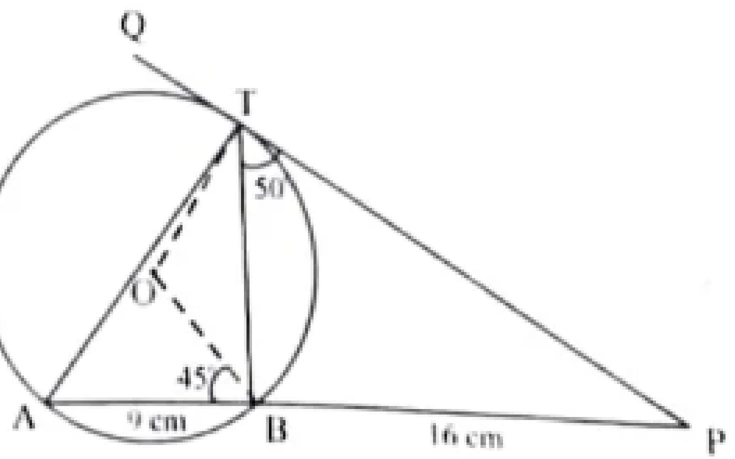
Circles

1. In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. Chord AB produced meets the tangent at P. AB = 9 cm, BP = 16 cm, ∠PTB = 50°, ∠OBA = 45°. Find:



(a) length of PT
(b) ∠BAT
(c) ∠BOT
(d) ∠ABT [2023]

Answer: (a) 20cm (b) 50° (c) 100° (d) 85°

Step-by-step Explanation:

(a) We know, $PT^2 = AP \times BP$ (When tangent and chord intersect externally, the product of the lengths of the segments of chord is equal to the square of the length of the tangent.)

$PT^2 = (16+9) \times 16$

$PT^2 = 25 \times 16$

$PT = \sqrt{25 \times 16}$

PT= 20 cm

(b) ∠BAT= ∠BTP = 50° (angle in the alternate segment)

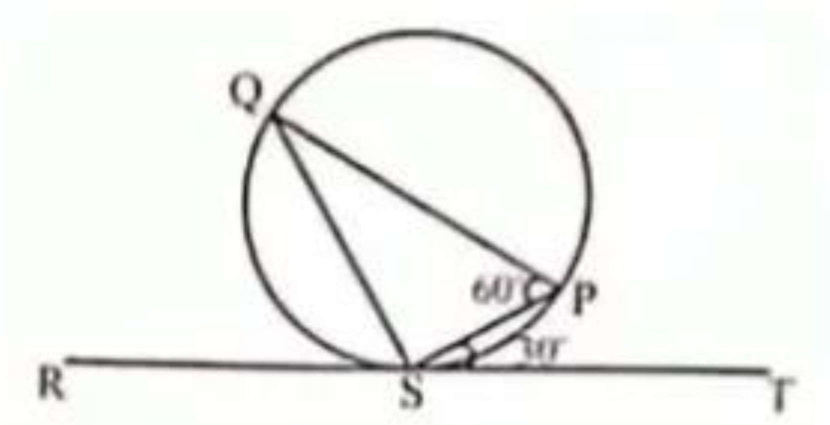
(c) ∠BOT =2∠BAT= 100° (Angle subtended by an arc at the center of a circle is double the angle subtended by it on remaining part of the circle.)

(d) In ΔBOT, OB = OT (radii of a circle)

∴∠OBT=∠BTO= 180°-100°/2= 40°

∴∠ABT= 45° + 40°= 85°

2. In the given diagram RT is a tangent touching the circle at S. If ∠PST= 50° and ∠SPQ = 60° then ∠PSQ is equal to:



(a) 40°
(b) 30°
(c) 60°
(d) 90° [2023]

Answer: (d)

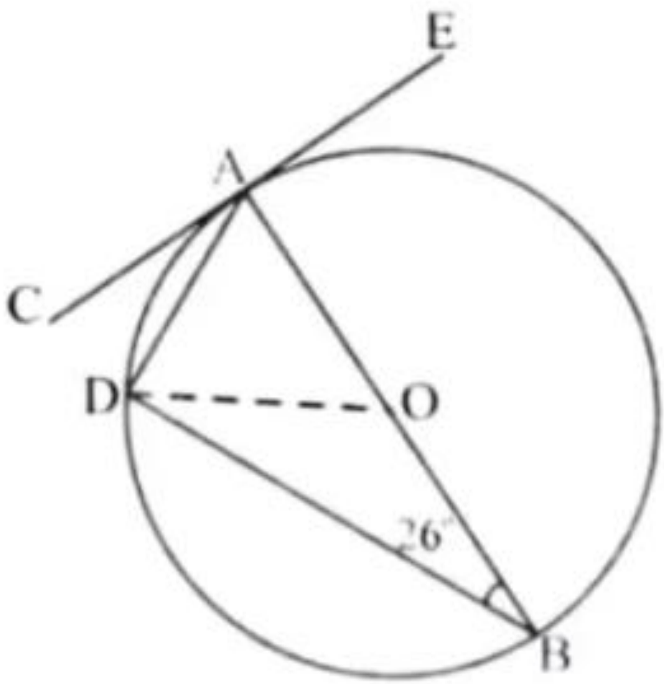
Step-by-step Explanation:

∠PQS=∠PST= 30°

In ΔPQS, ∠PQS +∠PSQ+∠QPS= 180°

∠PSQ= 180° – (60+30)°= 90°

3. In the given figure O, is the centre of the circle. CE is a tangent to the circle at A.
If ∠ABD=26°, then find



(a) ∠BDA
(b) ∠BAD
(c) ∠CAD
(d) ∠ODB [2023]

Answer: (a) 90° (b) 64° (c) 26° (d) 26°

Step-by-step Explanation:

(a) ∠BDA= 90° (angle in a semicircle is right angle.)

(b) ∠BAD= 180° – (90+26)° (sum of angles of a triangle is 180°

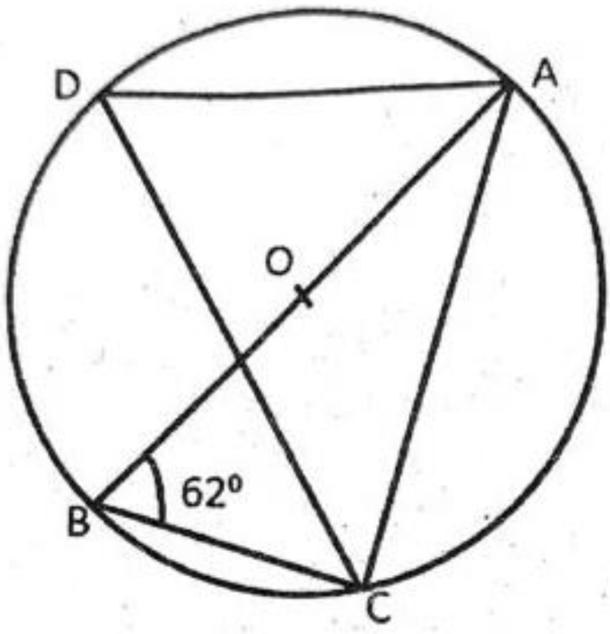
= 64°

(c) ∠CAD= 26° (angles in the alternate segments are equal.)

