3 Laws of Motion

Section-A : JEE Advanced/ IIT-JEE

A **1.** 5 **2.** $\rho L\alpha/2$

B **1.** F **2.** F **3.** T **4.** F

C **1.** (c) **2.** (a) **3.** (b) **4.** (a) **5.** (a)
6. (c) **7.** (d) **8.** (a) **9.** (a) **10.** (c)
11. (b) **12.** (d) **13.** (b) **14.** (a) **15.** (d)
16. (c)

D **1.** (b) **2.** (b, d) **3.** (b, c) **4.** (a) **5.** (a, c) **6.** (d)

E **1.** 71.05 N **2.** $f=\dfrac{(m_1\sin\alpha+m_2\sin\beta)\,g}{m_1\cos\alpha+m_2\cos\beta}$; $T=\dfrac{m_1m_2\,g\sin(\alpha-\beta)}{m_1\cos\alpha+m_2\cos\beta}$

3. $T=F\left(1-\dfrac{\ell}{L}\right)$ **4.** 4.2 Kg, 9.8 N

5. $mg\sin\theta$, $\tan^{-1}\mu$ **6.** 20N, 50N

7. $\dfrac{5\sqrt{3}}{8}g$, $\dfrac{3mg}{8}$ **8.** **(a)** $-1\,\text{m/s}$ **(b)** $\left(\dfrac{\pi}{3}+\dfrac{\sqrt{3}}{4}\right)$ sec

9. **(b)** F = 60 N; T = 18 N
$a=\dfrac{3}{5}\,\text{m/s}^2,\ f_1=15\,\text{N},\ f_2=30\,\text{N}$

10. $8\sqrt{2}$m, $7\sqrt{2}$ m, 2 sec. **11.** 10 m/s^2

F **1.** (d)

G **1.** (a) **2.** (b)

H **1.** (b) **2.** (b)

I **1.** 5

Section-B : JEE Main/ AIEEE

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (b)
7. (b)	**8.** (c)	**9.** (a)	**10.** (d)	**11.** (d)	**12.** (d)
13. (d)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (b)
19. (d)	**20.** (c)	**21.** (c)	**22.** (d)	**23.** (c)	**24.** (c)
25. (a)	**26.** (d)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (d)
31. (a)	**32.** (b)	**33.** (a)	**34.** (a)	**35.** (a)	

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. As seen by the observer on the ground, the frictional force is responsible to move the mass with an acceleration of 5 m/s^2.
Therefore, frictional force $= m\times a = 1\times 5 = 5\,N$.

2. Let A be the area of cross-section of the rod.
Consider the back half portion of the rod.

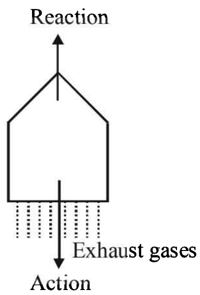
Mass of half portion of the rod $=\dfrac{\rho AL}{2}$

The force responsible for its acceleration is

$$f=\frac{\rho AL}{2}\times\alpha \qquad \therefore\ \text{Stress}=\frac{f}{A}=\frac{\rho L\alpha}{2}$$

B. True/ False

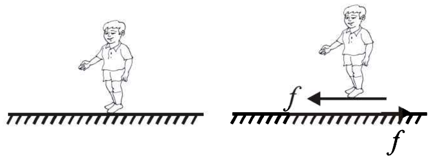
1. **KEY CONCEPT :** The rocket moves forward when the exhaust gases are thrown backward.
Here exhaust gases thrown backwards is action and rocket moving forward is reaction.



Note : This phenomenon takes place in the absence of air as well.

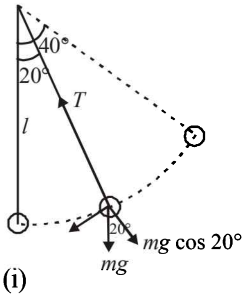
2. **KEY CONCEPT :** Friction force opposes the relative motion of the surface of contact.
When a person walks on a rough surface, the foot is the surface of contact. When he pushes the foot backward, the

motion of surface of contact tends to be backwards. Therefore the frictional force will act forward (in the direction of motion of the person)



3. As the angular amplitude of the pendulum is 40°, the bob will be in the mid of the equilibrium position and the extreme position as shown in the figure

Note : For equilibrium of the bob, $T - mg\cos 20° = \frac{mv^2}{l}$, where l is the length of the pendulum and is the velocity of the bob.



$\therefore \quad T = mg\cos 20° + \frac{mv^2}{l}$

$\frac{mv^2}{l}$ is always a positive quantity.

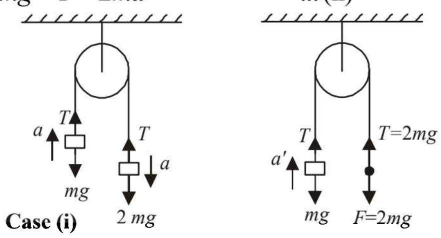
Hence, $T > mg\cos 20°$.

4. **Case (i)** For mass m

$T - mg = ma$ ... (i)

For mass $2m$

$2mg - T = 2ma$ ... (ii)



Case (i) Case (ii)

From (i) and (ii)

$a = g/3$

Case (ii) $T - mg = ma'$

$2mg - mg = ma'$ $[\because T = 2mg]$

$\therefore \quad a' = g$

Hence, $a < a'$

C. MCQs with ONE Correct Answer

1. **(c)** $F = ma$

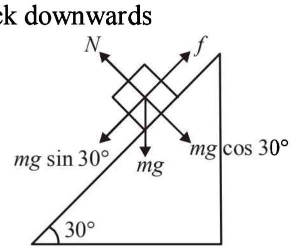
$\Rightarrow \quad a = \frac{F}{m} = \frac{5\times10^4}{3\times10^7} = \frac{5}{3}\times10^{-3}\,\text{ms}^{-2}$

Also, $v^2 - u^2 = 2as$

$\Rightarrow \quad v^2 - 0^2 = 2\times\frac{5}{3}\times10^{-3}\times3 = 10^{-2}$

$\Rightarrow \quad v = 0.1\ \text{ms}^{-1}$

2. **(a)** The force acting on the block along the incline to shift the block downwards



$= mg\sin\theta = 2\times9.8\sin30° = 9.8\,N$

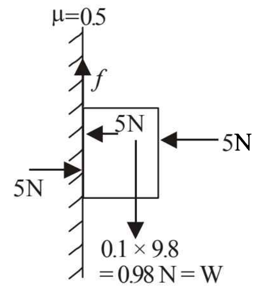
The limiting frictional force

$$f_l = \mu_s mg\cos\theta = 0.7\times2\times9.8\times\frac{\sqrt{3}}{2} = 11.8\,N$$

Note : The frictional force is never greater than the force tending to produce relative motion.

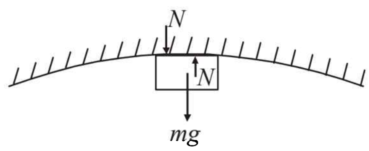
Therefore the frictional force is 9.8 N

3. **(b)** Limiting frictional force, $f_l = \mu_s N = 0.5\times5 = 2.5\,N$. But force tending to produce relative motion is the weight (W) of the block which is less than f_l. Therefore, the frictional force is equal to the weight, the magnitude of the frictional force f has to balance the weight 0.98 N acting downwards.



Therefore the frictional force = 0.98 N.

