Matter Wave

1 Mark Questions

1.Write the expression for the de-Broglie wavelength associated with a charged particle having charge q and mass m, when it is accelerated by a potential [All India 2014]
Ans.

A charged particle having charge q and mass m, then
kinetic energy of the particle is equal to the work done on it by the electric field.

i.e. $K = qV$

$$\Rightarrow \quad \frac{1}{2}mv^2 = qV$$

$$\Rightarrow \quad \frac{p^2}{2m} = qV$$

$$\Rightarrow \quad p = \sqrt{2mqV} \qquad \textbf{(1)}$$

2.State de-Broglie hypothesis. [Delhi 2012]

Ans.
de-Broglie hypothesis A Moving object sometimes acts as a wave and sometimes as a particle or a wave is associated with the moving particle which control this particle in every respect. This wave associated with the moving particle is called **matter wave** or de-Broglie wave. Its wavelength is given by

$$\nu = \frac{h}{mv}$$

where, $h =$ Planck constant, $m =$ mass of object, $v =$ velocity of the object. **(1)**

3.A proton and an electron have same kinetic energy. Which one has greater de-Broglie wavelength and why? [All India 2012]

Ans.

de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

where, K = KE

For given KE,

$$\Rightarrow \qquad \lambda \propto \frac{1}{\sqrt{m}}$$

Electron have smaller mass, $\lambda_e > \lambda_p$

$[\because m_e < m_p]$

For given kinetic energy, electrons have greater wavelength as these have smaller mass. **(1)**

4.Write the relationship of de-Broglie wavelength X associated with a particle of mass m in terms of its kinetic energy E . [Delhi 2011c]

Ans.

Kinetic energy, $K = \frac{p^2}{2m}$

p = momentum

m = mass

and K = kinetic energy

$$\Rightarrow \qquad p = \sqrt{2mK}$$

de-Broglie wavelength, $\lambda = \frac{h}{p}$

where, $p = \sqrt{2mK}$

$$\Rightarrow \qquad \lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2\,mE}} \quad [K = E]$$ **(1)**

5.Show graphically, the variation of de-Broglie wavelength (A,) with the potential (V) through which an electron is accelerated from rest. [Delhi 2011]

Ans.

> To plot the graph between the two quantities, first of all we have to find the relation between the two through the connecting formula.

Kinetic energy, $K = eV$

where, V = potential difference

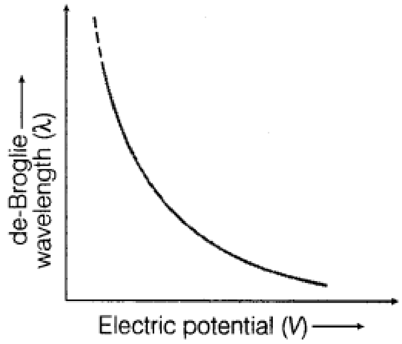
$\Rightarrow \quad p = \sqrt{2mK} = \sqrt{2meV}$

$\therefore$ de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

$$\lambda \propto \frac{1}{\sqrt{V}}$$

$\Rightarrow \quad V^2\lambda$ = constant



(1)

6.Name an experiment which shows wave nature of the electron. Which phenomenon was observed in this experiment using an electron beam? [Foreign 2010]

Ans.

Davisson-Germer experiment shows wave nature of electron.

The phenomenon of constructive interference was observed in Davisson-Germer experiment. **(1)**

7.An electron and a-particle have the same kinetic energy. How are the de-Broglie wavelengths associated with them related? [Delhi 2008]

Ans.

de-Broglie wavelength,

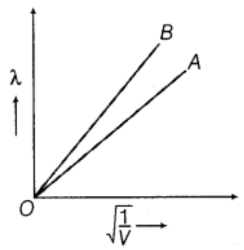
$$\lambda = \frac{h}{\sqrt{2mK}} \qquad (\because \; p = \sqrt{2mK})$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda_e}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_e}}$$

(1)

8. Two lines A and B in the given shows the variation of de-Broglie wavelength λ *versus* $\frac{1}{\sqrt{V}}$, where V is the accelerating potential difference for two particles carrying the same charge. Which one of the two represents a particle of smaller mass?