(d) ∠DOB= 2∠BAD= 2× 64= 128° (angle subtended by an arc at the center of a circle is double the angle subtended by it on any part on the remaining circle.)

∴ ∠ODB= 180° – (26+128)° =26° (sum of the angles of a triangle is 180°.)

4. In the given figure A, B, C and D are points on the circle with centre O. Given ∠ABC = 62°



Find:
(a) ∠ADC
(b) ∠CAB [2022 Semester-2]

Solution: (a) 62° (b) 28°

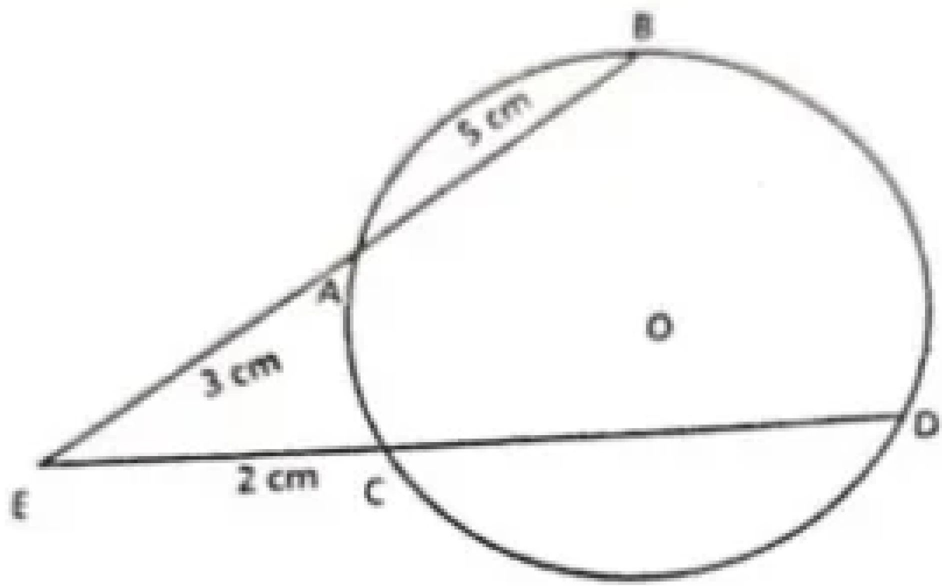
Step-by-step Explanation:

(a) ∠ADC= ∠ABC= 62° (Angles in the same segment are equal.)

(b) ∠ACB= 90° (angle in a semicircle is right angle.)

∴ ∠CAB= 180°– (62°+90°) =28° (sum of angles in a triangle is 180°.)

5. Two chords AB and CD of a circle intersect externally at E. If EC = 2 cm, EA = 3 cm and AB = 5 cm, Find the length of CD. [2022 Semester-2]



Answer: 10 cm

Step-by-step Explanation:

We know, AE × BE = CE × DE (when two chords intersect internally or externally, the products of the lengths of the segments of the chords are equal.)

3 × (3+5) = 2 × (2+CD)

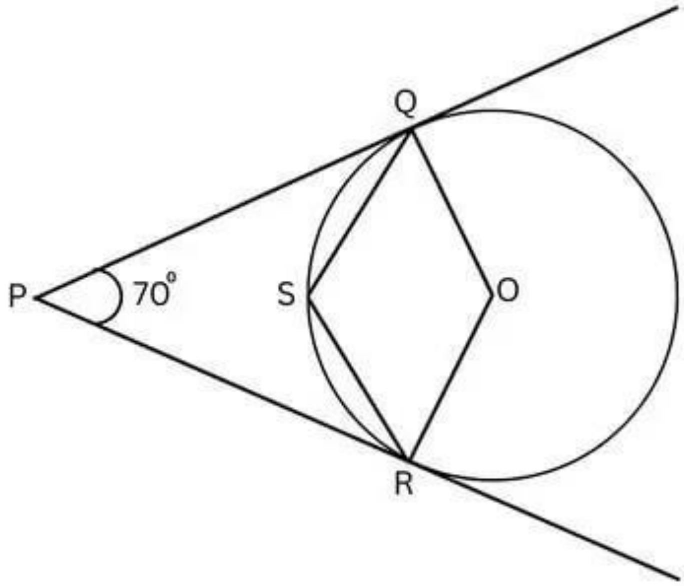
24/2 = 2 + CD

10 = CD

CD = 10 cm

6. In the given figure O is the centre of the circle. PQ and PR are tangents and ∠QPR = 70°. Calculate

(a) ∠QOR
(b) ∠QSR [2022 Semester-2]



Answer: (a) 110° (b) 125°

Step-by-step Explanation:

(a) ∠PQO= ∠PRO = 90° (tangent and the radius of a circle through the point of contact are perpendicular to each other.)

In Quadrilateral PQOR,

∠RPQ +∠PQO +∠QOR + ∠PRO = 360°

70° + 90° +∠QOR + 90° = 360°

∠QOR = 360° – 250° = 110°

(b) reflex ∠QOR = 360°-110° = 250°

$\angle QSR = 125°$ (angle subtended by an arc at the center of a circle is double the angle subtended by it on any part on the remaining circle.)