4. **(a)** Since the body presses the surface with a force N hence according to Newton's third law the surface presses the body with a force N. The other force acting on the body is its weight mg.



For circular motion to take place, a centripetal force is required which is provided by $(mg + N)$.

$\therefore \quad mg + N = \frac{mv^2}{r}$

where r is the radius of curvature at the top.

If the surface is smooth then on applying conservation of mechanical energy, the velocity of the body is always same at the top most point. Hence, N and r have inverse relationship. From the figure it is clear that r is minimum for first figure, therefore N will be maximum.

Note : If we do not assume the surface to be smooth, we cannot reach to a conclusion.

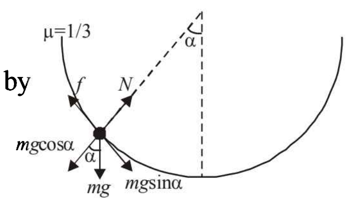
5. **(a)** **KEY CONCEPT :**

For the maximum possible value of α, $mg\sin\alpha$ will also be maximum and equal to the frictional force.

In this case f is the limiting friction. The two forces acting on the insect are mg and N. Let us resolve mg into two components.

$mg\cos\alpha$ balances N.

$mg\sin\alpha$ is balanced by the frictional force.



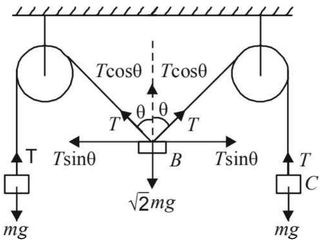
$\therefore \quad N = mg\cos\alpha$

$f = mg\sin\alpha$

But $f = \mu N = \mu\, mg\cos\alpha$

$\therefore \quad \mu\, mg\cos\alpha = mg\sin\alpha \Rightarrow \cot\alpha = \frac{1}{\mu} \Rightarrow \cot\alpha = 3$

6. **(c)** The tension in both strings will be same due to symmetry.



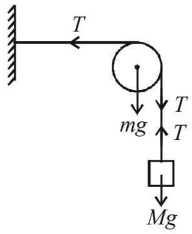
For equilibrium in vertical direction for body B we have

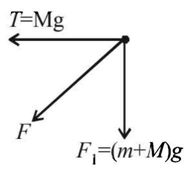
$$\sqrt{2}\,mg = 2T\cos\theta$$

$\therefore$ $\sqrt{2}\,mg = 2(mg)\cos\theta$ $\quad [\because T = mg, \text{(at equilibrium)}]$

$\therefore$ $\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

7. **(d)** At equilibrium $T = Mg$





F.B.D. of pulley

$F_1 = (m+M)g$

The resultant force on pulley is

$$F = \sqrt{F_1^2 + T^2} = [\sqrt{(m+M)^2 + M^2}\,]g$$

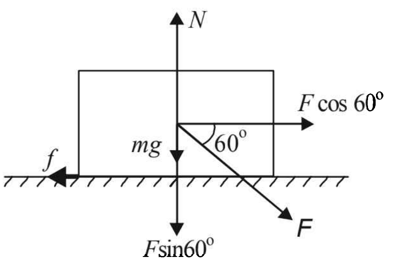
8. **(a)** The forces acting on the block are shown. Since the block is not moving forward for the maximum force F applied, therefore

$F\cos 60° = f = \mu N$ ... (i) (Horizontal Direction)

Note : For maximum force F, the frictional force is the limiting friction $= \mu N$]

and $F\sin 60° + mg = N$... (ii)

From (i) and (ii)



$$F\cos 60° = \mu\,[F\sin 60° + mg]$$

$$\Rightarrow F = \frac{\mu mg}{\cos 60° - \mu \sin 60°}$$

$$= \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{5}{\frac{1}{4}} = 20\,\text{N}$$

9. **(a)** Let ω be the angular frequency of the system.

The maximum acceleration of the system,

$$a = \omega^2 A = \left(\frac{k}{2m}\right) A \quad \left[\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}\right]$$

The force of friction provides this acceleration.

$$\therefore \quad f = ma = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

10. **(c)** In situation 1, the tension T has to hold both the masses $2m$ and m therefore,

$T = 3mg$

In situation 2, when the string is cut, the mass m is a freely falling body and its acceleration due to gravity is g.

For mass $2m$, just after the string is cut, T remains $3mg$ because of the extension of string.

$$\therefore \quad 3mg - 2mg = 2m \times a \quad \therefore \quad \frac{g}{2} = a$$

11. **(b)** The acceleration of mass m is due to the force $T\cos\theta$

$$\therefore \quad T\cos\theta = ma \quad \Rightarrow \quad a = \frac{T\cos\theta}{m} \qquad \text{... (i)}$$

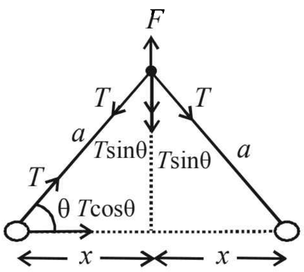
$$\text{also, } F = 2T\sin\theta \quad \Rightarrow \quad T = \frac{F}{2\sin\theta} \qquad \text{... (ii)}$$

From (i) and (ii)

$$a = \left(\frac{F}{2\sin\theta}\right)\frac{\cos\theta}{m}$$

$$= \frac{F}{2m\tan\theta} = \frac{F}{2m}\frac{x}{\sqrt{a^2 - x^2}}$$

$$\left[\because \tan\theta \frac{\sqrt{a^2 - x^2}}{x}\right]$$



12. **(d)** $\vec{p}(t) = A[\hat{i}\cos(kt) - \hat{j}\sin(kt)]$

$$\vec{F} = \frac{d\vec{p}}{dt} = Ak\,[-\hat{i}\sin(kt) - \hat{j}\cos(kt)]$$

Here, $\vec{F}.\vec{P} = 0$ But $\vec{F}.\vec{p} = Fp\cos\theta$

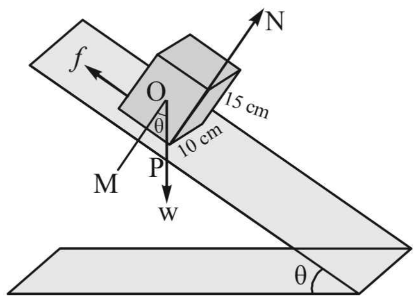
$\therefore$ $\cos\theta = 0 \Rightarrow \theta = 90°$.

13. **(b)** For the block to slide, the angle of inclination should be equal to the angle of repose, i.e.,

$\tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60°$.

Therefore, option (a) is wrong.

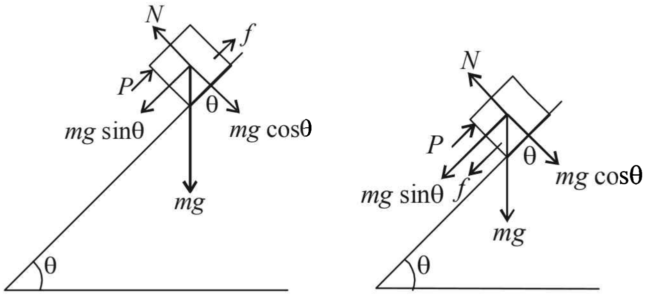
For the block to topple, the condition of the block will be as shown in the figure.



