# 4 CO-ORDINATE GEOMETRY

## EXERCISE

**361.** The image of P(a, b) on $y = -x$ is Q and the image of Q on the line $y = x$ is R. Then the mid point of R is

(A) (a + b, b + a) (B) $\left(\frac{a+b}{2}, \frac{b+a}{2}\right)$ (C) (a − b, b − a) (D) (0, 0)

**362.** The line $3x - 4y + 7 = 0$ is rotated through an angle $\frac{\pi}{4}$ in the clockwise direction about the point (−1, 1). The equation of the line in its new position is

(A) $7y + x - 6 = 0$ (B) $7y - x - 6 = 0$ (C) $7y + x + 6 = 0$ (D) $7y - x + 6 = 0$

**363.** Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

**364.** If $5a + 4b + 20c = t$, then the value of t for which the line $ax + by + c - 1 = 0$ always passes through a fixed point is

(A) 0 (B) 20 (C) 30 (D) none of these

**365.** The point A (2, 1) is translated parallel to the line $x - y = 3$ by a distance 4 units. If the new position A′ is in third quadrant, then the coordinates of A′ are

(A) $(2 + 2\sqrt{2}, 1 + 2\sqrt{2})$ (B) $(-2 + \sqrt{2}, -1 - 2\sqrt{2})$

(C) $(2 - 2\sqrt{2}, 1 - 2\sqrt{2})$ (D) none of these

**366.** If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

**367.** The lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 sq unit. The equation of this circle is ($\pi = 22/7$)

(A) $x^2 + y^2 + 2x - 2y = 62$ (B) $x^2 + y^2 + 2x - 2y = 47$
(C) $x^2 + y^2 - 2x + 2y = 47$ (D) $x^2 + y^2 - 2x + 2y = 62$

**368.** The range of values of 'a' such that the angle θ between the pair of tangents drawn from (a, 0) to the circle $x^2 + y^2 = 1$ satisfies $\frac{\pi}{2} < \theta < \pi$, is

(A) (1, 2) (B) $(1, \sqrt{2})$ (C) $(-\sqrt{2}, -1)$ (D) $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$

**369.** The locus of a point such that the tangents drawn from it to the circle $x^2 + y^2 - 6x - 8y = 0$ are perpendicular to each other is

(A) $x^2 + y^2 - 6x - 8y - 25 = 0$ (B) $x^2 + y^2 + 6x - 8y - 5 = 0$
(C) $x^2 + y^2 - 6x + 8y - 5 = 0$ (D) $x^2 + y^2 - 6x - 8y + 25 = 0$

**370.** The locus of the point $(\sqrt{(3h+2)}, \sqrt{3k})$. If (h, k) lies on $x + y = 1$ is

(A) a pair of straight lines (B) a circle
(C) a parabola (D) an ellipse

**371.** The shortest distance between the parabolas $y^2 = 4x$ and $y^2 = 2x - 6$ is

(A) 2 (B) $\sqrt{5}$ (C) 3 (D) none of these

**372.** A parabola is drawn with focus at (3, 4) and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is

(A) $x^2 - 6x - 8y + 25 = 0$ (B) $y^2 - 8x - 6y + 25 = 0$
(C) $x^2 - 6x + 8y - 25 = 0$ (D) $x^2 + 6x - 8y - 25 = 0$

**373.** Two perpendicular tangents PA and PB are drawn to $y^2 = 4ax$, minimum length of AB is equal to

(A) a (B) 4a (C) 8a (D) 2a

**374.** The locus of the points of trisection of the double ordinates of the parabola $y^2 = 4ax$ is

(A) $y^2 = ax$ (B) $9y^2 = 4ax$ (C) $9y^2 = ax$ (D) $y^2 = 9ax$

**375.** If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then PA. PB is equal to [where $P \equiv (\sqrt{3}, 0)$]

(A) $\frac{4(\sqrt{3}+2)}{3}$ (B) $\frac{4(2-\sqrt{3})}{3}$ (C) $\frac{4\sqrt{3}}{2}$ (D) $\frac{2(\sqrt{3}+2)}{3}$

**376.** The point, at shortest distance from the line $x + y = 7$ and lying on an ellipse $x^2 + 2y^2 = 6$, has coordinates

(A) $(\sqrt{2}, \sqrt{2})$ (B) $(0, \sqrt{3})$ (C) (2, 1) (D) $\left(\sqrt{5}, \frac{1}{\sqrt{2}}\right)$

**377.** The length of the common chord of the ellipse $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ and the circle $(x - 1)^2 + (y - 2)^2 = 1$ is

(A) zero (B) one (C) three (D) eight

**378.** The eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latusrectum is half of its minor axis is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) none of these

**379.** The locus of midpoints of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is

(A) $\frac{x^2}{a^2}+\frac{y^2}{b^2} = \frac{ex}{a}$ (B) $\frac{x^2}{a^2}-\frac{y^2}{b^2} = \frac{ex}{a}$ (C) $x^2 + y^2 = a^2 + b^2$ (D) none of these

**380.** An ellipse slides between two perpendicular straight lines. Then the locus of its centre is a/an
(A) parabola (B) ellipse (C) hyperbola (D) circle

**381.** If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2}$ = 1 and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of $b^2$ is
(A) 3 (B) 16 (C) 9 (D) 12

**382.** If $(a - 2) x^2 + ay^2 = 4$ represents rectangular hyperbola, then a equals
(A) 0 (B) 2 (C) 1 (D) 3

**383.** The eccentricity of the hyperbola whose asymptotes are $3x + 4y = 2$ and $4x - 3y + 5 = 0$ is
(A) 1 (B) 2 (C) $\sqrt{2}$ (D) none of these

**384.** The equation of the hyperbola whose foci are (6, 5), (−4, 5) and eccentricity 5/4 is

(A) $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$ (B) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(C) $\frac{(x-1)^2}{16} + \frac{(y-5)^2}{9} = 1$ (D) none of these

**385.** Area of the triangle formed by the lines $x - y = 0$, $x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is

(A) $|a|$ (B) $\frac{1}{2}|a|$ (C) $a^2$ (D) $\frac{1}{2}a^2$

**386.** The coordinates of a point on the line $\frac{x-1}{2} = \frac{y+1}{-3}$ = z at a distance $4\sqrt{14}$ from the point (1, −1, 0) nearer the origin are
(A) (9, −13, 4) (B) $(8\sqrt{14}, -12, -1)$ (C) $(-8\sqrt{14}, 12, 1)$ (D) (−7, 11, −4)

**387.** The distance of the point A(−2, 3, 1) from the line PQ through P(−3, 5, 2) which make equal angles with the axes is

(A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{\frac{14}{3}}$ (C) $\frac{16}{\sqrt{3}}$ (D) $\frac{5}{\sqrt{3}}$

**388.** The line joining the points (1, 1, 2) and (3, −2, 1) meets the plane $3x + 2y + z = 6$ at the point
(A) (1, 1, 2) (B) (3, −2, 1) (C) (2, −3, 1) (D) (3, 2, 1)

**389.** The point equidistant from the points (a, 0, 0), (0, b, 0), (0, 0, c) and (0, 0, 0) is

(A) $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$ (B) (a, b, c) (C) $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$ (D) none of these

**390.** The intercepts made on the axes by the plane which bisects the line joining the points (1, 2, 3) and (–3, 4, 5) at right angles are

(A) $\left(-\frac{9}{2},9,9\right)$ (B) $\left(\frac{9}{2},9,9\right)$ (C) $\left(9,-\frac{9}{2},9\right)$ (D) $\left(9,\frac{9}{2},9\right)$

**391.** A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The coordinates of the point A are

(A) $\left(\frac{13}{5},0\right)$ (B) $\left(\frac{5}{13},0\right)$ (C) (–7, 0) (D) none of these

**392.** The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0 lies in
(A) I quadrant (B) II Quadrant (C) III Quadrant (D) IV Quadrant

**393.** In $\Delta$ ABC if orthocentre be (1, 2) and circumcentre be (0, 0), then centroid of $\Delta$ABC is
(A) (1/2, 2/3) (B) (1/3, 2/3) (C) (2/3, 1) (D) none of these

**394.** If the point (a, a) fall between the lines |x + y| = 2, then

(A) |a| = 2 (B) |a| = 1 (C) |a| < 1 (D) |a| < $\frac{1}{2}$

**395.** If f (x + y) = f(x) f(y), $\forall$ x, y $\in$ R and f(1) = 2,then area enclosed by 3 |x| + 2 |y| $\leq$ 8 is

(A) f(4) sq unit (B) ($\frac{1}{2}$) f(6) sq unit (C) $\frac{1}{3}$ f(6) sq unit (D) $\frac{1}{3}$ f(5) sq unit

**396.** The four points of intersection of the lines (2x – y + 1) (x – 2y + 3) = 0 with the axes lie on a circle whose centre is at the point
(A) (–7/4, 5/4) (B) (3/4, 5/4) (C) (9/4, 5/4) (D) (0, 5/4)

