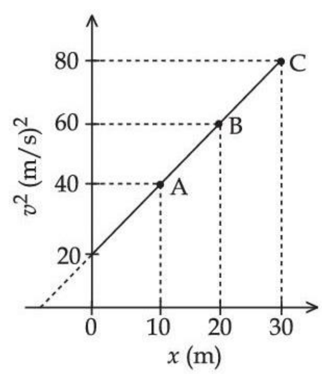
2

Motion

Numerical

Q.1 A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x(displacement) plot. The acceleration of the particle is ________ m/s^2.



31st Aug Evening Shift 2021

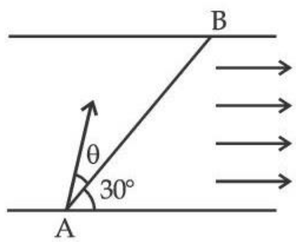
Q.2 If the velocity of a body related to displacement x is given by $\upsilon = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is .... m/s^2.

27st Aug Morning Shift 2021

Q.3 Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of .... m. (Take g = 10 m/s^2)

26st Aug Morning Shift 2021

Q.4 A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30∘∘ with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ________∘, so that the swimmer reaches point B.



27th July Evening Shift 2021

Q.5

Three particles P, Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving norma to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is ________________.

22th July Evening Shift 2021

Q.6 A person is swimming with a speed of 10 m/s at an angle of 120∘∘ with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is _________.

18th Mar Morning Shift 2021

Q.7 A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is ___________∘. (Round off to the Nearest Integer) (Find the angle in degrees)

16th Mar Evening Shift 2021

Q.8

If $\overrightarrow{P} \times \overrightarrow{Q} = \overrightarrow{Q} \times \overrightarrow{P}$, the angle between $\overrightarrow{P}$ and $\overrightarrow{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$. The value of '$\theta$' will be ___________$^\circ$.

25th Feb Evening Shift 2021

Numerical Answer Key

---

1. Ans. (1)

2. Ans. (12)

3. Ans. (50)

4. Ans. (30)

5. Ans. (3)

6. Ans. (5)

7. Ans. (120)

8. Ans. (180)

Numerical Explanation

Ans 1.

Given, $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

It is possible only when $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P} = 0$

$\Rightarrow$ We know, $\vec{P} \times \vec{Q}$ = PQsin θ

Only if $\vec{P} = 0$

or $\vec{Q} = 0$

or sin θ = 0 $\Rightarrow$ θ = 0 or 180^0

The angle b/w $\vec{P}$ & $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$

So, $\theta = 180^\circ$

Ans 2.

$V = \sqrt{5000 + 24x}$

$\frac{dV}{dx} = \frac{1}{2\sqrt{5000+24x}} \times 24 = \frac{12}{\sqrt{5000+24x}}$

Now, $a = V\frac{dV}{dx}$

$= \sqrt{5000 + 24x} \times \frac{12}{\sqrt{5000+24x}}$

a = 12 m/s^2

Ans 3.

When both balls will collied

$y_1 = y_2$

$35t - \frac{1}{2} \times 10 \times t^2 = 35(t-3) - \frac{1}{2} \times 10 \times (t-3)^2$

$35t - \frac{1}{2} \times 10 \times t^2 = 35t - 105 - \frac{1}{2} \times 10 \times t^2 - \frac{1}{2} \times 10 \times 3^2 + \frac{1}{2} \times 10 \times 6t$

0 = 150 − 30 t

t = 5 sec

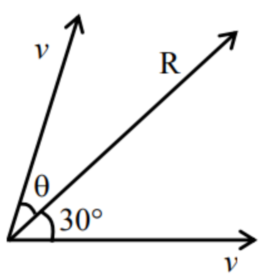
∴ Height at which both balls will collied

$h = 35t - \frac{1}{2} \times 10 \times t^2$

$= 35 \times 5 - \frac{1}{2} \times 10 \times 5^2$

h = 50 m

Ans 4.



Both velocity vectors are of same magnitude therefore resultant would pass exactly midway through them

$\theta = 30^\circ$

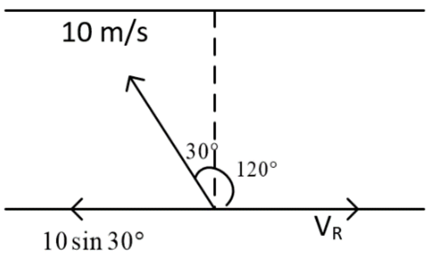
Ans 5.

$$\widehat{n}_1 \frac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\widehat{n}_2 \frac{\vec{A} \times \vec{C}}{\left|\vec{A} \times \vec{C}\right|} = \hat{k}$$

$$\cos\theta = \widehat{n}_1 . \widehat{n}_2 = \frac{1}{\sqrt{3}}$$

Ans 6.



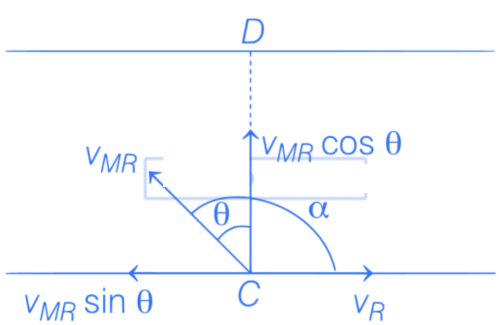
$$V_R = 10\sin 30^\circ$$

$$V_R = \frac{10}{2} = 5 \text{ m/s}$$

V_R = 5 m/s

Ans 7.

The situation is depicted in the following figure.



where, V_{MR} = velocity of man = 12 km/h

and v_R = velocity of water flow in river = 6 km/h

As, v_{MR} should be along CD.

$\Rightarrow v_R - v_{MR} \sin\theta = 0$

$\Rightarrow 6 - 12\sin\theta = 0 \Rightarrow \sin\theta = \frac{6}{12}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}(\sin 30^\circ)$ [$\because \sin 30^\circ = \frac{1}{2}$]

$\Rightarrow \theta = 30^\circ$

$\therefore \alpha = 90^\circ + \theta = 90^\circ + 30^\circ = 120^\circ$

$\Rightarrow \alpha = 120^\circ$

Ans 8.

Given, $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

It is possible only when $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$ = 0

$\Rightarrow$ We know, $\vec{P} \times \vec{Q}$ = PQsin θ

Only if $\vec{P} = 0$

or $\vec{Q} = 0$

or sin θ = 0 $\Rightarrow$ θ = 0 or 180^0

The angle b/w $\vec{P}$ & $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$

So, $\theta = 180^\circ$

MCQ (Single Correct Answer)

Q. 1 The ranges and heights for two projectiles projected with the same initial velocity at angles 42°° and 48°° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option:

A $R_1 > R_2$ and $H_1 = H_2$

B $R_1 = R_2$ and $H_1 < H_2$

C $R_1 < R_2$ and $H_1 < H_2$

D $R_1 = R_2$ and $H_1 = H_2$

1st Sep Evening Shift 2021

Q.2 A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

A $\sqrt{\frac{2ghv^2+1}{h^2}}$

B $\sqrt{2ghv^2 + h^2}$

C $\sqrt{\frac{2v^2h}{g} + h^2}$

D $\sqrt{\frac{2gh}{v^2} + h^2}$

31st Aug Morning Shift 2021

Q.3 A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45°° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take g = 10 ms^{-2})

A $h_{max} = 10$ m

$T = 2.5$ s

B $h_{max} = 15.625$ m

$T = 3.54$ s

C $h_{max} = 15.625$ m

$T = 1.77$ s

D $h_{max} = 3.54$ m

$T = 0.125$ s

27th Aug Evening Shift 2021

Q.4 Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

A 4.18 m

B 2.94 m

C 2.45 m

D 7.35 m

27th Aug Evening Shift 2021

Q.5 A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a:

(A) hyperbola

(B) parabola in the direction of motion of plane

(C) straight line vertically down the plane

(D) parabola in a direction opposite to the motion of plane

26th Aug Evening Shift 2021

Q.6 A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

(A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(B) $\frac{1}{3}$

(C) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(D) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

27th July Morning Shift 2021

Q.7 The instantaneous velocity of a particle moving in a straight line is given as V=αt+βt2V=αt+βt2, where αα and ββ are constants. The distance travelled by the particle between 1s and 2s is:

(A) $3\alpha + 7\beta$

(B) $\frac{3}{2}\alpha + \frac{7}{3}\beta$

(C) $\frac{\alpha}{2} + \frac{\beta}{3}$

(D) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

25th July Evening Shift 2021

Q.8 A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around:
(takes the value of gas 10 m/s^2)

- (A) 300 m
- (B) 200 m
- (C) 125 m
- (D) 250 m

25th July Evening Shift 2021

Q.9 The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is : (When v stands for velocity)

- (A) $2 mv^3$
- (B) $2 mnv^3$
- (C) $2nv^3$
- (D) $2n^2v^3$

25th July Evening Shift 2021

Q.10 Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4^{th} second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap? (Take g = 9.8 m/s^2)

A. 3 drops / 2 sconds

B. 2 drops / second

C. 1 drop / second

D. 1 drop / 7 seconds

25th July Morning Shift 2021

Q.11 A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

A. $\frac{t_1 t_2}{t_2 - t_1}$

B. $\frac{t_1 + t_2}{2}$

C. $\frac{t_1 t_2}{t_2 + t_1}$

D. $t_2 - t_1$

20th July Evening Shift 2021

Q.12 A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is :

A. $12\sqrt{2}$ m

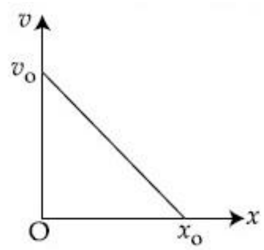
B. 20 m

C. 3 m

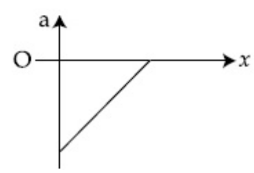
D. 15 m

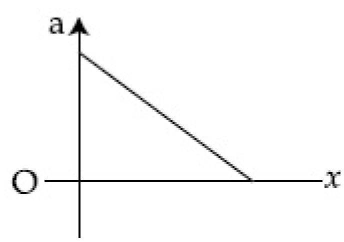
20th July Morning Shift 2021

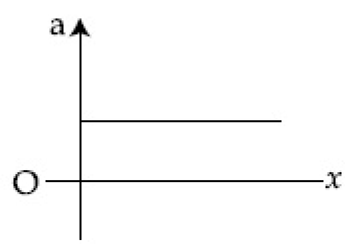
Q.13 The velocity – displacement graph of a particle is shown in the figure.

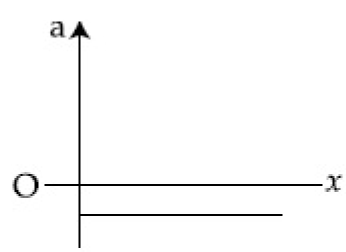


The acceleration – displacement graph of the same particle is represented by :

A


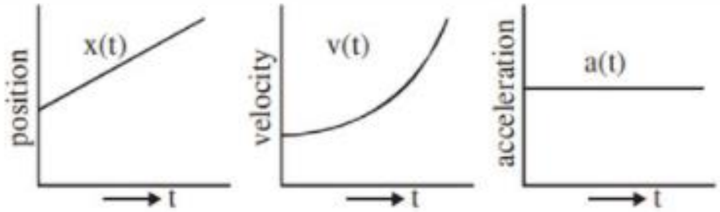
B


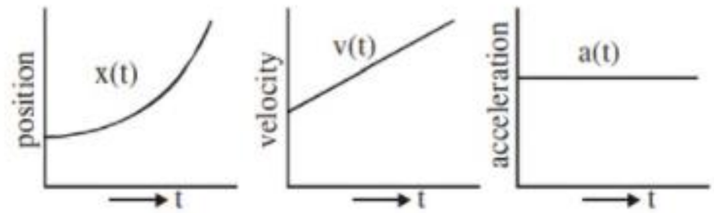
C


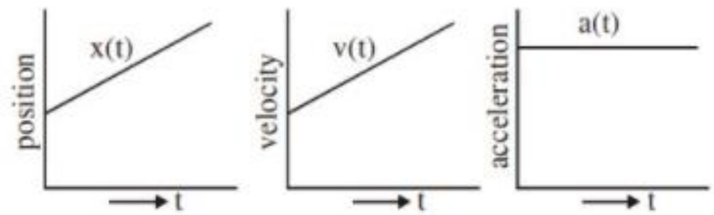
D


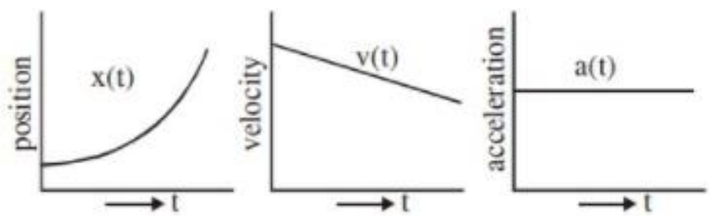
18th Mar Evening Shift 2021

Q.14 The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :

A


B


C


D


18th Mar Morning Shift 2021

Q.15

A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. (Take g = 10 ms^{-2})

(A) $2.50\ ms^{-1}$

(B) $3.0\ ms^{-1}$

(C) $2.0\ ms^{-1}$

(D) $3.50\ ms^{-1}$

17th Mar Evening Shift 2021

Q.16 The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is x = 0 at t = 0; then its displacement after time (t = 1) is :

(A) $v_0 + g + f$

(B) $v_0 + \frac{g}{2} + \frac{F}{3}$

(C) $v_0 + 2g + 3F$

(D) $v_0 + \frac{g}{2} + F$

17th Mar Evening Shift 2021

Q.17 A car accelerates from rest at a constant rate αα for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is:

A $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$

B $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$

C $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

D $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$

17th Mar Morning Shift 2021

Q.18 A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2 s?

