19 Electromagnetic Induction

MAGNETIC FLUX :

$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$ for uniform $\vec{B}$.

$\phi = \int \vec{B}\cdot d\vec{A}$ for non uniform $\vec{B}$.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION :

(i) An induced emf is setup whenever the magnetic flux linking that circuit changes.

(ii) The magnitude of the induced emf in any circuit is proportional to the rate of change of the magnetic flux linking the circuit, $\varepsilon \propto \frac{d\varphi}{dt}$.

LENZ'S LAWS :

The direction of an induced emf is always such as to oppose the cause producing it .

LAW OF EMI : $e = -\frac{d\varphi}{dt}$.

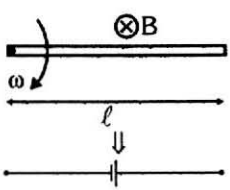
The negative sign indicates that the induced emf opposes the change of the flux.

EMF INDUCED IN A STRAIGHT CONDUCTOR IN UNIFORM MAGNETIC FIELD :

$$E = BLv\sin\theta$$

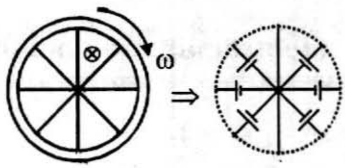
where B = flux density
L = length of the conductor
v = velocity of the conductor
θ = angle between direction of motion of conductor & B .

EMF INDUCED IN A ROD ROTATING PERPENDICULAR TO MAGNETIC FIELD



$$E = \frac{1}{2}B\omega\ell^2$$

For a wheel rotating in a earth magnetic field effective emf induced between the periphery & centre = $\frac{1}{2}B\omega\ell^2$



COIL ROTATION IN MAGNETIC FIELD SUCH THAT AXIS OF ROTATION IS PERPENDICULAR TO THE MAGNETIC FIELD :

Instantaneous induced emf. ω sin$E\omega t = E_0 \sin \omega t$

where N = number of turns in the coil

A = area of one turn

B = magnetic induction

ω = uniform angular velocity of the coil

E_0 = maximum induced emf

SELF INDUCTION & SELF INDUCTANCE :

When a current flowing through a coil is changed the flux linking with its own winding changes & due to the change in linking flux with the coil an emf is induced which is known as self induced emf & this phenomenon is known as self induction . This induced emf opposes the causes of Induction. The property of the coil or the circuit due to which it opposes any change of the current coil or the circuit is known as **SELF - INDUCTANCE** . It's unit is Henry.

Coefficient of Self inductance $L = \frac{\varphi_s}{i}$ or $\phi_s = Li$

i = current in the circuit .

ϕ_s = magnetic flux linked with the circuit due to the current i .

L depends only on ; **(i)** shape of the loop & **(ii)** medium

self induced emf $e_s = \frac{d\varphi_s}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$ (if L is constant)

Combination of inductors

- Series combination $L = L_1 + L_2 + \ldots\ldots\ldots$, i same, V in ratio of inductance, U in ratio of inductance, ϕ in ratio of inductance

- Parallel combination $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots\ldots$, V same, i in inverse ratio of inductance, U in inverse ratio of inductance, ϕ same

MUTUAL INDUCTION :

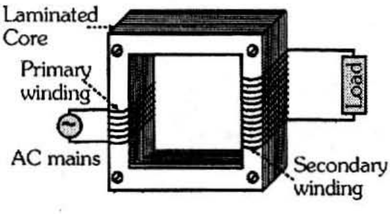
If two electric circuits are such that the magnetic field due to a current in one is partly or wholly linked with the other, the two coils are said to be electromagnetically coupled circuits . Then any change of current in one produces a change of magnetic flux in the other & the later opposes the change by inducing an emf within itself . This phenomenon is called **Mutual Induction** & the induced emf in the later circuit due to a change of current in the former is called **Mutually Induced EMF** . The circuit in which the current is changed, is called the primary & the other circuit in which the emf is induced is called the secondary. The co-efficient of mutual induction (mutual inductance) between two electromagnetically coupled circuit is the magnetic flux linked with the secondary per unit current in the primary.