[All India 2008]

Ans.

As, $\lambda = \frac{h}{\sqrt{2mqV}}$

The slope of λ *versus* $\frac{1}{\sqrt{V}}$ graph will be inversely proportional to the square root of the mass of the particles.

Now, slope of B is greater it represents that mass is smaller. **(1)**

2 Marks Questions

9. A proton and a deuteron are accelerated through the same accelerating potential. Which one of the two has

- greater value of de-Broglie wavelength associated with it and
- less momentum? Give reasons to justify your answer. [Delhi 2014]

Ans.

(i) de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mV_0q}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

[$\because$ V_0 and q are same, because proton and deuteron have been accelerated by same potential and have same charge].

Since, mass of proton is more as compared to a deuteron. So, it will have lesser value of de-Broglie wavelength associated with it. **(1)**

(ii) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$$

As, $\lambda_d > \lambda_p$

So, $p_d < p_p$ **(1)**

10.A deuteron and an a-particle are accelerated with the same accelerating potential. Which one of the two has

- greater value of de-Broglie wavelength, associated with it and
- less kinetic energy? Explain. [Delhi 2014]

Ans.

(i) de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mV_0q}} \Rightarrow \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\frac{\lambda_d}{\lambda_a} = \frac{\frac{1}{\sqrt{2me}}}{\frac{1}{\sqrt{4m2e}}} = \frac{2}{1}$$ **(1)**

wavelength of deuteron is two times the wavelength of α-particle.

(ii) $\frac{KE_d}{KE_a} = \frac{V_0e}{V_02e} = \frac{1}{2}$

KE of deuteron is half of KE of α-particle.

11.X-rays fall on a photosensitive surface to cause photoelectric emission. Assuming that the work-function of the surface can be neglected, find the relation between the de-Broglie wavelength (A,) of the electrons emitted to the energy (E_v) of the incident photons. Draw of the graph for A, as function of E_v. [Delhi 2014 C]

Ans.

From Einstein and photoelectric equation

$$K_{max} = h\nu - \phi_0$$

$$E = \phi_0 + K_{max} \qquad [\because E = h\nu]$$

According to the question, $\phi_0 = 0$

So, $E = K_{max}$

$$E = \frac{P^2}{2m}, \quad \left[\because K_{max} = \frac{P^2}{2m}\right]$$

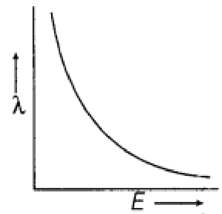
$$p = \sqrt{2mE} \qquad \text{... (i)}$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{p}$$

Substituting the value of p from Eq. (i), we get

$$\lambda = \frac{h}{\sqrt{2mE}}$$



(2)

12.An electron is revolving around the nucleus with a constant speed of 2.2 xlO^8 m/s. Find the de-Broglie wavelength associated with it. [All India 2014 C]

Ans.

Given, $v = 2.2 \times 10^8$ m/s

de-Broglie wavelength is given by

$$\lambda = \frac{h}{mv} \qquad \text{...(i)}$$

Here, $m = 9.1 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ kg-m^2-s

Substituting all values in Eq. (i), we get

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-3.1} \times 2.2 \times 10^8}$$

$$\lambda = 3.31 \times 10^{-12} \text{ m}$$

(2)

13.An a-particle and a proton are accelerated from rest by the same potential. Find the ratio of their de-Broglie wavelengths. [All India 2010; Foreign 2008]

Ans.