A $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis

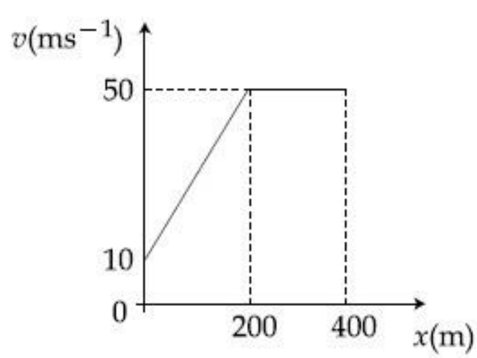
B $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

C $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

D $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

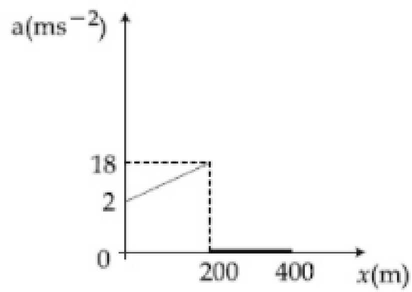
16th Mar Evening Shift 2021

Q.19 The velocity-displacement graph describing the motion of bicycle is shown in the figure.

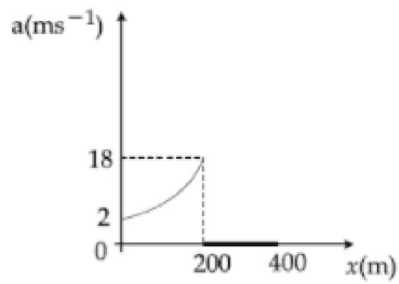


The acceleration-displacement graph of the bicycle's motion is best described by :

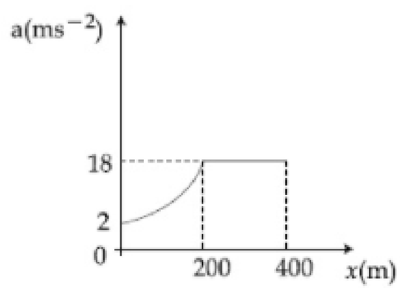
A



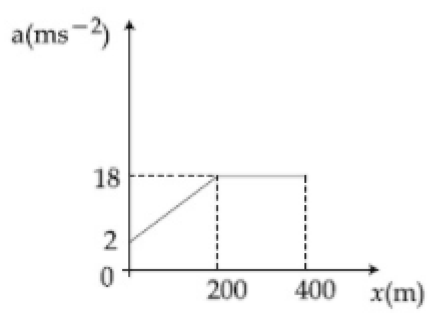
B



C



D



16th Mar Morning Shift 2021

Q.20

The trajectory of a projectile in a vertical plane is y = $\alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by :

A) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

B) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$

C) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$

D) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

26th Feb Evening Shift 2021

Q.21 A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ wil be :

A) $\frac{a_1+a_2}{a_2}$

B) $\frac{a_1+a_2}{a_1}$

C) $\frac{a_2}{a_1}$

D) $\frac{a_1}{a_2}$

26th Feb Evening Shift 2021

Q.22 A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is :

A) 50 m

B) 25 m

C) 45 m

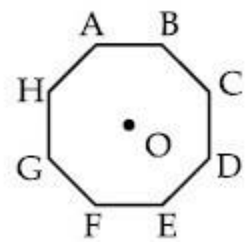
D) 35 m

25th Feb Evening Shift 2021

Q.23 In an octagon ABCDEFGH of equal side, what is the sum of

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH},$$

if, $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$



(A) $-16\hat{i} - 24\hat{j} + 32\hat{k}$

(B) $16\hat{i} + 24\hat{j} - 32\hat{k}$

(C) $16\hat{i} + 24\hat{j} + 32\hat{k}$

(D) $16\hat{i} - 24\hat{j} + 32\hat{k}$

25th Feb Morning Shift 2021

Q.24 An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

(A) $\frac{u+v}{2}$

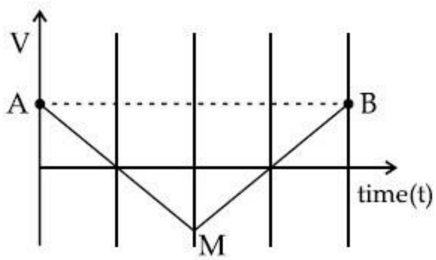
(B) $\sqrt{\frac{v^2-u^2}{2}}$

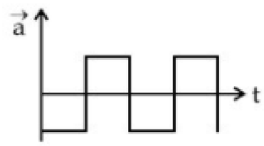
(C) $\frac{v-u}{2}$

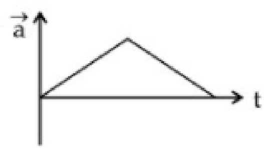
(D) $\sqrt{\frac{v^2+u^2}{2}}$

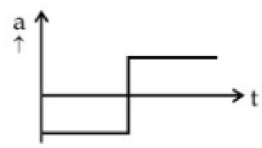
25th Feb Morning Shift 2021

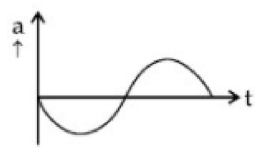
Q.25 If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



(A) 

(B) 

(C) 

(D) 

24th Feb Morning Shift 2021

MCQ Answer Key

1. Ans. (C)	10. Ans. (C)	19. Ans. (a)
2. Ans. (C)	11. Ans. (C)	20. Ans. (a)
3. Ans. (C)	12. Ans. (D)	21. Ans. (C)
4. Ans. (D)	13. Ans. (a)	22. Ans. (C)
5. Ans. (C)	14. Ans. (B)	23. Ans. (B)
6. Ans. (C)	15. Ans. (a)	24. Ans. (D)
7. Ans. (B)	16. Ans. (B)	25. Ans. (C)
8. Ans. (C)	17. Ans. (C)	

9. Ans. (a) **18. Ans. (a)**

MCQ Explanation

Ans 1.

Here, two projectiles are projected at angles 42°° and 48°° with same initial velocity.
As we know the expression of range of projectile,

$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

AT θ_1 = 42°,

$$\text{Range, } R_1 = \frac{u^2 \sin 2(42)^\circ}{g} = \frac{0.99u^2}{g}$$

At θ_2 = 48°

$$\text{Range, } R_2 = \frac{u^2 \sin 2(48)^\circ}{g} = \frac{0.99u^2}{g}$$

At θ_2 = 48°

$$\text{Range, } R_2 = \frac{u^2 \sin 2(48)^\circ}{g} = \frac{0.99u^2}{g}$$

The range of the projectile is same for the two projectiles.

Therefore, $R_1 = R_2$

Now, as we know the expression of height of the projectile,

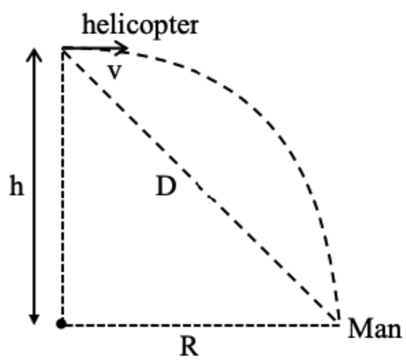
$$H_{\max} = \frac{u^2 \sin\theta}{2g}$$

At 42°, $H_{\max} = H_1 = \frac{u^2 \sin 42^\circ}{2g} = \frac{0.669u^2}{2g}$

At 48°, $H_{\max} = H_2 = \frac{u^2 \sin 48^\circ}{2g} = \frac{0.743u^2}{2g}$

Higher the value of θ higher the value of maximum height. Therefore, $H_1 < H_2$.

Ans 2.



$$R = \sqrt{\frac{2h}{g}}.\ v$$

$$D = \sqrt{R^2 + h^2}$$

$$= \sqrt{\left(\sqrt{\frac{2h}{g}}.\ v\right)^2 + h^2}$$

$$D = \sqrt{\frac{2hv^2}{g} + h^2}$$

Ans 3.

$H = \frac{U^2 \sin^2\theta}{2g}$

$= \frac{(25)^2.(\sin 45)^2}{2 \times 10}$

= 15.625 m

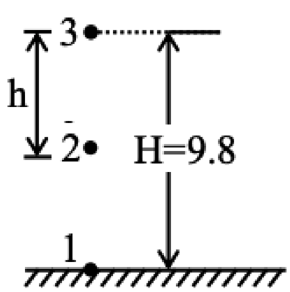
$T = \frac{U \sin\theta}{g}$

$= \frac{25 \times \sin 45^\circ}{10}$

= 2.5 $\times$ 0.7

= 1.77 s

Ans 4.



H = $\frac{1}{2}gt^2$

$\frac{9.8 \times 2}{9.8}$ = t^2

t = $\sqrt{2}$ sec

Δt : time interval between drops

h = $\frac{1}{2}g(\sqrt{2} - 2\Delta t)^2$

$\Delta t = \frac{1}{\sqrt{2}}$

h = $\frac{1}{2}g\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}\times 9.8\times\frac{1}{2} = \frac{9.8}{4} = 2.45$ m

H − h = 9.8 − 2.45

= 7.35 m

Ans 5.

Relative velocity of bomb w.r.t. observer in plane = 0.

Bomb will fall down vertically. So, it will move in straight line w.r.t. observer.

Ans 6.

$u=\sqrt{2gh}$

Now,

$S=\frac{h}{3}$

a = −g

$S=ut+\frac{1}{2}at^2$

$\frac{h}{3}=\sqrt{2gh}t+\frac{1}{2}(-g)t^2$

$t^2\left(\frac{g}{2}\right)-\sqrt{2gh}t+\frac{h}{3}=0$

From quadratic equation

$$t_1,t_2=\frac{\sqrt{2gh}\pm\sqrt{2gh-\frac{4g}{2}\frac{h}{3}}}{g}$$

$$\frac{t_1}{t_2}=\frac{\sqrt{2gh}-\sqrt{\frac{4gh}{3}}}{\sqrt{2gh}+\sqrt{\frac{4gh}{3}}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Ans 7.

$V = \alpha t + \beta t^2$

$\frac{ds}{dt} = \alpha t + \beta t^2$

$\int_{S_1}^{S_2} ds = \int_1^2 (\alpha t + \beta t^2) dt$

$S_2 - S_1 = \left[\frac{\alpha t^2}{2} + \frac{\beta t^3}{3}\right]_1^2$

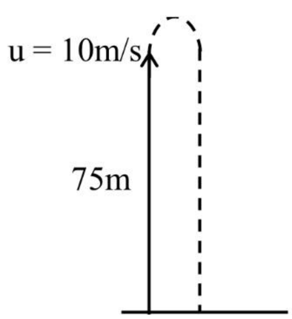
As particle is not changing direction

So distance = displacement

Distance = $\left[\frac{\alpha[4-1]}{2} + \frac{\beta[8-1]}{3}\right]$

$= \frac{3\alpha}{2} + \frac{7\beta}{3}$

Ans 8.



Object is projected as shown so as per motion under gravity

$S = ut + \frac{1}{2}at^2$

$-75 = +10t + \frac{1}{2}(-10)t^2 \Rightarrow t = 5$ sec

Object takes t = 5 s to fall on ground

Height of balloon from ground

H = 75 + ut

= 75 + 10×5 = 125 m

Ans 9.

$$t = mx^2 + nx$$

$$\frac{1}{v} = \frac{dt}{dx} = 2mx + n$$

$$v = \frac{1}{2mx+n}$$

$$\frac{dv}{dt} = -\frac{2m}{(2mx+n)^2}\left(\frac{dx}{dt}\right)$$

$$a = -(2m)v^3$$

Ans 10.

In 4 sec. 1^{st} drop will travel

$\Rightarrow \frac{1}{2} \times (9.8) \times (4)^2 = 78.4$ m

$\therefore 2^{nd}$ drop would have travelled

$\Rightarrow 78.4 - 34.3 = 44.1$ m.

Time for 2^{nd} drop

$\Rightarrow \frac{1}{2}(9.8)t^2 = 44.1$

t = 3 sec

∴ each drop have time gap of 1 sec

∴ 1 drop per sec

Ans 11.

L = Length of escalator

$$V_{b/esc} = \frac{L}{t_1}$$

When only escalator is moving.

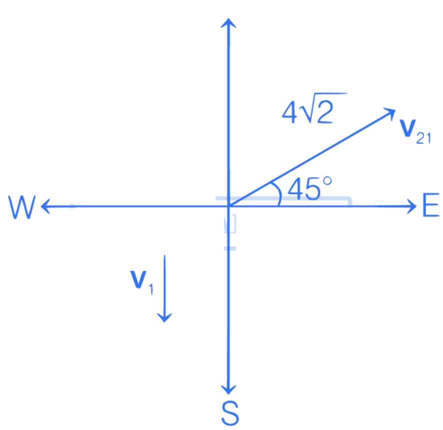
$$V_{esc} = \frac{L}{t_2}$$

when both are moving

$$V_{b/g} = V_{b/esc} + V_{esc}$$

$$V_{b/g} = \frac{L}{t_1} + \frac{L}{t_2} \Rightarrow \left[t = \frac{L}{V_{b/g}} = \frac{t_1 t_2}{t_1 + t_2}\right]$$

Ans 12.



In the above figure, v_1 is the speed of wind and v_{21} is the speed of butterfly with respect to wind.
So, v_{21} can be given as

$$v_{21} = 4\sqrt{2}\cos 45^{o}\,\hat{i} + 4\sqrt{2}\sin 45^{o}\,\hat{j}$$

$$= 4\sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + 4\sqrt{2} \times \frac{1}{\sqrt{2}}\hat{j} = 4\hat{i} + 4\hat{j}$$

and v_1 can be given as

$$v_1 = -\hat{j}$$

∴ Velocity of butterfly can be given as

$$v_2 = v_1 + v_{21} = 4\hat{i} + 4\hat{j} - \hat{j} = 4\hat{i} + 3\hat{j}$$

∴ Displacement of butterfly, $D = v_2 \times t$

$$= (4\hat{i} + 3\hat{j}) \times 3 = 12\hat{i} + 9\hat{j}$$

∴ Magnitude of displacement, $|D| = \sqrt{12^2 + 9^2} = 15$ m

Ans 13.

