Trigonometric Ratios

INTRODUCTION

The literal meaning of the word trigonometry is the 'science of triangle measurement'. The word trigonometry is derived from two Greek words trigon and metron which means measuring the sides of a triangle. It had its beginning more than two thousand years ago as a tool for astronomers. The Babylonians, Egyptians, Greeks and the Indians studied trigonometry only because it helped them in unravelling the mysteries of the universe. In modern times, it has gained wider meaning and scope. Presently, it is defined as that branch of mathematics which deals with the measurement of angles, whether of triangle or any other figure.

At present, trigonometry is used in surveying, astronomy, navigation, physics, engineering, etc.

Important Formulae and Results of Trigonometry

I. (i) $180^\circ = \pi$ radians.

(ii) $1^\circ = \dfrac{\pi}{180} = 0.01745$ radians (approximately).

(iii) $\pi = \dfrac{\text{circumference of a cirlce}}{\text{diameter of the circle}}$

$= \dfrac{22}{7} = 3.1416$ (approximately).

(iv) θ (in radian measure) $= \dfrac{l}{r}$.

(v) Each interior angle of a regular polygon of n sides $= \dfrac{n-2}{n} \times 180$ degrees.

II. (i) $\sin\theta \times \operatorname{cosec}\theta = 1$; $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$;

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$.

Also, $-1 \le \sin x \le 1$, $\operatorname{cosec} x \le -1$ or $\operatorname{cosec} x \ge 1$.

(ii) $\cos\theta \times \sec\theta = 1$; $\cos\theta = \dfrac{1}{\sec\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$.

Also, $-1 \le \cos x \le 1$, $\sec x \le -1$ or $\sec x \ge 1$.

(iii) $\tan\theta \times \cot\theta = 1$; $\tan\theta = \dfrac{1}{\cot\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$.

Also, $-\infty < \tan\theta < \infty$, $-\infty < \cot\theta < \infty$.

(iv) $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$; $\cos^2\theta = 1 - \sin^2\theta$

(v) $\sin^2\theta = 1 + \tan^2\theta$; $\sec^2\theta - \tan^2\theta = 1$; $\tan^2\theta = \sec^2\theta - 1$.

(vi) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$; $\operatorname{cosec}^2\theta - \cot^2\theta = 1$; $\cot^2\theta = \operatorname{cosec}^2\theta - 1$.

(vii) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

III. Values of trigonometrical ratios for particular angles

(i)

Angle	sine	cos	tan
0°	0	1	0
$30^\circ = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^\circ = \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60^\circ = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^\circ = \frac{\pi}{2}$	1	0	∞
$120^\circ = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	– –	$-\sqrt{3}$
$135^\circ = \frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1

$150° = \frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
$180° = \pi$	0	–1	0
$270° = \frac{3\pi}{2}$	–1	0	$-\infty$
$360° = 2\pi$	0	1	0

(ii) $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$; $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$;

$\tan 15° = 2-\sqrt{3}$.

(iii) $\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$;

$\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72°$.

(iv) $\cos 36° = \frac{\sqrt{5}+1}{4} = \sin 54°$;

$\sin 35° = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54°$.

(v) $\tan 7\frac{1°}{2} = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$;

$\cot 7\frac{1°}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$.

IV. Signs of trigonometrical ratios

Angle	sin	cos	tan
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$90° - \theta$ or $\frac{\pi}{2} - \theta$	$\cos\theta$	$\sin\theta$	$\cot\theta$
$90° + \theta$ or $\frac{\pi}{2} + \theta$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$
$180° - \theta$ or $\pi - \theta$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$
$180° + \theta$ or $\pi + \theta$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$
$270° - \theta$ or $\frac{3\pi}{2} - \theta$	$-\cos\theta$	$-\sin\theta$	$\cot\theta$
$270° + \theta$ or $\frac{3\pi}{2} + \theta$	$-\cos\theta$	$\sin\theta$	$-\cot\theta$
$360° - \theta$ or $2\pi - \theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$360° + \theta$ or $2\pi + \theta$	$\sin\theta$	$\cos\theta$	$\tan\theta$

V. Trigonometrical ratios for sum or difference of angles

(i) $\sin(A \pm B) = \sin A \times \cos B \pm \cos A \times \sin B$.

(ii) $\cos(A \pm B) = \cos A \times \cos B \sin A \times \sin B$.

(iii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \times \tan B}$.

(iv) $\cot(A \pm B) = \frac{\cot A \times \cot B \mp 1}{\cot B \pm \cot A}$.

(v) $\tan(A + B + C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)}$$

(vi) $\sin(A + B) \times \sin(A - B) = \sin^2 A - \sin^2 B$.

(vii) $\cos(A + B) \times \cos(A - B) = \cos^2 A - \sin^2 B$.

VI. Sum or difference of sine or cosine of angles into products

(i) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$.

(ii) $\sin C - \sin D = 2\cos\frac{C+D}{2} \cdot \sin\frac{C-D}{2}$.

(iii) $\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}$.

(iv) $\cos C - \cos D = 2\sin\frac{C+D}{2} \cdot \sin\frac{D-C}{2}$,

VII. Product of sines and cosines of angles into sum or difference of angles

(i) $2\sin A\cos B = \sin(A + B) + \sin(A - B)$.

(ii) $2\cos A\sin B = \sin(A + B) - \sin(A - B)$.

(iii) $2\cos A\cos B = \cos(A + B) + \cos(A - B)$.

(iv) $2\sin A\sin B = \cos(A - B) - \cos(A + B)$.

VIII. Trigonometrical ratios of multiple angles

(i) $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$.

(ii) $\cos^2 A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$= 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$.

(iii) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$.

(iv) $\sin^2 A = \dfrac{1-\cos 2A}{2}$; $\cos^2 A = \dfrac{1-\cos 2A}{2}$.

(v) $\tan A = \sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}} = \dfrac{1-\cos 2A}{\sin 2A}$.

(vi) $\sin 3A = 3 \sin A - 4 \sin^3 A$.

(vii) $\cos 3A = 4 \cos^3 A - 3 \cos A$.

(viii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$;

$\cot 3A = \dfrac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$.

IX. Trigonometrical ratios of submultiple angles

(i) $\sin A = 2 \sin = \dfrac{A}{2} \cos \dfrac{A}{2} = \dfrac{2\tan \frac{A}{2}}{1+\tan^2 \frac{A}{2}}$.

(ii) $\cos A = \cos^2 \dfrac{A}{2} - \sin^2 \dfrac{A}{2} = 2\cos^2 \dfrac{A}{2} - 1$

$= 1 - 2\sin^2 \dfrac{A}{2} = \dfrac{1-\tan^2 \frac{A}{2}}{1+\tan^2 \frac{A}{2}}$.

(iii) $\tan A = \dfrac{2\tan \frac{A}{2}}{1 \tan^2 \frac{A}{2}}$.

(iv) $\sin^2 \dfrac{A}{2} = \dfrac{1-\cos A}{2}$; $\cos^2 \dfrac{A}{2} = \dfrac{1+\cos A}{2}$

(v) $\tan \dfrac{A}{2} = \sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{1-\cos A}{\sin A}$.

(vi) $2 \sin \dfrac{A}{2} = \pm \sqrt{1+\sin A} \pm \sqrt{1-\sin A}$.

(vii) $2 \cos \dfrac{A}{2} = \pm \sqrt{1+\sin A} \mp \sqrt{1-\sin A}$.

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. If $\dfrac{1+\cos A}{1-\cos A} = \dfrac{m^2}{n^2}$, $\tan A =$

(*a*) $\pm \dfrac{2mn}{m^2+n^2}$ (*b*) $\pm \dfrac{2nm}{m^2+n^2}$

(*c*) $\dfrac{m^2+n^2}{m^2-n^2}$ (*d*) None of these

2. If $\sin 600^\circ \cos 30^\circ + \cos 120^\circ \sin 150^\circ = k$, then $k =$

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) None of these

3. If $\sin \theta + \cos \theta = a$ and $\sec \theta + \text{cosec}\, \theta = b$, then the value of $b(a^2 - 1)$ is equal to

(*a*) 2*a* (*b*) 3*a*

(*c*) 0 (*d*) 2*ab*

[Based on MAT, 2003]

4. If $7 \text{ cosec } \theta - 3 \cot \theta = 7$, then the value of $7 \cot \theta - 3 \text{ cosec } \theta$ is equal to:

(*a*) 5 (*b*) 3

(*c*) $\dfrac{7}{3}$ (*d*) $\dfrac{3}{7}$

[Based on MAT, 2003]

5. If $a \sec \theta + b \tan \theta = 1$ and $a^2 \sec^2\theta - b^2\tan^2\theta = 5$, then $a^2b^2 + 4a^2$ is equal to:

(*a*) $9b^2$ (*b*) $\dfrac{9}{a^2}$

(*c*) $\dfrac{-2}{b}$ (*d*) 9

[Based on MAT, 2003]

6. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then $\cos \theta - \sin \theta =$