7. ABCD is a cyclic quadrilateral. If $\angle BAD = (2x + 5)°$ and $\angle BCD = (x + 10)°$ then x is equal to:
(a) 65° (b) 45° (c) 55° (d) 5° [2022 Semester-2]

Answer: (c)

Step-by-step Explanation:

We know, by theorem, opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \angle BAD + \angle BCD = 180°$

$(2x + 5)° + (x + 10)° = 180°$

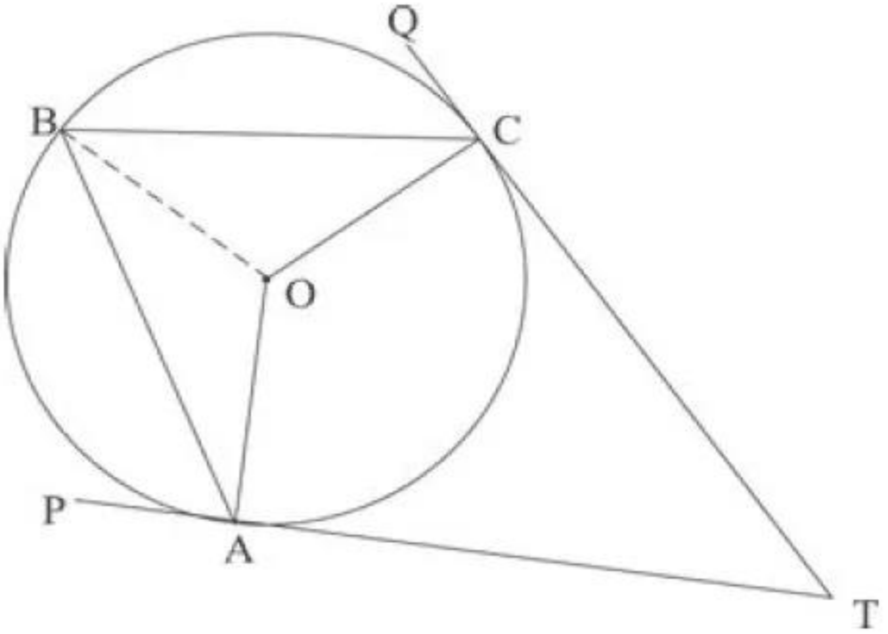
$3x + 15 = 180$

$3x = 165$

$x = 55°$

8. In the given figure TP and TQ are two tangents to the circle with centre O, touching at A and C, respectively. If $\angle BCQ = 55°$ and $\angle BAP = 60°$, find:

(i) $\angle OBA$ and $\angle OBC$
(ii) $\angle AOC$
(iii) $\angle ATC$ [2020]



Answer: (i) 30°, 35° (ii) 130° (iii) 50°

Step-by-step Explanation:

(i) PAT and QCT are tangents to the circle.

∴ ∠QCO = ∠PAO = 90° (tangent and the radius of a circle through the point of contact are perpendicular to each other.)

Now, ∠BCQ = 55°.

∴ ∠BCO = 90 – 55 = 35°

In ΔBOC, OB = OC (radii)

∴ ∠OBC = ∠OCB = 35°

Similarly,

∠BAO = 90 – 60 = 30°

In ΔOAB, $OA = OB$ (radii)

$\therefore \angle OBA = \angle BAO = 30°$

(ii) $\angle ABC = \angle OBA + \angle OBC = 30 + 35 = 65°$

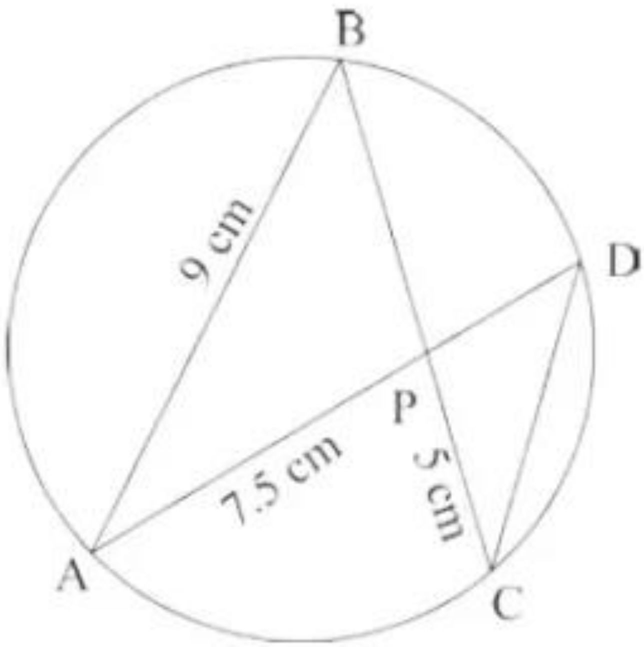
Hence, $\angle AOC = 2\angle ABC = 130°$ (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

(iii) $\angle ATC = 360° - (\angle TAO + \angle AOC + \angle TCO)$ (sum of angles of a quadrilateral is 360°.)

$\therefore \angle ATC = 360° - (90+130+90)° = 360° - 310° = 50°$

9. In the given figure AB = 9 cm, PA = 7.5 cm and PC = 5 cm. Chords AD and BC intersect at P.

(i) Prove that $\Delta PAB \sim \Delta PCD$
(ii) Find the length of the CD.
(iii) Find area of ΔPAB : area of ΔPCD [2020]



Answer: (ii) 6 cm (iii) 9 : 4

Step-by-step Explanation:

(i) Chords AD and BC intersect internally. Therefore according to the theorem, the product of the lengths of their segments are equal.

$\therefore AP \times PD = BP \times PC$

or, $AP/PC = BP/PD$

Now, In ΔPAB and ΔPCD

$\angle APB = \angle CPD$ (vertically opposite angles)

$AP/PC = BP/PD$ (proved above)

$\therefore \Delta PAB \sim \Delta PCD$ (S-A-S condition of similarity)

(ii) As $\Delta PAB \sim \Delta PCD$

$\therefore AP/PC = BP/PD = AB/CD$

$AP/PC = AB/CD$

$7.5/5 = 9/CD$

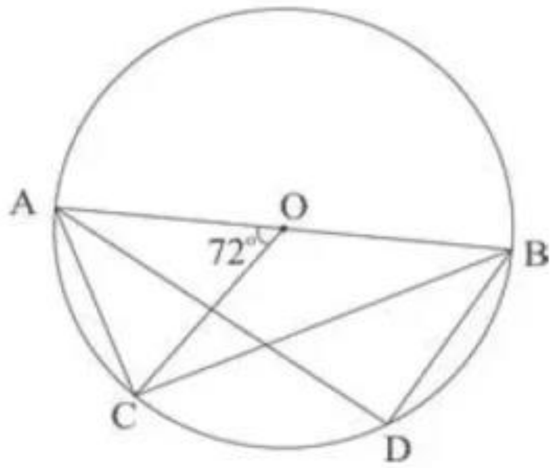
$CD = 9/1.5 = 6$ cm

(iii) Area of ΔPAB : Area of $\Delta PCD = (PA/PC)^2$ (ratio of areas of similar triangles is equal to the square of the ratio of their corresponding sides.)