$\because$ de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}} \qquad (\because K = qV)$$

Here, potential is kept constant. **(1)**

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}}$$

$$= \sqrt{\left(\frac{m_p}{m_\alpha}\right)\left(\frac{q_p}{q_\alpha}\right)} = \sqrt{\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2\sqrt{2}} \Rightarrow \lambda_\alpha : \lambda_p = 1 : 2\sqrt{2}$$ **(1)**

14.An electron is accelerated through a potential difference of 100 V. What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond? [Delhi 2010]

Ans.

Given, $V = 100$ V. Wavelength of accelerated electron beam from de-Broglie equation

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

For $V = 100$ V

$$\lambda = 1.227 \text{ Å}$$ (1)

This wavelength belongs to the X-ray part of electromagnetic radiation. (1)

15.An electron is accelerated through a potential difference of 144 V. What is the de-Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond? [Delhi 2010]

Ans.

Refer to ans. 14.

$$\lambda = 1 \text{ Å}$$ **(2)**

This wavelength belongs to X-ray part of electromagnetic spectrum.

16.An electron is accelerated through a potential difference of 64 V What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond? [Delhi 2010]

Ans. Refer to ans 14

17.Find the ratio of de-Broglie wavelengths associated with

- protons, accelerated through a potential of 128 V and
- a-particles, accelerated through a potential of 64 V. [HOTS; Delhi 2010C]

Ans.

To calculate the ratio of de-Broglie wavelengths of two particles, find the ratio in terms of symbols and then put the given numerical values.

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}} \quad (\because K = qV)$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{mqV}}$$

where, m = mass of charged particle,

q = charge, V = potential difference

$\therefore$ Ratio of de-Broglie wavelengths of proton and α-particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)} \quad \textbf{(1)}$$

Here, $\frac{m_\alpha}{m_p} = 4, \frac{q_\alpha}{q_p} = 2$

$$\frac{V_\alpha}{V_p} = \frac{64}{128} = \frac{1}{2}$$

($\because \alpha$ - particle is 4 times heavier than proton and it has double the charge than that of proton)

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{4 \times 2 \times \frac{1}{2}} = 2$$

$$\lambda_p : \lambda_\alpha = 2 : 1 \quad \textbf{(1)}$$

18.The ratio between the de-Broglie wavelengths associated with protons, accelerated through a potential of 512 V and a-particles, accelerated through a potential of X volt is found to be one. Find the value of X. [Delhi 2010c]

Ans.

de-Broglie wavelength of accelerated charged particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{mqV}}$$

Ratio of wavelengths of proton and α-particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)} \quad \textbf{(1)}$$

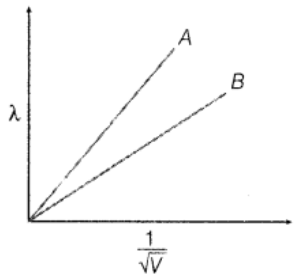
Here, $\frac{m_\alpha}{m_p} = 4, \quad \frac{q_\alpha}{q_p} = 2$

$$\frac{V_\alpha}{V_p} = \frac{X}{512}, \quad \frac{\lambda_p}{\lambda_\alpha} = 1$$

$$\Rightarrow \quad 1 = \sqrt{4 \times 2 \times \left(\frac{X}{512}\right)} = \frac{X}{64}$$

$$\Rightarrow \quad X = 64 \text{ V} \quad \textbf{(1)}$$

19. The two lines marked A and B in the given figure, show a plot of de-Broglie wavelength λ *versus* $\frac{1}{\sqrt{V}}$, where V is the accelerating potential for two nuclei $_1^2H$ and $_1^3H$.



(*i*) What does the slope of the lines represent?

(*ii*) Identify which of the lines corresponded to these nuclei.

[All India 2010]

Ans.

$\because$ de-Broglie wavelength of accelerated charged particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \quad \lambda\sqrt{V} = \frac{h}{\sqrt{2mq}} = \text{constant} \qquad \textbf{(1)}$$

(*i*) The slope of the line represents $\frac{h}{\sqrt{2mq}}$ where h = Planck's constant, q = charge and m = mass of charged particle.