(*a*) $\sqrt{2} \sin \theta$ (*b*) $2 \sin \theta$

(*c*) $-\sqrt{2} \sin \theta$ (*d*) None of these

7. For an acute angle θ, sin θ + cos θ takes the greatest value when 9 is

(*a*) 30 (*b*) 45

(*c*) 60 (*d*) 90

[Based on MAT, 2001]

8. If $b + c = 30$, then the value of $\cot \dfrac{B}{2} \cot \dfrac{C}{2} =$

(*a*) 3 (*b*) 2

(*c*) 9 (*d*) 7

[Based on MAT, 2005]

9. $\dfrac{3\pi}{5}$ radians is equal to:

(a) 108 (b) 54
(c) 100 (d) 81

[Based on MAT, 2001]

10. If sin A:cos A = 4:7, then the value of $\frac{7\sin A - 3\cos A}{7\sin A + 2\cos A}$ is:

(a) 3/14 (b) 3/2
(c) 1/3 (d) 1/6

[Based on MAT, 2001]

11. If cot θ + cos θ = p and cot θ – cos θ = 9, then $(p^2 - q^2)^2$ in terms of p and q is

(a) 16 pq (b) 8 pq
(c) 4 pq (d) 12 pq

12. Simplify: $\frac{\cos 60° + \sin 60°}{\cos 60° - \sin 60°}$.

(a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$
(c) $-(\sqrt{3} + 2)$ (d) 1

[Based on SCMHRD, 2002]

13. Find the value of $1/(1 + \tan^2 \theta) + 1/(1 + \cot^2 \theta)$.

(a) 1/2 (b) 2
(c) 1 (d) 1/4

[Based on IIFT, 2005]

14. If cos x/(1 – sin x) = 0.6, then (1 + cos x + sin x)/cos x equals:

(a) 0.6 (b) 5/8
(c) 8/5 (d) 1.2

[Based on IIFT, 2005]

15. If sin A = 3/5, tan B = 1/2 and $\pi/2 < A < \pi < B < 3\pi/2$, the value of 8 tan $A - \sqrt{5}$ see B =

(a) 7/2 (b) 5/2
(c) –5/2 (d) –7/2

16. If sec θ – tan θ = $\frac{a+1}{a-1}$, then cos θ =

(a) $\frac{a^2+1}{a^2-1}$ (b) $\frac{a^2-1}{a^2+1}$
(c) $\frac{2a}{a^2+1}$ (d) $\frac{2a}{a^2-1}$

17. If tan 20° = k, then $\frac{\tan 250° + \tan 340°}{\tan 200° - \tan 110°}$ =

(a) $\frac{1+k}{1-k}$ (b) $\frac{1-k}{1+k}$
(c) $\frac{1+k^2}{1-k^2}$ (d) $\frac{1-k^2}{1+k^2}$

18. The value of sin 780° sin 480° + cos 240° cos 300° =

(a) 1/2 (b) 1/4
(c) 1 (d) None of these

19. If tan θ + cot θ = 2, then sin θ =

(a) $\pm\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\pm\frac{1}{3}$ (d) None of these

20. If θ is in the first quadrant and tan θ = $\frac{3}{4}$, then $\frac{\tan(\pi/2-\theta)-\sin(\pi-\theta)}{\sin(3\pi/2+\theta)-\cot(2\pi-\theta)}$ =

(a) 8/11 (b) 6/11
(c) 11/8 (d) 11/6

21. If cot 20° = p, then $\frac{\tan 160° - \tan 110°}{1 + \tan 160° \tan 110°}$ =

(a) $\frac{p^2-1}{2p}$ (b) ——
(c) $\frac{1-p^2}{2p}$ (d) $\frac{2p}{1+p^2}$

22. The value of $\frac{\sin 150° - 5\cos 300° + 7\tan 225°}{\tan 135° + 3\sin 210°}$ is:

(a) 2 (b) 1
(c) –1 (d) –2

23. If cosec θ + cot θ = p, then cos θ =

(a) $\frac{p^2+1}{p^2-1}$ (b) $\frac{1+p^2}{1-p^2}$
(c) $\frac{p^2-1}{p^2+1}$ (d) $\frac{1-p^2}{1+p^2}$

24. If tan A + sin A = m and tan A – sin A = n, then $\frac{(m^2-n^2)^2}{mn}$ =

(a) 4 (b) 3
(c) 16 (d) 9

25. If cosec θ – sin θ = m and sec θ – cos θ = n then $(m^2n)^{2/3} + (mn^2)^{2/3}$ =

(a) –1 (b) 1
(c) 0 (d) None of these.

26. The value of cos 1° cos 2° cos 3° ... cos 179° =

(a) 1 (b) –1
(c) 0 (d) None of these.

27. Without using trigonometric tables, sin 48° sec 42° + cos 48° cosec 42° =

(a) 0 (b) 2
(c) 1 (d) None of these

28. $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ =$

(*a*) -1 (*b*) 1

(*c*) 1/2 (*d*) None of these.

29. $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + ... + \cos^2 90^\circ =$

(*a*) $8\frac{1}{2}$ (*a*) $6\frac{1}{2}$

(*c*) $7\frac{1}{2}$ (*d*) None of these.

30. The value of log tan 1° + log tan 2° + log tan 3° + ... + log tan 89° is equal to:

(*a*) 1 (*b*) 0

(*c*) 3 (*d*) None of these.

31. log sin 1° log sin 2° log sin 3° ... log sin 179° =

(*a*) 0 (*b*) 1

(*c*) $1/\sqrt{2}$ (*d*) None of these.

32. The value of cos 24° + cos 55° + cos 155° + cos 204° is

(*a*) 1 (*b*) -1

(*c*) 0 (*d*) None of these.

33. The value of cos 24° + cos 5° + cos 300° + cos 175° + cos 204° is:

(*a*) 0 (*b*) $-1/2$

(*c*) 1/2 (*d*) 1

34. $\sin^2\theta = \frac{(x+y)^2}{4xy}$ is possible only when:

(*a*) $x > 0, y > 0, x \neq y$ (*b*) $x > 0, y > 0, x = y$

(*c*) None of these. (*d*) $x > 0, y > 0, x \geq y$

35. If $7\sin^2\theta + 3\cos^2\theta = 4$, then $\tan\theta =$

(*a*) $\pm\frac{1}{3}$ (*b*) $\pm\frac{1}{2}$

(*c*) $\pm\frac{1}{\sqrt{3}}$ (*d*) $\pm\frac{1}{\sqrt{2}}$

36. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\frac{m^2-1}{n^2-1} =$

(*a*) $\cos^3\alpha$ (*b*) $\sin^3\alpha$

(*c*) $\sin^2\alpha$ (*d*) $\cos^2\alpha$

37. If $\sec A = a + (1/4a)$, then $\sec A + \tan A =$

(*a*) $2a$ or $1/2a$ (*b*) a or $1/a$

(*c*) $2a$ or $1/a$ (*d*) a or $1/2a$

38. The value of $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$ is:

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) None of these.

39. The value of tan 20° + tan 40° + tan 60° + ... + tan 180° is

(*a*) 1 (*b*) -1

(*c*) 0 (*d*) None of these

40. If $\cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\alpha = \frac{-3}{5}$, where θ does not lie in the third quadrant and α lies in the third quadrant, $\frac{2\tan\alpha + \sqrt{3}\tan\theta}{\cot^2\theta + \cos\alpha} =$

(*a*) $\frac{5}{22}$ (*b*) $\frac{3}{22}$

(*c*) $\frac{6}{25}$ (*d*) $\frac{8}{22}$

41. The value of cos 24° + cos 55° + cos 125° + cos 204° + cos 300° is

(*a*) 1/2 (*b*) $-1/2$

(*c*) 1 (*d*) -1

42. $\frac{\cot\theta - \operatorname{cosec}\theta + 1}{\cot\theta + \operatorname{cosec}\theta - 1}$ is equal to

(*a*) 1 (*b*) cot θ + cosec θ

(*c*) cosec θ – cot θ (*d*) None of these

43. If $90^\circ < \alpha < 180^\circ$, $\sin\alpha = \sqrt{3}/2$ and $180^\circ < \beta < 270^\circ$, $\sin\beta = -\sqrt{3}/2$, then $\frac{4\sin\alpha - 3\tan\beta}{\tan\alpha + \sin\beta} =$

(*a*) 2/3 (*b*) 0

(*c*) $-2/3$ (*d*) None of these

44. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} =$

(*a*) 2 sin θ (*b*) 2 cos θ

(*c*) $\frac{2}{|\cos\theta|}$ (*d*) $\frac{2}{|\sin\theta|}$

45. If $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \operatorname{cosec}\alpha + \cot\alpha$, then the quadrants in which α lies are:

(*a*) 1, 4 (*b*) 2, 3

(*c*) 1, 2 (*d*) 3, 4

46. If $\operatorname{cosec}\theta - \cot\theta = p$, then the value of $\operatorname{cosec}\theta =$

(*a*) $\frac{1}{2}\left(p + \frac{1}{p}\right)$ (*b*) $\frac{1}{2}\left(p - \frac{1}{p}\right)$

(*c*) $p + \frac{1}{p}$ (*d*) $p - \frac{1}{p}$

47. The value of tan 1° tan 2° ... tan 89° is

(*a*) -1 (*b*) 1

(*c*) 0 (*d*) None of these.