The slope of the given v versus x graph is $m = -\frac{v_0}{x_0}$ and intercept is c = + v_0. Hence, v varies with x as

$$v = -\left(\frac{v_0}{x_0}\right)x + v_0 \text{ ...... (1)}$$

where v_0 and x_0 are constants of motion. Differentiating with respect to time t, we have

$\frac{dv}{dt} = -\left(\frac{v_0}{x_0}\right)\frac{dx}{dt}$

or $a = -\left(\frac{v_0}{x_0}\right)v$ .... (2)

Using Eq. (1) in Eq. (2), we get

$a = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x + v_0\right)$

$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$

Thus the graph of a versus x is a straight line having a positive slope = $\left(\frac{v_0}{x_0}\right)^2$ and negative intercept = $-\frac{v_0^2}{x_0}$. Hence the correct choice is (a).

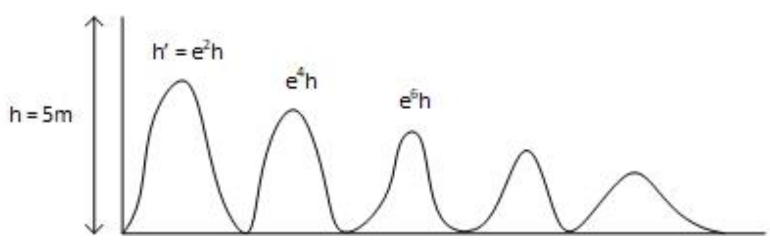
Ans 14.

Acceleration(a) is constant

∴ v ∝ t (straight line graph)

and x ∝∝ t^2 (parabolic graph)

Ans 15.



Total distance d = h + $2e^2h$ + $2e^4h$ + $2e^6h$ + $2e^8h$ + ...

d = h + $2e^2h$ (1 + e^2 + e^4 + e^6 + ...)

$$d = \frac{(1-e^2)h+2e^2h}{1-e^2} = \frac{h(1+e^2)}{1-e^2}$$

Total time = T + 2eT + $2e^2T$ + $2e^3T$ + ...

Total time = T + 2eT (1 + e + e^2 + e^3 + ...)

= $T + 2e.T\left(\frac{1}{1-e}\right)$

Total time $= \frac{T(1+e)}{1-e}$

Average speed of the ball

$$V_{avg} = \frac{h\frac{(1+e^2)}{(1-e^2)}}{T\left(\frac{1+e}{1-e}\right)}$$

$$= \frac{5}{1}\left(\frac{1+e^2}{(1+e)(1-e)}\frac{(1-e)}{(1+e)}\right)$$

$$V_{avg} = \frac{5(1+e^2)}{(1+e)^2}$$

$$\because h^1 = e^2 h$$

$$\frac{81}{100} = e^2$$

$$e = \frac{9}{10} = 0.9$$

$$V_{avg} = \frac{5\left(1+\frac{81}{100}\right)}{(1+0.9)^2}$$

= 2.50 m/sec.

Ans 16.

$$V = V_0 + gt + Ft^2$$

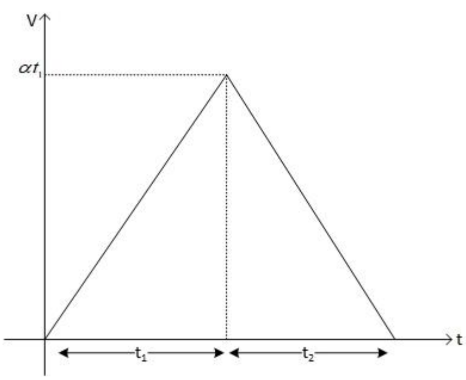
$$\frac{dx}{dt} = V_0 + gt + Ft^2$$

$$\int_{x=0}^{x} dx = \int_{t=0}^{t=1} \left(V_0 + gt + Ft^2\right) dt$$

$$\Rightarrow x = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3}\right]_{t=0}^{t=1}$$

$$\Rightarrow x = V_0 + \frac{g}{2} + \frac{F}{3}$$

Ans 17.



$$t_1 + t_2 = t, V' = 0 + \alpha t_1$$

$$V = u + at$$

$$0 = \alpha t_1 - \beta t_2$$

$$t_2 = \frac{\alpha}{\beta} t_1 = t$$

$t_1 = \left(\frac{\beta}{\alpha+\beta}\right) t$

Distance $= \frac{1}{2}(t_1 + t_2) \times \alpha t_1$ (area of triangle)

$= \frac{1}{2} t \times \alpha \left(\frac{\beta}{\alpha+\beta}\right) t$

$= \frac{\alpha\beta}{2(\alpha+\beta)} t^2$

Ans 18.

$\vec{v} = (0.5t^2 \hat{i} + 3t\hat{j} + 9\hat{k})$ m/s

At t = 2 s

$\vec{v} = (2\hat{i} + 6\hat{j} + 9\hat{k})$

Direction cosine along y-axis,

$cos\theta = \frac{(v.\hat{j})}{\sqrt{9^2+6^2+2^2}} = \frac{6}{\sqrt{121}} = \frac{6}{11}$

$\therefore \sin\theta = \frac{\sqrt{85}}{11}$

and $\tan\theta = \frac{\sqrt{85}}{6}$

$\therefore$ Mosquito make angle $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis.

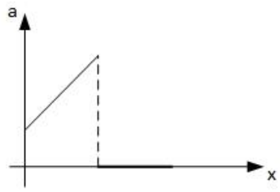
Ans 19.

We know that, $a = v\frac{dv}{dx}$

as slope is constant, so a ∝ v (from x = 0 to 200 m)

& slope = 0 so a = 0 (from x = 200 to 400 m)

$\therefore$



Ans 20.

y = αx − βx^2

comparing with trajectory equation

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$\beta = \frac{1}{2}\frac{g}{u^2\cos^2\theta}$

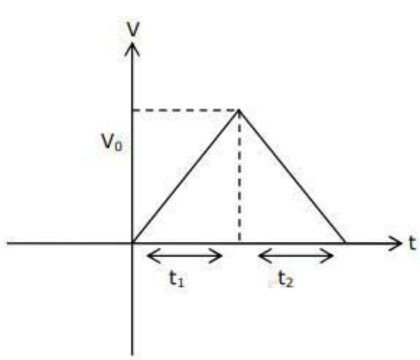
$\Rightarrow u^2 = \frac{g}{2\beta\cos^2\theta}$

Maximum height H :

$H = \frac{u^2\sin^2\theta}{2g} = \frac{g}{2\beta\cos^2\theta}\frac{\sin^2\theta}{2g}$

$\Rightarrow H = \frac{\tan^2\theta}{4\beta} = \frac{\alpha^2}{4\beta}$

Ans 21.



From given information :

$a_1 = \frac{v_0}{t_1}$

$v_0 = a_1 t_1$ .... (1)

For 2^{nd} interval

$a_2 = \frac{v_0}{t_2}$

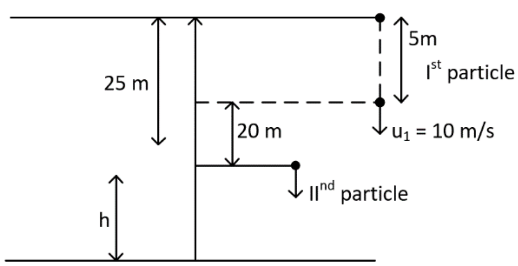
$v_0 = a_2 t_2$ ..... (2)

from (1) & (2)

$a_1 t_1 = a_2 t_2$

$\frac{t_1}{t_2} = \frac{a_2}{a_1}$

Ans 22.



For particle (1)

$20 + h = 10t + \frac{1}{2}gt^2$ ...... (i)

For particle (2)

$h = \frac{1}{2}gt^2$ ..... (ii)

put equation (ii) in equation (i)

$20 + \frac{1}{2}gt^2 = 10t + \frac{1}{2}gt^2$

t = 2 sec.

Put in equation (ii)

$h = \frac{1}{2}gt^2$

$= \frac{1}{2} \times 10 \times 2^2$

h = 20 m

The height of the building $= 25 + 20$ = 45 m

Ans 23.

We know,

$\because \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH} = \overrightarrow{0}$

By triangle law of vector addition, we can write

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB};\ \overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$$

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD};\ \overrightarrow{AE} = \overrightarrow{AO} + \overrightarrow{OE}$$

$$\overrightarrow{AF} = \overrightarrow{AO} + \overrightarrow{OF};\ \overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG}$$

$$\overrightarrow{AH} = \overrightarrow{AO} + \overrightarrow{OH}$$

Now

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$$

$$= (7\overrightarrow{AO}) + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}$$

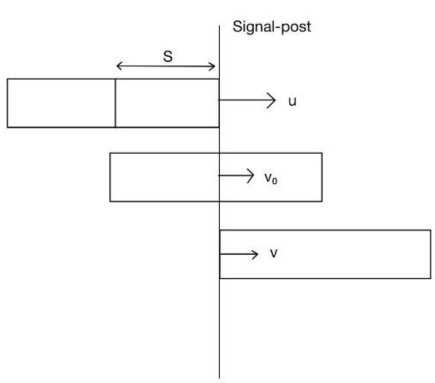
$$= (7\overrightarrow{AO}) + \overrightarrow{0} - \overrightarrow{OA}$$

$$= (7\overrightarrow{AO}) + \overrightarrow{AO}$$

$$= 8\overrightarrow{AO} = 8(2\hat{i} + 3\hat{j} - 4\hat{k})$$

$$= 16\hat{i} + 24\hat{j} - 32\hat{k}$$

Ans 24.



Let initial speed of train u. When midpoint of the train reach the signal post it's velocity becomes v0.

$\therefore v_0^2 = u^2 + 2as$ ....(1)

When train passes the signal post completely it's velocity becomes v.

$\therefore v^2 = v_0^2 + 2as$ ......(2)

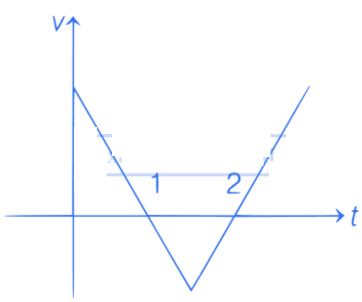
Subtracting (2) from (1) we get,

$v_0^2 - v^2 = u^2 - v_0^2$

$\Rightarrow v_0^2 + v_0^2 = u^2 + v^2$

$\Rightarrow v_0 = \sqrt{\frac{u^2+v^2}{2}}$

Ans 25.



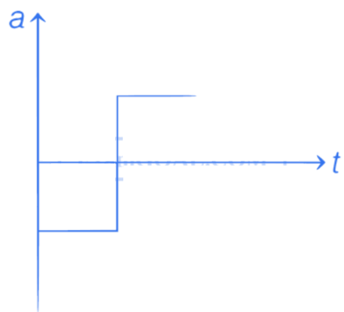
From the graph for first line, the slope is negative and intercept is positive.

So, equation of line is

$v = -mt + c$

$\Rightarrow a_1 = \frac{dv}{dt} = -m$

∴ The corresponding acceleration-time graph as shown below

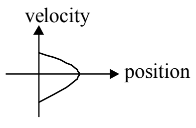

a
t

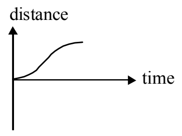
TOPIC 1 Distance, Displacement & Uniform Motion

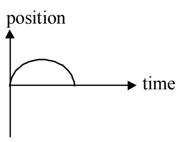
1. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0). **[12 Apr. 2019 II]**

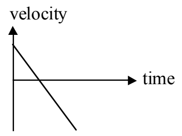
   (a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$ (c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[2018]**

   (a) 

   (b) 

   (c) 

   (d) 

3. A car covers the first half of the distance between two places at 40 km/h and other half at 60 km/h. The average speed of the car is **[Online May 7, 2012]**

   (a) 40 km/h (b) 45 km/h
   (c) 48 km/h (d) 60 km/h

4. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is **[2007]**

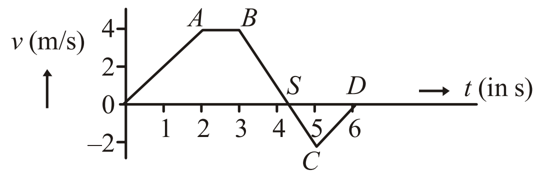
   (a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
   (c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$

5. A particle located at $x = 0$ at time $t = 0$, starts moving along with the positive x-direction with a velocity 'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[2006]**

   (a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

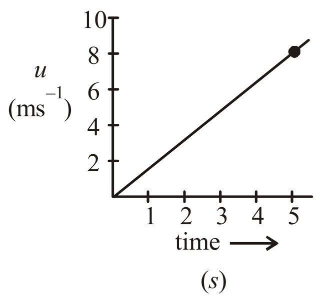
TOPIC 2 Non-uniform Motion

6. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is: **[05 Sep. 2020 (II)]**



   (a) $\frac{37}{3}$ m (b) 12 m (c) 11 m (d) $\frac{49}{4}$ m

7. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be ________. **[NA 4 Sep. 2020 (II)]**



8. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is ______. **[NA 9 Jan 2020 I]**

