Ratio and Proportion

Q1. Using Componendo and Dividendo solve for x: [2023]

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$$

Solution: x = 1

Step-by-step Explanation:

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$$

$$u\sin g\ componendo\ and\ dividendo,$$

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}+\sqrt{2x+2}-\sqrt{2x-1}}{\sqrt{2x+2}+\sqrt{2x-1}-\sqrt{2x+2}+\sqrt{2x-1}} = \frac{3+1}{3-1}$$

$$\frac{2\sqrt{2x+2}}{2\sqrt{2x-1}} = \frac{4}{2}$$

$$\frac{\sqrt{2x+2}}{\sqrt{2x-1}} = 2$$

$$squaring\ both\ sides,\ we\ get$$

$$\frac{2x+2}{2x-1} = 4$$

$$8x-4 = 2x+2$$

$$8x-2x = 2+4$$

$$6x = 6$$

$$x = 1$$

Q2. What number must be added to each of the numbers 4, 6, 8, 11 in order to get the four numbers in proportion? [2023]

Solution: 4

Step-by-step Explanation:

$$\text{Let } x \text{ be added to each of the numbers.}$$
$$\text{According to the problem,}$$
$$\frac{4+x}{6+x} = \frac{8+x}{11+x}$$
$$(4+x)(11+x) = (8+x)(6+x)$$
$$44 + 11x + 4x + x^2 = 48 + 6x + 8x + x^2$$
$$15x + 44 = 14x + 48$$
$$15x - 14x = 48 - 44$$
$$x = 4$$

Q3. The mean proportional between 4 and 9 is

(a) 4

(b) 6

(c) 9

(d) 36 [2023]

Solution: (b)

Step-by-step Explanation:

Let the mean proportional be x.

Therefore, $4/x = x/9$

$x^2 = 36$

x = 6

Q4. If x, y, z are in continued proportion then $(y^2 + z^2) : (x^2 + y^2)$ is equal to: [2]

(a) z: x

(b) x : z

(c) zx

(d) (y + z):(x + y) [2021 Semester-1]

Solution: (a)

Step-by-step Explanation:

$$\text{Given, } x, y, z \text{ are in continued proportion.}$$

$$\text{Therefore, } y^2 = zx$$

$$\text{Now, } \frac{y^2 + z^2}{x^2 + y^2}$$

$$= \frac{zx + z^2}{x^2 + zx}$$

$$= \frac{z(x + z)}{x(x + z)}$$

$$= \frac{z}{x}$$

Q5. If a, b, c, and d are proportional then (a+b)/(a-b) is equal to:
(a) c/d

(b) (c-d)/(c+d)

(c) d/c

(d) (c+d)/(c-d) [2021 Semester-1]

Solution: (d)

Step-by-step Explanation:

$$Given,\ \ a, b, c, d\ are\ in\ proportion.$$

$$Therefore,\ \frac{a}{b} = \frac{c}{d} = k(say)$$

$$so,\ a = bk\ and\ c = dk$$

$$Now,\ \frac{a+b}{a-b}$$

$$= \frac{b\left(\frac{a}{b}+1\right)}{b\left(\frac{a}{b}-1\right)}$$

$$= \frac{\left(\frac{c}{d}+1\right)}{\left(\frac{c}{d}-1\right)}$$

$$= \frac{\left(\frac{c+d}{d}\right)}{\left(\frac{c-d}{d}\right)}$$

$$= \frac{c+d}{c-d}$$

Q6. If x, 5.4, 5, 9 are in proportion then x is: [1]

(a) 3

(b) 9.72

(c) 25

(d) 25/3 [2021 Semester-1]

Solution: (a)

Step-by-step Explanation:

x/5.4 = 5/9

9x = 27

x = 3

$$Q7.\ If\ x=\frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}},\ prove\ that\ x^2-4ax+1=0 \qquad [2020]$$

