MATHEMATICS

Differential Equations

Speed TEST 83

Chapter-wise

No. of Questions	Maximum Marks	Time
30	**120**	**1 Hour**

GENERAL INSTRUCTIONS

- This test contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.
- You have to evaluate your Response Grids yourself with the help of solutions provided at the end of this book.
- Each correct answer will get you 4 marks and 1 mark shall be deduced for each incorrect answer. No mark will be given/ deducted if no bubble is filled. Keep a timer in front of you and stop immediately at the end of 60 min.
- The sheet follows a particular syllabus. Do not attempt the sheet before you have completed your preparation for that syllabus.
- After completing the sheet check your answers with the solution booklet and complete the Result Grid. Finally spend time to analyse your performance and revise the areas which emerge out as weak in your evaluation.

1. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) n^2y (b) $-n^2y$

(c) $-y$ (d) $2x^2y$

2. The solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2 \text{ is}$$

(a) $y = \log(x^2 + cx)$ (b) $\log y = x\left(cx^2 + \frac{1}{2}\right)$

(c) $x = \log y\left(cx^2 + \frac{1}{2}\right)$ (d) None of these

3. The solution of the differential equation $\frac{d^3y}{dx^3} - 8\frac{d^2y}{dx^2} = 0$

safisfying $y(0) = \frac{1}{8},\ y'(0) = 0$ and $y''(0) = 1$ is

(a) $y = \frac{1}{8}\left[\frac{e^{8x}}{8} - x + \frac{7}{8}\right]$ (b) $y = \frac{1}{8}\left[\frac{e^{8x}}{8} + x + \frac{7}{8}\right]$

(c) $y = \frac{1}{8}\left[\frac{e^{8x}}{8} + x - \frac{7}{8}\right]$ (d) $y = \frac{1}{8}\left[\frac{e^{8x}}{8} - x - \frac{7}{8}\right]$

4. The solution of $\left(\frac{x\,dx + y\,dy}{x\,dy - y\,dx}\right) = \sqrt{\left(\frac{a^2 - x^2 - y^2}{x^2 + y^2}\right)}$ is

(a) $\sqrt{(x^2+y^2)} = a \sin\{(\tan^{-1} y/x) + \text{constant}\}$

(b) $\sqrt{(x^2+y^2)} = a \cos\{(\tan^{-1} y/x) + \text{constant}\}$

(c) $\sqrt{(x^2+y^2)} = a\{\tan(\sin^{-1} y/x) + \text{constant}\}$

(d) $\sqrt{(x^2+y^2)} = a\{\tan(\cos^{-1} y/x) + \text{constant}\}$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. The gradient of the curve passing through (4, 0) is given by $\frac{dy}{dx}-\frac{y}{x}+\frac{5x}{(x+2)(x-3)}=0$ if the point $(5, a)$ lies on the curve, then the value of a is

(a) $\frac{67}{12}$ (b) $5\sin\frac{7}{12}$

(c) $5\log\frac{7}{12}$ (d) None of these

6. The differential equations of all conics whose axes coincide with the co-ordinate axis

(a) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$

(b) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2+x\frac{dy}{dx}=0$

(c) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

(d) $xy\frac{d^2y}{dx^2}-x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$

7. The equation of the curve satisfying the equation $(xy-x^2)\frac{dy}{dx}=y^2$ and passing through the point (–1, 1) is

(a) $y=(\log y-1)x$ (b) $y=(\log y+1)x$

(c) $x=(\log x-1)y$ (d) $x=(\log x+1)y$

8. If for the differential equation $y'=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$, the general solution is $y=\frac{x}{\log|Cx|}$, then $\phi(x/y)$ is given by