In ΔPOM, $\tan\theta = \frac{PM}{OM} = \frac{5\,\text{cm}}{7.5\,\text{cm}} = \frac{2}{3}$

For this, $\theta < 60°$. From this we can conclude that the block will topple at lesser angle of inclination. Thus the block will remain at rest on the plane up to a certain anlgle θ and then it will topple.

14. (a) As $\tan\theta > \mu$, the block has a tendency to move down the incline. Therfore a force P is applied upwards along the incline. Here, at equilibrium $P + f = mg\sin\theta \Rightarrow f = mg\sin\theta - P$



Now as P increases, f decreases linearly with respect to P.

When $P = mg\sin\theta, f = 0$.

When P is increased further, the block has a tendency to move upwards along the incline.

Therefore the frictional force acts downwards along the incline.

Here, at equilibrium $P = f + mg\sin\theta$

$\therefore \quad f = P - mg\sin\theta$

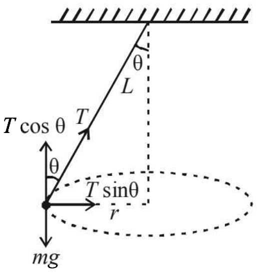
Now as P increases, f increases linearly w.r.t P.

This is represented by graph (a) .

15. (d) Here, the horizontal component of tension provides the necessary centripetal force.

$\therefore T\sin\theta = mr\omega^2$

From (i) and (ii)



$T \times \frac{r}{L} = mr\omega^2 \; [\because \sin\theta = \frac{r}{L}]$

$\therefore \omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5 \times 0.5}}$

$= \frac{18}{0.5} = 36$ rad/s

16. (c) For a plano convex lens

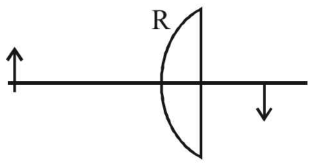
$$\frac{1}{f} = \frac{(\mu - 1)}{R} = \frac{1}{v} - \frac{1}{u} \quad ...(i)$$

Here $\mu = \frac{\lambda_a}{\lambda_m} = \frac{\lambda_a}{\frac{2}{3}\lambda_a} = \frac{3}{2} = 1.5$

Where λ_a = wavelength of light in air

λ_m = wavelength of light in water

v = 8m



Also $m = \frac{v}{u} = -\frac{1}{3}$

$\therefore u = -24$ cm.

From (i) $\frac{1.5 - 1}{R} = \frac{1}{8} - \left(\frac{1}{-24}\right) = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$

$\therefore$ R = 3m option (c) is correct

D. MCQs with ONE or MORE THAN ONE Correct

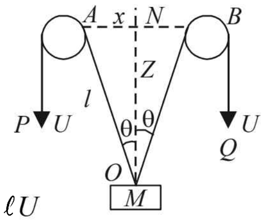
1. (b) This is a problem based on constraint motion. The motion of mass M is constraint with the motion of P and Q. Let $AN = x$, $NO = z$. Then velocity of mass is $\frac{dz}{dt}$. Also, let $OA = \ell$. then $\frac{d\ell}{dt} = U$

From ΔANO, using pythagorous theorem

$\therefore \quad x^2 + z^2 = \ell^2$

Here x is a constant.

Differentiating the above equation w.r.t to t



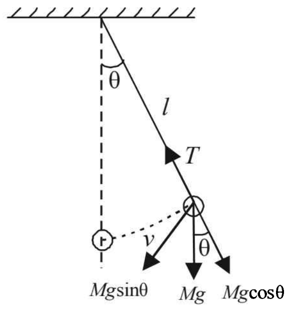
$0 + 2z\frac{dz}{dt} = 2\ell\frac{d\ell}{dt} \Rightarrow zv_M = \ell U$

$\Rightarrow v_M = \frac{\ell}{z}U = \frac{U}{z/\ell} = \frac{U}{\cos\theta} \quad \left(\because \cos\theta = \frac{z}{\ell}\right)$

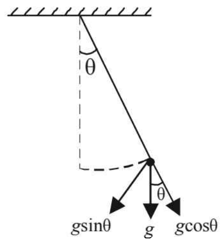
2. (b,d) Since earth is an accelerated frame and hence, cannot be an inertial frame.

Note : Strictly speaking Earth is accelerated reference frame. Earth is treated as a reference frame for practical examples and Newton's laws are applicable to it only as a limiting case.

3. (b, c) Since the body is moving in a circular path therefore it needs centripetal force $\left(\frac{Mv^2}{\ell}\right)$.

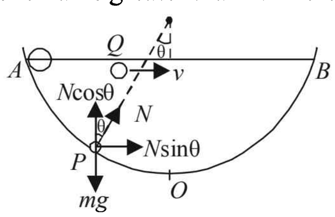


$\therefore \quad T - Mg\cos\theta = \frac{Mv^2}{\ell}$

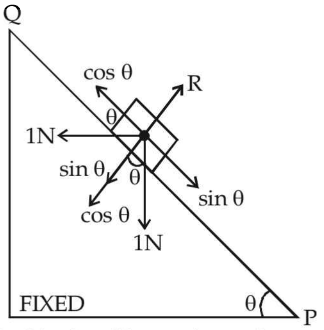


Also, the tangential acceleration acting on the mass is $g\sin\theta$.

4. (a) At A the horizontal speeds of both the masses is the same. The velocity of Q remains the same in horizontal as no force is acting in the horizontal direction. But in case of P as shown at any intermediate position, the horizontal velocity first increases (due to $N\sin\theta$), reaches a max value at O and then decreases. Thus it always remains greater than v. Therefore $t_P < t_Q$.



5. (a, c) The forces are resolved as shown in the figure. When $\theta = 45°$, $\sin\theta = \cos\theta$

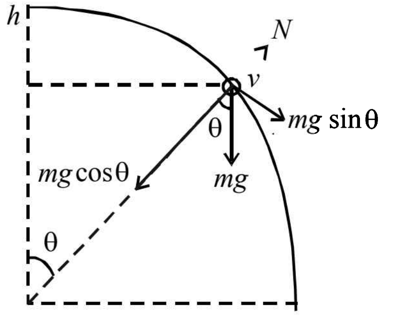


The block will remain stationary and the frictional force is zero.

When $\theta > 45°$, $\sin\theta > \cos\theta$

Therefore a frictional force acts towards Q.

6. (d)



As the bead is moving in the circular path

$$\therefore\ mg\cos\theta - N = \frac{mv^2}{R}$$

$$\therefore\ N = mg\cos\theta - \frac{mv^2}{R} \qquad ...(1)$$

By energy conservation, $\frac{1}{2}mv^2 = mg\left[R - R\cos\theta\right]$

$$\therefore\ \frac{v^2}{R} = 2g\left(1-\cos\theta\right) \qquad ..(2)$$

From (1) and (2)

$$N = mg\cos\theta - m\left[2g - 2g\cos\theta\right]$$

$$N = mg\cos\theta - 2mg + 2mg\cos\theta$$

$$N = 3mg\cos\theta - 2mg$$

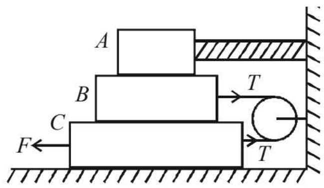
$$\Rightarrow \quad N = mg\left(3\cos\theta - 2\right)$$

Clearly N is positive (acts radially outwards) when $\cos\theta > \frac{2}{3}$

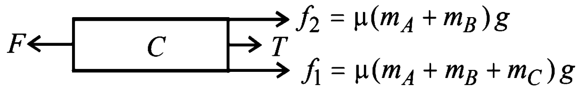
Similarly, N acts radially inwards if $\cos\theta < \frac{2}{3}$

