv)**, we were required to square the binomials $(x - 2)$ and $(x + 6)$.

We know that **we should use the identities $(a + b)^2 = a^2 + 2ab + b^2$ or $(a - b)^2 = a^2 - 2ab + b^2$ while squaring a binomial**.

Thus, as shown in column B, $(x - 2)^2 = (x)^2 - 2 \times x \times 2 + (2)^2 = x^2 - 4x + 4$

Also, as shown in column B, $(x + 6)^2 = (x)^2 + 2 \times x \times 6 + (6)^2 = x^2 + 12x + 36$

In **(v)**, we were required to multiply the binomials $(3a - 4)$ and $(a + 2)$.

We know that **when we multiply two expressions, each term of the first expression has to be multiplied with each term of the second expression**.

Thus, as shown in column B, $(3a - 4)(a + 2) = 3a \times a + 3a \times 2 - 4 \times a - 4 \times 2$

$= 3a^2 + 6a - 4a - 8$

$= 3a^2 + 2a - 8$

In **(vi)**, we were required to divide the expression $(4x + 7)$ by 4.

We know that **when we divide a polynomial by a monomial, we divide each term of the polynomial by the monomial**.

Thus, as shown in column B, $\frac{4x+7}{4} = \frac{4x}{4} + \frac{7}{4} = x + \frac{7}{4}$

The errors that we just saw are common errors made in algebraic expressions and they should be avoided by keeping in mind the rules that we have discussed above.

Let us try and solve the following examples by eliminating the aforementioned errors.

Example 1:

Find and correct the error in the expression $2x + x + 4x = 6x$.

Solution:

The coefficient of the middle term (x) is 1. However, it is not shown in the expression. It should be noted that when we add the three given terms, then we also need to include the coefficient of the middle term (x).

Hence, the expression will be written as $2x + 1x + 4x = 7x$.

Example 2:

Check if substituting $x = -2$ in $x^2 + 2x + 6$ gives $(-2)^2 + 2 - 2 + 6 = 4 + 2 - 2 + 6 = 10$.

Solution:

We should use brackets while substituting a negative value.

$x^2 + 2x + 6 = (-2)^2 + 2(-2) + 6 = 4 - 4 + 6 = 6$

Example 3:

Find and correct the errors in the following mathematical expressions:

(i) $\frac{11x^2}{11x^2} = 0$

(ii) $(y + 8)^2 = y^2 + 64$

(iii) $(x + y)(x + 2y) = x^2 + 2y^2$

(iv) $\frac{12x + 9}{9} = 12x + 1$

(v) $(4x)^2 + 7 = 4x^2 + 7$

(vi) $4a + 2a = 6a^2$

(vii) $(-1)^2 + 4(-1) = -1 - 4 = -5$

Solution:

(i) When the dividend and the divisor are the same, the quotient is always 1.

$$\frac{11x^2}{11x^2} = 1$$

(ii) We can find the square of a binomial using $(a + b)^2 = a^2 + 2ab + b^2$.

$(y + 8)^2 = (y)^2 + 2 \times y \times 8 + (8)^2 = y^2 + 16y + 64$

(iii) The correct expression for this problem is

$(x + y)\ (x + 2y) = x \times x + x \times 2y + y \times x + y \times 2y$

$= x^2 + 2xy + xy + 2y^2$

$= x^2 + 3xy + 2y^2$

$\therefore (x + y)\ (x + 2y) = x^2 + 3xy + 2y^2$

(iv) When dividing a polynomial by a monomial, we divide each term of the polynomial by the monomial.

$$\frac{12x+9}{9} = \frac{12x}{9} + \frac{9}{9}$$
$$= \frac{4x}{3} + 1$$
$$\therefore \frac{12x+9}{9} = \frac{4x}{3} + 1$$

(v) The correct expression is represented as

$(4x)^2 + 7 = 16x^2 + 7$

(vi) The correct expression is represented as

$4a + 2a = 6a$

(vii) The correct expression is represented as

$(-1)^2 + 4(-1) = 1 - 4 = -3$

The square of a negative number is always a positive number.