(a) $-x^2/y^2$ (b) y^2/x^2

(c) x^2/y^2 (d) $-y^2/x^2$

9. The solution to the differential equation $\frac{dy}{dx}=\frac{yf'(x)-y^2}{f(x)}$ where $f(x)$ is a given function is

(a) $f(x)=y(x+c)$ (b) $f(x)=cxy$

(c) $f(x)=c(x+y)$ (d) $yf(x)=cx$

10. The solution of the differential equation $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$ is:

(a) $\tan y \tan x = c$ (b) $\frac{\tan y}{\tan x}=c$

(c) $\frac{\tan^2 x}{\tan y}=c$ (d) None of these

11. The solution of the differential equation

$\log x\frac{dy}{dx}+\frac{y}{x}=\sin 2x$ is

(a) $y\log|x|=C-\frac{1}{2}\cos x$ (b) $y\log|x|=C+\frac{1}{2}\cos 2x$

(c) $y\log|x|=C-\frac{1}{2}\cos 2x$ (d) $xy\log|x|=C-\frac{1}{2}\cos 2x$

12. If $y=y(x)$ and $\frac{2+\sin x}{1+y}\left(\frac{dy}{dx}\right)=-\cos x$, $y(0)=1$,

then $y\left(\frac{\pi}{2}\right)$ equals

(a) 1/3 (b) 2/3

(c) –1/3 (d) 1

13. The solution of the differential equation

$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$ is given by

(a) $ye^{2\sqrt{x}}=2\sqrt{x}+c$ (b) $ye^{-2\sqrt{x}}=\sqrt{x}+c$

(c) $y=\sqrt{x}$ (d) $y=3\sqrt{x}$

14. The line normal to a given curve at each point (x, y) on the curve passes through the point (3, 0). If the curve contains the point (3, 4) then its equation is

(a) $x^2+y^2+6x-7=0$ (b) $2(x^2+y^2)-12x-7=0$

(c) $x^2+y^2-6x-7=0$ (d) None of these

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. Differential equation of all conics of the form $ax^2 + by = 1$, a and b being parameters is :

(a) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(b) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

(c) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(d) None of these

16. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is

(a) $x\,\phi(y/x) = k$ (b) $\phi(y/x) = kx$
(c) $y\,\phi(y/x) = k$ (d) $\phi(y/x) = ky$
(k is arbitrary constant)

17. The solution of $\cos y\frac{dy}{dx} = e^{x+\sin y} + x^2 e^{\sin y}$ is

(a) $e^x - e^{-\sin y} + \frac{x^3}{3} = C$ (b) $e^{-x} - e^{-\sin y} + \frac{x^3}{3} = C$

(c) $e^x + e^{-\sin y} + \frac{x^3}{3} = C$ (d) $e^x - e^{\sin y} - \frac{x^3}{3} = C$

18. Which of the following equations represents the curve for which the intercept cut-off by any tangent on y-axis is proportional to the square of the ordinate of the point of tangency?

(a) $x + y = cxy$ (b) $\frac{1}{x} + \frac{1}{y} = c$

(c) $\frac{A}{x} + \frac{B}{y} = xy$ (d) $\frac{A}{x} + \frac{B}{y} = 1$

19. The equation of the curve which is such that the portion of the axis of x-cut off between the origin and tangent at any point is proportional to the ordinate of that point is

(a) $x = y\,(a - b\log y)$ (b) $\log x = by^2 + a$
(c) $x^2 = y\,(a - b\log y)$ (d) None of these

20. If $y + \frac{d}{dx}(xy) = x\,(\sin x + \log x)$, then

(a) $y = \cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(b) $y = -\cos x - \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(c) $y = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x - \frac{x}{3}\log x - \frac{x}{9} + \frac{c}{x^2}$

(d) None of these

21. The equation of the curve passing through the point $\left(a, -\frac{1}{a}\right)$ and satisfying the differential equation $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ is

(a) $(x + a)(1 + ay) = -4a^2y$ (b) $(x + a)(1 - ay) = 4a^2y$

(c) $(x + a)(1 - ay) = -4a^2y$ (d) None of these

22. The real value of n for which the substitution $y = u^n$ will transform the differential equation $2x^4y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is

(a) 1/2 (b) 1
(c) 3/2 (d) 2

23. The solution of the differential equation

$\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$ is

(a) $y = \log(x^2 + cx)$ (b) $\log y = x\left(cx^2 + \frac{1}{2}\right)$

(c) $x = \log y\left(cx^2 + \frac{1}{2}\right)$ (d) None of these

24. The family of curves satisfying the differential equation $\frac{dy}{dx} - 2y\tan x + y^2\tan^4 x = 0$ is

(a) $y\sec^2 x = 5\tan^2 x + c$ (b) $y\sin^5 x + 5\cos^2 x$

(c) $5\sec^2 x = y(\tan^5 x + c)$ (d) None of these

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If a curve passes through the point $\left(2, \frac{7}{2}\right)$ and has slope $\left(1-\frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is – 2 is :

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

26. The curve that satisfies the differential equation $y' = \frac{x^2 + y^2}{2xy}$ and passes through (2, 1) is a hyperbola with eccentricity :

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) 2 (d) $\sqrt{5}$

27. The family of curves for which the area of the triangle formed by the x-axis, the tangent drawn at any point on the curve and radius vector of the point of tangency is constant equal to $2a^2$, is given by