48. If $\operatorname{cosec}^2\theta = \frac{4xy}{(x+y)^2}$, then

(*a*) $x = -y$ (*b*) $x = 1/y$

(*c*) $x = y$ (*d*) None of these.

49. The value of $\dfrac{\sin 300^\circ \tan 240^\circ \sec(-420^\circ)}{\cot(-315^\circ)\cos(210^\circ)\operatorname{cosec}(-315^\circ)}$ is

(*a*) $\sqrt{3}$ (*b*) $\sqrt{2}$

(*c*) $\sqrt{6}$ (*d*) $\sqrt{8}$

50. The length of an arc which subtends an angle 18° at the centre of the circle of radius 6 cm is:

(*a*) $(\pi/5)$ cm (*b*) $(2\pi/5)$ cm

(*c*) $(3\pi/5)$ cm (*d*) None of these

51. If x is real and $x + \dfrac{1}{x} = 2\cos\theta$, then $\cos\theta =$

(*a*) $\pm\dfrac{1}{2}$ (*b*) $\pm\dfrac{1}{3}$

(*c*) ± 1 (*d*) None of these

52. Which of the following is correct?

(*a*) $\sin 1^\circ > \sin 1$ (*b*) $\sin 1^\circ = \sin 1$

(*c*) $\sin 1^\circ < \sin 1$ (*d*) $\sin 1^\circ = \left(\dfrac{\pi}{180}\right)\sin 1$

53. Which one of the following is true?

(*a*) $\tan 1 = 1$ (*b*) $\tan 1 = \tan 2$

(*c*) $\tan 1 < \tan 2$ (*d*) $\tan 1 > \tan 2$

54. The value of $\cos^2\theta + \sec^2\theta$ is always

(*a*) less than 1 (*b*) equal to 1

(*c*) lies between 1 and 2 (*d*) greater than 2

55. If $\sin\alpha = \dfrac{2pq}{p^2+q^2}$, then $\sec\alpha - \tan\alpha =$

(*a*) $\dfrac{p-q}{p+q}$ (*b*) $\dfrac{pq}{p^2+q^2}$

(*c*) $\dfrac{p+q}{p-q}$ (*d*) None of these

56. If $13\sin A = 12$, $\pi/2 < A < \pi$ and $3\sec B = 5$, $3\pi/2 < B < 2\pi$ then $5\tan A + 3\tan^2 B =$

(*a*) 20/3 (*b*) –20/3

(*c*) 22/3 (*d*) –22/3

57. The value of $\sin 105^\circ + \cos 105^\circ$ is

(*a*) $1/\sqrt{2}$ (*b*) $-1/\sqrt{2}$

(*c*) 0 (*d*) None of these

58. If $\tan A = 1/2$ and $\tan B = 1/3$, the value of $A + B$ is

(*a*) $\pi/3$ (*b*) $\pi/4$

(*c*) $\pi/2$ (*d*) None of these

59. If $\tan(A - B) = 7/24$ and $\tan A = 4/3$ where A and B are acute, then $A + B =$

(*a*) $\pi/2$ (*b*) $\pi/3$

(*c*) $\pi/4$ (*d*) None of these

60. The value of $(\tan 69^\circ + \tan 66^\circ)/(1 - \tan 69^\circ \tan 66^\circ)$ is

(*a*) 1 (*b*) 0

(*c*) 2 (*d*) –1

61. The value of $\sin^2 75^\circ - \sin^2 15^\circ$ is:

(*a*) $\sqrt{3}/2$ (*b*) $-\sqrt{3}/2$

(*c*) 1/2 (*d*) None of these.

62. If $\sin\alpha = 8/17$, $0 < \alpha < 90^\circ$ and $\tan\beta = 5/12$, $0 < \beta < 90^\circ$, then $\cos(\alpha - \beta)$ is:

(*a*) 210/221 (*b*) 171/221

(*c*) 220/221 (*d*) None of these

63. The value of $\sin^2\theta + \sin^2(\theta + 60^\circ) + \sin^2(\theta - 60^\circ) =$

(*a*) 1/2 (*b*) 0

(*c*) 3/2 (*d*) None of these.

64. If $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$, then $\alpha + \beta =$

(*a*) $\pi/3$ (*b*) $\pi/2$

(*c*) $\pi/4$ (*d*) None of these

65. The value of $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ} =$

(*a*) 1 (*b*) 4

(*c*) 3 (*d*) None of these

66. The value of $\sqrt{2 + \sqrt{2(1 + \cos 4A)}}$ is equal to:

(*a*) $\cos A$ (*b*) $\sin A$

(*c*) $2\cos A$ (*d*) $2\sin A$

67. If $\tan A = \dfrac{1 - \cos B}{\sin B}$, then $\tan 2A =$

(*a*) $\tan B$ (*b*) $\cot B$

(*c*) $2\tan B$ (*d*) $2\cot B$

68. The value of $\dfrac{\cos 2\theta}{1 - \sin 2\theta} =$

(*a*) $\tan(\pi/4 - \theta)$ (*b*) $\cot(\pi/4 - \theta)$

(*c*) $\tan(\pi/4 + \theta)$ (*d*) $\cot(\pi/4 + \theta)$

69. The value of $\dfrac{\tan 40^\circ + \tan 20^\circ}{1 - \cot 70^\circ \cot 50^\circ}$ is equal to

(*a*) $\sqrt{3}$ (*b*) $\sqrt{2}$

(*c*) $1/\sqrt{3}$ (*d*) $1/\sqrt{2}$

70. The value of $\sqrt{3}\operatorname{cosec} 20^\circ - \sec 20^\circ =$

(*a*) 2 (*b*) 4

(*c*) 3 (*d*) None of these

71. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) None of these

72. $\tan 5x - \tan 3x - \tan 2x$ is equal to:

(a) $\tan 2x \tan 3x \tan 5x$ (b) $\frac{\sin 5x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$

(c) 0 (d) None of these

73. If $\tan A = \frac{n}{n+1}$ and $\tan B = \frac{1}{2n+1}$, the value of $\tan(A+B) =$

(a) –1 (b) 1

(c) 2 (d) None of these

74. If $\sin A = 1/\sqrt{10}$, $\sin B = 1/\sqrt{5}$ where A and B are positive and acute, $A + B =$

(a) $\pi/2$ (b) $\pi/4$

(c) $\pi/3$ (d) None of these

75. $\frac{1+\sin\theta - \cos\theta}{1+\sin\theta + \cos\theta}$ is equal to:

(a) $\cot(\theta/2)$ (b) $\tan(\theta/2)$

(c) $\sec(\theta/2)$ (d) $\operatorname{cosec}\theta/2$

76. The value of $\tan 57^\circ - \tan 12^\circ - \tan 57^\circ \tan 12^\circ =$

(a) –1 (b) 1

(c) 0 (d) None of these.

77. The value of $\tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ =$

(a) $\sqrt{3}$ (b) –1

(c) $1/\sqrt{3}$ (d) 1

78. The value of $\tan 56^\circ - \tan 11^\circ - \tan 56^\circ \tan 11^\circ$ is

(a) –1 (b) 0

(c) 1 (d) None of these.

79. If $A + B = 45^\circ$ and $(\cot A - 1)(\cot + B - 1) = 4K$, then $K =$

(a) 1/4 (b) 1/8

(c) 1/2 (d) None of these.

Difficulty Level-2 (Based on Memory)

1. A, B and C are three angles such that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$. Which of the following statements is always correct?

(a) ABC is a triangle, i.e., $A + B + C = \pi$

(b) $A = B = C$, i.e., ABC is an equilateral triangle

(c) $A + B = C$, i.e., ABC is a right-angled triangle

(d) None of these

2. If α lies in the second quadrant, then

$$\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} - \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} =$$

(a) $\tan\alpha$ (b) $2\tan\alpha$

(c) $2\cot\alpha$ (d) $\cot\alpha$

3. If $x = a\operatorname{cosec}^n\theta$ and $y = b\cot^n\theta$, then by eliminating θ:

(a) $(x/a)^{2/n} + (y/b)^{2/n} = 1$

(b) $(x/a)^{2/n} - (y/b)^{2/n} = 1$

(c) $(x/a)^2 - (y/b)^2 = 1$

(d) $(x/a)^{1/n} - (y/b)^{1/n} = 1$

4. If $\cot^2\theta - (1+\sqrt{3})\cot\theta + \sqrt{3} = 0$, what is the value of θ?

(a) $\frac{\pi}{2}, \frac{\pi}{3}$ (b) $\frac{\pi}{4}, \frac{\pi}{6}$

(c) $\frac{\pi}{4}, \frac{\pi}{2}$ (d) $\frac{\pi}{2}, \pi$

[Based on FMS, 2009]

5. Given that θ is an angle between 180° and 270°, what is the value of θ, if it satisfies the equation $3\cos^2\theta - \sin^2\theta = 1$?