Step-by-step Explanation:

$$x=\frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$$

$$u\sin g\ componendo\ and\ dividendo,$$

$$\frac{x+1}{x-1}=\frac{\sqrt{2a+1}+\sqrt{2a-1}+\sqrt{2a+1}-\sqrt{2a-1}}{\sqrt{2a+1}+\sqrt{2a-1}-\sqrt{2a+1}+\sqrt{2a-1}}$$

$$\frac{x+1}{x-1}=\frac{2\sqrt{2a+1}}{2\sqrt{2a-1}}$$

$$\frac{x+1}{x-1}=\frac{\sqrt{2a+1}}{\sqrt{2a-1}}$$

$$squaring\ both\ sides$$

$$\frac{(x+1)^2}{(x-1)^2}=\frac{2a+1}{2a-1}$$

$$\frac{x^2+2x+1}{x^2-2x+1}=\frac{2a+1}{2a-1}$$

$$u\sin g\ componendo\ and\ dividendo,$$
$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1}=\frac{2a+1+2a-1}{2a+1-2a+1}$$
$$\frac{2x^2+2}{4x}=\frac{4a}{2}$$
$$\frac{2(x^2+1)}{4x}=2a$$
$$x^2+1=4ax$$
$$x^2-4ax+1=0$$
$$Proved.$$

Q8. Using properties of proportion find $x : y$, given: [2020]

$$\frac{x^2+2x}{2x+4}=\frac{y^2+3y}{3y+9}$$

Solution: x : y= 2 : 3

Step-by-step Explanation:

$$\frac{x^2+2x}{2x+4}=\frac{y^2+3y}{3y+9}$$
$$U\sin g\ componendo\ and\ dividendo,$$
$$\frac{x^2+2x+2x+4}{x^2+2x-2x-4}=\frac{y^2+3y+3y+9}{y^2+3y-3y-9}$$
$$\frac{x^2+4x+4}{x^2-4}=\frac{y^2+6y+9}{y^2-9}$$
$$\frac{(x+2)^2}{(x+2)(x-2)}=\frac{(y+3)^2}{(y+3)(y-3)}$$
$$\frac{x+2}{x-2}=\frac{y+3}{y-3}$$
$$u\sin g\ componendo\ and\ dividendo,$$
$$\frac{x+2+x-2}{x+2-x+2}=\frac{y+3+y-3}{y+3-y+3}$$

$$\frac{2x}{4} = \frac{2y}{6}$$

$$By\ alternendo,$$

$$\frac{2x}{2y} = \frac{4}{6}$$

$$\frac{x}{y} = \frac{2}{3}$$

Q9. The following numbers, K + 3, K + 2, 3K – 7 and 2K – 3 are in proportion. Find K. [2019]

Solution: k= -1 or 5

Step-by-step Explanation:

$$\frac{k+3}{k+2} = \frac{3k-7}{2k-3}$$
$$\Rightarrow (k+3)(2k-3) = (3k-7)(k+2)$$
$$\Rightarrow 2k^2 + 6k - 3k - 9 = 3k^2 - 7k + 6k - 14$$
$$\Rightarrow -k^2 + 4k + 5 = 0$$
$$\Rightarrow k^2 - 4k - 5 = 0$$
$$\Rightarrow k^2 - 5k + k - 5 = 0$$
$$\Rightarrow k(k-5) + 1(k-5) = 0$$
$$\Rightarrow (k+1)(k-5) = 0$$
$$Either\ (k+1) = 0\ or\ (k-5) = 0$$
$$\Rightarrow k = -1\ or\ k = 5$$