**397.** Origin is a limiting point of a coaxial system of which $x^2 + y^2 - 6x - 8y + 1 = 0$ is a member. The other limiting point is

(A) (–2, –4) (B) $\left(\frac{3}{25},\frac{4}{25}\right)$ (C) $\left(-\frac{3}{25},-\frac{4}{25}\right)$ (D) $\left(\frac{4}{25},\frac{3}{25}\right)$

**398.** The shortest distance from the point (2, –7) to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is
(A) 1 (B) 2 (C) 3 (D) 4

**399.** The equation of the image of the circle $(x - 3)^2 + (y - 2)^2 = 1$ by the mirror x + y = 19 is
(A) $(x - 14)^2 + (y - 13)^2 = 1$ (B) $(x - 15)^2 + (y - 14)^2 = 1$
(C) $(x - 16)^2 + (y - 15)^2 = 1$ (D) $(x - 17)^2 + (y - 16)^2 = 1$

**400.** The locus of centre of a circle which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touch the y-axis is given by the equation
(A) $x^2 - 6x - 10y + 14 = 0$ (B) $x^2 - 10x - 6y + 14 = 0$
(C) $y^2 - 6x - 10y + 14 = 0$ (D) $y^2 - 10x - 6y + 14 = 0$

**401.** If tangents at A and B on the parabola $y^2 = 4ax$ intersect at point C, then ordinates of A, C and B are
(A) always in AP (B) always in GP (C) always in HP (D) none of these

**402.** Let P be any point on the parabola $y^2 = 4ax$ whose focus is S. If normal at P meet x-axis at Q. Then $\Delta$ PSQ is always

(A) isosceles (B) equilateral (C) right angled (D) None of these

**403.** The vertex of the parabola whose focus is (–1, 1) and directrix is $4x + 3y - 24 = 0$ is

(A) (0, 3/2) (B) (0, 5/2) (C) (1, 3/2) (D) (1/ 5/2)

**404.** The equation of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

(A) $x + 2y + 4 = 0$ (B) $2x + y - 4 = 0$ (C) $x - 2y - 4 = 0$ (D) $x - 2y + 4 = 0$

**405.** The locus of point of intersection of tangents to the parabolas $y^2 = 4(x + 1)$ and $y^2 = 8(x + 2)$ which are perpendicular to each other is

(A) $x + 7 = 0$ (B) $x - y = 4$ (C) $x + 3 = 0$ (D) $y - x = 12$

**406.** AB is a diameter of $x^2 + 9y^2 = 25$. The eccentric angle of A is $\pi/6$, then the eccentric angle of B is

(A) $5\pi/6$ (B) $-5\pi/6$ (C) $-2\pi/3$ (D) none of these

**407.** The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and parabola $y^2 = 4x$ above the x-axis is

(A) $\sqrt{3}\,y = 3x + 1$ (B) $\sqrt{3}\,y = -(x + 3)$ (C) $\sqrt{3}\,y = x + 3$ (D) $\sqrt{3}\,y = -(3x + 1)$

**408.** The tangent drawn at any point P to the parabola $y^2 = 4ax$ meets the directrix at the point K, then the angle which KP subtends at its focus is

(A) 30º (B) 45º (C) 60º (D) 90º

**409.** Sides of an equilateral $\Delta$ ABC touch the parabola $y^2 = 4ax$ then the points A, B and C lie on

(A) $y^2 = (x + a)^2 + 4ax$ (B) $y^2 = 3(x + a)^2 + ax$

(C) $y^2 = 3(x + a)^2 + 4ax$ (D) $y^2 = (x + a)^2 + ax$

**410.** The common tangent of the parabolas $y^2 = 4x$ and $x^2 = -8y$ is

(A) $y = x + 2$ (B) $y = x - 2$ (C) $y = 2x + 3$ (D) None of these

**411.** If a triangle is inscribed in a rectagular hyperbola, its orthocentre lies

(A) inside the curve (B) outside the curve (C) on the curve (D) none of these

**412.** Tangents drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$, then tangents are at angle

(A) $\pi/4$ (B) $\pi/2$ (C) $\pi/3$ (D) $2\pi/3$

**413.** A ray emanating from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8, then the equation of the reflected ray after first reflection is (P lies in first quadrant)

(A) $\sqrt{3}x - y + 7 = 0$ (B) $3\sqrt{3}x - 13y + 15\sqrt{3} = 0$

(C) $3\sqrt{3}x + 13y - 15\sqrt{3} = 0$ z (D) $\sqrt{3}x + y - 14 = 0$

**414.** The symmetric form of the equations of the line $x + y - z = 1$, $2x - 3y + z = 2$ is

(A) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ (B) $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{5}$ (C) $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{5}$ (D) $\frac{x}{3} = \frac{y}{2} = \frac{z}{5}$

**415.** The equation of the line passing through the point (1, 1, −1) and perpendicular to the plane x − 2y − 3z = 7 is

(A) $\frac{x-1}{-1} = \frac{y-1}{2} = \frac{z+1}{3}$ (B) $\frac{x-1}{-1} = \frac{y-1}{-2} = \frac{z+1}{3}$

(C) $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z+1}{-3}$ (D) none of these

**416.** The projections of a line on the axes are, 9, 12 and 8. The length of the line is
(A) 7 (B) 17 (C) 21 (D) 25

**417.** The three planes 4y + 6z = 5; 2x + 3y + 5z = 5; 6x + 5y + 9z = 10.
(A) meet in a point (B) have a line in common
(C) form a triangular prism (D) none of these

**418.** The foot of the perpendicular from P(1, 0, 2) to the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ is the point

(A) (1, 2, −3) (B) $\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$ (C) (2, 4, −6) (D) (2, 3, 6)

**419.** The plane containing the two lines $\frac{x-3}{1} = \frac{y-2}{4} = \frac{z-1}{5}$ and $\frac{x-2}{1} = \frac{y+3}{-4} = \frac{z+1}{5}$ is 11x + my + nz = 28, where
(A) m = −1, n = 3 (B) m = 1, n = −3 (C) m = −1, n = −3 (D) m = 1, n = 3

**420.** The projection of the line $\frac{x+1}{-1} = \frac{y}{2} = \frac{z-1}{3}$ on the plane x − 2y + z = 6 is the line of intersection of this plane with the plane
(A) 2x + y + 2 = 0 (B) 3x + y − z = 2 (C) 2x − 3y + 8z = 3 (D) none of these

**421.** If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 0$, then $((\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b}))))$ is equal to

(A) $48\vec{b}$ (B) $-48\vec{b}$ (C) $48\vec{a}$ (D) $-48\vec{a}$

**422.** Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $3\vec{a} + 4\vec{b} + 5\vec{c} = 0$. Then which of the following Statements is true ?

(A) $\vec{a}$ is parallel to $\vec{b}$ (B) $\vec{a}$ is perpendicular to $\vec{b}$

(C) $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$ (D) none of these

**423.** Given three vectors $\vec{a} = 6\hat{i} - 3\hat{j}$, $\vec{b} = 2\hat{i} - 6\hat{j}$ and $\vec{c} = -2\hat{i} + 21\hat{j}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$.

Then the resolutionof the vector $\vec{\alpha}$ into components with respect to $\vec{a}$ and $\vec{b}$ is given by

(A) $3\vec{a} - 2\vec{b}$ (B) $2\vec{a} - 3\vec{b}$ (C) $3\vec{b} - 2\vec{a}$ (D) none of these

**424.** If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed

(A) 4 (B) 9 (C) 8 (D) 6

**425.** If unit vector $\vec{c}$ makes an angle $\frac{\pi}{3}$ with $\hat{i} + \hat{j}$, then minimum and maximum values of $(\hat{i} \times \hat{j}).\vec{c}$ respectively are

(A) $0, \frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$ (C) $-1, \frac{\sqrt{3}}{2}$ (D) none of these

**426.** If the lines $x + ay + a = 0$, $bx + y + b = 0$ and $cx + cy + 1 = 0$ (a,b,c being distinct $\neq 1$) are concurrent, then the value of $\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1}$ is

(A) −1 (B) 0 (C) 1 (D) none of these

**427.** The equation of a line through the point (1, 2) whose distance from the point (3, 1) has the greatest possible value is

(A) $y = x$ (B) $y = 2x$ (C) $y = -2x$ (D) $y = -x$

**428.** If (−6, −4), (3, 5), (−2, 1) are the vertices of a parallelogram, then remaining vertex cannot be

(A) (0, −1) (B) (−1, 0) (C) (−11, −8) (D) (7, 10)

**429.** Length of the median from B on AC where, A (−1, 3) B (1, −1), C (5, 1) is

(A) $\sqrt{18}$ (B) $\sqrt{10}$ (C) $2\sqrt{3}$ (D) 4

**430.** The incentre of the triangle formed by the lines $x = 0$, $y = 0$ and $3x + 4y = 12$ is at

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) (1, 1) (C) $\left(1, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, 1\right)$