(a) $x = cy \pm \frac{a^2}{y}$ (b) $y = cx \pm \frac{a^2}{x}$

(c) $x^2 \pm ay^2 = cy$ (d) $a^2x^2 \pm y^2 = cy$

28. An integrating factor of the differential equation $\frac{dy}{dx} = y \tan x - y^2 \sec x$ is equal to :

(a) $\tan x$ (b) $\sec x$

(c) $\operatorname{cosec} x$ (d) $\cot x$

29. **Statement-1 :** The differential equation of the form $yf(xy)\,dx + x\phi(xy)\,dy = 0$ can be converted to homogeneous forms by substitution $xy = v$.

Statement-2 : All differential equation of first order and first degree become homogeneous, if we put $y = vx$.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement -1
(b) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1
(c) Statement -1 is False, Statement -2 is True
(d) Statement - 1 is True, Statement- 2 is False

30. **Statement-1 :** The general solution of $\frac{dy}{dx} + y = 1$ is $ye^x = e^x + c$

Statement-2 : The number of arbitrary constants in the general solution of the differential equation is equal to the order of differential equation.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

RESPONSE GRID	
	25. ⓐⓑⓒⓓ 26. ⓐⓑⓒⓓ 27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

MATHEMATICS CHAPTERWISE SPEED TEST-83			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	38	Qualifying Score	55
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Speed Test-83

1. (a) $y=(x+\sqrt{1+x^2})^n$

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{1}{2}(1+x^2)^{-1/2}.\,2x\right);$$

$$\frac{dy}{dx}=n(x+\sqrt{1+x^2})^{n-1}\frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}=\frac{n(\sqrt{1+x^2}+x)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\frac{dy}{dx}=ny$ or $\sqrt{1+x^2}y_1=ny$ $\quad(\because y_1=\frac{dy}{dx})$

Squaring, $(1+x^2)y_1{}^2=n^2y^2$

Differentiating, $(1+x^2)2y_1y_2+y_1{}^2.2x=n^2.2yy_1$

(Here, $y_2=\frac{d^2y}{dx^2}$) or $(1+x^2)y_2+xy_1=n^2y$

2. (c) Divide the equation by $y(\log y)^2$

$$\frac{1}{y(\log y)^2}\frac{dy}{dx}+\frac{1}{\log y}\cdot\frac{1}{x}=\frac{1}{x^2}$$

Put $\frac{1}{\log y}=z\Rightarrow\frac{-1}{y(\log y)^2}\frac{dy}{dx}=\frac{dz}{dx}$

Thus, we get, $-\frac{dz}{dx}+\frac{1}{x}\cdot z=\frac{1}{x^2}$, linear in z

$$\Rightarrow\frac{dz}{dx}+\left(-\frac{1}{x}\right)z=-\frac{1}{x^2}$$

$$\text{I.F.}=e^{-\int\frac{1}{x}dx}=e^{-\log x}\quad 1$$

$\therefore$ The solution is, $z\left(\frac{1}{x}\right)=\int\frac{-1}{x^2}\left(\frac{1}{x}\right)dx+c$

$$\Rightarrow\frac{1}{\log y}\left(\frac{1}{x}\right)=\frac{-x^{-2}}{-2}+c\ \Rightarrow x=\log y\left(cx^2+\frac{1}{2}\right)$$

3. (a) Let $\frac{d^2y}{dx^2}=t\Rightarrow\frac{d^3y}{dx^3}=\frac{dt}{dx}$ and the given equation recuces to $\frac{dt}{dx}=8t$.

Separating the variables, $\frac{dt}{t}=8dx$. Integrating we get.

$\ell n\,t=8x+c_1\Rightarrow\ell n\,y''=8x+c_1$

Put $x=0$, then $y''=1\Rightarrow C_1=0$

$\therefore\ell n\,y''=8x\Rightarrow y''=e^{8x}$,

Again integrate, we get $y'=\frac{e^{8x}}{8}+c_2$

Again putting $x=0$ and $y'=0\Rightarrow c_2=-\frac{1}{8}$

$$\therefore y'=\frac{e^{8x}}{8}-\frac{1}{8}\Rightarrow y=\frac{1}{8}\left(\frac{e^{8x}}{8}-x\right)+c_3$$

[After integration]

By giving values $x=0$, and $y=\frac{1}{8}$, we get $c_3=\frac{7}{64}$

Hence the final solution is $y=\frac{1}{8}\left[\frac{e^{8x}}{8}-x+\frac{7}{8}\right]$

4. (a) Taking $x = r\cos\theta$ and $y = r\sin\theta$, so that

$x^2 + y^2 = r^2$ and $\frac{y}{x} = \tan\theta$,

we have $x\,dx + y\,dy = r\,dr$

and $x\,dy - y\,dx = x^2\sec^2\theta d\theta = r^2\,d\theta$.

