FUNDAMENTALS OF MATHEMATICS # 02

SIMILAR TRIANGLES

We call two figures similar if one is simply a blown-up or scaled-up, and possibly rotated and/ or flipped, version of the other.

Sufficient Conditions For Similarity of Triangles.

Two triangles are said to be similar if any one of the following conditions is satisfied.

(i) their corresponding angles are equal.

(ii) their corresponding sides are proportional.

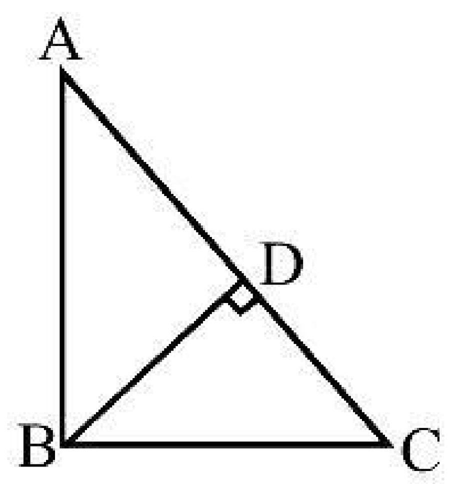
Based on the above, there are three axioms for similarity of two triangles.

1. A.A. (Angle-Angle) Axiom of Similarity.
2. S.A.S (Side-Angle-Side) Axiom of Similarity.
3. S.S.S. (Side-Side-Side) Axiom of Similarity.

Note :

(i) Congruent triangles are necessarily similar but the similar triangles may not be congruent.

(ii) If ABC is a triangle, right-angled at B and $BD \perp AC$, then $\Delta ABC \sim \Delta ADB \sim \Delta BDC$



(iii) If two triangles are similar to a third triangle, then they are similar to each other.

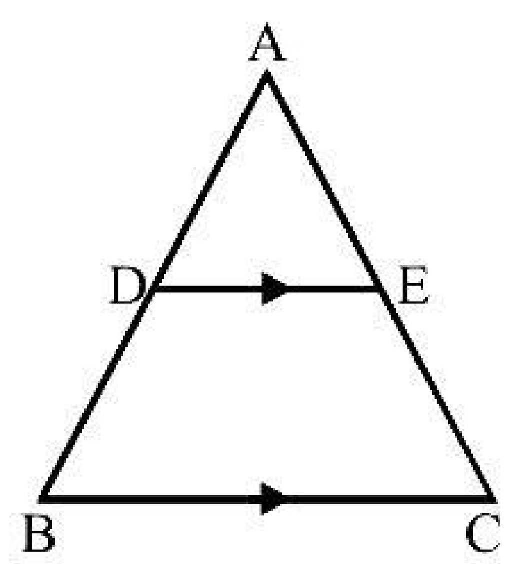
Similar Polygons

Two polygons with the same number of sides are said to be similar if their corresponding angles are equal as well as their corresponding sides are proportional.

Theorem 1 :

Basic Proportionality Theorem (B.P.T.)

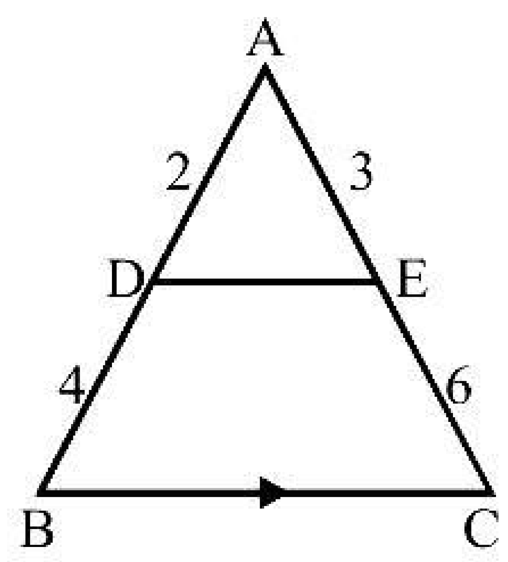
If a line is drawn parallel to a side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio. i.e., if in a ΔABC, $DE||BC$ then, $\frac{AD}{DB} = \frac{AE}{EC}$.



Theorem 2 :

Converse of B.P.T.

If a line divides any two sides of a triangle proportionally, then it must be parallel to the third side.



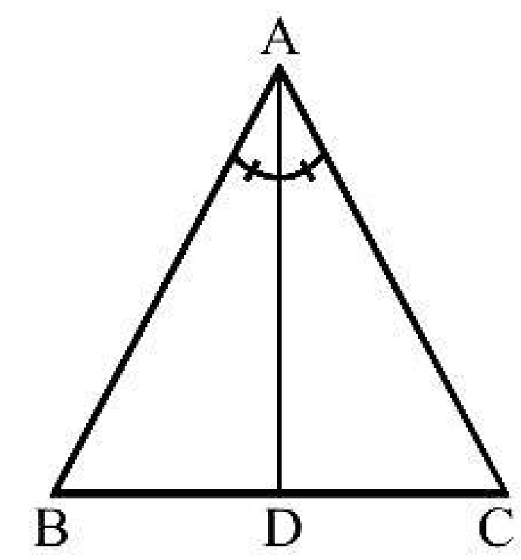
i.e., If in a ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$ then $DE \| BC$.

Theorem 3 :

Angle Bisector Theorems

(i) Angle Bisector Theorem (Internal)

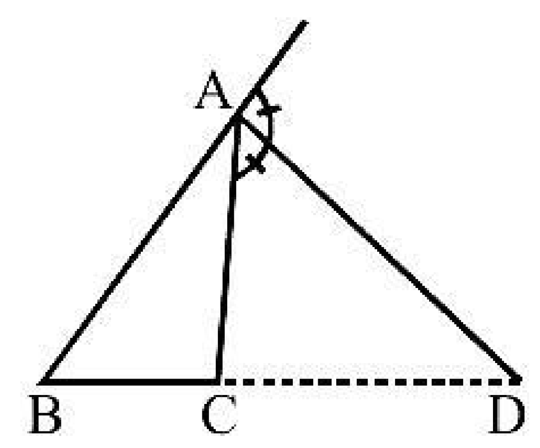
The internal bisector of an angle of a triangle divides the opposite side, internally, in the ratio of the sides containing the angle. i.e., If in a ΔABC, AD bisects $\angle A$,



then $\frac{AB}{AC} = \frac{BD}{DC}$.

(ii) Angle Bisector Theorem (External)

The bisector of an exterior angle of a triangle divides the opposite side (provided bisector and opposite side are not parallel) externally, in the ratio of the sides containing the angle.



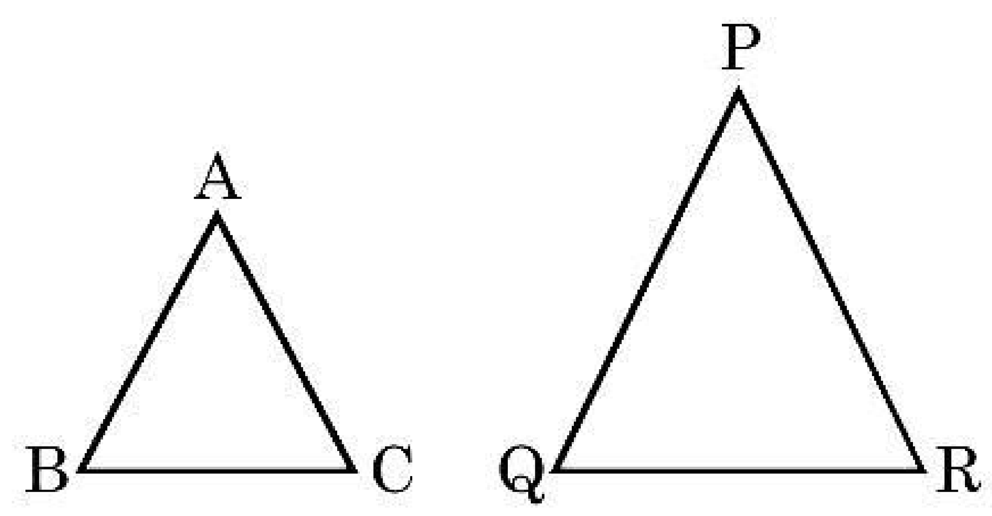
i.e., If in a ΔABC, AD bisects the exterior angle A and intersects side BC produced in D, then $\frac{BD}{CD} = \frac{AB}{AC}$

Converse of both the angle bisector theorems are also true.

Theorem 4 :

Proportion Applied to Area

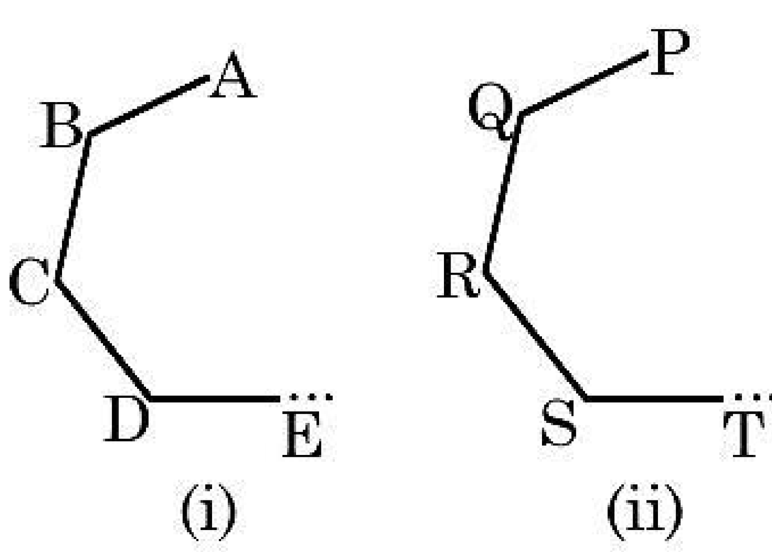
The areas of similar triangles are proportional to the squares of corresponding sides.



$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad (\Delta ABC \sim \Delta PQR)$$

Theorem 5 :

The areas of similar polygons are proportional to the squares of corresponding sides.



$$\frac{\text{Area } 1^{\text{st}} \text{ poly}}{\text{Area } 2^{\text{nd}} \text{ poly}} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CD^2}{RS^2} = \ldots$$

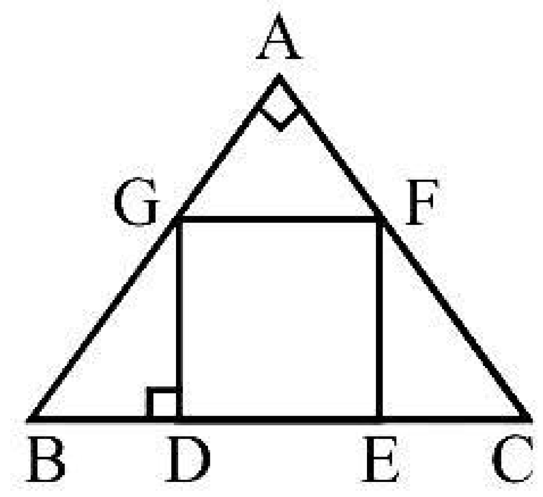
Illustration-1 : In the given figure, DEFG is a square and $\angle BAC = 90°$. Prove that :

(i) $\Delta AGF \sim \Delta DBG$

(ii) $\Delta AGF \sim \Delta EFC$

(iii) $\Delta DBG \sim \Delta EFC$

(iv) $DE^2 = BD \times EC$



Solution : (i) In Δs AGF and DBG, we have :

$\angle GAF = \angle BDG = 90°$ and $\angle AGF = \angle DBG$ [corresponding $\angle$s]

$\therefore \Delta AGF \sim \Delta DBG$ [by A.A. similarity]

(ii) In Δs AGF and EFC, we have :

$\angle FAG = \angle CEF = 90°$ and $\angle AFG = \angle ECF$ [corresponding $\angle$s]

$\therefore \Delta AGF \sim \Delta EFC$ [by A.A. similarity]

(iii) Since, $\Delta AGF \sim \Delta DBG$ and $\Delta AGF \sim \Delta EFC$, it follows that $\Delta DBG \sim \Delta EFC$.

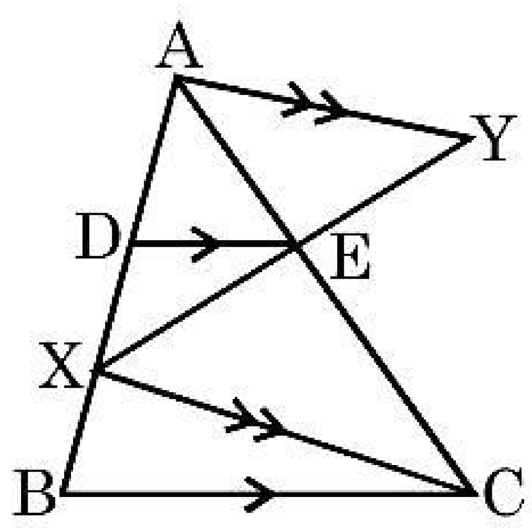
Note : If two triangles are similar to a third triangle, then they are similar to each other.

(iv) Since, $\Delta DBG \sim \Delta EFC$, we have :

$\frac{BD}{EF} = \frac{DG}{EC}$ or $\frac{BD}{DE} = \frac{DE}{EC}$ [$\because$ EF = DE, DG = DE (Sides of a sq.)]

Hence, $DE^2 = BD \times EC$.

Illustration-2 : Given the $\overline{DE} \parallel \overline{BC}$ and $\overline{AY} \parallel \overline{XC}$, prove that $\frac{EY}{EX} = \frac{AD}{DB}$.



Solution : The ratios of side lengths in the problem suggest we look for similar triangles.

Since $\overline{AY} \parallel \overline{XC}$, we have $\Delta AYE \sim \Delta CXE$.

$\Rightarrow$ EY/EX = AE/EC. All we have left is to show that AE/EC = AD/DB.

Since $\overline{DE} \parallel \overline{BC}$, we have $\Delta ADE \sim \Delta ABC$. Therefore, AD/AB = AE/AC, which is almost what we want! We break AB and AC into AD + DB and AE + EC, hoping we can do a little algebra to finish :

$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC}$$

If only we could get rid of the AD and AE in the denominators-then we would have AD/DB = AE/EC.

$$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$$

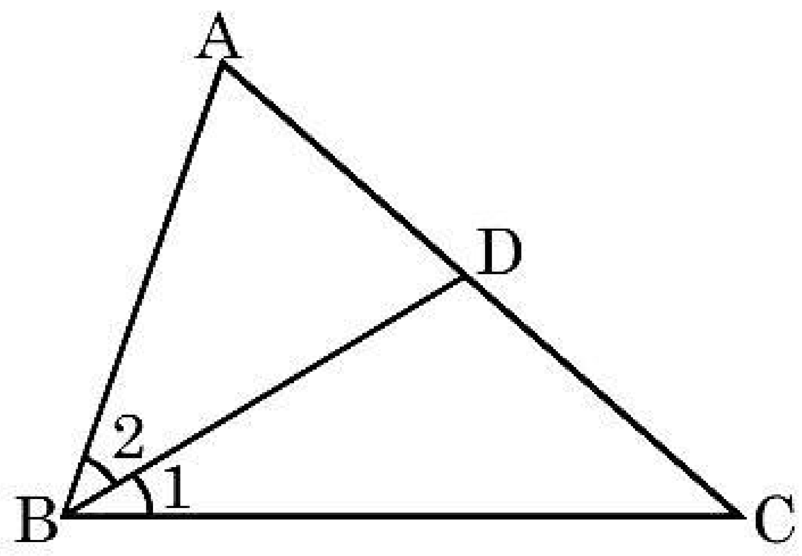
Therefore, $\frac{AD}{AD}+\frac{DB}{AD}=\frac{AE}{AE}+\frac{EC}{AE}$, so $1+\frac{DB}{AD}=1+\frac{EC}{AE}$, which gives us $\frac{DB}{AD}=\frac{EC}{AE}$

Flipping these fractions back over gives us AD/DB = AE/EC. Therefore, we have EY/EX = AE/EC = AD/DB, as desired.

Illustration-3 : In ΔABC, $\angle B = 2\angle C$ and the bisector of $\angle B$ intersects AC at D. Prove that $\frac{BD}{DA}=\frac{BC}{BA}$.

Solution : Given : In ΔABC, $\angle B = 2\angle C$ and BD is bisector of $\angle B$.

To Prove : $\frac{BD}{DA}=\frac{BC}{BA}$



BD is bisector of $\angle CBA$.

$\therefore \frac{BC}{BA}=\frac{CD}{AD}$ ...(1)

$2\angle C = \angle 1 + \angle 2$ [given]

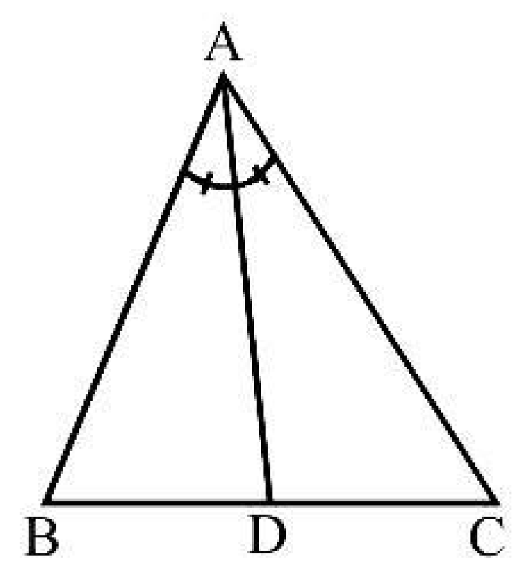
But $\angle 1 = \angle 2$

$\therefore$ $2\angle C = 2\angle 1$

$\Rightarrow$ $\angle C = \angle 1 \Rightarrow BD = CD$ ...(2)

from (1) & (2) $\frac{BC}{BA}=\frac{BD}{DA}$

Illustration-4 : In a ΔABC, AD is the bisector of $\angle BAC$.
If AB = 3.5 cm, AC = 4.2 cm and DC = 2.4 cm
Find BD.



Solution : Since, AD is the bisector $\angle A$, we have $\frac{BD}{DC}=\frac{AB}{AC}$

$\therefore \frac{BD}{2.4}=\frac{3.5}{4.2}\Rightarrow BD=\frac{2.4\times 3.5}{4.2}=2\text{cm}.$

Illustration-5 : O is any point inside a triangle ABC. The bisector of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in points D,E,F respectively. Prove that
AD ×BE × CF = DB × EC × FA.

Solution : Given : O is any point inside a ΔABC. The bisectors of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in points D, E, F respectively.

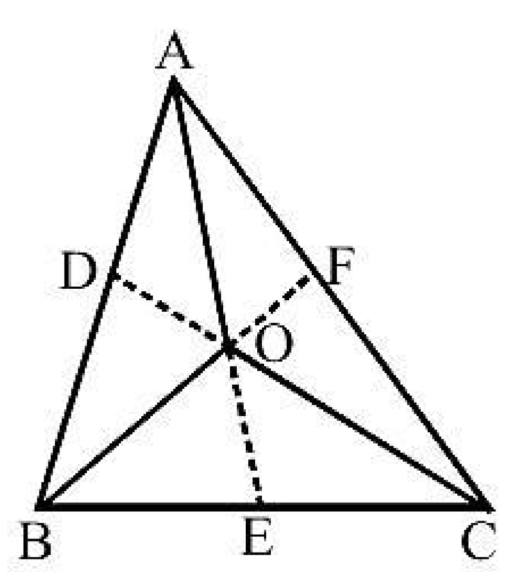
To prove : AD.BE.CF= DB.EC.FA

Proof : In ΔAOB, OD is the bisector of $\angle AOB$.

