[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.11 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Q.1 The complex number z which satisfies the equations $|z| = 1$ and $\left|\frac{z-\sqrt{2}(1+i)}{z}\right| = 1$ is

(A) 1 (B) $1+i$ (C) $\frac{1+i}{\sqrt{2}}$ (D) $\frac{-1-i}{\sqrt{2}}$

Q.2 If $z \in C$ then area enclosed by the curve $2\left|z^2+|z|^2\right| + 3\left|z^2-|z|^2\right| = 6|z|$, $z \neq 0$ is

(A) 3 sq. units (B) 6 sq. units (C) 3π sq. units (D) 6π sq. unit

Q.3 The complex numbers z_1, z_2 satisfies $\frac{z_2}{z_1} = \frac{1+i\sqrt{3}}{2}$, then origin, z_1 and z_2 are the vertices of a triangle which is

(A) of area zero
(B) right-angled isosceles
(C) equilateral
(D) obtuse-angled isosceles

Q.4 Let $z = \cos\theta + i\sin\theta$, then the value of $\sum_{n=0}^{9} \text{Re}\left(\frac{4i}{z^{2n+1}}\right)$ at $\theta = 3°$ is equal to

[**Note :** Re (z) denotes real part of z and $i = \sqrt{-1}$.]

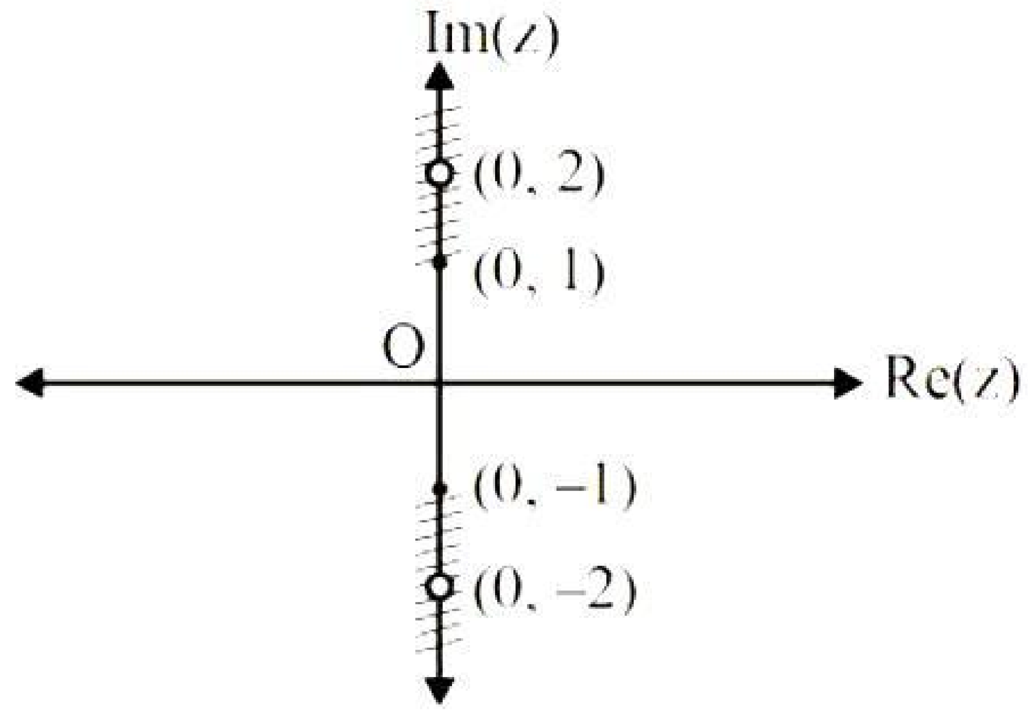
(A) $\frac{1}{2}$ cosec 3° (B) $\frac{1}{4}$ cosec 3° (C) cosec 3° (D) $\frac{1}{10}$ cosec 3°

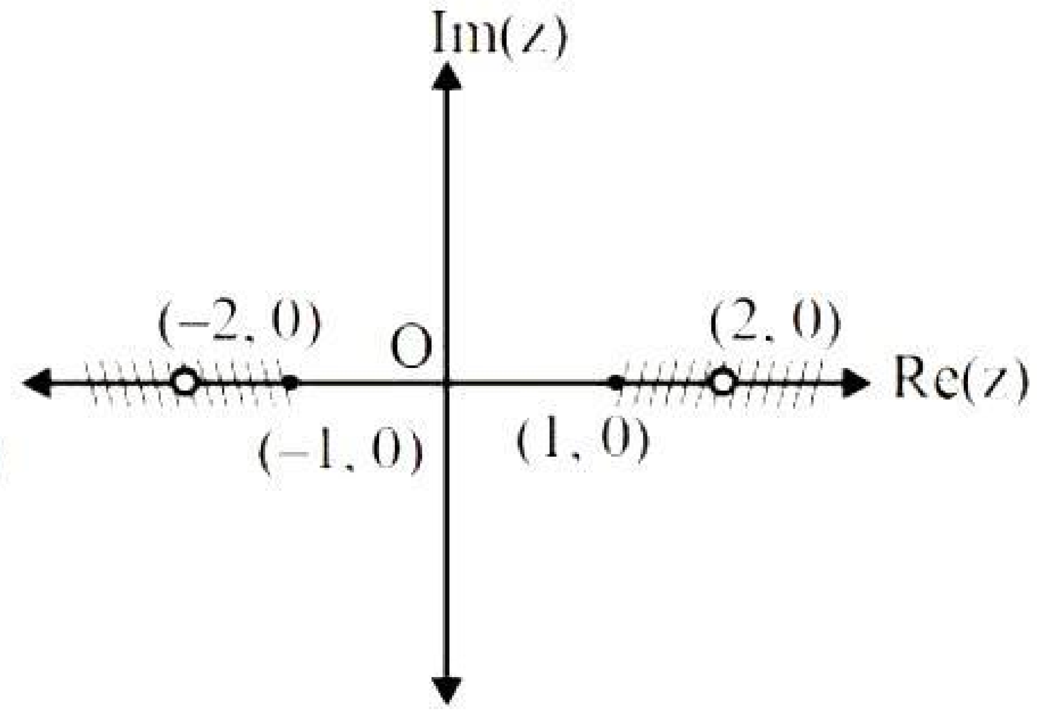
Q.5 Let $z = x + iy$ be a complex number where $x, y \in R$ and $i = \sqrt{-1}$. The area bounded by locus of $P(z)$ satisfying $|z-1| = 2\,I_m(z)$ and coordinate axes on the complex plane, is equal to