Area of ΔPAB : Area of $\Delta PCD = (7.5/5)^2 = 9 : 4$

10. In the figure given below, O is the centre of the circle and AB is a diameter.

If AC = BD and ∠AOC = 72°. Find:
(i) ∠ABC
(ii) ∠BAD
(iii) ∠ABD [2020]



Answer: (i) 36° (ii) 36° (iii) 54°

Step-by-step Explanation:

(i) ∠ABC = 1/2∠AOC =36° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

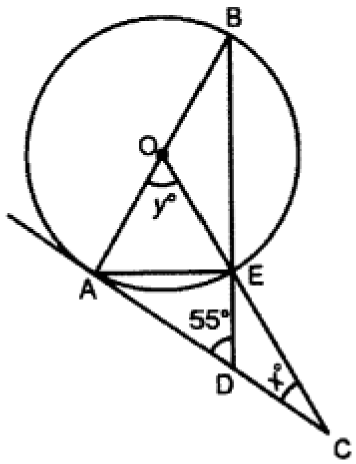
(ii) ∠BAD= ∠ABC = 36° (equal chords subtend equal angles.)

(iii) ∠ADB= 90° (angle in a semicircle is right angle.)

∴ ∠ABD= 180°- (∠BAD + ∠ADB) (sum of angles of a triangle is 180°.)

or, ∠ABD= 180° – 126°= 54°

11. In the given figure, AC is a tangent to the circle with center 0. If ∠ADB = 55°, find x and y. Give reasons for your answers. [3] [2019]



Answer: x= 20° , y= 70°

Step-by-step Explanation:

∠AEB= 90° (angle in a semicircle is right angle.)

∴ ∠AED= 90° (linear pair)

∠DAE= 180°-(90°+55°)= 35°

∴∠ABE= 35° (angles in the alternate segments are equal.)

∴ ∠AOE= y°= 70° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

∠OEB=∠OBE= 35° (isosceles triangle property)

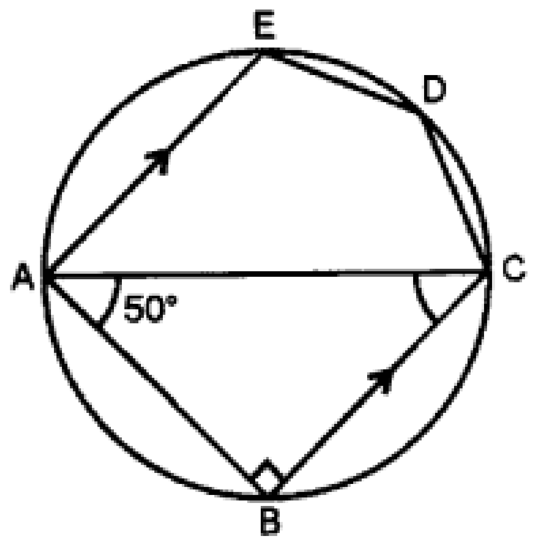
Hence, ∠DEC=∠OEB= 35°

∠EDC= 180 – 55=125° (linear pair)

Hence, x°= 180° – (125 + 35)° = 20°

12. In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC || AE. If ΔBAC = 50°, find giving reasons : [4]

(i) ∠ACB
(ii) ∠EDC
(iii) ∠BEC [2019]



Answer: (i) 40° (ii) 140° (iii) 50°

Step-by-step Explanation:

(i) ∠ABC= 90° (angle in a semicircle is right angle.)

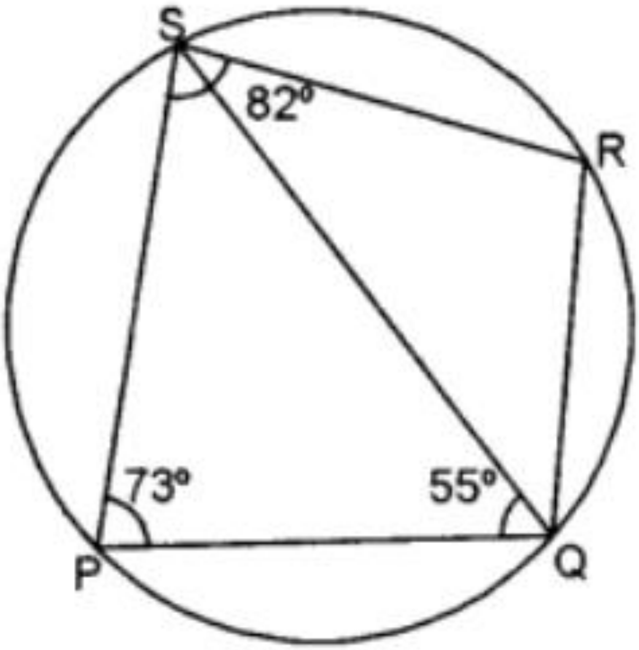
Hence, ∠ACB= 180° – (90+50)°= 40°

(ii) ∠CAE=∠ACB= 40°

Hence, ∠EDC= 180° – 40°= 140° (opposite angles of a cyclic quadrilateral are supplementary.)

(iii) ∠BEC= ∠BAC= 50° (angles in the same segment are equal.)

13. PQRS is a cyclic quadrilateral. Given ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate: [4]



(i) ∠QRS
(ii) ∠RQS
(iii) ∠PRQ [2018]

Answer: (i) 107° (ii) 43° (iii) 52°

Step-by-step Explanation:

(i) ∠QRS= 180°- 73° = 107° (opposite angles of a cyclic quadrilateral are supplementary.)

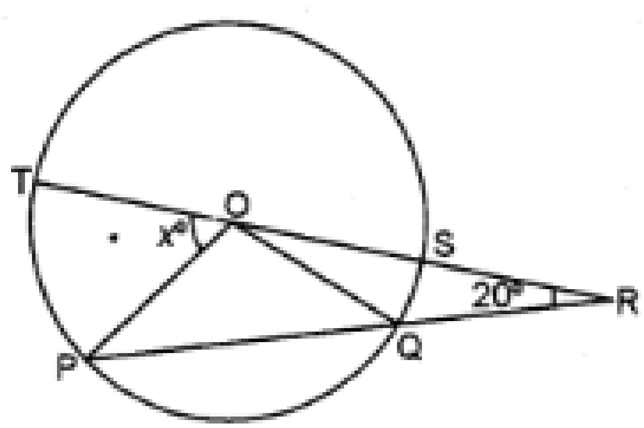
(ii) ∠PSQ= 180° – (73 +55)°=52°

∴ ∠RSQ= 82 – 52= 30°

Hence, ∠RQS= 180°- (107 + 30)°= 43°

(iii) ∠PRQ= ∠PSQ= 52° (angles in the same segment are equal.)