$\therefore \frac{OA}{OB}=\frac{AD}{DB}$ ...(i)

In ΔBOC, OE is the bisector of $\angle BOC$

$\therefore \frac{OB}{OC}=\frac{BE}{EC}$ ...(ii)



In ΔCOA, OF is the bisector of $\angle COA$.

$\therefore \quad \frac{OC}{OA} = \frac{CF}{FA}$ ...(iii)

Multiplying the corresponding sides of (i), (ii) and (iii), we get :

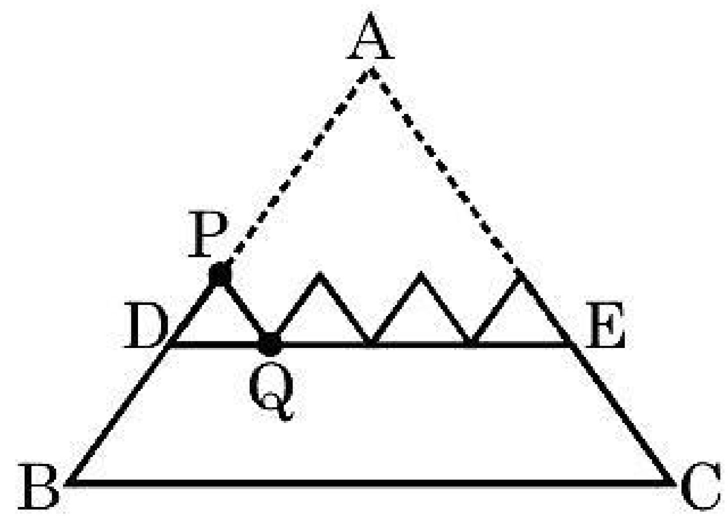
$$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

$\Rightarrow \quad 1 = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$

$\Rightarrow \quad DB \times EC \times FA = AD \times BE \times CF$

or $\quad AD \times BE \times CF = DB \times EC \times FA$.

Illustration-6 : All the triangles in the diagram below are similar to isosceles triangle ABC in which AB = AC. Each of the smaller triangle has area 1 unit^2 and ΔABC has area 40 unit^2, then area of trapezium DBCE is



(A) 16 (B) 8 (C) 24 (D) 32

Solution : $\Delta PDQ \sim \Delta ABC$

$$\frac{ar.\Delta PDQ}{ar.\Delta ABC} = \frac{DQ^2}{BC^2} = \frac{1}{40}$$

$\therefore \quad DQ = \frac{BC}{2\sqrt{10}}$

Also DE = 4.DQ

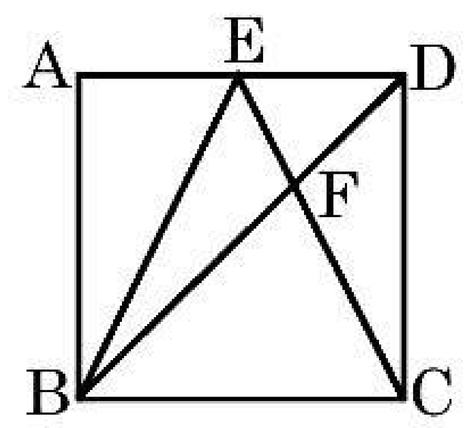
$DE = \frac{4.BC}{2\sqrt{10}}$ Hence $\frac{DE}{BC} = \frac{2}{\sqrt{10}}$

$$\frac{ar\Delta ADE}{ar\Delta ABC} = \frac{DE^2}{BC^2} = \frac{4}{10} = \frac{2}{5}$$

area $\Delta ADE = 16$

area trap DBCE = 40 – 16 = 24

Illustration-7 : ABCD is a square and E is mid point of AD. If area of square is given by A_1. Prove following :



(A) $\frac{\text{area of } \Delta EFD}{\text{area of } \Delta BFC} = \frac{1}{4}$

(B) $\frac{\text{area of } \Delta BEF}{\text{area of } \Delta DFC} = 1$

(C) area of $\Delta EFD = \frac{A_1}{12}$

(D) area of $\Delta BEF = \frac{A_1}{6}$

Solution : Let side of square = a

$A_1 = a^2$

area of $\Delta BED = \frac{1}{2} \times a \times \frac{a}{2} = \frac{1}{4}a^2 = \frac{A_1}{4}$

Since ΔEFD is similar to ΔCFB

$\Rightarrow \frac{ED}{BC} = \frac{1}{2} = \frac{EF}{FC} = \frac{BF}{FD}$

$\Rightarrow \frac{\text{area of } \Delta EFD}{\text{area of } \Delta BFC} = \frac{1}{4}$

Let area of $\Delta EFD = \Delta$

$\Rightarrow$ area of $\Delta BFC = 4\Delta$

Also let area of $\Delta BEF = x$ and let area of $\Delta DFC = y$

$\Rightarrow x + \Delta = \frac{A_1}{4}$ and $y + 4\Delta = \frac{A_1}{2}$

Area of ΔBED = Area of ΔCED

($\because$ triangles on the same base ED and between the same parallel lines ED and BC)

$\Rightarrow$ Area of ΔBEF + Area of ΔEFD = Area of ΔCDF + Area of ΔEFD

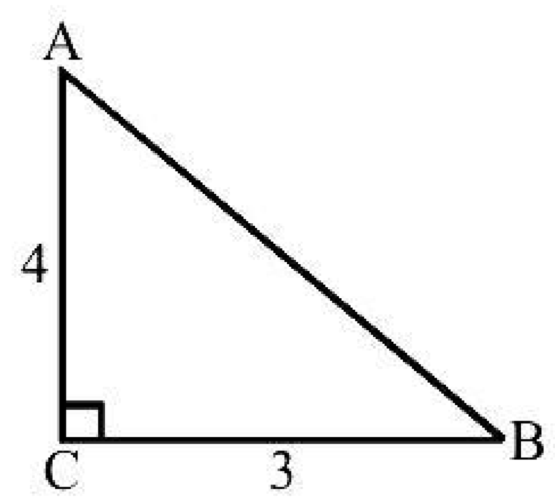
$\Rightarrow x + \Delta = y + \Delta \Rightarrow x = y$

Now x = y therfore $x + \Delta = \frac{A_1}{4}$ and $x + 4\Delta = \frac{A_1}{2}$ on solving these equations

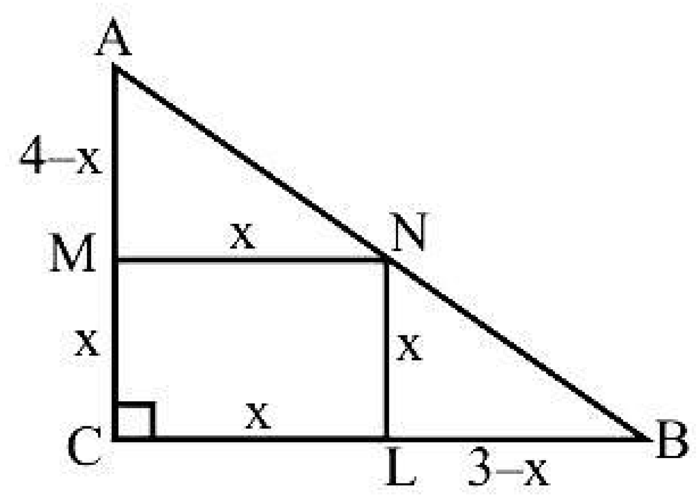
$\Rightarrow \Delta = \frac{A_1}{12} \Rightarrow \quad x = \frac{A_1}{6}$

Also $y + \Delta = \frac{A_1}{4} \Rightarrow y = \frac{A_1}{6}$

Illustration-8 : In a right angle triangle ABC. Let k is maximum possible area of a square that can be inscribed when one of its vertices coincide with the vertex of right angle of triangle then find the value of k



Solution : ΔAMN & ΔNLB are similar.



$\frac{4-x}{x} = \frac{x}{3-x}$

$\Rightarrow 12 - 7x + x^2 = x^2$

$x = \frac{12}{7}$

$k = \frac{144}{49}$

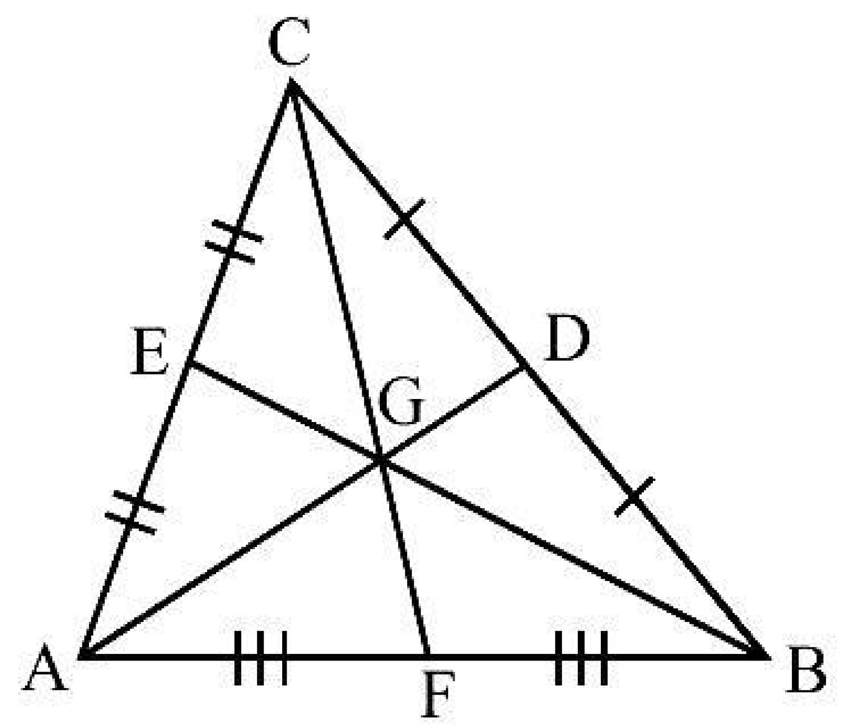
SOME IMPORTANT POINTS AND THEIR MEANING :

MEDIANS AND CENTROID :

A **median** of a triangle is a line segment from a vertex to the midpoint of the opposite side.

In the figure below, $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are all medians.

- The medians of a triangle are concurrent at a point called the **centroid** of the triangle. The centroid of the triangle is usually labelled by G.



- The medians of a triangle divide the triangle into six little triangles of equal area.
- The centroid of a triangle cuts its medians into 2 : 1 ratio. For example, for the triangle shown,

we have $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$.

- The median to the hypotenuse of a right triangle is equal in length to half the hypotenuse.

PERPENDICULAR BISECTORS AND CIRCUMCENTRE :

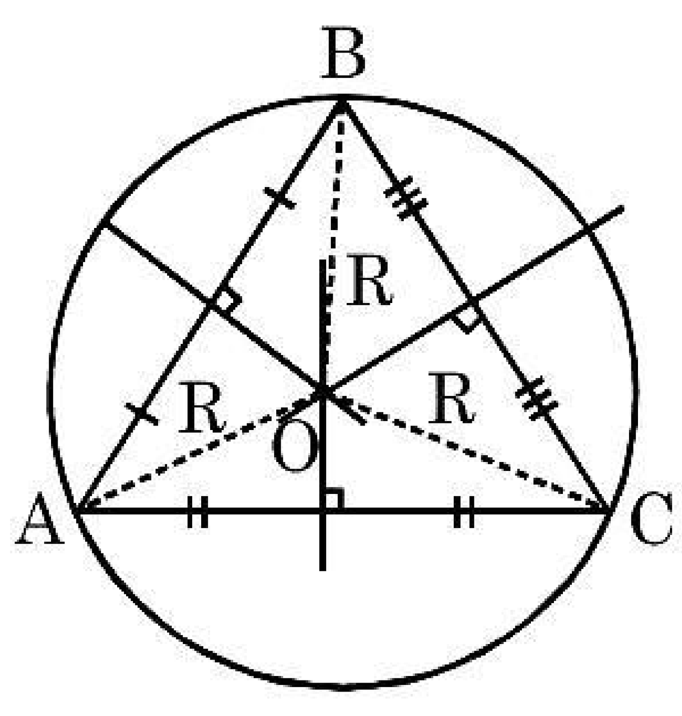
The perpendicular bisector of a segment is the straight line consisting of all the points that are equidistant from the end points of the segment.

Circumcentre :

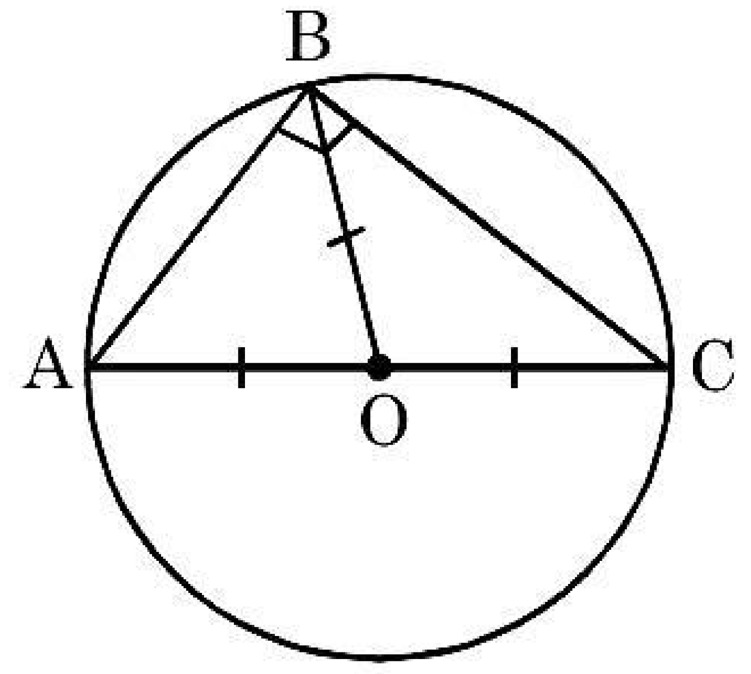
The perpendicular bisectors of the sides of a triangle are concurrent at a point called the **circumcenter**. The circle centered at the circumcenter that passes through the vertices of the triangle is called the **circumcircle** of the triangle, because it is **circumscribed** about the triangle (meaning it passes through all the vertices of the triangle).

Finally, the radius of this circle is called the **circumradius**, the circumcenter is usually labelled with the letter O, and the circumradius is usually denoted by R.

$$OA = OB = OC = R$$



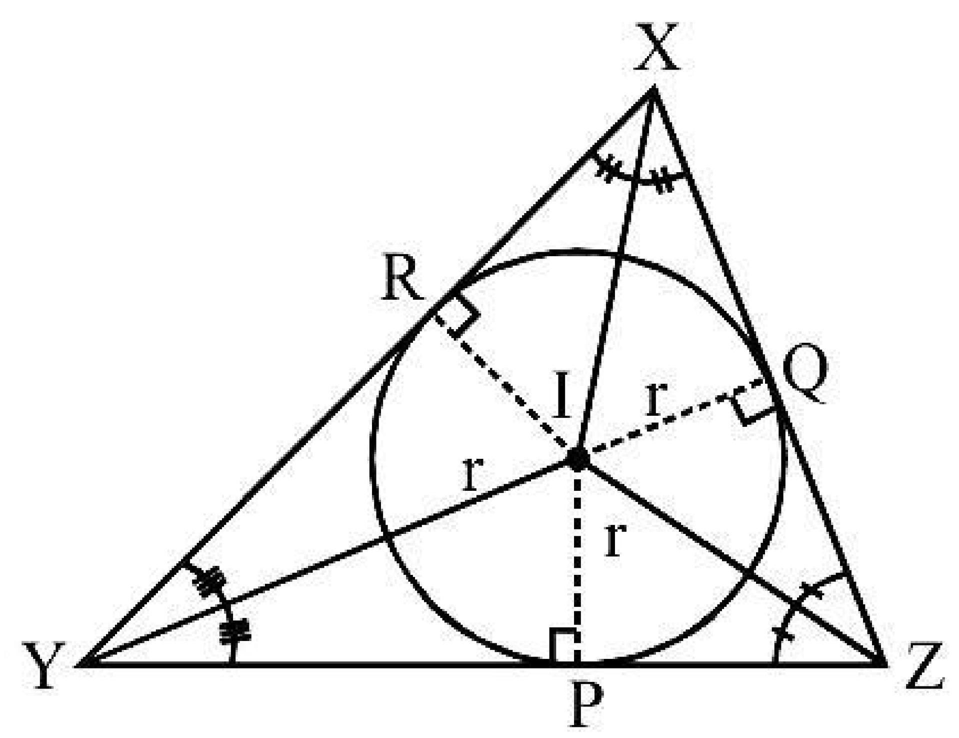
- The circumcenter of a right triangle is the midpoint of the hypotenuse and the circumradius equals half the length of the hypotenuse.



- Just as two points determine a line, we have now shown that three non-collinear points determine a circle. This means that given any three non-collinear points, there is exactly one circle that passes through all three.

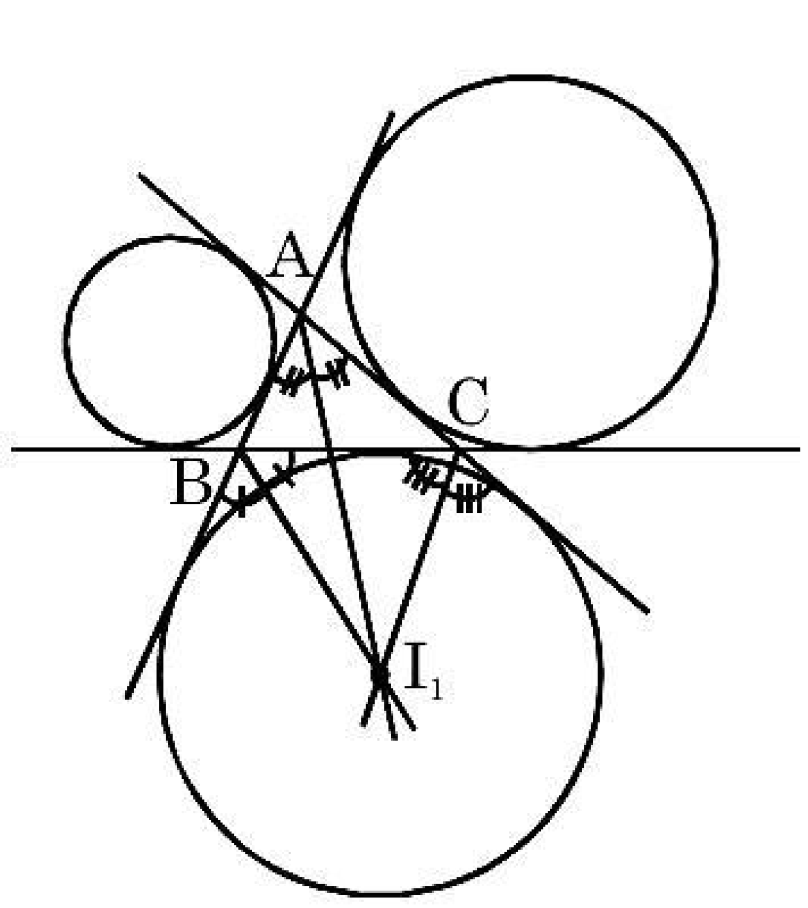
ANGLE BISECTORS, INCENTRE & EXCENTRE :

- The angle bisector of an angle consists of all points that are equidistant from the lines forming the angle.
- The angle bisectors of interior angles of a triangle are concurrent at a point called the **incentre**. This point is equidistant from the sides of the triangle. This common distance from the incentre to the sides of a triangle is called the **inradius**. Thus the circle with centre I and this radius is tangent to all three sides of the triangle. This circle is called the **incircle** because it is inscribed in the triangle (meaning it is tangent to all the sides of the triangle). The incentre is usually denoted I, and the inradius is usually written as r.