(*ii*) $\because$ $_1H^2$ and $_1H^3$ carry same charge (as they have same atomic number)

$$\therefore \quad \lambda\sqrt{V} \propto \frac{1}{\sqrt{m}}$$

The lighter mass, i.e. $_1H^2$ is represented by line of greater slope, i.e. A and similarly $_1H^3$ by line B. **(1)**

20.Derive an expression for the de-Broglie wavelength associated with an electron accelerated through a potential V. Draw a schematic diagram of a localised wave describing the wave nature of the moving electron. [Foreign 2009]

Ans.

Let an electron beam is accelerated by potential difference V from the position of rest.

$\therefore$ Kinetic energy of the electron, $K = eV$

Momentum of electron, $p = \sqrt{2mK}$

$$p = \sqrt{2meV}$$

where, m = mass of an electron

∴ By de-Broglied equation

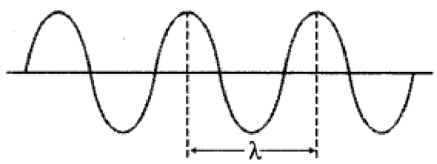
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

Here, $h = 6.63 \times 10^{-34}$ J-s

$e = 1.6 \times 10^{-19}$ C

$m = 9.1 \times 10^{-31}$ kg

$$\Rightarrow \quad \lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$ **(1)**



A matter wave associated with an electron of definite momentum has single wavelength and extends all over space. **(1)**

21.Calculate the ratio of the accelerating potential required to accelerate a proton and an cx-article to have the same de-Broglie wavelength associated with them. [Delhi 2009C]

Ans.

∵ de-Broglie matter wave equation for accelerating charged particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

where, h = Planck's constant

m = mass of charged particle

q = charge of charged particle

V = potential difference

Ratio of wavelengths of proton and α-particle,

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)}$$ **(1)**

$$\because \frac{\lambda_p}{\lambda_\alpha} = 1, \frac{m_\alpha}{m_p} = 4,$$

$$\frac{q_\alpha}{q_p} = 2, \frac{V_\alpha}{V_p} = ?$$

$$1 = \sqrt{4 \times 2 \times \left(\frac{V_\alpha}{V_p}\right)}$$

$$\Rightarrow 1 = 8 \times \frac{V_\alpha}{V_p} \Rightarrow \frac{v_p}{v_\lambda} = 8$$

$$\Rightarrow V_p : V_\alpha = 8 : 1$$ **(1)**

22.Calculate the ratio of the accelerating potential required to accelerate a deuteron and an a-particle to have the same de-Broglie wavelength associated with them. (Given, mass of deuteron = 3.2 x 1CT^{27} kg) [Delhi 2009C]

Ans.

∵ de-Broglie matter wave equation for accelerating charged particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

where, h = Planck's constant

m = mass of charged particle

q = charge of charged particle

V = potential difference

Ratio of wavelengths of proton and α-particle,

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)} \qquad \textbf{(1)}$$

$$\because \frac{\lambda_p}{\lambda_\alpha} = 1, \frac{m_\alpha}{m_p} = 4,$$

$$\frac{q_\alpha}{q_p} = 2, \frac{V_\alpha}{V_p} = ?$$

$$1 = \sqrt{4 \times 2 \times \left(\frac{V_\alpha}{V_p}\right)}$$

$$\Rightarrow 1 = 8 \times \frac{V_\alpha}{V_p} \Rightarrow \frac{v_p}{v_\lambda} = 8$$

$$\Rightarrow V_p : V_\alpha = 8 : 1$$

(1)

23.Crystal diffraction experiments can be performed either by using electrons accelerated through appropriate voltage or by using X-rays. If the wavelength of these probes (electrons or X-rays) is 1 A, estimate which of the two has greater energy. [All India 2009]

Ans.

