Chapter 4

MISCELLANEOUS EQUATIONS & INEQUATIONS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. The set of all values of the parameter k for which the inequatity $\left|\frac{x^2+kx+1}{x^2+x+1}\right|<3$ is satisfied for all real values of x is

(a) $-1<k<5$ (b) $-11<k<-1$
(c) $5<k<7$ (d) $k \in \boldsymbol{R}$

2. Number of real roots of the equation

$\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$ is

(a) 0 (b) 1
(c) 2 (d) 3

3. The solution set of $\left|\frac{x+1}{x}\right|+|x+1|=\frac{(x+1)^2}{|x|}$ is

(a) $\{x \mid x \ge 0\}$ (b) $\{x \mid x>0\} \cup \{-1\}$
(c) $\{-1, 1\}$ (d) $\{x \mid x \ge 1 \text{ or } x \le -1\}$

4. The system of equations $|x-1|+3y=4$, $x-|y-1|=2$ has

(a) No solution
(b) A unique solution
(c) Two solutions
(d) More than two solutions

5. The solution set of the inequatity

$|9^x-3^{x+1}-15|<2.9^x-3^x$ is

(a) $(-\infty, 1)$ (b) $(1, \infty)$
(c) $(-\infty, 1]$ (d) $(-\log_3 2, \infty)$

6. The solution set that satisfy the equation

$\left|\frac{x^2-8x+12}{x^2-10x+21}\right|=-\frac{x^2-8x+12}{x^2-10x+21}$ is

(a) $(-\infty, 2]$ (b) $[2,3) \cup [6,7)$
(c) $[6,7)$ (d) $\{6,7\}$

7. The set of all values of x satisfying the equation $x^2.2^{x+1}+2^{|x-3|+2}=x^2.\,2^{|x-3|+4}+2^{x-1}$ is

(a) $[3, \infty)$ (b) $\left\{-\frac{1}{2}, \frac{1}{2}\right\} \cup [3, \infty)$
(c) $\left(-\infty, -\frac{1}{2}\right)$ (d) None of these

8. If $(y^2-5y+3)(x^2+x+1)<2x$ for all $x \in \boldsymbol{R}$, then y lies in the interval

(a) $\left(\frac{5-\sqrt{13}}{2}, \frac{5+\sqrt{13}}{2}\right)$ (b) $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$
(c) $\left(\frac{5-\sqrt{13}}{2}, \frac{5+\sqrt{5}}{2}\right)$ (d) $\left[-\frac{2}{3}, \frac{2}{3}\right]$

9. The number of positive integers satisfying the equation $x+\log_{10}(2^x+1)=x\log_{10}5+\log_{10}6$ is

(a) 0 (b) 1
(c) 2 (d) infinite

10. The number of solutions of the equation

$4^x-3^{x-\frac{1}{2}}=3^{x+\frac{1}{2}}-2^{2x-1}, x \in \boldsymbol{R}$ is

(a) 0 (b) 1
(c) 2 (d) None of these

11. The values of a for which the equation $2(\log_3 x)^2-|\log_3 x|+a=0$ possess four real solutions satisfy

(a) $-2<a<0$ (b) $0<a<\frac{1}{8}$
(c) $0<a<5$ (d) None of these.

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ				

12. The number of ordered pairs (x, y) satisfying the system of equations :

$$6^x \left(\frac{2}{3}\right)^y - 3.2^{x+y} - 8.3^{x-y} + 24 = 0; xy = 2 \text{ is}$$

(a) 0 (b) 1
(c) 4 (d) 3

13. The number of ordered pairs (x, y) satisfying $3^x.5^y = 75$ and $3^y.5^x = 45$ is

(a) 0 (b) 1
(c) 3 (d) None of these

14. The number of real values of parameter k for which $(\log_{16} x)^2 - \log_{16} x + \log_{16} k = 0$ will have exactly one solution is

(a) 0 (b) 2
(c) 1 (d) 4

15. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(a) $(2, \infty)$ (b) $(1, 2)$
(c) $(-2, -1)$ (d) None of these

16. The equation $|x+1|^{\log_{x+1}(3+2x-x^2)} = (x-3)|x|$ has

(a) Unique Solution
(b) Two solutions
(c) No Solution
(d) More than two solutions

17. The number of real solutions of the equation

$$1 + |e^x - 1| = e^x (e^x - 2) \text{ is}$$

(a) 0 (b) 1
(c) 2 (d) infinitely many

18. If x and y are integers and $(x-8)(x-10) = 2^y$, then the number of solution of the pair (x, y) is

(a) 1 (b) 2
(c) 0 (d) 4

19. If x is a positive real number, then $\left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right] =$

(a) $\left[x + \frac{1}{2}\right]$ (b) $[x]$
(c) $\left[x + \frac{1}{4}\right]$ (d) $\left[2x + \frac{1}{4}\right]$

20. If $\log_{\frac{1}{2}} \frac{x^2+6x+9}{2(x+1)} < -\log_2(x+1)$, then x lies in the interval

(a) $(-1, -1+2\sqrt{2})$ (b) $(1-2\sqrt{2}, 2)$
(c) $(-1, \infty)$ (d) None of these

21. The least integer a, for which $1 + \log_5(x^2+1) \le \log_5(ax^2+4x+a)$ is true for all $x \in \boldsymbol{R}$ is

(a) 6 (b) 7
(c) 10 (d) 1

22. If 1 lies between the roots of equation $y^2 - my + 1 = 0$ and $[x]$ denotes greatest integer $\le x$ then $\left[\left(\frac{4[x]}{|x|^2+16}\right)^m\right]$ is equal to

(a) 0 (b) 1
(c) 2 (d) None of these

23. The solution set of the equation $4\{x\} = x + [x]$, where $\{x\}$ and $[x]$ denote the fractional and integral parts of a real number x respectively, is

(a) $\{0\}$ (b) $\left\{0, \frac{5}{3}\right\}$
(c) $[0, \infty)$ (d) None of these

24. Let **R** = the set of real numbers, **I** = the set of integers, **N** = the set of natural numbers. If S be the solution set of the equation $(x)^2 + [x]^2 = (x+1)^2 + [x+1]^2$, where (x) = the least integer greater than or equal to x and $[x]$ = the greatest integer less than or equal to x, then

(a) $S = \mathbf{R}$ (b) $S = \mathbf{R} - \mathbf{I}$
(c) $S = \mathbf{R} - \mathbf{N}$ (d) None of these.

25. Let $F(x)$ be a function defined by $F(x) = x - [x]$, $0 \ne x \in \mathbf{R}$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of $F(x) + F\left(\frac{1}{x}\right) = 1$ is

(a) 0 (b) infinite
(c) 1 (d) 2

26. If $[x]^2 = [x+2]$, where $[x]$ = the greatest integer less than or equal to x, then x must be such that

(a) $x = 2, -1$ (b) $x \in \{2, 3\}$
(c) $x \in [-1, 0) \cup [2, 3)$ (d) None of these

MARK YOUR RESPONSE					
	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ
	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ
	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ

27. If $5\{x\} = x + [x]$ and $[x] - \{x\} = \frac{1}{2}$ when $\{x\}$ and $[x]$ are fractional and integral part of x then x is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{7}{2}$

28. The solution of $x - 1 = (x - [x])(x - \{x\})$ (where $[x]$ and $\{x\}$ are the integral and fractional part of x) is

(a) $x \in \mathbf{R}$ (b) $x \in \mathbf{R} \sim [1, 2)$

(c) $x \in [1, 2)$ (d) $x \in \mathbf{R} \sim [1, 2]$

29. The solution set of $(x)^2 + (x + 1)^2 = 25$, where (x) is the least integer greater than or equal to x, is

(a) $(2, 4)$ (b) $(-5, -4] \cup (2, 3]$

(c) $[-4, -3) \cup [3, 4)$ (d) None of these

30. For $x \in \mathbf{R}$, $\|x\|$ is defined as follows

$: \|x\| = \begin{cases} x+1, & 0 \le x < 2 \\ |x-4|, & x \ge 2 \end{cases}.$

Then the solution set of the equation $\|x\|^2 + x = \|x\| + x^2$ is

(a) $\{-1, 1\}$ (b) $[2, \infty)$

(c) $[0, 2)$ (d) $\{0, 2\}$

31. If $f(x) = x^3 + 3x^2 + 6x + 2\sin x$, then the equation

$\frac{1}{x - f(1)} + \frac{2}{x - f(2)} + \frac{3}{x - f(3)} = 0$ has

(a) No real roots (b) 1 real root

(c) 2 real roots (d) More than 2 real roots

32. Solution set of the equation $|2^x - 1| + |4 - 2^x| < 3$ is

(a) ϕ (b) $(0, 2)$

(c) $(-\infty, \infty)$ (d) None of these

33. The values of 't' for which $\sin x(\sin x + \cos x) = [t]$, where [.] denotes greatest integral function, holds for all x, are

(a) $[0, 2)$ (b) $[0, 1] \cup [2, 3)$

(c) $[-1, 1) \cup [1, 2)$ (d) None of these

34. If $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, then

(a) both $g(x)$ and $h(x)$ are divisible by $(x - 1)$

(b) $h(x)$ is divisible but $g(x)$ is not divisible by $x - 1$

(c) $g(x)$ is divisible but $h(x)$ is not divisible by $x - 1$

(d) None of these

35. The equation $2^{|x^2 - 12|} = \sqrt{e^{|x| \log 4}}$ has

(a) no real solution

(b) only two real solutions whose sum in zero

(c) only two real solutions whose sum is non zero

(d) four real solutions whose sum is zero

36. Number of solution of the equation

$4^{\sin 2x + 2\cos^2 x} + 4^{1 - \sin 2x + 2\sin^2 x} = 65$ in $\left[0, \frac{\pi}{2}\right]$ is,

(a) zero (b) 1

(c) 2 (d) None of these

37. If $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive roots, then the minimum value of pr is equal to

(a) 5 (b) 25

(c) 80 (d) 100

38. In the rectangular cartesian plane, the equation of a curve is given by the equation $(x^2 + y^2)^2 = 4x^2y$. The exhaustive set of y-coordinates of the points on this curve is given by

(a) $[-1, 1]$ (b) $[0, \infty]$

(c) $[0, 1]$ (d) R

39. Let $f(x) = x^2 + bx + c$, where $b, c \in R$. If $f(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then the least value of $f(x)$ is

(a) 2 (b) 3

(c) $\frac{5}{2}$ (d) 4

40. If x, y, z are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then the exhaustive set of values of x is

(a) $\left[\frac{2}{3}, 2\right]$ (b) $\left[0, \frac{2}{3}\right]$

(c) $[0, 2]$ (d) $\left[-\frac{1}{3}, \frac{2}{3}\right]$

41. Let $f(x) = ax^3 + bx^2 + cx + d$, $a > 0$, $a, b, c, d \in R$ and $f(x) = 0$ has all roots of repeated nature.

If $g(x) = f'(x) - f''(x) + f'''(x)$ then $\forall\ x \in R$

(a) $g(x) > 0$ (b) $g(x) \ge 0$

(c) $g(x) < 0$ (d) $g(x) \le 0$

MARK YOUR RESPONSE					
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ
	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ
	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ

42. The set of values of 'a' for which the equation $x^3 - 3x + a = 0$ has three distinct real roots, is

(a) $(-\infty, \infty)$ (b) $(-2, 2)$

(c) $(-1, 1)$ (d) none of these

43. Let $p(x) = 0$ be a polynomial equation of least possible degree, with rational coefficients, having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then the product of all the roots of $p(x) = 0$ is

(a) 7 (b) 49

(c) 56 (d) 63

44. If all the solutions 'x' of $a^{\cos x} + a^{-\cos x} = 6$ $(a > 1)$ are real, then set of values of a is

(a) $[3 + 2\sqrt{2}, \infty]$ (b) $(6, 12)$

(c) $(1, 3 + 2\sqrt{2})$ (d) none of these.