Escribed Circle : It is a circle touching one side of a triangle internally and the other two externally. A triangle has three escribed circles. The centre of an escribed circle is the point of concurrence of the bisectors of the two exterior angles and the bisector of the third interior angle.

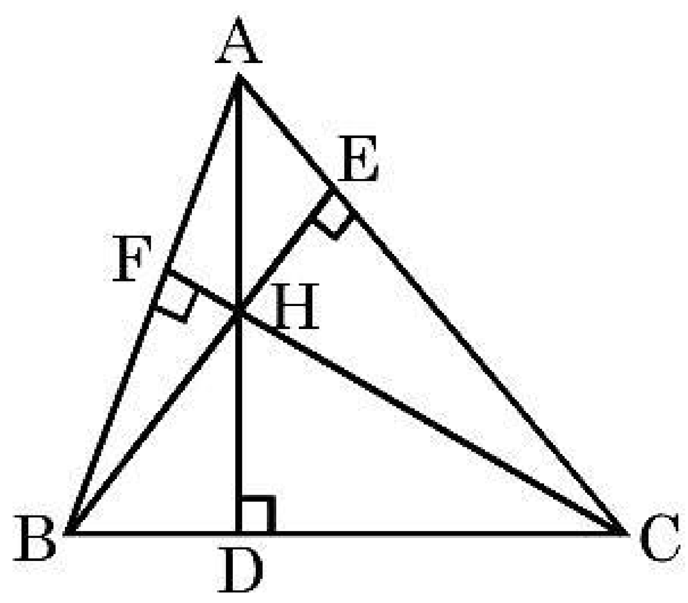
The centres of these escribed circles are called excentres and are labelled with I_1, I_2 and I_3. Also, their corresponding radii called exradii are denoted by r_1, r_2 and r_3.



ALTITUDES & ORTHOCENTRE :

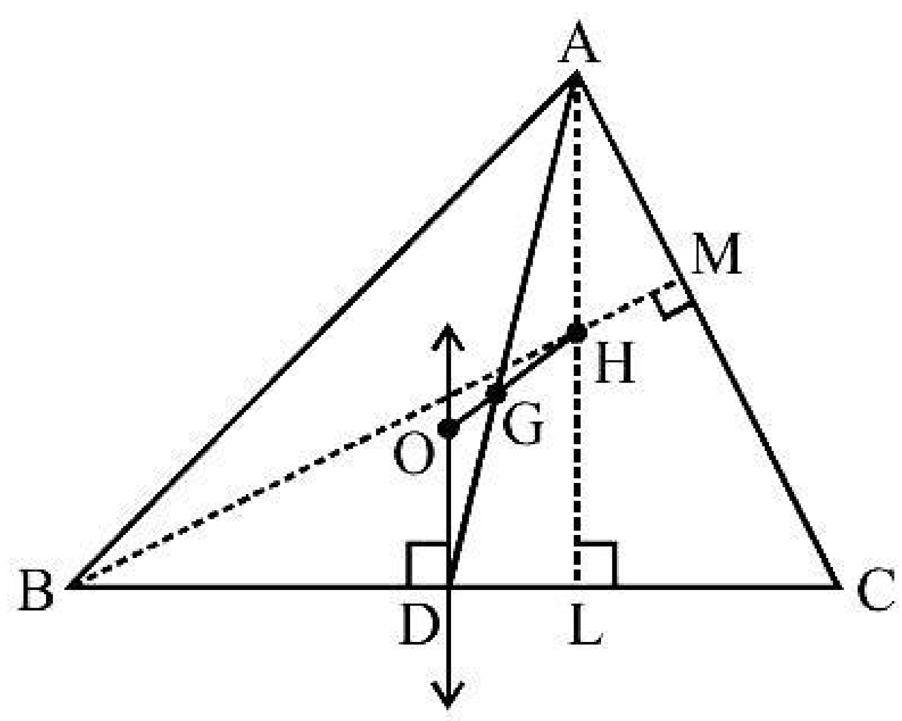
Altitude : It is the perpendicular dropped on any side of a triangle from the opposite vertex.

Orthocentre : The three altitudes of a triangle are concurrent and the point of concurrence is the orthocentre, which is denoted by H.



EULER'S LINE :

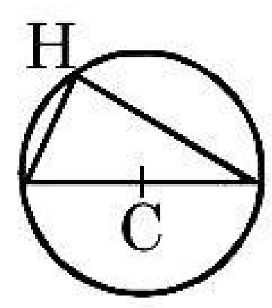
In the figure



- AD is a median and G the centroid.
  AG : GD = 2 : 1.
- BM is the altitude to AC from B.
  Here H is the orthocentre.
- OD is the perpendicular bisector of side BC.
  O is the circumcentre.

Note : O, G and H are collinear. This line is known as Euler's line and HG : GO = 2 : 1.

- In an equilateral triangle these four centres (centroid, orthocentre, incentre, circumcentre) are coincident.
- In an isosceles triangle they are collinear.
- The incentre and centroid are always within the triangle.
- In an acute angled triangle circumcentre and orthocentre also lie inside the triangle
- In a right triangle circumcentre will be the midpoint of the hypotenuse and the orthocentre is the vertex containing the right angle.



- In obtuse angled triangle orthocentre and circumcentre lie outside the triangle.

Illustration-9 : For any triangle, the area of triangle formed by medians of the triangle is $\left(\frac{3}{4}\right)^{th}$ the area of given triangle.

Solution : Consider ΔABC, AD, BE and CF are length of median say ℓ_1, ℓ_2 and ℓ_3 respectively

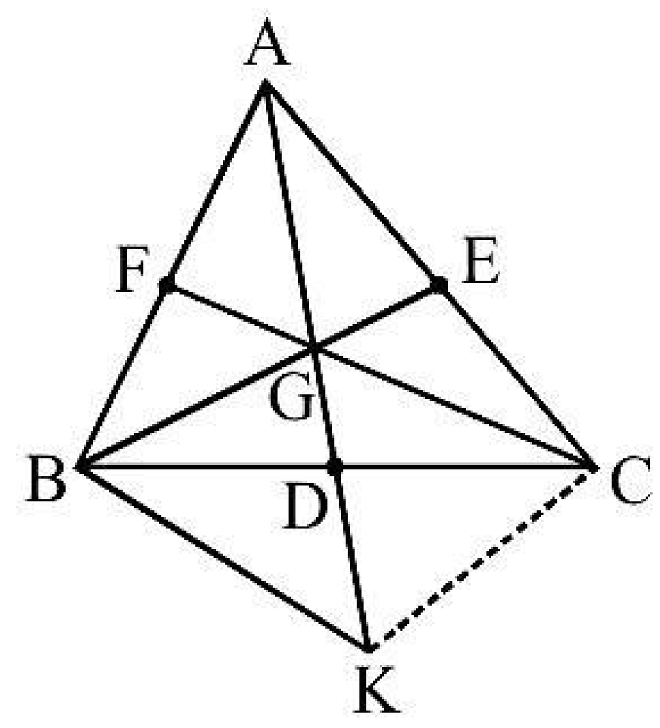
We have to find area of triangle, the length of whose sides are ℓ_1, ℓ_2 and ℓ_3.

Let G be the centroid of ΔABC

Let us produce AG to K

Such that GD = DK

and join CK and BK



We know that

BD = DC and GD = DK

$\therefore$ BKCG is a parallelogram and hence

BK = GC and CK = BG

$BK = \frac{2}{3}CF$ and $CK = \frac{2}{3}BE$

$BK = \frac{2}{3}\ell_3$ and $CK = \frac{2}{3}\ell_2$

and $GK = 2GD = 2.\left(\frac{1}{3}AD\right) = \frac{2}{3}\ell_1$

Now say PQR is triangle formed by length ℓ_1, ℓ_2 and ℓ_3 as sides. $\Delta PQR \sim \Delta GKC$

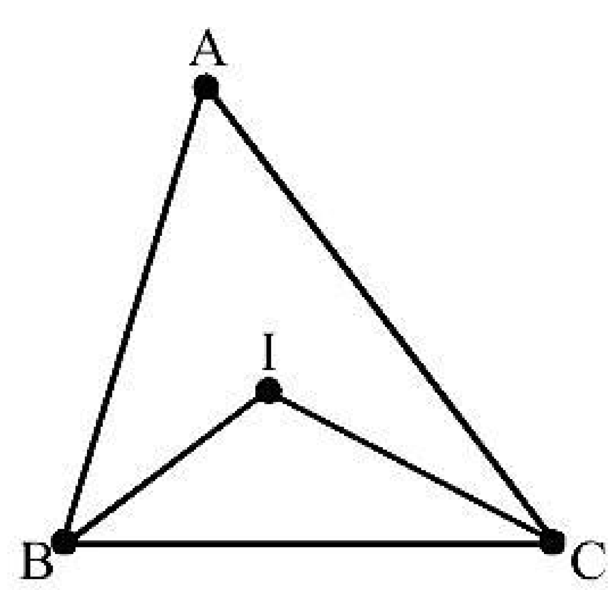
$$\frac{\text{area } \Delta PQR}{\text{area } \Delta GKC} = \frac{(\ell_1)^2}{\left(\frac{2\ell_1}{3}\right)^2} = \frac{9}{4}$$

$$\text{area } \Delta PQR = \frac{9}{4} \text{ area } \Delta GKC = \frac{9}{4} \times \frac{1}{3} \text{ area } \Delta ABC$$

$$\text{area } \Delta PQR = \frac{3}{4} \text{ area } \Delta ABC$$

Illustration-10 : If I is the incenter of ΔABC then $\angle BIC = 90° + \frac{1}{2}A$.

Solution : We have



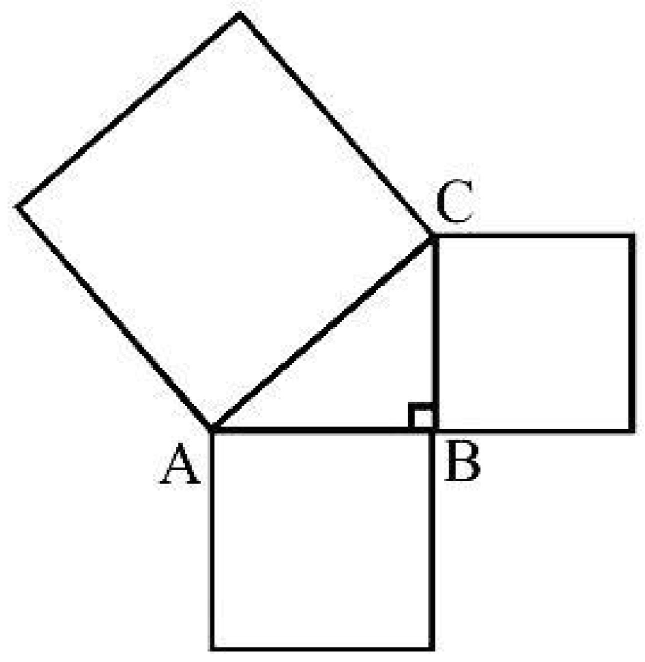
$\angle BIC = 180° - (\angle IBC + \angle ICB)$

$$= 180° - \frac{1}{2}(B + C)$$

$$= 180° - \frac{1}{2}(180° - A)$$

$$= 90° + \frac{1}{2}A.$$

SOME THEOREMS ON SIDE LENGTH OF A TRIANGLE



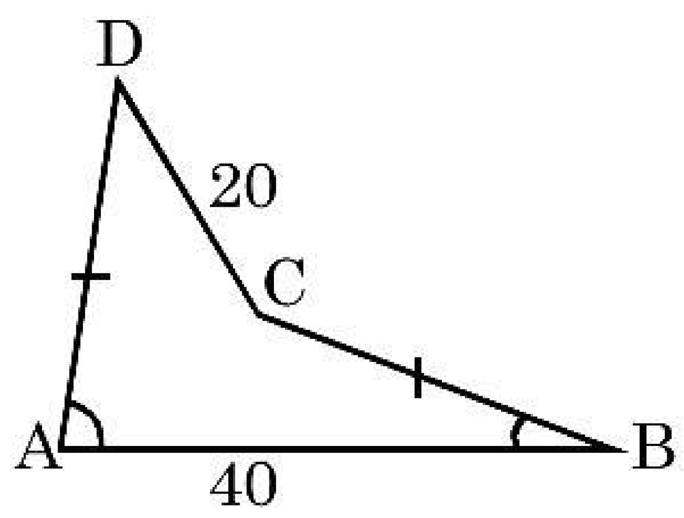
Baudhayana Theorem (Pythagorean/Pythagoras' Theorem) :

In a right-angled triangle the square described on the hypotenuse is equal to the sum of the squares described on the other two sides.

In Right Angle Triangle ABC, right angled at B,

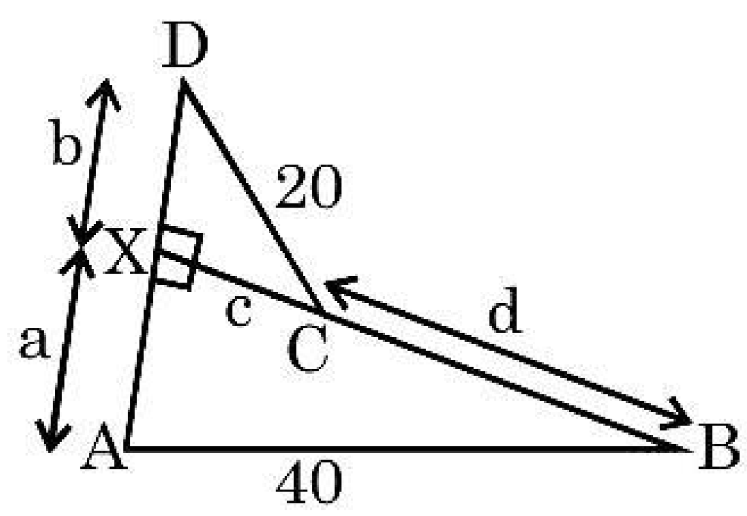
$AC^2 = AB^2 + BC^2$

Illustration-11 : ABCD is a quadrilateral (shown in figure) AD = BC, AB = 40, CD = 20. $\angle DAB + \angle ABC = 90°$. If area of quadrilateral is N then $\frac{N}{100}$ is



Solution : Given : AD = BC, AB = 40

CD = 20, $\angle DAB + \angle ABC = 90°$



Construction :

Extend BC to intersect AD in X.

$\Rightarrow \angle AXB = 180° - (\angle DAB + \angle ABC)$

$\Rightarrow \angle AXB = 180° - 90° = 90°$

Let CX = c, AX = a, XD = b

CB = d

BC = AD

BC = AX + XD

$\boxed{BC = a + b}$

$\therefore \boxed{d = a + b}$

area (quad) = $\frac{1}{2}a(c + d) + \frac{1}{2}bc$

$= \frac{1}{2}[ac + ad + bc]$

$= \frac{1}{2}[ad + c(a + b)]$

$= \frac{1}{2}[ad + cd]$

In ΔBXA

$$a^2 + (c + d)^2 = 40^2 \quad ...(1)$$

In ΔCXD

$$c^2 + b^2 = 20^2 \quad ...(2)$$

On substracting (2) from (1)

$$a^2 + 2cd + d^2 - b^2 = 1200$$

$$a^2 + 2cd + (a + b)^2 - b^2 = 1200 \quad (d = a + b)$$

$$2a^2 + 2cd + 2ab = 1200$$

$$2a(a + b) + 2cd = 1200$$

$$2ad + 2cd = 1200$$

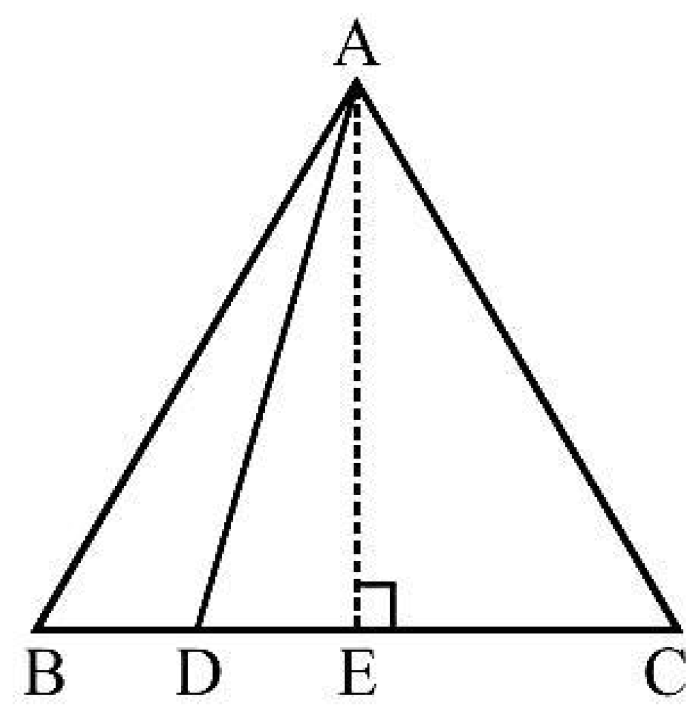
$$\frac{1}{2}(ad + cd) = 300 = \text{Area of quad. ABCD}$$

Illustration-12 : In an equilateral triangle ABC, the side BC is trisected at D. Prove that : $9\,AD^2 = 7\,AB^2$

Solution : Given : An equilateral ΔABC, in which the side BC is trisected at D.

To Prove : $9\,AD^2 = 7AB^2$

Const. : Draw $AE \perp BC$



Proof : We know that in an equilateral triangle perpendicular from a vertex bisects the base.

$$\therefore\ BE = EC = \frac{1}{2}BC \quad ... (i)$$

Also, the side BC is trisected at D. [given]

$$\Rightarrow BD = \frac{1}{3}BC \quad \text{[from figure]} \quad ... (ii)$$

Now, in a right-triangle AED, We have :

$$AD^2 = AE^2 + DE^2$$

$$= AE^2 + (BE - BD)^2 \quad [\because DE = BE - BD]$$

$$= AE^2 + BE^2 + BD^2 - 2\,BE.BD$$

$$= AB^2 + BD^2 - 2\,BE.BD \quad [\because AB^2 = AE^2 + BE^2]$$

$$= BC^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{1}{2}BC\right).\left(\frac{1}{3}BC\right)$$

$$\left[\because AB = BC,\ BD = \frac{1}{3}\ BC \text{ and } BE = \frac{1}{2}\ BC \text{ see (i) and (ii)}\right]$$

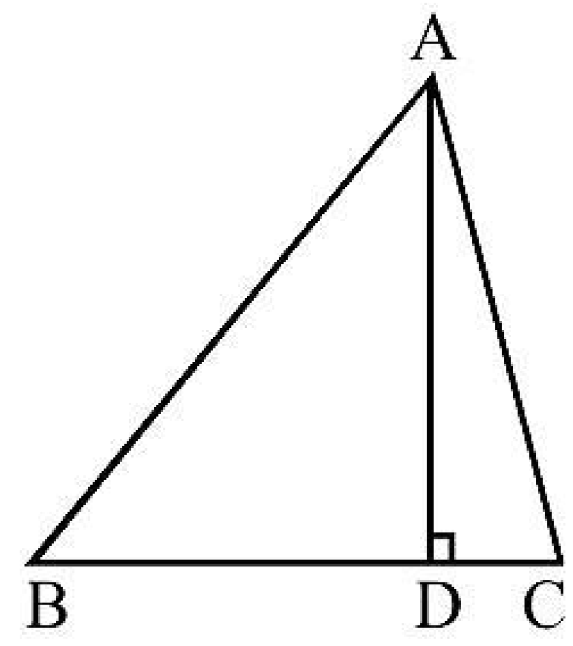
$$= BC^2 + \frac{BC^2}{9} - \frac{BC^2}{3} = \frac{9BC^2 + BC^2 - 3BC^2}{9}$$

$\Rightarrow AD^2 = \frac{7BC^2}{9}$ or $9AD^2 = 7BC^2$

or $9AD^2 = 7AB^2$ (AB = BC = CA)

Illustration-13 : The perpendicular AD on the base BC of a ΔABC intersects BC at D so that DB = 3 CD. Prove that : $2AB^2 = 2AC^2 + BC^2$.