For an accelerated electron beam, the de-Broglie matter wave equation states that $\lambda = \frac{h}{\sqrt{2emV}} = \frac{h}{\sqrt{2mK}}$

$$\Rightarrow \quad K = \frac{h^2}{2m\lambda^2} \qquad \ldots(i)$$

For X-ray of same wavelength, $\lambda = 1\,\mathring{A}$.

$$E' = h\nu = \frac{hc}{\lambda} \qquad \ldots(ii)$$

$$\frac{K}{E'} = \frac{h^2}{2m\lambda^2} \Big/ \frac{hc}{\lambda}$$

$$\frac{K}{E'} = \frac{h^2}{2m\lambda^2} \times \frac{\lambda}{hc} = \frac{h}{2mc\lambda} \qquad \textbf{(1)}$$

where, $h = 6.6 \times 10^{-34}$ J-s, $c = 3 \times 10^8$ m/s

$m = 9.1 \times 10^{-31}$ kg

$\lambda = 1 \text{Å} = 1 \times 10^{-10}$ m

$$\frac{K}{E'} = \frac{6.6 \times 10^{-34}}{2 \times 9.1 \times 10^{-31} \times 3 \times 10^8 \times 1 \times 10^{-10}}$$

$$\frac{K}{E'} = 0.012 \Rightarrow \frac{K}{E'} = \frac{11}{911} < 1$$

$\Rightarrow K < E'$

$\Rightarrow$ Energy possess by X-ray is more than electron. **(1)**

24.In Davisson-Germer experiment, state the observations which led to

- show the wave nature of electrons and
- confirm the de-Broglie relation. [Delhi 2008]

Ans.

(*i*) The existence of peak corresponding to sharp diffraction maximum in the electron distribution at a voltage 54 V and scattering angle 50° led to show wave nature of electron. **(1)**

The existence of peak depicts the wave properties like interference.

(*ii*) As per Bragg's law for first order diffraction (maximum),

$2d \sin\theta = \lambda$

Here, $d = 0.914$ Å, $\theta = 65°$

$\Rightarrow \lambda = 1.65$ Å ...(i)

By de-Broglie equation

$$\lambda = \frac{h}{p} = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} \cong 1.65 \text{ Å} \quad \text{...(ii)}$$

$\Rightarrow$ Bragg's law confirms the de-Broglie equation. **(1)**

25.For what kinetic energy of a neutron will the associated de-Broglie wavelength be 1.32 x 10^{-10} m? [All India 2008]

Ans.

From de-Broglie matter wave equation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad (\because\ p = \sqrt{2mK})$$

$$\Rightarrow \quad K = \frac{h^2}{2m\lambda^2} \quad ...(i)$$

(1)

where, $m = 1.66 \times 10^{-27}$ kg

$\lambda = 1.32 \times 10^{-10}$ m

$h = 6.63 \times 10^{-34}$ J-s

$\therefore$

$$K = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.66 \times 10^{-27} \times (1.32 \times 10^{-10})^2}$$

$$K = 7.5 \times 10^{-21}\ \text{J}$$

(1)

26.An electron and a-particle have the same de-Broglie wavelength with them. How are their kinetic energies related to each other? [Delhi 2008]

Ans.

From de-Broglie matter wave equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \Rightarrow mK = \frac{h^2}{2\lambda^2} =$$

constant

(for given wavelength)

$$\Rightarrow \quad K \propto \frac{1}{m}$$

(1)

$$\Rightarrow \quad \frac{K_e}{K_\alpha} = \frac{m_\alpha}{m_e}$$

where, m_e and m_α are masses of electron and α-particles, respectively **(1)**

27.For what kinetic energy of a neutron, will the associated de-Broglie wavelength be 1.32 x $10^{-1°}$ m? Given that the mass of a neutron = 1.675 xlO $^{-27}$ kg. [All India 2008C]

Ans.