9. A bullet of mass 20g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to : **[10 Apr. 2019 II]**

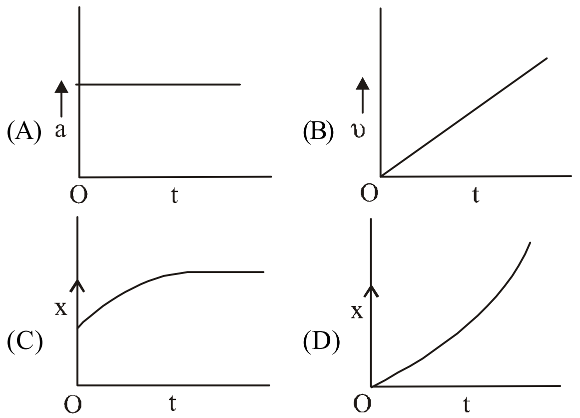
   (a) 0.1 ms^{-1} (b) 0.7 ms^{-1}
   (c) 0.3 ms^{-1} (d) 0.4 ms^{-1}

10. The position of a particle as a function of time t, is given by
$x(t) = at + bt^2 - ct^3$
where, a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: **[9 Apr. 2019 II]**

(a) $a+\frac{b^2}{4c}$ (b) $a+\frac{b^2}{3c}$

(c) $a+\frac{b^2}{c}$ (d) $a+\frac{b^2}{2c}$

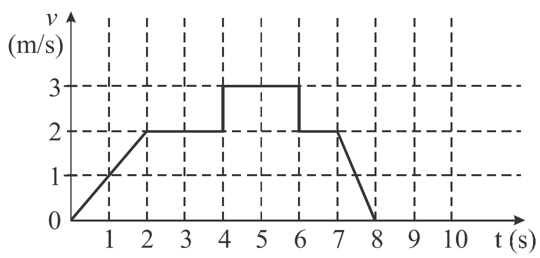
11. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represents the motion qualitatively (a = acceleration, v = velocity, x = displacement, t = time) **[8 Apr. 2019 II]**



(a) (B), (C) (b) (A)

(c) (A), (B), (C) (d) (A), (B), (D)

12. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s? **[10 Jan. 2019 II]**



(a) 10 m (b) 6 m
(c) 3m (d) 9m

13. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to: **[9 Jan. 2019 II]**

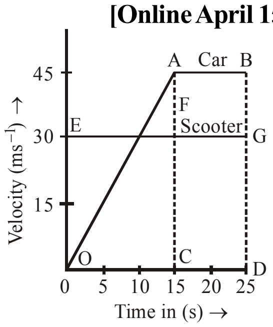
(a) $\frac{2a_1 a_2}{a_1 + a_2} t$ (b) $\sqrt{2a_1 a_2}\ t$

(c) $\sqrt{a_1 a_2}\ t$ (d) $\frac{a_1 + a_2}{2} t$

14. An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding) **[Online April 15, 2018]**

(a) 75m (b) 160m (c) 100m (d) 150m

15. The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively **[Online April 15, 2018]**



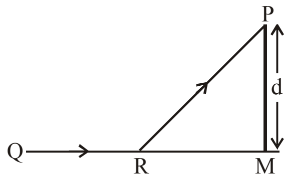
(a) 337.5m and 25s

(b) 225.5m and 10s

(c) 112.5m and 22.5s

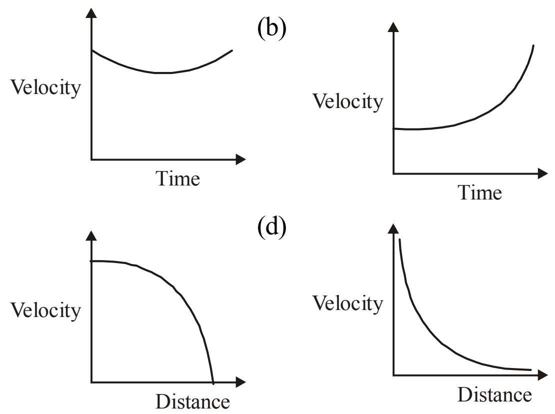
(d) 11.2.5m and 15s

16. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum? **[Online April 15, 2018]**

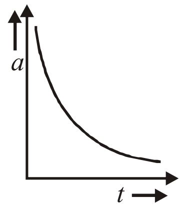


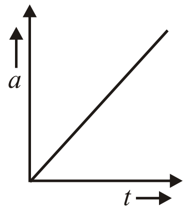
(a) $\frac{d}{\sqrt{3}}$ (b) $\frac{d}{2}$ (c) $\frac{d}{\sqrt{2}}$ (d) d

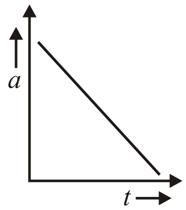
17. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ? **[Online April 8, 2017]**

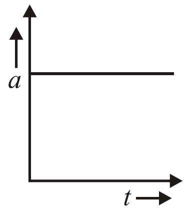


18. The distance travelled by a body moving along a line in time t is proportional to t^3.
The acceleration-time (a, t) graph for the motion of the body will be **[Online May 12, 2012]**

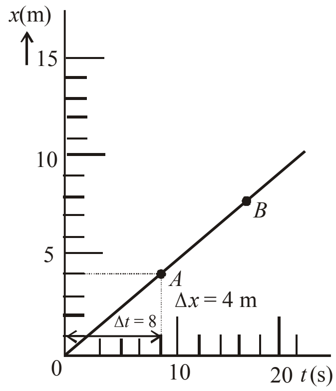
(a) 

(b) 

(c) 

(d) 

19. The graph of an object's motion (along the x-axis) is shown in the figure. The instantaneous velocity of the object at points A and B are v_A and v_B respectively. Then **[Online May 7, 2012]**



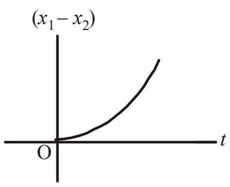
(a) $v_A = v_B = 0.5$ m/s (b) $v_A = 0.5$ m/s $< v_B$
(c) $v_A = 0.5$ m/s $> v_B$ (d) $v_A = v_B = 2$ m/s

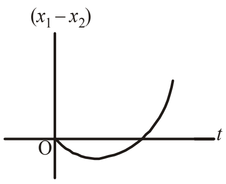
20. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

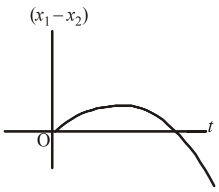
$\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be: **[2011]**

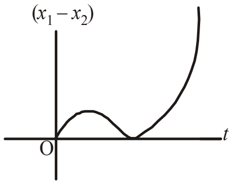
(a) 2 s (b) 4 s (c) 8 s (d) 1 s

21. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't'; and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't'? **[2008]**

(a) 

(b) 

(c) 

(d) 

22. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then **[2005]**

(a) $S = \frac{1}{6}ft^2$ (b) $S = ft$
(c) $S = \frac{1}{4}ft^2$ (d) $S = \frac{1}{72}ft^2$

23. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is **[2005]**

(a) $\frac{1}{2}$ ms^{-2} towards north
(b) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north - east
(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north - west
(d) zero

24. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is **[2005]**

(a) $2bv^3$ (b) $-2abv^2$ (c) $2av^2$ (d) $-2av^3$

25. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20m. If the car is going twice as fast i.e., 120 km/h, the stopping distance will be **[2004]**

(a) 60m (b) 40m (c) 20m (d) 80m

26. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is **[2003]**

(a) 12m (b) 18m (c) 24m (d) 6m

27. If a body looses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[2002]**

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm.

28. Speeds of two identical cars are u and $4u$ at the specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is **[2002]**

(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

TOPIC 3 Relative Velocity

29. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8

km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible) **[2 Sep. 2020 (I)]**

(a) 29.5 ms^{-1} (b) 28.5 ms^{-1}
(c) 31.5 ms^{-1}q (d) 30.5 ms^{-1}

30. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/h. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in same direction, and (ii) in the opposite directions is: **[12 Jan. 2019 II]**

(a) $\frac{11}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{25}{11}$

31. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60º with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is: **[12 Jan. 2019 II]**

(a) $\frac{\sqrt{3}}{2}v$ (b) $\frac{2v}{\sqrt{3}}$ (c) v (d) $\frac{v}{2}$

32. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after a time of : **[Online April 9, 2017]**

(a) $\sqrt{110}$ s (b) $\sqrt{120}$ s
(c) $10\sqrt{2}$ s (d) 15 s

33. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator? **[Online April 12, 2014]**

(a) 37 s (b) 27 s (c) 24 s (d) 45 s

34. A goods train accelerating uniformly on a straight railway track, approaches an electric pole standing on the side of track. Its engine passes the pole with velocity u and the guard's room passes with velocity v. The middle wagon of the train passes the pole with a velocity. **[Online May 19, 2012]**

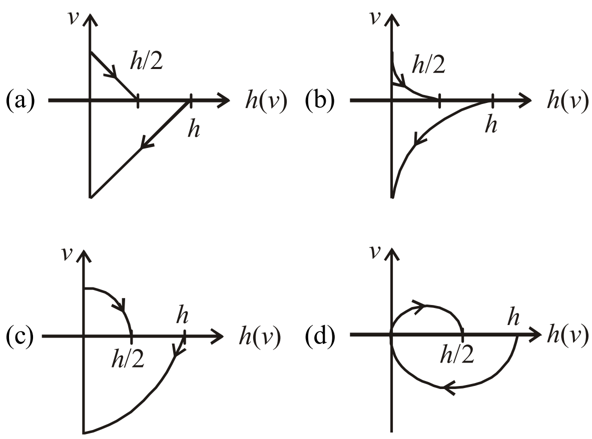
(a) $\frac{u+v}{2}$ (b) $\frac{1}{2}\sqrt{u^2+v^2}$
(c) $\sqrt{uv}$ (d) $\sqrt{\left(\frac{u^2+v^2}{2}\right)}$

TOPIC 4 Motion Under Gravity

35. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the accelertion due to gravity] : **[5 Sep. 2020 (I)]**

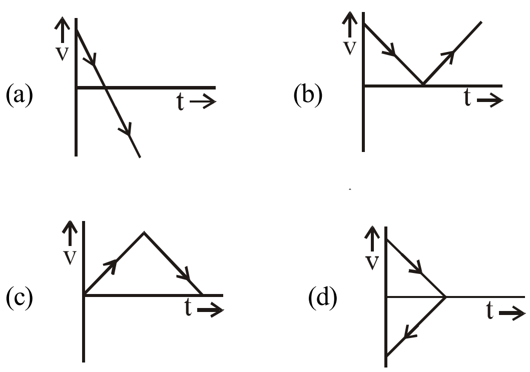
(a) $t=\frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (b) $t=1.8\sqrt{\frac{h}{g}}$
(c) $t=3.4\sqrt{\left(\frac{h}{g}\right)}$ (d) $t=\sqrt{\frac{2h}{3g}}$

36. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by :
(graph are drawn schematically and on not to scale) **[4 Sep. 2020 (I)]**



37. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is _______. **[NA 8 Jan. 2020 II]**

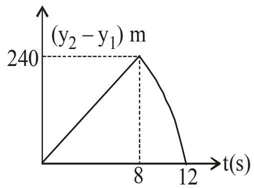
38. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[2017]**

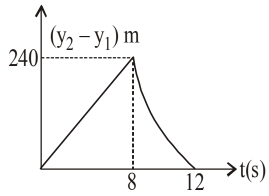


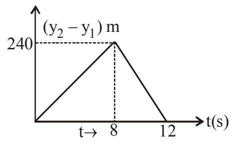
39. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

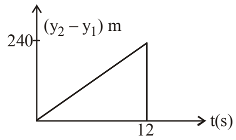
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/ s^2) **[2015]**

(*The figures are schematic and not drawn to scale*)

(a) 

(b) 

(c) 

(d) 

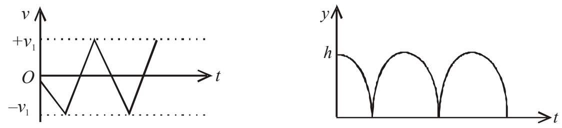
40. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is: **[2014]**

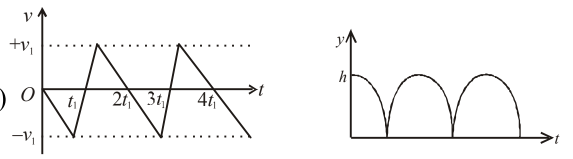
(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2 u^2d$

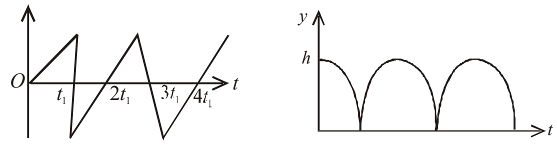
(c) $2gH = nu^2 (n-2)$ (d) $gH = (n-2)u^2$

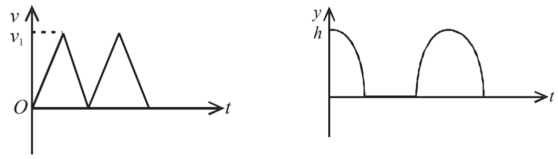
41. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.

Then the velocity as a function of time and the height as a function of time will be : **[2009]**

(a) 

(b) 

(c) 

(d) 

42. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2 . He reaches the ground with a speed of 3 m/s. At what height, did he bail out ? **[2005]**

(a) 182 m (b) 91 m

(c) 111m (d) 293m

43. A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What is the position of the ball at $\frac{T}{3}$ second **[2004]**

(a) $\frac{8h}{9}$ meters from the ground

(b) $\frac{7h}{9}$ meters from the ground

(c) $\frac{h}{9}$ meters from the ground

(d) $\frac{17h}{18}$ meters from the ground

44. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then **[2002]**

(a) $v_B > v_A$

(b) $v_A = v_B$

(c) $v_A > v_B$

(d) their velocities depend on their masses.