The given equation can be transformed into

$$\frac{r\,dr}{r^2 d\theta} = \sqrt{\left(\frac{a^2 - r^2}{r^2}\right)}$$

$$\Rightarrow \frac{dr}{d\theta} = \sqrt{(a^2 - r^2)}$$

$$\Rightarrow \frac{dr}{\sqrt{(a^2 - r^2)}} = d\theta$$

Integrating both sides, then we get

$$\sin^{-1}\left(\frac{r}{a}\right) = \theta + c$$

$$\Rightarrow \sin^{-1}\left(\frac{\sqrt{(x^2+y^2)}}{a}\right) = \tan^{-1}\left(\frac{y}{x}\right) + c \quad \text{...(i)}$$

$$\Rightarrow \sqrt{(x^2+y^2)} = a\ \sin\{\tan^{-1}(y/x) + c\}$$

$$\Rightarrow \sqrt{(x^2+y^2)} = a\ \sin\{\tan^{-1}(y/x) + \text{constant}\}$$

5. (c) The differential equation is $\frac{dy}{dx} - \frac{y}{x} = -\frac{5x}{(x+2)(x-3)}$

$$\text{I. F} = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\ln x} = \frac{1}{x}$$

Solution is $y\left(\frac{1}{x}\right) = \int\left(\frac{1}{x}\right) \times \frac{5x}{(x+2)(x-3)}dx = \ln\left(\frac{x+2}{x-3}\right) + C$

It passes through (4, 0), so $C = -\ln 6$

$$\therefore\ y = x\ln\left\{\frac{(x+2)}{6(x-3)}\right\}$$

Putting (5, a), we get $a = 5\ln\left(\frac{7}{12}\right)$

6. (c) Any conic whose axes coincide with co-ordinate axis is

$ax^2 + by^2 = 1$ ..(i)

Diff. both sides w.r.t. 'x', we get

$$2ax + 2by\frac{dy}{dx} = 0 \text{ i.e. } ax + by\frac{dy}{dx} = 0 \quad \text{..(ii)}$$

$$\text{Diff. again, } a + b\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = 0 \quad \text{..(iii)}$$

$$\text{From (ii), } \frac{a}{b} = -\frac{y\,dy/dx}{x}$$

$$\text{From (iii), } \frac{a}{b} = -\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right)$$

$$\therefore\ \frac{y\frac{dy}{dx}}{x} = y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

7. (a) We have, $(xy - x^2)\frac{dy}{dx} = y^2$

$$\Rightarrow y^2\frac{dx}{dy} = xy - x^2 \Rightarrow \frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}\cdot\frac{1}{y} = \frac{-1}{y^2}$$

Putting $\frac{1}{x} = u$ so that $\frac{-1}{x^2}\frac{dx}{dy} = \frac{du}{dy}$

We obtain $\frac{du}{dy} + \frac{u}{y} = \frac{1}{y^2}$. Which is linear.

$$\text{I.F.} = e^{\int\frac{1}{y}dy} = e^{\log y} = y$$

Hence the solution is $uy = \int\frac{1}{y^2}.y\,dy + C$

or $\frac{y}{x} = \log y + C$ or $y = x(\log y + C)$

This passes through the point (–1, 1),

$\therefore\ 1 = -1(\log 1 + C)$ i.e $C = -1$

Thus, the equation of the curve is $y = x(\log y - 1)$.

8. (d) Putting $v = y/x$ so that $x\frac{dv}{dx} + v = \frac{dy}{dx}$

We have $x\frac{dv}{dx} + v = v + \phi(1/v)$

$$\Rightarrow \frac{dv}{\phi(1/v)} = \frac{dx}{x}$$

$$\Rightarrow \log|Cx| = \int\frac{dv}{\phi(1/v)}$$

(C being constant of integration)

But $y = \frac{x}{\log|Cx|}$ is the general solution,

So $\frac{x}{y} = \frac{1}{v} = \log|Cx| = \int\frac{dv}{\phi(1/v)}$

$\Rightarrow\ \phi(1/v) = -1/v^2$

(differentiating w.r.t. v both sides)