Solution : Given : In figure, AD ⊥ BC and DB = 3CD.



To Prove : $2AB^2 = 2AC^2 + BC^2$.

Proof : From right triangles ADB and ADC, we have :

$AB^2 = AD^2 + BD^2$

and $AC^2 = AD^2 + CD^2$

$\therefore AB^2 - AC^2 = BD^2 - CD^2$

$= (3\,CD)^2 - CD^2$ [∵ BD = 3 CD]

$= 9CD^2 - CD^2 = 8CD^2 = 8\times\left(\frac{1}{4}BC\right)^2 = 8\times\frac{BC^2}{16} = \frac{BC^2}{2}$

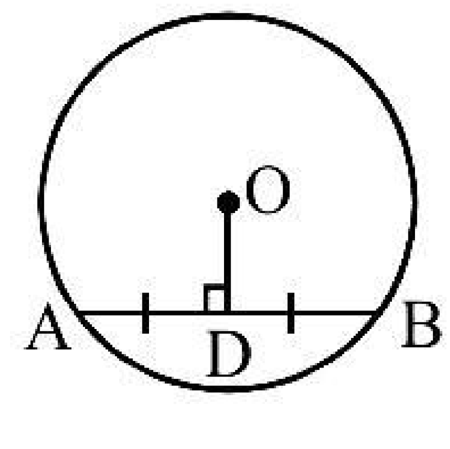
$\left(\because \frac{BD}{CD} = \frac{3}{1} \Rightarrow \frac{BD+CD}{CD} = \frac{3+1}{1} \Rightarrow \frac{BC}{CD} = \frac{4}{1} \Rightarrow CD = \frac{1}{4}BC\right)$

$\therefore 2AB^2 - 2AC^2 = BC^2$

Hence, $2AB^2 = 2AC^2 + BC^2$.

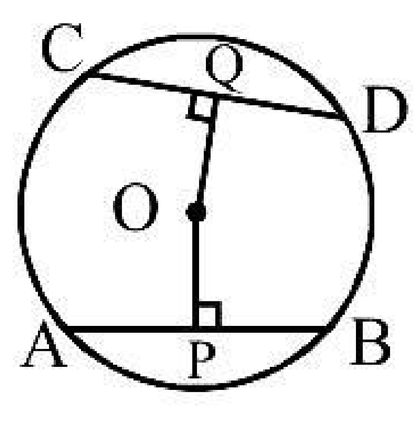
CIRCLE

Theorem 1 :



If a straight line drawn from the center of a circle bisects a chord, not passing through the centre, then it cuts the chord at right angles. Conversely, if it cuts the chord at right angles, then it bisects the chord.

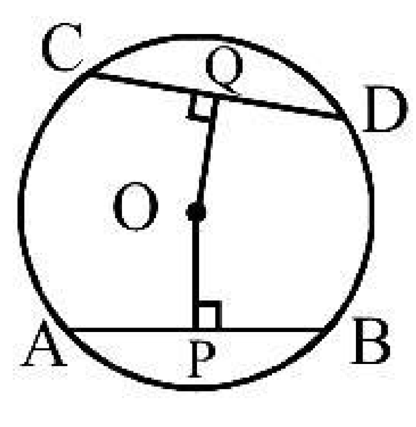
Theorem 2 :



Equal chords of a circle are equidistant from the center.

AB = CD, OP ⊥ AB and OQ ⊥ CD ⇒ OP = OQ

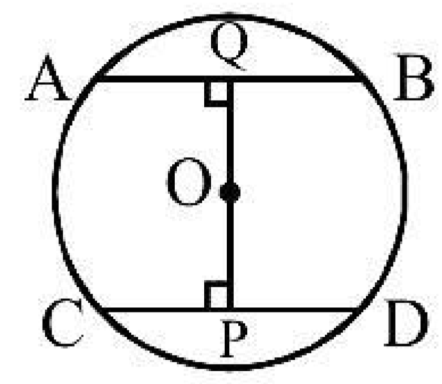
Conversely,



Chords which are equidistant from the centre are equal.

OP = OQ, OP ⊥ AB and OQ ⊥ CD ⇒ AB = CD

Theorem 3 :

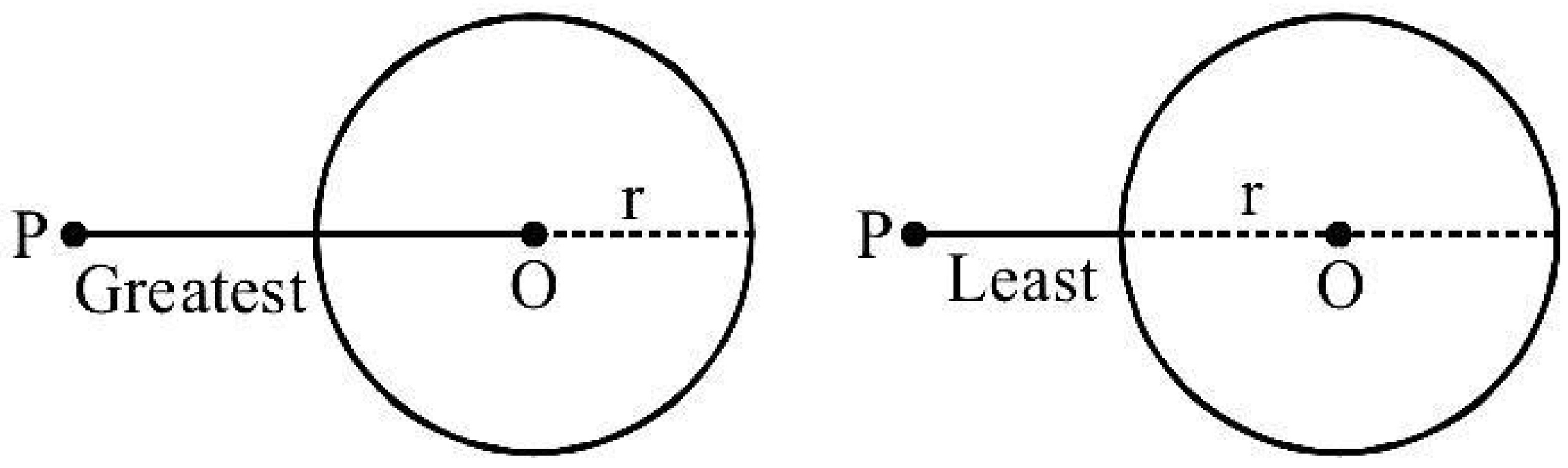


Of any two chords of a circle, that which is nearer to the centre is greater than one lying remote. Conversely, the greater of two chords is nearer to the centre.

$OP > OQ \Rightarrow CD < AB$

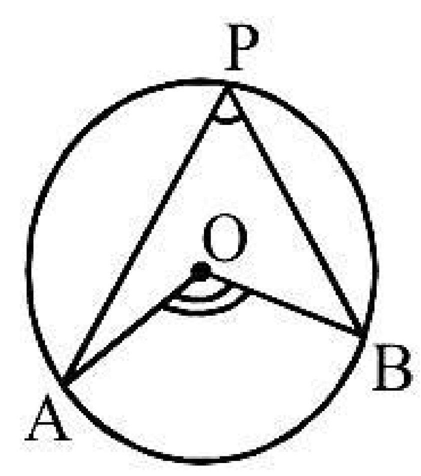
Theorem 4 :

If P be any point (interior, exterior or on the circle) then greatest distance between point P and circumference of the circle is PO +r and the smallest distance between point P and circumference of the circle is |PO –r| (where r is radius and O is centre)



ANGLES IN A CIRCLE

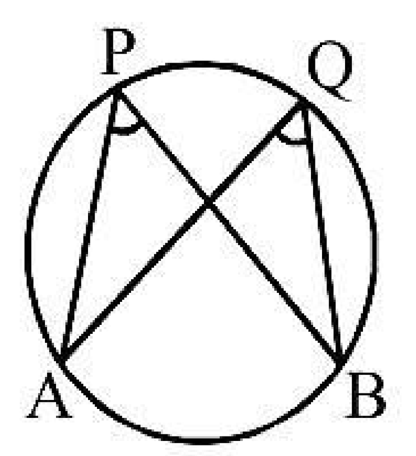
Theorem 1 :



The angle at the centre of a circle is double of an angle at the circumference subtended by the same arc.

$\angle AOB = 2\angle APB$

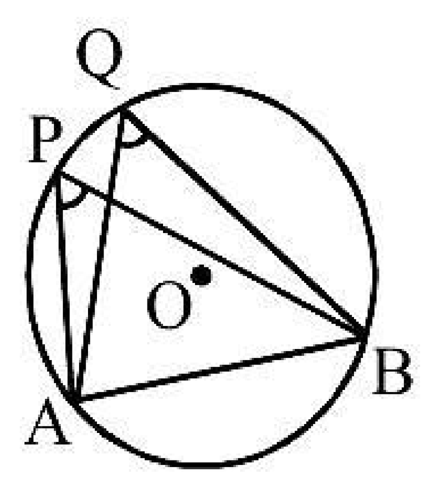
Theorem 2 :



Angles in the same segment of a circle are equal.

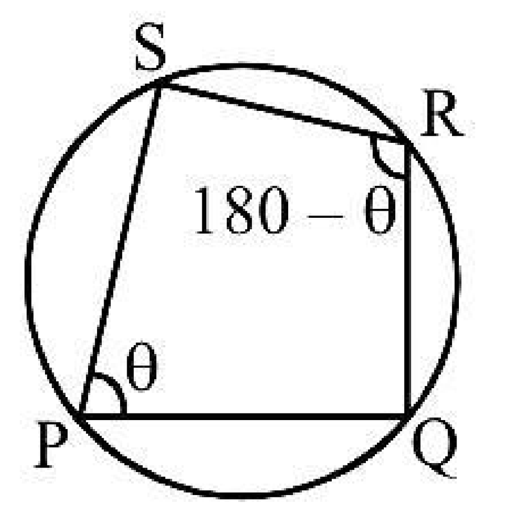
$\angle APB = \angle AQB$

Converse of Theorem 2 :



If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, then four points lie on a circle (i.e. they are concyclic). If $\angle APB = \angle AQB \Rightarrow$ AB is a chord of circle and A, P, Q, B are concyclic points

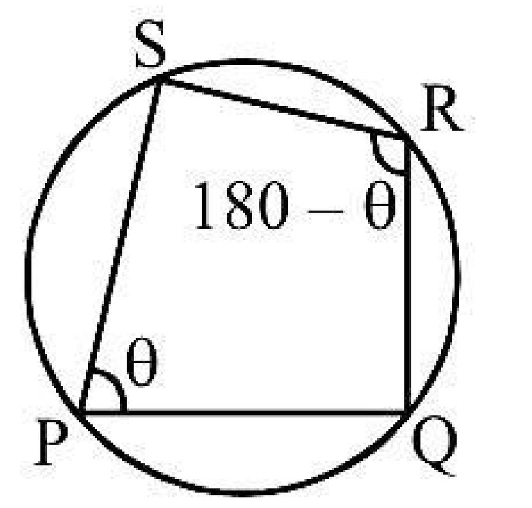
Theorem 3 :



The opposite angles of any quadrilateral inscribed in a circle are together equal to two right angles.

$\angle P + \angle R = 180°$ and $\angle Q + \angle S = 180°$

Converse of Theorem 3 :

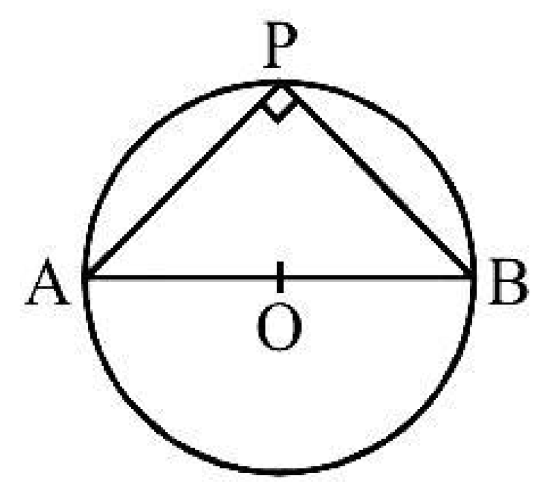


If a pair of opposite angles of a quadrilateral are supplementary, then its vertices are concyclic.

If $\angle P + \angle R = 180°$ or $\angle Q + \angle S = 180°$

$\Rightarrow$ Then points P, Q, R and S are concyclic.

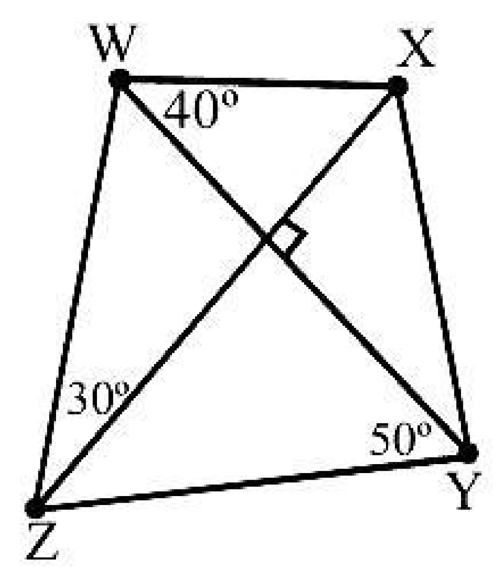
Theorem 4 :



The angle in a semi-circle is a right angle.

$\angle APB = 90°$

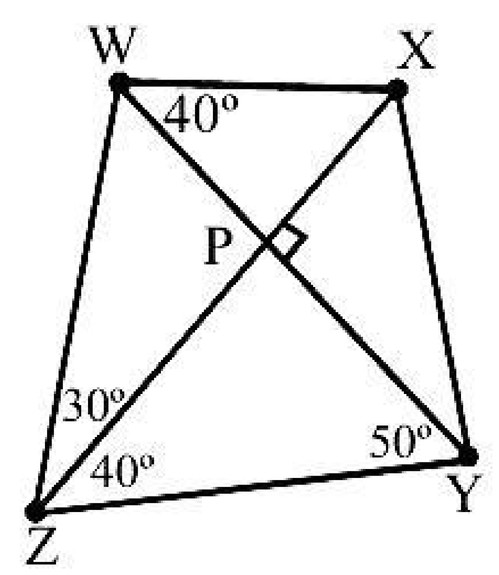
Illustration-14 : In quadrilateral WXYZ with perpendicular diagonals, we are given $\angle WZX = 30°$, $\angle XWY = 40°$, and $\angle WYZ = 50°$.



(a) Compute $\angle Z$.

(b) Compute $\angle X$.

Solution: Let P be the intersection of the diagonals.



In ΔPZY

$\angle PZY = 90° - 50° = 40°$.

Now consider angles.

$\angle XWY$ and $\angle XZY$

Since $\angle XWY = \angle XZY = 40°$

Using theorem : WXYZ is cyclic quadrilateral.

$\angle Z = \angle WZX + \angle PZY$

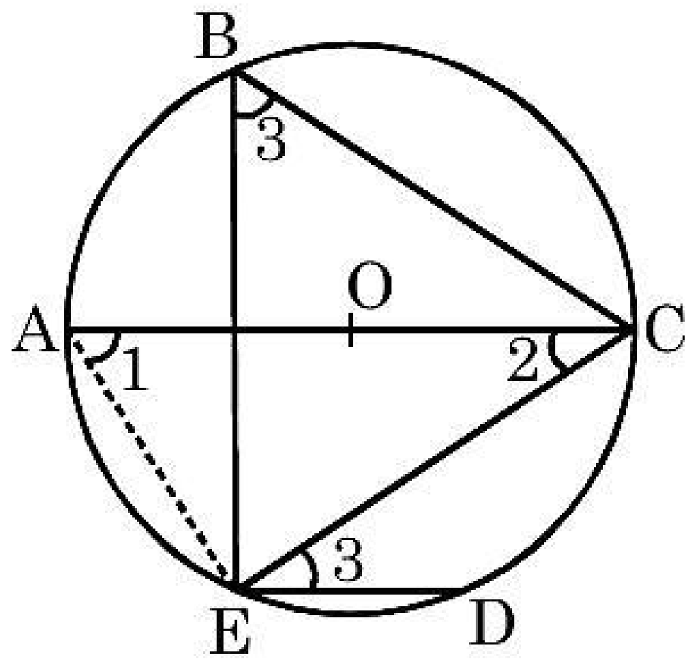
$= 30° + 40° = 70°$

Now $\angle Z + \angle X = 180°$ {opposite angle of cyclic quadrilateral}

$\angle X = 180° - 70° = 110°$

Illustration-15 : In the given figure, the chord ED is parallel to the diameter AC and ∠CBE = 50°. Find ∠CED.