E. Subjective Problems

1. When force F is applied on C, the block C will move towards left.

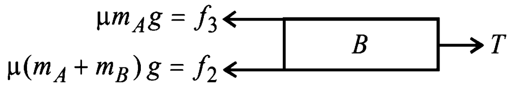


The F.B.D. for mass C is



As C is moving with constant speed $F = f_1 + f_2 + T$ ... (i)

F.B.D. for mass B is



As B is moving with constant speed $f_2 + f_3 = T$ (ii)

Subtracting (ii) from (i)

$$F - (f_2 + f_3) = f_1 + f_2 + T - T = f_1 + f_2$$

$$\Rightarrow \quad F = f_1 + 2f_2 + f_3 = \mu(m_A + m_B + m_C)g + 2\mu(m_A + m_B)g + \mu m_A g$$

$$F = \mu(4m_A + 3m_B + m_C)g$$

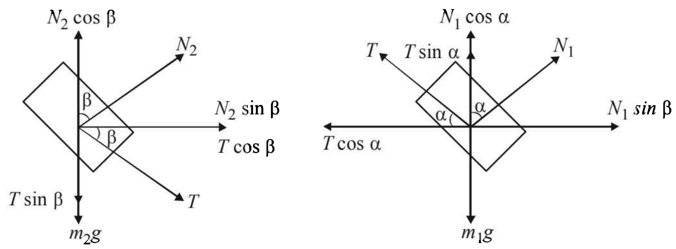
$$= 0.25\,[4\times3 + 3\times4 + 5]\times 9.8 = 71.05\,N$$

2. Without Pseudo Force

F.B.D for mass m_2

$$N_2\cos\beta = T\sin\beta + m_2 g \qquad ...(i)$$

and $(N_2\sin\beta + T\cos\beta) = m_2 f$ ...(ii)



FBD for mass m_1

$$N_1\cos\alpha + T\sin\alpha = m_1 g \qquad ...(iii)$$

and $(N_1\sin\alpha - T\cos\alpha) = m_1 f$ ...(iv)

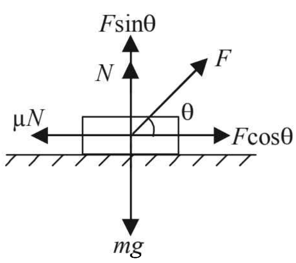
On solving the four equations, we get the above results.

3. From equation (i) $T = \frac{M}{L}(L-\ell)a$

Also, $F = Ma \quad \therefore \frac{T}{F} = \left(\frac{L-\ell}{L}\right) \Rightarrow T = F\left(1 - \frac{\ell}{L}\right)$

4. (a) If M_1, M_2 and M_3 are considered as a system, then the force responsible to more them is $M_1 g$ and the retarding force is $(M_2 g\sin\theta + \mu M_2 g\cos\theta + \mu M_3 g)$. These two should be equal as the system is moving with constant velocity.

5. Let F be the force applied to move the body at an angle θ to the horizontal.



The body will move when

$$F\cos\theta = \mu N \quad ...(i)$$

Applying equilibrium of forces in the vertical direction we get

$$F\sin\theta + N = mg$$

$$\Rightarrow \quad N = mg - F\sin\theta \quad ...(ii)$$

$\Rightarrow$ From (i) and (ii)

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \quad ...(iii)$$

Differentiating the above equation w.r.t. θ, we get

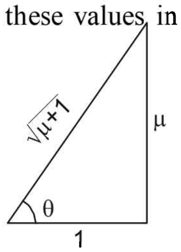
$$\frac{dF}{d\theta}=\frac{\mu mg}{(\cos\theta+\mu\sin\theta)^2}[-\sin\theta+\mu\cos\theta]=0$$

$\Rightarrow \quad \theta=\tan^{-1}\mu$

This is the angle for minimum force.

To find the minimum force substituting these values in equation (iii)

$$\sin\theta=\frac{\mu}{\sqrt{\mu^2+1}},\quad \cos\theta=\frac{1}{\sqrt{\mu^2+1}}$$



$$F=\frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}}+\frac{\mu}{\sqrt{\mu^2+1}}\times\mu}$$

$$\Rightarrow \quad F=\frac{\mu mg(\sqrt{\mu^2+1})}{\mu^2+1}=\frac{\mu mg}{\sqrt{\mu^2+1}}$$

$\Rightarrow \quad F=mg\sin\theta$

6. Let λ be the mass per unit length of lower wire.

Let us consider the dotted portion as a system and the tension T accelerates the system upwards

$\therefore \quad T-(m_1+\lambda\ell)g=(m_1+\lambda\ell)a$

$\therefore \quad T=(m_1+\lambda\ell)(a+g)$

$=(1.9+0.2\times0.5)(9.8+0.2)=2\times10=20\,N$

To find tension T' Let us consider the dotted portion given in figure (2)

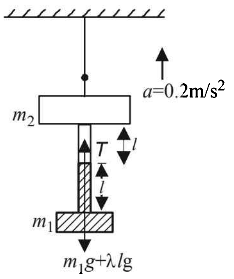


Fig. (1)

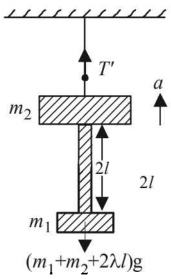


Fig. (2)

$T'-(m_2g+\lambda\times2\ell g+m_1g)=(m_1+\lambda2\ell+m_2)a$

$\therefore \quad T'=(m_1+\lambda2\ell+m_2)(a+g)$

$=(1.9+0.2\times1+2.9)(10)=5\times10=50\,N.$

Alternatively considering m_1, m_2 and lower wire as a system

$T'-5g=5a$

7.

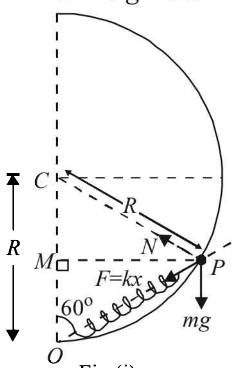


Fig (i)

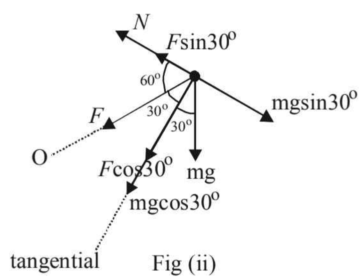


Fig (ii)

In ΔOCP, $OC=CP=R$

$\therefore \quad \angle COP=\angle CPO=60^\circ \Rightarrow \angle OCP=60^\circ$

$\therefore \quad \Delta OCP$ is an equilateral triangle $\Rightarrow OP=R$

$\therefore \quad$ Extension of string $=R-\frac{3R}{4}=\frac{R}{4}=x$

The forces acting are shown in the figure (i)

The free body diagram of the ring is shown in fig. (ii)

Force in the tangential direction

$=F\cos30^\circ+mg\cos30$

$=[kx+mg]\cos30^\circ$

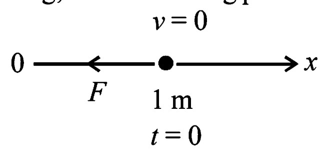
$F_t=\frac{5mg}{8}\sqrt{3} \quad \therefore F_t=ma_t \quad \Rightarrow \quad a_t=\frac{5\sqrt{3}}{8}g$