**431.** Centre of the circle whose radius is 3 and which touches internally the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point (−1, −1) is

(A) $\left(\frac{7}{5}, \frac{-4}{5}\right)$ (B) $\left(\frac{4}{5}, \frac{7}{5}\right)$ (C) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (D) $\left(\frac{7}{5}, \frac{3}{5}\right)$

**432.** The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at

(A) (6, 7) (B) (−6, 7) (C) (6, −7) (D) (−6, −7)

**433.** The locus of a point which moves so that the ratio of the length of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is 2 : 3 is

(A) $5x^2 + 5y^2 + 60x - 7 = 0$ (B) $5x^2 + 5y^2 - 60x - 7 = 0$
(C) $5x^2 + 5y^2 + 60x + 7 = 0$ (D) $5x^2 + 5y^2 + 60x + 12 = 0$

**434.** The set of values of ‘c’ so that the equations $y = |x| + c$ and $x^2 + y^2 - 8|x| - 9 = 0$ have no solution, is

(A) $(-\infty, -3) \cup (3, \infty)$ (B) $(-3, 3)$
(C) $(-\infty, 5\sqrt{2}) \cup (5\sqrt{2}, \infty)$ (D) $(5\sqrt{2} - 4, \infty)$

**435.** PQ is any focal chord of the parabola $y^2 = 32x$. The length of PQ can never be less than
(A) 40 (B) 45 (C) 32 (D) 48

**436.** If $(\alpha^2, \alpha - 2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points (2, 2) and (8, −4), then $\alpha$ belongs to the interval
(A) $(-2 + 2\sqrt{2}, 2)$ (B) $(-2 + 2\sqrt{2}, \infty)$ (C) $(-2 - 2\sqrt{2}, \infty)$ (D) none of these

**437.** If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then
(A) $-2 \le T \le 2$ (B) $T \in (-\infty, -8) \cup (8, \infty)$
(C) $T^2 < 8$ (D) $T^2 \ge 8$

**438.** If tangents at extremities of a focal chord AB of the parabola $y^2 = 4ax$ intersect at a point C, then $\angle$ ACB is equal to
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

**439.** The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is
(A) 27/4 sq unit (B) 9 sq unit (C) 27/2 sq unit (D) 27 sq unit

**440.** If the tangent at the point $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then the value of $\phi$ is
(A) $\pm \pi/2$ (B) $\pm \pi/4$ (C) $\pm \pi/3$ (D) $\pm \pi/6$

**441.** If roots of quadratic equation $ax^2 + 2bx + c = 0$ are not real, then
$ax^2 + 2bxy + cy^2 + dx + ey + f = 0$ represents a/an
(A) ellipse (B) circle (C) parabola (D) hyperbola

**442.** The distance between the foci of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$ is
(A) 4 (B) 6 (C) 10 (D) 8

**443.** Equation of the rectangular hyperbola whose focus is (1, −1) and the corresponding directrix $x - y + 1 = 0$ is
(A) $x^2 - y^2 = 1$ (B) $xy = 1$
(C) $2xy - 4x + 4y + 1 = 0$ (D) $2xy + 4x - 4y - 1 = 0$

**444.** If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then PA. PB is equal to [where $P \equiv (\sqrt{3}, 0)$]
(A) $\frac{4(\sqrt{3}+2)}{3}$ (B) $\frac{4(2-\sqrt{3})}{3}$ (C) $\frac{4\sqrt{3}}{2}$ (D) $\frac{2(\sqrt{3}+2)}{3}$

**445.** The point (−2m, m + 1) is an interior point of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$. Then m belongs to the interval
(A) $-5 - 2\sqrt{6} < m < 1$ (B) $0 < m < 4$
(C) $-1 < m < \frac{3}{5}$ (D) $-1 < m < -5 + 2\sqrt{6}$

**446.** Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta = 120°$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, then

$\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$ is equal to

(A) 225 (B) 275 (C) 325 (D) 300

**447.** If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}|$ is equal to

(A) 3 (B) 4 (C) 5 (D) 6

**448.** for non-zero vectros $\vec{a}, \vec{b}, \vec{c}$ $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds iff

(A) $\vec{a}\cdot\vec{b} = 0,\ \vec{b}\cdot\vec{c} = 0$ (B) $\vec{b}\cdot\vec{c} = 0,\ \vec{c}\cdot\vec{a} = 0$

(C) $\vec{c}\cdot\vec{a} = 0,\ \vec{a}\cdot\vec{b} = 0$ (D) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

**449.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$, and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linear dependent vectros and $|\vec{c}| = \sqrt{3}$, then

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$ (C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

**450.** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]$ is equal to

(A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

**451.** $(\vec{r}\cdot\hat{i})(\vec{r} \times \hat{i}) + (\vec{r}\cdot\hat{j})(\vec{r} \times \hat{j}) + (\vec{r}\cdot\hat{k})(\vec{r} \times \hat{k})$ is equal to

(A) $3\vec{r}$ (B) $\vec{r}$ (C) $\vec{0}$ (D) none of these

**452.** Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30º, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) 3

**453.** Let $\vec{a}$ and $\vec{b}$ are two vectors making angles $\theta$ with each other, then unit vectors along bisector of $\vec{a}$ and $\vec{b}$ is

(A) $\pm\frac{\hat{a}+\hat{b}}{2}$ (B) $\pm\frac{\hat{a}+\hat{b}}{2\cos\theta}$ (C) $\pm\frac{\hat{a}+\hat{b}}{2\cos\theta/2}$ (D) $\pm\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$

**454.** Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \neq 0$ and $\vec{a} \times \vec{b} = 2\vec{a} \times \vec{c}$, $|\vec{a}| = |\vec{c}| = 1$, $|\vec{b}|=4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$, if $\vec{b} - 2\vec{c} = \lambda\vec{a}$. Then $\lambda$ equals

(A) 1 (B) –1 (C) 2 (D) –4

**455.** Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero and non-coplanar vectros and $\vec{p}$, $\vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}$, $\vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{r} = \vec{a} - 4\vec{b} + 2\vec{c}$. If the volume of the parallelopiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$ is $V_1$ and that of the parallelopiped determined by $\vec{p}$, $\vec{q}$ and $\vec{r}$ is $V_2$ then $V_2 : V_1$ is equal to

(A) 3 : 1 (B) 7 : 1 (C) 11 : 1 (D) 15 : 1

**456.** The line joining the points $6\vec{a} - 4\vec{b} - 5\vec{c}$, $-4\vec{c}$ and the line joining the points $-\vec{a} - 2\vec{b} - 3\vec{c}$, $\vec{a} + 2\vec{b} - 5\vec{c}$ intersect at

(A) $2\vec{c}$ (B) $-4\vec{c}$ (C) $8\vec{c}$ (D) none of these

**457.** A vector $\vec{a} = (x, y, z)$ makes an obtuse angle with y-axis, equal angles with $\vec{b} = (y, -2z, 3x)$

and $\vec{c}$ =(2z, 3x, – y) and $\vec{a}$ is perpendicular to $\vec{d}$ = (1, –1, 2) if $|\vec{a}|$ = 2, then vector $\vec{a}$ is
(A) (1, 2, 3) (B) (2, –2, –2) (C) (–1, 2, 4) (D) none of these

**458.** The position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ of four points A, B, C and D on a plane are such that $(\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$, then the point D is
(A) centroid of $\Delta$ ABC (B) orthocentre of $\Delta$ ABC
(C) circumcentre of $\Delta$ ABC (D) none of these

**459.** Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is $\vec{a} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector
(A) $-9\hat{i} + 5\hat{j} + 2\hat{k}$ (B) $9\hat{i} + 5\hat{j} - 2\hat{k}$ (C) $9\hat{i} - 5\hat{j} - 2\hat{k}$ (D) $9\hat{i} + 5\hat{j} + 2\hat{k}$

**460.** Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a}$ and $\vec{b}$ is -
(A) $\frac{19}{5\sqrt{43}}$ (B) $\frac{19}{3\sqrt{43}}$ (C) $\frac{19}{2\sqrt{45}}$ (D) $\frac{19}{6\sqrt{43}}$

**461.** If 3a + 2b + 6c = 0, then family of straight lines ax + by + c = 0 passes through a fixed point whose coordinates are given by
(A) (1/2, 1/3) (B) (2, 3) (C) (3, 2) (D) (1/3, 1/2)

**462.** Equation of a straight line passing through the point of intersection of x – y + 1 = 0 and 3x + y – 5 = 0 are perpendicular to one of them is
(A) x + y + 3 = 0 (B) x + y – 3 = 0 (C) x – 3y – 5 = 0 (D) x + 3y + 5 = 0

**463.** The points (p + 1,1), (2p + 1,3) and (2p + 2, 2p) are collinear, if
(A) p = –1 (B) p = 1/2 (C) p = 2 (D) $p = -\frac{1}{3}$