Hints & Solutions

1. **(b)** Given, $v = b\sqrt{x}$

or $\frac{dx}{dt} = b\,x^{1/2}$

or $\int_0^x x^{-1/2}dx = \int_0^t b\,dt$

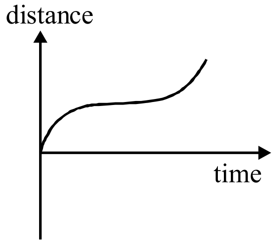
or $\frac{x^{1/2}}{1/2} = 6t$ or $x = \frac{b^2t^2}{4}$

Differentiating w. r. t. time, we get

$\frac{dx}{dt} = \frac{b^2 \times 2t}{4}$ $(t = \tau)$

or $v = \frac{b^2\tau}{2}$

2. **(b)** Graphs in option (c) position-time and option (a) velocity-position are corresponding to velocity-time graph option (d) and its distance-time graph is as given below. Hence distance-time graph option (b) is incorrect.



3. **(c)** Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{x}{T}$

$= \frac{x}{\frac{x}{2\times 40} + \frac{x}{2\times 60}} = 48$ km/h

4. **(c)** We know that, $v = \frac{dx}{dt}$

$\Rightarrow dx = v\,dt$

Integrating, $\int_0^x dx = \int_0^t v\,dt$

or $x = \int_0^t (v_0 + gt + ft^2)\,dt = \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$

or, $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$, $x = v_0 + \frac{g}{2} + \frac{f}{3}$.

5. **(a)** $v = \alpha\sqrt{x}$,

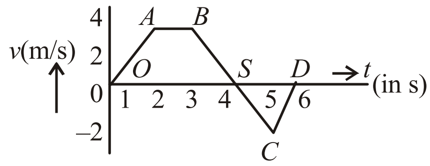
$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha\,dt$

Integrating both sides,

$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt$; $\left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t$

$\Rightarrow 2\sqrt{x} = \alpha t \Rightarrow x = \frac{\alpha^2}{4}t^2$

6. **(a)**



$OS = 4 + \frac{1}{3} = \frac{13}{3}$

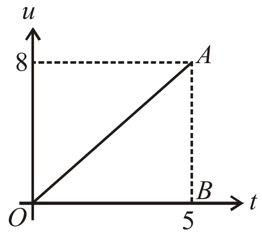
$SD = 2 - \frac{1}{3} = \frac{5}{3}$

Distance covered by the body = area of v-t graph

= ar ($OABS$) + ar (SCD)

$= \frac{1}{2}\left(\frac{13}{3}+1\right)\times 4 + \frac{1}{2}\times\frac{5}{3}\times 2 = \frac{32}{3} + \frac{5}{3} = \frac{37}{3}$ m

7. **(20)**



Distance travelled = Area of speed-time graph

$= \frac{1}{2}\times 5\times 8 = 20$ m

8. **(3)** Distance X varies with time t as $x^2 = at^2 + 2bt + c$

$\Rightarrow 2x\frac{dx}{dt} = 2at + 2b$

$\Rightarrow x\frac{dx}{dt} = at + b \Rightarrow \frac{dx}{dt} = \frac{(at+b)}{x}$

$\Rightarrow x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = a$

$\Rightarrow \frac{d^2x}{dt^2} = \frac{a - \left(\frac{dx}{dt}\right)^2}{x} = \frac{a - \left(\frac{at+b}{x}\right)^2}{x}$

$= \frac{ax^2 - (at+b)^2}{x^3} = \frac{ac - b^2}{x^3}$

$\Rightarrow$ $a \propto x^{-3}$ Hence, n = 3

9. **(b)** From the third equation of motion

$v^2 - u^2 = 2aS$

But, $a = \frac{F}{m}$

$\therefore v^2 = u^2 - 2\left(\frac{F}{m}\right)S$

$\Rightarrow v^2 = (1)^2 - (2)\left[\frac{2.5\times10^{-2}}{20\times10^{-3}}\right]\frac{20}{100}$

$\Rightarrow v^2 = 1 - \frac{1}{2}$

$\Rightarrow v = \frac{1}{\sqrt{2}}\text{m/s} = 0.7\text{m/s}$

10. **(b)** $x = at + bt^2 - ct^3$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(at + bt^2 + ct^3)$

$= a + 2bt - 3ct^2$

Acceleration, $\frac{dv}{dt} = \frac{d}{dt}(a + 2bt - 3ct^2)$

or $0 = 2b - 3c \times 2t$ $\quad \therefore t = \left(\frac{b}{3c}\right)$

and $v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = \left(a + \frac{b^2}{3c}\right)$

11. **(d)** For constant acceleration, there is straight line parallel to t-axis on $\vec{a} - t$.

Inclined straight line on $\vec{v} - t$, and parabola on $\vec{x} - t$.

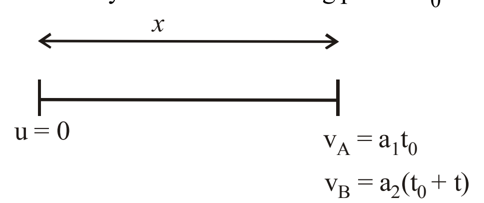
12. **(d)** Position of the particle,

S = area under graph (time t = 0 to 5s)

$= \frac{1}{2}\times2\times2 + 2\times2 + 3\times1 = 9\text{m}$

13. **(c)** Let time taken by A to reach finishing point is t_0

$\therefore$ Time taken by B to reach finishing point $= t_0 + t$



$v_A - v_B = v$

$\Rightarrow v = a_1 t_0 - a_2(t_0 + t) = (a_1 - a_2)t_0 - a_2t$ ...(i)

$x_B = x_A = \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t)$

$\Rightarrow \left(\sqrt{a_1} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}t$

$\Rightarrow t_0 = \frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}}$

Putting this value of t_0 in equation (i)

$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}} - a_2t$

$= \left(\sqrt{a_1} + \sqrt{a_2}\right)\sqrt{a_2}t - a_2t = \sqrt{a_1a_2}t + a_2t - a_2t$

or, $v = \sqrt{a_1a_2}t$

14. **(b)** According to question, $u_1 = 40$ km/h, $v_1 = 0$ and $s_1 = 40$ m

using $v^2 - u^2 = 2as$; $0^2 - 40^2 = 2a \times 40$ ...(i)

Again, $0^2 - 80^2 = 2as$ ...(ii)

From eqn. (i) and (ii)

Stopping distance, $s = 160$ m

15. **(c)** Using equation, $a = \frac{v - u}{t}$ and $S = ut + \frac{1}{2}at^2$

Distance travelled by car in 15 sec $= \frac{1}{2}\frac{(45)}{15}(15)^2$

$= \frac{675}{2}$ m

Distance travelled by scooter in 15 seconds = 30 × 15 = 450

($\because$ distance = speed × time)

Difference between distance travelled by car and scooter in 15 sec, 450 – 337.5 = 112.5 m

Let car catches scooter in time t;

$\frac{675}{2} + 45(t - 15) = 30t$

337.5 + 45t – 675 = 30t $\Rightarrow$ 15t = 337.5

$\Rightarrow$ t = 22.5 sec

16. **(a)** Let the car turn of the highway at a distance 'x' from the point M. So, RM = x

And if speed of car in field is v, then time taken by the car to cover the distance $QR = QM - x$ on the highway,

$t_1 = \frac{QM - x}{2v}$ .....(i)

Time taken to travel the distance 'RP' in the field

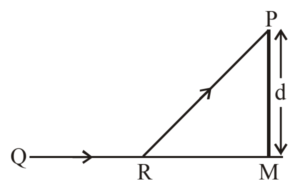
$t_2 = \frac{\sqrt{d^2 + x^2}}{v}$ ..... (ii)

Total time elapsed to move the car from Q to P

$t = t_1 + t_2 = \frac{QM - x}{2v} + \frac{\sqrt{d^2 + x^2}}{v}$

For 't' to be minimum $\frac{dt}{dx} = 0$

$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$



or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$

17. **(c)** According to question, object is moving with constant negative acceleration i.e., a = – constant (C)

$$\frac{vdv}{dx} = -C$$

$vdv = -Cdx$

$$\frac{v^2}{2} = -Cx + k \qquad x = -\frac{v^2}{2C} + \frac{k}{C}$$

Hence, graph (3) represents correctly.

18. **(b)** Distance along a line i.e., displacement (s) $= t^3$ ($\because s \propto t^3$ given)

By double differentiation of displacement, we get acceleration.

$$V = \frac{ds}{dt} = \frac{dt^3}{dt} = 3t^2 \text{ and } a = \frac{dv}{dt} = \frac{d3t^2}{dt} = 6t$$

$a = 6t$ or $a \propto t$

Hence graph (b) is correct.

19. **(a)** Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$

From graph, $v_A = \frac{\Delta x_A}{\Delta t_A} = \frac{4m}{8s} = 0.5$ m/s

and $v_B = \frac{\Delta x_B}{\Delta t_B} = \frac{8m}{16s} = 0.5$ m/s

i.e., $v_A = v_B = 0.5$ m/s

20. **(a)** Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\Rightarrow \frac{dv}{\sqrt{v}} = -2.5\, dt$$

Integrating,

$$\int_{6.25}^{0} v^{-1/2} dv = -2.5\int_0^t dt$$

$$\Rightarrow \left[\frac{v^{+1/2}}{(1/2)}\right]_{6.25}^{0} = -2.5[t]_0^t$$

$\Rightarrow -2(6.25)^{1/2} = -2.5t$

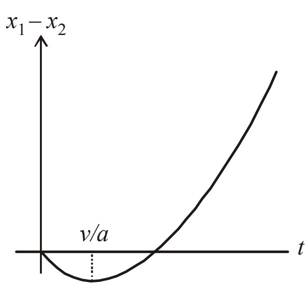
$\Rightarrow -2 \times 2.5 = -2.5t$

$\Rightarrow t = 25$

21. **(b)** **For the body starting from rest, distance travelled (x_1) is given by**

$$x_1 = 0 + \frac{1}{2}at^2$$

$$\Rightarrow x_1 = \frac{1}{2}at^2$$



For the body moving with constant speed

$x_2 = vt$

$$\therefore x_1 - x_2 = \frac{1}{2}at^2 - vt$$

at $t = 0, x_1 - x_2 = 0$

This equation is of parabola.

For $t < \frac{v}{a}$; the slope is negative

For $t = \frac{v}{a}$; the slope is zero

For $t > \frac{v}{a}$; the slope is positive

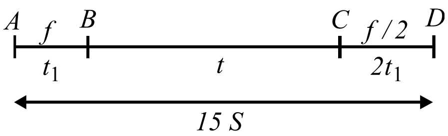
These characteristics are represented by graph (b).

22. **(d)** Let car starts from A from rest and moves up to point B with acceleration f.

Distance, $AB = S = \frac{1}{2}ft_1^2$

Distance, $BC = (ft_1)t$

Distance, $CD = \frac{u^2}{2a} = \frac{(ft_1)^2}{2(f/2)} = ft_1^2 = 2S$



Total distance, $AD = AB + BC + CD = 15S$

$AD = S + BC + 2S$

$\Rightarrow S + f\,t_1 t + 2S = 15S$

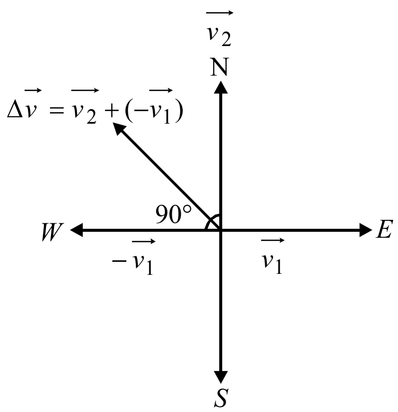
$\Rightarrow f\,t_1 t = 12S$ .... (i)

$\frac{1}{2}f\,t_1^2 = S$ .... (ii)

Dividing (i) by (ii), we get $t_1 = \frac{t}{6}$

$$\Rightarrow S = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$$

23. **(c)**



Initial velocity, $\vec{v_1} = 5\hat{i}$, Final velocity, $\vec{v_2} = 5\hat{j}$,

Change in velocity $\Delta\vec{v} = (\vec{v}_2 - \vec{v}_1)$

$= \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos 90}$

$= \sqrt{5^2 + 5^2 + 0} = 5\sqrt{2}$m/s

[As $|v_1| = |v_2| = 5$ m/s]

Avg. acceleration $= \frac{\Delta\vec{v}}{t}$

$= \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2

$\tan\theta = \frac{5}{-5} = -1$

which means θ is in the second quadrant. (towards north-west)

24. **(d)** Given, $t = ax^2 + bx$;

Diff. with respect to time (t)

$\frac{d}{dt}(t) = a\frac{d}{dt}(x^2) + b\frac{dx}{dt} = a.2x\frac{dx}{dt} + $ b.v.

$\Rightarrow 1 = 2axv + bv = v(2ax + b)$($v$ = velocity)

$2ax + b = \frac{1}{v}$.

Again differentiating, we get

$2a\frac{dx}{dt} + 0 = -\frac{1}{v^2}\frac{dv}{dt}$

$\Rightarrow a = \frac{dv}{dt} = -2av^3 \qquad \left(\because \frac{dx}{dt} = v\right)$

25. **(d)** In first case speed,

$u = 60 \times \frac{5}{18}$ m/s $= \frac{50}{3}$ m/s

$d = 20$m,

Let retardation be a then

$(0)^2 - u^2 = -2ad$

or $u^2 = 2ad$ ...(i)

In second case speed, $u' = 120 \times \frac{5}{18}$

$= \frac{100}{3}$ m/s

and $(0)^2 - u'^2 = -2ad'$

or $u'^2 = 2ad'$ ...(ii)

(ii) divided by (i) gives,

$4 = \frac{d'}{d} \Rightarrow d' = 4 \times 20 = 80$m

26. **(c)** **Fir first case :** Initial velocity,

$u = 50 \times \frac{5}{18}$ m/s,

$v = 0, s = 6\text{m}, a = a$

Using, $v^2 - u^2 = 2as$

$\Rightarrow 0^2 - \left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$

$\Rightarrow -\left(50 \times \frac{5}{18}\right)^2 = 2 \times a \times 6$

$a = -\frac{250 \times 250}{324 \times 2 \times 6} \approx = -16\ \text{ms}^{-2}$.