(a) 180° (b) 220°

(c) 225° (d) 240°

[Based on FMS, 2009]

6. If each α, β, γ is a positive acute angle such that $\sin(\alpha + \beta - \gamma) = 1/\sqrt{2}$, $\operatorname{cosec}(\beta + \gamma - \alpha) = 2/\sqrt{3}$ and $\tan(\gamma + \alpha - \beta) = 1/\sqrt{3}$. What are the values of α, β, γ?

(a) $\left(37\frac{1}{2}^\circ, 52\frac{1}{2}^\circ, 45^\circ\right)$ (b) (37°, 53°, 45°)

(c) $\left(45^\circ, 37\frac{1}{2}^\circ, 52\frac{1}{2}^\circ\right)$ (d) $\left(34\frac{1}{2}, 55\frac{1}{2}^\circ, 45^\circ\right)$

[Based on IIFT, 2010]

7. The minimum value of $3^{\sin x} + 3^{\cos x}$ is:

(a) 2 (b) $2(3^{-1/\sqrt{2}})$

(c) $3^{i-1/\sqrt{2}}$ (d) None of these

[Based on IIFT, 2010]

8. What is the maximum possible value of $(21\sin X + 72\cos X)$?

(a) 21 (b) 57

(c) 63 (d) 75

[Based on XAT, 2011]

9. If $\tan\theta = p/q$, then $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} =$

(*a*) $(p^2 + q^2) / (p^2 - q^2)$ (*b*) $(p^2 - q^2) / (p^2 + q^2)$
(*c*) $(p^2 + q^2) / (p^2 - q^2)$ (*d*) None of these

10. If A lies in the second quadrant and B lies in the third quadrant and $\cos A = -\sqrt{3}/2$, $\sin B = -3/5$, then $\frac{2\tan B + \sqrt{3}\tan A}{\cot^2 A + \cos B} =$

(*a*) 5/21 (*b*) 5/24
(*c*) 5/22 (*d*) None of these

11. If $f(x) = \cos^2 x + \sec^2 x$, its value always is

(*a*) $f(x) < 1$ (*b*) $f(x) = 1$
(*c*) $2 > f(x) > 1$ (*d*) $f(x) \geq 2$

12. If $\sin\theta = -7/25$ and θ is in the third quadrant, then $\frac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} =$

(*a*) 17/31
(*b*) 16/31
(*c*) 15/31
(*d*) None of these

13. $\tan 7\frac{1^\circ}{2}$ is equal to:

(*a*) $\frac{2\sqrt{2} - (1 + \sqrt{3})}{\sqrt{3} - 1}$ (*b*) $\frac{1 \quad \sqrt{3}}{1 \quad \sqrt{3}}$
(*c*) $\frac{1}{\sqrt{3}} + \sqrt{3}$ (*d*) $2\sqrt{2} + \sqrt{3}$

14. If $\frac{\cos 3A + \sin 3A}{\cos A - \sin A} = 1 - K \sin 2A$, the value of K is:

(*a*) –2 (*b*) 2
(*c*) 3 (*d*) 4

15. If $180^\circ < \theta < 270^\circ$, then the value of $\sqrt{4\sin^4\theta + \sin^2 2\theta} + 4\cos^2(\pi/4 - \theta/2)$ is:

(*a*) 2 (*b*) 4
(*c*) 3 (*d*) None of these

16. For all θ, the value of $\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} =$

(*a*) $\sec\theta - \tan\theta$ (*b*) $(\sec\theta + \tan\theta)^2$
(*c*) $(\sec\theta - \tan\theta)^2$ (*d*) $\sec\theta + \tan\theta$

Answer Keys

Difficulty Level-1

1. (*b*)	**2.** (*c*)	**3.** (*a*)	**4.** (*b*)	**5.** (*a*)	**6.** (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*a*)	**10.** (*d*)	**11.** (*a*)	**12.** (*c*)	**13.** (*c*)
14. (*c*)	**15.** (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*a*)	**19.** (*b*)	**20.** (*c*)	**21.** (*a*)	**22.** (*d*)	**23.** (*c*)	**24.** (*c*)	**25.** (*b*)	**26.** (*c*)
27. (*b*)	**28.** (*b*)	**29.** (*a*)	**30.** (*b*)	**31.** (*a*)	**32.** (*c*)	**33.** (*c*)	**34.** (*b*)	**35.** (*c*)	**36.** (*d*)	**37.** (*a*)	**38.** (*c*)	**39.** (*c*)
40. (*a*)	**41.** (*a*)	**42.** (*c*)	**43.** (*a*)	**44.** (*d*)	**45.** (*c*)	**46.** (*a*)	**47.** (*b*)	**48.** (*c*)	**49.** (*c*)	**50.** (*c*)	**51.** (*c*)	**52.** (*c*)
53. (*d*)	**54.** (*d*)	**55.** (*a*)	**56.** (*b*)	**57.** (*a*)	**58.** (*b*)	**59.** (*a*)	**60.** (*d*)	**61.** (*a*)	**62.** (*c*)	**63.** (*c*)	**64.** (*c*)	**65.** (*b*)
66. (*c*)	**67.** (*a*)	**68.** (*b*)	**69.** (*a*)	**70.** (*b*)	**71.** (*c*)	**72.** (*a*)	**73.** (*b*)	**74.** (*b*)	**75.** (*b*)	**76.** (*b*)	**77.** (*d*)	**78.** (*c*)
79. (*c*)												

Difficulty Level-2

1. (*a*)	**2.** (*b*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)	**7.** (*b*)	**8.** (*d*)	**9.** (*b*)	**10.** (*c*)	**11.** (*d*)	**12.** (*a*)	**13.** (*a*)
14. (*a*)	**15.** (*a*)	**16.** (*d*)										

Explanatory Answers

DIFFICULTY LEVEL-1

1. (*b*) $n^2 + n^2 \cos A = m^2 - m^2 \cos A$

$$\Rightarrow \quad \cos A = \frac{m^2 - n^2}{m^2 + n^2}$$

$$\sin^2 A = 1 - \cos^2 A$$

$$= 1 - \frac{(m^2 - n^2)^2}{(m^2 + n^2)^2} = \frac{4m^2n^2}{(m^2 + n^2)^2}$$

$$\Rightarrow \quad \sin A = \pm \frac{2mn}{m^2 + n^2}$$

$$\therefore \quad \tan A = \pm \frac{2mn}{m^2 - n^2}.$$

2. (*c*) $K = \sin 240^\circ \cos 30^\circ + \cos 120^\circ \sin 150^\circ$

$= -\sin 60^\circ \cos 30^\circ + (-\cos 60^\circ)(\sin 30^\circ)$

$= -\left(\sqrt{3}/2\right)\left(\sqrt{3}/2\right) + (-1/2)(1/2)$

$= -\frac{3}{4} - \frac{1}{4} = -1.$

3. (*a*) $b(a^2 - 1) = (\sec\theta + \csc\theta)$

$[(\sin\theta + \cos\theta)^2 - 1]$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta$$

$= 2(\sin\theta + \cos\theta) = 2a.$

4. (*b*) $7\csc\theta = 3\cot\theta + 7$

$\Rightarrow 49\csc^2\theta = 9\cot^2\theta + 49 + 42\cot\theta$

$\Rightarrow 49(1 + \cot^2\theta) = 9\cot^2\theta + 49 + 42\cot\theta$

$\Rightarrow \quad 40\cot^2\theta = 42\cot\theta$

$$\Rightarrow \quad \cot\theta = \frac{21}{20}$$

Since, $\csc^2\theta - \cot^2\theta = 1$, therefore

$\csc^2\theta = 1 + \cot^2\theta$

$$= 1 + \frac{441}{400} = \frac{841}{400}$$

$$\Rightarrow \quad \csc\theta = \frac{29}{20}$$

$$\therefore 7\cot\theta - 3\csc\theta = 7 \times \frac{21}{20} - 3 \times \frac{29}{20}$$

$$= \frac{147}{20} - \frac{87}{20} = 3.$$

5. (*a*) $\sec\theta = 1 - b\tan\theta$

$\Rightarrow \quad a^2\sec^2\theta = 1 + b^2\tan^2\theta - 2b\tan\theta$ (1)

Also, $a^2\sec^2\theta = 5 + b^2\tan^2\theta$ (2)

$\therefore$ (1) and (2)

$\Rightarrow 1 - 2b\tan\theta = 5 \Rightarrow b\tan\theta = -2$

$\therefore \quad a\sec\theta = 3 \Rightarrow a = 3\cos\theta$

$\Rightarrow \quad a^2 = 9\cos^2\theta$ (3)

$$b\tan\theta = -2 \Rightarrow b^2\frac{\sin^2\theta}{\cos^2\theta} = 4$$

$$\Rightarrow \quad \cos^2\theta = \frac{b^2}{b^2 + 4} \qquad (4)$$

From (3) and (4), we get

$$a^2 = \frac{9b^2}{b^2 + 4} \Rightarrow (b^2 + 4)a^2 = 9b^2.$$

6. (*a*) Given $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$= \left(\sqrt{2} - 1\right)\cos\theta$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2} - 1}\sin\theta$$

$$= \frac{(\sqrt{2} + 1)\sin\theta}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$$

$= \sqrt{2}\sin\theta + \sin\theta$

$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2}\sin\theta.$

7. (*b*) For $\theta = 30^\circ$, $\sin\theta + \cos\theta$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

For $\theta = 45^\circ$, $\sin\theta + \cos\theta$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

For $\theta = 60^\circ$, $\sin\theta + \cos\theta$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

For $\theta = 90^\circ$, $\sin\theta + \cos\theta = 1 + 0 = 1$.

