Coulomb's Law

On the basis of "Torsion balance" experiment "Charles Augustine Coulomb" put a quantitative law for the force of attraction or repulsion on the charges which states that–

"The force of attraction or repulsion on one charge q_2 placed at some separation from another charge q_1 (whose dimensions are small compared to their distance of separation) in infite homogeneous medium, is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them."

$$F \propto |q_1||q_2|$$

$$\propto \frac{1}{r^2}$$

$$F \propto \frac{|q_1||q_2|}{r^2} \Rightarrow F = k\,\frac{q_1 q_2}{r^2}$$

Where k is a constant of proportionality. In C.G.S. Unit i.e. if F is measured in dyne, q_1 and q_2 in stat coulomb and r in cm, then k = 1

In c.g.s. unit $\quad F = \frac{q_1 q_2}{r^2}$

But if the force is measured in newton, q_1 and q_2 in coulomb and r in metre then

Then $\quad k = \frac{1}{4\pi \epsilon_0}$ in air or vacuum and $k = \frac{1}{4\pi \epsilon}$ in an infinite homogeneous material medium other than air.

So if the intervening medium between the charges is air or it is vacuum then in S.I. Units

$$F = \frac{1}{4\pi \epsilon_0}\,\frac{q_1 q_2}{r^2}$$

ϵ_0 is called permittivity of free space.

ϵ is called absolute permittivity of the given material medium.

The ratio of the absolute permittivity of a given medium and that of the permittivity of free space is called relative permittivity of that medium or its dielectric constant (represented by symbol ϵ_r or K)

ϵ_r or $K = \frac{\epsilon}{\epsilon_0}$ So, $\epsilon = \epsilon_0 \epsilon_r$ or $K \epsilon_0$

Hence, $\quad F_{medium} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} = \frac{F}{K}$

Note

(i) K, the dielectric constant of the medium (also called relative permittivity) being ratio of two like quantities, is a dimensionless constant.

(ii) Air or vacuum has minimum relative permittivity ($K = \varepsilon_0/\varepsilon_0 = 1$). The relative permittivity of all other media is greater than 1 usually and K = infinite for a conducting medium.

(iii) When the charges are placed in infite dielectric medium then dielectric medium is getting polarized and force on q_1 or q_2 is not simply due to q_1 or q_{2i} but due to polarized charges also and net force on q_i or q_j becomes $\frac{1}{\epsilon_r}$ times. (See next Illustration)

Illustration :

Two equal point charges (10^{-3}C)are placed 1 cm apart in medium of dielectric constant K = 5
(a) Find the interaction force between the point charges.
(b) Net force on any of the charge.

Sol.

(a) Interaction force between point charges

$$F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2} = 9 \times 10^9 \frac{(10^{-3})^2}{(10^{-2})^2}$$

$$= 9 \times 10^7 \text{ N}$$

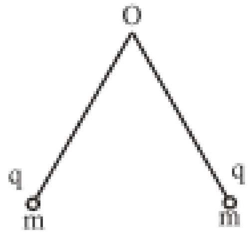
(b) Net force

$$F' = \frac{1}{4\pi \in_0 k} \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9}{5} \frac{(10^{-3})^2}{(10^{-2})^2}$$

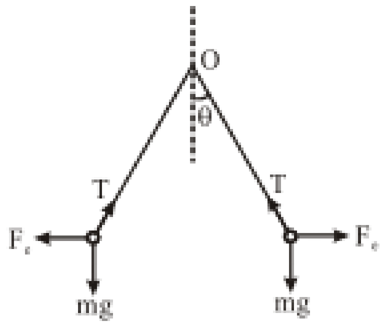
$$= 18 \times 10^6 \text{ N}$$

Illustration :

Two small balls each of mass m and charge q on each of them are suspended through two light insulating string of length l from a point. Find the expression for angle θ made by any of the string with vertical when under static equilibrium.



Sol. *Let angle of any string with vertical be θ as shown*



$$F_e = \frac{1}{4\pi \in_0} \frac{q^2}{(2\ell \sin\theta)^2} = T\sin\theta \qquad \text{... (i) for horizontal direction}$$

$$T\cos\theta = mg \qquad \text{... (ii) for vertical direction}$$

Dividing (i) by (ii)

$$\tan\theta = \frac{F_e}{mg}$$

(i) The force of electrostatic interaction between two charges is operative along the line joining the charges.

(ii) The force obeys inverse square law

(iii) If $q_1q_2 > 0$ (it means the product of the two charges is positive) this implies that charges are similar, i.e., either both positive or both negative. Hence, repulsion will result.