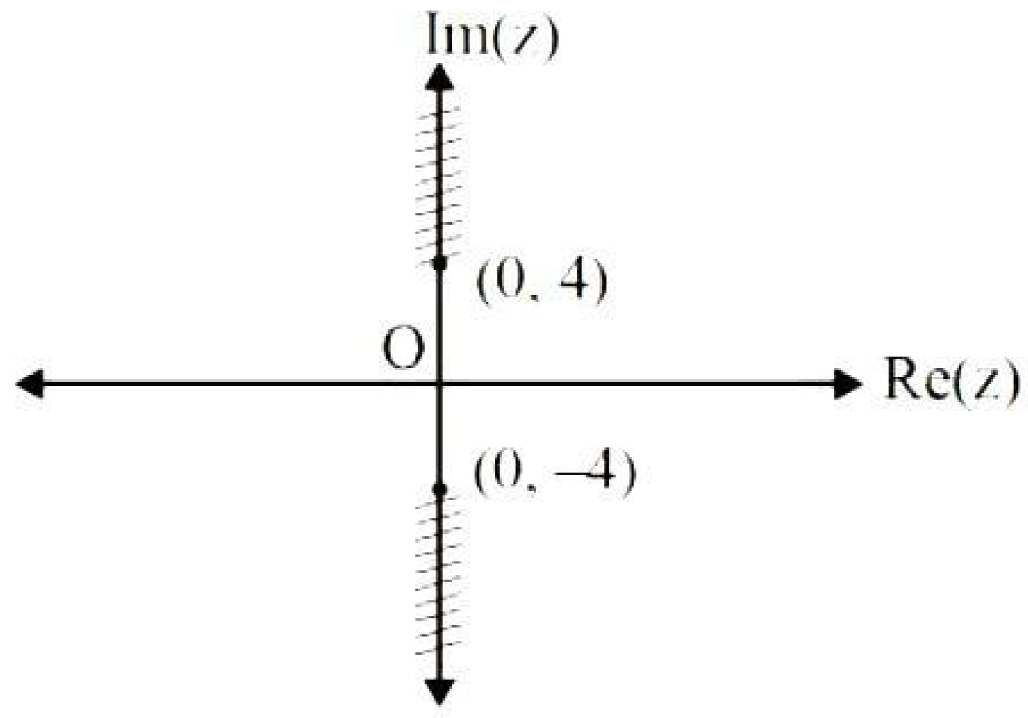
(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

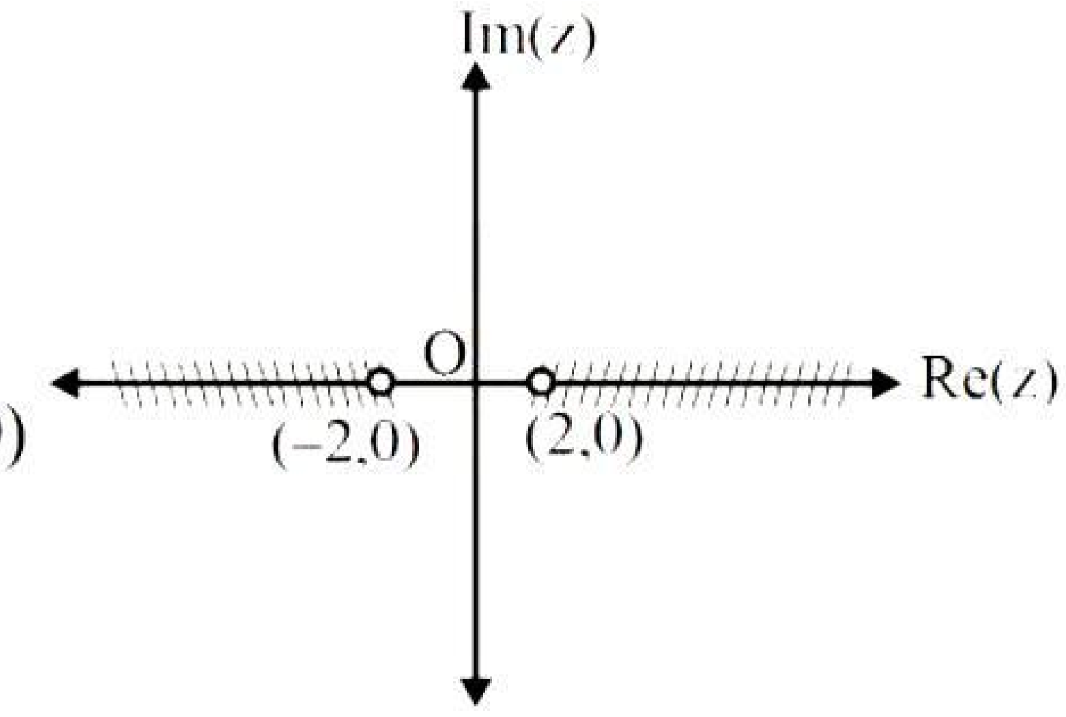
[**Note :** $I_m(z)$ denotes imaginary part of z.]

Q.6 The set $\left\{\left(\frac{-4iz}{4-z^2}\right) : z \text{ is a complex number}, |z| = 2, z \neq \pm 2\right\}$ is best represented by

(A)


(B)


(C)


(D)


Q.7 The area enclosed by the curves C_1 : $\arg(z-2) = \frac{\pi}{4}$, C_2 : $\arg(z-2) = \frac{3\pi}{4}$ and $C_3 : (\text{Re}(z-2))^2 = \left(\frac{z-\bar{z}}{2i}\right)$ on the complex plane is

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{5}{6}$ (D) $\frac{4}{3}$

Q.8 The number of complex numbers z satisfying simultaneously $|z+1-i| = 2$ and $\text{Re}(z) \geq 1$ equals

(A) 0 (B) 1
(C) 2 (D) infinite

Q.9 The complex number z satisfies the condition $\left|z - \frac{25}{z}\right| = 24$. The maximum distance from the origin of co-ordinates to the point z is

(A) 25 (B) 30 (C) 32 (D) 16

Q.10 If $z \in C$ and satisfies the equation $(z-\bar{z})^2 = 4|z|^2 - 12$ then the maximum value of $|z|$ is

(A) $\sqrt{6}$ (B) $2\sqrt{3}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$

Q.11 Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin. Then n must be of the form

(A) 4k + 1 (B) 4k + 2 (C) 4k + 3 (D) 4k

[PARAGRAPH TYPE]

Q.12 to Q.14 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Paragraph for question nos. 12 to 14

Let two curves $C_1 : x^2 + y^2 = 2$ and C_2 : locus of z which satisfies $\left| |z + 3\sqrt{2}| - |z - 3\sqrt{2}| \right| = 2\sqrt{2}$.

Q.12 If z_0 lies on C_1 then maximum value of $|z_0 + 4 + 4i|$ is

(A) $5\sqrt{2}$ (B) $6\sqrt{2}$ (C) $8\sqrt{2}$ (D) $16\sqrt{2}$

Q.13 If $z_1\ (\text{Re}(z_1) > 0)$ lies on C_1 and C_2 both and z_2 is obtained by rotating z_1 about origin in anticlockwise direction through $135°$ and $z_3 = \bar{z}_2$ then real part of $(z_1 + z_2 + z_3)$ equals

(A) $-2-\sqrt{2}$ (B) $-\sqrt{2}+1$ (C) $-2+\sqrt{2}$ (D) $-\sqrt{2}-1$

Q.14 If locus of z satisfying $|\arg(z-1)| = \tan^{-1}(4)$ meets the curve C_2 at A and B then area of the triangle formed by A, B and C where complex number corresponding to C is $e^{i\,2\pi}$, is

(A) 4 sq. units (B) $8\sqrt{2}$ sq. units (C) 16 sq. units (D) $16\sqrt{2}$ sq. units

[MULTIPLE CORRECT CHOICE TYPE]

Q.15 to Q.16 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct.