It is given that $\lambda = 1.32 \times 10^{-10}$ m

and $m_n = 1.675 \times 10^{-27}$ kg

$$\text{As,} \quad \lambda = \frac{h}{\sqrt{2m_n K}} \Rightarrow K = \frac{h^2}{2m_n\lambda^2}$$

$\therefore$ Kinetic energy of neutron

$$K = \frac{(6.63 \times 10^{-34})^2}{2 \times (1.675 \times 10^{-27}) \times (1.32 \times 10^{-10})^2}$$

$$= 7.53 \times 10^{-21}\ \text{J}$$

(2)

3 Marks Questions

28.An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. Taking other factors, such as numerical aperture, etc., to be same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light? [All india 2014]

Ans.

Given, $V = 50\text{ kV} = 50 \times 10^3\text{ V}$

$\therefore$ de-Broglie wavelength,

$$\lambda = \frac{12.27}{\sqrt{V}}\text{ Å} = \frac{12.27}{\sqrt{50 \times 10^3}}\text{ Å} = 0.0526\text{ Å}$$

Resolving power of a microscope

$$R = \frac{2\mu \sin\theta}{\lambda}$$

From the formula, it is clear that if other factors remains same, then resolving power is inversely proportional to wavelength of the radiation used. The wavelength of moving electron is very small as compared to that of yellow light; so it has greater resolving power than optical microscope.

29.(i) Describe briefly how the Davission-Germer experiment demonstrated the wave nature of electrons.

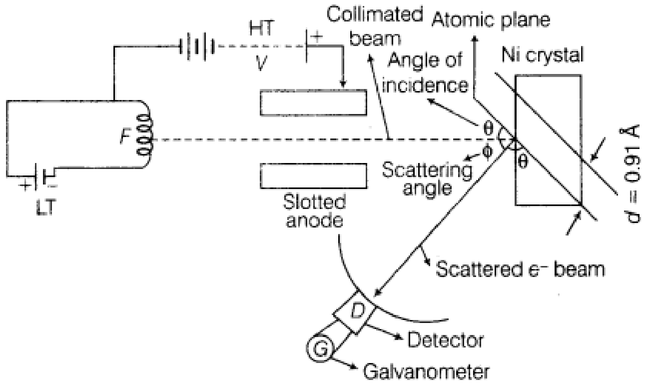
(ii) An electron is accelerated from rest through a potential V, obtain the expression for the de-Broglie wavelength associated with it.

Ans.

(*i*) The wave nature of electron was verified by Davisson-Germer experiment in 1927. The experimental arrangement is shown in the figure. It consists of an electron gun which comprises of a tungsten filament *F* coated with barium oxide and heated by a low voltage power supply. Electrons emitted by the filament are accelerated to a desired velocity by applying suitable potential from a high voltage power supply.

They are made to pass through a cylinder with free holes along its axis producing a fine collimated beam. The beam is made to fall on the surface of a nickel crystal. The electrons are scattered in all directions by the atoms of the crystal.

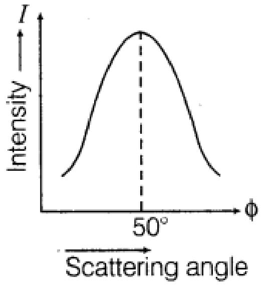
A beam of electrons emitted by the electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle.



The scattered beam of electrons is received by detector which can be rotated at any angle.

The energy of the incident beam of electrons varied by changing the applied voltage to the electron.

Intensity of scattered beam of electrons is found to maximum when angle of scattering is 50° and accelerating potential is 54 V.



Here, $\theta + 50° + \theta = 180°$, i.e. $\theta = 65°$

For Ni crystal, lattice spacing $(d) = 0.91$ Å

For first principle maximum, $n = 1$

Electron's diffraction is similar to X-ray diffraction.