Q10. Using properties of proportion solve for x, given [2019]

$$\frac{\sqrt{5x} + \sqrt{2x-6}}{\sqrt{5x} - \sqrt{2x-6}} = 4$$

Solution: x = 30

Step-by-step Explanation:

$$\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}}=4$$

$$\textit{By componendo and dividendo,}$$

$$\frac{\sqrt{5x}+\sqrt{2x-6}+\sqrt{5x}-\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}-\sqrt{5x}+\sqrt{2x-6}}=\frac{4+1}{4-1}$$

$$\Rightarrow \frac{2\sqrt{5x}}{2\sqrt{2x-6}}=\frac{5}{3}$$

$$\Rightarrow \frac{\sqrt{5x}}{\sqrt{2x-6}}=\frac{5}{3}$$

$$\textit{squaring both sides,}$$

$$\frac{5x}{2x-6}=\frac{25}{9}$$

$$\Rightarrow 50x-150=45x$$

$$\Rightarrow 5x=150$$

$$\Rightarrow x=30$$

Q11. Using properties of proportion, solve for x. Given that x is positive : [3] [2018]

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

Solution: x = 5/8

Step-by-step Explanation:

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

by componendo and dividendo,

$$\frac{2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1}}{2x+\sqrt{4x^2-1}-2x+\sqrt{4x^2-1}}=\frac{4+1}{4-1}$$

$$\frac{4x}{2\sqrt{4x^2-1}}=\frac{5}{3}$$

$$\frac{2x}{\sqrt{4x^2-1}}=\frac{5}{3}$$

squaring both sides,

$$\frac{4x^2}{4x^2-1}=\frac{25}{9}$$

$$100x^2-25=36x^2$$

$$64x^2=25$$

$$x^2=\frac{25}{64}$$

$$x=\pm\sqrt{\frac{25}{64}}$$

$$x=\frac{5}{8}$$

Q12. If b is the mean proportion between a and c, show that: [3] [2017]

$$\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4}=\frac{a^2}{c^2}$$

Step-by-step Explanation:

$Given,\ b\ is\ the\ mean\ proportional\ between\ a\ and\ c.$

$$Therefore,\ b^2 = ac$$

$$Now,$$

$$L.H.S. = \frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$$

$$= \frac{a^4 + a^2.ac + (ac)^2}{(ac)^2 + ac.c^2 + c^4}$$

$$= \frac{a^4 + a^3c + a^2c^2}{a^2c^2 + ac^3 + c^4}$$

$$= \frac{a^2(a^2 + ac + c^2)}{c^2(a^2 + ac + c^2)}$$

$$= \frac{a^2}{c^2}$$

$$= R.H.S.$$

$proved.$

Q13. If (7m+2n)/(7m-2n)=5/3 , use properties of proportion to find :

(i) m : n

(ii) $(m^2 + n^2)/(m^2 - n^2)$[3] [2017]

Solution: (i) 8:7 (ii) 113/15

Step-by-step Explanation:

$$(i)\ \frac{7m+2n}{7m-2n}=\frac{5}{3}$$

$$by\ componendo\ and\ dividendo,$$

$$\frac{7m+2n+7m-2n}{7m+2n-7m+2n}=\frac{5+3}{5-3}$$

$$\Rightarrow \frac{14m}{4n}=\frac{8}{2}$$

$$\Rightarrow \frac{7m}{2n}=4$$

$$\Rightarrow 7m=8n$$

$$\Rightarrow \frac{m}{n}=\frac{8}{7}$$

$$(ii)\ \frac{m}{n}=\frac{8}{7}$$

$$squaring\ both\ sides,$$

$$\Rightarrow \frac{m^2}{n^2}=\frac{64}{49}$$

$$by\ componendo\ and\ dividendo,$$

$$\Rightarrow \frac{m^2+n^2}{m-n^2}=\frac{64+49}{64-49}$$

$$\Rightarrow \frac{m^2+n^2}{m-n^2}=\frac{113}{15}$$

Q14. If (3a + 2b) : (5a + 3b) = 18 : 29. Find a : b. [3] [2016]