(iv) If $q_1q_2 < 0$, it means the product of the magnitude of the charges is negative. In other words, these are unlike charges, i.e., one charge is positive and the other charge is negative. Hence, the electrostatic force between them is attractive.

Like charges repel each other unlike charges attract

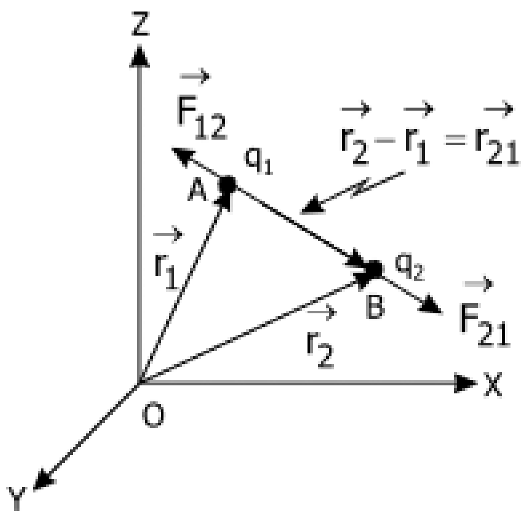
(vi) The force on q_1 due to q_2 is equal and opposite to the force on q_2 due to q_1

$$\vec{F}_{12} = -\vec{F}_{21}$$

i.e. The force of electrostatic interaction between two charges obey Newton's 3rd law. It should, however, be noted that the equality $\vec{F}_{12} = -\vec{F}_{21}$ breaks down when one charge is accelerated towards the other i.e., Newton's 3rd law doesn't hold . This is why Newton's 3rd law is supposed to be a weak law of physics.

Force Between Two Charges in Terms of Their Position Vectors :

Consider two like charge q_1 and q_2 located in vacuum at positions A and B respectively. Let the positions of A and B with reference tothe orign O of the coordinate frame be given by position vectors



$\vec{r}_1$ and $\vec{r}_2$ respectively, i.e., $\vec{OA} = \vec{r}_1$ and $\vec{OB} = \vec{r}_2$

Now, $\vec{OA} + \vec{AB} = \vec{OB}$ (triangle law of vectors)

$$\therefore \quad \vec{AB} = \vec{OB} - \vec{OA} = \vec{r}_2 - \vec{r}_1 = \vec{r}_{21}$$

According to Coulomb's law, force on q_2 due to q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^3}\vec{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Similarly, $\vec{F}_{12} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$

It should be noted that :

(i) For coulomb's law to give precise result the spatial extent of charges should be very small in comparison to their separation.

(ii) Coulomb's law is valid for wide range of distance .

Illustration :

Two point charges A and B have charges respectively $\frac{1}{2}$ C and 2 C with their position vectors respectively as $(\hat{i}+\hat{j}+\hat{k})$ and $(-\vec{i}-\vec{j}+3\vec{k})$. *Find the force on charge at A due to B.*

Sol.

$$q_A = \frac{1}{2}C \qquad \vec{r}_A = \hat{i}+\hat{j}+\hat{k}$$

$$q_B = 2C \qquad \vec{r}_B = -\hat{i}-\hat{j}+3\hat{k}$$

$$\vec{F}_{AB} = \frac{1}{4\pi\in_0}\frac{q_A q_B}{|\overrightarrow{r_{AB}}|^2}\hat{r}_{AB}$$

$$= \frac{1}{4\pi\in_0}\frac{q_A q_B}{\left|\overrightarrow{r_{AB}}\right|^3}\left(\overrightarrow{r_A} - \overrightarrow{r_B}\right) = (9\times10^9)\times\frac{\frac{1}{2}\times1}{\left|2\hat{i}+2\hat{j}-2\hat{k}\right|^3}(2\hat{i}+2\hat{j}-2\hat{k})$$

$$= \frac{9\times10^9\times(\hat{i}+\hat{j}-k)}{24\sqrt{3}}N.$$

SUPERPOSITION OF ELECTROSTATIC FORCES

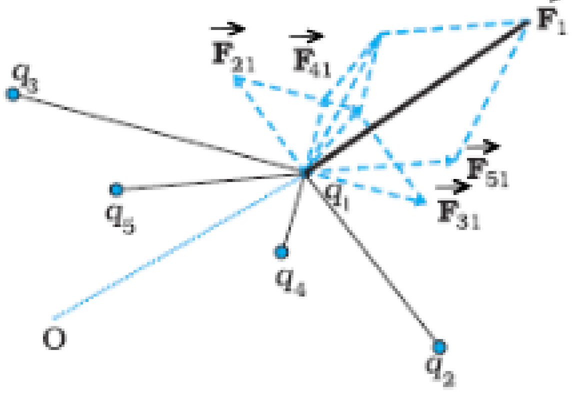
Experimentally it is verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the principle of superposition.