Case-2 : Initial velocity, $u = 100$ km/hr

$= 100 \times \frac{5}{18}$ m/sec

$v = 0, s = s, a = a$

As $v^2 - u^2 = 2as$

$\Rightarrow 0^2 - \left(100 \times \frac{5}{18}\right)^2 = 2as$

$\Rightarrow -\left(100 \times \frac{5}{18}\right)^2 = 2 \times (-16) \times 5$

$s = \frac{500 \times 500}{324 \times 32} = 24$m

27. **(a)** In first case

$u_1 = u$; $v_1 = \frac{u}{2}$, $s_1 = 3$ cm, $a_1 = ?$

Using, $v_1^2 - u_1^2 = 2a_1s_1$ ...(i)

$\left(\frac{u}{2}\right)^2 - u^2 = 2 \times a \times 3$

$\Rightarrow a = \frac{-u^2}{8}$

In second case: Assuming the same retardation

$u_2 = u/2$; $v_2 = 0$; $s_2 = ?$; $a_2 = \frac{-u^2}{8}$

$v_2^2 - u_2^2 = 2a_2 \times s_2$ ...(ii)

$\therefore 0 - \frac{u^2}{4} = 2\left(\frac{-u^2}{8}\right) \times s_2$

$\Rightarrow s_2 = 1$ cm

28. **(d)** **For first car**

$u_1 = u, v_1 = 0, a_1 = -a, s_1 = s_1$

As $v_1^2 - u_1^2 = 2a_1s_1$

$\Rightarrow -u^2 = -2as_1$

$\Rightarrow u^2 = 2as_1$

$\Rightarrow s_1 = \frac{u^2}{2a}$ ...(i)

For second car

$u_2 = 4u, v_1 = 0, a_2 = -a, s_2 = s_2$

$\therefore v_2^2 - u_2^2 = 2a_2s_2$

$\Rightarrow -(4u)^2 = 2(-a)s_2$

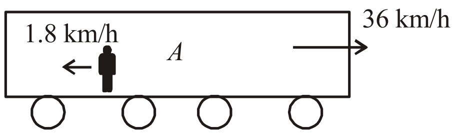
$\Rightarrow \quad 16u^2 = 2as_2$

$\Rightarrow \quad s_2 = \dfrac{8u^2}{a}$ ...(ii)

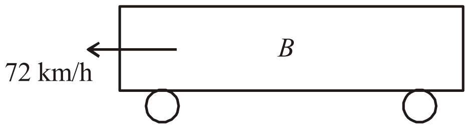
Dividing (i) and (ii),

$$\frac{s_1}{s_2} = \frac{u^2}{2a} \cdot \frac{a}{8u^2} = \frac{1}{16}$$

29. (a) According to question, train *A* and *B* are running on parallel tracks in the opposite direction.



$V_A = 36 \text{ km/h} = 10 \text{ m/s}$



$V_B = -72 \text{ km/h} = -20 \text{ m/s}$

$V_{MA} = -1.8 \text{ km/h} = -0.5 \text{ m/s}$

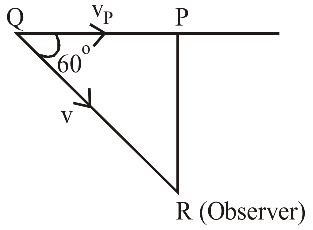
$V_{\text{man}, B} = V_{\text{man}, A} + V_{A, B}$

$= V_{\text{man}, A} + V_A - V_B = -0.5 + 10 - (-20)$

$= -0.5 + 30 = 29.5 \text{ m/s}.$

30. (a)

31. (d)



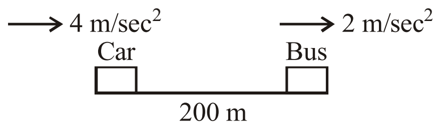
Distance, PQ = $v_p \times t$ (Distance = speed × time)

Distance, QR = V.t

$$\cos 60° = \frac{PQ}{QR}$$

$$\frac{1}{2} = \frac{v_P \times t}{V.t} \Rightarrow v_P = \frac{v}{2}$$

32. (c)



Given, $u_C = u_B = 0$, $a_C = 4 \text{ m/s}^2$, $a_B = 2 \text{ m/s}^2$

hence relative acceleration, $a_{CB} = 2 \text{ m/sec}^2$

Now, we know, $s = ut + \dfrac{1}{2}at^2$

$200 = \dfrac{1}{2} \times 2t^2 \quad \because u = 0$

Hence, the car will catch up with the bus after time

$t = 10\sqrt{2}$ second

33. (c) Person's speed walking only is $\dfrac{1}{60} \dfrac{\text{"escalator"}}{\text{second}}$

Standing the escalator without walking the speed is $\dfrac{1}{40} \dfrac{\text{"escalator"}}{\text{second}}$

Walking with the escalator going, the speed add.

So, the person's speed is $\dfrac{1}{60} + \dfrac{1}{40} = \dfrac{15}{120} \dfrac{\text{"escalator"}}{\text{second}}$

So, the time to go up the escalator $t = \dfrac{120}{5} = 24$ second.

34 (d) Let '*S*' be the distance between two ends '*a*' be the constant acceleration

As we know $v^2 - u^2 = 2aS$

or, $aS = \dfrac{v^2 - u^2}{2}$

Let *v* be velocity at mid point.

Therefore, $v_c^2 - u^2 = 2a\dfrac{S}{2}$

$v_c^2 = u^2 + aS$

$v_c^2 = u^2 + \dfrac{v^2 - u^2}{2}$

$v_c = \sqrt{\dfrac{u^2 + v^2}{2}}$

35. (c) For upward motion of helicopter,

$v^2 = u^2 + 2gh \Rightarrow v^2 = 0 + 2gh \Rightarrow v = \sqrt{2gh}$

Now, packet will start moving under gravity.

Let '*t*' be the time taken by the food packet to reach the ground.

$s = ut + \dfrac{1}{2}at^2$

$\Rightarrow -h = \sqrt{2gh}\,t - \dfrac{1}{2}gt^2 \Rightarrow \dfrac{1}{2}gt^2 - \sqrt{2gh}\,t - h = 0$

or, $t = \dfrac{\sqrt{2gh} \pm \sqrt{2gh + 4 \times \dfrac{g}{2} \times h}}{2 \times \dfrac{g}{2}}$

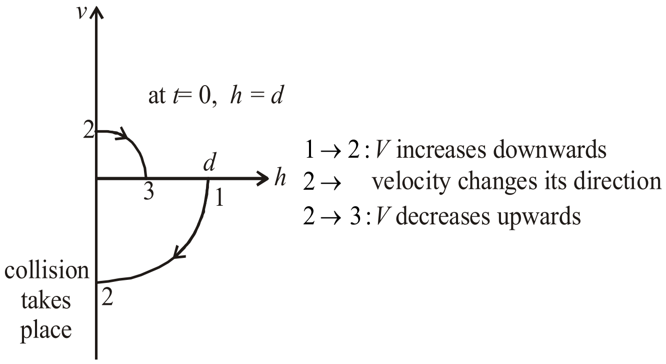
or, $t = \sqrt{\dfrac{2gh}{g}}(1 + \sqrt{2}) \Rightarrow t = \sqrt{\dfrac{2h}{g}}(1 + \sqrt{2})$

or, $t = 3.4\sqrt{\dfrac{h}{g}}$

36. (c) For uniformly accelerated/ deaccelerated motion :

$v^2 = u^2 \pm 2gh$

As equation is quadratic, so, *v*-*h* graph will be a parabola



Initially velocity is downwards (–ve) and then after collision it reverses its direction with lesser magnitude, i.e. velocity is upwards (+ve).

Note that time $t = 0$ corresponds to the point on the graph where $h = d$.

Next time collision takes place at 3.

37. **(08.00)** Let the ball takes time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2$

$\Rightarrow S = 0 \times t + \frac{1}{2}gt^2$

$\Rightarrow \quad 200 = gt^2 \qquad [\because 2S = 100m]$

$\Rightarrow t = \sqrt{\frac{200}{g}} \qquad \ldots(i)$

In last $\frac{1}{2}s$, body travels a distance of 19 m, so in $\left(t - \frac{1}{2}\right)$

distance travelled = 81

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

$\therefore \; g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$

$\Rightarrow \left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$

$\therefore \; \frac{1}{2} = \frac{1}{\sqrt{g}}(\sqrt{200} - \sqrt{81 \times 2}) \qquad$ using (i)

$\Rightarrow \sqrt{g} = 2(10\sqrt{2} - 9\sqrt{2})$

$\Rightarrow \sqrt{g} = 2\sqrt{2}$

$\therefore$ g = 8 m/s^2

38. **(a)** For a body thrown vertically upwards acceleration remains constant ($a = -g$) and velocity at anytime t is given by $V = u - gt$

During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other.

Hence graph (a) correctly depicts velocity versus time.

39. **(b)** $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

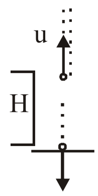
for $y_1 = -240m, t = 8s$

$\therefore \quad y_2 - y_1 = 30t$ for $t \le 8s$.

for $t > 8s$,

$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$

40. **(c)** Speed on reaching ground



$v = \sqrt{u^2 + 2gh}$

Now, $v = u + at$

$\Rightarrow \quad \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \frac{u}{g}$,

$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$

(from question)

$\Rightarrow \quad 2gH = n(n-2)u^2$

41. **(b)** For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \frac{1}{2}at^2$

We get $y - h = -\frac{1}{2}gt^2 \Rightarrow y = h - \frac{1}{2}gt^2$

The graph between y and t is a parabola with $y = h$ at $t = 0$. As time increases y decreases.

For upward motion.

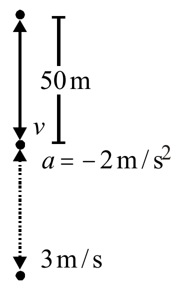
The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocity just after collision. As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2}gt^2$

All these characteristics are represented by graph (b).

42. **(d)** Initial velocity of parachute after bailing out,



$u = \sqrt{2gh}$

$u = \sqrt{2 \times 9.8 \times 50} = 14\sqrt{5}$

The velocity at ground,

$v = 3m/s$

$S = \frac{v^2 - u^2}{2 \times 2} = \frac{3^2 - 980}{4} \approx 243$ m

Initially he has fallen 50 m.

$\therefore$ Total height from where he bailed out = 243 + 50 = 293 m

43. **(a)** We have $s = ut + \frac{1}{2}gt^2$,

$\Rightarrow h = 0 \times T + \frac{1}{2}gT^2$

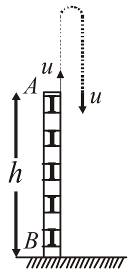
$\Rightarrow h = \frac{1}{2}gT^2$

Vertical distance moved in time $\frac{T}{3}$ is

$h' = \frac{1}{2}g\left(\frac{T}{3}\right)^2 \Rightarrow h' = \frac{1}{2} \times \frac{gT^2}{9} = \frac{h}{9}$

$\therefore$ Position of ball from ground $= h - \frac{h}{9} = \frac{8h}{9}$

44. **(b)** Ball A is thrown upwards with velocity u



from the building. During its downward journey when it comes back to the point of throw, its speed is equal to the speed of throw

(u). So, for the journey of both the balls from point A to B.

We can apply $v^2 - u^2 = 2gh$.

As u, g, h are same for both the balls, $v_A = v_B$

TOPIC 1 Vectors

1. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m. The value of x is ______.

[NA Sep. 05, 2020 (I)]

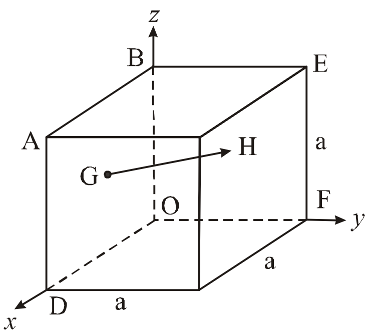
2. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of 2 $\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is _______.