Solution:



∠CBE = ∠CAE (∵ angles in the same segment)

∠CAE = ∠1 = 50° ...(1) (∵ ∠CBE = 50°)

∠AEC = 90° ...(2) (∵ Angle in a semi circle is a right angle)

Now in ΔAEC

∠1 + ∠AEC + ∠2 = 180° [∵ sum of angles of a Δ = 180°]

∴ 50° + 90° + ∠2 = 180°

⇒ ∠2 = 40° ...(3)

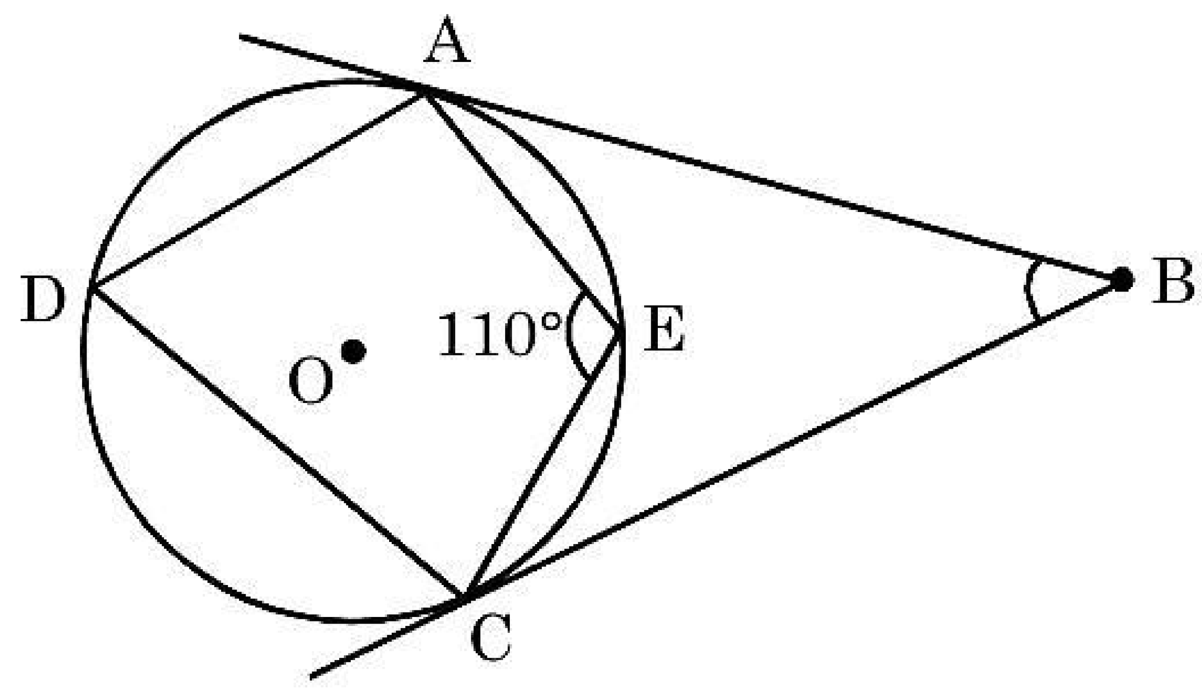
Also, ED || AC (Given)

∴ ∠2 = ∠3 (Alternate angles)

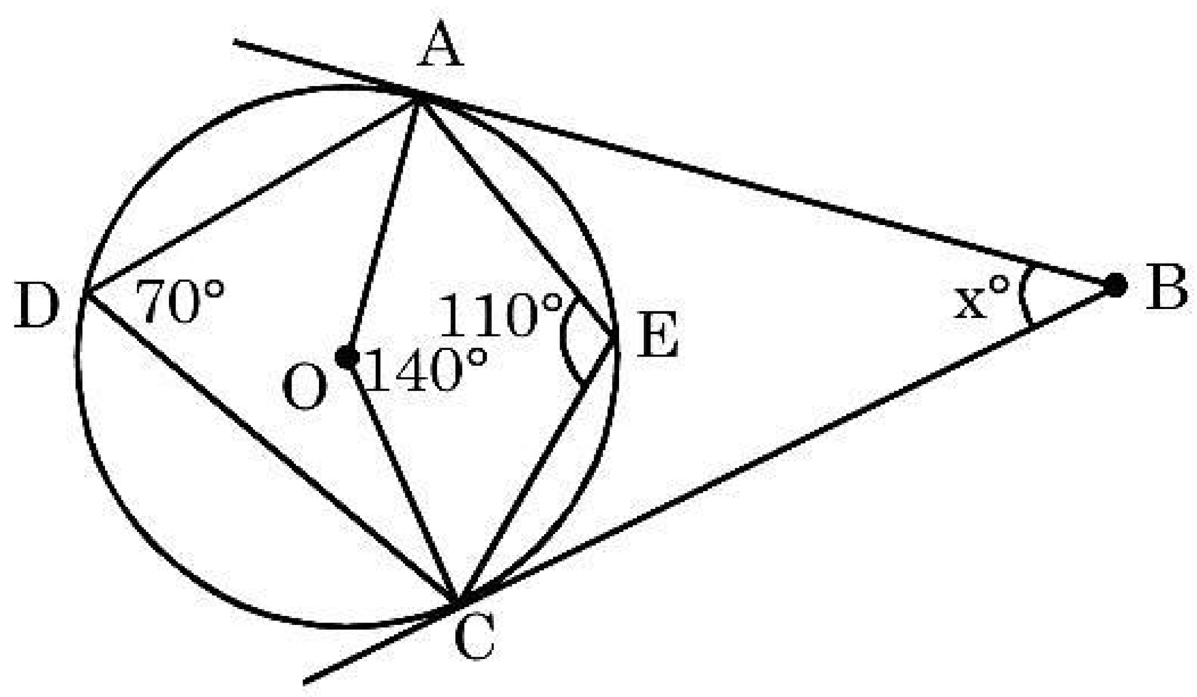
∴ 40° = ∠3 i.e., ∠3 = 40°

Hence ∠CED = 40°

Illustration-16 : In the figure below, BA and BC are tangents to the circle at points A and C, respectively. If ∠AEC = 110°. Find ∠ABC.



Solution :



Consider the diagram to the right.

• Opposite angles of a cyclic quadrilateral sum up to 180°.

So ∠ADC = 180° – 110° = 70°.

• From the inscribed angle theorem, we can see that

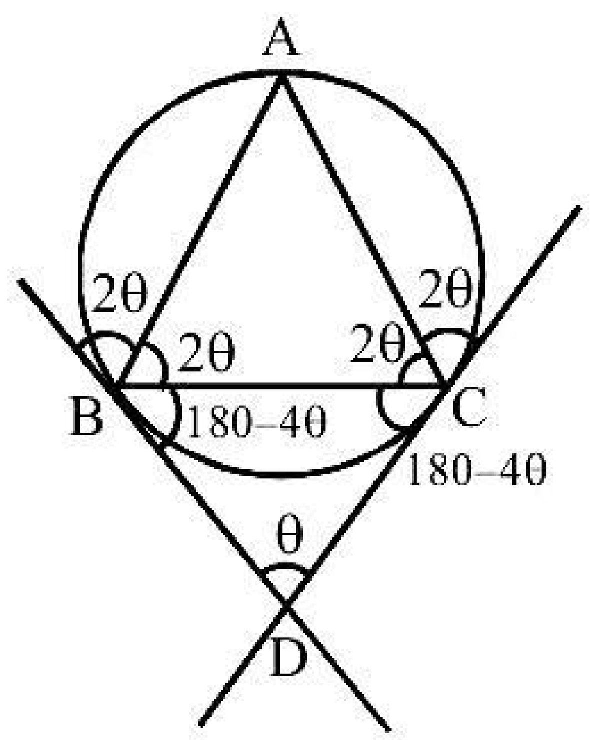
$\angle AOC = 2\angle ADC = 2(70°) = 140°$

Since the sum of interior angle of a quadrilateral is 360°, we have

$$\angle ABC = 360° - \angle BCO - \angle BAO - \angle AOC$$
$$= 360° - 90° - 90° - 140°$$
$$= 40°$$

Illustration-17 : An acute isosceles triangle, ABC is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$, find the measure of $\angle A$.

Solution : Let $\angle D = \theta$



$\Rightarrow \angle ABC = \angle ACB = 2\theta$

In ΔBCD.

$180^o - 4\theta + 180^o - 4\theta + \theta = 180^o$

$\Rightarrow 7\theta = 180^o$

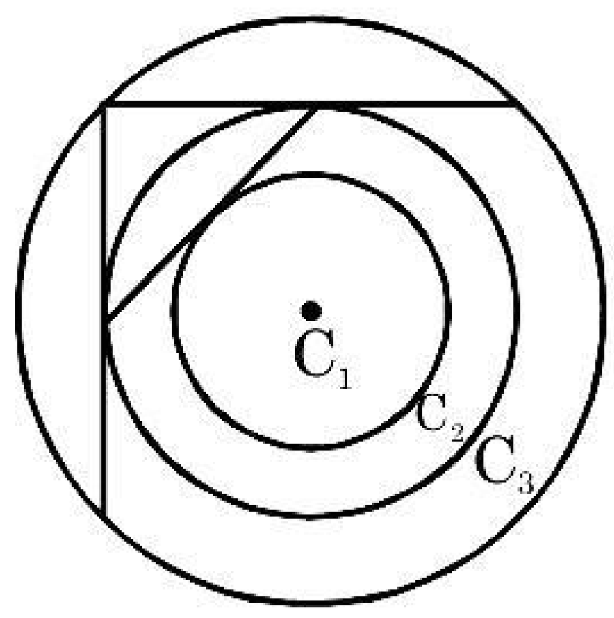
$$\Rightarrow \theta = \frac{180°}{7}$$

$\therefore \angle A = 180^o - 4\theta$

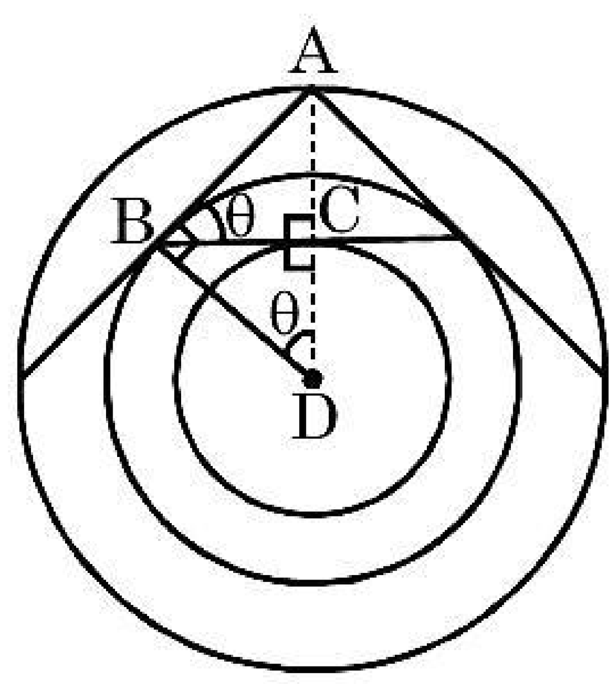
$$= 180° - \frac{4 \times 180°}{7}$$

$$\angle A = \frac{3 \times 180°}{7}$$

Illustration-18 : In the given figure if C_1, C_2, C_3 are three concentric circles such that radius of C_1 and C_2 is 1 and 3 unit respectively, then radius of C_3 is



Solution : Let $\angle BDC = \theta$, then



$$\angle DBC = \frac{\pi}{2} - \theta$$

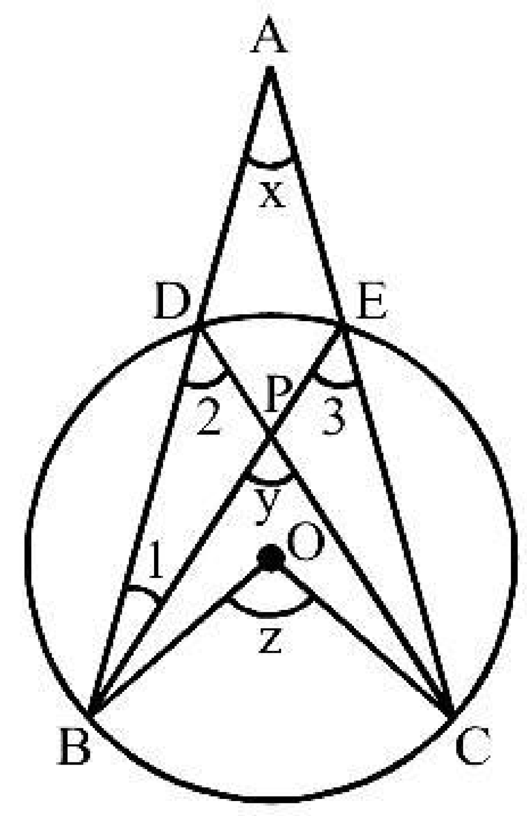
$\therefore \angle CBA = \theta$, So

$\Delta ABD \sim \Delta BCD$, so

$$\frac{AD}{BD} = \frac{BD}{CD}$$

$$\Rightarrow R_3 = \frac{(3)^2}{1} = 9$$

Illustration-19 : In the given fig., O is the centre of the circle. Prove that $\angle x + \angle y = \angle z$.



Solution : Given : In fig., O is the centre of the circle.

To Prove : $\angle x + \angle y = \angle z$

Proof : $\angle z = 2\angle 2$ [angle at the centre is double to the angle in the remaining part.]

or $\angle z = \angle 2 + \angle 2$

But, $\angle 2 = \angle 3$ [angles in the same segment]

$\Rightarrow \angle z = \angle 2 + \angle 3$ ... (i)

Now, we will determine the values of $\angle 2$ and $\angle 3$ in terms of $\angle x$ and $\angle y$.

$\because$ $\angle 3$ is an exterior angle of ΔAEB,

$\therefore$ $\angle 3 = \angle 1 + \angle x$ ... (ii) [ext. $\angle$ = sum of two int. opp. $\angle$s]

Again, $\angle y$ is an exterior angle of ΔDEP

$\therefore \angle y = \angle 1 + \angle 2 \quad \Rightarrow \angle 2 = \angle y - \angle 1$ ... (iii)

(i), (ii) and (iii)

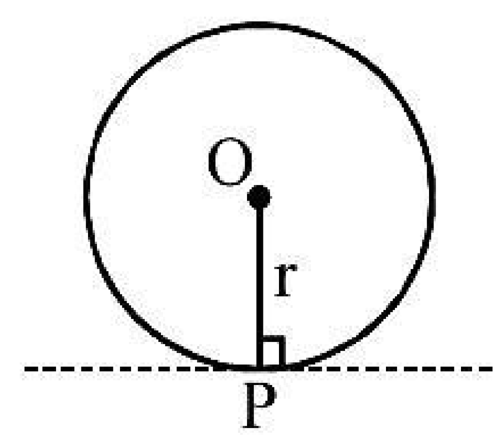
$\Rightarrow \angle z = (\angle y - \angle 1) + (\angle 1 + \angle x) = \angle x + \angle y$

Hence, $\angle x + \angle y = \angle z$.

TANGENT

Theorem 1 :

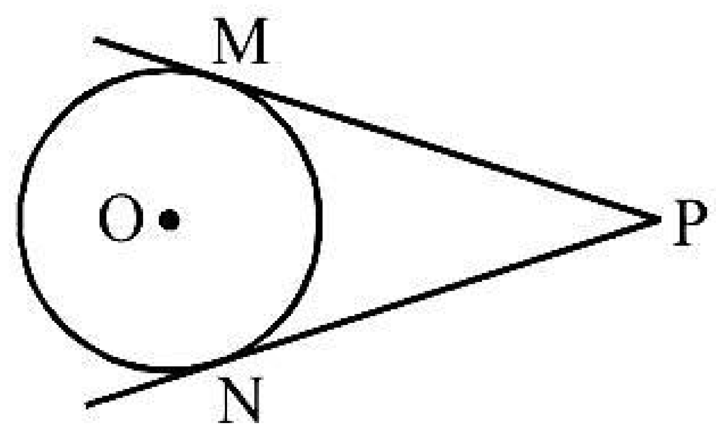
The tangent at any point of a circle is perpendicular to the radius drawn to the point of contact.



Theorem 2 :

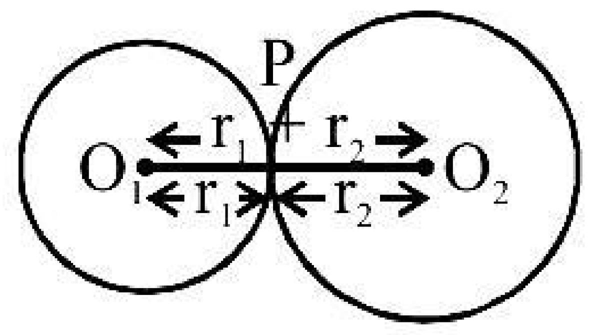
Two tangents can be drawn to a circle from an external point and length of these tangents are equal.

So, PM = PN

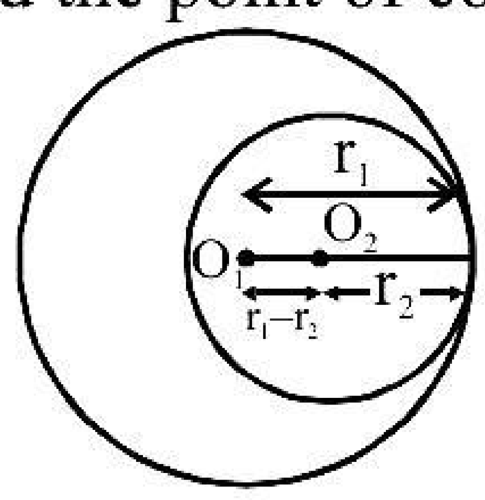


Theorem 3 :

If two circles touch one another, the centres and the point of contact are collinear.



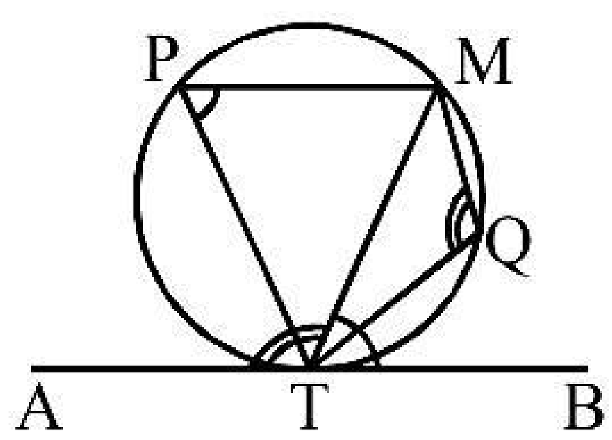
$O_1O_2 = r_1 + r_2$



$O_1O_2 = |r_1 - r_2|$

Theorem 4 (Alternate Segments Theorem) :

The angles made by a tangent to a circle with a chord drawn from the point of contact are respectively equal to the angles in the alternate segments of the circle.

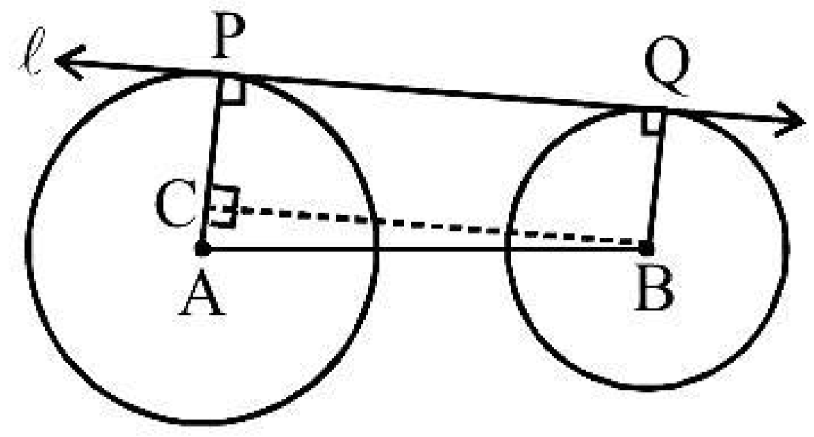


$\angle MTB = \angle TPM$, $\angle MTA = \angle MQT$

LENGTH OF COMMON TANGENTS

Direct Common Tangent (External Common Tangent) :

Consider two circle with centre A and B and line 'ℓ' touching the two circles at P and Q. Let radius of the circles be r_1 and r_2.



Now draw perpendicular from 'B' to AP meeting at 'C'.