The principle of superposition says that in a system of charges $q_1, q_2, ..., q_n$, the force on q_1 due to q_2 is the same as given by Coulomb's law, i.e., it is unaffected by the presence of the other charges $q_3, q_4, ..., q_n$. The total force $\vec{F}_1$ on the charge q_1, due to all other charges, is then given by the vector sum of the forces $\vec{F}_{12}, \vec{F}_{13}, ..., \vec{F}_{1n}$:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ... + \vec{F}_{1n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + ... + \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

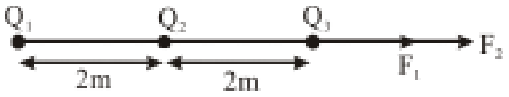
$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{j=2}^{n}\frac{q_j}{r_{1j}^2}\hat{r}_{1j}$$



The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

Illustration :

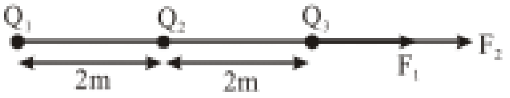
Three charges each of $20\mu C$ are placed along a straight line, successive charges being 2 m apart as shown in Figure. Calculate the force on the charge on the right end.



Sol.

$$F = F_1 + F_2$$

$$F_1 = \frac{kQ_1Q_2}{r^2} = \frac{(9x10^9)(20x10^{-6})^2}{4^2} = 0.225N$$



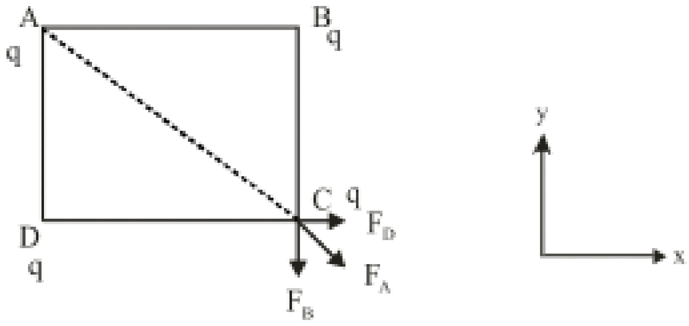
$$F_2 = \frac{kQ_2Q_3}{r^2} = \frac{(9x10^9)(20x10^{-6})^2}{2^2} = 0.9\,N$$

$$F = 0.225 + 0.9 = 1.125 \text{ N to the right}$$

Illustration:

Four identical point charges each of magnitude q are placed at the corners of a square of side a. Find the net electrostatic force on any of the charge.

Sol.



Let the concerned charge be at C then charge at C will experience the force due to charges at A, B and D. Let these forces respectively be $\vec{F}_A, \vec{F}_B$ and $\vec{F}_D$ *thus forces are given as*

$$\vec{F}_A = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{AC^2} \text{ along } AC = \frac{q^2}{4\pi\varepsilon_0 2a^2}\left(\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}\right)$$

$$\vec{F}_B = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{BC^2} \text{ along } BC = \frac{q^2}{4\pi\varepsilon_0 a^2}(-\hat{j})$$

$$\vec{F}_D = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{DC^2} \text{ along } DC = \frac{q^2}{4\pi\varepsilon_0 a^2}(\hat{i})$$

$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_D$$

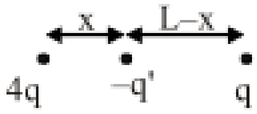
$$= \frac{q^2}{4\pi\varepsilon_0 a^2}\left[\hat{i}\left(\frac{1}{2\sqrt{2}} + 1\right) - \hat{j}\left(\frac{1}{2\sqrt{2}} + 1\right)\right]$$

$$\left|\vec{F}_{net}\right| = \sqrt{2}\left(\frac{1}{2\sqrt{2}} + 1\right)\frac{q^2}{4\pi\varepsilon_0 a^2} \qquad = \left(\frac{1}{2} + \sqrt{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}.$$

Illustration:

Two point charges +4q and +q are placed at a distance L apart. A third charge is so placed that all the three charges are in equilibrium. Find the location, magnitude and nature of third charge. Discuss also, whether the equilibrium of the system is stable, unstable or neutral.

Sol.