8. (*b*) $\cot\frac{B}{2}\cot\frac{C}{2} = \frac{s}{s-a} = \frac{2s}{2s-2a}$

$$= \frac{a+b+c}{b+c-a} = \frac{4a}{2a} = 2.$$

9. (*a*) $\frac{3\pi}{5}$ radians $= \frac{3}{5}\times 180 = 108^\circ$.

10. (*d*) $\frac{\sin A}{\cos A} = \frac{4}{7}$.

Therefore,

$$\frac{7\sin A - 3\cos A}{7\sin A + 2\cos A} = \frac{7\frac{\sin A}{\cos A} - 3}{7\frac{\sin A}{\cos A} + 2}$$

$$= \frac{4-3}{4+2} = \frac{1}{6}.$$

11. (*a*) $p^2 - q^2 = 4\cos\theta\cot\theta = 4\cos^2\theta/\sin\theta$

$\Rightarrow$ $(p^2-q^2)^2 = 16\cos^4\theta/\sin^2\theta$

$pq = \cot^2\theta - \cos^2\theta$

$$= \cos^2\theta\left(\frac{1-\sin^2\theta}{\sin^2\theta}\right) = \frac{\cos^4\theta}{\sin^2\theta}$$

$\therefore$ $(p^2-q^2)^2 = 16\,pq$.

12. (*c*) $\frac{\cos 60^\circ + \sin 60^\circ}{\cos 60^\circ - \sin 60^\circ} = \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{\sqrt{3}}{2}}$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{1-3} = -(2+\sqrt{3}).$$

13. (*c*) $\frac{1}{1+\tan^2\theta} + \frac{1}{1+\cot^2\theta}$

$$= \frac{1}{\sec^2\theta} + \frac{1}{\text{cosec}^2\theta}$$

[$\because$ $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$]

$= \cos^2\theta + \sin^2\theta = 1$.

14. (*c*) $\frac{\cos x}{1-\sin x} = 0.6$

$\Rightarrow$ $\frac{1}{\sec x - \tan x} = 0.6$

$\because$ $\sec^2 x - \tan^2 x = 1$

$\therefore$ $\sec x - \tan x = \frac{1}{\sec x - \tan x}$

and, $(1 + \cos x + \sin x)/\cos x = \sec x + \tan x + 1$

$\therefore$ $\sec x + \tan x + 1 = 1 + \frac{1}{\sec x - \tan x}$

$$= 1 + \frac{6}{10} = \frac{8}{5}.$$

15. (*d*) $\sin A = 3/5 \Rightarrow \tan A = -3/4$ $\left[\because \frac{\pi}{2} < A < \pi\right]$

$\tan B = 1/2$, $\sec B = -\sqrt{5}/2$

$\therefore$ $8\tan A - \sqrt{5}\sec B = 8\left(\frac{-3}{4}\right) - \sqrt{5}\left(-\sqrt{5}/2\right)$

$$= -6 + \frac{5}{2} = -7/2.$$

16. (*b*) $\sec\theta - \tan\theta = \frac{a+1}{a-1}$

$\Rightarrow$ $\sec\theta + \tan\theta = \frac{a-1}{a+1}$

$\Rightarrow$ $\sec\theta + \tan\theta = \frac{a-1}{a+1}$

Adding, $2\sec\theta = \frac{(a+1)^2 + (a-1)^2}{a^2-1}$

$$= \frac{2(a^2+1)}{a^2-1}$$

$\Rightarrow$ $\sec\theta = \frac{a^2+1}{a^2-1}$

$\therefore$ $\cos\theta = \frac{a^2-1}{a^2+1}$.

17. (*d*) $\frac{\tan(270^\circ - 20^\circ) + \tan(360^\circ - 20^\circ)}{\tan(180^\circ + 20^\circ) - \tan(90^\circ + 20^\circ)}$

$$= \frac{\cot 20^\circ - \tan 20^\circ}{\tan 20^\circ + \cot 20^\circ} = \frac{1/k - k}{k + 1/k} = \_\_\_\_.$$

18. (*a*) $\sin 60^\circ \sin 120^\circ + \cos 240^\circ \cos 300^\circ$

$= \sin 60^\circ \sin 60^\circ - \cos 60^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

19. (*b*) $\tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = \pi/4$

$\therefore \quad \sin\theta = \sin\pi/4 = 1/\sqrt{2}$.

20. (*c*) $\dfrac{\cot\theta - \sin\theta}{-\cos\theta + \cot\theta}$

$= \dfrac{(4/3)-(3/5)}{-(4/5)+(4/3)}$ $\left[\begin{array}{l}\therefore \theta < 90^\circ \text{ and } \tan\theta = 3/4 \\ \sin\theta = 3/5,\ \cos\theta = 4/5\end{array}\right]$

$= 11/8.$

21. (*a*) $\dfrac{\tan 160^\circ - \tan 110^\circ}{1+\tan 160^\circ \tan 110^\circ}$

$= \dfrac{\tan(180^\circ-20^\circ) - \tan(90^\circ+20^\circ)}{1+\tan(180^\circ-20^\circ)\tan(90^\circ+20^\circ)}$

$= \dfrac{-\tan 20^\circ + \cot 20^\circ}{1+(-\tan 20^\circ)(-\cot 20^\circ)}$

$= \dfrac{-1/p+p}{1+1/p \cdot p}$

$= \dfrac{p^2-1}{2p}.$ $(\because \cot 20^\circ = p)$

22. (*d*) $\dfrac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ}$

$= \dfrac{\sin(180^\circ-30^\circ) - 5\cos(360^\circ-60^\circ) + 7\tan(180^\circ+45^\circ)}{\tan(180^\circ-45^\circ)+3\sin(180^\circ+30^\circ)}$

$= \dfrac{\sin 30^\circ - 5\cos 60^\circ + 7\tan 45^\circ}{-\tan 45^\circ - 3\sin 30^\circ}$

$= \dfrac{1/2 - 5/2 + 7}{-1-3/2} = -2.$

23. (*c*) Given $\operatorname{cosec}\theta + \cot\theta = p$

$\Rightarrow \operatorname{cosec}\theta - \cot\theta = 1/p$

$\Rightarrow \quad \operatorname{cosec}\theta = \dfrac{1}{2}\left(p+\dfrac{1}{p}\right) = \dfrac{p^2+1}{2p}$

$\cot\theta = \dfrac{p^2-1}{2p}$

$\therefore \quad \cos\theta = \dfrac{\cot\theta}{\operatorname{cosec}\theta} = \dfrac{p^2-1}{p^2+1}.$

24. (*c*) $\dfrac{(m^2-n^2)^2}{mn} = \dfrac{(4\tan A\sin A)^2}{\tan^2 A - \sin^2 A}$

$= \dfrac{16\sin^4 A}{\cos^2 A} \times \dfrac{\cos^2 A}{\sin^2 A(1-\cos^2 A)}$

$= \dfrac{16\sin^4 A}{\sin^2 A\sin^2 A} = 16.$

25. (*b*) $m = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

$n = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$(m^2n)^{2/3} + (mn^2)^{2/3} = \left(\dfrac{\cos^4\theta}{\sin^2\theta}\cdot\dfrac{\sin^2\theta}{\cos\theta}\right)^{2/3}$

$+ \left(\dfrac{\cos^2\theta}{\sin\theta}\cdot\dfrac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$

$= \cos^2\theta + \sin^2\theta$

$= 1.$

26. (*c*) $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 178^\circ \cos 179^\circ$

$= 0.$ $[\therefore \cos 90^\circ = 0]$

27. (*b*) $\sin(90^\circ - 42^\circ)\sec 42^\circ + \cos(90^\circ - 42^\circ)\operatorname{cosec} 42^\circ$

$= \cos 42^\circ \sec 42^\circ + \sin 42^\circ \operatorname{cosec} 42^\circ$

$= 1 + 1 = 2.$

28. (*b*) $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$

$= \tan 5^\circ \tan 25^\circ .1. \tan(90^\circ - 25^\circ)\tan(90^\circ - 50^\circ)$

$= \tan 5^\circ \tan 25^\circ .1. \cot 25^\circ \cot 5^\circ$

$= 1.$

29. (*a*) $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + \ldots + \cos^2 90^\circ$