Thus, CPQB is a rectangle and hence PQ = CB; PC = BQ

Now, in ΔACB

$AB^2 = AC^2 + BC^2$

$\Rightarrow BC^2 = AB^2 - AC^2$

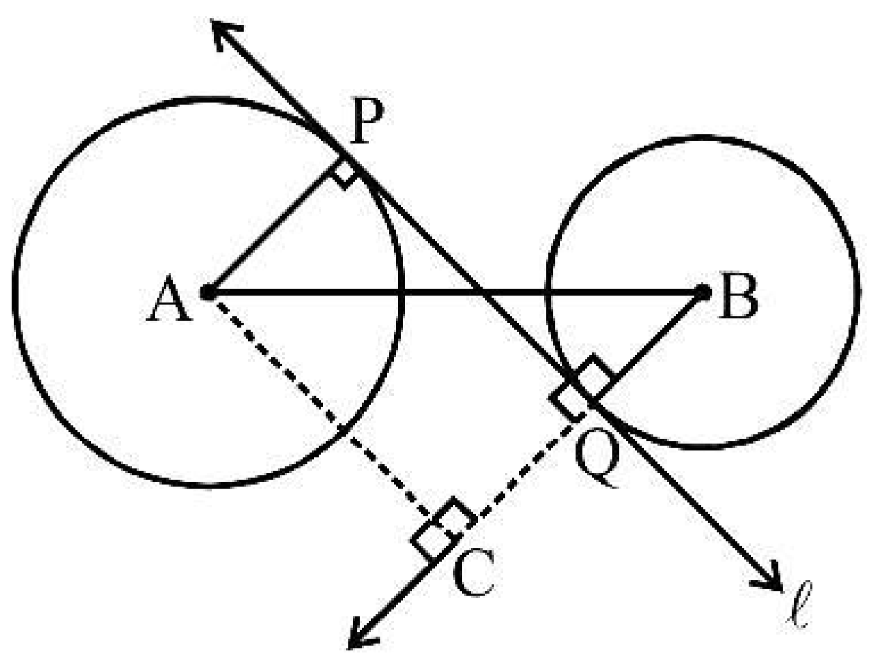
$\Rightarrow PQ^2 = AB^2 - AC^2$ [BC = PQ]

$\Rightarrow PQ^2 = AB^2 - (r_1 - r_2)^2$ [$AC = AP - PC = r_1 - r_2$]

Hence, length of direct common tangent

$PQ = \sqrt{d^2 - (r_1 - r_2)^2}$ [where AB = d = distance between the centres]

Transverse Common Tangent (Internal Common Tangent) :



Drop perpendicular from A to meet BQ produced in 'C'

Now $AP = r_1$; $BQ = r_2$; AB = d

AP = QC and PQ = AC

Also BC = BQ + QC

So $BC = r_2 + r_1$

Now in ΔACB

$AB^2 = AC^2 + BC^2$

$\Rightarrow d^2 = AC^2 + (r_1 + r_2)^2$

$\Rightarrow AC^2 = d^2 - (r_1 + r_2)^2$

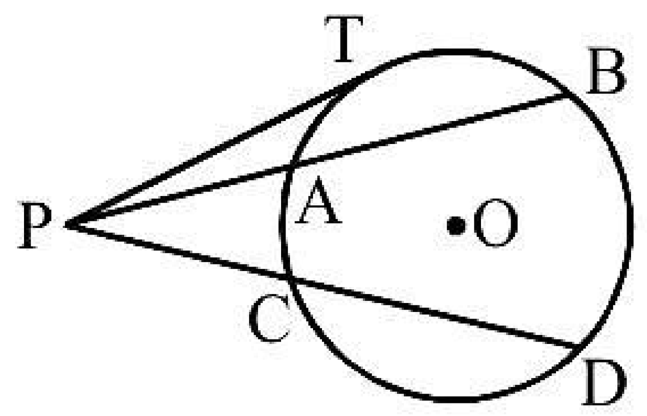
$\Rightarrow PQ^2 = d^2 - (r_1 + r_2)^2$

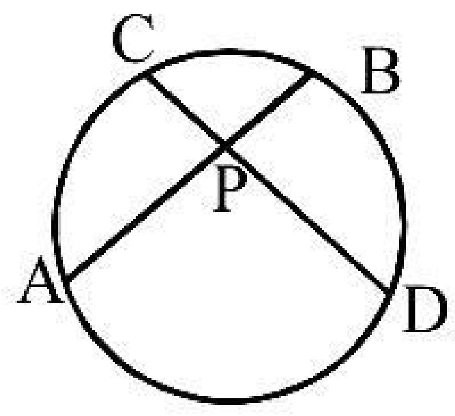
$\Rightarrow PQ = \sqrt{d^2 - (r_1 + r_2)^2}$

Length of transverse common tangent is $\sqrt{d^2 - (r_1 + r_2)^2}$

POWER OF A POINT

Suppose a line through a point P intersects a circle in two points, A and B. The **power of a point theorem** states that for all such lines, the product (PA)(PB) is constant. We call this product the power of point P.





P lies outside the circle

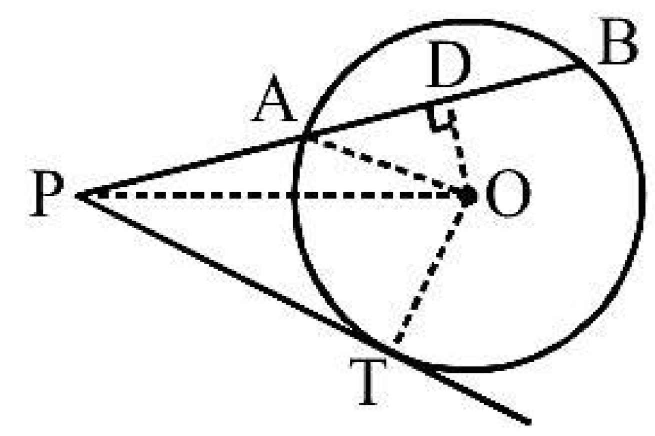
$PT^2 = (PA)(PB) = (PC)(PD)$

= The power of point P

P lies inside the circle

$PA \cdot PB = PC \cdot PD$

Proof : Given : A secant PAB to a circle C(O, r) intersecting it in points A and B and PT is a tangent segment of the circle.



To Prove : $PA \times PB = PT^2$.

Construction : Draw $OD \perp AB$. Join OP, OT and OA.

Proof : $OD \perp AB \Rightarrow AD = DB$

Now, $PA \times PB = (PD - AD)(PD + DB)$

$= (PD - AD)(PD + AD)$ [$\because$ DB = AD]

$= PD^2 - AD^2$

$= (OP^2 - OD^2) - AD^2$ [$OP^2 = OD^2 + PD^2$] [Pythagoras Theorem]

$= OP^2 - (OD^2 + AD^2)$

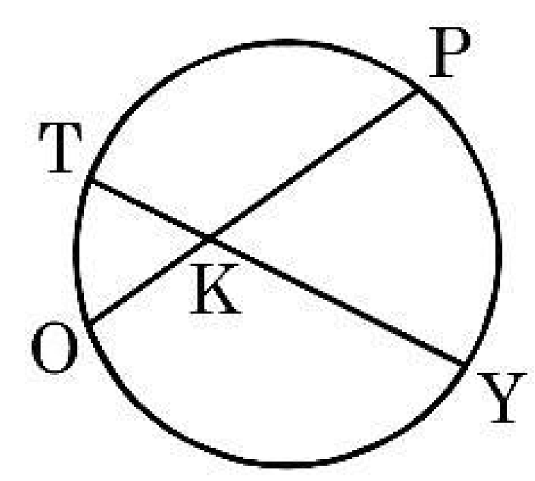
$= OP^2 - OA^2$ [$\because OA^2 = OD^2 + AD^2$]

$= OP^2 - OT^2$ [$\because$ OA = OT] [radii of same circle]

$= PT^2$ [$\because \angle OTP = 90°$, $OP^2 = OT^2 + PT^2$]

Illustration-20 : Chords $\overline{TY}$ and $\overline{OP}$ meet at point K such that TK = 2, KY = 16 and KP = 2(KO). Find OP.

Solution : A quick sketch suggests how to apply power of a point.



From the power of point K, we have

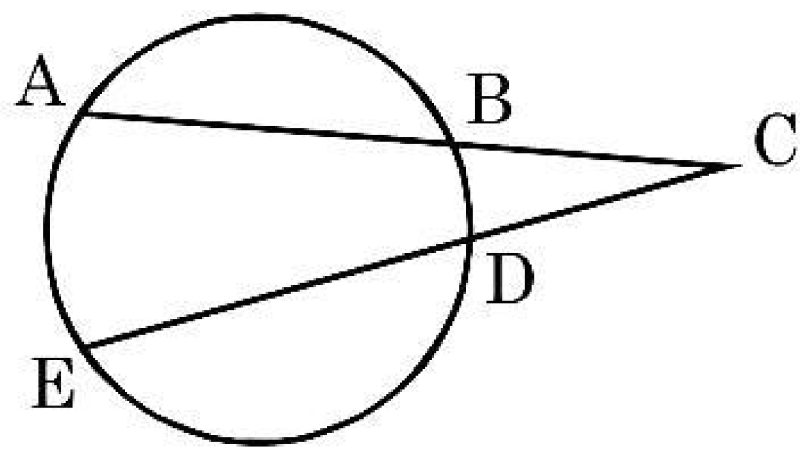
$$(KP)(KO) = (KT)(KY).$$

Substituting the given information in this equation yields

$$2(KO)(KO) = 2(16),$$

from which we find KO = 4. Therefore, KP = 2KO = 8, so OP = KO + KP = 12.

Illustration-21 : In the diagram, CB = 9, BA = 11 and CE = 18. Find DE.

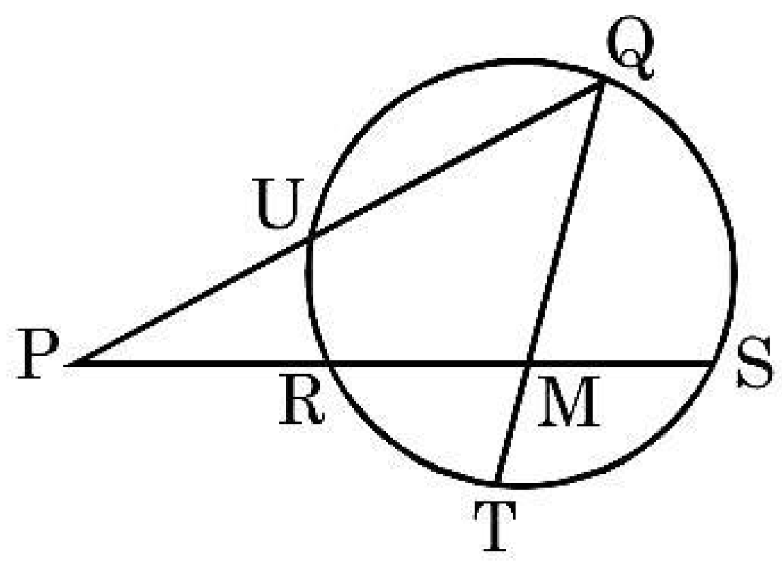


Solution : We have two intersecting secants, so we apply power of a point, which gives

$$(CD)(CE) = (CB)(CA).$$

Therefore, (CD)(18) = 9(20), so CD = 10 and DE = CE – CD = 8.

Illustration-22 : Points R and M trisect $\overline{PS}$, so PR = RM = MS. Point U is the midpoint of $\overline{PQ}$, TM = 2 and MQ = 8. Find PU.



Solution : The power of point M gives us

$$(MR)(MS) = (MT)(MQ).$$

We know that RM = MS, so substitution gives $MR^2 = (2)(8)$, i.e., MR = 4. Therefore, PR = MR = 4 and PS = 3(MR) = 12. Since U is the midpoint of $\overline{PQ}$, we have PQ = 2PU.

Now we can apply the power of point P to find :

$$(PU)(PQ) = (PR)(PS).$$

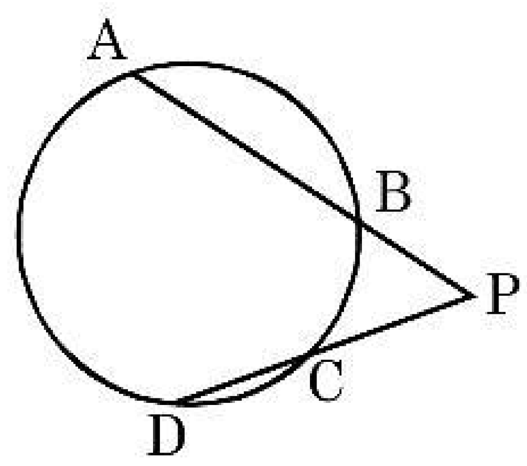
Substitution gives (PU)(2PU) = 4(12), so PU = $2\sqrt{6}$.

Illustration-23 : In the diagram,

we have BP = 8, AB = 10,

CD = 7 and ∠APC = 60°.

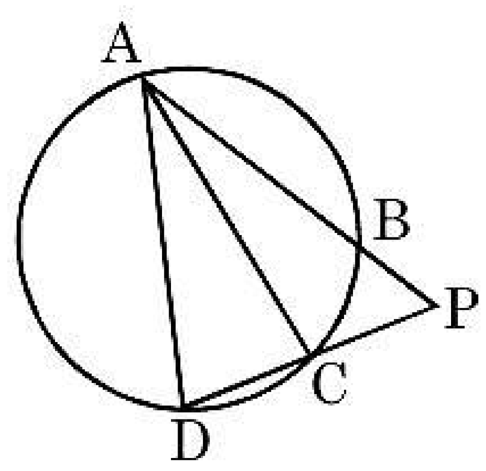
Find the area of the circle.



Solution : It's not immediately obvious how

we will find the radius,

so we start by finding what we can. The power of a point P gives us



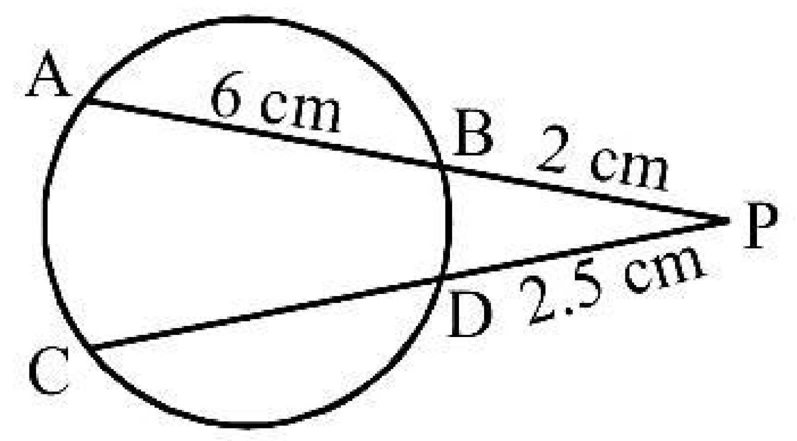
(PC)(PD) = (PB)(PA),

so (PC)(PC + 7) = 144. Therefore, $PC^2 + 7PC - 144 = 0$, so $(PC + 16)(PC - 9) = 0$. PC must be positive, so PC = 9.

Seeing that $\angle APC = 60°$ makes us wonder if there are any equilateral or 30-60-90 triangle lurking about. Since CP = AP/2 and the angle between these sides is 60°, the sides adjacent to the 60° angle in ΔACP are in the same ratio as the sides adjascent to the 60° angle in a 30-60-90 triangle with right angle at $\angle ACP$! $\Rightarrow \angle ACP = 90° \Rightarrow \angle ACD = 90°$

Since $\angle ACD$ is right and inscribed in $\overset{\frown}{AD}$, we know $\overset{\frown}{AD}$ is a semicircle. Therefore, $\overline{AD}$ is a diameter of the circle. Since $AC = CP\sqrt{3} = 9\sqrt{3}$ from our 30-60-90 triangle, we have $AD = \sqrt{AC^2 + CD^2} = \sqrt{243 + 49} = 2\sqrt{73}$. Finally, the radius of the circle is $AD/2 = \sqrt{73}$, so the area is $(\sqrt{73})^2 \pi = 73\pi$.

Illustration-24 : Chords AB and CD of a circle intersect each other at P as shown in fig., if AB = 6 cm, PB = 2 cm, PD = 2.5 cm, find CD.



Solution : Since chords AB and CD of a circle meet in P (when produced),

$\therefore$ PA.PB = PC.PD

$\Rightarrow$ (6 + 2).2 = PC (2.5)

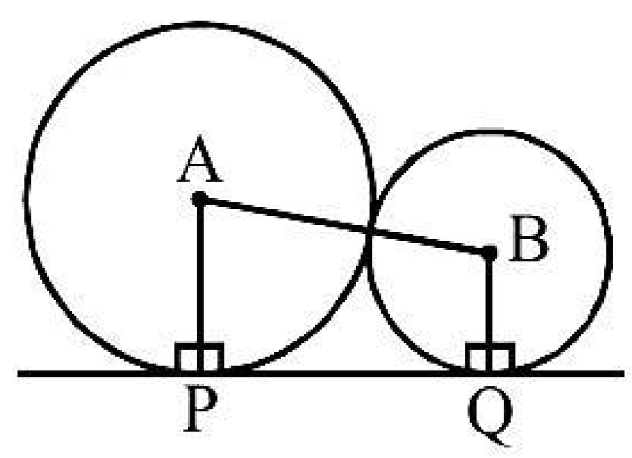
$\Rightarrow$ $PC = \frac{16}{2.5} = \frac{16 \times 2}{5} = \frac{32}{5} = 6.4$ cm

$\therefore$ CD = PC – PD = 6.4 – 2.5 = 3.9 cm.

Illustration-25 : If two circles of radius 1 cm and 4 cm touch each other, then find the length of common tangent.

Solution : Distance between centres $AB = r_1 + r_2 = 5$

Length of tangent



$$PQ = \sqrt{d^2 - (r_1 - r_2)^2}$$

$$= \sqrt{5^2 - (4-1)^2}$$

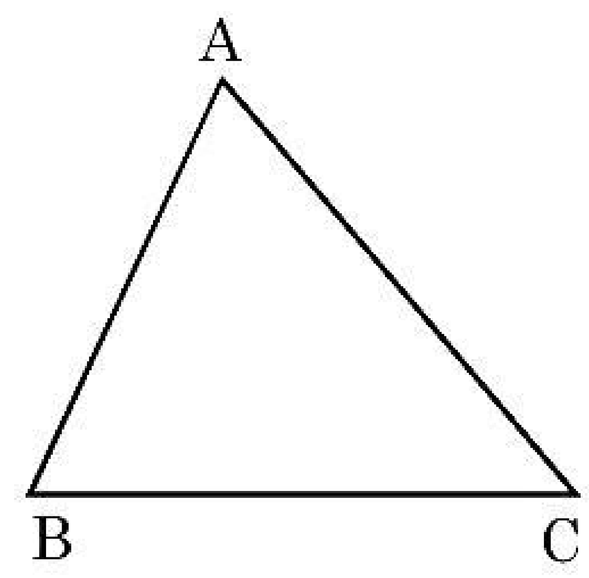
PQ = 4 cm

INEQUALITIES IN TRIANGLE

In any triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle. The middle side, of course, is therefore opposite the middle angle.

In other words, in ΔABC, $AB \geq AC \geq BC$ if and only if $\angle C \geq \angle B \geq \angle A$.

Important :



- $\angle C$ of ΔABC is acute if and only if $AB^2 < AC^2 + BC^2$.

  $\angle C$ of ΔABC is right if and only if $AB^2 = AC^2 + BC^2$.