**464.** If $m_1$ and $m_2$ are the roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$ and if area of the triangle formed by the lines $y = m_1 x$, $y = m_2 x$, and y = c is $(a + b)c^2$, then the value of 2008 $(a^2 + b^2)$ must be
(A) 5050 (B) 2255 (C) 5522 (D) none of these

**465.** If the lines x = a + m, y = –2 and y = mx are concurrent, the least value of |a| is :
(A) 0 (B) $\sqrt{2}$ (C) $2\sqrt{2}$ (D) none of these

**466.** A focal chord of $y^2 = 4ax$ meets in P and Q. If S is the focus, then $\frac{1}{SP} + \frac{1}{SQ}$ is equal to
(A) $\frac{1}{a}$ (B) $\frac{2}{a}$ (C) $\frac{4}{a}$ (D) none

**467.** If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2 - 5)}$ represents an ellipse with major axis as y-axis and f is a decreasing function, then
(A) $a \in (-\infty, 1)$ (B) $a \in (5, \infty)$ (C) $a \in (1, 4)$ (D) $a \in (-1, 5)$

**468.** The set of positive value of m for which a line with slope m is a comon tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and parabola $y^2 = 4ax$ is given by

(A) (2, 0) (B) (3, 5) (C) (0, 1) (D) none of these

**469.** From a point on the line y = x + c, c (parameter), tangents are drawn to the hyperbola $\frac{x^2}{2}-\frac{y^2}{1}=1$ such that chords of contact pass through a fixed point $(x_1, y_1)$. Then $\frac{x_1}{y_1}$ is equal to

(A) 2 (B) 3 (C) 4 (D) none

**470.** If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of $b^2$ is

(A) 3 (B) 16 (C) 9 (D) 12

**471.** Column 2 contains curves satisfying the condition in column I.

| **Column – I** | **Column – II** |
|---|---|
| (a) Subtangent is constant | (p) Parabola |
| (b) Subnormal is constant | (q) Ellipse |
| (c) Subtangent is equal to twice the abscissa | (r) Hyperbola |
| (d) Subnormal is equal to twice the abscissa | (s) Exponential curve |

(A) a → s, b → p, c → p, d → r (B) a → p, b → p, c → r, d → r

(C) a → s, b → r, c → p, d → r (D) None of these

**472.** The distance of the point (2, 1, −2) from the line $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-3}$ measured parallel to the plane x + 2y + z = 4 is

(A) $\sqrt{10}$ (B) $\sqrt{20}$ (C) $\sqrt{5}$ (D) $\sqrt{30}$

**473.** The shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (C) $3\sqrt{6}$ (D) $5\sqrt{6}$

**474.** The equation of the line passing through (1, 2, 3) and parallel to the planes x − y + 2z = 5 and 3x + y + z = 6 is

(A) $\frac{x-1}{-3}=\frac{y-2}{5}=\frac{z-3}{4}$ (B) $\frac{x-1}{-3}=\frac{y-2}{-5}=\frac{z-1}{4}$

(C) $\frac{x-1}{-3}=\frac{y-2}{-5}=\frac{z-1}{-4}$ (D) None

**475.** The line $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}$ and the plane 4x + 5y + 3z − 5 = 0 intersect at a point

(A) (3, −1, 1) (B) (3, −2, 1) (C) (2, −1, 3) (D) (−1, −2, −3)

**476.** A line makes angles of 45º and 60º with the positive axes of X and Y respectively. The angle made by the same line with the positive Z axis is

(A) 30º or 60º (B) 60º or 90º (C) 90º or 120º (D) 60º or 120º

**477.** The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ meets the co-ordinate axes in A, B, C. The centroid of the triangle ABC is

(A) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (B) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$ (C) $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$ (D) (a, b, c)

**478.** The line $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, **if**

(A) k = 0 or –1 (B) k = 0 or 1 (C) k = 0 or –3 (D) k = 3 or –3

**479.** The position vectors of the points P and Q with respet to the origin O are $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$, respectively. If M is a point on PQ, such that OM is the bisector of POQ, then $\overrightarrow{OM}$ is

(A) $2(\hat{i} - \hat{j} + \hat{k})$ (B) $2\hat{i} + \hat{j} - 2\hat{k}$ (C) $2(-\hat{i} + \hat{j} - \hat{k})$ (D) $2(\hat{i} + \hat{j} + \hat{k})$

**480.** If $\vec{b}$ is vector whose initial point divides the join of $5\hat{i}$ and $5\hat{j}$ in the ratio k : 1 and terminal point is origin and $|\vec{b}| \le \sqrt{37}$, then k lies in the interval

(A) [–6, –1/6] (B) $(-\infty, -6] \cup [-1/6, \infty)$ (C) [0, 6] (D) None of these

**481.** Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then one equatin of the bisector of the angle PQR is

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

**482.** If the vertices of a triangle have integral co-ordinates, the triangle can not be

(A) an equilateral triangle (B) a right angled triangle
(C) an isosceles triangle (D) none of the above

**483.** If $P(a_1, b_1)$ and $Q(a_2, b_2)$ are two points, then OP, OQ cos ($\angle$ POQ) is (O is origin)

(A) $a_1a_2 + b_1b_2$ (B) $a_1^2 + a_2^2 + b_1^2 + b_2^2$

(C) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (D) none of tehse

**484.** If points A $(x_1, y_1)$, B$(x_2, y_2)$ and C$(x_3, y_3)$ are such that $x_1, x_2, x_3$, and $y_1, y_2, y_3$ are in G.P., with same common ratio then

(A) A,B and C are concyclic points (B) A,B and C are collinear points
(C) A,B and C are vertices of an equilateral triangle (D) None of the above

**485.** The equation of the line parallel to lines $L_1 \equiv x + 2y - 5 = 0$ and $L_2 \equiv x + 2y + 9 = 0$ and dividing the distance between $L_1$ and $L_2$ in the ratio 1 : 6 (internally), is

(A) x + 2y – 3 = 0 (B) x + 2y + 2 = 0 (C) x + 2y + 7 = 0 (D) None of these

**486.** Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the three vectors having magnitudes 1, 5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is $\theta$ and $\vec{a} \times (\vec{a} \times \vec{b}) = \vec{c}$, then tan $\theta$ is equal to

(A) 0 (B) 2/3 (C) 3/5 (D) 3/4

**487.** $\vec{a}$ and $\vec{c}$ are unit vectors and $|\vec{b}| = 4$. If angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$ and $\vec{a} \times \vec{b} = 2\vec{a} \times \vec{c}$, then $\vec{b} = \lambda\vec{a} + 2\vec{c}$, where $\lambda$ is equal to

(A) $\pm \frac{1}{4}$ (B) $\pm \frac{1}{2}$ (C) $\pm 1$ (D) $\pm 4$

**488.** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors, then the triple product $[\vec{a} + \vec{b} + \vec{c} \quad \vec{a} + \vec{b} \quad \vec{b} + \vec{c}]$ equals

(A) 0 (B) 1 or –1 (C) 1 (D) 3

**489.** If $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = a\vec{\delta}$ and $\vec{\beta} + \vec{\gamma} + \vec{\delta} = b\vec{\alpha}$ and $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ are non-coplanar and $\vec{\alpha}$ is not parallel to $\vec{\delta}$, then $\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta}$ equals

(A) $a\vec{\alpha}$ (B) $b\vec{\delta}$ (C) 0 (D) $(a + b)\vec{\gamma}$

**490.** The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ if c is equal to

(A) $\pm 1$ (B) $\pm \frac{1}{3}$ (C) $\pm \sqrt{5}$ (D) None

**Questions based on statements (Q. 491 - 500)**

**Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.**

**(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.**

**(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.**

**(C) If Statement-I is true but Statement - II is false.**

**(D) If Statement-I is false but Statement - II is true.**

**491.** **Statement–I :** The lines $(a + b)x + (a – b)y – 2ab = 0$, $(a – b)x + (a + b)y – 2ab = 0$ and $x + y = 0$ form an isosceles triangle.

**Statement–II :** If internal bisector of any angle of triangle is perpendiuclar to the oppoiste side, then the given triangle is isosceles.

**492.** **Statement–I :** The chord of contact of tangent from three points A, B, C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B, C will be collinear.

**Statement–II :** A, B, C always lies on the normal to the circle $x^2 + y^2 = a^2$

**493.** Let $C_1$ be the circle with centre $O_1$ (0, 0) and radius 1 and $C_2$ be the circle with centre $O_2$ $(t, t^2 + 1)$ $(t \in R)$ and radius 2.

**Statement–I :** Circles $C_1$ and $C_2$ always have at least one common tangent for any value of t.

**Statement–II:** For the two circles, $O_1 O_2 \geq |r_1 – r_2|$, where $r_1$ and $r_2$ are their radii for any value of t.