14. In the figure given below 'O' is the center of the circle. If QR = OP and ∠ORP= 20°. Find the value of 'x ' giving reasons. [3] [2018]



Answer: 60°

Step-by-step Explanation:

OP=QR (given) and OP= OQ (radii)

Hence, OQ = QR

∴ ∠QOR =∠ORQ= 20°

∴ ∠OQR= 180° – 40°= 140° (angle sum property of triangle)

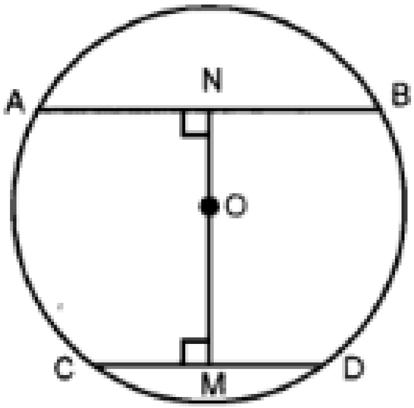
∴ ∠OQP= 180° – 140° =40°

∴ ∠OPQ= 40°

∴ ∠POQ= 180° – 80° = 100°

∴ x°= 180°- (100 +20)° = 60° (angles in a straight line)

15. AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords. [3] [2017]



Answer: 17 cm

Step-by-step Explanation:

Join OB and OD,

NB= 1/2 AB= 12 cm and MD= 1/2 CD= 5 cm (perpendicular drawn from the center of a circle to the chord bisects it.)

In ΔONB, By pythagoras theorem,

$ON = \sqrt{OB^2 - NB^2}$

$ON = \sqrt{169 - 144} = 5$ cm

In ΔOMD, By pythagoras theorem,

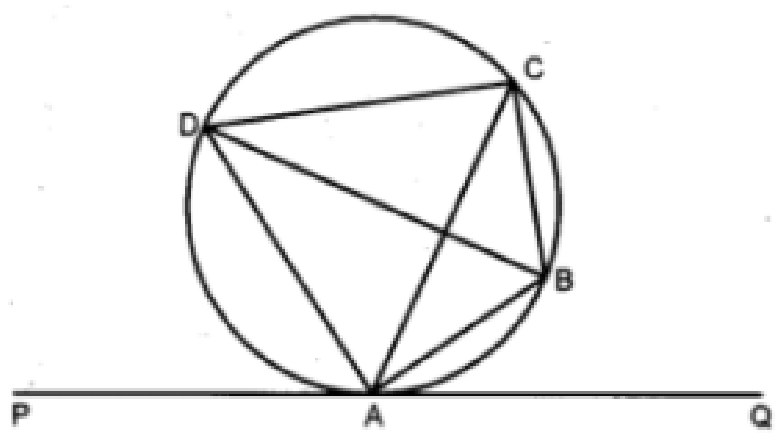
$OM = \sqrt{OD^2 - MD^2}$

$ON = \sqrt{169 - 25} = 12$ cm

∴ MN= 5 + 12= 17 cm

16. In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of ∠CAQ and ∠PAC. If ∠BAQ = 30° prove that :

(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle. [2017]



Step-by-step Explanation:

(i) Given that AB and AD are bisectors of ∠CAQ and ∠PAC.

Let ∠CAB= ∠BAQ= x° and ∠CAD= ∠DAP= y°.

∴ ∠BAQ +∠CAB +∠CAD +∠DAP= $(2x + 2y)°$

$(2x + 2y)°$= 180° (angles in a straight line.)

$2(x+y) = 180°$

$x + y = 90°$

or, ∠BAD = 90°

Hence, BD is the diameter of the circle. (angle in a semicircle is right angle.)

(ii) ∠ACB = ∠BAQ = x° (angles in the alternate segments are equal.)

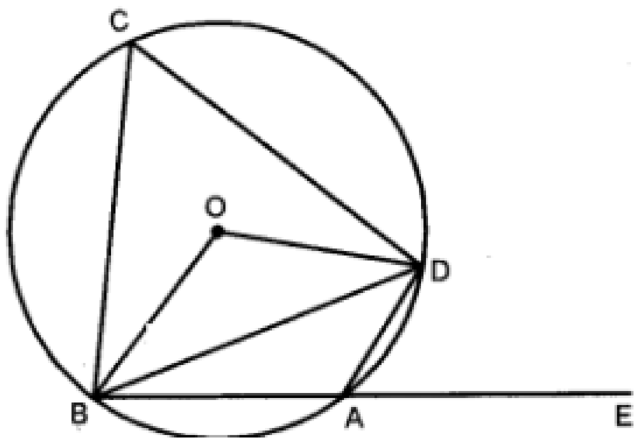
∠CAB = x°

∴ AB = BC

Hence, ABC is an isosceles triangle.

17. In the figure given, O is the center of the circle. ∠DAE = 70°. Find giving suitable reasons, the measure of: [4]

(i) ∠BCD
(ii) ∠BOD
(iii) ∠OBD [2017]



Answer: (i) 70° (ii) 140° (iii) 20°

Step-by-step Explanation:

(i) ∠BCD = ∠DAE = 70° (exterior angle of a cyclic quadrilateral is equal to opposite interior angle.)

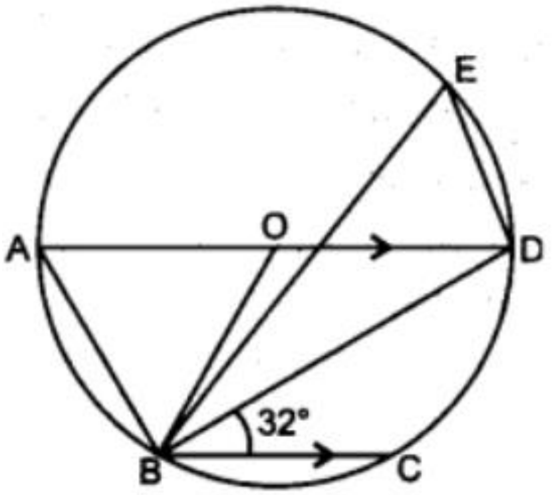
(ii) ∠BOD = 2∠BCD = 140° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

(iii) In ΔBOD, OB = OD (radii)

∴ ∠OBD = ∠ODB = 180 – ∠BOD / 2 = 20°

18. In the given figure below, AD is a diameter. O is the centre of the circle. AD is parallel to BC and ∠CBD = 32°.

Find :
(i) ∠OBD
(ii) ∠AOB
(iii) ∠BED [4] [2016]



Answer: (i) 32° (ii) 64° (iii) 58°

Step-by-step Explanation:

(i) Since AD is parallel to BC,

∠ODB = ∠CBD = 32° (alternate interior angles)

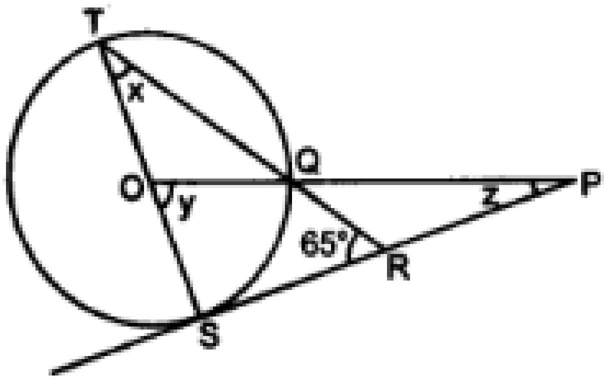
∠OBD =∠ODB = 32° (property of isosceles triangle.)