Also, when the ring is just released

$N+F\sin30^\circ=mg\sin30^\circ$

$\Rightarrow \quad N=(mg-F)\sin30^\circ=\left(mg-\frac{mg}{4}\right)\times\frac{1}{2}=\frac{3mg}{8}$

8. $m=10^{-2}$ kg, motion is along positive X-axis



$F(x)=-\frac{K}{2x^2}$, $K=10^{-2}\,Nm^2$; At $t=0, x=1.0$ m and $V=0$

(a) $F(x)=\frac{-K}{2x^2}$ or $m\left(\frac{dV}{dx}\right)V=-\frac{K}{2x^2}$

or $m\int_0^v VdV=-\int_1^x\frac{K}{2x^2}dx$

or $\frac{mV^2}{2}=\left[\frac{K}{2x}\right]_1^x=\frac{K}{2}\left(\frac{1}{x}-1\right)$

or $V^2=\frac{K}{m}\left(\frac{1}{x}-1\right)$ or $|\vec{V}|=\pm\sqrt{\frac{K}{m}\left(\frac{1}{x}-1\right)}$ ... (i)

Initially the particle was moving in $+X$ direction at $x=1$. When the particle is at $x=0.5$, obviously its velocity will be in $-X$ direction. The force acting in $-X$ direction first decreases the speed of the particle, bring it momentarily at rest and then changes the direction of motion of the particle.

When x = 0.5 m : $|\vec{V}|=-\sqrt{\frac{K}{m}\left(\frac{1}{0.5}-1\right)}$

$$=-\sqrt{\frac{K}{m}}=-\sqrt{\frac{10^{-2}}{10^{-2}}}=-1\,m/s$$

(b) As $\frac{K}{m}=1$ m/s, hence from (i)

$$V=\frac{dx}{dt}=-\sqrt{\frac{1-x}{x}}$$

Note : We have chosen –ve sign because force tends to decrease the displacement with time

$$\sqrt{\frac{x}{1-x}}dx=-dt;\int_1^{0.25}\sqrt{\frac{x}{1-x}}dx=\int_0^t-dt$$

Put $x = \sin^2\theta$, $dx = 2\sin\theta\cos\theta\, d\theta$

So, $\int_{\pi/2}^{\pi/6} 2\sin^2\theta\, d\theta = -t$

$\cos 2\theta = 1 - 2\sin^2\theta; \quad 2\sin^2\theta = 1 - \cos 2\theta$

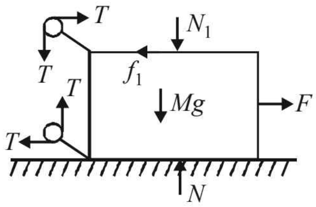
$\int_{\pi/2}^{\pi/6}(1-\cos 2\theta)\,d\theta = -t; \left[\theta - \sin\frac{2\theta}{2}\right]_{\pi/2}^{\pi/6} = -t$

$\frac{\pi}{6} - \frac{1}{2}\sin\frac{\pi}{3} - \frac{\pi}{2} - \frac{1}{2}\sin\pi = -t$

$\therefore \quad t = \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)$ sec.

9. Given m_1 = 20 kg, m_2 = 5 kg, M = 50 kg, μ = 0.3 and $g = 10\,m/s^2$

(A) Free body diagram of mass M is



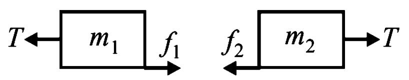
(B) The maximum value of f_1 is

$(f_1)_{max} = (0.3)(20)(10) = 60\,N$

The maximum value of f_2 is

$(f_2)_{max} = (0.3)(5)(10) = 15\,N$

Forces on m_1 and m_2 in horizontal direction are as follows :



Note : There are only two possibilities.

(1) Either both m_1 and m_2 will remain stationary (w.r.t. ground) or (2) both m_1 and m_2 will move (w.r.t. ground). First case is possible when.

$T \le (f_1)_{max}$ or $T \le 60\,N$

and $T \le (f_2)_{max}$ or $T \le 15\,N$

These conditions will be satisfied when $T \le 15\,N$ say T = 14 then $f_1 = f_2 = 14\,N$.

Therefore the condition $f_1 = 2f_2$ will not be satisfied.

Thus m_1 and m_2 both can't remain stationary.

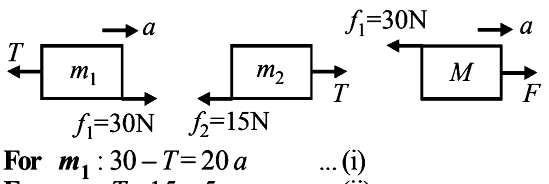
In the second case, when m_1 and m_2 both move

$f_2 = (f_2)$ max $= 15\,N$

Therefore, $f_1 = 2f_2 = 30\,N$

Note : Since $f_1 < (f_1)_{max}$, there is no relative motion between m_1 and M, i.e., all the masses move with same acceleration, say 'a'.

Free body diagrams and equations of motion are as follows:



For m_1 : $30 - T = 20a$ ...(i)

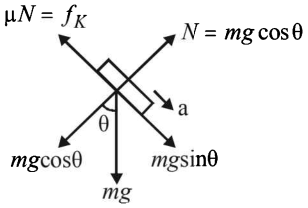
For m_2 : $T - 15 = 5a$ ...(ii)

For M : $F - 30 = 50a$ ...(iii)

Solving these three equations, we get,

$F = 60\,N$, $T = 18\,N$ and $a = \frac{3}{5}\,m/s^2$.

10.



$a = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m}$

$\therefore \quad a_A = g\sin\theta - \mu_{k,A}\, g\cos\theta$ ...(i)

and $a_B = g\sin\theta - \mu_{k,B}\, g\cos\theta$ ...(ii)

Putting values we get

$a_A = 4\sqrt{2}\,m/s^2$ and $a_B = 3.5\sqrt{2}\,m/s^2$

Let a_{AB} is relative acceleration of A w.r.t. B. Then

$a_{AB} = a_A - a_B$

$L = \sqrt{2}$ m

[where L is the relative distance between A and B]

Then $L = \frac{1}{2}a_{AB}t^2$

or $t^2 = \frac{2L}{a_{AB}} = \frac{2L}{a_A - a_B}$

Putting values we get, $t^2 = 4$ or $t = 2s$.

Distance moved by B during that time is given by

$S = \frac{1}{2}a_B t^2 = \frac{1}{2} \times 3.5\sqrt{2} \times 4 = 7\sqrt{2}$ m

Similarly for $A = 8\sqrt{2}$ m.