$= (\cos^2 5^\circ + \cos^2 85^\circ) + (\cos^2 10^\circ + \cos^2 80^\circ)$

$+ \ldots + (\cos^2 40^\circ + \cos^2 50^\circ) + \cos^2 45^\circ + \cos^2 90^\circ$

$= (1 + 1 + \ldots 8 \text{ times}) + \dfrac{1}{2} + 0$

$= 8\dfrac{1}{2}.$

30. (*b*) $\log(\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 88^\circ \tan 89^\circ)$

$= \log(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ) \ldots \tan 45^\circ$

$= \log(\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ) \ldots \tan 45^\circ$

$= \log(1.\ 1.\ 1.\ \ldots\ 1) = \log 1 = 0.$

31. (*a*) $\log \sin 1^\circ \log \sin 2^\circ \ldots \log \sin 90^\circ \ldots \log \sin 179^\circ$

$= \log \sin 1^\circ \log \sin 2^\circ \ldots (0) \log \sin 91^\circ \ldots \log \sin 179^\circ$

$= 0.$

32. (*c*) cos 24° + cos 55° + cos 155° + cos 204°

$= \cos 24° + \cos 55° + \cos(180° - 25°) + \cos(180° + 24°)$

$= \cos 24° + \cos 55° - \cos 25° - \cos 24° = 0.$

33. (*c*) cos24° + cos5° + cos300° + cos 175° + cos 204°

$= \cos 24° + \cos 5° + \cos(360° - 60°) + \cos(180° - 5°) + \cos(180° + 24°)$

$= \cos 24° + \cos 5° + \cos 60° - \cos 5° - \cos 24°$

$= 1/2.$

34. (*b*) $\sin^2\theta \geq 1$

$\Rightarrow \dfrac{(x+y)^2}{4xy} \geq 1$

$\Rightarrow (x+y)^2 \geq 4xy$

$\Rightarrow (x+y)^2 - 4xy \geq 0$

$\Rightarrow (x-y)^2 \geq 0$

$(x-y)^2 > 0$ is true.

But $(x-y)^2 = 0$ is true only when $x = y$.

35. (*c*) Dividing by $\cos^2\theta$

$7\tan^2\theta + 3 = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = 1/3$

$\Rightarrow \tan\theta = \pm\dfrac{1}{\sqrt{3}}.$

36. (*d*) $m^2 - 1 = \dfrac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta}$

$n^2 - 1 = \dfrac{\tan^2\alpha - \tan^2\beta}{\tan^2\beta}$

$= \dfrac{\sin^2\alpha\ \cos^2\beta - \sin^2 13\ \cos^2\alpha}{\cos^2\alpha\ \cos^2\beta} \times \dfrac{\cos^2\alpha}{\sin^2\beta}$

$= \dfrac{\sin^2\alpha\ (1-\sin^2\beta) - \sin^2\beta(1-\sin^2\alpha)}{\sin^2\beta\cos^2\alpha}$

$= \dfrac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta\ \cos^2\alpha}$

$\therefore \dfrac{m^2-1}{n^2-1} = \dfrac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta} \times \dfrac{\cos^2\alpha\ \sin^2\beta}{\sin^2\alpha - \sin^2\beta}$

$= \cos^2\alpha.$

37. (*a*) $\tan^2 A = \sec^2 A - 1$

$= (a + 1/4a)^2 - 1$

$= (a - 1/4a)^2$

$\Rightarrow \tan A = \pm\left(a - \dfrac{1}{4a}\right)$

$\therefore \sec A + \tan A = a + \dfrac{1}{4a} + a - \dfrac{1}{4a}$

or, $a + \dfrac{1}{4a} - a + \dfrac{1}{4a} = 2a$ or $\dfrac{1}{2a}.$

38. (*c*) $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$

$= \dfrac{(\sin A + \cos A)(\sin^2 A + \cos^2 A + \sin A\ \cos A}{\sin A + \cos A}$

$+ \dfrac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \sin A\ \cos A}{\cos A - \sin A}$

$= 2(\sin^2 A + \cos^2 A) = 2.$

39. (*c*) tan 20° + tan 40° + tan 60° + ... + tan 180°

$= \tan 20° + \tan 40° + ... + \tan(180° - 40°) + \tan(180° - 20°) + \tan 180°$

$= \tan 20° + \tan 40° + ... + \tan(180° - 40°) + \tan(180° - 20°) + 0$

$= \tan 20° + \tan 40° + ... - \tan 40° - \tan 20°$

$= (\tan 20° - \tan 20°) + (\tan 40° - \tan 40°) + ...$

$= 0 + 0 + ... = 0.$

40. (*a*) $\cos\theta = -\sqrt{3}/2,\ 90° < \theta < 180°$

$\Rightarrow \tan\theta = -1/\sqrt{3}$

and, $\sin\alpha = \dfrac{-3}{5}$ and $180° < \alpha < 270°$

$\Rightarrow \tan\alpha = \dfrac{3}{4},\ \cos\alpha = \dfrac{-4}{5}$

$\therefore$ Given expression $= \dfrac{2(3/4) + \sqrt{3}(-1/\sqrt{3})}{(-\sqrt{3})^2 + (-4/5)}$

$= \dfrac{1/2}{11/5} = \dfrac{5}{22}.$

41. (*a*) Given expression

$= \cos 24° + \cos 55° - \cos 55° - \cos 24° + \cos 60°$

$= 1/2.$

42. (*c*) Given expression

$$= \frac{(\cot\theta - \operatorname{cosec}\theta) + (\operatorname{cosec}^2\theta - \cot\theta)}{\cot\theta + \operatorname{cosec}\theta - 1}$$

$$= \frac{(\operatorname{cosec}\theta - \cot\theta)[\operatorname{cosec}\theta + \cot\theta - 1]}{\cot\theta + \operatorname{cosec}\theta - 1}$$

$= \operatorname{cosec}\theta - \cot\theta.$

43. (*a*) Given $\sin\alpha = \sqrt{3}/2,\ 90° < \alpha < 180°$

$\Rightarrow \tan\alpha = -\sqrt{3}$

and, $\sin\beta = -\sqrt{3}/2,\ 180° < \beta < 270°$

$\Rightarrow \tan\beta = \sqrt{3}$

$$\text{Given expression} = \frac{4(\sqrt{3}/2) - 3(\sqrt{3})}{-\sqrt{3} - (\sqrt{3}/2)}$$

$$= \frac{-\sqrt{3}(2)}{-3\sqrt{3}} = \frac{2}{3}.$$

44. (*d*) $$\text{Given expression} = \frac{1+\cos\theta + 1 - \cos\theta}{\sqrt{1-\cos^2\theta}}$$

$$= \frac{2}{|\sin\theta|}.$$

45. (*c*)

46. (*a*) Given $\operatorname{cosec}\theta - \cot\theta = p$

$\Rightarrow \operatorname{cosec}\theta + \cot\theta = 1/p$

$\Rightarrow 2\operatorname{cosec}\theta = p + \dfrac{1}{p}$

$\therefore \operatorname{cosec}\theta = \dfrac{1}{2}\left(p + \dfrac{1}{p}\right).$

47. (*b*) Given expression

$= \tan 1° \tan 2° \ldots \tan 45° \ldots \tan 88° \tan 89°$

$= \tan 1° \tan 2° \ldots 1 \ldots \tan(90° - 2°) \tan(90° - 1°)$

$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots 1$

$= 1.\ 1.\ 1.\ \ldots 1 = 1.$

48. (*c*) We know that

$$\operatorname{cosec}^2\theta \ge 1 \Rightarrow \frac{4xy}{(x+y)^2} \ge 1$$

$\Rightarrow 4xy - (x+y)^2 \ge 0$

$\Rightarrow -(x-y)^2 \ge 0 \Rightarrow (x-y)^2 \le 0$

But $(x-y)^2$ cannot be negative

$\therefore (x-y)^2 = 0$ is possible only when $x = y$.

49. (*c*) Given expression

$$= \frac{\sin(360°-60°)\ \tan(270°-30°)\ \sec(360°+60°)}{[-\cot(270°+45°)]\ \cos(180°+30°)\ [-\operatorname{cosec}(270°+45°)]}$$

$$= \frac{(-\sin 60°)(\cot 30°)(\sec 60°)}{(\tan 45°)(-\cos 30°)(\sec 45°)}$$

$$= \frac{-(\sqrt{3}/2)(\sqrt{3})(2)}{(1)(-\sqrt{3}/2)(\sqrt{2})} = \frac{6}{\sqrt{6}} = \sqrt{6}.$$

50. (*c*) We have $l = r\,\theta$, where θ is in radians

Given $\theta = 18° = 18° \times \pi/180°$

$= \dfrac{\pi}{10}$ radians

$\therefore$ Length of the arc $= 6(\pi/10) = \dfrac{3\pi}{5}$.

51. (*c*) Given $x + \dfrac{1}{x} = 2\cos\theta$

$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$

Since x is real, discriminant ≥ 0

$\Rightarrow 4\cos^2\theta - 4 \ge 0$

$\Rightarrow \cos^2\theta \ge 1 \Rightarrow \cos\theta = \pm 1$

As $\cos\theta$ cannot be > 1 or < -1, $\cos\theta = \pm 1$.