**494.** **Statement–I :** If end points of two normal chords AB and CD (normal at A and C) of a parabola $y^2=4ax$ are concyclic, then the tangents at A and C will intersect on the axis of the parabola.
**Statement–II :** If four point on the parabola $y^2 = 4ax$ are concyclic, then sum of their ordinates is zero.

**495.** **Statement–I :** Locus o fthe centre of a variable circle touching two cicles $(x-1)^2+(y-2)^2=25$ and $(x - 2)^2 + (y - 1)^2 = 16$ is an ellipse.
**Statement–II :** If a circle $S_2 = 0$ lies completely inside the circle $S_1 = 0$, then locus of the centre of a variable circle $S = 0$ that touches both the circles is an ellipse.

**496.** **Statement–I :** Given the base BC of the triagle and the ratio radius of the ex-circles opposite to the angles B and C. Then locus of the vertex A is hyperbola.
**Statement–II :** $|S'P - SP| = 2a$, where S and S′ are the two foci, 2a = length of the transverse axis and P be any point on the hyperbola.

**497.** Let $\vec{r}$ be a non-zero vector satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ for given non-zero vectors $\vec{a}.\vec{b}$ and $\vec{c}$.

**Statement–I :** $[\vec{a}-\vec{b} \;\; \vec{b}-\vec{c} \;\; \vec{c}-\vec{a}] = 0$

**Statement–II :** $[\vec{a}\,\vec{b}\,\vec{c}] = 0$

**498.** **Statement–I :** $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are non-coplanar. If $[\vec{d}\,\vec{b}\,\vec{c}] = [\vec{d}\,\vec{a}\,\vec{b}] = [\vec{d}\,\vec{c}\,\vec{a}] = 1$, then $\vec{d} = \vec{a} + \vec{b} + \vec{c}$.

**Statement–II :** $[\vec{d}\,\vec{b}\,\vec{c}] = [\vec{d}\,\vec{a}\,\vec{b}] = [\vec{d}\,\vec{c}\,\vec{a}] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

**499.** The equation of two straight lines are $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{-3}$ and $\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z+3}{2}$.

**Statement–I :** The given lines are coplanar.
**Statement–II :** The equations $2x_1 - y_1 = 1$, $x_1 + 3y_1 = 4$ and $3x_1 + 2y_1 = 5$ are consistent.

**500.** **Statement–I :** There exist two points on the line $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+2}{2}$ which are at a distance of 2 units from point (1, 2, –4).

**Statement–II :** Perpendicular distance of point (1, 2, –4) from the line $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+2}{2}$ is 1 unit.

# CO–ORDINATE GEOMETRY

| | | | | | |
|---|---|---|---|---|---|
| **361.** D | **362.** A | **363.** D | **364.** B | **365.** C | **366.** C |
| **367.** C | **368.** D | **369.** A | **370.** B | **371.** B | **372.** A |
| **373.** B | **374.** B | **375.** A | **376.** C | **377.** A | **378.** C |
| **379.** A | **380.** D | **381.** B | **382.** C | **383.** C | **384.** A |
| **385.** A | **386.** D | **387.** B | **388.** B | **389.** C | **390.** A |
| **391.** A | **392.** A | **393.** B | **394.** C | **395.** C | **396.** A |
| **397.** B | **398.** B | **399.** D | **400.** D | **401.** A | **402.** A |
| **403.** D | **404.** D | **405.** C | **406.** B | **407.** C | **408.** D |
| **409.** C | **410.** D | **411.** C | **412.** B | **413.** B | **414.** C |
| **415.** C | **416.** B | **417.** B | **418.** B | **419.** C | **420.** A |
| **421.** A | **422.** D | **423.** B | **424.** B | **425.** B | **426.** C |
| **427.** B | **428.** A | **429.** B | **430.** B | **431.** B | **432.** D |
| **433.** C | **434.** D | **435.** C | **436.** A | **437.** D | **438.** C |
| **439.** D | **440.** C | **441.** A | **442.** D | **443.** C | **444.** A |
| **445.** D | **446.** D | **447.** C | **448.** D | **449.** D | **450.** A |
| **451.** C | **452.** B | **453.** C | **454.** D | **455.** D | **456.** B |
| **457.** B | **458.** B | **459.** B | **460.** A | **461.** A | **462.** B |
| **463.** C | **464.** C | **465.** C | **466.** A | **467.** D | **468.** C |
| **469.** A | **470.** B | **471.** A | **472.** D | **473.** B | **474.** A |
| **475.** A | **476.** D | **477.** D | **478.** C | **479.** B | **480.** B |
| **481.** C | **482.** A | **483.** A | **484.** B | **485.** A | **486.** D |
| **487.** D | **488.** B | **489.** C | **490.** C | **491.** A | **492.** C |
| **493.** A | **494.** A | **495.** D | **496.** D | **497.** B | **498.** B |
| **499.** A | **500.** C | | | | |

# HINTS & SOLUTIONS : CO-ORDINATE GEOMETRY

**361. D**

The image of P(a, b) on y=−x is Q(−b, −a) (interchange and change signs) and the image of Q(−b, −a) on y = x is R(−a, −b) (merely interchange)

$\therefore$ The mid point of PR is (0, 0).

**362. A**

As (−1, 1) is a point on 3x − 4y + 7 = 0, the rotation is possible.

Slope of the given line = $\frac{3}{4}$.

Slope of the line in its new position$=\dfrac{\frac{3}{4}-1}{1+\frac{3}{4}}=-\dfrac{1}{7}$

The required equation is $y-1=-\frac{1}{7}(x+1)$

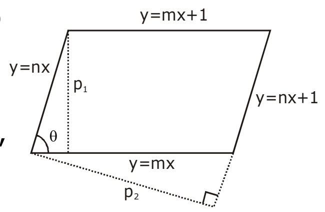
or 7y + x − 6 = 0

**363. D**

If $p_1$ and $p_2$ be the distance between parallel sides and $\theta$ be the angle between adjacent sides, than Required area

$= p_1\, p_2 \operatorname{cosec}\theta$

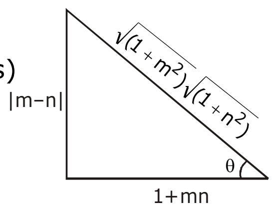
Where,




$p_1 = \dfrac{1}{\sqrt{(1+m^2)}}$,

$p_2 = \dfrac{1}{\sqrt{(1+n^2)}}$

(distance between || lines)




and $\tan\theta = \dfrac{|m-n|}{|1+mn|}$

$\therefore$ Required area

$$=\frac{1}{\sqrt{(1+m^2)}\sqrt{(1+n^2)}}\cdot\frac{\sqrt{(1+m^2)}\sqrt{(1+n^2)}}{|m=n|}=\frac{1}{|m-n|}$$

**364. B**

Equation of line $\frac{ax}{c-1}$ and $\frac{by}{c-1}+1=0$ has two independent parameters. It can pass through a fixed point if it contains only one independent parameter. Now there must be one relation between $\frac{a}{c-1}$ and $\frac{b}{c-1}$ independent of a,b and c so that $\frac{a}{c-1}$ can be expressed in terms of $\frac{b}{c-1}$ and straight line contains only one independent parameter. Now that given relation can be expressed as

$$\frac{5a}{c-1}+\frac{4b}{c-1}=\frac{t-20c}{c-1}$$

RHS is independent of c if t = 20.

**365. C**

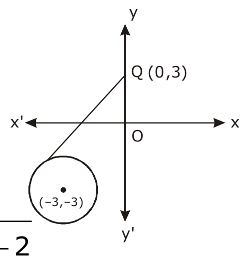
$$\frac{x-2}{\cos 45^\circ}=\frac{y-1}{\sin 45^\circ}=-4$$

$\Rightarrow x-2=-2\sqrt{2},\ y-1=-2\sqrt{2}$

$\therefore\ x=2-2\sqrt{2},\ y=1-2\sqrt{2}$

$\Rightarrow\ A'=(2-2\sqrt{2},\ 1-2\sqrt{2})$

**366. C**




$\because$ Q lies on y-axis,

Put x = 0

in 5x − 2y + 6 = 0

$\therefore$ y = 3

$\Rightarrow$ Q(0, 3)

$\therefore$ PQ

$=\sqrt{0^2+3^2+0+6\times 3-2}$

$=\sqrt{25}=5$

**367. C**

The centre of the circle is the point of intersection of the given diameters 2x−3y=5 and 3x − 4y = 7. Which is (1, −1) and the radius is r, where $\pi r^2=154 \Rightarrow r^2$

$=154\times\frac{7}{22}\Rightarrow r=7$ and hence the required equation of the circle is $(x-1)^2+(y+1)^2=7^2$

$\Rightarrow\ x^2+y^2-2x+2y=47$

**368. D**

Equation of pair of tangents by $SS_1=T^2$ is $(a^2-1)y^2-x^2+2ax-a^2=0$

If $\theta$ be the angle between the tangents, then

$$\tan\theta=\frac{2\sqrt{(h^2-ab)}}{a+b}=\frac{2\sqrt{-(a^2-1)(-1)}}{a^2-2}=\frac{2\sqrt{a^2-1}}{a^2-2}$$