Q.15 Let z be a variable complex number satisfying $\frac{2z - \bar{z}}{3} = a - \frac{1}{3} + ib$ where $a, b \in R$ and $a^2 - b^2 = 1$. If locus of z is the curve 'C' then

(A) director circle of the curve 'C' is $(x+1)^2 + y^2 = 10$.

(B) area of the triangle formed by the straight line $x - 3y + 1 = 0$ and tangent at one of the vertices and the x-axis is $\frac{3}{2}$ sq. units.

(C) distance between directrices of the curve 'C' is $\frac{18}{\sqrt{10}}$.

(D) equation of the circle passing through the points (2, 1) and both foci of the curve 'C' is $x^2 + y^2 + 2x - 9 = 0$.

Q.16 Let P(z) satisfies $z\bar{z} + (4-5i)\bar{z} + (4+5i)z = 40$. If $a = \max.|z + 2 - 3i|$ and $b = \min.|z + 2 - 3i|$, then

(A) $a + b = 18$ (B) $a + b = 9$ (C) $a - b = 4\sqrt{2}$ (D) $ab = 73$

[INTEGER TYPE]

Q.17 to Q.20 are "Integer Type" questions. (The answer to each of the questions **are upto <u>4 digits</u>**)

Q.17 Consider the circle $|z - 5i| = 3$ and two points z_1 and z_2 on it are such that $|z_1| < |z_2|$ and $\arg. z_1 = \arg. z_2 = \frac{\pi}{3}$. A tangent is drawn at z_2 to the circle, which cuts the real axis at z_3, then find $|z_3|^2$.

Q.18 Consider $\alpha_k = \cos\frac{2k\pi}{13} + i\sin\frac{2k\pi}{13}$, $k = 0, 1, 2, \ldots\ldots, 12$, if $S = \frac{\sum_{k=1}^{12}\left|\overline{\alpha}_k + \alpha_{12-k}\right|}{\sum_{k=1}^{6}\left|\alpha_{2k-1} + \alpha_{2k}\right|}$, then find the value of S.

Q.19 Let $A(z_1)$ be the point of intersection of curves $\arg(z - 2 + i) = \frac{3\pi}{4}$ and $\arg(z + \sqrt{3}i) = \frac{\pi}{3}$, $B(z_2)$ be the point on $\arg(z + \sqrt{3}i) = \frac{\pi}{3}$ such that $|z_2 - 5|$ is minimum and $C(z_3)$ be the centre of circle $|z - 5| = 3$. If Δ denotes area of triangle ABC, then find the value of Δ^2.

Q.20 Let $P(z) = z^3 + az^2 + bz + c$ where a, b and c are real. There exists a complex number ω such that the three roots of $P(z)$ are $\omega + 3i$, $\omega + 9i$ and $2\omega - 4$ where $i^2 = -1$. Find the value of $|a + b + c|$.

ANSWER KEY

Q.1	C	Q.2	A	Q.3	C	Q.4	C	Q.5	D
Q.6	B	Q.7	B	Q.8	B	Q.9	A	Q.10	C
Q.11	D	Q.12	A	Q.13	C	Q.14	A	Q.15	BCD
Q.16	ACD	Q.17	[11]	Q.18	[2]	Q.19	[12]	Q.20	[136]

[HINT SOLUTION]

Q.1 $\frac{1+i}{\sqrt{2}}$

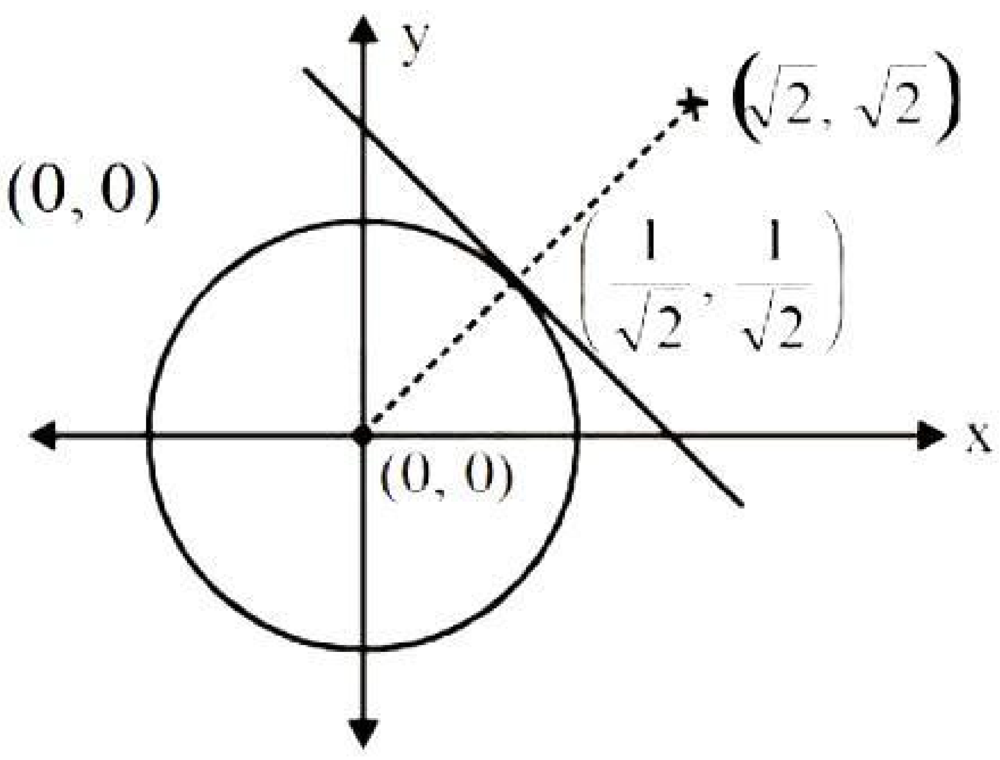
Sol. $\because \left|\frac{z-\sqrt{2}(1+i)}{z}\right| = 1 \Rightarrow \left|z-\sqrt{2}(1+i)\right| = |z-0|$

$\therefore$ z is on perpendicular bisector of line joining $(\sqrt{2}, \sqrt{2})$ and $(0, 0)$

$\therefore$ (0, 0) is centre of circle $|z| = 1$

$\therefore$ point z will be $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore z = \frac{1+i}{\sqrt{2}}$