(ii) ∠AOB = 2∠ODB = 64° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

(iii) ∠BOD = 180° – ∠AOB = 180° – 64° = 116°

Hence, ∠BED = 116/2 = 58° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

19. In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the value of x, y and z. [4] [2015]



Answer: x=25°, y= 50°, z= 40°

Step-by-step Explanation:

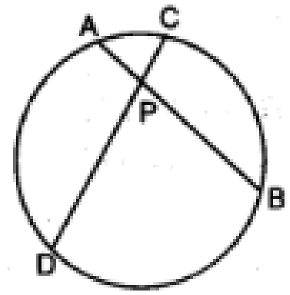
∠RST= 90° (tangent and the radius of a circle through the point of contact are perpendicular to each other.)

Hence, ∠RTS= x= 180° – (65 + 90)°= 25°

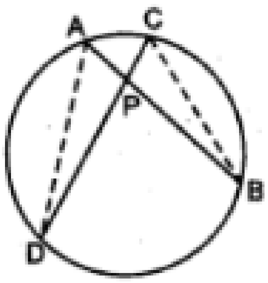
y = ∠SOQ= 2∠RST= 50° (angle subtended by an arc at the center is double the angle subtended by it on the remaining part on the circle.)

z= ∠OPS= 180° – (90 + 50)°= 40°

20. AB and CD are two chords of a circle intersecting at P. Prove that AP × PB = CP × PD. [3] [2015]



Step-by-step Explanation:



Let us join AD and BC.

Let us join AD and BC.

Now, In ΔAPD and ΔCPB,

∠A =∠C (angles in the same segment are equal.)

∠APD = ∠BPC (vertically opposite angles)

∴ ΔAPD ~ ΔCPB (A-A condition of similarity)

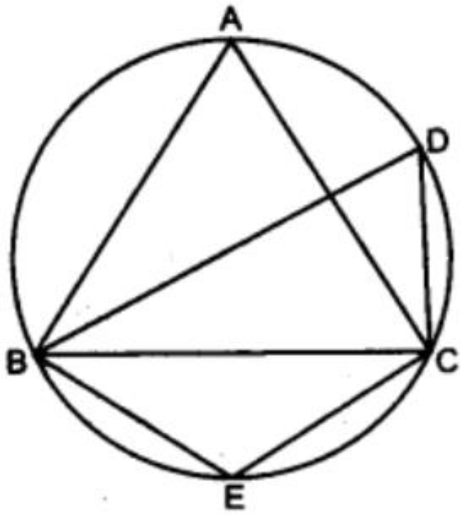
∴ AP/CP = PD/PB

or, AP × PB = CP × PD

Proved.

21. In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate: [3]

(i) ∠BDC
(ii) ∠BEC
(iii) ∠BAC [2014]



Answer: (i) 32° (ii) 148° (iii) 32°

Step-by-step Explanation:

(i) ∠BCD= 90° (angle in a semicircle is right angle.)

∴ ∠BDC= 180°– (∠BCD + ∠DBC)

=180° – (90 + 58)°

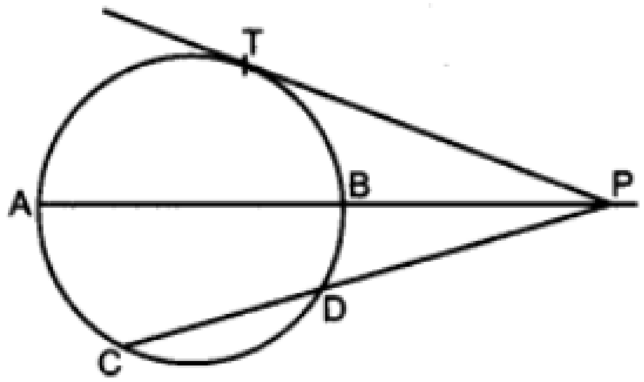
= 180° – 148° = 32°

(ii) ∠BEC = 180° – ∠BDC = 180° – 32°= 148° (opposite angles of a cyclic quadrilateral are supplementary.)

(iii) ∠BAC = ∠BDC = 32° (angles in the same segment are equal.)

22. In the figure given below, diameter AB and CD of a circle meet at P. PT is a tangent to the circle at T. CD=7.8 cm, PD=5 cm, PB=4 cm.

Find:
(i) AB.
(ii) the length of tangent PT. [3][2014]



Answer: (i) 12 cm (ii) 8 cm

Step-by-step Explanation:

(i) We know, When two chords intersect internally or externally, the product of the lengths of the segments of the chords are equal.

∴ AP × PB = CP × PD

or, (AB +4) × 4 = (7.8 + 5) × 5

or, AB + 4 = 12.8 × 5/4

or, AB = 16 – 4 = 12 cm

(ii) We know, When a tangent and a chord of a circle intersect externally, the product of the lengths of the segments of the chord is equal to the square of the length of the tangent.

∴ $PT^2 = AP \times PB$

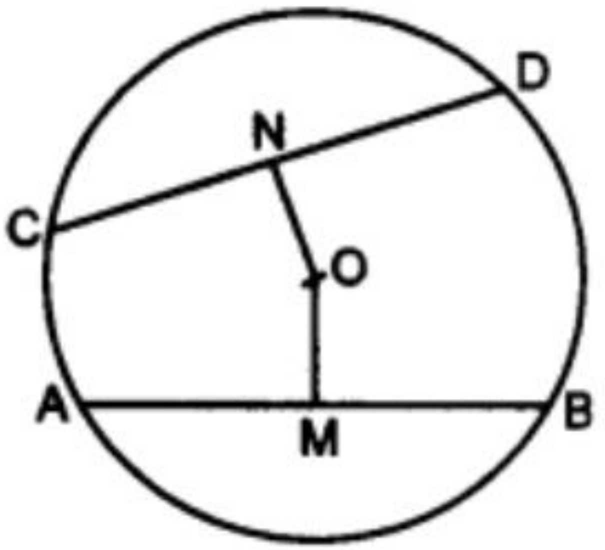
or, $PT^2 = 16 \times 4$

or, $PT = \sqrt{64}$

PT = 8

23. In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD. AB = 24 cm, OM = 5 cm, ON = 12 cm.

Find the :
(i) radius of the circle.
(ii) length of chord CD. [3] [2014]



Answer: (i) 13 cm (ii) 10 cm

Step-by-step Explanation:

Let us join CO and AO.