According to Bragg's equation,

$$2d \sin\theta = n\lambda \quad \text{gives} \quad \lambda = 1.65 \text{ Å}$$

According to de-Broglie hypothesis,

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å} = \frac{12.27}{\sqrt{54}}$$ **(2)**

$\therefore$ de-Broglie wavelength of moving electron at $V = 54$ is $1.67 Å$ which is in close agreement with 1.675 Å

This proves the existence of de-Broglie waves for slow moving electrons.

(*ii*) Energy of electron, $E = eV$

Momentum is given by

$$p = \sqrt{2mE} = \sqrt{2meV} \qquad ...(i)$$

de-Broglie wavelength

$$\lambda = \frac{h}{p} \qquad ...(ii)$$

From Eq. (i) substituting the value of p in Eq. (ii)

$$\lambda = \frac{h}{\sqrt{2meV}}$$

30.Determine the de-Broglie wavelength of a proton whose kinetic energy is equal to the rest mass energy of an electron. Mass of a proton 1836 times that of electron, (ii) In which region of electromagnetic spectrum does this wavelength lie? [All India 2011C]

Ans.

> In these types of questions, we have to mind that in full-wave rectifier, full cycle of the input will be used.

(*i*) de-Broglie matter wave equation is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \qquad (\because p = \sqrt{2mK})$$

where, m = mass of proton

K = kinetic energy of proton.

According to the question, kinetic energy of proton, $K = m_e c^2$

(Einstein's mass-energy relation)

$$\Rightarrow \lambda = \frac{h}{\sqrt{2m(m_e c^2)}} \qquad \textbf{(1)}$$

$$\lambda = \frac{h}{\sqrt{2}\, c\sqrt{mm_e}} = \frac{h}{\sqrt{2}c \times (m_e)\sqrt{1836}}$$

$$(\because m = 1836\, m_e)$$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.414 \times (3 \times 10^8) \times 9.1 \times 10^{-31} \times 42.8}$$

$$\lambda = 4 \times 10^{-14} \text{ m} \qquad \textbf{(1)}$$

(*ii*) This region of electromagnetic spectrum is X-ray. **(1)**

31.The mass of a particle moving with velocity 5 x 10^6 m/s has de-Broglie wavelength associated with it to be 0.135 nm. Calculate its mass.

(ii) In which region of the electromagnetic spectrum does this wavelength lie? [All India 2011C]

Ans.

(i) From de-Broglie matter wave equation,

$$\lambda = \frac{h}{mv} \Rightarrow m = \frac{h}{\lambda v} \quad \textbf{(1)}$$

Here, $\lambda = 0.135 \times 10^{-9}$ m

$v = 5 \times 10^{6}$ m/s

$$\therefore \quad m = \frac{6.63 \times 10^{-34}}{0.135 \times 10^{-9} \times 5 \times 10^{6}}$$

$$= 9.82 \times 10^{-31} \text{ kg} \quad \textbf{(1)}$$

(ii) This wavelength 0.135 nm falls in the region of X-ray of electromagnetic spectrum. **(1)**

32. (i) A particle is moving three times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}.
Calculate the particle's mass and identify the particle.

(ii) An electron and a proton have the same kinetic energy. Which of the two will have larger de-Broglie wavelength? Give reason. **[All India 2011C]**

Ans.

Given, $v_{\text{particle}} = 3\, v_{\text{electron}}$ ...(i)

and $\lambda_{\text{particle}} = 1.813 \times 10^{-4}\, \lambda_{\text{electron}}$

(i) As, $\lambda = \frac{h}{mv}$ (de-Broglie equation)

$$\Rightarrow \frac{m_{\text{particle}}}{m_{\text{electron}}} = \frac{\lambda_{\text{electron}} \times v_{\text{electron}}}{\lambda_{\text{particle}} \times v_{\text{particle}}}$$

$\therefore \quad m_{\text{particle}} = 1839\, m_{\text{electron}}$ [From Eq. (i)]

$m_{\text{particle}} = 1839 \times 9.1 \times 10^{-31}$

$= 1.673 \times 10^{-27}$ kg **(2)**

Particle is either a proton or a neutron.