$\Rightarrow\ \phi(x/y) = -y^2/x^2$

9. (a) We have $\frac{dy}{dx} = \frac{f'(x)}{f(x)}y - \frac{y^2}{f(x)}$

$$\Rightarrow \frac{dy}{dx} - \frac{f'(x)}{f(x)}y = -\frac{y^2}{f(x)}$$

Divide by y^2

$$y^{-2}\frac{dy}{dx} - y^{-1}\frac{f'(x)}{f(x)} = -\frac{1}{f(x)}$$

$$\frac{dy}{dx} = \frac{dz}{dx}$$

$$-\frac{dz}{dx}-\frac{f'(x)}{f(x)}(z)=-\frac{1}{f(x)} \Rightarrow \frac{dz}{dx}+\frac{f'(x)}{f(x)}z=\frac{1}{f(x)}$$

$$\text{I.F.}=e^{\int\frac{f'(x)}{f(x)}dx}=e^{\log f(x)}=f(x)$$

$\therefore$ The solution is $z(f(x))=\int\frac{1}{f(x)}(f(x))dx+c$

$$\Rightarrow y^{-1}(f(x))=x+c \Rightarrow f(x)=y(x+c)$$

10. (a) Given differential equation is
$\sec^2 x \tan y\, dx+\sec^2 y \tan x\, dy=0$
On separating the variables
(dividing the equation by $\tan x$ tany)

$$\Rightarrow \frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy$$

On integrating both sides, we get

$$\int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy$$

Put $\tan x=u \Rightarrow \sec^2 x\,.dx=du$
and $\tan y=v \Rightarrow \sec^2 y.dy=dv$

$$\therefore \int\frac{du}{u}=-\int\frac{dv}{v} \Rightarrow \log u=-\log v+\log c$$

$$\Rightarrow u=\frac{c}{v} \Rightarrow u.v=c$$

$\therefore$ Required solution is $\tan x.\tan y=c$

11. (c) $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\sin 2x}{\log x}$

$$\text{I.F.}=e^{\int\frac{dx}{x\log x}}$$

$$\therefore \text{I.F.}=e^{\int\frac{1}{t}dt}=e^{\log t}=t=\log|x|$$

solution is given by

$$y(\text{I.F.})=\int Q.(\text{I.F.})\,dx+C$$

$$y\log|x|=\int\frac{\sin 2x}{\log|x|}(\log|x|)\,dx+C$$

$$=-\frac{\cos 2x}{2}+C$$

12. (a) $\frac{dy}{dx}\left(\frac{2+\sin x}{1+y}\right)=-\cos x, y(0)=1$

$$\Rightarrow \frac{dy}{(1+y)}=\frac{-\cos x}{2+\sin x}dx$$

Integrating both sides

$$\Rightarrow \ln(1+y)=-\ln(2+\sin x)+C$$

Put $x=0$ and $y=1$

$$\Rightarrow \ln(2)=-\ln 2$$

Put $x=\frac{\pi}{2}$

$$\ln(1+y)=-\ln 3+\ln 4=\ln\frac{4}{3} \Rightarrow y=\frac{1}{3}$$

13. (a) $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{\sqrt{x}}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \qquad \ldots(i)$$

$$\Rightarrow \frac{dy}{dx}+P(y)=Q$$

This is linear differential equation.

Here, $P=\frac{1}{\sqrt{x}}$ and $Q=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$\therefore$ I.F $=e^{\int Pdx}=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

$\therefore$ Solution is

$$y.e^{2\sqrt{x}}=\int e^{2\sqrt{x}}.\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx \Rightarrow y.e^{2\sqrt{x}}=2\sqrt{x}+c$$

14. (c) The equation of normal to a curve at a point (x, y) is

$$(Y-y)\frac{dy}{dx}+(X-x)=0$$

Since it passes throgh the point (3, 0), we have

$$(0-y)\frac{dy}{dx}+(3-x)=0 \Rightarrow y\frac{dy}{dx}=(3-x) \Rightarrow ydy=(3-x)dx$$

Integrating, we get $\frac{y^2}{2}=3x-\frac{x^2}{2}+C$

$\Rightarrow x^2+y^2-6x-2c=0$

Since the curve passes through (3, 4), we have

$$9+16-18-2c=0 \Rightarrow c=\frac{7}{2}.$$

$\therefore x^2+y^2-6x-7=0$ is the required equation of the curve.

15. (a) The given equation is $ax^2+by^2=1$.