45. The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is

(a) 0 (b) 1

(c) no least value (d) none of these

46. Let S be the set of values of a for which $(a-4)\sec^4 x + (a-3)\sec^2 x + 1 = 0$ has real solutions. Then S is

(a) R (b) $(-\infty, 3]$

(c) $(4, \infty)$ (d) $[3, 4)$

47. If $\frac{x^2}{3} - 4x + 13 = \sin\frac{a}{x}$, for some real x, then a is equal to

(a) $(2n+1)\frac{\pi}{2}$ (b) $3(4n+1)\frac{\pi}{2}$

(c) $3(1+4n)\pi$ (d) None of these

MARK YOUR RESPONSE					
	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ
	47. ⓐⓑⓒⓓ				

B COMPREHENSION TYPE

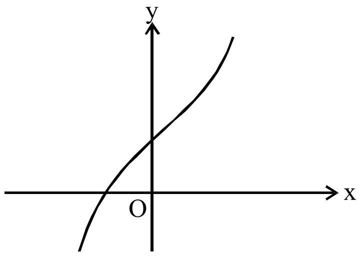
This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

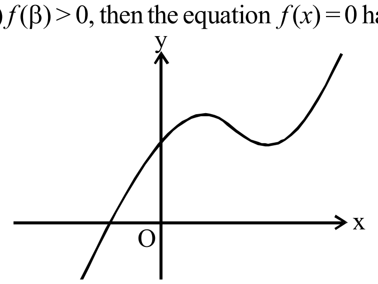
$f(x) = ax^3 + bx^3 + cx + d$

Let $f(x) = ax^3 + bx^2 + cx + d$ and $a > 0$, then $f'(x) = 3ax^3 + 2bx + c$

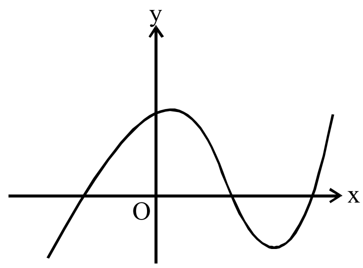
CASE (1) : The equation $f'(x) = 0$ has no real roots . Then $f(x)$ always increases as x increases and the equation $f(x) = 0$ has one real root.



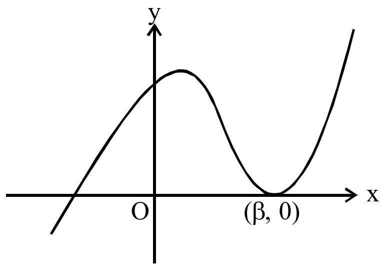
CASE (2) : The equation $f'(x) = 0$ has two distinct roots α and β $(\alpha < \beta)$. If $f(\alpha)f(\beta) > 0$, then the equation $f(x) = 0$ has one real root.



CASE (3) : The equation $f'(x) = 0$ has two distinct roots α and β and $f(\alpha)\, f(\beta) < 0$, then the equation $f(x) = 0$ has three real roots.



CASE (4) : The equation $f'(x) = 0$ has two distinct roots α and β but $f(\beta) = 0$ also. In this case $f(x) = 0$ has repeated root and one other root.



In general, the equation $g(x) = 0$ has a repeated root $x = \alpha$, if $g(\alpha) = g'(\alpha) = 0$.

Now answer the following questions:

1. The equation $8x^3 - 20x^2 + 6x + 9 = 0$ has
(a) one repeated root
(b) all distinct real roots
(c) all roots repeated
(d) only one real root which is not repeated

2. The equation $x^3 - 3x + 1 = 0$ has
(a) three distinct real root
(b) one repeated and one different root
(c) only one real root which is not repeated
(d) only one repeated root

3. If the roots of $x^3 - 3x + 1 = 0$ be α, β, γ then the value of $[\alpha]+[\beta]+[\gamma]$ is
(a) 0
(b) 1
(c) -1
(d) not defined as all roots are not real

PASSAGE-2

Let $ax^3 + bx^2 + cx + d = 0$ be a cubic equation having roots α, β and γ then sum of roots, i.e. $\alpha+\beta+\gamma = -\frac{b}{a}$

Sum of product of roots in pairs, i.e., $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$; Product of roots, i.e., $\alpha\beta\gamma = -\frac{d}{a}$

Consider a cubic equation $x^3 - x^2 + \beta x + \gamma = 0$, where β and γ are real.

4. If the roots of the equation are in *A.P.* then β lies in the interval
(a) $\left(-\infty, \frac{1}{3}\right]$ (b) $\left[-\frac{1}{27}, \infty\right)$
(c) $\left(\frac{1}{3}, \infty\right)$ (d) $(0, \infty)$

5. If the roots of the equation are in *A.P.* then γ lies in the interval
(a) $\left(-\infty, \frac{1}{3}\right]$ (b) $\left[-\frac{1}{27}, \infty\right)$
(c) $\left(-\infty, -\frac{1}{27}\right)$ (d) $(-\infty, 0)$

6. If $\beta+\gamma=0$, none being zero then the equation has all three roots real if
(a) $\beta > 0$ (b) $\beta < 0$
(c) $\gamma < 0$ (d) $\gamma < -1$

PASSAGE-3

Let $f(x) = a_0x^n + a_1x^{n-1} + ... + a_n; (a_0 \neq 0)$. If $f(a)$ and $f(b)$ are of opposite sign; (where $a < b$) i.e. $f(a)f(b) < 0$ then at least one or in general odd number of roots of the equation $f(x) = 0$ lie between a and b.

7. If $0 \le p \le 16$, then the equation $x^3 - 12x - p = 0$ has one root in
(a) $(2,3)$ (b) $(3,4)$
(c) $(4,5)$ (d) none of these

8. The equation $2\sin^2\theta\, x^2 - 3\sin\theta\, x + 1 = 0; \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ has one root lying in the interval.
(a) $(0,1)$ (b) $(1,2)$
(c) $(2,3)$ (d) $(-1,0)$

9. If $f(x) = ax^2 + bx + c$, such that $c < 0$ and $a - 2b + 4c > 0$, then $f(x)$ has
(a) one root in the interval $\left(0, \frac{1}{2}\right)$
(b) one root in the interval $\left(-\frac{1}{2}, 0\right)$
(c) both roots are positive
(d) none of these

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. **Statement-1** : If $1 \le a \le 2$ then $\sqrt{a+2\sqrt{a-1}}+\sqrt{a-2\sqrt{a-1}}=2$

Statement-2 : If $1 \le a \le 2$ then $\sqrt{a-2\sqrt{a-1}}=\sqrt{a-1}-1$

2. **Statement-1** : $\log_2 7$ = is an irrational number

Statement-2 : $\log_2 7=\dfrac{\log 7}{\log 2}$ and ratio of two irrational numbers can not be rational.

3. **Statement-1** : If one root is $\sqrt{3}-\sqrt{2}$ then the equation of the lowest degree with rational coefficients is $x^4-10x^2+1=0$

Statement-2 : For a polynomial equation with rational coefficient irrational roots occur in pairs.

4. **Statement-1** : 2 is a multiple root of order '2' of the equation $x^3-3x^2+4=0$

Statement-2 : If $f(x)=x^3-3x^2+4$ then $f''(2)=0$

5. **Statement-1** : The remainder obtained when the polynomial $1+x+x^3+x^9+x^{27}+x^{81}+x^{243}$ is divided by $(x-1)$ is 7

Statement-2 : If $f(a)=0$ then $x-a$ is a factor of $f(x)$

6. **Statement-1** : Number of triplets (x, y, z) forming the solution set of $x+y=2, xy-z^2=1$, $x, y, z \in R$ is infinite.

Statement-2 : Number of unknowns are more than the number of independent equations.

7. **Statement-1** : The least natural number 'a' for which $x+ax^{-2}>2\ \forall x\in(0,\infty)$ is 2.

Statement-2 : $a>\dfrac{32}{27}$

8. **Statement-1** : The number of pairs of positive integers (x, y) where x and y are prime numbers and $x^2-2y^2=1$ is 1.

Statement-2 : 2 is the only even prime number.

9. **Statement-1** : If $f(x)=ax^3+bx^2+cx+d$ and $f'(x)=0$ has a repeated root $x=\alpha$ (say), then the curve $y=f(x)$ has an inflexion with a tangent parallel to the x-axis at $x=\alpha$,

Statement-2 : $f'(x)=0$ has a repeated root $x=\alpha$, so the equation $f(x)=0$ has all three equal roots, each equal to α and hence $f(\alpha)=f'(\alpha)=f''(\alpha)=0$.

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. The equation $|x+1||x-1|=a^2-2a-3$ can have real solution in x if a belongs to

(a) $[1-\sqrt{5},-1]$ (b) $[-1,3]$
(c) $[3,1+\sqrt{5}]$ (d) $[1+\sqrt{3},1+\sqrt{5}]$

2. The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$ has

(a) at least one real root
(b) exactly three real solutions
(c) exactly one irrational solution
(d) exactly one integral solution

3. If $b^2>4ac$, then $a(x^2+4x+4)^2+b(x^2+4x+4)+c=0$ have all roots real and distinct if

(a) $b<a<0<c$ (b) $a<b<0<c$
(c) $b<0<a<c$ (d) $a<c<0<b$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

4. The real solution of simultaneous equations : $xy + 3y^2 - x + 4y - 7 = 0$ and $2xy + y^2 - 2x - 2y + 1 = 0$ is
(a) $x = 2, y = -3$
(b) $x \in \mathbf{R}, y = 1$
(c) $x = 1, y = 1$
(d) $x = -1, y = 1$

5. If $|k-1|^{\log_3 k^2 - 2\log_k 9} = (k-1)^7$, then which of the following is/are correct?
(a) $0 < \sin^{-1} k + \cos^{-1}(k-1) < \frac{\pi}{4}$
(b) $0 < \tan^{-1} k < \frac{\pi}{6}$
(c) $0 < \cot^{-1} k < \frac{\pi}{6}$
(d) $\frac{\pi}{4} < \tan^{-1} k < \frac{\pi}{2}$

6. The equation $||x-1| + a| = 4$ can have real solutions for x if a belongs to the interval
(a) $(-\infty, +\infty)$
(b) $(-\infty, -4]$
(c) $(4, +\infty)$
(d) $[-4, 4]$

7. The integral value(s) of a for which the equation
$$(x^2 + x + 2)^2 - (a-3)(x^2 + x + 2)(x^2 + x + 1) + (a-4)(x^2 + x + 1)^2 = 0$$
have at least one real root is/are
(a) 5
(b) 6
(c) 4
(d) None of these

8. Let x_1, x_2, x_3 are distinct positive numbers forming a G.P. and satisfy the cubic equation $x^3 - x^2 + \beta x + \gamma = 0$, then
(a) $\beta > \frac{1}{3}$
(b) $0 < \beta < \frac{1}{3}$
(c) $-\frac{1}{27} \le \gamma < 0$
(d) $-\frac{1}{27} < \gamma < 0$

9. If the equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ have four positive roots, then
(a) roots are necessarily integers
(b) $a + b = 2$
(c) $ab = -24$
(d) $ax^2 + bx - 2 = 0$ has rational roots

10. The polynomial equation $x^6 + 2x^3 + 5 + ax^3 + a = 0$ has
(a) no real root is $|a| < 4$
(b) all roots real if $|a| > 4$
(c) at most two real roots for all $a \in R$
(d) at least two non-real roots for all $a \in R$

11. If the equations $ax^3 + (a+b)x^2 + (b+c)x + c = 0$ and $2x^3 + x^2 + 2x - 5 = 0$ have a common root, then $a + b + c$ can be equal to $(a, b, c \in R, a \ne 0)$
(a) $5a$
(b) $3b$
(c) $2c$
(d) 0

12. If $x + ay + a^2z = a^3$, $x + by + b^2z = b^3$ and $x + cy + c^2z = c^3$ then
(a) $x = abc$
(b) $y = ab + bc + ca$
(c) $z = a + b + c$
(d) none of these

13. If the reciprocal of every root of $x^3 + x^2 + ax + b = 0$ is also a root then
(a) $a = b = 1$
(b) $a = b = -1$
(c) $a = 1, b = -1$
(d) $a = -1, b = 1$

14. If a, b, c be the sides of a triangle and the equation
$$x^2 + y^2 - 2x - 4y - 4 + (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 0$$
in variables x and y has real solution then
(a) the triangle is equilateral
(b) the triangle is of constant area
(c) exactly one ordered pair (x, y) of solution is possible
(d) the equation can never have real solution

15. The solutions of the equations $x^2 + y^2 - 8x - 8y = 20$ and $xy + 4x + 4y = 40$ satisfy the following equation(s).
(a) $x + y = 10$
(b) $|x + y| = 10$
(c) $|x - y| = 10$
(d) $x + y = -10$

16. Values of x for which $5^{\log_5(x^2 - 9x + 24)} > x - 1$ belong to
(a) $x \in \boldsymbol{R}$
(b) $x \in (0, \infty)$
(c) $x \in (-\infty, 0)$
(d) none of these

17. If the equation $x^4 + ax^3 - 13x^2 + bx - 4 = 0$ has one repeated root and one more root being $2 + \sqrt{5}$, then
(a) $a = 0, b = 10$
(b) $a = 0, b = -20$
(c) repeated root is – 2
(d) repeated root is 2