Mutual inductance = $M = \frac{\varphi_m}{I_p} = \frac{\text{flux linked with secondary}}{\text{current in the primary}}$

mutually induced emf : $E_m = \frac{d\varphi_m}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$ (If M is constant)

M depends on (1) geometry of loops (2) medium (3) orientation & distance of loops .

- **If two coils of self inductance L_1 and L_2 are wound over each other, the mutual inductance** $M = K\sqrt{L_1L_2}$ where K is called coupling constant.
- **For two coils wound in same direction and connected in series**
  $L = L_1 + L_2 + 2M$
- **For two coils wound in opposite direction and connected in series**
  $$L = L_1 + L_2 - 2M$$
- **For two coils in parallel** $L = \frac{L_1L_2 - M^2}{L_1 + L_2 \pm 2M}$
- **Transformer**

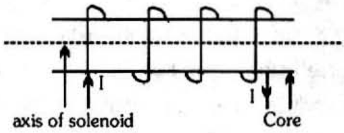


$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

❑ For ideal transformer $\frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$ ❑ Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100\%$

SOLENOID :

There is a uniform magnetic field along the axis of the solenoid



$B = \mu\ ni$ (ideal : length >> diameter)

where μ = magnetic permeability of the core material

n = number of turns in the solenoid per unit length

i = current in the solenoid

Self inductance of a solenoid $L = \mu_0 n^2 A l$

A = area of cross section of solenoid .

SUPER CONDUCTION LOOP IN MAGNETIC FIELD :

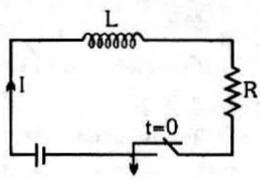
$R = 0$; $\varepsilon = 0$. Therefore ϕ_{total} = constant. Thus in a superconducting loop flux never changes. (or it opposes 100%)

(i) **ENERGY STORED IN AN INDUCTOR** : $W = \frac{1}{2} LI^2$.

(ii) Energy of interaction of two loops $U = I_1\phi_2 = I_2\phi_1 = MI_1I_2$ where M is mutual inductance

GROWTH OF A CURRENT IN AN L – R CIRCUIT :

$I = \frac{E}{R}(1 - e^{-Rt/L})$. [If initial current = 0]



$\frac{L}{R}$ = time constant of the circuit.

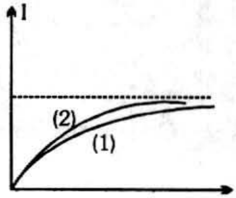
$I_0 = \frac{E}{R}$.

(i) L behaves as open circuit at t = 0 [If $i = 0$]

(ii) L behaves as short circuit at t = ∞ always.

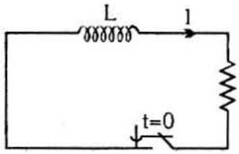
Curve (1) $\longrightarrow$ $\frac{L}{R}$ Large

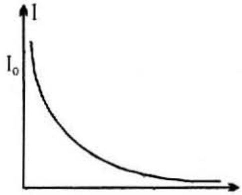
Curve (2) $\longrightarrow$ $\frac{L}{R}$ Small



DECAY OF CURRENT :

Initial current through the inductor = I_0 ; Current at any instant $i = I_0 e^{-Rt/L}$





KEY POINTS

- An emf is induced in a closed loop where magnetic flux is varied. The induced electric field is not conservative field because for induced electric field, the line integral $\oint \vec{E}.d\vec{\ell}$ around a closed path is non-zero.
- Acceleration of a magnet falling through a long solenoid decrease because the induced current produced in a circuit always flows in such direction that it opposes the change or the cause that produces it.
- The mutual inductance of two coils is doubled if the self inductance of the primary and secondary coil is doubled because mutual inductance $M \propto \sqrt{L_1 L_2}$.
- The possibility of an electric bulb fusing is higher at the time of switching ON and OFF because inductive effects produce a surge at the time of switch-off and switch-on.
- Motional emf : If a conductor is moved in a magnetic field then motional emf will be $E = B\ell_{eff}v$

  Here $v \perp \ell_{eff}$ & $v \perp B$ & $B \perp \ell_{eff}$

  $\ell_{eff} \rightarrow$ effective length between the end points of conductor which is perpendicular to the velocity.