Differentiating we get, $2ax+2by\frac{dy}{dx}=0$

$$\Rightarrow ax+by\frac{dy}{dx}=0, \qquad \text{....(1)}$$

Differentiating again, $a+b\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)=0, \qquad$ ....(2)

From eqs. (1) and (2), we get

$$a=-\frac{by}{x}\frac{dy}{dx}=-b\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)$$

$$\frac{d^2y}{\quad}\quad\left(\frac{dy}{\quad}\right)^2\quad\frac{dy}{\quad}=0$$

16. **(b)** Let $\frac{y}{x}=v$

$\therefore \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

Then, $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi(y/x)}{\phi'(y/x)}$ reduces to

$v+x\frac{dv}{dx}=v+\frac{\phi(v)}{\phi'(v)}$

$\Rightarrow \quad \frac{\phi'(v)}{\phi(v)}dv=\frac{dx}{x}$

On integrating, we get

$\log\phi(v)=\log x+\log k$

$\Rightarrow \quad \phi(v)=xk$

$\Rightarrow \quad \phi\left(\frac{y}{x}\right)=kx$

17. **(c)** $\cos y\frac{dy}{dx}=e^x.e^{\sin y}+x^2e^{\sin y}=e^{\sin y}(x^2+e^x)$

$\Rightarrow \quad \frac{\cos y}{e^{\sin y}}\frac{dy}{dx}=(x^2+e^x)\Rightarrow\int\frac{\cos y}{e^{\sin y}}dy=\int(x^2+e^x)dx$

$\sin y=t$

$\cos y\, dy=dt$

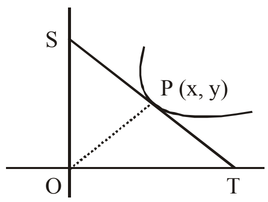
$\Rightarrow \int e^{-t}dt=\frac{x^3}{3}+e^x+C'$

$\Rightarrow \frac{e^{-t}}{-1}=\frac{x^3}{3}+e^x+C' \Rightarrow e^x+e^{-\sin y}+\frac{x^3}{3}=C$

18. **(d)** The equation of tangent at any point P(x,y) is

$Y-y=\frac{dy}{dx}(X-x)$

For Y intercept put $X=0$



$\Rightarrow Y=y-x\frac{dy}{dx}$

Given $Y\propto y^2 \Rightarrow y-x\frac{dy}{dx}=ky^2$

[k proportionality constant]

$\Rightarrow\frac{dy}{dx}-\frac{1}{x}y=-\frac{ky^2}{x} \quad \Rightarrow y^{-2}\frac{dy}{dx}-\frac{1}{x}y^{-1}=-\frac{k}{x}$

Put $y^{-1}=z\Rightarrow -y^{-2}\frac{dy}{dx}=\frac{dz}{dx}$.

Then $-\frac{dz}{dx}-\frac{1}{x}z=-\frac{k}{x} \quad \Rightarrow\frac{dz}{dx}+\left(\frac{1}{x}\right)z=\frac{k}{x}$

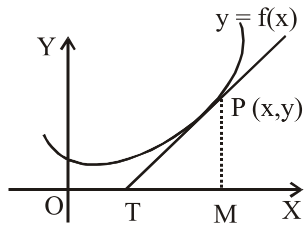
I.F. $=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

The solution is $z(x)=\int\frac{k}{x}(x)dx+c$

$y^{-1}x=kx+c \Rightarrow \frac{x}{y}=kx+c \Rightarrow \frac{1}{ky}=1+\frac{c}{kx}$

$\Rightarrow\frac{\left(\frac{1}{k}\right)}{y}+\frac{\left(-\frac{c}{k}\right)}{x}=1 \Rightarrow \frac{A}{x}+\frac{B}{y}=1$

19. **(a)** Let the equation of the curve be $y=f(x)$.