18. The values of x satisfying the inequality $|x^3 - 1| \ge 1 - x$ belong to
(a) $(-\infty, -1]$
(b) $[0, 1]$
(c) $[1, \infty)$
(d) $(-\infty, \infty)$

19. If S is the set of all real x such that $\frac{2x - 1}{2x^3 + 3x^2 + x}$ is positive, then S contains
(a) $\left(-\infty, -\frac{3}{2}\right)$
(b) $\left(\frac{1}{2}, 0\right)$
(c) $\left(\frac{1}{2}, 3\right)$
(d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

MARK YOUR RESPONSE					
	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ				

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

1. Observe the following Column :

Column-I		**Column-II**
(A) The positive integer x, for which $\dfrac{2x-1}{2x^3+3x^2+x}$ is positive can be equal to	p.	1
(B) If the quadratic equation $3x^2 + 2(a^2 + 1)x + (a^2 - 3a + 2) = 0$ possesses roots of opposite sign then $[a]$, where $[a]$ represents integral part of a can be equal to	q.	2
(C) The roots of the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ can be equal to	r.	5
(D) If the roots of the equation $x^4 - 8x^3 + bx^2 - cx + 16 = 0$ are all positive then one of the roots is	s.	8
	t.	10

2. Let α, β and γ are three real numbers such that $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$ and $\alpha^3 + \beta^3 + \gamma^3 = 8$. Now match the entries from the following two columns :

Observe the following Column :

Column-I		**Column-II**
(A) The value of $\alpha^4 + \beta^4 + \gamma^4$ is equal to	p.	20
(B) $(1 - \alpha)(1 - \beta)(1 - \gamma)$ is equal to	q.	18
(C) If $-1 < x < 1$, then $(x - \alpha)(x - \beta)(x - \gamma)$ is	r.	a positive quantity
(D) $(1 + \alpha^2)(1 + \beta^2)(1 + \gamma^2)$ is equal to	s.	a negative quantity
	t.	zero

3. Observe the following Column :

Column-I		**Column-II**
(A) a, b, c, d are four distinct real numbers and they are in A.P. If $2(a-b) + x(b-c)^2 + (c-a)^3 = 2(a-d) + (b-d)^2 + (c-d)^3$ then the value of x can be	p.	A rational number
(B) If roots of equation $ax^2 + bx + c = 0$, $a \neq 0$ are α and β and the roots of the equation $a^5x^2 + ba^2c^2x + c^5 = 0$ are 4 and 8 then $\lvert \alpha\beta \rvert$ is	q.	An irrational number
(C) If $\log_3(\log_5 x) + \log_{1/3}(\log_{1/5} y) = 1$ and $x^2y = 1$ $(x, y \in R)$ then the value of $5x + y$ is	r.	2
(D) Let a, b, c are rational numbers $a \neq -1$ and x_1, x_2 and x_1x_2 are the real roots of the equation $x^3 - ax^2 + bx - c = 0$ then $x_1 x_2$ is	s.	26

MARK YOUR RESPONSE

1.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

2.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

3.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

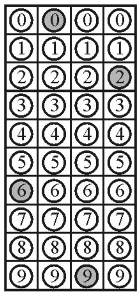
F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.



1. The number of negative integers belonging to the solution set of the equation $2^{|x+2|} - |2^{x+1} - 1| = 2^{x+1} + 1$ is equal to

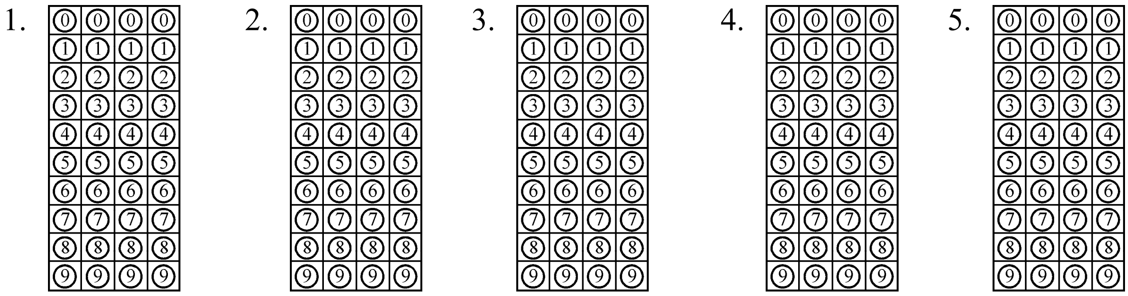
2. The least positive integer x, which satisfies the inequality $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$ is equal to

3. The number of solutions of the equation $|[x] - 2x| = 4$, where $[x]$ is the greatest integer $\le x$, is equal to

4. Let $p(x)$ be a real polynomial function given by $p(x) = ax^3 + bx^2 + cx + d$, such that If $|p(x)| \le 1$ for all x such that $|x| \le 1$ then the greatest value of $|a| + |b| + |c| + |d|$ is

5. Let x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in G.P. where $(x_1, x_2, x_3 > 0)$, then the maximum value of $[\beta] + [\gamma] + 2$ is ______, where [.] denotes the greatest integer function.

MARK YOUR RESPONSE



Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(a)	**11**	(b)	**21**	(b)	**31**	(c)	**41**	(a)
2	(b)	**12**	(d)	**22**	(a)	**32**	(a)	**42**	(b)
3	(b)	**13**	(b)	**23**	(b)	**33**	(a)	**43**	(c)
4	(b)	**14**	(c)	**24**	(b)	**34**	(a)	**44**	(a)
5	(b)	**15**	(a)	**25**	(b)	**35**	(d)	**45**	(b)
6	(b)	**16**	(c)	**26**	(c)	**36**	(b)	**46**	(d)
7	(b)	**17**	(b)	**27**	(b)	**37**	(c)	**47**	(c)
8	(b)	**18**	(b)	**28**	(c)	**38**	(c)		
9	(b)	**19**	(b)	**29**	(b)	**39**	(d)		
10	(b)	**20**	(a)	**30**	(d)	**40**	(a)		

B COMPREHENSION TYPE

1	(a)	**3**	(c)	**5**	(b)	**7**	(b)	**9**	(b)
2	(a)	**4**	(a)	**6**	(b)	**8**	(b)		

C REASONING TYPE

1	(c)	**3**	(c)	**5**	(b)	**7**	(a)	**9**	(a)
2	(c)	**4**	(c)	**6**	(d)	**8**	(b)		

D MULTIPLE CORRECT CHOICE TYPE

1	(a, c)	**6**	(b,d)	**11**	(a, c, d)	**16**	(a,b,c)
2	(a,b,c,d)	**7**	(b)	**12**	(a,c)	**17**	(b, c)
3	(c,d)	**8**	(b, d)	**13**	(a,b)	**18**	(a,b,c)
4	(a,b,c,d)	**9**	(a,b,c,d)	**14**	(a,c)	**19**	(a,b,c)
5	(c,d)	**10**	(a, d)	**15**	(a,b,d)		

E MATRIX-MATCH TYPE

1. **A - p, q, r, s; B - p; C - r, s; D - q**
2. **A - q, r; B -t; C - r; D - p, r**
3. **A - p, q, s; B - p, r; C - p, s; D - p**

F NUMERIC/INTEGER ANSWER TYPE

1	2	**2**	8	**3**	4	**4**	7	**5**	1

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(a)** We have $\left|\dfrac{x^2+kx+1}{x^2+x+1}\right|<3 \Rightarrow -3<\dfrac{x^2+kx+1}{x^2+x+1}<3$

$\Rightarrow -3(x^2+x+1)<x^2+kx+1<3(x^2+x+1)$

$[\because x^2+x+1>0\ \forall x\in \mathbf{R}]$

$\Rightarrow 4x^2+(k+3)x+4>0$ and $2x^2-(k-3)x+2>0$

Since, the coefficients of x^2 in both inequations are positive, therefore above inequatities hold if discriminants of both are negative, i.e.,

$(k+3)^2-4.4.4<0$ and $(k-3)^2-4.2.2<0$

$\Rightarrow -11<k<5$ and $-1<k<7$

$\therefore$ The common values of k are $-1<k<5$.

2. **(b)** $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$

$\Rightarrow \sqrt{x-\sqrt{1-x}}=1-\sqrt{x} \Rightarrow x-\sqrt{1-x}=1+x-2\sqrt{x}$

$\Rightarrow -\sqrt{1-x}=1-2\sqrt{x} \Rightarrow 1-x=1+4x-4\sqrt{x}$

$\Rightarrow 4\sqrt{x}=5x$

$\therefore x=\dfrac{16}{25}$ or, 0. Now $x=0$ does not sastisfy but $x=\dfrac{16}{24}$ satisfies the equation.

[NOTE : On squaring the equation,there is always chance of erroneous roots, hence the solution must be checked with original equation]. The only solution is $x=\dfrac{16}{25}$*].*

3. **(b)** We have $\left|\dfrac{x+1}{x}\right|+|x+1|=\dfrac{(x+1)^2}{|x|}$

$\Rightarrow \dfrac{|x+1|}{|x|}+|x+1|=\dfrac{(x+1)^2}{|x|}, x\neq 0$

$\Rightarrow \dfrac{|x+1|+|x||x+1|}{|x|}=\dfrac{(x+1)^2}{|x|}$

$\Rightarrow |x+1|(1+|x|)=|x+1|^2$

$\Rightarrow |x+1|(1+|x|-|x+1|)=0$

$\Rightarrow x=-1$ or, $1+|x|=|x+1|$,

$\Rightarrow$ Now $|x+y|=|x|+|y|$ if and only if x and y have the same sign.

$\therefore$ $|x+1|=|x|+1$ if $x>0$

$\therefore$ Solution set is $x=-1$ or $x>0$

$\Rightarrow x\in\{-1\}\cup(0,\infty)$

ALTERNATIVELY:

$\because \dfrac{x+1}{x}+(x+1)=\dfrac{(x+1)^2}{x}$

$\therefore$ Given equation is equivalent to $|x|+|y|=|x+y|$ which holds if and only if $xy\geq 0$

$\Rightarrow \left(\dfrac{x+1}{x}\right)(x+1)\geq 0$

$\Rightarrow x=-1$ or $x>0$

4. **(b)** The given equations are

$|x-1|+3y=4 \Rightarrow \begin{cases} x+3y=5, x\geq 1 ....(1)\\ -x+3y=3, x<1........(2)\end{cases}$

and

$x-|y-1|=2 \Rightarrow \begin{cases} x-y=1, y\geq 1........(3)\\ x+y=3, y<1........(4)\end{cases}$

Solving (1) and (3) we get $x=2, y=1$

Solving (1) and (4) we get $x=2, y=1$

$(\because x\geq 1, y<1)$, so no solution.

Solving (2) and (3) we get $x=3, y=2$

$(\because x<1, y\geq 1)$, so no solution.

Solving (2) and (4) we get $x=\dfrac{5}{2}, y=\dfrac{3}{2}$

$(\because x<1, y\geq 1)$, again no solution.

Hence solution is $x=2, y=1$ (a unique solution)

ALTERNATIVELY:

$|x-1|+3y=4$ ...(1)

$x-|y-1|=2$ ...(2)

Eliminating x, we get $||y-1|+1|+3y=4$

$\Rightarrow |y-1|+3y=3 \Rightarrow |y-1|=3(1-y) \Rightarrow y=1$

Putting $y=1$ in (2) we get $x=2$

$\Rightarrow x=2, y=1$ is the only solution.

5. **(b)** Let $3^x = y$, then the inequality is

$|y^2 - 3y - 15| < 2y^2 - y$ ...(1)

The inequality holds if $2y^2 - y > 0 \Rightarrow y < 0$ or $y > \frac{1}{2}$.

$\because\ y = 3^x \not\leq 0 \Rightarrow y > \frac{1}{2}$

Now the inequality on solving,

$-(2y^2 - y) < y^2 - 3y - 15 < 2y^2 - y$

$\Rightarrow 3y^2 - 4y - 15 > 0$ and $y^2 + 2y + 15 > 0$

Solution of first inequality

$3y^2 - 4y - 15 > 0$ is $y < -\frac{5}{3}$ or $y > 3$.