(i) In ΔAMO, AM = 24/2 = 12 cm. (Perpendicular drawn from the centre of a circle to the chord bisects it.)

∴ by pythagoras theorem,

$AO = \sqrt{AM^2 + OM^2}$

$AO = \sqrt{12^2 + 5^2}$

$AO = \sqrt{169} = 13$ cm

∴ radius = 13 cm

(ii) In ΔCNO,

$CN = \sqrt{CO^2 - ON^2}$

$CN = \sqrt{13^2 - 12^2}$

$CN = \sqrt{25} = 5$ cm

∴ CD = 2 CN = 10 cm

24. In the given figure,

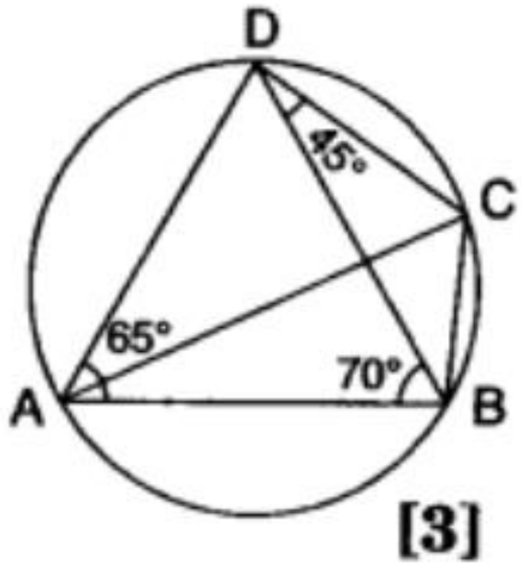
∠BAD = 65°,

∠ABD = 70°,

∠BDC = 45°

i.)Prove that AC is a diameter of the circle.

ii.)Find ∠ACB. [2013]



Answer: (ii) 45°

Step-by-step Explanation:

(i) In ΔADB,

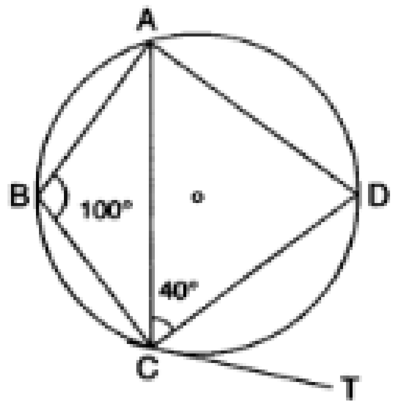
∠ADB = 180°- (65+70)°= 45°

∴∠ADC = (45+45)°= 90°

Hence, AC is the diameter of the circle. (angle in a semicircle is a right angle.)

(ii) ∠ACB =∠ADB = 45°

25. In the given circle with centre O, ∠ABC = 100°, ∠ACD = 40° and CT is a tangent to the circle at C.
Find ∠ADC and ∠DCT. [2013]



Answer: ∠ADC = 80° and ∠DCT= 60°

Step-by-step Explanation:

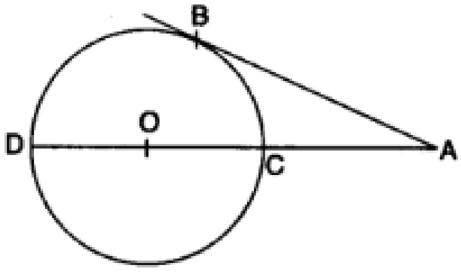
∠ADC= 180°-∠ABC (opposite angles of a cyclic quadrilateral are supplementary.)

∠ADC= (180-100)° = 80°

∠DAC= 180°-(40 + 80)° = 60°

∴∠DCT = ∠DAC= 60° (angles in the alternate segment are equal.)

26. In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle. [3] [2012]



Answer: 11.25 cm

Step-by-step Explanation:

We know, When a tangent and a chord of a circle intersect externally, the product of the lengths of the segments of the chord is equal to the square of the length of the tangent.

$\therefore AB^2 = AC \times AD$

or, $15^2 = 7.5 \times (CD + 7.5)$

or, $CD + 7.5 = 225/7.5$

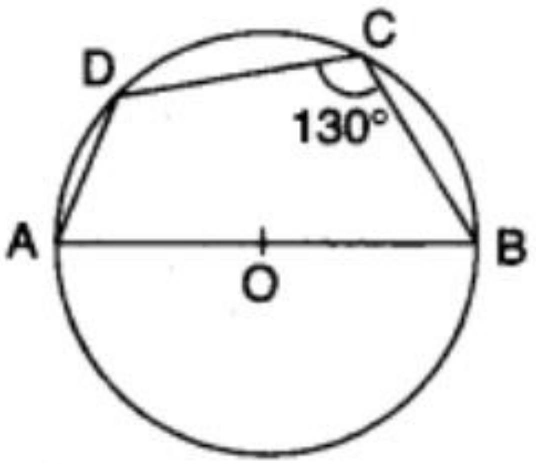
or, $CD = 30 - 7.5 = 22.5$ cm

CD is the diameter of the circle.

$\therefore$ radius= 22.5/2= 11.25 cm

27. In the given figure AB is the diameter of a circle with centre O. ∠BCD = 130°. Find:

(i) ∠DAB
(ii) ∠DBA. [3] [2012]



Answer: (i) ∠DAB=50° (ii) ∠DBA= 40°

Step-by-step Explanation:

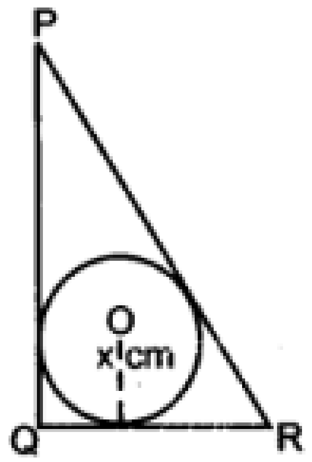
(i) ∠DAB= 180°-130°= 50° (opposite angles of a cyclic quadrilateral are supplementary.)

(i) ∠ADB= 90° (angle in a semicircle is right angle.)