11. Applying pseudo force ma and resolving it.

Applying $F_{net} = ma_r$

$ma\cos\theta - (f_1 + f_2) = ma_r$

$ma\cos\theta - \mu N_1 - \mu N_2 = ma_r$

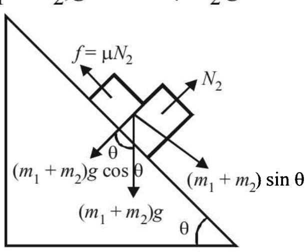
$ma\cos\theta - \mu ma\sin\theta - \mu\, mg = ma_r$

$\Rightarrow \quad a_r = a\cos\theta - \mu a\sin\theta - \mu g$

$= 25 \times \frac{4}{5} - \frac{2}{5} \times 25 \times \frac{3}{5} - \frac{2}{5} \times 10 = 10\,m/s^2$

F. Match the Following

1. **(d)** If $(m_1 + m_2)\sin\theta < \mu N_2$ the bodies will be at rest i.e., $(m_1 + m_2)g\sin\theta < \mu m_2\, g\cos\theta$



$\tan\theta < \frac{\mu m_2 g}{(m_1 + m_2)g}$

$\Rightarrow \quad \tan\theta < \frac{\mu m_2}{m_1 + m_2}$

$\Rightarrow \quad \tan\theta < \frac{0.3 \times 2}{1 + 2}$

$\Rightarrow \quad \tan\theta < 0.2$

i.e., If the angle $\theta < 11.5°$ the frictional force is less than

$\mu N_2 = \mu m_2 g = 0.3 \times 2 \times g = 0.6\,g$

and is equal to $(m_1 + m_2)g \sin\theta$

At $\theta = 11.5°$ the bodies are on the verge of moving, $f = 0.6\,g$

At $\theta > 11.5°$ the bodies start moving and $f = 0.6$ g

The above relationship is true for (d).

G. Comprehension Based Questions

1. (a) Force on the block along slat $= m r\omega^2 = m v \dfrac{dv}{dr}$

$$\therefore \int_0^v V dv = \int_{R/2}^{r} \omega^2 r dr \Rightarrow V = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\therefore \int_{R/4}^{r} \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega\, dt$$

On solving we get

$$r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{wt}$$

$$\text{or } r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2wt} + r^2 - 2r\frac{R}{2}e^{wt}$$

$$\therefore \quad r = \frac{R}{4}\left(e^{wt} + e^{-wt}\right)$$

2. (b) $\vec{F}_{rot} = \vec{F}_{in} + 2m\left(V_{rot}\hat{i}\right) \times w\hat{k} + m\left(w\hat{k} \times r\hat{i}\right) \times w\hat{k}$

$$\therefore m r\omega^2 \hat{i} = \vec{F}_{in} + 2m\, V_{rot}\omega\left(-\hat{j}\right) + m\omega^2 r\hat{i}$$

$$\vec{F}_{in} = mrV_{rot}\,\omega\hat{j} \qquad -(i)$$

$$\text{But } r = \frac{R}{4}\left[e^{wt} + e^{-wt}\right]$$

$$\therefore \frac{dr}{dt} = V_r = \frac{R}{4}\left[\omega e^{wt} - \omega e^{-wt}\right] - (ii)$$

From (i) and (ii)

$$\vec{F}_{in} = 2m\frac{R\omega}{4}\left(e^{wt} - e^{-wt}\right)\omega\hat{j}$$

$$\therefore \vec{F}_{in} = \frac{mR\omega^2}{2}\left(e^{wt} - e^{-wt}\right)\hat{j}$$

$$\therefore \vec{F}_{reaction} = \frac{mR\omega^2}{2}\left(e^{wt} - e^{-wt}\right)\hat{j} + mg\,\hat{K}$$

H. Assertion & Reason Type Questions

1. (b) **Statement 1 :** Cloth can be pulled out without dislodging the dishes from the table because of inertia. Therefore, statement – 1 is true.

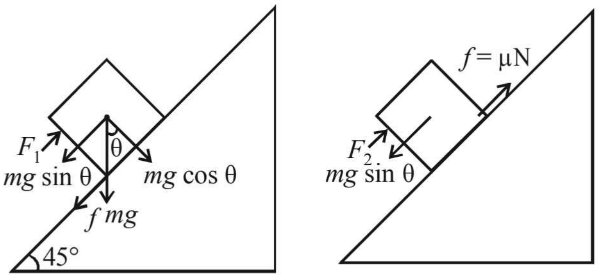
Statement 2 : This is Newton's third law and hence true. But statement 2 is not a correct explanation of statement 1.

2. (b) It is easier to pull a heavy object than to push it on a level ground. Statement-1 is true. This is because the normal reaction in the case of pulling is less as compared by pushing. ($f = \mu$ N). Therefore the frictional force is small in case of pulling.

statement-2 is true but is not the correct explanation of statement-1.

I. Integer Value Correct Type

1. 5



The pushing force $F_1 = mg\sin\theta + f$

$\therefore F_1 = mg\sin\theta + \mu\, mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$

The force required to just prevent it from sliding down

$F_2 = mg\sin\theta - \mu N = mg(\sin\theta - \mu\cos\theta)$

Given, $F_1 = 3F_2$

$\therefore \quad \sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$

$\therefore \quad 1 + \mu = 3(1 - \mu) \quad [\because \sin\theta = \cos\theta]$

$\therefore \quad 4\mu = 2 \qquad \therefore \mu = 0.5$

$\therefore \quad N = 10\,\mu = 5$

Section-B JEE Main/ AIEEE

1. (a) $W = \Delta K = FS$

$$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = F \times 3 \quad ...(i)$$

$$\frac{1}{2}m\left(\frac{v}{2}\right)^2 - 0 = F \times S \quad ...(ii)$$

On dividing

$$\frac{1/4}{3/4} = S/3$$

$\therefore$ S = 1 cm

2. (c)
- For the man standing in the left, the acceleration of the ball

  $\vec{a}_{bm} = \vec{a}_b - \vec{a}_m \Rightarrow a_{bm} = g - a$

  Where 'a' is the acceleration of the mass (because the acceleration of the lift is 'a')
- For the man standing on the ground, the acceleration of the ball

  $\vec{a}_{bm} = \vec{a}_b - \vec{a}_m \Rightarrow a_{bm} = g - 0 = g$

3. (a) When F_1, F_2 and F_3 are acting on a particle then the particle remains stationary. This means that the resultant of F_1 F_2 and F_3 is zero. When F_1 is removed, F_2 and F_3 will remain. But the resultant of F_2 and F_3 should be equal and opposite to F_1. i.e. $|\vec{F}_2 + \vec{F}_3| = |\vec{F}_1|$

$$\therefore \quad a = \frac{|\vec{F}_2 + \vec{F}_3|}{m} \Rightarrow a = \frac{F_1}{m}$$

4. **(b)** Let the two forces be F_1 and F_2 and let $F_2 < F_1$. R is the resultant force.

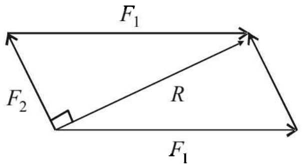
Given $F_1 + F_2 = 18$ ...(i)

From the figure $F_2^2 + R^2 = F_1^2$

$F_1^2 - F_2^2 = R^2$

$\therefore \quad F_1^2 - F_2^2 = 144$ ...(ii)

Only option (b) follows equation (i) and (ii).