$\because$ $\theta$ lies in II quadrant, then $\tan\theta<0$

$\therefore\ \dfrac{2\sqrt{a^2-1}}{a^2-2}<0$

$\Rightarrow a^2 - 1 > 0$ and $a^2 - 2 < 0 \Rightarrow 1 < a^2 < 2$

$\Rightarrow a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$

**369. A**

Given circle is $(x - 3)^2 + (y - 4)^2 = 25$
Since, locus of point of intersection of two perpendicular tangents is director circle, then its equation is
$(x - 3)^2 + (y - 4)^2 = 50$
$\Rightarrow x^2 + y^2 - 6x - 8y - 25 = 0$

**370. B**

Let $x_1 = \sqrt{3h+2}$ and $y_1 = \sqrt{3k}$

$\therefore \quad x_1^2 + y_1^2 = 3(h + k) + 2 = 3(1) + 2$

$(\because h + k = 1)$

$\Rightarrow x_1^2 + y_1^2 = 5$ Locus of $(x_1, y_1)$ is $x^2 + y^2 = 5$

**371. B**

Shortest distance between two curves occured along the common normal.
Normal to $y^2 = 4x$ at $(m^2, 2m)$ is
$y + mx - 2m - m^3 = 0$

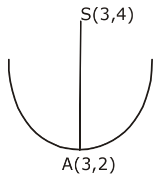
Normal to $y^2 = 2(x - 3)$ at $\left(\frac{m^2}{2} + 3, m\right)$ is

$y + m(x - 3) - m - \frac{m^3}{2} = 0$

Both are same if $-2m - m^3 = -4m - \frac{1}{2}m^3$
$\Rightarrow m = 0, \pm 2$ So, points will be (4, 4) and (5, 2) or (4, −4) and (5, −2)

Hence, shortest distance will be $\sqrt{(1+4)} = \sqrt{5}$

**372. A**



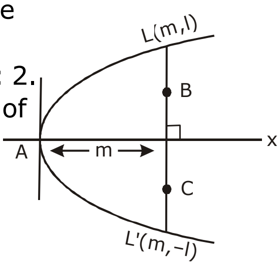

$y^2 - 12x - 4y + 4 = 0$
$\Rightarrow (y - 2)^2 = 12x$
It vertex is (0, 2) and a = 3,
its focus = (3, 2)
Hence, for the required parabola ;
focus is (3, 4) vertex = (3, 2) and a = 2
Hence, the equation of the parabola is
$(x - 3)^2 = 4(2)(y - 2)$ or $x^2 - 6x - 8y + 25 = 0$

**373. B**

Chord of contact of mutually perpendicular tangents is always a focal chord.
Therefore minimum length of AB is 4a

**374. B**

Let B and C be the trisected points.
B divides LL' in 1 : 2.
Then, coordinate of

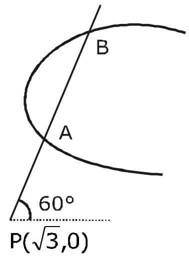



$B\left(\frac{1.m + 2.m}{1+2}, \frac{-\ell + 2\ell}{3}\right)$

or B $(m, \ell/3)$
Let $x_1 = m$, $y_1 = \ell/3$
$\therefore \quad (m, \ell) = (x_1, 3y_1)$
but $(m, \ell)$ lie on parabola, $(3y_1)^2 = 4ax_1$

$\Rightarrow 9y_1^2 = 4ax_1 \quad \therefore$ Locus $9y^2 = 4ax$

**375. A**




$\because P \equiv (\sqrt{3}, 0)$

$\frac{x - \sqrt{3}}{\cos 60^\circ} = \frac{y - 0}{\sin 60^\circ} = r$

or $x = \sqrt{3} + \frac{r}{2}$, $y = \frac{r\sqrt{3}}{2}$

or $\left(\sqrt{3} + \frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ lie on $y^2 = x + 2$, then

$\frac{3r^2}{4} = \sqrt{3} + \frac{r}{2} + 2 \Rightarrow \frac{3r^2}{4} - \frac{r}{2} - (2 + \sqrt{3}) = 0$

$\therefore PA.\, PB = r_1 r_2 = \left|\frac{-(2+\sqrt{3})}{\frac{3}{4}}\right| = \frac{4}{3}(2 + \sqrt{3})$

**376. C**

The tangent at the point of shortest distance from the line x + y = 7 parallel to the given line. Any point on the given ellipse is $(\sqrt{6}\cos\theta, \sqrt{3}\sin\theta)$.
Equation of the tangent is

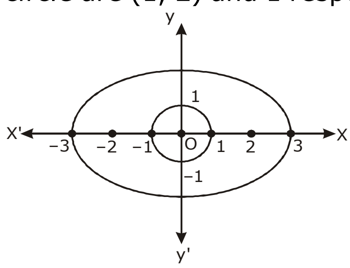
$\frac{x\cos\theta}{\sqrt{6}} + \frac{y\sin\theta}{\sqrt{3}} = 1$. it is parallel to x+y=7

$\Rightarrow \frac{\cos\theta}{\sqrt{6}} = \frac{\sin\theta}{\sqrt{3}} \Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{\sin\theta}{1} = \frac{1}{\sqrt{3}}$

The required point is (2, 1).

**377. A**

Centre of the ellipse is (1, 2) and length of major axis and minor axis are 6 and 4 respectively and centre and radius of the circle are (1, 2) and 1 respectively.




Hence, ellipse and circle do not touch or cut. $\quad \therefore$ Common chord impossible.
$\therefore$ Hence, length of common chord = 0

**378. C**

$\because$ Latusrectum $= \frac{1}{2}$ (minor axis)

$\Rightarrow \frac{2b^2}{a} = \frac{1}{2}(2b) \Rightarrow 2b = a \Rightarrow 4b^2 = a^2$

$\Rightarrow 4a^2(1 - e^2) = a^2 \Rightarrow 4 - 4e^2 = 1$

$\therefore e^2 = \frac{3}{4} \Rightarrow e = \frac{\sqrt{3}}{2}$

**379. A**

Let mid point of focal chord is $(x_1, y_1)$ then equation of a chord whose mid point $(x_1, y_1)$ is $T = S_1$

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$

$\Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$

Since, it is a focal chord, then its passes through focus $(\pm ae, 0)$, then

$\pm \frac{ex_1}{a} + 0 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$

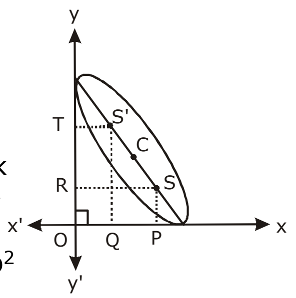
$\therefore$ Locus of mid point of focal chord is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \pm \frac{ex}{a}$

**380. D**

Let $S \equiv (x_1, y_1)$, $S' \equiv (x_2, y_2)$

Let $C \equiv (h, k)$




$\therefore \frac{x_1 + x_2}{2} = h$

$\Rightarrow x_1 + x_2 = 2h$

and $y_1 + y_2 = 2k$

$\because$ SP. S' Q $= b^2$

$\Rightarrow y_1y_2 = b^2$

and SR. S' T $= b^2$

$\Rightarrow x_1x_2 = b^2$

Distance between foci SS' = 2ae

$\Rightarrow \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = (2ae)$

$\Rightarrow (x_1 - x_2)^2 + (y_1 - y_2)^2 = 4a^2e^2$

$\Rightarrow (x_1+x_2)^2 - 4x_1x_2 + (y_1 + y_2)^2 - 4y_1y_2 = 4a^2e^2$

$\Rightarrow 4h^2 - 4b^2 + 4k^2 - 4b^2 = 4(a^2 - b^2)$

$\Rightarrow h^2 + k^2 - 2b^2 = a^2 - b^2$

$\therefore h^2 + k^2 = a^2 + b^2$

Locus of centre is $x^2 + y^2 = a^2 + b^2$ which is a circle

**381. B**

If eccentricities of ellipse and hyperbola are e and $e_1$

$\therefore$ Foci $(\pm ae, 0)$ and $(\pm a_1e_1, 0)$

Here, $ae = a_1e_1$

$a^2e^2 = a_1^2 e_1^2$

$a^2\left(1 - \frac{b^2}{a^2}\right) = a_1^2\left(1 + \frac{b_1^2}{a_1^2}\right)$

$\Rightarrow a^2 - b^2 = a_1^2 + b_1^2$

$\Rightarrow 25 - b^2 = \frac{144}{25} + \frac{81}{25} = 9 \therefore b^2 = 16$

**382. C**

For rectangular hyperbola

$a - 2 = -a$ $\qquad\therefore a = 1$

**383. C**

Since, asymptotes $3x + 4y = 2$ and $4x - 3y + 5 = 0$ are perpendicular to each other. Hence, hyperbola is rectangular hyperbola but we know that the eccentricity of rectangular hyperbola is $\sqrt{2}$.