Solution: a : b= 4 : 3

Step-by-step Explanation:

$$\frac{3a+2b}{5a+3b}=\frac{18}{29}$$
$$\Rightarrow 90a+54b=87a+58b$$
$$\Rightarrow 90a-87a=58b-54b$$
$$\Rightarrow 3a=4b$$
$$\Rightarrow \frac{a}{b}=\frac{4}{3}$$

Q15. If x/a = y/b = z/c show that $x^3/a^3 + y^3/b^3 + z/c^3 =$ 3xyz/abc [3] [2016]

Step-by-step Explanation:

$$\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k\ (say)$$
$$x=ak,\ y=bk,\ z=ck$$
$$L.H.S.$$
$$=\frac{x^3}{a^3}+\frac{y^3}{b^3}+\frac{z^3}{c^3}$$
$$=\frac{a^3k^3}{a^3}+\frac{b^3k^3}{b^3}+\frac{c^3k^3}{c^3}$$
$$=k^3+k^3+k^3$$
$$=3k^3$$
$$R.H.S.$$
$$=\frac{3xyz}{abc}$$
$$=\frac{3.ak.bk.ck}{abc}$$
$$=3k^3$$
$$L.H.S.=R.H.S.$$
$$Proved.$$

Q16. If a, b, c are in continued proportion, prove that (a + b + c) (a – b + c) = $a^2+b^2+c^2$. [2015]

Step-by-step Explanation:

$$a,\ b,\ c \ are\ in\ continued\ proportion.$$

$$\frac{a}{b} = \frac{b}{c} = k\ (say)$$

$$a = bk = ck.k = ck^2$$

$$b = ck$$

$$L.H.S.$$

$$(a+b+c)(a-b+c)$$

$$= (ck^2+ck+c)(ck^2-ck+c)$$

$$= c(k^2+k+1)c(k^2-k+1)$$

$$= c^2[(k^2+1)^2 - k^2]$$

$$= c^2\ (k^4+2k^2+1-k^2)$$

$$= c^2(k^4+k^2+1)$$

$$R.H.S.$$

$$a^2+b^2+c^2$$

$$= c^2k^4+c^2k^2+c^2$$

$$= c^2(k^4+k^2+1)$$

$$L.H.S. = R.H.S.$$

$$Proved.$$

Q17. Given $(x^3+12x)/(6x^2+8)=(y^3+27y)/(9y^2+27)$. Using Componendo and Dividendo find x : y. [2015]

Solution: x : y= 2 : 3

Step-by-step Explanation:

$$\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$$

$$\textit{By componendo and dividendo,}$$

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8}=\frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\Rightarrow\frac{(x+2)^3}{(x-2)^3}=\frac{(y+3)^3}{(y-3)^3}$$

$$\Rightarrow\frac{x+2}{x-2}=\frac{y+3}{y-3}$$

$$\textit{by componendo and dividendo,}$$

$$\Rightarrow\frac{x+2+x-2}{x+2-x+2}=\frac{y+3+y-3}{y+3-y+3}$$

$$\Rightarrow\frac{2x}{4}=\frac{2y}{6}$$

$$\Rightarrow\frac{x}{2}=\frac{y}{3}$$

$$\textit{by alternendo}$$

$$\Rightarrow\frac{x}{y}=\frac{2}{3}$$

Q18. If $(x^2+y^2)/(x^2-y^2)=17/8$,then find the value of: [3]

(i) x : y

(ii) $(x^3 + y^3)/(x^3 - y^3)$ [2014]