It is given that OT $\propto y$

$\Rightarrow$ OT $=by$

$\Rightarrow$ OM – TM $=by$

$\Rightarrow \quad x-\frac{y}{dy/dx}=by$ [$\because$ TM = Length of the subtangent]

$\Rightarrow \quad x-y\frac{dx}{dy}=b.y \quad \Rightarrow \quad \frac{dx}{dy}-\frac{x}{y}=-b$

It is linear differential equation. Its solution is given by

$\frac{x}{y}=-b\log y+a\Rightarrow x=y(a-b\log y)$

20. **(d)** The given differential equation can be written as

$x\frac{dy}{dx}+2y=x(\sin x+\log x)$

$\Rightarrow \quad \frac{dy}{dx}+\frac{2}{x}.\ y=\sin x+\log x$

which is linear in y i.e. of the type $\frac{dy}{dx}+Py=Q$

Hence $P=\frac{2}{x}$

$\therefore \quad \int P\,dx=2\log x=\log x^2$

$\therefore \quad e^{\int P\,dx}=e^{\log x^2}=x^2$

$\therefore \quad$ Sol. is $y.x^2=\int x^2(\sin x+\log x)dx+c$

$=-x^2\cos x+2x\sin x+2\cos x+\frac{x^2}{3}\log x-\frac{x^2}{9}+c$

i.e. $y=-\cos x+\frac{2}{x}\sin x+\frac{2}{x^2}\cos x+\frac{x}{3}\log x-\frac{x}{9}+\frac{c}{x^2}$

21. **(c)** We have $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$\Rightarrow ydx - xdy = ay^2dx + ady$

$\Rightarrow y(1-ay)dx = (x+a)dy$

$\Rightarrow \frac{dx}{x+a} - \frac{dy}{y(1-ay)} = 0$

Integrating, we get

$\log(x+a) - \log y + \log(1-ay) = \log C$

or $\log\frac{(a+x)(1-ay)}{y} = \log C$ i.e. $(x+a)(1-ay) = Cy$

Since the curve passes through $\left(a, -\frac{1}{a}\right)$

$\therefore\ 2a\times(1+1) = -\frac{C}{a}$ i.e $C = -4a^2$

So, $(x+a)(1-ay) = -4a^2y$

22. **(c)** $\because y = u^n$

$\therefore \frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$

On substituting the values of y and $\frac{dy}{dx}$ in the given equation, then

$2x^4.u^n.nu^{n-1}\frac{du}{dx} + u^{4n} = 4x^6$

$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4n}}{2nx^4u^{2n-1}}$

Since, it is homogeneous. Then, the degree of $4x^6 - u^{4n}$ and $2nx^4\ u^{2n-1}$ must be same.

$\therefore\ 4n = 6$ and $4 + 2n - 1 = 6$

Then, we get $n = \frac{3}{2}$

23. **(c)** Divide the equation by $y(\log y)^2$

$\frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{1}{\log y}\cdot\frac{1}{x} = \frac{1}{x^2}$

Put $\frac{1}{\log y} = z \Rightarrow \frac{-1}{y(\log y)^2}\frac{dy}{dx} = \frac{dz}{dx}$

Thus, we get, $-\frac{dz}{dx} + \frac{1}{x}\cdot z = \frac{1}{x^2}$, linear in z

$\Rightarrow \frac{dz}{dx} + \left(-\frac{1}{x}\right)z = -\frac{1}{x^2}$

$\text{I.F.} = e^{-\int\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

$\therefore$ The solution is, $z\left(\frac{1}{x}\right) = \int\frac{-1}{x^2}\left(\frac{1}{x}\right)dx + c$

$\Rightarrow \frac{1}{\log y}\left(\frac{1}{x}\right) = -$

24. **(c)** Divide the equation by y^2, we get

$y^{-2}\frac{dy}{dx} - (2\tan x)y^{-1} = -\tan^4 x$ [see the Bernoulli's equation]

Put $y^{-1} = z \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$

Hence, $-\frac{dz}{dx} - (2\tan x).z = -\tan^4 x$

$\Rightarrow \frac{dz}{dx} + (2\tan x)z = \tan^4 x$

Which is linear in z, Integrating factor,

$\text{I.F.} = e^{\int 2\tan x dx} = e^{2\log|\sec x|} = \sec^2 x$

The solution is $z(\sec^2 x) = \int(\tan^4 x)\sec^2 x dx + a$

$y^{-1}(\sec^2 x) = \frac{1}{5}\tan^5 x + a \Rightarrow 5\sec^2 x = y(\tan^5 x + c),\ C = 5a$

25. **(a)** Slope $= \frac{dy}{dx} = 1 - \frac{1}{x^2} \Rightarrow \int dy = \int\left(1 - \frac{1}{x^2}\right)dx$

$\Rightarrow y = x + \frac{1}{x} + C$, which is the equation of the curve

since curve passes through the point $\left(2, \frac{7}{2}\right)$

$\therefore\ \frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$

$\therefore\ y = x + \frac{1}{x} + 1$

when $x = -2$, then $y = -2 + \frac{1}{-2} + 1 = \frac{-3}{2}$

26. **(a)** Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, the given equation reduces to the form