5. **(d)** $\Delta K = FS$

$\frac{1}{2}mu^2 = F \times S_1$ ...(i)

$\frac{1}{2}m(4u)_2 = FS_2$ ...(ii)

Dividing (i) and (ii),

$$\frac{u^2}{16u^2} = \frac{2as_1}{2as_2} \Rightarrow \frac{1}{16} = \frac{s_1}{s_2}$$

6. **(b)** **For mass m_1**

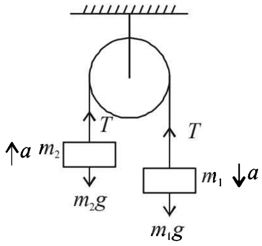
$m_1 g - T = m_1 a$

For mass m_2

$T - m_2 g = m_2 a$

Adding the equations we get

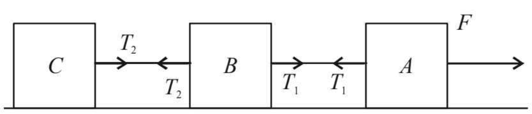
$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$



$$\therefore \frac{1}{8} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} \Rightarrow \frac{m_1}{m_2} + 1 = 8\frac{m_1}{m_2} - 8 \Rightarrow \frac{m_1}{m_2} = \frac{9}{7}$$

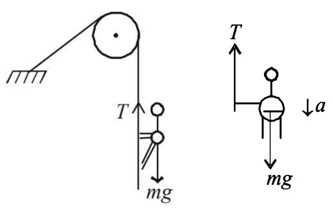
7. **(b)** $F = (m + m + m) \times a \quad \therefore a = \frac{10.2}{6} \text{m/s}^2$

$\therefore T_2 = ma = 2 \times \frac{10.2}{6} = 3.4\text{N}$



8. **(c)** $mg - T = ma$

$$\therefore \quad a = g - \frac{T}{m} = 10 - \frac{360}{60} = 4m/s^2$$

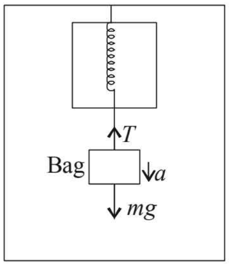


9. **(a)** For the bag accelerating down

$mg - T = ma$

$\therefore \quad T = m(g - a)$

$= \frac{49}{10}(10 - 5) = 24.5\text{ N}$

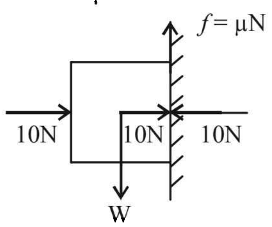


10. **(d)** As shown in the figure, the three forces are represented by the sides of a triangle taken in the same order.

Therefore the resultant force is zero. $\vec{F}_{net} = m\vec{a}$. Therefore acceleration is also zero ie velocity remains unchanged.

11. **(d)** For the block to remain stationary with the wall

$f = W \quad \therefore \mu N = W$



$0.2 \times 10 = W \Rightarrow W = 2N$

12. **(d)** $u = 6$ m/s, $v = 0$, $t = 10s$,

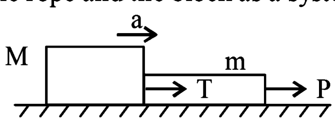
$$a = -\frac{f}{m} = \frac{-\mu mg}{m} = -\mu g = -10\mu$$

$v = u + at$

$0 = 6 - 10\mu \times 10$

$\therefore \mu = 0.06$

13. **(d)** Taking the rope and the block as a system



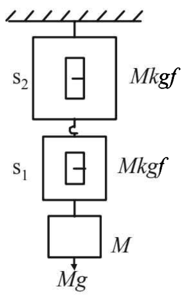
we get $P = (m + M)\, a \quad \therefore a = \frac{P}{m + M}$

Taking the block as a system, we get $T = Ma$

$$\therefore \quad T = \frac{MP}{m + M}$$

14. **(a)** The Earth pulls the block by a force Mg. The block in turn exerts a force Mg on the spring of spring balance S_1 which therefore shows a reading of M kgf.

The spring S_1 is massless. Therefore it exerts a force of Mg on the spring of spring balance S_2 which shows the reading of M kgf.

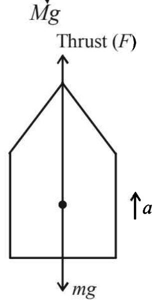


15. **(b)** As shown in the figure $F - mg = ma$

$\therefore \quad F = m\,(g + a)$

$= 3.5 \times 10^4\,(10 + 10)$

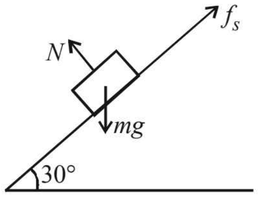
$= 7 \times 10^5$N



16. **(c)** Acceleration $a = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) g$

$$= \frac{(5-4.8)\times 9.8}{(5+4.8)} \text{m/s}^2 = 0.2 \text{ m/s}^2$$

17. **(c)**

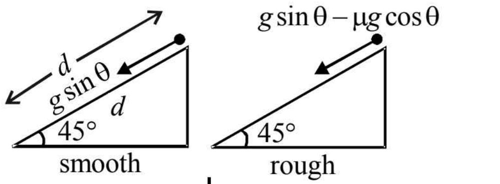


mg $\sin\theta = f_s$ (for body to be at rest)

$\Rightarrow m \times 10 \times \sin 30° = 10$

$\Rightarrow m \times 5 = 10 \Rightarrow m = 2.0$ kg

18. **(b)**



When surface is smooth	When surface is rough
$d = \frac{1}{2}(g\sin\theta)t_1^2$,	$d = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)t_2^2$
$t_1 = \sqrt{\frac{2d}{g\sin\theta}}$,	$t_2 = \sqrt{\frac{2d}{g\sin\theta - \mu g\cos\theta}}$

According to question, $t_2 = nt_1$

$$n\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{\frac{2d}{g\sin\theta - \mu g\cos\theta}}$$

$$n = \frac{1}{\sqrt{1-\mu_k}} \qquad \left(\because \cos 45° = \sin 45° = \frac{1}{\sqrt{2}}\right)$$

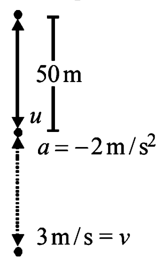
$$n^2 = \frac{1}{1-\mu_k} \text{ or } 1-\mu_k = \frac{1}{n^2} \text{ or } \mu_k = 1 - \frac{1}{n^2}$$

19. **(d)** The velocity of parachutist when parachute opens is

$u = \sqrt{2gh} = \sqrt{2\times 9.8 \times 50} = \sqrt{980}$

The velocity at ground, $v = 3$m/s

$\therefore S = \dfrac{v^2 - u^2}{2\times(-2)} = \dfrac{3^2 - 980}{-4} \approx 243$ m



Initially he has fallen 50 m.

$\therefore$ Total height from where he bailed out = 243 + 50 = 293 m

20. **(c)** Let K be the initial kinetic energy and F be the resistive force. Then according to work-energy theorem, $W = \Delta K$

i.e., $3F = \dfrac{1}{2}mv^2 - \dfrac{1}{2}m\left(\dfrac{v}{2}\right)^2$ ...(1)

For *B* to *C*:

$Fx = \dfrac{1}{2}m\left(\dfrac{v}{2}\right)^2 - \dfrac{1}{2}m(0)^2$ ... (2)

Dividing eqns. (1) and (2) we get $\dfrac{x}{3} = \dfrac{1}{3}$

or $x = 1$ cm

21. **(c)** Force experienced by the particle, $F = m\omega^2 R$

$\therefore \dfrac{F_1}{F_2} = \dfrac{R_1}{R_2}$

22. **(d)** According to work-energy theorem, $W = \Delta K = 0$

(Since initial and final speeds are zero)