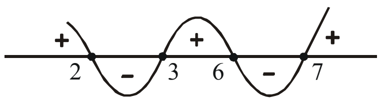
Solution of second inequality $y^2 + 2y + 15 > 0$ is $y \in \mathbf{R}$

The common solution is $y > 3 \Rightarrow 3^x > 3 \Rightarrow x > 1 \Rightarrow x \in (1, \infty)$

6. **(b)** We know that $|f(x)| = -f(x)$, if $f(x) \leq 0$

$\therefore \left|\frac{x^2 - 8x + 12}{x^2 - 10x + 21}\right| = -\frac{x^2 - 8x + 12}{x^2 - 10x + 21}$

$\Rightarrow \frac{x^2 - 8x + 12}{x^2 - 10x + 21} \leq 0$



$\Rightarrow \frac{(x-2)(x-6)}{(x-3)(x-7)} \leq 0, x \neq 3, 7$

$\Rightarrow (x-2)(x-3)(x-6)(x-7) \leq 0, x \neq 3, 7$

$\Rightarrow 2 \leq x < 3$ or, $6 \leq x < 7 \Rightarrow x \in [2, 3) \cup [6, 7)$

7. **(b)** The problem contains one absolute value term $|x - 3|$. Thus, we consider two cases

CASE : (I)

Let $x < 3$, then $|x - 3| = -(x - 3)$, the equation becomes

$x^2 . 2^{x+1} + 2^{-x+5} = x^2 . 2^{-x+7} + 2^{x-1}$

$\Rightarrow x^2 (2^{x+1} - 2^{-x+7}) = 2^{x-1} - 2^{-x+5}$

$\Rightarrow x^2 . 2^{-x+7}(2^{2x-6} - 1) = 2^{-x+5}(2^{2x-6} - 1)$

$\Rightarrow (2^{2x-6} - 1)(x^2 . 2^{-x+7} - 2^{-x+5}) = 0$

$\therefore\ 2^{2x-6} - 1 = 0 \Rightarrow 2x - 6 = 0$

$\Rightarrow x = 3$, refected $(\because x < 3)$

or $x^2 . 2^{-x+7} - 2^{-x+5} = 0$

$\Rightarrow 2^{-x+5}(2^2 . x^2 - 1) = 0 \Rightarrow x = \pm\frac{1}{2}$

$\Rightarrow x \in \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

CASE : (II)

Let $x \geq 3$, then $|x - 3| = x - 3$, the equation becomes

$x^2 . 2^{x+1} + 2^{x-1} = x^2 . 2^{x+1} + 2^{x-1}$, which is identity.

$\therefore$ All x, such that $x \geq 3$ is the solution of the equation.

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\} \cup [3, \infty)$

8. **(b)** Given $(y^2 - 5y + 3)(x^2 + x + 1) < 2x$

$\Rightarrow y^2 - 5y + 3 < \frac{2x}{x^2 + x + 1}$ ...(1)

$[\because x^2 + x + 1 > 0\ \forall x \in \mathbf{R}]$

Let $\frac{2x}{x^2 + x + 1} = z \Rightarrow zx^2 + (z-2)x + z = 0$

$\therefore\ x \in \mathbf{R} \Rightarrow (z-2)^2 - 4z.z \geq 0$

$\Rightarrow 3z^2 + 4z - 4 \leq 0 \Rightarrow -2 \leq z \leq \frac{2}{3}$

$\Rightarrow -2 \leq \frac{2x}{x^2 + x + 1} \leq \frac{2}{3}$

Clearly, the inequatity (1) hods if $y^2 - 5y + 3 < -2$

$\Rightarrow y^2 - 5y + 5 < 0 \Rightarrow \frac{5 - \sqrt{5}}{2} < y < \frac{5 + \sqrt{5}}{2}$

Alternatively :

$(y^2 - 5y + 3)(x^2 + x + 1) < 2x$

$\Rightarrow (y^2 - 5y + 3)x^2 + (y^2 - 5y + 1)x + (y^2 - 5y + 3) < 0\ \forall x \in R$

$\therefore\ y^2 - 5y + 3 < 0$ and $(y^2 - 5y + 1)^2 - 4(y^2 - 5y + 3)^2 < 0$

$\Rightarrow y^2 - 5y + 3 < 0$ and $(-y^2 + 5y - 5)(3y^2 - 15y + 7) < 0$

$\Rightarrow y^2 - 5y + 3 < 0$ and $(y^2 - 5y + 5)(3y^2 - 15y + 7) > 0$

$\Rightarrow \quad \frac{5-\sqrt{13}}{2} < y < \frac{5+\sqrt{13}}{2}$ and $y < \frac{15-\sqrt{141}}{6}$

or $\frac{5-\sqrt{5}}{2} < y < \frac{5+\sqrt{5}}{2}$ or $y > \frac{15+\sqrt{141}}{6}$

$\therefore \frac{5-\sqrt{5}}{2} < y < \frac{5+\sqrt{5}}{2}$

9. **(b)** The equation is

$x[1-\log_{10}5]+\log_{10}(2^x+1)=\log_{10}6$

$\Rightarrow x[\log_{10}10-\log_{10}5]+\log_{10}(2^x+1) = \log_{10}6$

$\Rightarrow x\log_{10}2 + \log_{10}(2^x+1) = \log_{10}6$

$\Rightarrow \log_{10}2^x + \log_{10}(2^x+1) = \log_{10}6$

$\Rightarrow \log_{10}2^x(2^x+1)=\log_{10}6$

$\Rightarrow (2^x)^2 + 2^x - 6 = 0 \Rightarrow 2^x = 2$ or $2^x = -3$

$\because \quad 2^x \neq -3 \Rightarrow 2^x = 2 \Rightarrow x=1$, which is positive integer.

10. **(b)** $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$\Rightarrow 4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$

$\Rightarrow 2^{2x}(1+\frac{1}{2}) = 3^x\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

$\Rightarrow 2^{2x}\left(\frac{3}{2}\right) = 3^x\left(\frac{4}{\sqrt{3}}\right) \Rightarrow 2^{2x-3} = 3^{x-\frac{3}{2}}$

$\Rightarrow 2^{2x-3} = (\sqrt{3})^{2x-3}$, which holds if $2x-3=0$

$\Rightarrow x = \frac{3}{2}$.

The solution set is $\left\{\frac{3}{2}\right\}$.

11. **(b)** The given equation is rewritten as

$: 2|\log_3 x|^2 - |\log_3 x| + a = 0.$

For two real and distinct values of $|\log_3 x|$, we must have $(-1)^2 - 4.2.a > 0 \Rightarrow a < \frac{1}{8}$

Again, $|\log_3 x| \geq 0$. Therefore the above equation must have positive roots for which $a > 0$.

[$ax^2 + bx + c = 0$ has positive roots if a and c have same sign and b has opposite sign]

$\therefore$ Required values are $0 < a < \frac{1}{8}$

12. **(d)** The first equation is

$2^x . 3^x . \left(\frac{2}{3}\right)^y - 3.2^{x+y} - 8.3^{x-y} + 24 = 0$

$\Rightarrow \quad 2^{x+y} . 3^{x-y} - 3.2^{x+y} + 8.3^{x-y} + 24 = 0$

$\Rightarrow (2^{x+y}-8)(3^{x-y}-3)=0$

Either $2^{x+y}-8=0$ or $3^{x-y}=3$

$\Rightarrow x+y=3$ or $x-y=1$

If $x+y=3$ and $xy=2 \Rightarrow x=1, y=2$ or $x=2, y=1$

If $x-y=1$ and $xy=2 \Rightarrow x=2, y=1$ or $x=-1, y=-2$

$\therefore (x, y) \in \{(1,2),(2,1),(-1,-2)\}$.

Hence, three solutions.

13. **(b)** The equations are $3^x \cdot 5^y = 75$ ...(1)

and $3^y \cdot 5^x = 45$ ...(2)

Dividing the two equations, we get

$\left(\frac{3}{5}\right)^{x-y} = \frac{75}{45} = \frac{5}{3} \Rightarrow x-y=-1$ ...(3)

Multiplying equation (1) and (2), we get

$(15)^{x+y} = 45 \times 75 = (15)^3 \Rightarrow x+y=3$ ...(4)

Solving (3) and (4), we get $x = 1, y = 2$. So, $(x, y) = (1, 2)$.

14. **(c)** The equation is $(\log_{16} x)^2 - \log_{16} x + \log_{16} k = 0$.

Clearly $x > 0$.

Solving the equation, we get

$\log_{16} x = \frac{1 \pm \sqrt{1-4(\log_{16} k)}}{2}$

For exactly one solution $1 - 4\log_{16} k = 0 \Rightarrow k^4 = 16$

$\Rightarrow k = \pm 2$ [taking real values]

Now $\log_{16} k$ is defined if $k > 0 \Rightarrow k = 2$.

15. **(a)** $\log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$

$\Rightarrow \log_{0.3}(x-1) < \frac{1}{2}\log_{0.3}(x-1)$

$\Rightarrow \log_{0.3}(x-1)^2 < \log_{0.3}(x-1)$

$\Rightarrow (x-1)^2 > (x-1)$ $[\because \text{ base} < 1]$

$\Rightarrow (x-1)(x-2) > 0 \Rightarrow x < 1$ or $x > 2$.

But $\log(x-1)$ is defined if $x-1>0 \Rightarrow x>1$

$\therefore$ The common values of x are $x > 2$

$\Rightarrow x \in (2, \infty)$

16. (c) $|x+1|^{\log_{x+1}(3+2x-x^2)} = (x-3)|x|$ ...(1)

The equation (1) is valid if

$x+1>0,\ x+1 \neq 1$ and $3+2x-x^2>0$

$\Rightarrow x>-1,\ x\neq 0$ and $-1<x<3 \Rightarrow x\in(-1,3)-\{0\}$

Further, the left hand side of the equation (1) is exponential, hence positive, therefore

RHS $>0 \Rightarrow (x-3)|x|>0 \Rightarrow x-3>0$ $[\because |x|>0]$

$\therefore x>3.$

Clearly it has no common value with interval obtained earlier. Hence $x \in \Phi$

17. (b) $2+|e^x-1| = e^{2x}-2e^x+1 = |e^x-1|^2$ *i.e.*,

$|e^x-1|^2 - |e^x-1| - 2 = 0$

$\Rightarrow |e^x-1| = 2$ or $-1 \Rightarrow |e^x-1| = 2$

(negative value not admissible)

$\Rightarrow e^x - 1 = \pm 2 \Rightarrow e^x = 3$ or $-1 \Rightarrow e^x = 3$

(negative value not admissible)

$\Rightarrow x = \log_e 3 \Rightarrow$ only one solution.

18. (b) Since 2^y is positive for all values of y, $(x-8)(x-10)$ should be positive. Therofore $x > 10$ or < 8.

Since 2^y is a power of 2, $x-10$ and $x-8$ should be both powers of 2.

$\therefore x = 12$ and $x = 6 \Rightarrow (12, 3)$ and $(6, 3)$ are the only solutions.

19. (b) If $x = 2m + y$ where m is an integer and $0 \le y < 1$, then $[x] = 2m$ and

$$\left[\frac{x}{2}\right] = m,\ \left[\frac{x+1}{2}\right] = \left[\frac{2m+y+1}{2}\right] = m$$

$$\left(\because \frac{1}{2} < \frac{1+y}{2} < 1\right)$$

$$\therefore \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right] = [x].$$

Also if $x = (2m+1)+y$, then

$$\left[\frac{x}{2}\right] = m,\ \left[\frac{x+1}{2}\right] = \left[\frac{2m+y+1}{2}\right] = m+1$$

But $[x] = 2m+1 = \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right]$

$\therefore \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right] = [x]$ for every real number.

20. (a) The log functions are defined if

$\frac{x^2+6x+9}{2(x+1)} > 0$ and $x+1>0$

$\Rightarrow \frac{(x+3)^2}{2(x+1)} > 0$ and $x+1>0 \Rightarrow x>-1$

Now the inequality is

$$\log_{2^{-1}} \frac{x^2+6x+9}{2(x+1)} < -\log_2(x+1)$$

$$\Rightarrow -\log_2 \frac{x^2+6x+9}{2(x+1)} < -\log_2(x+1)$$

$$\Rightarrow \log_2 \frac{x^2+6x+9}{2(x+1)} > \log_2(x+1)$$

$$\Rightarrow \frac{x^2+6x+9}{2(x+1)} > (x+1) \Rightarrow \frac{-x^2+2x+7}{2(x+1)} > 0$$

$\Rightarrow (x+1)(x^2-2x-7) < 0$

$\Rightarrow x^2-2x-7 < 0$ $[\because x+1>0]$

$\Rightarrow -1-2\sqrt{2} < x < -1+2\sqrt{2}$, but $x > -1$

$\Rightarrow -1 < x < -1+2\sqrt{2}$.