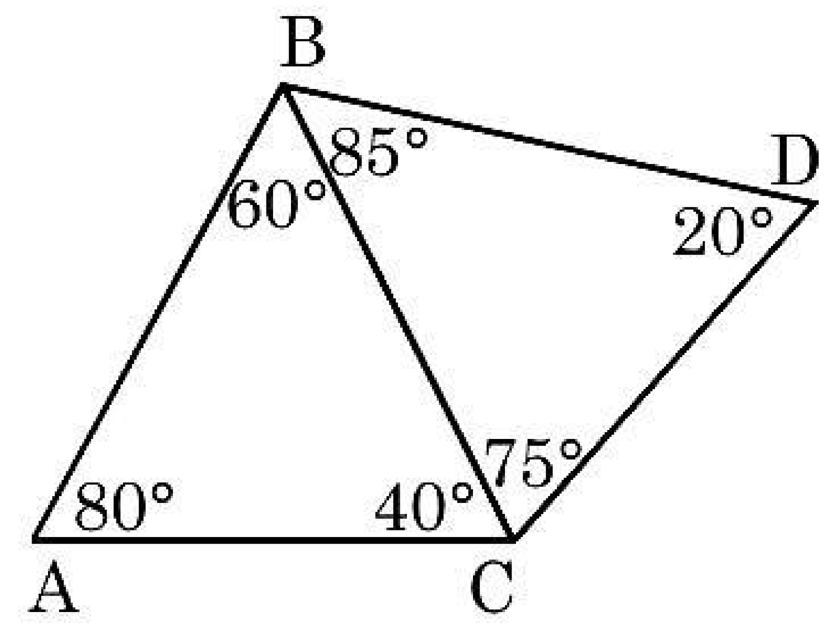
  $\angle C$ of ΔABC is obtuse if and only if $AB^2 > AC^2 + BC^2$.

- The Triangular Inequality states that for any three points, A, B and C, we have

  $$AB + BC \geq AC,$$

  where equality holds if and only if B lies on $\overline{AC}$. Therefore, for nondegenerate triangles (i.e., those in which the vertices are not collinear), $AB + BC > AC$.

Illustration-26 : Given the angles as shown, order the lengths AB, BD, CD, BC and AC from greatest to least. (Note : the diagram is not drawn to scale!)

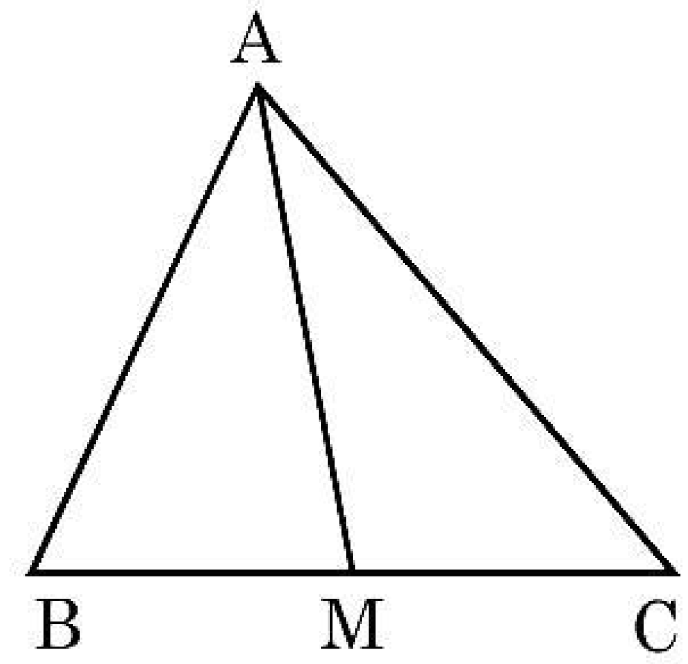


Solution : First we focus on ΔABC. Since $\angle A > \angle B > \angle C$, we have $BC > AC > AB$. Then, we turn to ΔBDC. Since $\angle B > \angle C > \angle D$, we have $CD > BD > BC$. BC is the smallest in one inequality string and the largest in the other, so we can put the inequalities together :

$$CD > BD > BC > AC > AB.$$

Illustration-27 : In ΔABC the median $\overline{AM}$ is longer than BC/2. Prove that $\angle BAC$ is acute.

Solution : We start with a diagram. We'd like to prove something about an angle, but all we are given is an inequality regarding lengths. So, we use the length inequality to get some angle inequalities to work with. Specifically, since $AM > BM$ in ΔABM and $AM > MC$ in ΔACM (because $AM > BC/2$ and $BM = MC = BC/2$), we have



$$\angle B > \angle BAM$$
$$\angle C > \angle CAM.$$

We want to prove something about $\angle BAC$, which equals $\angle BAM + \angle CAM$, so we add these two inequalities to give $\angle B + \angle C > \angle BAC$. Since $\angle B + \angle C + \angle BAC = 180°$, we can write $\angle B + \angle C > \angle BAC$ as

$$180° - \angle BAC > \angle BAC.$$

This gives us $180° > 2\angle BAC$, so $90° > \angle BAC$. Therefore, $\angle BAC$ is acute.

Illustration-28 : In ΔXYZ, we have $XY = 11$ and $YZ = 14$. For how many integer values of XZ is ΔXYZ acute ?

Solution : In order for ΔXYZ to be acute, all three of its angles must be acute. Using the given side lengths and our inequalities above, we see that we must have

$$121 + 196 > XZ^2$$

$$196 + XZ^2 > 121$$

$$121 + XZ^2 > 196$$

Simplifying these three yields :

$$317 > XZ^2$$

$$XZ^2 > -75$$

$$XZ^2 > 75$$

The middle inequality is clearly always true. (We really didn't even have to include it – clearly 11 couldn't ever be the largest side!) Combining the other two add noting that we seek integer values of XZ, we have $9 \le XZ \le 17$ (since $8^2 < 75 < 9^2$ and $18^2 > 317 > 17^2$). So, there are 9 integer values of XZ such that ΔXYZ is acute.

Illustration-29 : In how many ways can we choose three different numbers from the set {1, 2, 3, 4, 5, 6} such that the three could be the sides of a nondegenerate triangle ?

(Note : The order of the chosen numbers doesn't matter; we consider {3, 4, 5} to be the same as {4, 3, 5}.)

Solution : We first notice that if we have three numbers to consider as possible side lengths of a triangle, we only need to make sure that the sum of the smallest two is greater than the third. (Make sure you see why!) We could just start listing all the ones we see that work, but we should take an organized approach to make sure we don't miss any. We can do so by classifying sets of three numbers by the smallest number.

Case 1 : Smallest side has length 1. No triangles can be made with three different lengths from our set if we include one of length 1.

Case 2 : Smallest side has length 2. The other two sides must be 1 apart, giving the sets {2, 3, 4}, {2, 4, 5}, and {2, 5, 6}.

Case 3 : Smallest side has length 3. There are only three possibilities and they all work : {3, 4, 5}, {3, 4, 6}, {3, 5, 6}.

Case 4 : Smallest side has length 4. The only possibility is {4, 5, 6}, which works.

Adding them all up, we have 7 possibilities.

We sometimes have to use some other tools in addition to triangle inequality.

Illustration-30 : Can the lengths of the altitudes of a triangle be in the ratio 2 : 5 : 6 ? Why or why not ?

Solution : We don't know anything about how the lengths of the altitudes of a triangle are related to each other. We do, however, know a whole lot about how the lengths of the sides of a triangle are related to each other. Therefore, we turn the problem from one involving altitudes into one involving side lengths. We let the area be K, let the side lengths be a, b, c and the lengths of the altitudes to these sides be h_a, h_b, h_c, respectively. Therefore, we have $K = ah_a/2 = bh_b/2 = ch_c/2$, so the sides of the triangle have lengths

$$\frac{2K}{h_a}, \frac{2K}{h_b}, \frac{2K}{h_c}.$$

If our heights are in the ratio 2 : 5 : 6, then for some x, our heights are 2x, 5x and 6x. Then, our sides are

$$\frac{2K}{2x}, \frac{2K}{5x}, \frac{2K}{6x}.$$

However, the sum of the smallest two sides is then

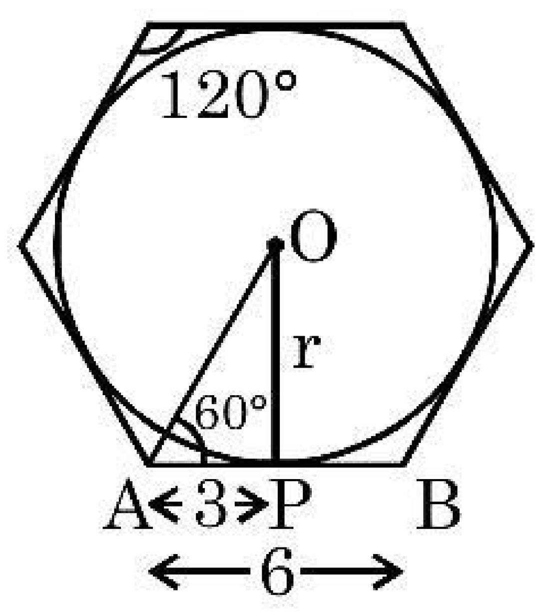
$$\frac{2K}{5x} + \frac{2K}{6x} = \frac{12K}{30x} + \frac{10K}{30x} = \frac{22K}{30x},$$

which is definitely less than the largest side, which is $2K/2x = K/x$. Therefore, the sides don't satisfy the triangle inequality, which means it is impossible to have a triangle with heights in the ratio 2 : 5 : 6.

Concept : When facing problems involving lengths of altitudes of a triangle, consider using area as a tool.

Illustration-31 : If a circle is inscribed in regular hexagon of side 6 cm, then area of inscribed circle.

Solution : Since we know that sum of angles of polygon = $(n - 2)180°$



Each interior angle of polygon = $\frac{(n-2)180°}{n}$

For hexagon n = 6

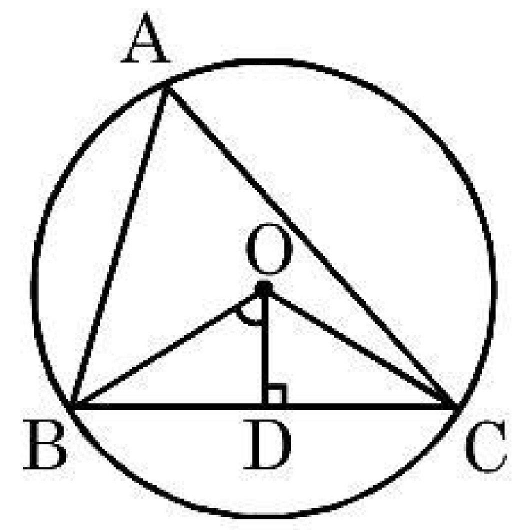
$\therefore$ Each angle = $\frac{(6-2)180°}{6} = 120°$

In ΔOPA

$$\frac{r}{3} = \tan 60^\circ \therefore r = 3\sqrt{3}$$

Area of circle $= \pi r^2 = \pi(3\sqrt{3})^2 = 27\pi$

Illustration-32 : In given circle with centre O a triangle ABC is inscribed, $\angle A = 60^\circ$. OD is perpendicular to BC, then what is $\angle BOD$?



Solution : In ΔOBD and ΔODC

$OD \perp BC$

$\therefore \angle ODB = \angle ODC = 90^\circ$

$OB = OC$ (radius of circle)

$BD = DC$ [$\perp$ from centre bisects chord]

$\therefore \Delta OBD \cong \Delta OCD$

Hence $\angle BOD = \angle COD = \frac{1}{2}\angle BOC$ .... (1)

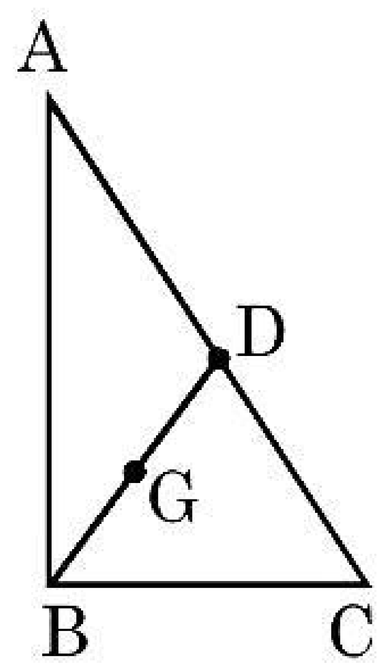
Again $\angle BOC = 2\angle BAC$

$\angle BOC = 2[\angle A]$ .... (2)

From (1) and (2)

$$\angle BOD = \frac{1}{2}(2\angle A) = 60^\circ$$

Illustration-33 : In right ΔABC right angled at 'B' (As in figure). Prove that distance between circumcentre and centroid $GD = \frac{1}{3}(BD) = \frac{1}{3}\left(\frac{1}{2}AC\right)$. If AC = 10, then find GD

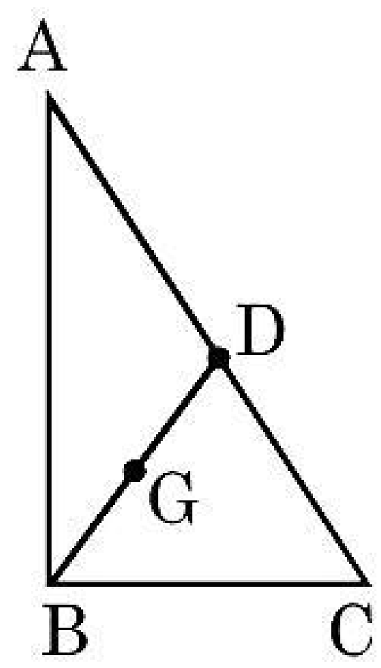


Solution. : **Property :** We know that mid-point of hypotenuse is equidistance from all the three vertices of triangle

$\therefore AD = CD = BD$

$\therefore BD = \frac{1}{2}AC$

$BD = \frac{1}{2}(10) = 5$

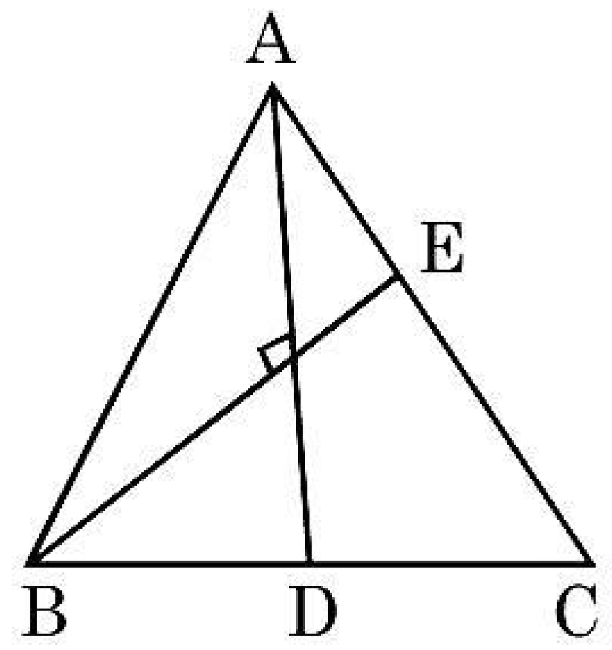


Again centroid divides median in ratio 2 : 1

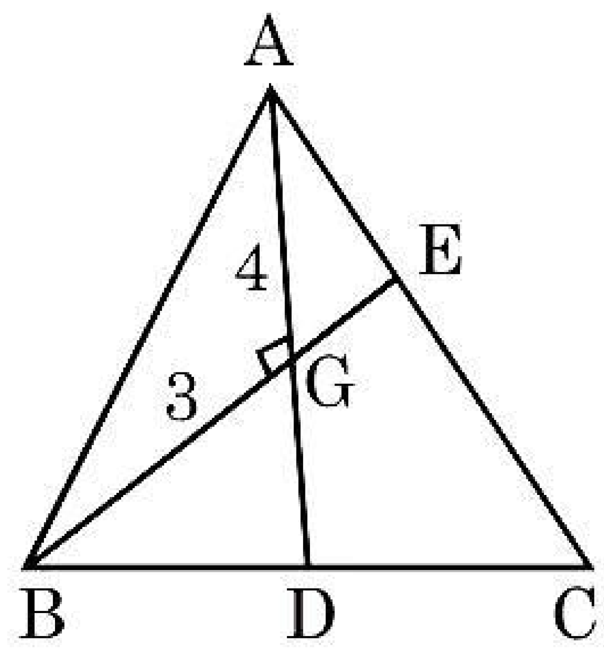
$\therefore GD = \frac{1}{3}BD = \frac{5}{3}$

[D is circumcentre of ΔABC since mid-point of hypotenuse is circumcentre of triangle]

Illustration-34 : In ΔABC, AD and BE are medians such that they intersect at right angles. If AD = 6 and $BE = \frac{9}{2}$ then area of ΔABC is



Solution. : Let G be the centroid of ΔABC i.e. point of intersection of medians AD and BE



Now $AG = \frac{2}{3}AD = \frac{2}{3}[6] = 4$

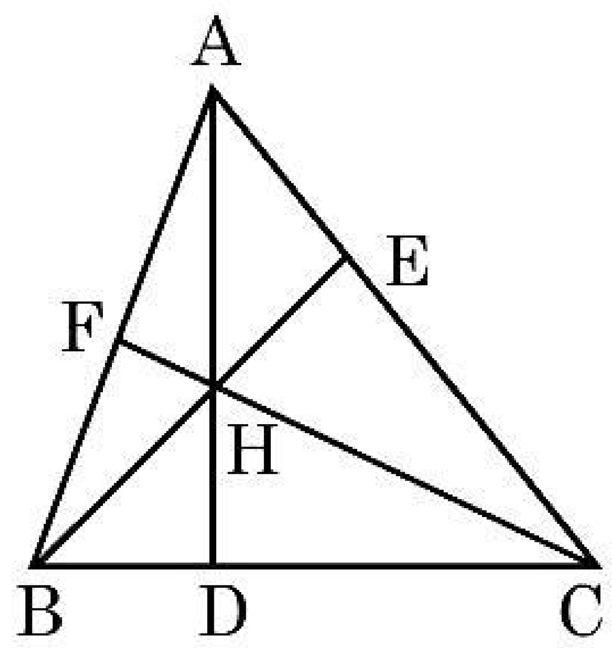
$BG = \frac{2}{3}BE = \frac{2}{3} \times \frac{9}{2} = 3$

Thus Area of ΔAGB is $\frac{1}{2} \times 3 \times 4 = 6$ unit^2

Now we know that area of ΔABC

$= 3 \times$ Area $(\Delta AGB) = 3(6) = 18$ unit^2

Illustration-35 : In, ΔABC, AD, BE and CF are altitudes and H is orthocentre. If $\angle A = 40°$ then find $\angle FDE$.