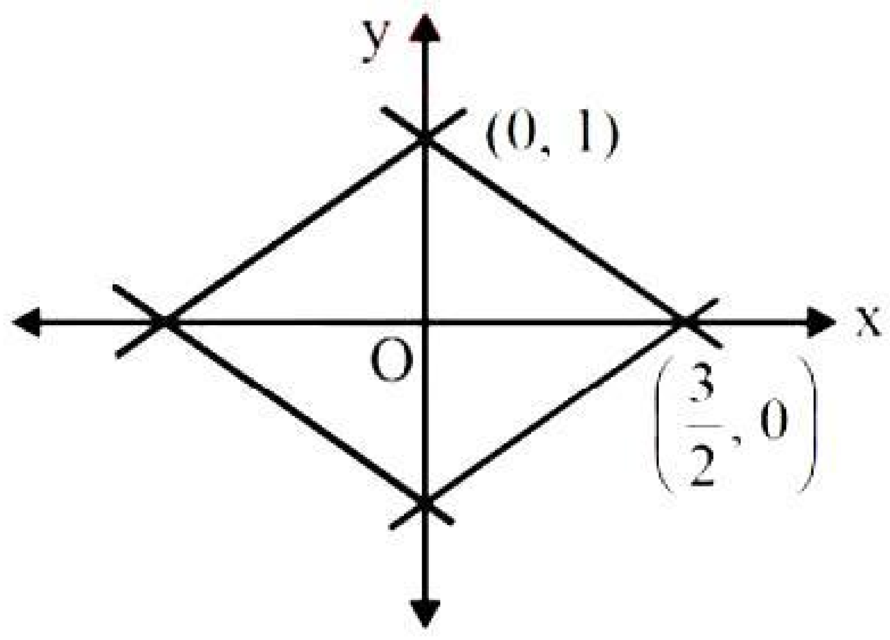


Q.2 3 sq. units

Sol. $2|z + \bar{z}| + 3|z - \bar{z}| = 6.$

$4|x| + 6|y| = 6$

$2|x| + 3|y| = 3$

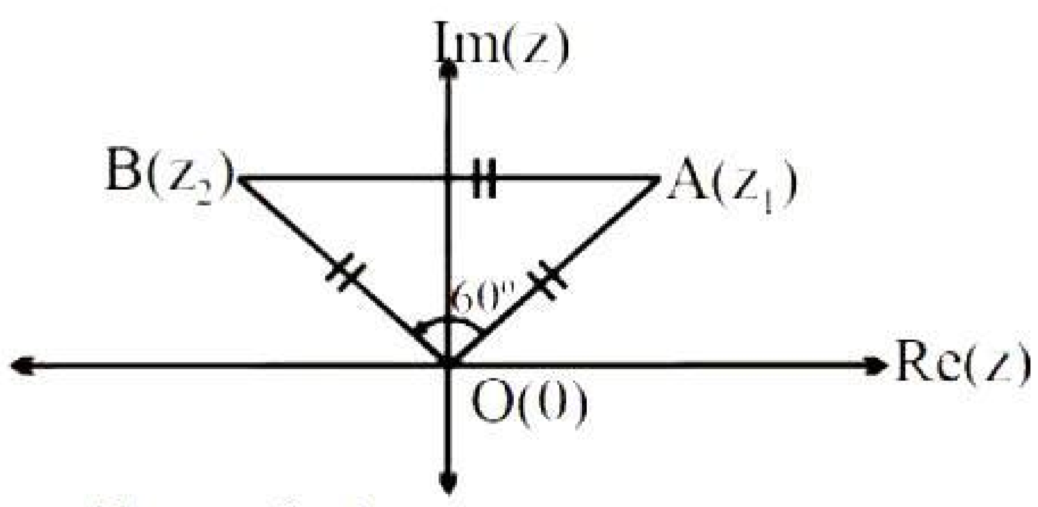


Required area $= 4 \times \frac{1}{2} \times \frac{3}{2} \times 1 = 3.$

Q.3 equilateral

Sol. Given, $\frac{z_2 - 0}{z_1 - 0} = e^{\frac{i\pi}{3}} \Rightarrow \left|\frac{z_2-0}{z_1-0}\right| = \left|e^{\frac{i\pi}{3}}\right| = 1$

$\Rightarrow |z_2 - 0| = |z_1 - 0|$



Also, $\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{3} \Rightarrow \angle z_1oz_2 = \frac{\pi}{3} \Rightarrow$ Triangle is equilateral. **Ans.**

Aliter :

$\frac{z_2}{z_1} = \frac{1+i\sqrt{3}}{2}$

Subtract 1 on b.t.s., we get

$$\Rightarrow \frac{z_2}{z_1} - 1 = \frac{-1 + i\sqrt{3}}{2} \quad \Rightarrow |z_2 - z_1| = |z_1| = |z_2|$$

$\Rightarrow$ Triangle is equilateral.

Q.4 cosec 3° (D) $\frac{1}{10}$ cosec 3°

Sol. As, $\frac{1}{z^{2n+1}} = (\cos(2n+1)\theta - i\sin(2n+1)\theta)$

$$\therefore \frac{4i}{z^{2n+1}} = 4i\left(\cos(2n+1)\theta - i\sin(2n+1)\theta\right) \Rightarrow \text{Re}\left(\frac{4i}{z^{2n+1}}\right) = 4\sin(2n+1)\theta$$

$$\therefore \sum_{n=0}^{9} \text{Re}\left(\frac{4i}{z^{2n+1}}\right) = 4\sum_{n=0}^{9} \sin(2n+1)\theta = 4(\sin\theta + \sin 3\theta + \sin 5\theta + \ldots\ldots + \sin 19\theta)$$

$$= 4\left(\frac{\sin(10\theta)\cdot\sin(10\theta)}{\sin\theta}\right) = 4 \times \frac{\sin^2(30°)}{\sin 3°} = \text{cosec } 3°.$$

Q.5 $\frac{1}{2\sqrt{3}}$

Sol. Given $|z - 1| = 2\,I_m(z)$

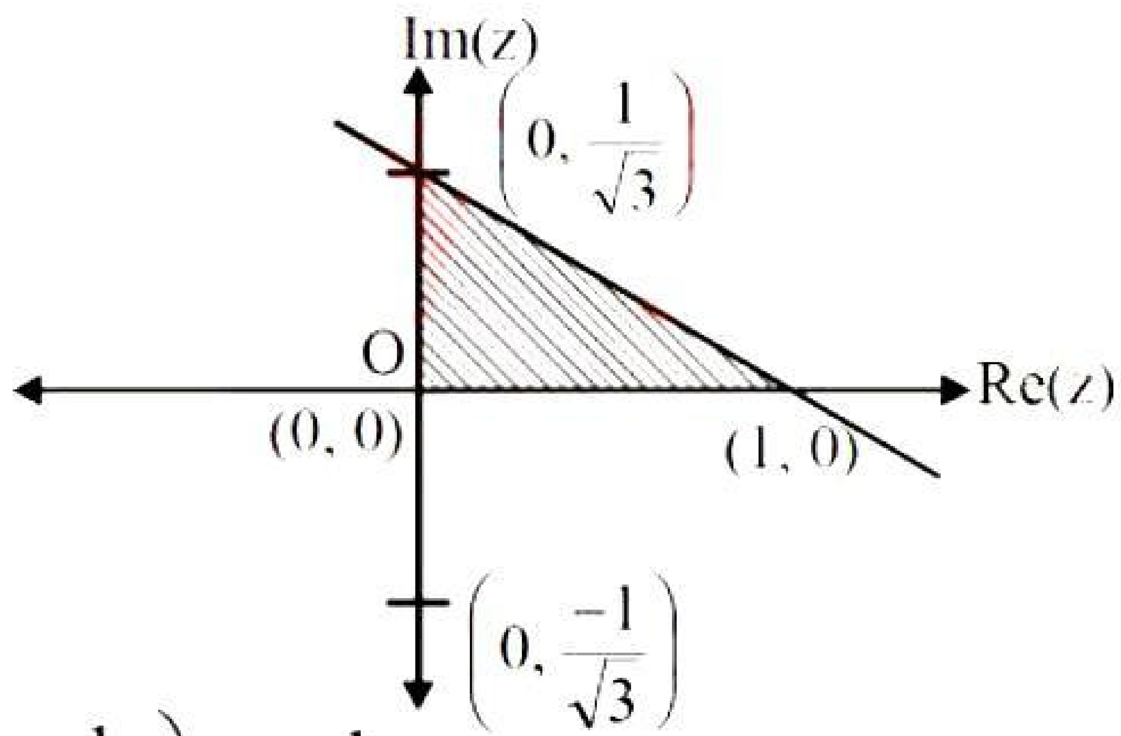
$\therefore (x-1)^2 + y^2 = 4y^2 \Rightarrow 3y^2 = (x-1)^2$