Third charge should be placed between 4q and q so that force on third charge to be zero (let at distance x from 4q). Third charge should be –ve (let –q') for the equilibrium of other charges
For equilibrium of third charge

$$\frac{K(4q)(q')}{x^2} = \frac{K(q)(q')}{(L-x)^2} \Rightarrow x = \frac{2L}{3}$$

for equilibrium 4q

$$\frac{K(4q)(q')}{\left(\frac{2L}{3}\right)^2} = \frac{K(4q)(q)}{L^2} \qquad \Rightarrow \qquad q' = \frac{4q}{9}$$

Practice Exercise

Q.1 Three particles, each of mass 1 g and carrying a charge q, are suspended from a common point by insulating massless strings, each 100 cm long. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 3 cm, calculate the charge q on each particle. ($g = 10 m/s^2$).

Q.2 Five point charges, each of value +q are placed on five vertices of a regular hexagon of side L m. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of hexagon?

Q.3 Two equal point charges q are fixed at x = – a and x = a along the x-axis. A particle of mass m and charge q/2 is brought to the origin and given a small displacement along the (a) x-axis and (b) y-axis. Describe the motion in the two cases.

Answers

Q.1 3.16×10^{-9} C Q.2 $\frac{1}{4\pi \epsilon_0}\left[\frac{q}{R}\right]^2$ Q.3 (a) Accelerated motion (b) SHM

The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

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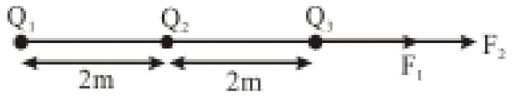
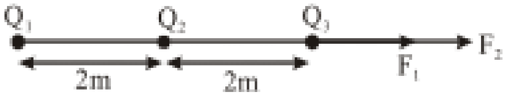


Sol.

$$F = F_1 + F_2$$



$$F_1 = \frac{kQ_1Q_2}{r^2} = \frac{\left(9x10^9\right)\left(20x10^{-6}\right)^2}{4^2} = 0.225\text{N}$$

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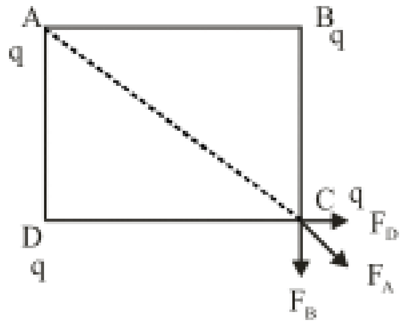

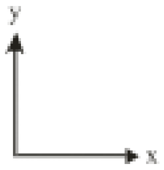

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Practice Exercise

Q.1 A particle having a charge of 2.0 10 6 C and a mass of 100 g is placed at the bottom of a smooth inclined plane of inclination 30 . Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium?

Q.2 A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 g/mol. Let us now take two pieces of copper each weighing 10 g. Let us transfer one electron from one piece to another for every 1000 atoms in a piece. What will be the coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart? [e= 1.6×10^{-19}C, $(1/4\pi\epsilon_0) = 9\times10^9$ and Avogadro's number $=6\times10^{23}$ per mol]

Q.3 Two spherical conductors B and C having equal radii and carrying equal charges with them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both.What is the new force of repulsion between B and C ? (When two conductors of identical geometry having charges q_1 and q_2 if touched have final charges $\frac{q_1+q_2}{2}$ on each conductor)

Q.4 A ring of radius 0.1 m is made our of a metallic wire of area of cross– section 10^{-6} m^2. The ring has a uniform charge of π coulomb. Find the change in the radius of the ring when a charge of 10^{-8} coulomb is placed at the centre of the ring.

Young's modulus of the metal is 2×10^{11} N/ m^2.

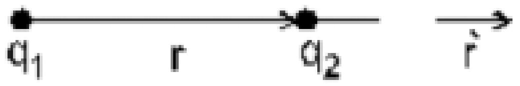
Answers

Q.1 27 cm from the bottom Q.2 2.08×10^{14}N Q.3 3F/8

Q.4 2.25×10^{-13} m

Vector form of coulomb's law :

By stating coulomb's law in vector form more information can be packed in it.



Let the position vector of charge q_2 relative to charge q_1 be $\vec{r}$ and $\hat{r}$ is a unit vector in the direction of $\vec{r}$

so, $\vec{r} = |\vec{r}|\,\hat{r} = r\,\hat{r}$ hence, $\hat{r} = \frac{\vec{r}}{r}$

Coulomb's law, in vector form, may be written as

$$\vec{F} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^3}\vec{r}$$

From the above form of the coulomb's law, It may be justified that,

Illustration:

Two point charges +4q and +q are placed at a distance L apart. A third charge is so placed that all the three charges are in equilibrium. Find the location, magnitude and nature of third charge. Discuss also, whether the equilibrium of the system is stable, unstable or neutral.

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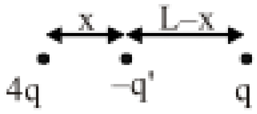

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