21. (b) For the validity of inequality $ax^2+4x+a>0$, which is possible if $a>0$ and

$16-4a^2<0 \Rightarrow a>2$ ...(1)

Futher, the inequality can be rewritten as

$$\log_5 5 + \log_5(x^2+1) \le \log_5(ax^2+4x+a)$$

$$\Rightarrow 5(x^2+1) \le ax^2+4x+a$$

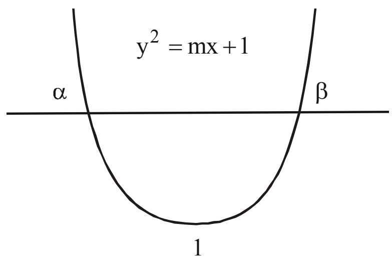
$$\Rightarrow (a-5)x^2+4x+(a-5) \ge 0.$$

It holds if $a-5>0$ and $16-4(a-5)^2 \le 0$

$\Rightarrow a>5$ and $a \le 3$ or $a \ge 7 \Rightarrow a \ge 7$ ...(2)

Combining the results of (1) and (2) for common values, we get $a \in [7, \infty)$.

22. **(a)** Since 1 lies between the roots of $y^2 - my + 1 = 0$



$\therefore$ Let $f(y) = y^2 - my + 1$

$\therefore$ $f(1) < 0 \Rightarrow 2 - m < 0 \Rightarrow m > 2$

Now, A.M. $\geq$ G.M.

$\Rightarrow \dfrac{|x|^2 + 16}{2} \geq 4|x|, \dfrac{1}{2} \geq \dfrac{4|x|}{|x|^2 + 16}$

$\Rightarrow 0 \leq \dfrac{4|x|}{|x|^2 + 16} \leq \dfrac{1}{2}$

$\Rightarrow 0 \leq \left(\dfrac{4|x|}{|x|^2 + 16}\right)^m < 1 \Rightarrow \left[\dfrac{4|x|}{|x|^2 + 16}\right] = 0$

23. **(b)** Let $x = [x] + \{x\}$, the equation becomes

$4\{x\} = [x] + \{x\} + [x] \Rightarrow 3\{x\} = 2[x]$

$\Rightarrow \{x\} = \dfrac{2}{3}[x]$ ...(1)

$\because$ $0 \leq \{x\} < 1 \Rightarrow 0 \leq \dfrac{2}{3}[x] < 1$

$\Rightarrow 0 \leq [x] < \dfrac{3}{2}$ and $[x]$ is integer.

$\therefore$ $[x] = 0$ or 1, from (1) we get $\{x\} = 0$ if $[x] = 0$ and

$[x] = \dfrac{2}{3}$ if $[x] = 1$

$\therefore$ $x = 0 + 0$ or $1 + \dfrac{2}{3} \Rightarrow x = 0$ or $\dfrac{5}{3}$.

The solution set is $x \in \left\{0, \dfrac{5}{3}\right\}$.

24. **(b)** $(x-1)^2 = \{(x) - 1\}^2 = (x)^2 - 2(x) + 1$ and

$[x+1]^2 = \{[x] + 1\}^2 = [x]^2 + 2[x] + 1$

$\therefore$ The equation becomes, $[x] - (x) + 1 = 0$ ...(1)

Let $x = n \in \mathbf{I}$, then $[x] = (x) = n$, and equation (1) becomes $1 = 0$, not possible.

Let $x = n + f$, where $n \in \mathbf{I}$ and $0 < f < 1$, then $[x] = n$ and $(x) = n + 1$, the equation (1) becomes

$n - n - 1 + 1 = 0 \Rightarrow 0 = 0$, an identity.

$\therefore$ All x such that $x \notin \mathbf{I}$ is the solution of given equation.

$\therefore$ Solution set is $S = \mathbf{R} - \mathbf{I}$

25. **(b)** Given $F(x) = x - [x]$ So, $F(x) + F\left(\dfrac{1}{x}\right) = 1$

$\Rightarrow x - [x] + \dfrac{1}{x} - \left[\dfrac{1}{x}\right] = 1$

$\Rightarrow x + \dfrac{1}{x} = [x] + \left[\dfrac{1}{x}\right] + 1$ ...(1)

$\because$ RHS is an integer.

$\therefore$ LHS must be integer.

Let $x + \dfrac{1}{x} = [x] + \left[\dfrac{1}{x}\right] + 1 = k, k \in \mathbf{I}$, the equation is

$x + \dfrac{1}{x} = k \Rightarrow x^2 - kx + 1 = 0$,

For real x, $k^2 - 4 \geq 0 \Rightarrow k \leq -2$ or $k \geq 2$ and $k \in \mathbf{I}$, we can get infinitety many values of k and we get solution for each value of k.

26. **(c)** $\because$ $[x + n] = [x] + n$, if $n \in \mathbf{I}$, therefore the equation becomes,

$[x]^2 = [x] + 2 \Rightarrow [x]^2 - [x] - 2 = 0$

$\Rightarrow ([x] + 1)([x] - 2) = 0 \Rightarrow [x] = -1$ or 2

If $[x] = -1$ then $-1 \leq x < 0$

If $[x] = 2$, then $2 \leq 3$

$\therefore$ $x \in [-1, 0) \cup [2, 3)$.

27. **(b)** $5\{x\} = x + [x]$ and $[x] - \{x\} = \dfrac{1}{2}$, since $x = [x] + \{x\}$

$\Rightarrow 4\{x\} = 2[x]$ and $[x] - \{x\} = \dfrac{1}{2}$.

After solving $[x] = 1$ and $\{x\} = \dfrac{1}{2}$ $\therefore$ $x = 1 + \dfrac{1}{2} = \dfrac{3}{2}$.

28. **(c)** $\because$ $(x-1) = (x - [x])(x - \{x\}) \Rightarrow x = 1 + \{x\}[x]$

$\Rightarrow [x] + \{x\} = 1 + \{x\}[x] \Rightarrow (\{x\} - 1)([x] - 1) = 0$

$\{x\} - 1 \neq 0$ $\therefore$ $[x] - 1 = 0$

$[x] = 1$ $\Rightarrow x \in [1, 2)$

29. (b) $(x)^2 + (x+1)^2 = 25 \Rightarrow (x)^2 + \{(x)+1\}^2 = 25$

$\Rightarrow 2(x)^2 + 2(x) - 24 = 0$

$\Rightarrow (x)^2 + (x) - 12 = 0 \Rightarrow (x) = -4$ or 3

Now $(x) = -4 \Rightarrow -5 < x \le -4$ and $(x) = 3$

$\Rightarrow 2 < x \le 3.$

$\therefore$ Solution set is $(-5, -4] \cup (2, 3]$

30. (d) **Case 1 :** Let $0 \le x < 2$, then $\|x\| = x + 1$ and the equation becomes

$(x+1)^2 + x = (x+1) + x^2 \Rightarrow 2x = 0$

$\Rightarrow 2x = 0 \Rightarrow x = 0$

Case 2 : Let $x \ge 2$, then $\|x\| = |x-4|$ and the equation becomes

$|x-4|^2 + x = |x-4| + x^2$

$\Rightarrow x^2 - 8x + 16 + x = |x-4| + x^2 \Rightarrow |x-4| = 16 - 7x$

$\therefore x - 4 = \pm(16 - 7x)$, provided $16 - 7x \ge 0$

$\therefore x = \frac{5}{2}$ or 2, but for $x = \frac{5}{2}$, $16 - 7x < 0$, hence rejected

$\therefore x = 2$. The solution set is $\{0, 2\}$.

31. (c) $f'(x) = 3x^2 + 6x + 6 + 2\cos x$

$= 3(x+1)^2 + 3 + 2\cos x > 0$

for all x, so $f(x)$ is an increasing function.

Thus $f(1) < f(2) < f(3)$

Let $f(1) = a, f(2) = b$ and $f(3) = c$, then $a < b < c$

Given equation is $\frac{1}{x-a} + \frac{2}{x-b} + \frac{3}{x-c} = 0$

$\Rightarrow (x-b)(x-c) + 2(x-a)(x-c) + 3(x-a)(x-b) = 0$

Let $g(x) = (x-b)(x-c) + 2(x-a)(x-c) + 3(x-a)(x-b)$

$g(a) = (a-b)(a-c) + 2.0 + 3.0 > 0$;

$g(b) = 2(b-a)(b-c) < 0$ and

$g(c) = 3(c-a)(c-b) > 0$

Hence, the equation $g(x) = 0$ has a root in (a, b) and another in (b, c).

32. (a) $|2^x - 1| + |4 - 2^x| \ge |2^x - 1 + 4 - 2^x| = 3;$

Hence the given inequality has no solution.

33. (a) $\sin^2 x + \sin x \cos x = [t]$

$\frac{1-\cos 2x}{2} + \frac{\sin 2x}{2} = [t]$

$\Rightarrow \sin 2x - \cos 2x = 2[t] - 1$

$\sqrt{2}\{\sin 2x \cos \pi/4 - \cos 2x \sin \pi/4\} = 2[t] - 1$

$\Rightarrow \sqrt{2}\{\sin(2x - \pi/4)\} = 2[t] - 1$

$\Rightarrow -\sqrt{2} \le \{2[t] - 1\} \le \sqrt{2}$

$\Rightarrow \frac{1-\sqrt{2}}{2} \le [t] \le \frac{1+\sqrt{2}}{2} \Rightarrow [t] = 0, 1 \Rightarrow t \in [0, 2)$

34. (a) $f(x) = g(x^3) + xh(x^3)$

Let $f_1(x) = 1 + x + x^2$

Clearly roots of $f_1(x) = 0$ are ω, ω^2, where ω is non-real cube root of unity

$\therefore f(\omega) = 0, f(\omega^2) = 0$

$\Rightarrow g(\omega^3) + \omega h(\omega^3) = 0$ and

$g(\omega^6) + \omega^2 h(\omega^6) = 0$

$\Rightarrow g(1) + \omega h(1) = 0$ ... (i)

$g(1) + \omega^2 h(1) = 0$ ... (ii)

Adding (i) and (ii), $2g(1) + h(1)(\omega + \omega^2) = 0$

$\Rightarrow h(1) = 2g(1)$

From (i), $g(1) + 2\omega g(1) = 0 \Rightarrow g(1)(1 + 2\omega) = 0$

$\Rightarrow g(1) = 0 \Rightarrow h(1) = 0$

$\Rightarrow g(x)$ and $h(x)$ are divisible by $(x - 1)$

35. (d) $\sqrt{e^{|x|\log 4}} = e^{\frac{|x| 2\log 2}{2}} = e^{|x|\log 2} = (e^{\log 2})^{|x|} = 2^{|x|}$

$\therefore$ Given equation is $2^{|x^2-12|} = 2^{|x|}$

$\Rightarrow |x^2 - 12| = |x|$

$\Rightarrow x^4 - 25x^2 + 144 = 0$

$\Rightarrow x^2 = 16, 9$

$\Rightarrow x = \pm 3, \pm 4$

36. (b) $4^{\sin 2x + 2\cos^2 x} + 4^{3 - (\sin 2x + 2\cos^2 x)} = 65$

Put $y = 4^{\sin 2x + 2\cos^2 x} \Rightarrow y + \frac{64}{y} - 65 = 0$

$\therefore y^2 - 65y + 64 = 0 \Rightarrow y = 1$ or $y = 64$

$\Rightarrow \sin 2x + 2\cos^2 x = 0 \Rightarrow \cos^2 x + \sin x \cos x = 0$

$\Rightarrow \cos x(\cos x + \sin x) = 0 \Rightarrow x = \frac{\pi}{2}$

Also, $\sin 2x + 2\cos^2 x = 3 \Rightarrow \sin 2x + \cos 2x = 2$ is not possible as maximum of $\sin 2x + \cos 2x = \sqrt{2}$

37. **(c)** Let $\alpha, \beta, \gamma, \delta$ be the four positive roots then

$\alpha+\beta+\gamma+\delta=-p,\ \alpha\beta+\beta\gamma+\gamma\delta+\alpha\gamma+\alpha\delta+\beta\delta=q,$

$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=-r$ and $\alpha\beta\gamma\delta=5$

Now $\left(\frac{\alpha+\beta+\gamma+\delta}{4}\right)\left(\frac{\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta}{4}\right)$

$\geq \sqrt[4]{\alpha\beta\gamma\delta}\ \sqrt[4]{\alpha^3\beta^3\gamma^3\delta^3}$

$\Rightarrow \left(\frac{-p}{4}\right)\left(\frac{-r}{4}\right) \geq 5 \Rightarrow pr \geq 80$

38. **(c)** The equation is $x^4+2(y^2-2y)x^2+y^4=0.$

Put $x^2=t$ then $t^2+2(y^2-2y)t+y^4=0$ ...(i)

Equation (i) will have non-negative roots if

$4(y^2-2y)^2-4y^4 \geq 0 \Rightarrow y \leq 1$

Also, $y^4 \geq 0$ and $0 \leq -(y^2-2y)$.