[NA 7 Jan. 2020 II]

3. Let $|\overrightarrow{A_1}| = 3$, $|\overrightarrow{A_2}| = 5$ and $|\overrightarrow{A_1} + \overrightarrow{A_2}| = 5$. The value of $(2\overrightarrow{A_1} + 3\overrightarrow{A_2}) \bullet (3\overrightarrow{A_1} - 2\overrightarrow{A_2})$ is : **[8 April 2020 II]**

(a) – 106.5 (b) – 99.5
(c) –112.5 (d) –118.5

4. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be: **[10 Jan. 2019 I]**



(a) $\frac{1}{2}a(\hat{k} - \hat{i})$ (b) $\frac{1}{2}a(\hat{i} - \hat{k})$
(c) $\frac{1}{2}a(\hat{j} - \hat{i})$ (d) $\frac{1}{2}a(\hat{j} - \hat{k})$

5. Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: **[10 Jan. 2019 II]**

(a) 120° (b) 60°
(c) 90° (d) 30°

6. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is: **[10 Jan. 2019 II]**

(a) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (b) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$
(c) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (d) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

7. Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by **[Online April 16, 2018]**

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$
(c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

8. A vector $\vec{A}$ is rotated by a small angle Δθ radian (Δθ << 1) to get a new vector $\vec{B}$. In that case $|\vec{B} - \vec{A}|$ is : **[Online April 11, 2015]**

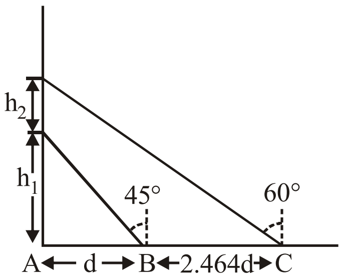
(a) $|\vec{A}|\Delta\theta$ (b) $|\vec{B}|\Delta\theta - |\vec{A}|$
(c) $|\vec{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$ (d) 0

9. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is **[2004]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) π (d) $\frac{\pi}{4}$

TOPIC 2 Motion in a Plane with Constant Acceleration

10. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45º with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60º with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30º = 0.5774): **[Sep. 05, 2020 (I)]**



(a) 1.464 d (b) 0.732 d
(c) 0.464 d (d) d

11. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ ms^{-1}, a particle moves in the x–y plane with a constant acceleration of $(10\hat{i} + 4\hat{j})$ ms^{-2}. At time t, its coordiantes are (20 m, y_0 m). The values of t and y_0 are, respectively : **[Sep. 04, 2020 (I)]**
(a) 2 s and 18 m (b) 4 s and 52 m
(c) 2 s and 24 m (d) 5 s and 25 m

12. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **[9 April 2019 II]**
(a) 40 (b) 25 (c) 100 (d) 50

13. A particle moves from the point $(2.0\hat{i} + 4.0\hat{j})$m, at t = 0, with an initial velocity $(5.0\hat{i} + 4.0\hat{j})$ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j})$ms^{-2}. What is the distance of the particle from the origin at time 2s? **[11 Jan. 2019 II]**
(a) 15m (b) $20\sqrt{2}$m
(c) 5m (d) $10\sqrt{2}$m

14. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[9 Jan. 2019 I]**
(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) $y^2 = x^2$ + constant (d) xy = constant

15. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s². The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is: **[9 Jan. 2020 II]**
(a) 32 (b) 50 (c) 60 (d) 40

16. A particle is moving along the x-$axis$ with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is____ **[NA 8 Jan. 2020 I]**

17. A particle moves such that its position vector $\vec{r}(t) = \cos \omega t\,\hat{i} + \sin \omega t\,\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle: **[8 Jan. 2020 II]**
(a) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin
(b) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$
(c) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$
(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

18. A particle is moving with velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**
(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) xy = constant (d) $y^2 = x^2$ + constant

19. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is : **[2009]**
(a) $7\sqrt{2}$ units (b) 7 units
(c) 8.5 units (d) 10 units

20. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by **[2003]**
(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$
(c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

TOPIC 3 Projectile Motion

21. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: **[9 Jan. 2020 II]**

(a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

22. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$): **[12 April 2019 I]**

(a) $\theta_0 = \sin^{-1}\frac{1}{\sqrt{5}}$ and $v_0 = \frac{5}{3}\ ms^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{9}{3}\ ms^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

23. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is : **[12 April 2019 I]**

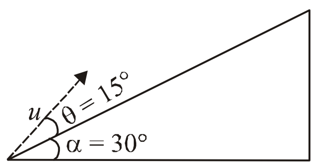
(a) R/4g (b) R/g (c) R/2g (d) 2R/g

24. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? **[12 April 2019 II]**

(a) $R^2 = 4\,h_1h_2$ (b) $R^2 = 16\,h_1h_2$

(c) $R^2 = 2\,h_1h_2$ (d) $R^2 = h_1h_2$

25. A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to : (Take $g = 10\ ms^{-2}$) **[10 April 2019 II]**



(a) 20 cm (b) 18 cm (c) 26 cm (d) 14 cm

26. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is: **[11 Jan. 2019 I]**

(a) 10.3 m (b) 2.8 m

(c) 2.5 m (d) 5.1 m

27. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is: **[10 Jan. 2019 I]**

(a) 1 : 16 (b) 1 : 2 (c) 1 : 4 (d) 1 : 8

28. The initial speed of a bullet fired from a rifle is 630 m/s. The rifle is fired at the centre of a target 700 m away at the same level as the target. How far above the centre of the target ? **[Online April 11, 2014]**

(a) 1.0 m (b) 4.2 m (c) 6.1 m (d) 9.8 m

29. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = \left(40\hat{i} + 50\hat{j}\right)$ m at $t = 2$s. If the projectile was launched at an angle θ from the horizontal, then θ is (take g = 10 ms^{-2}) **[Online April 9, 2014]**

(a) $\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\frac{3}{2}$

(c) $\tan^{-1}\frac{7}{4}$ (d) $\tan^{-1}\frac{4}{5}$

30. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ m/s^2$, the equation of its trajectory is : **[2013]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

31. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity v = 20 m/s, on a horizontal surface ? (g = 10 m/s^2) **[Online April 25, 2013]**

(a) 30° (b) 60° (c) 75° (d) 45°

32. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is : **[Online April 22, 2013]**

(a) 4.4 m (b) 2.4 m (c) 3.6 m (d) 1.6 m

33. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[2012]**

(a) $20\sqrt{2}$ m (b) 10 m

(c) $10\sqrt{2}$ m (d) 20 m

34. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is: **[2011]**

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

35. A projectile can have the same range 'R' for two angles of projection. If 'T_1' and 'T_2' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to. **[2004]**

(a) R (b) $\frac{1}{R}$ (c) $\frac{1}{R^2}$ (d) R^2

36. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ? **[2004]**

(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45°

37. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? **[2003]**

$[g = 10\text{m/s}^2,\ \sin 30^o = \frac{1}{2},\ \cos 30^o = \frac{\sqrt{3}}{2}]$

(a) 5.20m (b) 4.33m (c) 2.60m (d) 8.66m

TOPIC 4 Relative Velocity in Two Dimensions & Uniform Circular Motion

38. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of: **[Sep. 06, 2020 (I)]**

(a) 10^{-3} (b) 10^{-4}
(c) 10^{-2} (d) 10^{-1}

39. When a carsit at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60º from the horizontal. On furter increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45º. The value of β is close to: **[Sep. 06, 2020 (II)]**

(a) 0.50 (b) 0.41
(c) 0.37 (d) 0.73

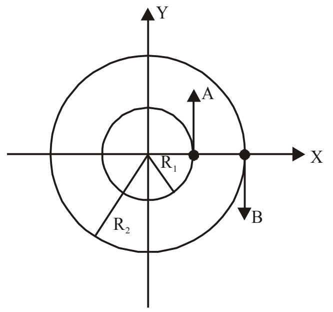
40. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[9 April 2019 I]**

(a) 90° (b) 150°
(c) 120° (d) 60°

41. Ship A is sailing towards north-east with velocity km/hr where points east and , north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in: **[8 April 2019 I]**

(a) 4.2 hrs. (b) 2.6 hrs.
(c) 3.2 hrs. (d) 2.2 hrs.

42. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure : **[12 Jan. 2019 II]**



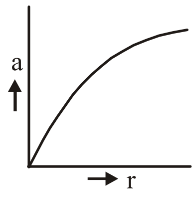
The relative velocity $\vec{v_A} - \vec{v_B}$ and $t = \frac{\pi}{2\omega}$ is given by:

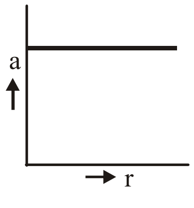
(a) $\omega(R_1 + R_2)\hat{i}$ (b) $-\omega(R_1 + R_2)\hat{i}$
(c) $\omega(R_2 - R_1)\hat{i}$ (d) $\omega(R_1 - R_2)\hat{i}$

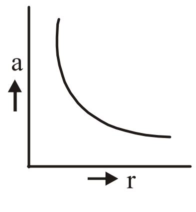
43. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[Online April 10, 2015]**

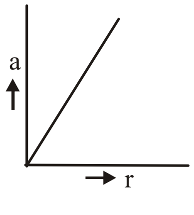
(a) $10\sqrt{3}$m/s (b) zero
(c) $10\sqrt{2}$m/s (d) 10 m/s

44. If a body moving in circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes relation between acceleration and radius? **[Online April 10, 2015]**

(a) 

(b) 

(c) 

(d) 

Hints & Solutions

1. **(195)**

Given : $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N

And, $\vec{r} = [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})] = 3\hat{i} + \hat{j} - 2\hat{k}$

Torque, $\tau = \vec{r} \times \vec{F} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

Magnitude of torque, $|\vec{\tau}| = \sqrt{195}$.

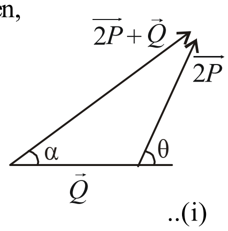
2. **(90)** Given,

$|\vec{R}| = |\vec{P}| \Rightarrow |\vec{P} + \vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ.\cos\theta = P^2$

$\Rightarrow Q + 2P\cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{Q}{2P}$ ..(i)



$$\tan\alpha = \frac{2P\sin\theta}{Q + 2P\cos\theta} = \infty \ (\because 2\,P\cos\theta + Q = 0)$$

$\Rightarrow \alpha = 90°$

3. **(d)** Using,

$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2 \times 3 \times 5\cos\theta$

or $\cos\theta = -0.3$

$\left(2\vec{A_1} + 3\vec{A_2}\right).\left(3\vec{A_1} - 2\vec{A_2}\right) = 2A_1 \times 3A_1$

$+ (3A_2)(3A_1)\cos\theta - (2A_1)(2A_2)\cos\theta - 3A_2 \times 2A_2$

$= 6A_1^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2^2$

$= 6A_1^2\ 6A_2^2 + 5A^{-}_1A_2\cos\theta$

$= 6 \times 3^2 - 6 \times 5^2 + 5 \times 3 \times 5(-0.3)$

$= -118.5$

4. **(c)** From figure,

$\vec{r}_G = \dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}$

$\vec{r}_H = \dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}$

$\therefore \vec{r}_H - \vec{r}_G = \left(\dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}\right) - \left(\dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}\right) = \dfrac{a}{2}\left(\hat{j} - \hat{i}\right)$

5. **(a)** Using, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$4F^2 + 9F^2 + 12F^2\cos\theta = R^2$

When forces Q is doubled,

$4F^2 + 36F^2 + 24F^2\cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2\cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$\therefore \cos\theta = -\dfrac{12F^2}{24F^2} = -\dfrac{1}{2} \quad \Rightarrow \theta = 120^{\circ}$

6. **(a)** Let magnitude of two vectors $\vec{A}$ and $\vec{B}$ = a

$|\vec{A} + \vec{B}| = \sqrt{a^2 + a^2 + 2a^2\cos\theta}$ and

$|\vec{A} - \vec{B}| = \sqrt{a^2 + a^2 - 2a^2\left[\cos(180° - \theta)\right]}$

$= \sqrt{a^2 + a^2 - 2a^2\cos\theta}$

and accroding to question,

$|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$

or, $\dfrac{a^2 + a^2 + 2a^2\cos\theta}{a^2 + a^2 - 2a^2\cos\theta} = n^2$

$\Rightarrow \dfrac{a^2(1 + 1 + 2\cos\theta)}{a^2(1 + 1 - 2\cos\theta)} n^2 \Rightarrow \dfrac{(1 + \cos\theta)}{(1 - \cos\theta)} = n^2$

using componendo and dividendo theorem, we get

$\theta = \cos^{-1}\left(\dfrac{n^2 - 1}{n^2 + 1}\right)$

7. **(a)** If $\vec{C} = a\hat{i} + b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ ..... (i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ..... (ii)

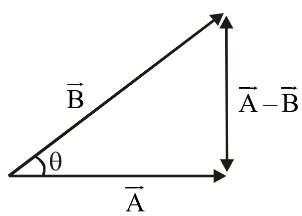
Solving equation (i) and (ii) we get

$a = \dfrac{1}{3}, b = \dfrac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $|\vec{C}| = \sqrt{\dfrac{1}{9} + \dfrac{4}{9}} = \sqrt{\dfrac{5}{9}}$

8. (a) Arc length = radius × angle

So, $|\vec{B} - \vec{A}| = |\vec{A}|\Delta\theta$



9. **(c)** $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

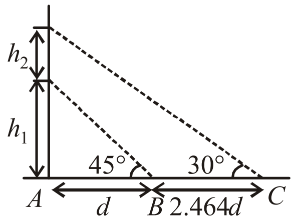
$\therefore \vec{A} \times \vec{B} = 0$

Angle between them is 0, π, or 2 π

from the given options, $\theta = \pi$

10. **(d)** From figure/ trigonometry,

$\frac{h_1}{d} = \tan 45°$ $\quad \therefore h_1 = d$



And, $\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$

$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46d$

$\Rightarrow (h_1 + h_2) = \frac{3.46d}{\sqrt{3}} \Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$

$\therefore h_2 = d$

11. **(a)** Given : $\vec{u} = 5\hat{j}$ m/s

Acceleration, $\vec{a} = 10\hat{i} + 4\hat{j}$ and final coordinate $(20, y_0)$ in time t.