52. (*c*) $1^c = \dfrac{180}{\pi} = \dfrac{180 \times 7}{22} = 57°$ (approx.)

$\Rightarrow \sin 1 = \sin 57°$

$\therefore \sin 45° < \sin 1 < \sin 60°$

$\Rightarrow \dfrac{1}{\sqrt{2}} < \sin 1 < \sqrt{3}/2$

$\Rightarrow 0.7 < \sin 1 < 0.8$

Also, $\sin 0° < \sin 1° < \sin 30°$

$\Rightarrow 0 < \sin 1° < 0.5$

$\therefore \sin 1° < \sin 1.$

53. (*d*) $1^c = 180/\pi = 57°$ (approx.)

$\Rightarrow \tan 1 = \tan 57° > 0$

Also, $\tan 2 = \tan 114° < 0$

$\therefore \tan 1 > \tan 2.$

54. (*d*) $\cos^2\theta + \sec^2\theta = (\cos\theta - \sec\theta)^2 + 2\cos\theta\sec\theta$

$= (\cos\theta - \sec\theta)^2 + 2$

As $(\cos\theta - \sec\theta)^2$ being a perfect square is always positive, $\cos^2\theta + \sec^2\theta$ is always greater than 2.

55. (*a*) Given $\sin\alpha = \dfrac{2pq}{p^2+q^2}$

$\Rightarrow \quad \sec\theta = \dfrac{p^2+q^2}{p^2-q^2}, \tan\alpha = \dfrac{2pq}{p^2-q^2}$

Given expression

$$\frac{p^2+q^2}{p^2-q^2} = -\frac{2pq}{p^2-q^2}$$

$$= \frac{(p-q)^2}{p^2-q^2} = \frac{p-q}{p+q}.$$

56. (*b*) Given $\sin A = 12/13$, A lies in the second quadrant and $\sec B = 5/3$, B lies in the fourth quadrant.

$\Rightarrow \quad \tan A = -12/5, \tan B = -4/3.$

Given expression $= 5\,(-12/5) + 3\,(16/9)$

$= -12 + 16/3 = -20/3.$

57. (*a*) Given expression

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

58. (*b*) $\tan(A+B) = \dfrac{1/2+1/3}{1-(1/2)(1/3)}$

$$= \frac{5/6}{5/6} = 1$$

$\Rightarrow \quad A + B = \pi/4.$

59. (*a*) $\dfrac{\tan A - \tan B}{1+\tan A\,\tan B} = \dfrac{7}{24}$

$\Rightarrow \quad \dfrac{4/3-\tan B}{1+\dfrac{4}{3}\tan B} = \dfrac{7}{24}$

$\Rightarrow \quad \tan B = 3/4$

$\cot(A+B) = \dfrac{\cot A\,\cot B - 1}{\cot B + \cot A}$

$$= \frac{(3/4)(4/3)-1}{(4/3)+(3/4)} = 0$$

$\Rightarrow \quad A + B = \pi/2.$

60. (*d*) Given expression $= \tan(69° + 66°)$

$= \tan(135°)$

$= \tan(180° - 45°)$

$= -\tan 45° = -1.$

61. (*a*) Given expression $= \sin(75° + 15°)\sin(75° - 15°)$

$= \sin 90° \sin 60°$

$= 1 \times \sqrt{3}/2 = \sqrt{3}/2.$

62. (*c*) $\cos\alpha = 15/17, \cos\beta = 12/13, \sin\beta = 5/13$

$\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$= \left(\frac{15}{17}\right)\left(\frac{12}{13}\right) + \left(\frac{8}{17}\right)\left(\frac{5}{13}\right)$$

$$= \frac{220}{221}.$$

63. (*c*) Given expression

$= \sin^2\theta + (\sin\theta\cos 60° + \cos\theta\sin 60°)^2$

$+ (\sin\theta\cos 60° - \cos\theta\sin 60°)^2$

$= \sin^2\theta + 2(\sin^2\theta\cos^2 60° + \cos^2\theta\sin^2 60°)$

$= \sin^2\theta + \dfrac{1}{2}\sin^2\theta + \dfrac{3}{2}\cos^2\theta$

$= \dfrac{3}{2}(\sin^2\theta + \cos^2\theta) = \dfrac{3}{2}.$

64. (*c*) $\tan(\alpha+\beta) = \dfrac{m/m+1 + 1/2m+1}{1-(m/m+1)(1/2m+1)}$

$$= \frac{2m^2+m+m+1}{2m^2+2m+m+1-m}$$

$$= \frac{2m^2+2m+1}{2m^2+2m+1} = 1$$

$\therefore \quad \alpha + \beta = \pi/4.$

65. (*b*) Given expression

$$= \frac{\cos 10° - \sqrt{3}\sin 10°}{\sin 10° \cos 10°}$$

$$= \frac{[2(1/2)\cos 10° - (\sqrt{3}/2)\sin 10°]}{(1/2)\sin 20°}$$

$$= 2(\sin 30°\cos 10° - \cos 30°\sin 10°) \times \frac{2}{\sin 20°}$$

$$= \frac{4\sin 20°}{\sin 20°} = 4.$$

66. (*c*) Given expression $= \sqrt{2+\sqrt{2(2\cos^2 2A)}}$

$= \sqrt{2+2\cos 2A}$

$= \sqrt{4\cos^2 A} = 2\cos A.$

67. (*a*) Given $\tan A = \dfrac{2\sin^2(B/2)}{2\sin(B/2)\cos(B/2)}$

$= \tan(B/2)$

$\Rightarrow \quad A = B/2 \Rightarrow 2A = B$

$\Rightarrow \quad \tan 2A = \tan B.$

68. (*b*) Given expression

$$= \frac{\sin\,[\pi/2-2\theta]}{1-\cos(\pi/2-2\theta)}$$

$$= \frac{2\sin(\pi/4-\theta)\,\cos\,(\pi/4-\theta)}{2\sin^2(\pi/4-\theta)}$$

$= \cot\,(\pi/4 - \theta)$.

69. (*a*) Given expression

$$= \frac{\tan 40^\circ + \tan 20^\circ}{1-\cot\,(90^\circ-20^\circ)\,\cot\,(90^\circ-40^\circ)}$$

$$= \frac{\tan 40^\circ + \tan 20^\circ}{1-\tan 20^\circ \tan 40^\circ}$$

$= \tan\,(40^\circ + 20^\circ) = \tan 60^\circ = \sqrt{3}$.

70. (*b*) Given expression

$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ\ \cos 20^\circ}$$

$$= \frac{2\left[\left(\frac{\sqrt{3}}{2}\right)\cos 20^\circ - \left(\frac{1}{2}\right)\sin 20^\circ\right]}{1/2\ \sin 40^\circ}$$

$$= \frac{2[\cos 30^\circ\ \cos 20^\circ - \sin 30^\circ\ \sin 20^\circ]}{1/2\ \sin 40^\circ}$$

$$= \frac{4\ \cos 50^\circ}{\sin 40^\circ} = \frac{4\ \sin 40^\circ}{\sin 40^\circ} = 4.$$

71. (*c*) Given expression

$= (\tan 81^\circ + \tan 9^\circ) - (\tan 63^\circ + \text{and } 27^\circ)$

$= (\cot 9^\circ + \tan 9^\circ) - (\cot 27^\circ + \tan 27^\circ)$

$$= \frac{1}{\sin 9^\circ\ \cos 9^\circ} - \frac{1}{\sin 27^\circ\ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{}{\sin 54^\circ} = \frac{2(4)}{\sqrt{5}-1} - \frac{2(4)}{\sqrt{5}+1}$$

$$= \frac{8(\sqrt{5}+1-\sqrt{5}+1)}{4} = 4.$$

72. (*a*) We have $5x = 3x + 2x$

$$\Rightarrow \quad \tan 5x = \frac{\tan 3x + \tan 2x}{1-\tan 3x\ \tan 2x}$$

$\Rightarrow \tan 5x - \tan 5x \tan 3x \tan 2x$

$= \tan 3x + \tan 2x$

$\Rightarrow \tan 5x - \tan 3x - \tan 2x$

$= \tan 5x \tan 3x \tan 2x.$

73. (*b*)

$$\tan\,(A+B) = \frac{\frac{n}{n+1}+\frac{1}{2n+1}}{1-\frac{n}{n+1}\times\frac{1}{2n+1}}$$

$$= \frac{2n^2+n+n+1}{2n^2+2n+n+1-n}$$

$$= \frac{2n^2+2n+1}{2n^2+2n+1} = 1.$$

74. (*b*) A, B are positive and each less than 90°

$\therefore \quad \cos A = 3/\sqrt{10}$, $\cos B = 2/\sqrt{5}$

$$\therefore\ \sin\,(A+B) = \left(\frac{1}{\sqrt{10}}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{5}}\right)$$

$$= \frac{5}{\sqrt{50}} = \frac{}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore \quad A + B = \pi/4.$