(ii) Now, $\lambda = \frac{h}{\sqrt{2mK}}$

Therefore, it is clear that KE of electron is more than that of proton or neutron. **(1)**

33.An electron and a photon each have a wavelength lnm. Find [Ail India 2011C]

- their momenta
- the energy of the photon and
- the kinetic energy of electron.

Ans.

(*i*) For electron or photon, momentum

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-9}} = 6.63 \times 10^{-25}\ \text{m}$$

(*ii*) $E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^{8})}{10^{-9} \times (1.6 \times 10^{-19})}$

$$= 1243\ \text{eV}$$

(*iii*) As, $E = \frac{p^2}{2m} = 2.9 \times 10^{-31} \times (1.6 \times 10^{-19})$

$$= 1.52\ \text{eV}$$

34.A proton and an a-particle are accelerated through the same potential. Which of the two has '

- greater value of de-Broglie wavelength associated with it and
- less kinetic energy? Justify your answer. [Delhi 2009]

Ans.

From de-Broglie matter wave equation,

$$\lambda = \frac{h}{p}$$

But, $p = \sqrt{2mK}$ and $K = qV$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

where, m = mass of charged particle

q = charge

V = potential difference

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{mq}}$$

(for same accelerating voltage) **(1/2)**

(*i*) Ratio of wavelengths of proton and α-particle.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)} \quad \textbf{(1/2)}$$

But, $\frac{m_\alpha}{m_p} = 4, \quad \frac{q_\alpha}{q_p} = 2$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{(4) \times 2} = 2\sqrt{2}$$

$$\Rightarrow \lambda_p : \lambda_\alpha = 2\sqrt{2} : 1$$

Proton have greater de-Broglie wavelength associated with it. **(1)**

(*ii*) $\because$ Kinetic energy, $K = qV$

$$\Rightarrow \quad \frac{K_p}{K_\alpha} = \left(\frac{q_p}{q_\alpha}\right)$$

(for same accelerating voltage)

$$\frac{K_p}{K_\alpha} = \frac{1}{2} \quad \Rightarrow \quad K_p = \frac{1}{2} K_\alpha$$

Proton have less KE. (1)

35.An electron and a proton are accelerated through the same potential. Which one of the two has

- greater value of de-Broglie wavelength associated with it and
- less momentum? Justify your answer. [Delhi 2009]

Ans.

(*i*) From de-Broglie equation

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

As, $\quad p = \sqrt{2mK}$

and $\quad K = qV$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mqV}} \quad \ldots\text{(i)}$$

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

Ratio of wavelengths of electron and proton

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\left(\frac{m_p}{m_e}\right)\left(\frac{q_p}{q_e}\right)}$$

$\because$ Ratio of mass of proton and electron

$$\frac{m_p}{m_e} = 1836 \quad \text{(constant)}$$

$$\frac{q_p}{q_e} = 1$$

(Both electron and proton have same charge)

$$\Rightarrow \quad \frac{\lambda_e}{\lambda_p} = \sqrt{1836 \times 1}$$

$$\lambda_e \approx 42.8\lambda_p \text{ nearly}$$

Electron have greater wavelength associated with it than that of proton. $\left(1\frac{1}{2}\right)$

(*ii*) $\because \quad \lambda = \frac{h}{p}$ (de-Broglie equation)

$$\Rightarrow \quad p = \frac{h}{\lambda}$$

$$\Rightarrow \quad p \propto \frac{1}{\lambda} \Rightarrow \frac{p_e}{p_p} = \frac{\lambda_p}{\lambda_e}$$

But from Eq. (i), we get

$$\frac{\lambda_p}{\lambda_e} = \frac{1}{42.8}$$

$$\Rightarrow \quad \frac{p_e}{p_p} = \frac{\lambda_p}{\lambda_e} \doteq \frac{1}{42.8}$$

Momentum of proton is nearly 42.8 times to that of momentum of electron. (1)