$\Rightarrow 0 \leq y \leq 2$. So $y \in [0,1]$

39. **(d)** $f(x)$ will also be a factor of

$3(x^4+6x^2+25)-(3x^4+4x^2+28x+5)$, which

equals $14(x^2-2x+5)$. So, $f(x)=x^2-2x+5 \geq 4$

40. **(a)** We have $y+z=4-x$ and $y^2+z^2=6-x^2$.

Also, $yz=\frac{1}{2}[(y+z)^2-(y^2+z^2)]=x^2-4x+5$.

Therefore y, z must be roots of the equation

$t^2-(4-x)t+x^2-4x+5=0.$ As y and z are real, so

$(4-x)^2-4(x^2-4x+5) \geq 0 \Rightarrow \frac{2}{3} \leq x \leq 2$

41. **(a)** Clearly the equation $f'(x)=0$ must also have repeated roots. So, $f'(x) \geq 0 \forall x$.

Let $f'(x) = a_1x^2+b_1x+c_1$, where $a_1>0$ and

$b_1^2-4a_1c_1=0$, then

$g(x) = a_1x^2+(b_1-2a_1)x+2a_1-b_1+c_1$

Its discriminant =

$b_1^2-4a_1c_1-4a_1^2<0 \Rightarrow g(x)>0\ \ \forall\ x \in R$

42. **(b)** Let $f(x)=x^3-3x+a$

$f'(x)=3x^2-3$

For three distinct real roots $f'(x)=0$ should have two distinct real roots α and β such that $f(\alpha)f(\beta)<0$

Here $\alpha=1, \beta=-1$

Now $f(\alpha)f(\beta)<0$

$\Rightarrow (1-3+a)(-1+3+a)<0$

$\Rightarrow (a-2)(a+2)<0$

$\Rightarrow -2<a<2.$

43. **(c)** $x=7^{1/3}+7^{2/3}$

$x^3=7+7^2+3.7x \Rightarrow x^3-21x-56=0$

$\therefore$ Product of roots = 56

44. **(a)** Let $a^{\cos x}=t \Rightarrow t+\frac{1}{t}=6 \Rightarrow t^2-6t+1=0$

$\Rightarrow t=\frac{6\pm\sqrt{36-4}}{2}=3\pm2\sqrt{2} \Rightarrow a^{\cos x}=3\pm2\sqrt{2}$

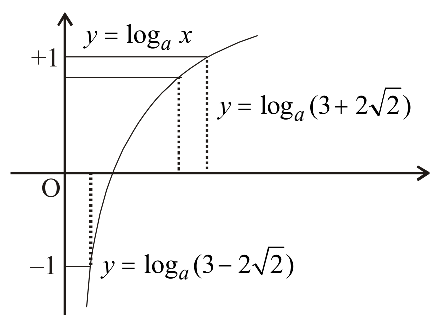
$\Rightarrow \cos x=\log_a(3\pm2\sqrt{2})$

Since $a>1$, for all the roots to be real,

We must have $\log_a(3+2\sqrt{2}) \leq 1$ and

$\log_a(3-2\sqrt{2}) \geq -1,$

Both are true for $a \geq 3+2\sqrt{2}$.



45. **(b)** Let $f(x,y,z)=x^2+4y^2+3z^2-2x-12y-6z+14$

$=(x-1)^2+(2y-3)^2+3(z-1)^2+1$

For least value of $f(x,y,z)$

$x-1=0; 2y-3=0$ and $z-1=0$

$\therefore \quad x=1; y=\frac{3}{2}; z=1$

Hence least value of $f(x,y,z)$ is $f\left(1,\frac{3}{2},1\right)=1.$

46. **(d)** $\sec^2 x=\frac{1}{4-a},-1$

Rejecting the negative value

$\sec^2 x=\frac{1}{4-a} \Rightarrow 4-a>0 \Rightarrow a<4$ ...(1)

Also, $\frac{1}{4-a}\geq 1 \Rightarrow 4-a\leq 1 \Rightarrow a\geq 3$ ...(2)

Combining (1) and (2), we get the solution [3, 4).

47. **(c)** LHS $=\frac{x^2-12x+39}{3}=\frac{(x-6)^2+3}{3}$

$=\frac{(x-6)^2}{3}+1=\sin\frac{a}{x}$

Since, $\sin\frac{a}{x}\leq 1, x=6$

$\Rightarrow \sin\frac{a}{6}=1 \Rightarrow a=3(1+4n)\pi$

B COMPREHENSION TYPE

1. **(a)** $f(x)=8x^3-20x^2+6x+9$, then

$f'(x)=24x^2-40x+6$. Now, $f'(x)=0$

$\Rightarrow 24x^2-40x+6=0$ gives $x=\frac{3}{2},\frac{1}{6}$ But $f\left(\frac{3}{2}\right)=0$.

Therefore $x=\frac{3}{2}$ is a repeated root of $f(x)=0$

2. **(a)** $f(x)=x^3-3x+1 \Rightarrow f'(x)=3(x^2-1)$

Now $f'(x)=0 \Rightarrow 3(x^2-1)=0$ gives $x=\pm 1$

We have $f(1)=-1$ and $f(-1)=3$, which are of opposite sign. Hence, $f(x)=0$ has all different real roots.

3. **(c)** We have $f(-1)=3>0,\ f(1)=-1<0$

Also $f(0)=1>0, f(2)=3>0$ and $f(-2)=-1<0$

Thus the question $f(x)=0$ has three distinct roots α,β,γ such that $-2<\alpha<-1$, $0<\beta<1$ and $1<\gamma<2$.

Clearly none of α,β,γ can be integer.

As obtained in Q.4, we have $-2<\alpha<-1$, $0<\beta<1$ and $1<\gamma<2$.

$\therefore [\alpha]=-2, [\beta]=0$ and $[\gamma]=1$; $[\alpha]+[\beta]+[\gamma]=-1$

4. **(a)** Let the roots of the equation be $a-d, a, a+d$ then

$(a-d)+a+(a+d)=1 \Rightarrow a=\frac{1}{3}$

$(a-d)a+(a-d)(a+d)+a(a+d)=\beta$

$\Rightarrow 3a^2-d^2=\beta \Rightarrow d^2=\frac{1}{3}-\beta$

$\because d^2\geq 0 \qquad$ So, $\beta\leq\frac{1}{3}$

5. **(b)** Again $(a-d)a(a+d)=\gamma \Rightarrow a(a^2-d^2)=-\gamma$

or $\frac{1}{3}\left(\frac{1}{9}-d^2\right)=-\gamma \Rightarrow \frac{1}{27}+\gamma=\frac{1}{3}d^2$

So, $\gamma+\frac{1}{27}\geq 0 \Rightarrow \gamma\geq -\frac{1}{27}$

6. **(b)** Put $\gamma=-\beta$ in the given equation, then we have

$x^3-x^2+\beta x-\beta=0 \Rightarrow (x-1)(x^2+\beta)=0$

Clearly for all three real roots $\beta<0$ and so, $\gamma\geq 0$.

7. **(b)** $f(x)=x^3-12x-p$

$f'(x)=3x^2-12=3(x^2-4)>0$

$\Rightarrow f(x)$ is strictly increasing for $x>2$ and $x<-2$

$f(3)=-9-p<0$

$f(4)=16-p>0 \Rightarrow$ one root lies between (3, 4).

8. **(b)** $f(x)=2\sin^2\theta x^2-3\sin\theta.x+1$

$f(1)=2\sin^2\theta-3\sin\theta+1=(2\sin\theta-1)(\sin\theta-1)<0$

$f(2)=8\sin^2\theta-6\sin\theta+1$

$=(4\sin\theta-1)(2\sin\theta-1)>0$

$f(1)f(2)<0 \Rightarrow$ one real root in the interval (1,2).

9. **(b)** $f(x)=ax^2+bx+c$, $\qquad f(0)=c<0$

$f\left(-\frac{1}{2}\right)=\frac{a-2b+4c}{4}>0$

$\Rightarrow$ one root lies in the interval $\left(-\frac{1}{2},0\right)$.

C REASONING TYPE

1. **(c)** If $1 \le a \le 2 \Rightarrow 0 \le a-1 \le 1$

$\Rightarrow \sqrt{a+2\sqrt{a-1}} + \sqrt{a-2\sqrt{a-1}}$

$= \sqrt{1} + \sqrt{a-1} + \sqrt{1} - \sqrt{a-1} = 2$

Statement-1 is true but Statement-2 is false.
Hence (c) is correct choice.

2. **(c)** Let $\log_7 2 = \frac{p}{q}$, p and q being two coprime positive integers

$\Rightarrow 2 = 7^{p/q} \Rightarrow 2^q = 7^p$

Clearly for no combination of p and q the above equation can hold.
$\Rightarrow \log_2 7$ is an irrational number

3. **(c)** $x = \sqrt{3} - \sqrt{2}$ or $x^2 = 5 - 2\sqrt{6}$

$(x^2 - 5)^2 = 24$

$x^4 - 10x^2 + 25 = 24 \Rightarrow x^4 - 10x^2 + 1 = 0$

For a complete polynomial equation with rational coefficients irrational roots occur in pairs.

4. **(c)** $f(x) = x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x = 0 \Rightarrow x = 0, 2$

$f''(x) = 6x - 6 = 0 \Rightarrow x = 1$

$f(0) = 4$
$f(2) = 8 - 12 + 4 = 0$
$f(2) = 0$ and $f'(2) = 0$
$\Rightarrow$ 2 is a multiple root of order 2
Statement-1 is true but Statement - 2 is false

5. **(b)** Remainder $= f(1) = 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$
If $f(a) = 0$ then $(x - a)$ is a factor of $f(x)$
Statement-1 and Statement-2 are true but Statement-2 is not correct Reason of Statement-1

6. **(d)** Statements – 2 is obviously correct.
$y = 2 - x \Rightarrow xy - z^2 = 1 \Rightarrow 2x - x^2 - z^2 = 1$

$\Rightarrow z^2 + (x-1)^2 = 0 \Rightarrow z = 0, x = 1$ and hence y = 1

7. **(a)** $f(x) = x + ax^{-2} \Rightarrow f'(x) = 1 - 2ax^{-3} = 0$
$\Rightarrow x = (2a)^{1/3}$

$f''(x) = 6ax^{-4} > 0 \; \forall x \in (0, \infty)$ as $a > 0$

$\therefore x = (2a)^{1/3}$ is a point of global minima

Thus $(2a)^{1/3} + a(2a)^{-2/3} > 2 \Rightarrow a > \frac{32}{27}$

***ALTERNATIVELY*:**

$x + ax^{-2} > 2 \Rightarrow x^3 - 2x^2 + a > 0$

Let $f(x) = x^3 - 2x^2 + a \Rightarrow f'(x) = 3x^2 - 4x = 0$

$\Rightarrow x = \frac{4}{3}$ $\quad (\because x > 0)$

$f''(x) = 6x - 4 > 0$ if $x = \frac{4}{3}$

$\Rightarrow f(x)$ has minima at $x = \frac{4}{3}$

$\therefore f\left(\frac{4}{3}\right) > 0 \Rightarrow a > \frac{32}{27}$

8. **(b)** Clearly x is odd thus $x^2 = 8k + 1 \Rightarrow y^2 = 4k$
$\Rightarrow y = 2$ as y is prime $\Rightarrow x = 3$

9. **(a)** As $f'(\alpha) = 0$, so $y = f(x)$ has a tangent parallel to x-axis at $x = \infty$ but $f''(\alpha) = 0$ and $f''(\alpha) \neq 0$, so $x = \alpha$ represents a point of inflexion.

D MULTIPLE CORRECT CHOICE TYPE

1. **(a, c)** Equation has solution only if $a^2 - 2a - 3 \ge 0$
$\Rightarrow a \le -1$ or $a \ge 3$
Then equation becomes

$x^2 - 1 = \pm(a^2 - 2a - 3)$

$\Rightarrow x^2 = a^2 - 2a - 2$ or $x^2 = -a^2 + 2a + 4$
For real solution

$a^2 - 2a - 2 \ge 0$ or $-a^2 + 2a + 4 \ge 0$

$\Rightarrow a \le 1 - \sqrt{3}$ or $a \ge 1 + \sqrt{3}$ or $1 - \sqrt{5} \le a \le 1 + \sqrt{5}$

$\therefore a \in [1 - \sqrt{5}, -1] \cup [3, 1 + \sqrt{5}]$

2. **(a,b,c,d)** The L.H.S. of th equation

$x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ is defined if $x > 0$ and $x \ne 1$.