**384. A**

Centre of hyperbola is $\left(\frac{6-4}{2}, \frac{5+5}{2}\right)$

ie, (1, 5)

Distance between foci = 2ae

$10 = 2ae \Rightarrow 5 = a \times \frac{5}{4} \therefore a = 4$

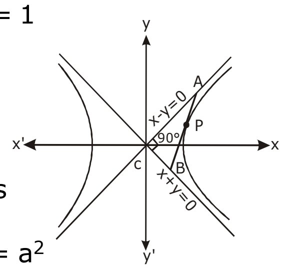
$b^2 = a^2(e^2 - 1) = 16\left(\frac{25}{16} - 1\right)$

$= 25 - 16 = 9$

$\therefore$ Equation of hyperbola is

$\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

**385. A**




$\because x - y = 0$

and $x + y = 0$

are the asymptotes of the rectangular hyperbola $x^2 - y^2 = a^2$

Equation of tangent at $P(a \sec\phi, a\tan\phi)$ of $x^2 - y^2 = a^2$ is

$ax\sec\phi - ay\tan\phi = a^2$

or $x\sec\phi - y\tan\phi = a$ .....(i)

Solving $y = x$ and $y = -x$ with Eq. (i), then we get and

$$\left\{\begin{array}{l} A(a(\sec\varphi + \tan\varphi), a(\sec\varphi + \tan\varphi)) \\ \text{and } B(a(\sec\varphi - \tan\varphi), a(\tan\varphi - \sec\varphi)) \end{array}\right\}$$

$\therefore$ Area of

$\Delta CAB = \frac{1}{2}\left|a(\tan^2\phi - \sec^2\phi) - a(\sec^2\phi - \tan^2\phi)\right|$

$= \frac{1}{2}|-a - a| = |-a| = |a|$

**386. D**

Any point on given line is (2r + 1, −3r − 1, r), its distance from (1, −1, 0).

$\Rightarrow (2r)^2 + (-3r)^2 + r^2 = (4\sqrt{14})^2$

$\Rightarrow r = \pm 4$

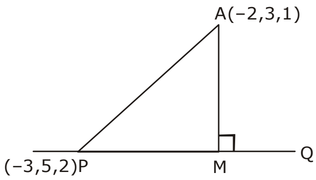
$\Rightarrow$ Coordinates are (9, −13, 4) and (−7, 11, −4) and nearer to the origin is (−7, 11, −4).

**387. B**

Here, $\alpha = \beta = \gamma$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \cos\alpha = \frac{1}{\sqrt{3}}$




DC's of PQ are

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

PM = Projection of AP on PQ

$= \left|(-2+3)\frac{1}{\sqrt{3}} + (3-5).\frac{1}{\sqrt{3}} + (1-2).\frac{1}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$

and AP $= \sqrt{(-2+3)^2 + (3-5)^2 + (1-2)^2} = \sqrt{6}$

AM $= \sqrt{(AP)^2 - (PM)^2} = \sqrt{6 - \frac{4}{3}} = \sqrt{\frac{14}{3}}$

**388. B**

The straight line joining the points (1,1,2) and (3, −2, 1) is

$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-2}{-1} = r$ (say)

$\therefore$ Point is (2r + 1, 1 − 3r, 2 − r)

which lies on 3x + 2y + z = 6

$\therefore 3(2r + 1) + 2(1 - 3r) + 2 - r = 6$

$\therefore$ r = 1 Required point is (3, −2, 1)

**389. C**

Let point is $(\alpha,\beta,\gamma)$

$\therefore (\alpha - a)^2 + \beta^2 + \gamma^2 = \alpha^2 + (\beta - b)^2 + \gamma^2$

$= \alpha^2 + \beta^2 + (\gamma - c)^2 = \alpha^2 + \beta^2 + \gamma^2$

we get, $\alpha = \frac{a}{2}$, $\beta = \frac{b}{2}$ and $\gamma = \frac{c}{2}$

$\therefore$ Required point is $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

**390. A**

Let plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ ....(i)

mid point of P(1, 2, 3) and Q(−3, 4, 5) ie, (−1, 3, 4) lie on Eq. (i)

$\therefore -\frac{1}{a} + \frac{3}{b} + \frac{4}{c} = 1$ ....(ii)

Also, PQ is parallel to normal of the plane (i)

$\frac{1/a}{-4} = \frac{1/b}{2} = \frac{1/c}{2}$

$\Rightarrow \frac{1}{-2a} = \frac{1}{b} = \frac{1}{c} = \lambda$ (say)

$\therefore \frac{1}{a} = -2\lambda, \frac{1}{b} = \lambda, = \frac{1}{c} = \lambda$

$\therefore$ From Eq. (ii), $2\lambda + 3\lambda + 4\lambda = 1 \therefore \lambda = \frac{1}{9}$

$a = -\frac{1}{2\lambda}, b = \frac{1}{\lambda}, c = \frac{1}{\lambda}$

$\therefore a = -\frac{9}{2}, b = 9, c = 9$

Intercepts are $\left(-\frac{9}{2}, 9, 9\right)$

**391. A**

Let the coordinates of A be (a, 0).

Then the slope of the reflected ray is

$\frac{3-0}{5-a} = \tan\theta$ (say) ....(i)

Then the slope of the incident ray

$= \frac{2-0}{1-a} = \tan(\pi - \theta)$ ...(ii)

From Eqs. (i) and (ii), $\tan\theta + \tan(\pi - \theta) = 0$

$\Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0$

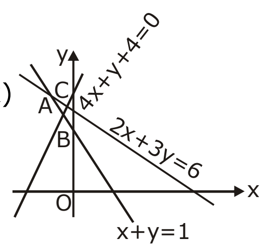
$\Rightarrow 3 - 3a + 10 - 2a = 0$ ; $a = \frac{13}{5}$

Thus, the coordinate of A is $\left(\frac{13}{5}, 0\right)$

**392. A**

Coordinates of A and B are

(−3, 4) and $\left(-\frac{3}{5}, \frac{8}{5}\right)$




If orthocentre P(h, k)

Then, slope of PA × slope of BC= −1

$\Rightarrow \frac{k-4}{h+3} \times 4 = -1$

$\Rightarrow 4k - 16 = -h - 3$

$\Rightarrow h + 4k = 13$ ....(i)

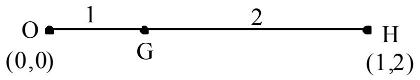
and slope of PB × slope of AC = −1

$\Rightarrow \frac{k - \frac{8}{5}}{h + \frac{3}{5}} \times -\frac{2}{3} = -1 \Rightarrow \frac{5k-8}{5h+3} \times \frac{2}{3} = 1$

$\Rightarrow 10k - 16 = 15h + 9 \Rightarrow 15h - 10k + 25 = 0$

$\Rightarrow 3h - 2k + 5 = 0$ ....(ii)

**393.** **[B]**




$G \equiv \left(\frac{1\times1+2\times0}{3}, \frac{2\times1+2\times0}{3}\right) = \left(\frac{1}{3},\frac{2}{3}\right)$

**394.** **C**

$\because$ (a, a) fall between the lines $|x + y| = 2$,

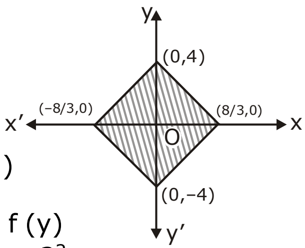
then $\frac{a+a-2}{a+a+2} < 0$

$\Rightarrow \frac{a-1}{a+1} < 0$ or $-1 < a < 1$ $\quad \therefore |a| < 1$

**395.** **C**

Required area

$= 4 \times \frac{1}{2}\left(\frac{8}{3}\times 4\right)$




$= \frac{64}{3} = \frac{2^6}{3}$ ...(i)

$\because f(x + y) = f(x)\, f(y)$

$\therefore f(2) = f(1)\, f(1) = 2^2$

$f(3) = f(1 + 2) = f(1)\, f(2) = 2^3$

.........................................

$\therefore f(n) = 2^n$

$\therefore$ Area $= \frac{2^6}{3} = \frac{f(6)}{3}$ sq unit.

**396.** **A**

Equation of conic is

$(2x - y + 1)(x - 2y + 3) + \lambda xy = 0$

for circle coefficient of xy = 0

ie, $-5 + \lambda = 0$, $\therefore \lambda = 5$

$\therefore$ Circle is $2x^2 + 2y^2 + 7x - 5y + 3 = 0$

$\therefore x^2 + y^2 + \frac{7}{2}x - \frac{5}{2}y + \frac{3}{2} = 0$

$\therefore$ Center is $\left(-\frac{7}{4},\frac{5}{4}\right)$

**397.** **B**

We know, that, limiting points of the co-axial system of circles

$x^2 + y^2 + 2gx + 2fy + c=0$

(other than origin) are given by

$\left(-\frac{gc}{g^2+f^2},\frac{-fc}{g^2+f^2}\right)$

Here given circle $x^2 + y^2 - 6x - 8y + 1 = 0$, then $g = -3$, $f = -4$, $c = 1$.