$S_x = u_x t + \frac{1}{2} a_x t^2$ $\quad [\because u_x = 0]$

$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2$ s

$S_y = u_y \times t + \frac{1}{2} a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18$ m

12. **(d)** $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$\therefore a = \sqrt{30^2 + 40^2} = 50 \text{ m/s}^2$

13. **(b)** As $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4$

$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$

[as $\vec{s}$ = change in position = $\vec{r}_f - \vec{r}_i$]

$\vec{r}_r = 20\hat{i} + 20\hat{j}$

$|\vec{r}_r| = 20\sqrt{2}$

14. **(c)** From given equation,

$\vec{V} = K\left(y\hat{i} + x\hat{j}\right)$

$\frac{dx}{dt} = ky$ and $\frac{dy}{dt} = kx$

Now $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y} = \frac{dy}{dx}, \Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

15. **(c)** Using $S = ut + \frac{1}{2}at^2$

$y = u_y t + \frac{1}{2} a_y t^2$ (along y Axis)

$\Rightarrow \quad 32 = 0 \times t + \frac{1}{2}(4)t^2$

$\Rightarrow \quad \frac{1}{2} \times 4 \times t^2 = 32$

$\Rightarrow t = 4\ s$

$S_x = u_x t + \frac{1}{2} a_x t^2$ (Along x Axis)

$\Rightarrow \quad x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60$

16. **(580)**

For pariticle 'A'	For particle 'B'
$X_A = -3t^2 + 8t + 10$	$Y_B = 5 - 8t^3$
$\vec{V}_A = (8 - 6t)\hat{i}$	$\vec{V}_B = -24t^2\hat{j}$
$\vec{a}_A = -6\hat{i}$	$\vec{a}_B = -48t\hat{j}$

At $t = 1$ sec

$\vec{V}_A = (8 - 6t)\hat{i} = 2\hat{i}$ and $\vec{v}_B = -24\hat{j}$

$\therefore \vec{V}_{B/A} = -\vec{v}_A + \vec{v}_B = -2\hat{i} - 24\hat{j}$

$\therefore$ Speed of B w.r.t. A, $\sqrt{v} = \sqrt{2^2 + 24^2}$

$= \sqrt{4 + 576} = \sqrt{580}$

$\therefore v = 580$ (m/s)

17. **(d)** Given, Position vector,

$\vec{r} = \cos\omega t\hat{i} + \sin\omega t\,\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\hat{i} + \cos\omega t\,\hat{j})$

Acceleration,

$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\,\hat{j})$

$\vec{a} = -\omega^2\vec{r}$

$\therefore \vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}.\vec{r} = 0 \quad \therefore \vec{v} \perp \vec{r}$

Thus, the particle is performing uniform circular motion.

18. **(d)** $v = k(yi + xj)$

$v = kyi + kxj$

$$\frac{dx}{dt} = ky, \ \frac{dy}{dt} = kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{kx}{ky}$$

$$ydy = xdx \qquad ...(i)$$

Integrating equation (i)

$$\int ydy = \int x \cdot dx$$

$$y^2 = x^2 + c$$

19. **(a)** Given $\vec{u} = 3\hat{i} + 4\hat{j}, \ \vec{a} = 0.4\hat{i} + 0.3\hat{j}, t = 10\text{s}$

From 1st equatoin of motion.

$$a = \frac{v-u}{t}$$

$$\therefore v = at\ tu$$

$$\Rightarrow v = (0.4\hat{i} + 0.3\hat{j}) \times 10 + (3\hat{i} + 4\hat{j})$$

$$\Rightarrow \quad 4\hat{i} + 3\hat{j} + 3\hat{j} + 4\hat{j}$$

$$\Rightarrow \quad v = 7\hat{i} + 7\hat{j}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ unit.}$$

20. **(b)** Coordinates of moving particle at time 't' are $x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore \ v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

21. **(a)** Using principal of conservation of linear momentum for horizontal motion, we have

$2mv_x = mu + mu \cos 60°$

$$v_x = \frac{3u}{4}$$

For vertical motion

$$h = 0 + \frac{1}{2}gT^2 \ \Rightarrow \ T = \sqrt{\frac{2h}{g}}$$

Let R is the horizontal distance travelled by the body.

$$R = v_x T + \frac{1}{2}(0)(T)^2 \ \text{(For horizontal motion)}$$

$$R = v_x T = \frac{3u}{4} \times \sqrt{\frac{2h}{g}}$$

$$\Rightarrow \ R = \frac{3\sqrt{3}u^2}{8g}$$

22. **(c)** Given, $y = 2x - 9x^2$

On comparing with,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

We have,

$$\tan\theta = 2 \text{ or } \cos\theta = \frac{1}{\sqrt{5}}$$

$$\text{and } \frac{g}{2u^2\cos^2\theta} = 9 \ \text{ or } \ \frac{10}{2u^2(1/\sqrt{5})^2} = 9$$

$$\therefore \qquad u = 5/3 \text{ m/s}$$

23. **(d)** R will be same for θ and $90° - \theta$.

Time of flights:

$$t_1 = \frac{2u\sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\text{Now, } t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right)$$

$$= \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$$

24. **(b)** For same range, the angle of projections are : θ and $90° - \theta$. So,

$$h_1 = \frac{u^2\sin^2\theta}{2g} \text{ and}$$

$$h_2 = \frac{u^2\sin^2(90° - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$\text{Also, } R = \frac{u^2\sin 2\theta}{g}$$

$$h_1 h_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g}$$

$$= \frac{u^2}{16}\frac{u^2(2\sin\theta\cos\theta)^2}{g^2}$$

$$= \frac{R^2}{16}$$

or $R^2 = 16\, h_1 h_2$

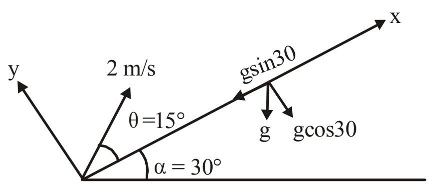
25. **(a)** On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

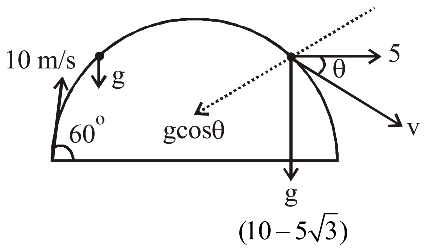
$$\text{Distance, S} = (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$$



$$= (2\cos 15°)\frac{4}{10}\frac{\sin 15°}{10\cos 30°} - \left(\frac{1}{2}\times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\text{m} \simeq 20\text{cm}$$

26. **(b)**



Horizontal component of velocity

$v_x = 10\cos 60° = 5$ m/s

vertical component of velocity

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s

After t = 1 sec.

Horizontal component of velocity $v_x = 5$ m/s

Vertical component of velocity

$v_y = |(5\sqrt{3}-10)|$ m/s $= 10-5\sqrt{3}$

Centripetal, acceleration $a_n = \frac{v^2}{R}$

$$\Rightarrow R = \frac{v_x^2+v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10\cos\theta} \quad ...(i)$$

From figure (using (i))

$$\tan\theta = \frac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100(2-\sqrt{3})}{10\cos 15} = 2.8\text{m}$$

27. **(a)** As we know, range $R = \frac{u^2\sin 2\theta}{g}$

and, area $A = \pi R^2$

$\therefore A \propto R^2$ or, $A \propto u^4$

$$\therefore \frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

28. **(c)** Let 't' be the time taken by the bullet to hit the target.

$\therefore$ 700 m = 630 ms^{-1} t

$$\Rightarrow t = \frac{700\text{m}}{630\text{ms}^{-1}} = \frac{10}{9}\text{sec}$$

For vertical motion,

Here, $u = 0$

$$\therefore h = \frac{1}{2}gt^2$$

$$= \frac{1}{2}\times 10\times\left(\frac{10}{9}\right)^2$$

$$= \frac{500}{81}\text{m} = 6.1\text{ m}$$

Therefore, the rifle must be aimed 6.1 m above the centre of the target to hit the target.

29. **(c)** From question,

Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20\text{m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$$\Rightarrow u_y \times 2 + \frac{1}{2}(-10)\times 4$$

or, $50 = 2u_y - 20$

or, $u_y = \frac{70}{2} = 35$m/s

$$\therefore \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$$

$$\Rightarrow \text{Angle } \theta = \tan^{-1}\frac{7}{4}$$

30. **(b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$

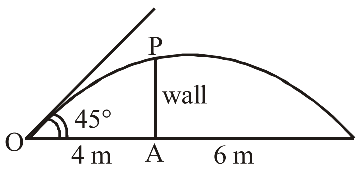
$\Rightarrow x = t$ ...(i)

$y = 2t - \frac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

31. **(b)**

32. **(b)**



As ball is projected at an angle 45° to the horizontal therefore Range = 4H

or $10 = 4H \Rightarrow H = \frac{10}{4} = 2.5$ m

($\because$ Range = 4 m + 6 m = 10m)

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

$$\therefore u^2 = \frac{H\times 2g}{\sin^2\theta} = \frac{2.5\times 2\times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$$

or, $u = \sqrt{100} = 10$ ms^{-1}

Height of wall PA

$$= OA\tan\theta - \frac{1}{2}\frac{g(OA)^2}{u^2\cos^2\theta}$$

$$= 4 - \frac{1}{2}\times\frac{10\times 16}{10\times 10\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}} = 2.4\text{ m}$$

33. **(d)** $R = \frac{u^2 \sin^2 \theta}{g}$, $H = \frac{u^2 \sin^2 \theta}{2g}$

H_{max} at $2\theta = 90°$

$$H_{max} = \frac{u^2}{2g}$$

$$\frac{u^2}{2g} = 10 \Rightarrow u^2 = 10g \times 2$$

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R_{max} = \frac{u^2}{g}$$

$$R_{max} = \frac{10 \times g \times 2}{g} = 20 \text{ metre}$$

34. **(a)** Let, total area around fountain

$A = \pi R_{max}^2$ ...(i)

Where $R_{max} = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin 90°}{g} = \frac{v^2}{g}$ ...(ii)

From equation (i) and (ii)

$$A = \pi \frac{v^4}{g^2}$$

35. **(a)** A projectile have same range for two angle
Let one angle be θ, then other is 90° – θ

$$T_1 = \frac{2u \sin \theta}{g}, T_2 = \frac{2u \cos \theta}{g}$$

then, $T_1 T_2 = \frac{4u^2 \sin \theta \cos \theta}{g} = 2R$

$(\because R = \frac{u^2 \sin^2 \theta}{g})$

Thus, it is proportional to R. (Range)

36. **(c)** Yes, Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man.

$$\frac{v_o}{2} = v_o \cos \theta$$

or $\theta = 60°$

37. **(d)** Horizontal range is required

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30°)}{10} = 5\sqrt{3} = 8.66 \text{ m}$$

38. **(a)** Here, $R = 0.1$ m

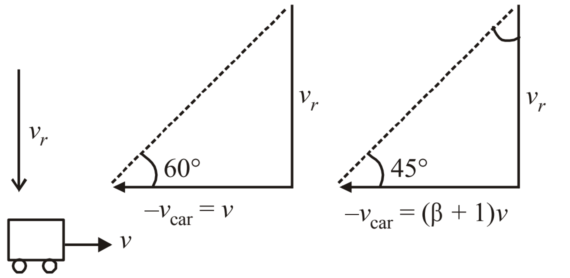
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105 \text{ rad/s}$$

Acceleration of the tip of the clock second's hand,

$$a = \omega^2 R = (0.105)^2 (0.1) = 0.0011 = 1.1 \times 10^{-3} \text{ m/s}^2$$

Hence, average acceleration is of the order of 10^{-3}.

39. **(d)** The given situation is shown in the diagram. Here v_r be the velocity of rain drop.



When car is moving with speed v,

$\tan 60° = \frac{v_r}{v}$ ...(i)

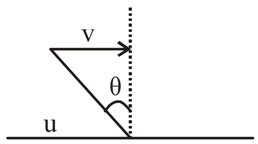
When car is moving with speed $(1+\beta)v$,

$\tan 45° = \frac{v_r}{(\beta+1)v}$ ...(ii)

Dividing (i) by (ii) we get,

$$\sqrt{3}v = (\beta+1)v \Rightarrow \beta = \sqrt{3} - 1 = 0.732.$$

40. **(c)** $\sin \theta = \frac{u}{v} = \frac{2}{4} = \frac{1}{2}$

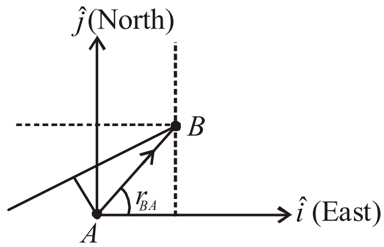


or $\theta = 30°$

with respect to flow,

$= 90° + 30° = 120°$

41. **(b)**



$$\vec{v}_A = 30\hat{i} + 50\hat{j} \text{ km/hr}$$

$$\vec{v}_B = (-10\hat{i}) \text{ km/hr}$$

$$r_{BA} = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = 40\hat{i} - 50\hat{j}$$

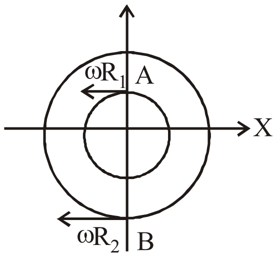
$$t_{minimum} = \frac{|(\vec{r}_{BA}) \cdot (\vec{v}_{BA})|}{|(\vec{v}_{BA})|^2}$$

$$= \frac{|(80\hat{i} + 150\hat{j})(-40\hat{i} - 50\hat{j})|}{(10\sqrt{41})^2}$$

$$\therefore t = \frac{10700}{10\sqrt{41} \times 10\sqrt{41}} = \frac{107}{41} = 2.6 \text{ hrs.}$$

42. **(c)** From, $\theta = \omega t = \omega \frac{\pi}{2\omega} = \frac{\pi}{2}$

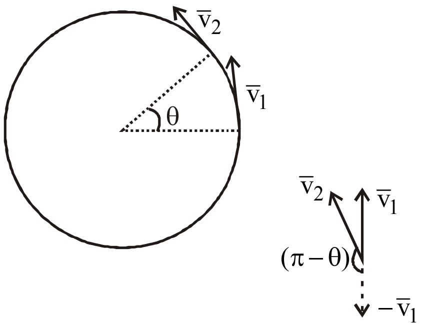
So, both have completed quater circle



Relative velocity,

$v_A - v_B = \omega R_1(-\hat{i}) - \omega R_2(-i) = \omega(R_2 - R_1)i$

43. **(d)**



Change in velocity,

$|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos(\pi - \theta)}$

$= 2v\sin\frac{\theta}{2} \qquad (\because |\vec{v}_1| = |\vec{v}_2|) = v$

$= (2 \times 10) \times \sin(30°) = 2 \times 10 \times \frac{1}{2}$

$= 10$ m/s

44. **(c)** Speed, V = constant (from question)

Centripetal acceleration,

$a = \frac{V^2}{r}$

ra = constant

Hence graph (c) correctly describes relation between acceleration and radius.