Taking log of both the sides at the base 2,

$\left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right] \log_2 x = \log_2 2^{\frac{1}{2}}$

$\Rightarrow \frac{3}{4} t^3 + t^2 - \frac{5}{4} t = \frac{1}{2}$, where $t = \log_2 x$

$\Rightarrow\ 3t^3+4t^2-5t-2=0$

$\Rightarrow\ (t-1)(t+2)(3t+1)=0 \Rightarrow\ t=1,-2 \text{ or } -\frac{1}{3}$

$\therefore\ \log_2 x=1,-2 \text{ or } -\frac{1}{3} \Rightarrow x=2,\ 2^{-2} \text{ or } 2^{-1/3}$

$\Rightarrow x=2,\ \frac{1}{4} \text{ or } \frac{1}{\sqrt[3]{2}}$

$\therefore$ The equation has exactly three real solutions, hence at least one real solution. Also there is exactly one irrational solution.

3. **(c,d)** Let $x^2+4x+4=y$, then the equation becomes

$ay^2+by+c=0$ ... (1)

$\because\ b^2-4ac>0 \Rightarrow$ equation (1) has two distinct real roots, say α and β.

$\therefore\ y=\alpha$ or $\beta \Rightarrow x^2+4x+4=\alpha$ or β

$\Rightarrow\ (x+2)^2=\alpha$ or β ... (2)

Clearly equation (2) will give four distinct real values of x if α and β are positive. That is, if equation (1) has positive roots. For this a and c should have the same sign and the sign of b should be opposite. Only options (c) and (d) satisfy this condition.

4. **(a,b,c,d)** The equations are $xy+3y^2-x+4y-7=0$ ... (1)

and $2xy+y^2-2x-2y+1=0$ ... (2)

Multiply (1) by 2 and subtract from (2),

$5y^2+10y-15=0 \Rightarrow y^2+2y-3=0$

$\Rightarrow\ y=-3$ or $y=1$

If $y=-3$, then from any of (1) and (2), $x=2$.

If $y=1$, then both (1) and (2) reduce to identity.

$\therefore$ Solution is $x=2, y=-3$ and if $y=1, x\in \boldsymbol{R}$.

5. **(c,d)** Since LHS $>0 \Rightarrow k>1$

Now $2\log_3 k-4\log_k 3=7$

Let $\log k=t>0$ as $k>1$. Then, $2t-4/t=7$

$\Rightarrow\ 2t^2-7k-4=0 \Rightarrow t=-\frac{1}{2}, 4$

But $t>0$, so $\log_3 k=4 \Rightarrow k=3^4=81$

$\text{Sin}^{-1} k+\cos^{-1}(k-1)$ is not defined *for* $k=81$.

$\pi/4<\tan^{-1}k<\pi/2$ because $k>1$.

Also, $0<\cot^{-1}k<\frac{\pi}{6}$ *as* $k>\sqrt{3}$

6. **(b,d)** $||x-1|+a|=4 \Rightarrow |x-1|+a=\pm 4$

$\Rightarrow |x-1|=-a\pm 4$

The above equation holds if

$-a+4\geq 0$ or $-a-4\geq 0$

$\Rightarrow a\leq 4$ or $a\leq -4 \Rightarrow a\in(-\infty, 4]$

7. **(b)** Put $x^2+x+1=y$, the equation reduces to

$(y+1)^2-(a-3)(y+1)y+(a-4)y^2=0$

$\Rightarrow\ (a-5)y-1=0 \Rightarrow y=\frac{1}{a-5}$

$\therefore\ x^2+x+1=\frac{1}{a-5}$.

Now $x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\geq\frac{3}{4}$

$\therefore\ \frac{1}{a-5}\geq\frac{3}{4} \Rightarrow \frac{4-3a+15}{4(a-5)}\geq 0$

$\Rightarrow\ \frac{3a-19}{a-5}\leq 0 \Rightarrow (a-5)(3a-19)\leq 0, a\neq 5$

$\Rightarrow 5<a\leq\frac{19}{3}\ \because\ 6<\frac{19}{3}<7$

$\therefore$ Only integral value of a, such that

$a\in\left(5,\frac{19}{3}\right)$ is $a=6$.

8. **(b,d)** Let $f(x)=x^3-x^2+\beta x+\gamma$, then $f(x)=0$ has different roots, so $f'(x)=0$ must have two distinct roots $\Rightarrow 3x^2-2x+\beta=0$ has positive and distinct roots

$\Rightarrow \beta>0$ and $4-12\beta>0 \Rightarrow 0<\beta<\frac{1}{3}$

Now product of the roots

$=x_1x_2x_3=-\gamma \Rightarrow -\gamma>0 \Rightarrow \gamma<0$

Also, $\frac{x_1+x_2+x_3}{3}>(x_1\,x_2\,x_3)^{1/3}$

$(\because x_1, x_2, x_3$ are unequal$)$

$\Rightarrow\ x_1\,x_2\,x_3<\left(\frac{1}{3}\right)^3 \Rightarrow -\beta<\frac{1}{27} \Rightarrow \beta>-\frac{1}{27}$

CAUTION : *Mostly students will opt for option (c). It should be carefully noted that AM ~ GM equality holds if and only if numbers are equal*

9. **(a,b,c,d)** Let the roots be x_1, x_2, x_3, x_4 then

$x_1 + x_2 + x_3 + x_4 = 4$ and $x_1 x_2 x_3 x_4 = 1$

$\Rightarrow$ A.M. *of* x_1, x_2, x_3, x_4 = G.M. of

$x_1, x_2, x_3, x_4 \Leftrightarrow x_1 = x_2 = x_3 = x_4$

$\therefore \; x_1 = x_2 = x_3 = x_4 = 1 \Rightarrow x_1, x_2, x_3, x_4$ in A.P. as well as G.P. and in H.P.

Also $x^4 - 4x^3 + ax^2 + bx + 1$

$= (x-1)^4 \Rightarrow a = 6, b = -4$

10. **(a, d)** The equation is $(x^3+1)^2 + a(x^3+1) + 4 = 0$

$\Rightarrow t^2 + at + 4 = 0$ ...(i)

Where $t = x^3 + 1$. If $D < 0 \Rightarrow a^2 - 16 < 0$ then above equation has no real roots and all six roots are imaginary. If $D \geq 0,$ then equation (i) has two roots. So if $a^2 - 16 \geq 0,$ then the given equation has two real roots and other imaginary, except when $t = 1$. That is when $a = -5$, and equation is $x^3(x^3 - 3) = 0$, which has three repeated, one real and two imaginary roots.

11. **(a, c, d)** $2x^3 + x^2 + 2x - 5 = 0$

$\Rightarrow (x-1)(2x^2 + 3x + 5) = 0$

So, one root is 1 and other roots are imaginary.

Also, $ax^3 + (a+b)x^2 + (b+c)x + c = 0$

$\Rightarrow (x+1)(ax^2 + bx + c) = 0$

Clearly the two equations have either 1 as common root or both roots must be common. So,

$a + b + c = 0$ or $\frac{a}{2} = \frac{b}{3} = \frac{c}{5} \Rightarrow a + b + c = 0$

or $a + b + c = 2c$ or $5a$

12. **(a,c)** a, b, c satisfy the cubic $t^3 = t^2 z + ty + x$ or $t^3 - zt^2 - yt - x = 0$ has roots a, b, c thus $a + b + c = z, ab + bc + ca = -y$ and $abc = x$

13. **(a,b)** The equation is reciprocal so, $f(x) = 0$

and $f\left(\frac{1}{x}\right) = 0$ represent same equation

$\therefore x^3 + x^2 + ax + b = 0$ and $\frac{1}{x^3} + \frac{1}{x^2} + \frac{a}{x} + b = 0$

or $bx^3 + ax^2 + x + 1 = 0$ are indentical

$\therefore \; \frac{1}{b} = \frac{1}{a} = \frac{a}{1} = \frac{b}{1} \Rightarrow a^2 = 1$ and $a = b$

$\Rightarrow a = b = \pm 1$

14. **(a,c)** $\frac{a+b+c}{3} \geq \frac{3}{\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)}$

$\Rightarrow (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$

$\therefore \; (x-1)^2 + (y-2)^2 - 9$

$+ (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 0$

So the equality holds if and only if $x - 1 = 0, y - 2 = 0$ and $a = b = c$

15. **(a,b,d)** We have, $x^2 + y^2 - 8x - 8y = 20$ ....(i)

and $xy + 4x + 4y = 40$ ....(ii)

Multiplying (ii) by 2 an adding it to (i) we get $x^2 + y^2 + 2xy = 100$ or $(x + y)^2 = 100$

$\Rightarrow x + y = \pm 10 \;\Rightarrow |x + y| = 10$

16. **(a,b,c)** $5^{\log_5(x^2 - 9x + 24)} > x - 1$

$\Rightarrow \; x^2 - 9x + 24 > x - 1$

$\Rightarrow \; x^2 - 10x + 25 > 0 \Rightarrow (x-5)^2 > 0.$

Which is true for $\forall x \in R - \{5\}$.

Also the inequality holds for $x = 5$

17. **(b, c)** $f(x) = x^4 + ax^3 - 13x^2 + bx - 4 = 0$

If $2 + \sqrt{5}$ is one root then other root has to be $2 - \sqrt{5}.$

Let α be repeated root then

$(2+\sqrt{5})(2-\sqrt{5})\alpha^2 = -4$

$\Rightarrow \quad \alpha^2 = 4 \text{ or } \alpha = \pm 2$ ...(1)

Also, $\sum \alpha\beta = -13$

$\therefore$

$-1 + (4 - 2\sqrt{5})\alpha + (4 + 2\sqrt{5})\alpha + \alpha^2 = -13$

$\alpha^2 + 8\alpha + 12 = 0$ or $(\alpha + 6)(\alpha + 2) = 0$

$\alpha = -2, -6$ ...(2)

From (1) and (2), $\alpha = -2$

$\Rightarrow \quad 2 + \sqrt{5} + 2 - \sqrt{5} + 2(-2) = a$

or $a = 0$ or $f(-2) = 0$

$\therefore \quad 16 - 52 - 2b - 4 = 0 \;\Rightarrow\; \boldsymbol{b} = -20$

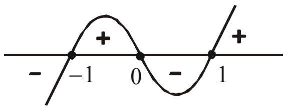
18. (a,b,c) $|x^3-1| \ge 1-x \Rightarrow |x-1|(x^2+x+1) \ge 1-x$

$[\because x^2+x+1>0]$

Let $x<1$, then we get $(1-x)(x^2+x+1) \ge 1-x$

$\Rightarrow (x-1)(x^2+x+1-1) \le 0$

$\Rightarrow x(x+1)(x-1) \le 0.$



Solving by method of intervals, we get

$x \in (-\infty, -1] \cup [0, 1]$

Let $x \ge 1$, then we get $(x-1)(x^2+x+1) \ge 1-x$

$\Rightarrow (x-1)(x^2+x+2) \ge 0$

$\Rightarrow x \ge 1 \qquad [\because x^2+x+2>0 \ \forall x \in \mathbf{R}]$

$\Rightarrow x \in [1, \infty)$

Combining the two solutions, we get

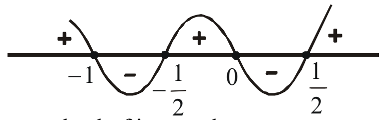
$x \in (\infty, -1] \cup [0, 1] \cup [1, \infty)$

or $x \in (-\infty, -1] \cup [0, \infty)$

19. (a,b,c) $\dfrac{2x-1}{2x^3+3x^2+x} > 0$

$\Rightarrow \dfrac{2x-1}{x(2x^2+3x+1)} > 0 \Rightarrow \dfrac{2x-1}{x(x+1)(2x+1)} > 0$

$\Rightarrow x(x+1)(2x+1)(2x-1) > 0, x \ne 0, -1, -\dfrac{1}{2}$



Using method of intervals, we get

$x<-1$ or $-\dfrac{1}{2} < x < 0$ or $x > \dfrac{1}{2}$.

So $x \in (-\infty, -1) \cup \left(-\dfrac{1}{2}, 0\right) \cup \left(\dfrac{1}{2}, \infty\right)$.