Solution: (i) 5 : 3 (ii) 76/49

Step-by-step Explanation:

$$(i)\ \frac{x^2+y^2}{x^2-y^2}=\frac{17}{8}$$

by componendo and dividendo,

$$\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2}=\frac{17+8}{17-8}$$

$$\Rightarrow \frac{2x^2}{2y^2}=\frac{25}{9}$$

$$\Rightarrow \frac{x^2}{y^2}=\frac{25}{9}$$

$$\Rightarrow \frac{x}{y}=\frac{5}{3}$$

$$(ii)\ \frac{x}{y}=\frac{5}{3}$$

cubing both sides,

$$\frac{x^3}{y^3}=\frac{125}{27}$$

by componendo and dividendo

$$\Rightarrow \frac{x^3+y^3}{x^3-y^3}=\frac{125+27}{125-27}$$

$$\Rightarrow \frac{x^3+y^3}{x^3-y^3}=\frac{152}{98}$$

$$\Rightarrow \frac{x^3+y^3}{x^3-y^3}=\frac{76}{49}$$

Q19. If (x – 9) : (3x + 6) is the duplicate ratio of 4 : 9, find the value of x. [3] [2014]

Solution: x= 25

Step-by-step Explanation:

$$\frac{x-9}{3x+6} = \left(\frac{4}{9}\right)^2$$
$$\Rightarrow \frac{x-9}{3x+6} = \frac{16}{81}$$
$$\Rightarrow 81x - 729 = 48x + 96$$
$$\Rightarrow 33x = 825$$
$$\Rightarrow x = \frac{825}{33}$$
$$\Rightarrow x = 25$$

Q20. What number must be added to each of the number 6, 15, 20 and 43 to make them proportional? [3] [2013]

Solution: 3

Step-by-step Explanation:

$$let\ x\ be\ added\ to\ the\ numbers.$$
$$Therefore,$$
$$\frac{6+x}{15+x} = \frac{20+x}{43+x}$$
$$\Rightarrow 258 + 43x + 6x + x^2 = 300 + 15x + 20x + x^2$$
$$\Rightarrow 49x - 35x = 300 - 258$$
$$\Rightarrow 14x = 42$$
$$\Rightarrow x = 3$$

Q21. Using the properties of proportion, solve for x, given [3]

$(x^4+1)/(2x^2)=17/8$ [2013]

Solution: x= ± 2

Step-by-step Explanation:

$$\frac{x^4+1}{2x^2}=\frac{17}{8}$$

by componendo and dividendo,

$$\Rightarrow \frac{x^4+1+2x^2}{x^4+1-2x^2}=\frac{17+8}{17-8}$$

$$\Rightarrow \frac{(x^2+1)^2}{(x^2-1)^2}=\frac{25}{9}$$

$$\Rightarrow \frac{x^2+1}{x^2-1}=\frac{5}{3}$$

$$\Rightarrow 5x^2-5=3x^2+3$$

$$\Rightarrow 2x^2=8$$

$$\Rightarrow x^2=4$$

$$\Rightarrow x=\pm 2$$

Q22. The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves Rs.80 every month, find their monthly pocket money. [2012]

Solution: Ravi- Rs 200, Sanjeev- Rs 280

Step-by-step Explanation:

$$\text{Let monthly pocket money of Ravi and Sanjeev be } 5x \text{ and } 7x \text{ respectively.}$$
$$\text{They save Rs. } 80 \text{ per month.}$$
$$\text{Therefore, by the problem}$$
$$\frac{5x-80}{7x-80} = \frac{3}{5}$$
$$\Rightarrow 25x - 400 = 21x - 240$$
$$\Rightarrow 4x = 160$$
$$\Rightarrow x = 40$$
$$\text{Therefore, Ravi's pocket money} = 5 \times 40 = Rs.200$$
$$\text{and Sanjeev's pocket money} = 7 \times 40 = Rs.\ 280$$

$Q23.\ If\ x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, using properties of proportion show that $x^2 - 2ax + 1 = 0$ [2012]