$\Rightarrow \pm\sqrt{3}\,y = (x-1)$

$\therefore L_1 : x - \sqrt{3}\,y - 1 = 0$ (Rejected) think?

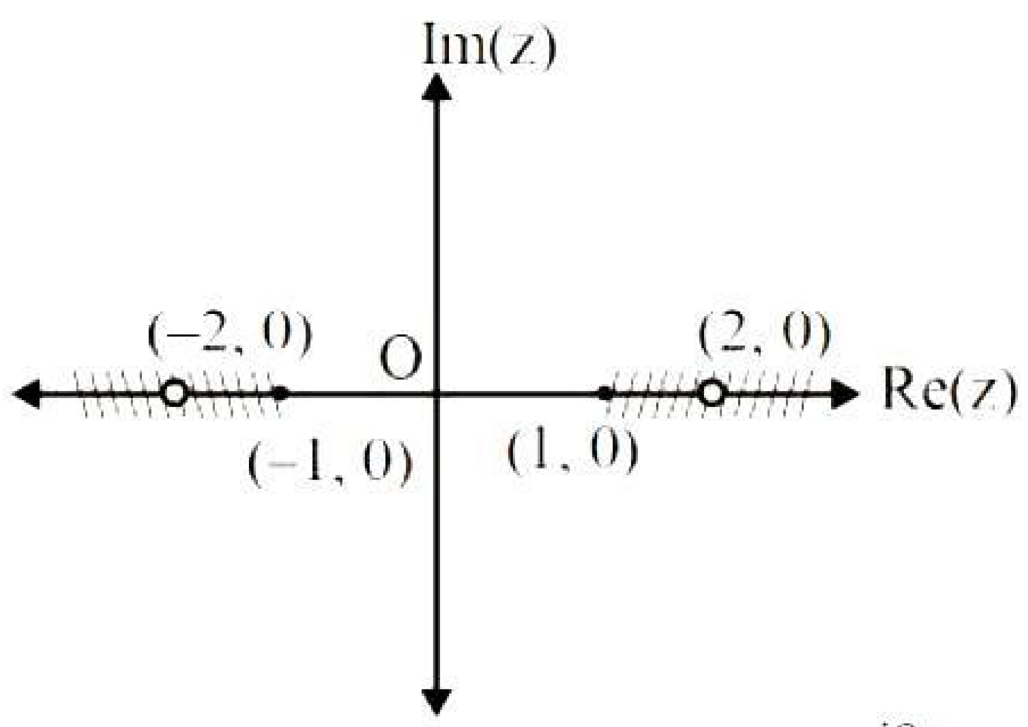
and $L_2 : x + \sqrt{3}\,y - 1 = 0$.

Also, $L_3 : x = 0$ and $L_4 : y = 0$



So, area of triangle formed by L_2, L_3 and L_4 is $= \left(\frac{1}{2} \times 1 \times \frac{1}{\sqrt{3}}\right) = \frac{1}{2\sqrt{3}}$.

Q.6



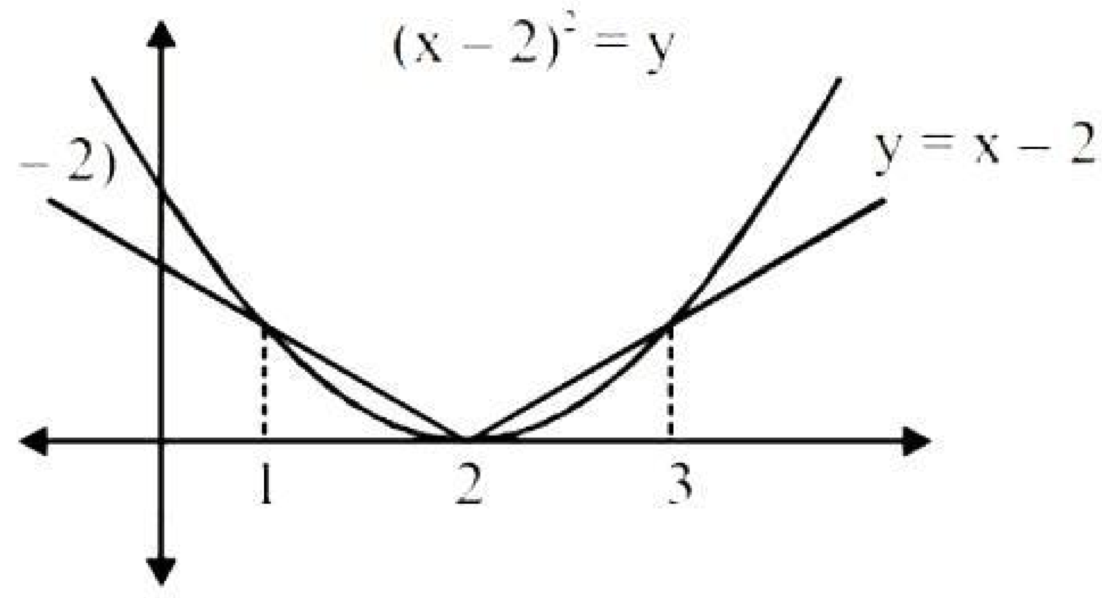
Sol. Consider $\frac{-4iz}{4 - z^2} = \frac{(-4i)(2e^{i\theta})}{4 - 4e^{i2\theta}}$ (As $|z| = 2 \Rightarrow z = 2e^{i\theta}$)

$$= \frac{-2i(\cos\theta + i\sin\theta)}{(1 - \cos 2\theta) + (-i\sin\theta)} = \frac{-2i(\cos\theta + i\sin\theta)}{-2i^2\sin^2\theta - 2i\sin\theta\cdot\cos\theta}$$

$$= \frac{\cos\theta + i\sin\theta}{\sin\theta(\cos\theta + i\sin\theta)} = \text{cosec}\,\theta \in (-\infty, -1] \cup [1, \infty) \Rightarrow \text{Option (B) is correct.}$$