$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x(vx)} = \frac{1+v^2}{2v} \Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1-v^2}{2v}$

$\Rightarrow \frac{2v\,dv}{1-v^2} = \frac{dx}{x} \Rightarrow -\log(1-v^2) = \log x - a$

$\Rightarrow \log(1-v^2)x = a \Rightarrow x(1-v^2) = c$, where $c = e^a$

When $x = 2,\ v = \frac{1}{2}$, [when $x = 2,\ y = 1,\ v = \frac{1}{2}$]

so that $c = \frac{3}{2}$

So the equation of the curve is $x^2 - y^2 = \frac{3x}{2}$

$\Rightarrow \left(x - \frac{3}{4}\right)^2 - y^2 = \frac{9}{16}$

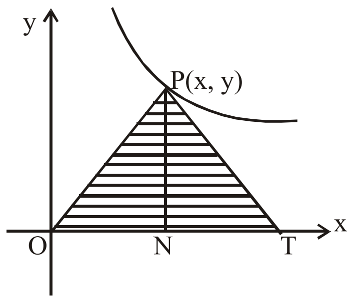
which is a rectangular hyperbola with eccentricity $\sqrt{2}$.

$\sqrt{2}$]

27. (a) Tangent drawn at any point P (x, y) is

$$Y - y = \frac{dy}{dx}(X - x) \qquad ...(i)$$

The triangle , whose area is given is Δ OPT (see the shaded region in the adjacent figure) If coordinates of T are (X, 0) then



$X = x - y\frac{dx}{dy}$ (Put Y = 0 in (i)) $\therefore$ area of Δ OPT

$= \frac{1}{2}|X \cdot y| = 2a^2$ (given)

$\therefore \frac{1}{2}|xy - y^2\frac{dx}{dy}| = 2a^2 \Rightarrow xy - y^2\frac{dx}{dy} = \pm 2a^2$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = \pm\frac{2a^2}{y^2}$, which is linear in x

The integrating factor is I.F. $= e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$

The solution is $x\left(\frac{1}{y}\right) = \int \pm\frac{2a^2}{y^2}\left(\frac{1}{y}\right)dy + c$

$\Rightarrow \frac{x}{y} = \pm\frac{2a^2y^{-2}}{-2} + c \Rightarrow x = cy \pm \frac{a^2}{y}$

28. (b) Consider the differential equation

$$\frac{dy}{dx} = y\tan x - y^2\sec x$$

Divide by y^2 on both the sides, we get

$$\frac{1}{y^2}\left(\frac{dy}{dx}\right) = \frac{\tan x}{y} - \sec x \quad ...(1)$$

Let $\frac{1}{y} = z$

Diff both side, we get

$$\frac{-1}{y^2}\cdot\frac{dy}{dx} = \frac{dz}{dx}$$

Put value of $\frac{1}{y^2}\frac{dy}{dx}$ in the equation(1) , we get

$$-\left(\frac{dz}{dx}\right) - (\tan x)z = -\sec x$$

$$\Rightarrow \left(\frac{dz}{dx}\right) + (\tan x)z = \sec x$$

This is the linear diff equation in 'z' i.e.

This is of the form $\frac{dz}{dx} + P.z = Q$

then integrating factor = $e^{\int Pdx}$

$\therefore$ In the given question

I.F. = $e^{\int \tan x\,dx} = e^{\log(\sec x)} = \sec x$

29. (c) $\because xy = v$

$\therefore \quad x\frac{dy}{dx} + y = \frac{dv}{dx}$

Then, the given equation reduces to

$$\frac{v}{x}f(v) + x\phi(v)\left(\frac{1}{x}\left(\frac{dv}{dx} - y\right)\right) = 0$$

$$\Rightarrow \frac{v}{x}f(v) + \phi(v)\frac{dv}{dx} - y\phi(v) = 0$$

$$\Rightarrow \left\{\frac{v(f(v) - \phi(v))}{x}\right\} + \phi(v)\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{dx}{x} + \frac{\phi(v)dv}{vf(v) - \phi(v)} = 0$$

Which is variable seperable form.

30. (c) $\frac{dy}{dx} + y = 1 \Rightarrow \frac{dy}{1-y} = dx$

$$\int\frac{dy}{1-y} = \int dx - \log(1-y) = x$$

$1 - y = e^{-x}$, $ye^x = e^x + c$

Order of differential equation is the number of orbitarary constants.

Both are true but Statement 2 is not correct reason.