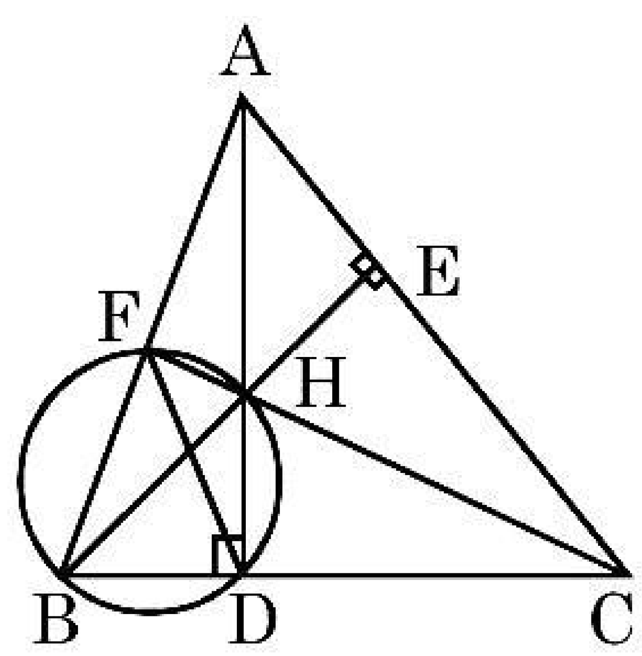
Solution. : It is very clear that $\angle HFB = 90°$ and $\angle HDB = 90°$

(Since CF and AD are altitudes)

Thus for quadrilateral HFBD

sum of opposite angle is 180°

Hence it is cyclic



In ΔAEB

$\angle ABE = 90° - A = \angle FBH$

and using the property of circle

$\angle FBH = \angle FDH$

$\therefore \angle FDH = 90° - A$

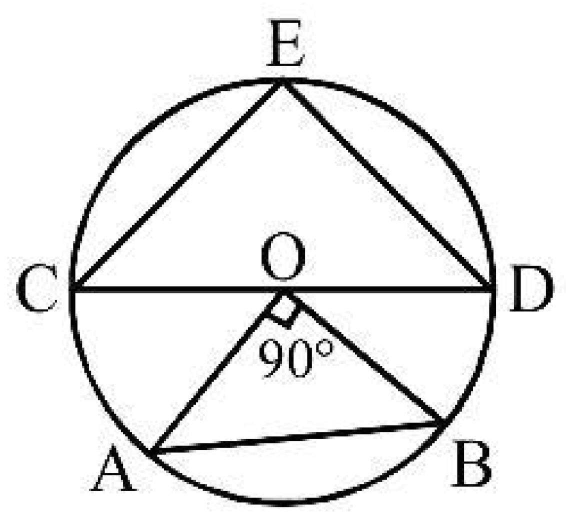
Similarly $\angle HDE = 90° - A$

$\therefore \angle FDE = 180° - 2A$

$\angle FDE = 180° - 2(40°) = 100°$

EXERCISE

1. In the diagram 'O' is the centre of the circle. The point E lies on the circumference of the circle such that the area of the ΔECD is maximum. If $\angle AOB$ is a right angle, then the ratio of the area of ΔECD to the area of the ΔAOB is :



(A) 2 : 1 (B) 3 : 2 (C) 4 : 3 (D) 4 : 1

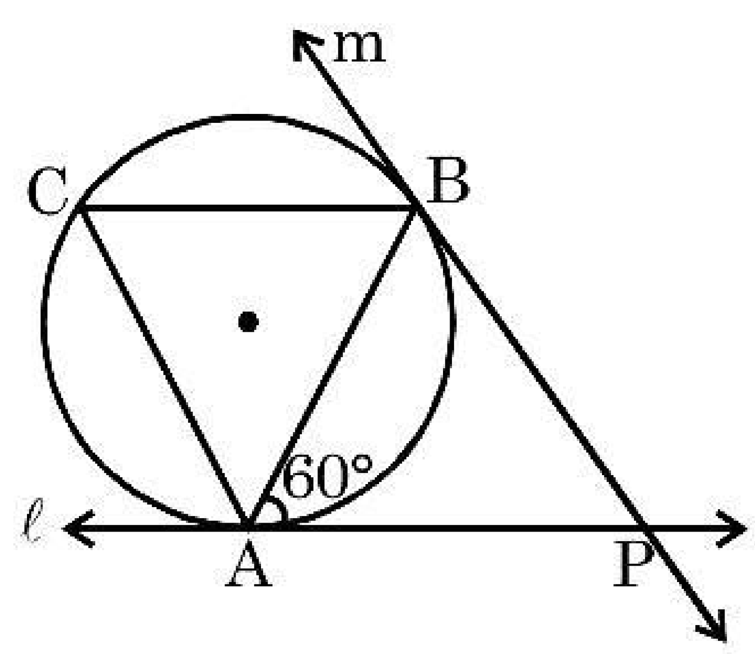
GM0001

2. Let ΔXOY be a right angled triangle with $\angle XOY = 90°$. Let M and N be the midpoints of legs OX and OY, respectively. Given that XN = 19 and YM = 22, the length XY is equal to

(A) 24 (B) 26 (C) 28 (D) 34

GM0002

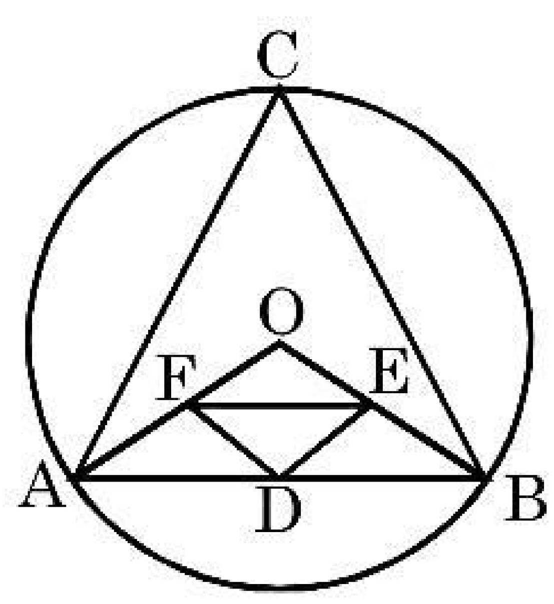
3. In the diagram below, if ℓ and m are two tangents and AB is a chord making an angle of 60° with the tangent ℓ, then the angle between ℓ and m is



(A) 45° (B) 30° (C) 60° (D) 90°

GM0003

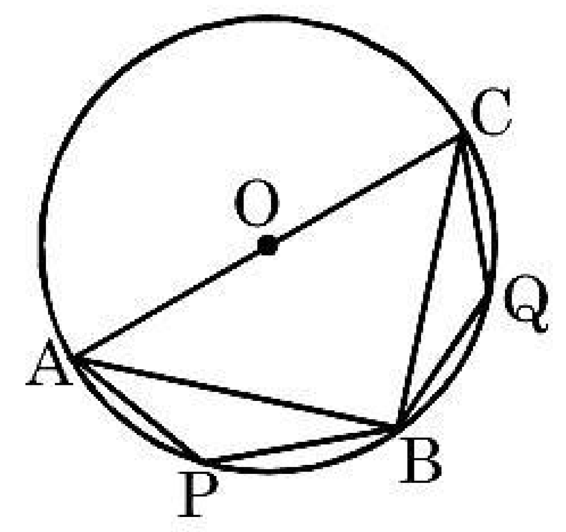
4. In the diagram, O is the centre of the circle and D, E and F are mid points of AB, BO and OA respectively. If $\angle DEF = 30°$, then $\angle ACB$ is



(A) 30° (B) 60° (C) 90° (D) 120°

GM0004

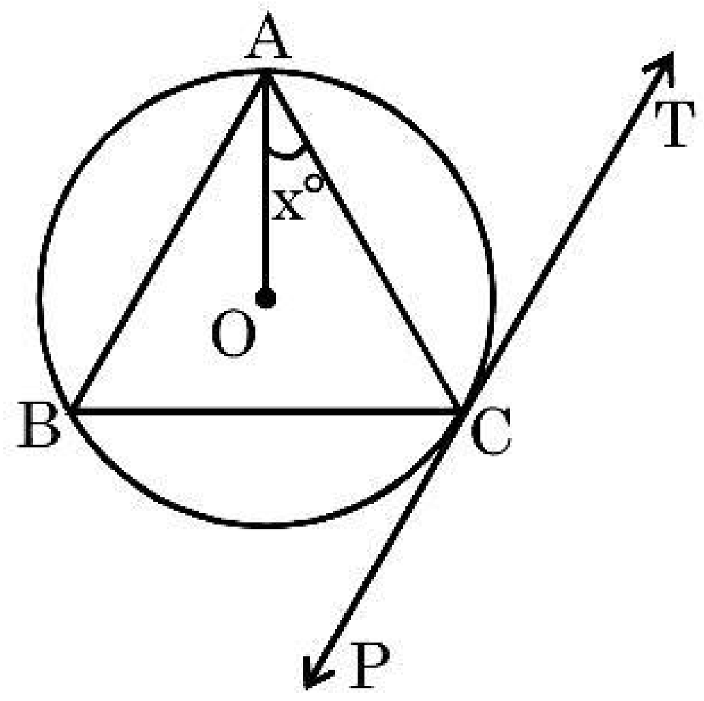
5. In the below diagram, O is the centre of the circle, AC is the diameter and if $\angle APB = 120°$, then $\angle BQC$ is



(A) 30° (B) 150° (C) 90° (D) 120°

GM0005

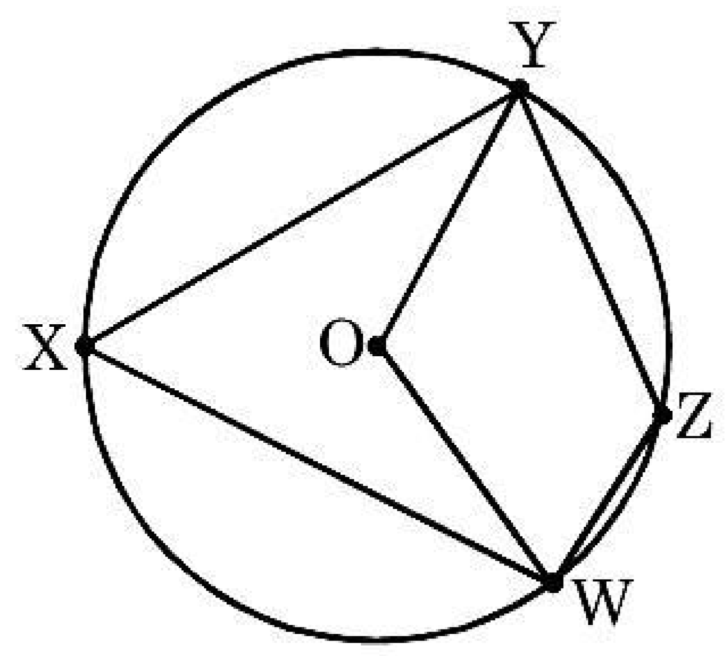
6. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumcentre of ΔABC. If $\angle ACP = 118°$, then the measure of $\angle x$ is



(A) 28° (B) 32° (C) 42° (D) 38°

GM0006

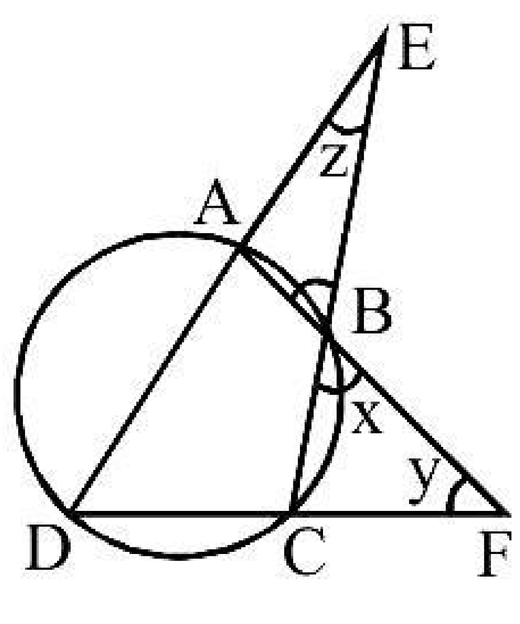
7. In the cyclic quadrilateral WXYZ on the circle centered at O, $\angle ZYW = 10°$ and $\angle YOW = 100°$. What is the measure of $\angle YWZ$?



(A) 30° (B) 40° (C) 50° (D) 60°

GM0007

8. In the adjacent figure, if $\angle y + \angle z = 100°$ then the measure of $\angle x$ is :



(A) 50° (B) 40° (C) 45° (D) Cannot be determined

GM0008

9. In a ΔABC if P be the point of intersection of the interior angle bisectors and Q be the point of intersection of the exterior angle bisectors of angles B and C respectively, then the figure BPCQ is a :

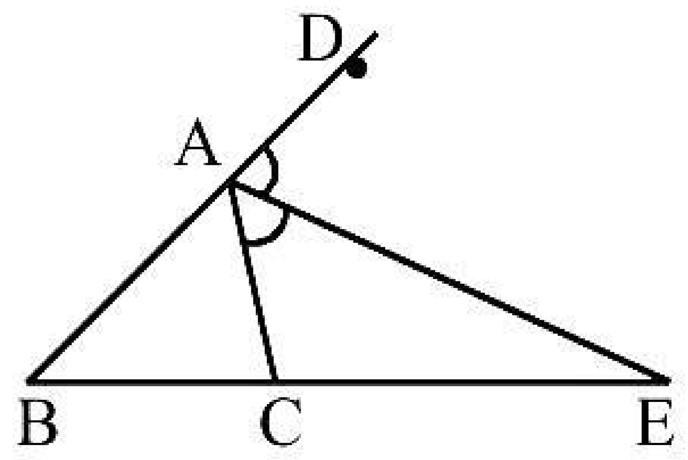
(A) Parallelogram (B) Rhombus (C) Trapezium (D) Cyclic quadrilateral

GM0009

10. D, E and F are points on sides BC, CA and AB respectively of ΔABC such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. If AB = 5 cm, BC = 8 cm and CA = 4 cm, determine AF, CE and BD.

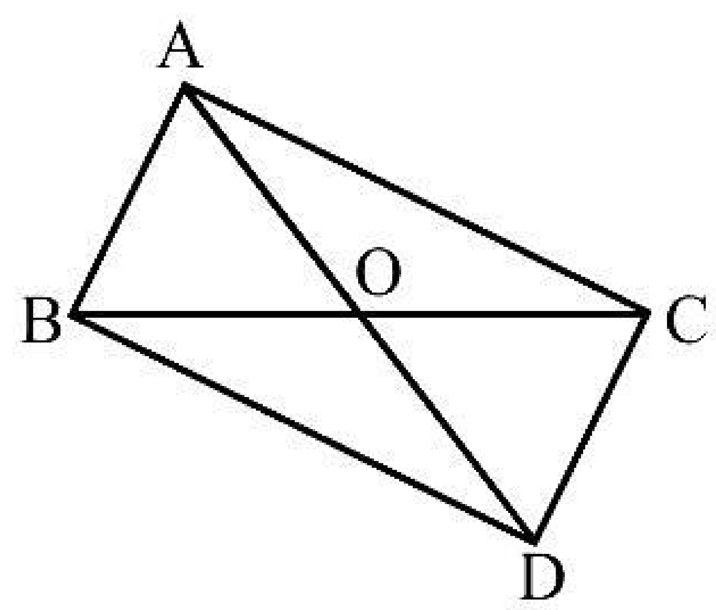
GM0010

11. In the adjoining figure, AE is the bisector of exterior $\angle CAD$ meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, find CE.



GM0011

12. In fig. ΔABC and ΔDBC are two triangles on the same base BC. Prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

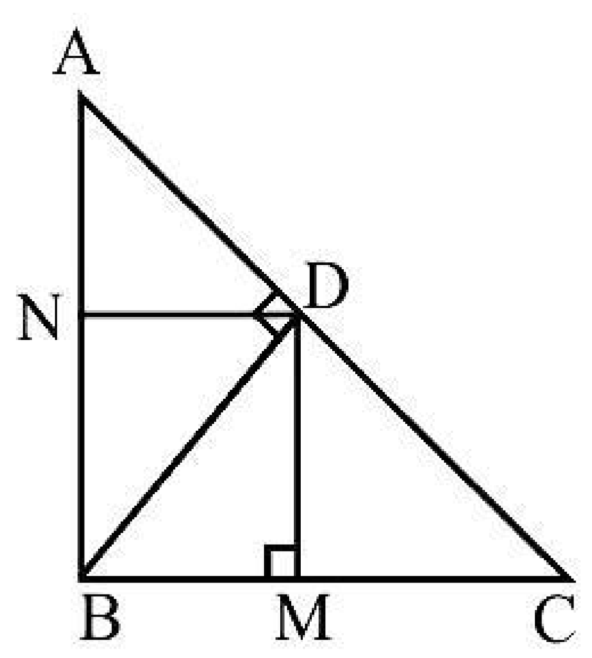


GM0012

13. ABC is a right-triangle with $\angle ABC = 90°$, $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that

(i) $DM^2 = DN \times MC$

(ii) $DN^2 = DM \times AN$

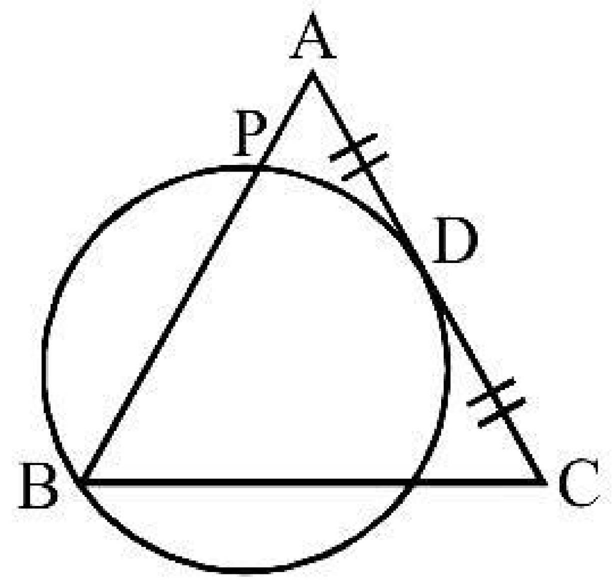


GM0013

14. Prove that sum of squares of diagonals of a parallelogram is equal to sum of squares of its sides.

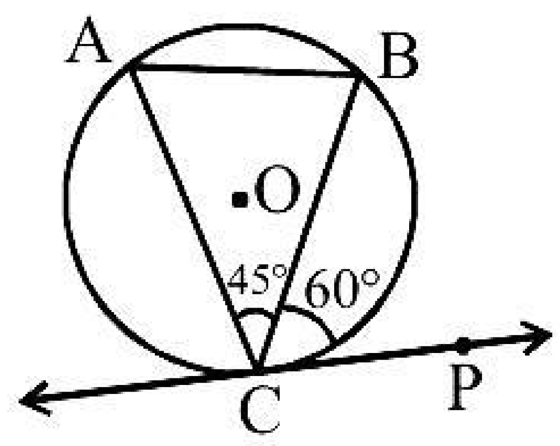
GM0014

15. In fig., ABC is a triangle in which AB = AC. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, show that 4 AP = AB.



GM0015

16. In fig., CP is a tangent to a circle. If $\angle PCB = 60°$ and $\angle BCA = 45°$, find $\angle ABC$.



GM0016

17. The inscribed circle of ΔABC touches BC, CA and AB at X, Y and Z respectively. If $\angle A = 64°$, $\angle C = 52°$, find $\angle XYZ$ and $\angle XZY$.

GM0017

18. Let $\overline{AM}$ be a median of ΔABC. Prove that AM > (AB + AC – BC)/2.

GM0018

ANSWER KEY

1.	A	**2.**	B	**3.**	C	**4.**	B	**5.**	B	**6.**	A
7.	B	**8.**	B	**9.**	D	**10.**	$AF = \frac{5}{3}, CE = \frac{32}{13}, BD = \frac{40}{9}$				
11.	CE = 18 cm	**16.**	$\angle ABC = 75°$	**17.**	$\angle XYZ = 58°, \angle XZY = 64°$						