Q.7 $\frac{1}{3}$

Sol. $A = 2\int_2^3 \left(x - 2 - (x-2)^2\right)dx$

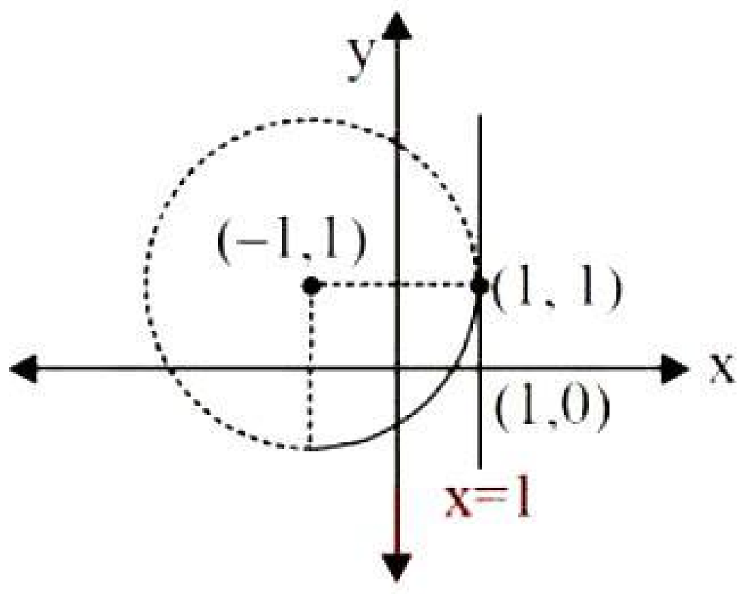


$= 2\left[\frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}\right]_2^3$

$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{1}{3}$.

Q.8 1

Sol. $z = 1 + i$, only satisfies both .



Q.9 25

Hint : $||z_1| - |z_2|| \le |z_1 + z_2|$ (Using left side of triangle inequality.)

Here, $1 \le |z| \le 25$.

Q.10 $\sqrt{3}$

Sol. Let $z = x + iy$

$-4y^2 = 4(x^2 + y^2) - 12$

$\Rightarrow$ $x^2 + 2y^2 = 3 \Rightarrow \frac{x^2}{3} + \frac{y^2}{3/2} = 1$

$x = \sqrt{3}\cos\theta,\ y = \sqrt{\frac{3}{2}}\sin\theta$

$x^2 + y^2 = 3\cos^2\theta + \frac{3}{2}\sin^2\theta = \frac{3}{2}(2\cos^2\theta + \sin^2\theta) = \frac{3}{2}(1 + \cos^2\theta)$

$|z|^2\Big|_{max} = \frac{3}{2} \times 2 = 3$

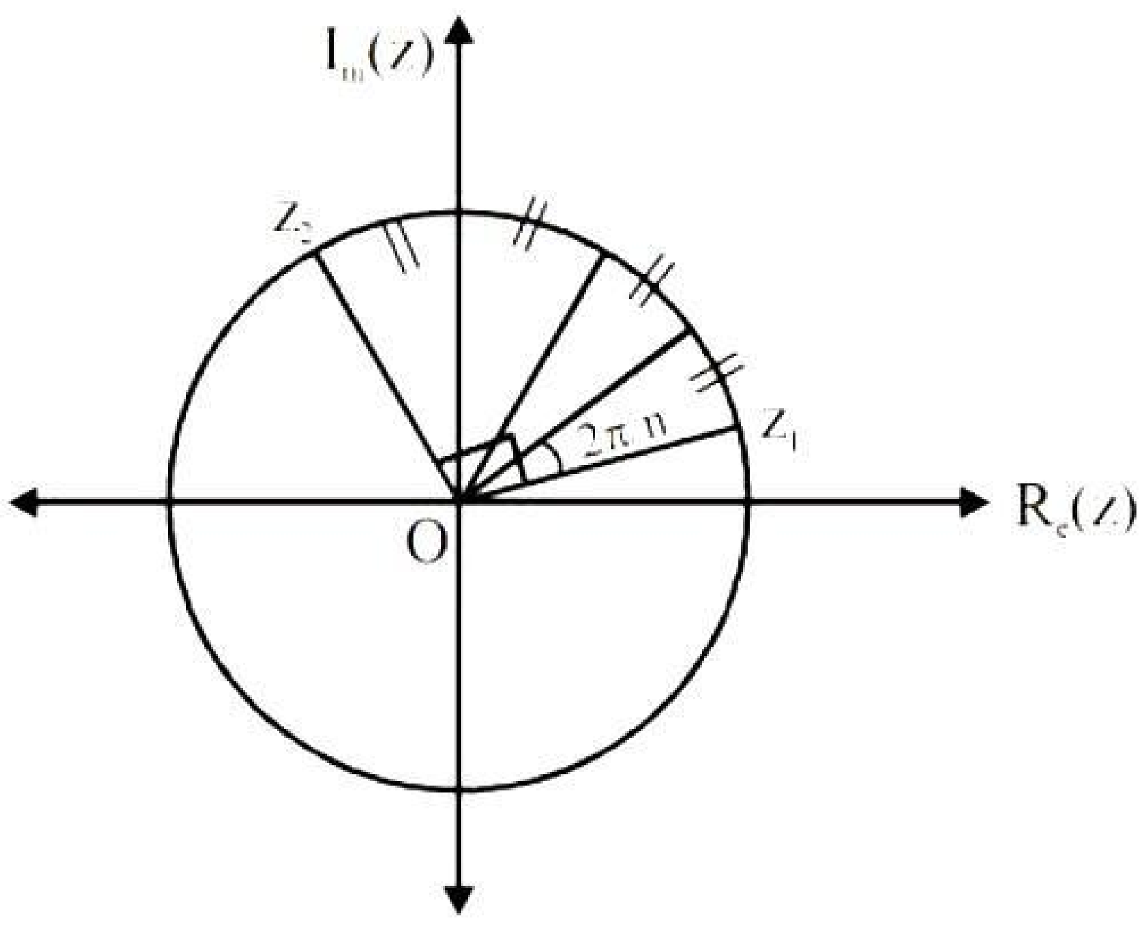
$\Rightarrow$ $|z|_{max} = \sqrt{3}$

Q.11 4k

Sol. **[D]**

n^{th} root of unity

$$\cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}$$



Let for $m = k$, root of unity makes an angle of $90°$

with $\frac{2k\pi}{n} = \frac{\pi}{2} \Rightarrow \quad n = 4k$

for $m = 0 \quad z_1 = 1$

for $m = k \quad z_2$ such that $z_2Oz_1 = 90°$

Aliter :

$$\therefore \left(\frac{2\pi}{n}\right)k = \frac{\pi}{2} \quad \Rightarrow \quad n = 4k$$

