[SINGLE CORRECT CHOICE TYPE]

1. Which of the following conditions imply that the real number x is rational ?

I $x^{1/2}$ is rational **II** x^2 and x^5 are rational **III** x^2 and x^4 are rational

(A) I and II only (B) I and III only (C) II and III only (D) I, II and III

2. Let $n=\sqrt{6+\sqrt{11}}+\sqrt{6-\sqrt{11}}-\sqrt{22}$, then

(A) $n \geq 1$ (B) $0 < n < 1$ (C) $n = 0$ (D) $-1 < n < 0$

3. Number of real distinct x satisfying the equation $|x-2| + |x-3| = |x-1|$ is

(A) 1 (B) 2 (C) 3 (D) more than 3

4. The sum of the solutions of the equation $9^x - 6 \cdot 3^x + 8 = 0$ is

(A) $\log_3 2$ (B) $\log_3 6$ (C) $\log_3 8$ (D) $\log_3 4$

5. If $x=\left(2^{\sqrt{5}}\right)\left(5^{\sqrt{2}}\right)$, then $\log_{10} x=\left(\sqrt{A}-\sqrt{B}\right)(\log_{10} 2)+\sqrt{B}$. The value of (A + B) equals

(A) 7 (B) 9 (C) 11 (D) 13

6. a = log12, b = log21, c = log11 and d = log22 then $\log\left(\frac{1}{7}\right)$ can be expressed in this form

P(a – b) + Q(c – d) where P and Q are integers then the value of (7P – Q) equals

(A) 5 (B) 9 (C) 13 (D) 15

7. Given $\log_2 a = p$, $\log_4 b = p^2$ and $\log_{c^2}(8)=\frac{2}{p^3+1}$. If $\log_2\left(\frac{c^8}{ab^2}\right) = (\alpha p^3 - \beta p^2 - \gamma p + \delta)$ where $\alpha,\beta,\gamma,\delta \in N$, then find the value of $(\alpha + \beta + \gamma + \delta)$.

(A) 17 (B) 12 (C) 15 (D) 96

8. Let $\log_M N = \alpha + \beta$, where α is an integer and β is non negative fraction. If M and α are prime and $\alpha + M = 7$ then $N \in [a,b)$, Then the sum of all possible value(s) of $|b - 5a|$ is

(A) 0 (B) 24 (C) 48 (D) 96

[MATRIX TYPE]

Q.9 & Q.10 has **four** statements (A,B,C and D) given in **Column-I** and **four** statements (P, Q, R and S) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II.**

9.

Column-I	Column-II
(A) The expression $x=\log_2\log_9\sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}$ simplifies to	(P) an integer
(B) The number $N=2^{(\log_2 3.\log_3 4.\log_4 5...\log_{99}100)}$ simplifies to	(Q) a prime
(C) The expression $\frac{1}{\log_5 3}+\frac{1}{\log_6 3}-\frac{1}{\log_{10} 3}$ simplifies to	(R) a natural
(D) The number $N=\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$ simplifies to	(S) a composite

10. **Column-I** and **column-II** contains **four** entries each. Entry of column-I are to be uniquely matched with only one entry of column-II.

Column-I	**Column-II**
(A) $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+........+\frac{1}{\sqrt{49}+\sqrt{48}}$	(P) 3
(B) Let $A=\log_{\sqrt{3}}8\cdot\log_4 81;\ B=\log_{\sqrt{6}}3\cdot\log_3 36$ Then the value of (A – B) equals	(Q) 6
(C) Let $A=\log^2_{\sqrt{2}}\left(\frac{1}{4}\right);\ B=\log^3_{2\sqrt{2}}(8);\ C=-\log_5\log_3\sqrt{\sqrt[5]{9}}$. Then the value of $\left(\frac{A}{B}+C\right)$ equals	(R) 7
(D) $\left(\sqrt[3]{4}-\sqrt[3]{10}+\sqrt[3]{25}\right)\left(\sqrt[3]{2}+\sqrt[3]{5}\right)$	(S) 8

[COMPREHENSION TYPE]

Paragraph for question nos. 11 to 13

Let A denotes the sum of the roots of the equation $\frac{1}{5-4\log_4 x}+\frac{4}{1+\log_4 x}=3.$

B denotes the value of the product of m and n, if $2^m = 3$ and $3^n = 4$.

C denotes the product of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right)+(\log_3 x)^2=1$.

11. The value of A + B equals

(A) 10 (B) 6 (C) 8 (D) 4

12. The value of B + C equals

(A) 6 (B) 2 (C) 4 (D) 5

13. The value of A ÷ C + B equals

(A) 4 (B) 8 (C) 7 (D) 5

[SUBJECTIVE TYPE]

14. Find all integral solution of the equation, $4\log_{x/2}\left(\sqrt{x}\right)+2\log_{4x}\left(x^2\right)=3\log_{2x}\left(x^3\right)$

15. **(i)** Prove that if $x = \log_c b + \log_b c$, $y = \log_a c + \log_c a$, $z = \log_b a + \log_a b$ then $xyz = x^2 + y^2 + z^2 - 4$.

(ii) $y=a^{\frac{1}{(1-\log_a x)}}$ and $z=a^{\frac{1}{(1-\log_a y)}}$, prove that $x=a^{\frac{1}{(1-\log_a z)}}$

Answers

1. (A) **2.** (C) **3.** (B) **4.** (C) **5.** (A) **6.** (A) **7.** (A) **8.** (D)

9. A-P ; B-PRS ; C-PR ; D-PQR **10.** A-Q, B-S, C-P, D-R **11.** (C) **12.** (D) **13.** (A) **14.** 1,4