∴ In ΔADB,

∠DBA= 180°- (90+50)°=40°

28. In triangle PQR, PQ = 24 cm, QR = 7 cm and ∠PQR = 90°. Find the radius of the inscribed circle. [2012]



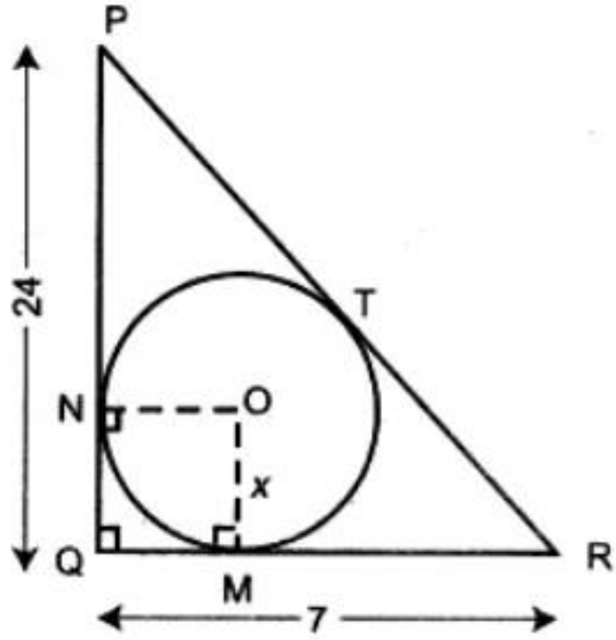
Answer: 3cm

Step-by-step Explantion:

$PR^2 = QR^2 + PQ^2$

$PR^2 = 7^2 + 24^2$

PR= √625 = 25 cm



ON and OM are joined.

We know, tangent to a circle and radius through the point of contact are perpendicular to each other.

∴ ∠ONQ= 90° and ∠OMQ= 90°

QM = QN (tangents drwan from an external point to a circle are equal in length.)

∴ OMQN is a square with each side x cm.

∴ MR= TR=(7-x) cm, PN=PT= (24-x) cm

Now, PR= PT + TR

(24-x) + (7-x) = 25

or, 31 – 2x = 25

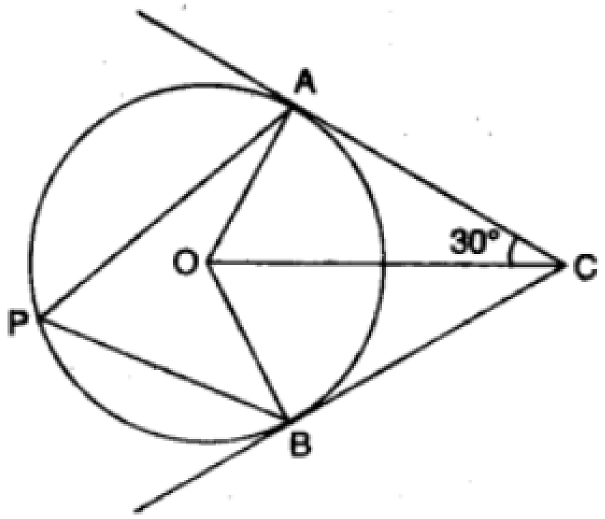
or, 2x = 6

or, x= 3 cm

Hence, radius= 3cm

29. In the given figure O is the centre of the circle. Tangents at A and B meet at C. If ∠AOC = 30°, find

(i) ∠BCO
(ii) ∠AOB
(iii) ∠APB [3] [2011]



Answer: (i) 30° (ii) 120° (iii) 60°

Step-by-step Explanation:

(i) ∠BCO = ∠ACO = 30° (two tangents drawn from an external point to a circle are equally inclined to the line segment joining the centre to that point.)

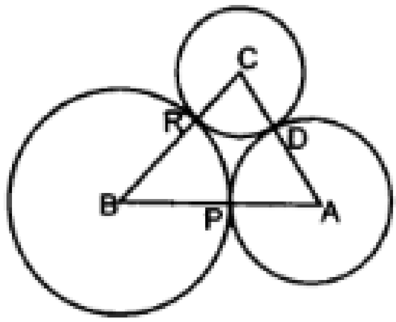
(ii) ∠OAC = ∠OBC = 90° (tangent to a circle and radius through the point of contact are perpendicular to each other.)

∴ ∠AOB = 360°- (90 +30 + 30 +90)° = 120°

(iii) ∠APB = 1/2 of ∠AOB = 1/2 of 120°= 60°

30. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other

with the vertices as their centres. Find the radii of the three circles. [3][2011]



Answer: 6 cm, 2 cm, 4 cm

Step-by-step Explanation:

Let the radii of the three circles be r_1 , r_2 , and r_3 respectively.

so, BC= $r_1 + r_2 = 8$ …… (1)

AC= $r_2 + r_3 = 6$ ….. (2)

AB= $r_1 + r_3 = 10$ ….. (3)

Adding (1), (2) and (3) we get,

$2(r_1 + r_2 + r_3)= 24$

$r_1 + r_2 + r_3 =12$ …. (4)

subtracting (1) from (4), we get,

$(r_1 + r_2 + r_3)-(r_1 + r_2)=12- 8$

$r_3 =4$ cm

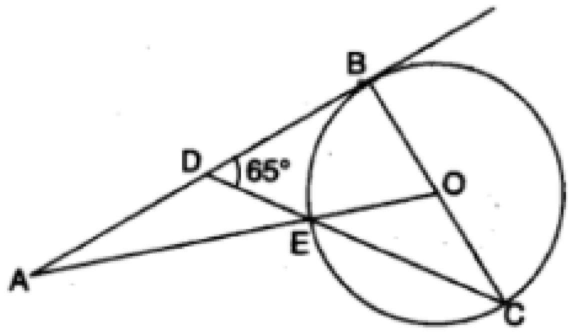
Similarly, subtracting (2) from (4), we get,

$r_1 = 6$ cm and

subtracting (3) from (4), we get,

$r_2 = 2$ cm

31. In the following figure O is the centre of the circle and AB is a tangent to it at point B. ∠BDC = 65°. Find ∠BAO. [3] [2010]



Answer: 40°

Step-by-step Explanation:

In ΔCBD, ∠ABO= 90° (tangent to a circle and radius through the point of contact are perpendicular to each other.)

∴ ∠BCD= 180°-(65 + 90)°= 25°

∴ ∠BOA= 2∠BCD= 50° (angle subtended by an arc at the centre of a circle is double the angle subtended by it on the remaining part of the circle.)

In ΔAOB, ∠BAO= 180°-(90 + 50)°= 40°