Q.12 $5\sqrt{2}$

Q.13 $-2+\sqrt{2}$

Q.14 4 sq. units

Sol. Using triangle inequality

$|(z^2 - 2iz) - (z^2 - 9z + 7iz)| \le 18$

$|9z - 9iz| \le 18 \Rightarrow |z| \le \sqrt{2}$

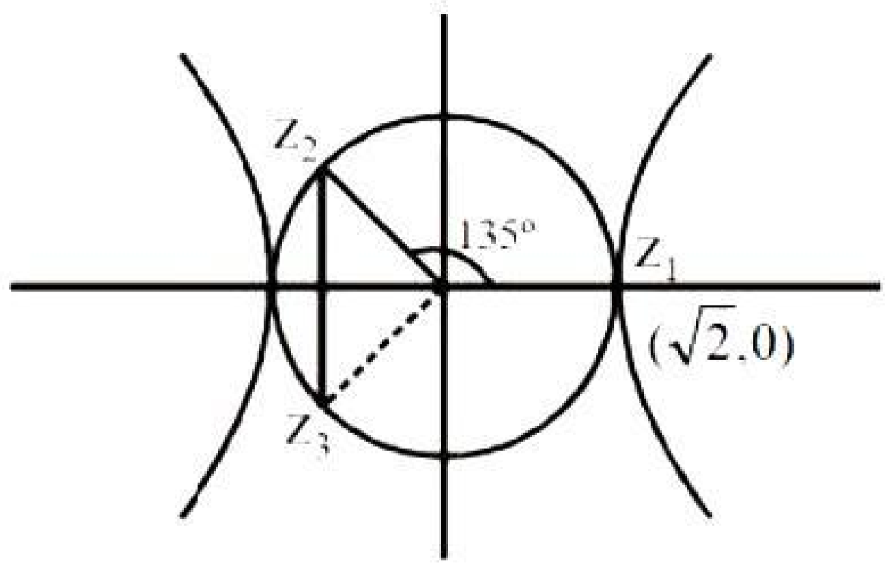
$C_1 : x^2 + y^2 = 2$

$C_2 : \frac{x^2}{2} - \frac{y^2}{16} = 1$

(i) $|z_0 + 4 + 4i| \le |z_0| + |4 + 4i|$

$\le \sqrt{2} + 4\sqrt{2}$

$\le 5\sqrt{2}$

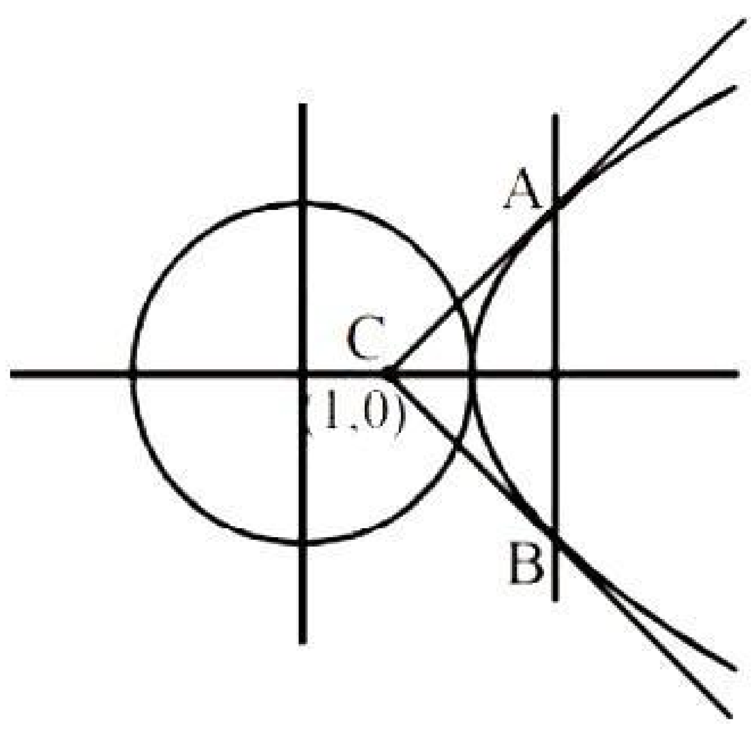


(ii) $z_1 = \sqrt{2}$

$z_2 = -1 + i$

$z_3 = \bar{z}_2 = -1 - i$

$\therefore \text{Re}(z_1 + z_2 + z_3) = -2 + \sqrt{2}$



(iii) Line AB

$y = 4x - 4$

which is tangent to thecurve C_2.

$A \equiv (2, 4)$, $B \equiv (2, -4)$ and $C \equiv (1, 0)$

$\therefore \text{Ar}(\Delta ABC) = \frac{1}{2} \times 1 \times 8 = 4$ sq. units.

Q.15 (B) area of the triangle formed by the straight line $x - 3y + 1 = 0$ and tangent at one of the vertices and the x-axis is $\frac{3}{2}$ sq. units.

(C) distance between directrices of the curve 'C' is $\frac{18}{\sqrt{10}}$.

(D) equation of the circle passing through the points (2, 1) and both foci of the curve 'C' is $x^2 + y^2 + 2x - 9 = 0$.

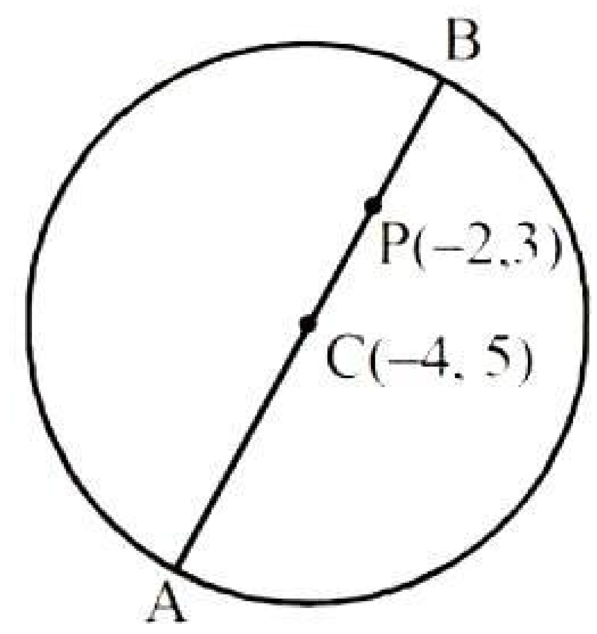
Sol. 40570/cn/MORE Locus of z is

$$\frac{(x+1)^2}{9} - \frac{y^2}{1} = 1$$

Now verify all options.

Q.16 (A) $a + b = 18$ (C) $a - b = 4\sqrt{2}$ (D) $ab = 73$

Sol. Centre of circle is $(-4, 5)$.