Thus, $S = (-\infty, -1) \cup \left(\dfrac{1}{2}, 0\right) \cup \left(\dfrac{1}{2}, \infty\right)$

E MATRIX-MATCH TYPE

1. A - p, q, r, s; B - p; C - r, s; D - q

(A) $\dfrac{2x-1}{2x^3+3x^2+x} > 0 \Rightarrow \dfrac{2x-1}{x(2x+1)(x+1)} > 0$

$\therefore \ x \in (-\infty, -1) \cup \left(-\dfrac{1}{2}, 0\right) \cup \left(\dfrac{1}{2}, \infty\right)$

(B) We have $a^2-3a+2<0 \Rightarrow 1<a<2$

(C) Let $\sqrt{x-1}=t \Rightarrow x=t^2+1$, so the equation is

$\sqrt{t^2+4-4t}+\sqrt{t^2+9-6t}=1 \Rightarrow |t-2|+|t-3|=1$

whose solution is $2 \le t \le 3 \Rightarrow 5 \le x \le 10$

(D) Let $\alpha, \beta, \gamma, \delta$ be the roots, then

$\alpha+\beta+\gamma+\delta=8$ and $\alpha\beta\gamma\delta=16$

$\Rightarrow \dfrac{\alpha+\beta+\gamma+\delta}{4} = (\alpha\beta\gamma\delta)^{1/4} = 2$

$\therefore \ \alpha=\beta=\gamma=\delta=2$

or $x^4-8x^3+bx^2-cx+16=(x-2)^4 \Rightarrow b=24, c=32$

2. A - q, r; B -t; C - r; D - p, r

$(\Sigma\alpha)^2 = \Sigma\alpha^2 - 2\Sigma\alpha\beta \Rightarrow \Sigma\alpha\beta = -1$

Also, $\Sigma\alpha^3 - 3\alpha\beta\gamma = \Sigma(\alpha)(\Sigma\alpha^2 - \Sigma\alpha\beta) \Rightarrow \alpha\beta\gamma = -2$

(A) $(\Sigma\alpha^2)^2 = \Sigma\alpha^4 + 2\Sigma\alpha^2\beta^2 = \Sigma\alpha^4$

$+2[(\Sigma\alpha\beta)^2 - 2\alpha\beta\gamma(\Sigma\alpha)]$

$\therefore \ \Sigma\alpha^4 = 18$

(B) Clearly α, β, γ are roots of the equation.

$x^3-2x^2-x+2=0.$

So, $(x-\alpha)(x-\beta)(x-\gamma) = x^3-2x^2-x+2$

Put $x=1$, then $(1-\alpha)(1-\beta)(1-\gamma)=0$

(C) As the roots of $x^3-2x^2-x+2=0$ are -1, 1 and 2, so, for $-1<x<1$, $x^3-2x^2-x+2>0$

(Local max of $f(x)=x^3-2x^2-x+2$ is less than 18)

(D) As α, β, γ are equal to -1, 1 and 2, so

$(1+\alpha^2)(1+\beta^2)(1+\gamma^2)=20$

3. A - p, q, s; B - p, r; C - p, s; D - p

(A) Let the common difference of AP be t then

$2(-t)+x(-t)^2+(2t)^3 = 2(-3t)+(-2t)^2+(-t)^3$

$\Rightarrow 9t^3+(x-4)t^2+4t=0$ and $t \ne 0$

$\Rightarrow 9t^2+(x-4)t+4=0$

$t \in R \Rightarrow (x-4)^2-144 \ge 0$

$\Rightarrow x \le -8$ or $x \ge 16$

(B) $\alpha+\beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

If roots of other equation be γ and δ then

$$\gamma+\delta = \left(-\frac{b}{a}\right)\left(\frac{c}{a}\right)^2 = (\alpha+\beta)(\alpha^2\beta^2) = \alpha^3\beta^2 + \alpha^2\beta^3$$

$$\gamma\,\delta = \left(\frac{c}{a}\right)^5 = (\alpha\beta)^5 = (\alpha^3\beta^2)(\alpha^2\beta^3)$$

So the roots of the second equation are $\alpha^3\beta^2$ and $\alpha^2\beta^3$

$\Rightarrow (\alpha^3\beta^2)(\alpha^2\beta^3) = 4 \times 8 \Rightarrow |\alpha\beta| = 2$

(C) $\log_3(\log_5 x) + \log_{1/3}(\log_{1/5} y) = 1$

$$\Rightarrow \log_3\left(\frac{\log_5 x}{\log_{1/5} y}\right) = 1 \Rightarrow \frac{\log_5 x}{\log_{1/5} y} = 3$$

$$\Rightarrow \log_5\left(\frac{x}{y}\right) = -3 \Rightarrow \frac{x}{y} = 5^{-3}$$

Also, $x^2 y = 1 \Rightarrow x = \frac{1}{5}$ and $y = 5^2 \Rightarrow 5x + y = 26$

(D) $x_1 + x_2 + x_1x_2 = a$ ...(1)

$x_1x_2 + x_1^2x_2 + x_1x_2^2 = b$ ....(2)

$x_1^2x_2^2 = c$ ...(3)

From equation (2),

$x_1x_2(1+x_1+x_2) = b \Rightarrow x_1x_2(1+a-x_1x_2) = b$

$$\Rightarrow x_1x_2(1+a) - c = b \Rightarrow x_1x_2 = \frac{b+c}{1+a},$$

which is a rational number.

F NUMERIC/INTEGER ANSWER TYPE

1. Ans : 2

$|x+2|$ vanishes at $x = -2$ and $|2^{x+1} - 1|$ vanishes at $x = -1$, hence we divide the problem into three intervals :

(i) If $x < -2$, then $|x+2| = -(x+2)$

Also $x+1 < -1 \Rightarrow 2^{x+1} < 2^{-1} = \frac{1}{2}$

$\Rightarrow 2^{x+1} < 1 \Rightarrow |2^{x+1} - 1| = -(2^{x+1} - 1)$

$\therefore$ Equation is $2^{-x-2} + 2^{x+1} - 1 = 2^{x+1} + 1$

$\Rightarrow 2^{-x-2} = 2 \Rightarrow x = -3$

(ii) If $-2 \le x < -1$, then $|x+2| = x+2$

Also, $x+1 < 0 \Rightarrow 2^{x+1} < 1$

$\Rightarrow |2^{x+1} - 1| = -(2^{x+1} - 1)$

$\therefore$ Equation is $2^{x+2} + 2^{x+1} - 1 = 2^{x+1} + 1$

$\Rightarrow 2^{x+2} = 2 \Rightarrow x = -1 \notin [-2, -1)$

(iii) If $x \ge -1$, then $|x+2| = x+2$ and

$|2^{x+1} - 1| = 2^{x+1} - 1$

$\therefore$ Equation is $2^{x+2} - 2^{x+1} + 1 = 2^{x+1} + 1$

$\Rightarrow 2^{x+2} = 2^{x+2}$, which is identity.

$\therefore$ All x such that $x \ge -1$ satisfy the equation.

Hence, the solution set is $x \in \{-3\} \cup [-1, \infty)$

2. Ans : 8

The inequatity is $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$ ... (1)

The L. H. S. is valid if

(i) $x^2 - 10x + 22 > 0 \Rightarrow x < 5 - \sqrt{3}$ or $x > 5 + \sqrt{3}$

(ii) $\frac{x}{2} > 0 \Rightarrow x > 0$

Now the inequatity (1) will be solved for two cases of $\log_2\left(\frac{x}{2}\right)$.

Case1 : $0 < \log_2\left(\frac{x}{2}\right) < 1 \Rightarrow 1 < \frac{x}{2} < 2 \Rightarrow 2 < x < 4$

The inequatity in this case is

$$\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$$

$\Rightarrow x^2 - 10x + 22 < 1 \Rightarrow x^2 - 10x + 21 < 0 \Rightarrow 3 < x < 7.$

The common solution is $3 < x < 4$

Case2 : $\log_2\left(\frac{x}{2}\right) > 1 \Rightarrow \frac{x}{2} > 2 \Rightarrow x > 4$. The inequatity is then $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$

$\Rightarrow\ x^2 - 10x + 22 > 1 \Rightarrow\ x^2 - 10x + 21 > 0$

$\Rightarrow\ x < 3$ or $x > 7$.

The common solution is $x > 7$.

$\therefore$ The values of x from two cases are $x \in (3, 4) \cup (7, \infty)$

Now taking intersection with intital values of x, we get

$x \in \left(3, 5-\sqrt{3}\right) \cup (7, \infty)$

3. Ans : 4

$|[x] - 2x| = 4 \Rightarrow [x] - 2x = \pm 4 \Rightarrow [x] - 2[x] - 2\{x\} = \pm 4$

$\Rightarrow -[x] - 2\{x\} = \pm 4 \Rightarrow \{x\} = \dfrac{-[x] \mp 4}{2}$

If $\{x\} = \dfrac{-[x]-4}{2}$, then $0 \le \{x\} < 1 \Rightarrow 0 \le -[x] - 4 < 2$

$\Rightarrow -6 < [x] \le -4$

$\therefore [x] = -5$ or $-4 \Rightarrow \{x\} = \dfrac{1}{2}$ or 0

$\therefore x = -5 + \dfrac{1}{2}$ or $-4 + 0 \Rightarrow x = -\dfrac{9}{2}$ or -4

If $\{x\} = \dfrac{-[x]+4}{2}$, then $0 \le \{x\} < 1$

$\Rightarrow 0 \le -[x] + 4 < 2 \Rightarrow 2 < [x] \le 4$

$\therefore [x] = 3$ or $4 \Rightarrow \{x\} = \dfrac{1}{2}$ or 0

$\therefore x = 3 + \dfrac{1}{2}$ or $4 + 0 \Rightarrow x = \dfrac{7}{2}$ or 4

$\therefore$ Solution set is $x \in \left\{-\dfrac{9}{2}, -4, \dfrac{7}{2}, 4\right\}$

4. Ans : 7

Without loss of generality we can assume that $a, b \ge 0$.

Now

If $c, d \ge 0$ then $p(1) = a + b + c + d \le 1$

$\Rightarrow |a| + |b| + |c| + |d| \le 1$

If $c \ge 0$, $d < 0$ then $|a| + |b| + |c| + |d|$

$= a + b + c - d = (a + b + c + d) - 2d$

$= p(1) - 2p(0) \le 1 + 2 = 3$

If $c < 0$, $d \ge 0$ then $|a| + |b| + |c| + |d| = a + b - c + d$

$= \dfrac{4}{3}p(1) - \dfrac{1}{3}p(-1) - \dfrac{8}{3}p\left(\dfrac{1}{2}\right) + \dfrac{8}{3}p\left(-\dfrac{1}{2}\right) \le \dfrac{4}{3} + \dfrac{1}{3} + \dfrac{8}{3} + \dfrac{8}{3} = 7$

Finally : $d < 0, c < 0$ then

$|a| + |b| + |c| + |d| = a + b - c - d$

$= \dfrac{5}{3}p(1) - 4p\left(\dfrac{1}{2}\right) + \dfrac{4}{3}p\left(-\dfrac{1}{2}\right) \le \dfrac{5}{3} + 4 + \dfrac{4}{3} = 7$

$\therefore |a| + |b| + |c| + |d| \le 7$

5. Ans : 1

Let $f(x) = x^3 - x^2 + \beta x + \gamma$ ...(1)

$f(x) = 0$ has three positive real roots in G.P.

$\Rightarrow\ f'(x) = 0$ will have two distinct real roots

$\Rightarrow\ 3x^2 - 2x + \beta = 0$ has two distinct roots

$\therefore\ D > 0$

$4 - 12\beta > 0 \Rightarrow \beta < \dfrac{1}{3}$ ...(2)

Also from the equation (1)

$x_1x_2 + x_2x_3 + x_3x_1 = \beta$

$x_1 + x_2 + x_3 = 1$

$x_1x_2x_3 = -\gamma$

$\Rightarrow\ x_2^3 = -\gamma > 0$ $\quad (\because\ x_2^2 = x_1x_3)$

$\Rightarrow\ \gamma < 0$

Also, $x_2(x_1 + x_3) + x_2^2 = \beta$

$\Rightarrow\ x_2(1 - x_2) + x_2^2 = \beta \Rightarrow x_2 = \beta > 0$

From (2) and (4), we get $0 < \beta < \dfrac{1}{3}$ and $\gamma < 0$.

$[\beta]_{max.} = 0, [\gamma]_{max.} = -1$.

Hence $[\beta] + [\gamma] + 2 = 1$.