75. (*b*) Given expression

$$= \frac{2\ \sin^2\theta/2 + 2\ \sin\theta/2\ \cos\theta/2}{2\ \cos^2\theta/2 + 2\ \sin\theta/2\ \cos\theta/2}$$

$$= \frac{2\ \sin\theta/2\,[\sin\theta/2 + \cos\theta/2]}{2\ \cos\theta/2\,[\sin\theta/2 + \cos\theta/2]}$$

$= \tan\,\theta/2.$

76. (*b*) $\tan 45^\circ = \tan\,(57^\circ - 12^\circ)$

$$1 = \frac{\tan 57^\circ - \tan 12^\circ}{1+\tan 57^\circ \tan 12^\circ}$$

$\Rightarrow \tan 57^\circ - \tan 12^\circ = 1 + \tan 57^\circ \tan 12^\circ$

$\Rightarrow \tan 57^\circ - \tan 12^\circ - \tan 57^\circ \tan 12^\circ = 1.$

77. (*d*) $\tan 225^\circ = \tan\,(100^\circ + 125^\circ)$

$$1 = \frac{\tan\ 100^\circ + \tan\ 125^\circ}{1-\tan\ 100^\circ\ \tan\ 125^\circ}$$

$\Rightarrow \tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1.$

78. (*c*) $\tan 45^\circ = \tan\,(56^\circ - 11^\circ)$

$$1 = \frac{\tan 56^\circ - \tan 11^\circ}{1+\tan 56^\circ \tan 11^\circ}$$

$1 + \tan 56^\circ \tan 11^\circ = \tan 56^\circ - \tan 11^\circ$

$\therefore\ \tan 56^\circ - \tan 11^\circ - \tan 56^\circ \tan 11^\circ = 1.$

79. (*c*) $\cot\,(A+B) = \cot 45^\circ$

$$\Rightarrow \quad \frac{\cot\ A \cot\ B - 1}{\cot\ B + \cot\ A} = 1$$

$\Rightarrow \quad \cot A \cot B - \cot A - \cot B + 1 = 1 + 1$

$\Rightarrow \quad (\cot A - 1)(\cot B - 1) = 2$

$\Rightarrow \quad K = 1/2.$

Difficulty Level-2

1. (*a*) tan [(*A* + *B*) + *C*]

$$= \frac{\tan(A+B)+\tan C}{1-\tan(A+B)\tan C} = \frac{\frac{\tan A+\tan B}{1-\tan A\tan B}+\tan C}{1-\frac{\tan A+\tan B}{1-\tan A\cdot\tan B}\cdot\tan C}$$

$$= \frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{\text{Denominator}}$$

$= 0$

$[\because \tan A + \tan B + \tan C = \tan A \tan B \tan C]$

$\therefore$ $A + B + C = \pi$, i.e., *A*, *B*, *C* is a triangle.

2. (*b*) The given expression

$$= \frac{(1-\sin\alpha)-(1+\sin\alpha)}{\sqrt{1-\sin^2\alpha}} = \frac{-2\sin\alpha}{|\cos\alpha|}$$

$$= \frac{-2\sin\alpha}{-\cos\alpha} \qquad \left[\because \frac{\pi}{2} < \alpha < \pi\right]$$

$= 2\tan\alpha.$

3. (*b*) $\operatorname{cosec}\theta = (x/a)^{1/n}$, $\cot\theta = (y/b)^{1/n}$

But $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow$ $(x/a)^{2/n} - (y/b)^{2/n} = 1.$

4. (*b*) It is better to go through options.

When $\theta = \frac{\pi}{4}$

$$\cot^2\frac{\pi}{4} - (1+\sqrt{3})\cot\frac{\pi}{4} + \sqrt{3} = 0 \Rightarrow \theta = 0$$

When, $\theta = \frac{\pi}{6}$

$$\cot^2\frac{\pi}{6} - (1+\sqrt{3})\cot\frac{\pi}{6} + \sqrt{3} = 0 \Rightarrow \theta = 0$$

5. (*c*) $3\cos^2\theta - \sin^2\theta = 1$

$\Rightarrow$ $3(1-\sin^2\theta) - \sin^2\theta = 1$

$\Rightarrow$ $3 - 3\sin^2\theta - \sin^2\theta = 1$

$\Rightarrow$ $3 - 4\sin^2\theta = 1$

$\Rightarrow$ $-4\sin^2\theta = -2$

$\Rightarrow$ $\sin^2\theta = \frac{1}{2}$

$\sin\theta = -\frac{1}{\sqrt{2}}$

(Taking only negative value)

$\therefore$ $\theta = 225°$

6. (*a*) $\sin(\alpha+\beta-\gamma) = \frac{1}{\sqrt{2}}$

$\sin(\alpha+\beta-\gamma) = \sin 45°$

$\therefore$ $\alpha+\beta-\gamma = 45°$ (1)

Similarly, $\beta+\gamma-\alpha = 60°$ (2)

$\gamma+\alpha-\beta = 30°$ (3)

On solving Eqs. (1), (2) and (3),

$$\alpha = 37\frac{1}{2}°,\ \beta = 52\frac{1}{2}°,\ \gamma = 45°$$

7. (*b*) We know that, AM(*a*, *b*) ≥ GM(*a*, *b*)

$$\text{AM}(3^{\sin x}, 3^{\cos x}) \geq \text{GM}(3^{\sin x}, 3^{\cos x})$$

$$\frac{3^{\sin x}+3^{\cos x}}{2} \geq \sqrt{3^{\sin x}\times 3^{\cos x}}$$

$$3^{\sin x}+3^{\cos x} \geq 2\sqrt{3^{\sin x+\cos x}}$$

$$3^{\sin x}+3^{\cos x} \geq 2\sqrt{3^{-\sqrt{2}}}$$

(The minimum value of $\sin x + \cos x = -\sqrt{2}$)

So, $3^{\sin x}+3^{\cos x} \geq 2\times 3^{-\frac{\sqrt{2}}{2}}$

$$3^{\sin x}+3^{\cos x} \geq 2\times 3^{-\frac{1}{\sqrt{2}}}$$

So, the minimum value of the given function is $2\times 3^{-\frac{1}{\sqrt{2}}}$.

8. (*d*) The maximum value of

$$E = a\cos x + b\sin x = \sqrt{a^2+b^2}$$

$$a = 72 = 3(24),\ b = 21 = 3(7)$$

$$\sqrt{a^2+b^2} = 3\times 25 = 75$$

9. (*b*)

$$\frac{\sin\theta}{\cos\theta} = \frac{p}{q} \Rightarrow \frac{p\sin\theta}{q\cos\theta} = \frac{p^2}{q^2}$$

$$\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \frac{p^2-q^2}{p^2+q^2}.$$

10. (*c*) A lies in second quadrant, $\tan A = -1/\sqrt{3}$

B lies in third quadrant, $\tan B = 3/4$, $\cos B = -4/5$

$$\therefore \frac{2\tan B + \sqrt{3}\tan A}{\cot^2 A + \cos B} = \frac{2(3/4) + \sqrt{3}(-1/\sqrt{3})}{(-\sqrt{3})^2 + (-4/5)}$$

$$= \frac{3/2 - 1}{3 - 4/5} = \frac{5}{22}.$$

11. (*d*) $f(x) = \cos^2 x + \sec^2 x = (\cos x - \sec x)^2 + 2$

$\Rightarrow$ $f(x) \geq 2.$

12. (*a*) Given $180° < \theta < 270° \Rightarrow \tan\theta = 7/24$

$$\therefore \frac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} = \frac{7(24/7) - 24(7/24)}{7(24/7) + 24(7/24)}$$

$$= \frac{24-7}{7+24} = \frac{17}{31}.$$

13. (*a*) We have $\tan A = \dfrac{1-\cos 2A}{\sin 2A}$. Put $A = 7\dfrac{1}{2}^{\circ}$

$$\tan 7\frac{1}{2}^{\circ} = \frac{1-\cos 15°}{\sin 15°} = \frac{1 - \dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{2\sqrt{2} - (\sqrt{3}+1)}{\sqrt{3}-1}.$$

14. (*a*) $\dfrac{4\cos^3 A - 3\cos A + 3\sin A - 4\sin^3 A}{\cos A - \sin A}$

$= 1 - K\sin 2A$

$\Rightarrow 1 + 2\sin 2A = 1 - K\sin 2A$

$\Rightarrow K = -2.$

15. (*a*) Given expression

$= \sqrt{4\sin^4\theta + 4\sin^2\theta\cos^2\theta} + 2[1 + \cos(\pi/2 - \theta)]$

$= \sqrt{4\sin^2\theta(\sin^2\theta + \cos^2\theta)} + 2(1 + \sin\theta)$

$= 2|\sin\theta| + 2 + 2\sin\theta$

$= -2\sin\theta + 2 + 2\sin\theta$

$= 2$ (since $180° < \theta < 270° \Rightarrow |\sin\theta| = -\sin\theta$)

16. (*d*) Given expression

$$= \sqrt{\frac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}} = \frac{1+\sin\theta}{\cos\theta}$$

$= \sec\theta + \tan\theta.$