$\therefore$ Workdone by friction + Work done by gravity = 0

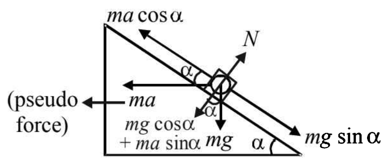
i.e., $-(\mu\, mg\cos\phi)\dfrac{\ell}{2} + mg\ell\sin\phi = 0$

or $\dfrac{\mu}{2}\cos\phi = \sin\phi$ or $\mu = 2\tan\phi$

23. **(c)** Mass $(m) = 0.3$ kg $\Rightarrow F = m.a = 15x$

$a = -\dfrac{15}{0.3}x = \dfrac{150}{3}x = 50x$ $a = 50 \times 0.2 = 10\,\text{m/s}^2$

24. **(c)** From diagram,



For block to remain stationary,

$mg\sin\alpha = ma\cos\alpha \Rightarrow a = g\tan\alpha$

25. **(a)** $v^2 - u^2 = 2as$ or $0^2 - u^2 = 2(-\mu_k g)s$

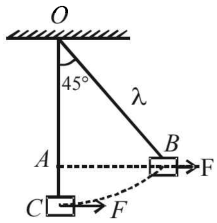
$-100^2 = 2 \times -\dfrac{1}{2} \times 10 \times s \Rightarrow$ s = 1000 m

26. **(d)** Work done by tension + Work done by force (applied) + Work done by gravitational force = change in kinetic energy

Work done by tension is zero

$\Rightarrow 0 + F \times AB - Mg \times AC = 0$

$\Rightarrow \text{F} = \text{Mg}\left(\dfrac{\text{AC}}{\text{AB}}\right) = \text{Mg}\left[\dfrac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right]$



$[\because AB = \ell\sin 45° = \dfrac{\ell}{\sqrt{2}}$

and $AC = OC - OA = \ell - \ell\cos 45° = \ell\left(1 - \dfrac{1}{\sqrt{2}}\right)$

where ℓ = length of the string.]

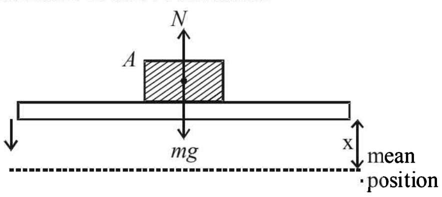
$\Rightarrow F = Mg(\sqrt{2} - 1)$

27. **(d)** $W_{hand} + W_{gravity} = \Delta K$

$\Rightarrow F(0.2) - (0.2)(10)(2.2) = 0 \Rightarrow F = 22\ N$

28. **(c)** $F = \dfrac{m(v-u)}{t} = \dfrac{0.15(0-20)}{0.1} = 30\,N$

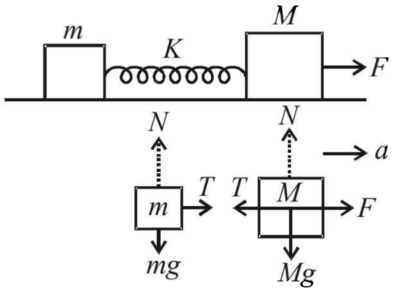
29. **(b)** For block A to move in SHM.



$mg - N = m\omega^2 x$
where x is the distance from mean position
For block to leave contact $N = 0$

$$\Rightarrow mg = m\omega^2 x \Rightarrow x = \frac{g}{\omega^2}$$

30. (d) Drawing free body-diagrams for m & M,



we get $T = ma$ and $F - T = Ma$
where T is force due to spring
$\Rightarrow F - ma = Ma$ or, $F = Ma + ma$

$$\therefore \quad a = \frac{F}{M+m}.$$

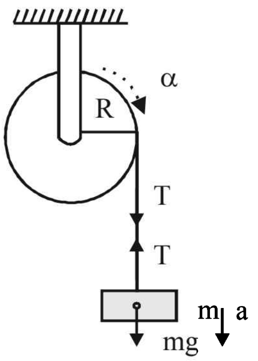
Now, force acting on the block of mass m is

$$ma = m\left(\frac{F}{M+m}\right) = \frac{mF}{m+M}.$$

31. (a) $mg \sin\theta = ma \quad \therefore a = g \sin\theta$
where a is along the inclined plane
$\therefore$ vertical component of acceleration is $g \sin^2\theta$
$\therefore$ relative vertical acceleration of A with respect to B is

$g(\sin^2 60 - \sin^2 30] = \frac{g}{2} = 4.9$ m/s² in vertical direction

32. (b) From figure,
Acceleration $a = R\alpha$ ...(i)
and $mg - T = ma$ ...(ii)
From equation (i) and (ii)



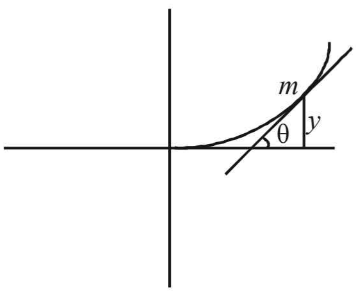
$$T \times R = mR^2\alpha = mR^2\left(\frac{a}{R}\right)$$

or $T = ma$
$\Rightarrow \quad mg - ma = ma$

$$\Rightarrow \quad a = \frac{g}{2}$$

33. (a) At limiting equilibrium, $\mu = \tan\theta$

$$\tan\theta = \mu = \frac{dy}{dx} = \frac{x^2}{2} \text{ (from question)}$$



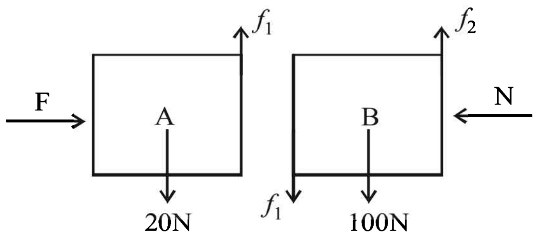
$\because$ Coefficient of friction $\mu = 0.5$

$$\therefore \quad 0.5 = \frac{x^2}{2}$$

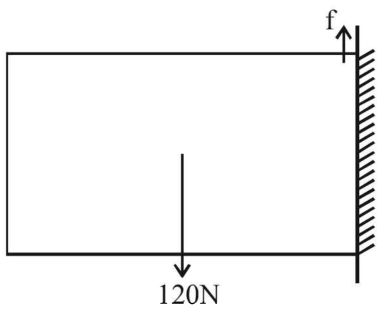
$\Rightarrow \quad x = \pm 1$

$$\text{Now, } y = \frac{x^3}{6} = \frac{1}{6} m$$

34. (a)



Assuming both the blocks are stationary
$N = F$
$f_1 = 20\text{ N}$
$f_2 = 100 + 20 = 120\text{N}$



Considering the two blocks as one system and due to equilibrium f = 120N

35. (a) Loss in P.E. = Work done against friction from $p \to Q$ + work done against friction from $Q \to R$
$mgh = \mu(mg\cos\theta)\,PQ + \mu mg\,(QR)$
$h = \mu \cos\theta \times PQ + \mu(QR)$

$$2 = \mu \times \frac{\sqrt{3}}{2} \times \frac{2}{\sin 30°} + \mu x$$

$2 = 2\sqrt{3}\,\mu + \mu x$ ---(i)

$$[\sin 30° = \frac{2}{PQ}]$$

Also work done $P \to Q$ = work done $Q \to R$

$\therefore 2\sqrt{3}\,\mu = \mu x$

$\therefore x \approx 3.5\text{m}$

From (i) $2 = 2\sqrt{3}\,\mu + 2\sqrt{3}\,\mu = 4\sqrt{3}\,\mu$

$$\mu = \frac{2}{4\sqrt{3}} = \frac{1}{2 \times 1.732} = 0.29$$