Step-by-step Explanation:

$$x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$
$$\text{By componendo and dividendo,}$$
$$\frac{x+1}{x-1} = \frac{\sqrt{a+1}+\sqrt{a-1}+\sqrt{a+1}-\sqrt{a-1}}{\sqrt{a+1}+\sqrt{a-1}-\sqrt{a+1}+\sqrt{a-1}}$$
$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$
$$\text{squaring both sides,}$$
$$\frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$$
$$\Rightarrow \frac{x^2+2x+1}{x^2-2x+1} = \frac{a+1}{a-1}$$
$$\text{By componendo and dividendo,}$$
$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{a+1+a-1}{a+1-a+1}$$
$$\Rightarrow \frac{2x^2+2}{4x} = \frac{2a}{2}$$
$$\Rightarrow \frac{2\left(x^2+1\right)}{4x} = a$$
$$\Rightarrow x^2 + 1 = 2ax$$
$$\Rightarrow x^2 - 2ax + 1 = 0$$
$$\text{Proved.}$$

Q24. Using componendo and dividendo, find the value of x.
[3] [2011]

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=9$$

Solution: x= 7

Step-by-step Explanation:

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}}=9$$

$$u\sin g\ componendo\ and\ dividendo,$$

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}}=\frac{9+1}{9-1}$$

$$\Rightarrow \frac{2\sqrt{3x+4}}{2\sqrt{3x-5}}=\frac{10}{8}$$

$$\Rightarrow \frac{\sqrt{3x+4}}{\sqrt{3x-5}}=\frac{5}{4}$$

$$squaring\ both\ sides$$

$$\Rightarrow \frac{3x+4}{3x-5}=\frac{25}{16}$$

$$\Rightarrow 75x-125=48x+64$$

$$\Rightarrow 27x=189$$

$$\Rightarrow x=\frac{189}{27}$$

$$\Rightarrow x=7$$

Q25. 6 is the mean proportion between two numbers x and y and 48 is the third proportional of x and y. Find the numbers. [3] [2011]

Solution: x = 3, y = 12

Step-by-step Explanation:

$$6 \text{ is the mean proportional between } x \text{ and } y.$$
$$Therefore,\ xy = 36$$
$$\Rightarrow x = \frac{36}{y}$$
$$48 \text{ is the third proportional of } x \text{ and } y.$$
$$Therefore,\ y^2 = 48x$$
$$\text{putting the value of } x, \text{ we get}$$
$$y^2 = \frac{48 \times 36}{y}$$
$$y^3 = 48 \times 36$$
$$y = \sqrt[3]{48 \times 36}$$
$$y = 2 \times 6$$
$$y = 12$$
$$Hence,\ x = \frac{36}{12} = 3$$

Q26. If x, y, z are in continued proportion, prove that

$\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$. [2010]

Step-by-step Explanation:

$$x,\ y,\ z\ are\ in\ continued\ proportion.$$

$$Therefore,\ y^2 = xz$$

$$L.H.S.$$

$$= \frac{(x+y)^2}{(y+z)^2}$$

$$= \frac{x^2+2xy+y^2}{y^2+2yz+z^2}$$

$$= \frac{x^2+2xy+xz}{xz+2yz+z^2}$$

$$= \frac{x(x+2y+z)}{z(x+2y+z)}$$

$$= \frac{x}{z}$$

$$= R.H.S.$$

$$Proved.$$

$$Q27.\ Given,\ x = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}},\ use\ componendo$$

$and\ dividendo\ to\ prove\ that\ b^2 = \frac{2a^2x}{x^2+1}$. [2010]

Step-by-step Explanation:

$$x = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$$

$$U\sin g\ componendo\ and\ dividendo,$$

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2-b^2}}$$

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2-b^2}}$$

squaring both sides,

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2}$$

$$\frac{x^2+2x+1}{x^2-2x+1} = \frac{a^2+b^2}{a^2-b^2}$$

$u\sin g$ *componendo and dividendo,*

$$\frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1} = \frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

$$\frac{2x^2+2}{4x} = \frac{2a^2}{2b^2}$$

$$\frac{x^2+1}{2x} = \frac{a^2}{b^2}$$

$$b^2(x^2+1) = 2a^2x$$

$$b^2 = \frac{2a^2x}{x^2+1}$$

Proved.