Also, radius = $\sqrt{(-4)^2 + (5)^2 + 40} = 9$

$\therefore$ Distance of centre $(-4, 5)$ from $(-2, 3) = 2\sqrt{2}$

So, $a = \max. |z - (-2 + 3i)| = 9 + 2\sqrt{2}$

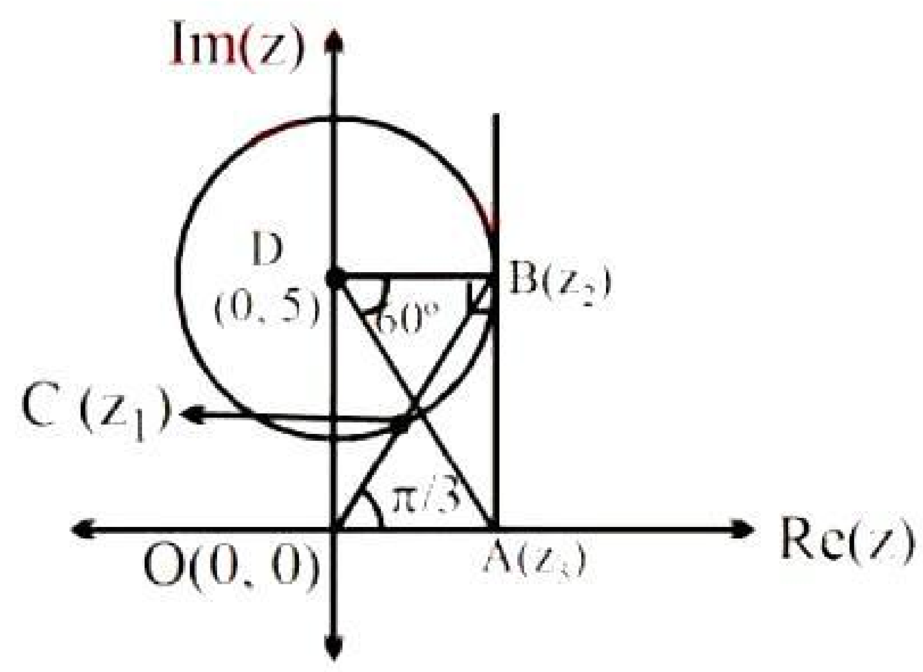
and $b = \min. |z - (-2 + 3i)| = 9 - 2\sqrt{2}$

Hence, $a + b = 18$

Also, $(a - b) = 4\sqrt{2}$ and $ab = \frac{(a+b)^2 - (a-b)^2}{4} = \frac{324 - 32}{4} = \frac{292}{4} = 73$.

Q.17 11

Sol. As, O, A, B and D are concyclic,



so $\cos 60° = \frac{BD}{AD} = \frac{3}{AD}$

$\therefore AD = 6$ and $OD = 5$

Hence, $|z_3| = OA = \sqrt{36 - 25} = \sqrt{11}$

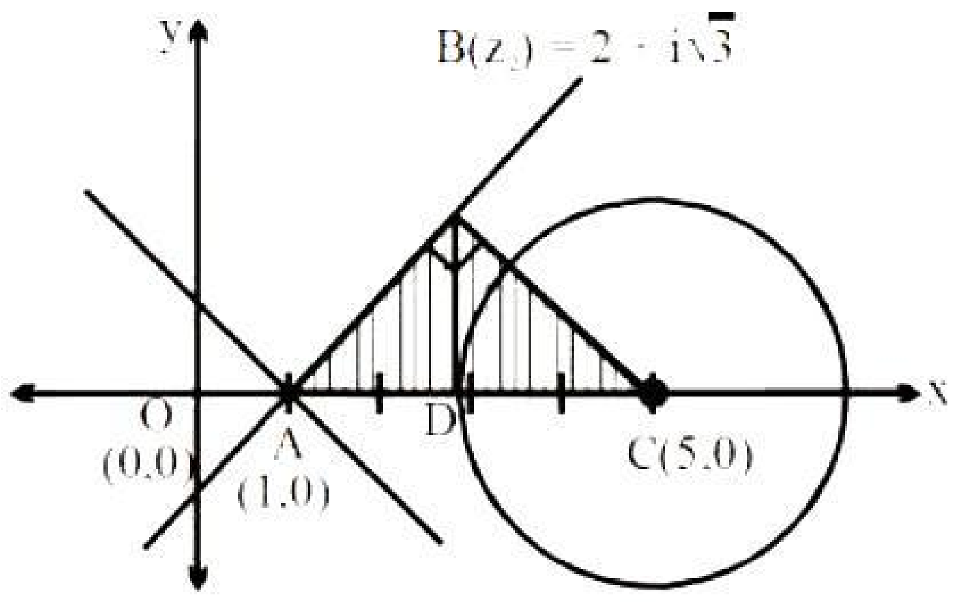
Hence $|z_3|^2 = 11$.

Q.18 2

Sol. $$S = \frac{\sum_{k=1}^{12} |\overline{\alpha}_k + \alpha_{12-k}|}{\sum_{k=1}^{6} |\alpha_{2k-1} + \alpha_{2k}|} = \frac{\sum_{k=1}^{12} |\alpha_{13-k} + \alpha_{12-k}|}{\sum_{k=1}^{6} |\alpha_{2k-1}(1+\alpha_1)|} = \frac{\sum_{k=1}^{12} |\alpha_{12-k}(\alpha_1 + 1)|}{\sum_{k=1}^{6} |\alpha_{2k-1}||(1+\alpha_1)|} = \frac{\sum_{k=1}^{12} |\alpha_1 + 1|}{\sum_{k=1}^{6} |\alpha_1 + 1|} = 2$$

Q.19 12

Sol.



Clearly $A(z_1)$ is the point of intersection of $\arg(z - 2 + i) = \frac{3\pi}{4}$ and $\arg(z + \sqrt{3}\, i) = \frac{\pi}{3}$

$\Rightarrow \; z_1 = 1$

Also, $B(z_2)$ is the point on $\arg(z + \sqrt{3}\, i) = \frac{\pi}{3}$ such that $|z_2 - 5|$ is minimum, so $z_2 = 2 + i\sqrt{3}$.

also, $C(z_3)$ be the centre of the circle $|z - 5| = 3$, so $z_3 = 5$.

Hence, area of $\Delta ABC = \frac{1}{2}(AC) \times (BD) = \frac{1}{2}(4) \times \left(\sqrt{3}\right) = 2\sqrt{3}$ (square unit.)

$\therefore \quad \Delta = 2\sqrt{3} \quad \Rightarrow \quad \Delta^2 = 12$

Q.20 136

Sol. It is to be noted that two of the numbers need to be conjugates and one number must be real, as the coefficients of the cubic are all real.

Three roots are, $\omega + 3i$, $\omega + 9i$ and $2\omega - 4$

let $\omega + 3i$ is real, hence $\omega = \alpha - 3i$ where $\alpha \in R$

then $(\alpha - 3i + 9i)$ and $(2\alpha - 6i - 4)$

i.e. $\alpha + 6i$ and $(2\alpha - 4) - 6i$ must be complex conjugate $\Rightarrow \alpha = 2\alpha - 4$

$\therefore \quad \alpha = 4$

Hence the roots are

$4, 4 + 6i, 4 - 6i$ (other options not possible)

the equation is

$(z - 4)(z^2 - 8z + 5z) \Rightarrow z^3 - 12z^2 + 84z - 208$

Hence $|a + b + c| = |-12 + 84 - 208| = 136.$