Hence other limiting point is $\left(\frac{3}{25},\frac{4}{25}\right)$

**398.** **B**

Centre $C \equiv (7, 5)$ and radius

$r = \sqrt{(49+25+151)} = 15$

If P(2, –7) The shortest distance

$= |CP - r| = \left|\sqrt{25+144}-15\right| = |13 - 15| = 2$

**399.** **D**

The image of the circle has same radius butcentre different. If centre is $(\alpha, \beta)$,

then $\frac{\alpha-3}{1} = \frac{\beta-2}{1} = \frac{-2(3+2-19)}{1^2+1^2}$

$\Rightarrow \alpha - 3 = \beta - 2 = 14 \therefore \alpha = 17, \beta = 16$

$\therefore$ Required circle is $(x - 17)^2 + (y - 16)^2 =1$

**400.** **D**

Let centre of the circle be (h, k)

$\because$ circle touch y-axis $\therefore$ radius = |h|

Equation of Circle is $(x - h)^2+(y - k)^2=h^2$ ....(i)

Given circle (i) touches externally the circle

$x^2 + y^2 - 6x - 6y + 14 = 0$

$\therefore$ Distance between centres = sum of radii

$\Rightarrow \sqrt{(h-3)^2+(k-3)^2} = |h| + 2$

$\Rightarrow (h - 3)^2 + (k - 3)^2 = h^2 + 4 + 4|h|$

$\Rightarrow k^2 - 6h - 4|h| - 6k + 14 = 0$

Here h always +ve

$\therefore \quad k^2 - 10h - 6k + 14 = 0$

$\therefore$ Locus of centre of circle is

$y^2 - 10x - 6y + 14 = 0$

**401.** **A**

Let $A \equiv (at_1^2, 2at_1)$, $B\ (at_2^2, 2at_2)$

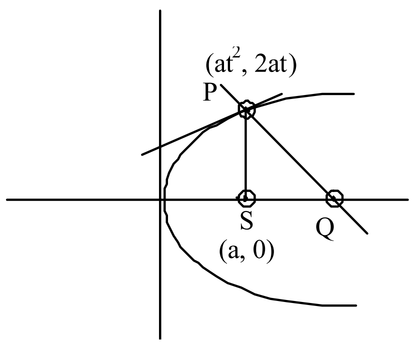
then, $C \equiv (at_1t_2, a(t_1 + t_2))$

$\because$ ordinates of A,B,C are $2at_1$, $2at_2$ and $a(t_1 + t_2)$ also,

$\frac{\text{ordinate of A} + \text{ordinate of B}}{2}$ = ordinate of C

Hence, ordinates of A, C and B are in AP.

**402.** **[A]**



$y^2 = 4ax$

$2y \frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Equation of normal at P
$y - 2at = -t(x - at^2)$
Q $(2a + at^2, 0)$
$PS = x + a = at^2 + a$
$SQ = at^2 + a$
$PS = SQ$
Isosceles

**403. D**

Let $(\alpha,\beta)$ be the feet of perpendicular from $(-1, 1)$ on directrix $4x + 3y - 24 = 0$, then

$\frac{\alpha+1}{4} = \frac{\beta-1}{3} = -\left(\frac{-4+3-24}{4^2+3^2}\right) = 1$

or $\alpha = 3, \beta = 4$ $\therefore (\alpha, \beta) \equiv (3, 4)$
Hence, vertex is the mid point of (3, 4) and (−1, 1) ie, (1, 5/2)

**404. D**

Equation of tangent in terms of slope of parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$ ....(i)

$\because$ Eq. (i), is also tangent of $x^2 = -32y$

then $x^2 = -32\left(mx + \frac{1}{m}\right)$

$\Rightarrow x^2 + 32mx + \frac{32}{m} = 0$

$\therefore B^2 = 4AC$ (Condition of tangency)

$\Rightarrow (32m)^2 = 4.1. \frac{32}{m} \Rightarrow m^3 = \frac{1}{8}$ or $m = \frac{1}{2}$

From Eq. (i), $y = \frac{x}{2} + 2 \Rightarrow x - 2y + 4 = 0$

**405. C**

$y = m(x + 1) + (1/m)$

or $y = mx + \left(m + \frac{1}{m}\right)$ ...(i)

is a tangent to the first parabola

and $y = m'(x + 2) + \frac{2}{m'}$

$= m'x + 2\left(m' + \frac{1}{m'}\right)$ ....(ii)

is a tangent to the second parabola given

$m.m' = -1$ or $m' = -\frac{1}{m}$ Then, from Eq. (ii)

$y = -\frac{x}{m} + 2\left(-\frac{1}{m} - m\right)$

$\Rightarrow y = -\frac{x}{m} - 2\left(m + \frac{1}{m}\right)$ ...(iii)

Substracting Eq. (iii) from Eq. (i), then

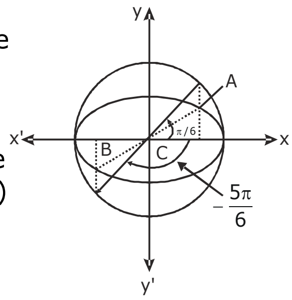
$x\left(m + \frac{1}{m}\right) + 3\left(m + \frac{1}{m}\right) = 0$ or $x + 3 = 0$

**406. B**

$\because$ Eccentric angle of A is $\frac{\pi}{6}$

$\therefore$ Eccentric angle of B is $-(\pi - \pi/6)$

ie, $-\frac{5\pi}{6}$




**407. C**

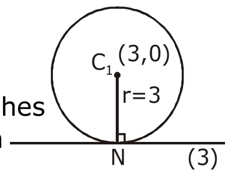
$y^2 = 4x$ ....(1)
$(x - 3)^2 + y^2 = 9$ ....(2)

$\left\{\begin{array}{l} y^2 = 4ax \\ 4a = 4 \\ a = 1 \end{array}\right.$

Equation of tangent of parabola (1) is

$y = mx + \frac{1}{m}$ ...(3)

$\Rightarrow$ line (3) is also touches the circle (2), then
$\Rightarrow C_1N = r \Rightarrow m = ?$




**408. D**

Do yourself

**409. C**

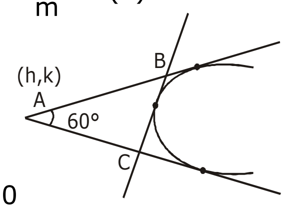
$y^2 = 4ax$ ...(1)
Equation of any tangent to the parabola is $y = mx + \frac{a}{m}$ ....(2)

If passes through A(h, k)

$\Rightarrow k = mh + \frac{}{m}$

$\Rightarrow m^2h - mk + a = 0$
Let $m_1$ & $m_2$ be the roots, then

$\Rightarrow m_1 + m_2 = \frac{k}{h}$ & $m_1m_2 = \frac{a}{h}$




$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \Rightarrow 3(x + a)^2 = y^2 - 4ax$

**410. D**

$y^2 = 4x$....(1), $x^2 = -8y$ ....(2)
Equation of tangent of the parabola (1) is

$y = mx + \frac{1}{m}$ . ...(3) this is also the tangent to the parabola (2), then $x^2 = -8\left(mx + \frac{1}{m}\right)$

$\Rightarrow mx^2 + 8m^2x + 8 = 0$ has equal roots

$\therefore 64m^4 = 32m \Rightarrow m = \left(\frac{1}{2}\right)^{1/3}$

$\therefore$ By (3) $\Rightarrow y = \left(\frac{1}{2}\right)^{1/3} + (2)^{1/3}$

**411. C**

Let rectangular hyperbola

$xy = c^2$ ...(i)

Let three points on Eq. (i) are

$A\left(ct_1, \frac{c}{t_1}\right)$, $B\left(ct_2, \frac{c}{t_2}\right)$, $C\left(ct_3, \frac{c}{t_3}\right)$

Let orthocentre is P(h, k) then slope of AP × slope of BC = –1

$$\Rightarrow \frac{k - \frac{c}{t_1}}{h - ct_1} \times \frac{\frac{c}{t_3} - \frac{c}{t_2}}{ct_3 - ct_2} = -1$$

$$\Rightarrow \frac{k - \frac{c}{t_1}}{h - ct_1} \times -\frac{1}{t_2\, t_3} = -1$$

$\Rightarrow k - \frac{c}{t_1} = ht_2\, t_3 - ct_1t_2t_3$ .....(ii)

Similarly, $BP \perp AC$

then $k - \frac{c}{t_2} = ht_3t_1 - ct_1t_2t_3$ ......(iii)

Substracting Eq. (iii) from Eq. (ii), then we get $h = -\frac{c}{t_1t_2t_3}$

Substituting the value h in Eq. (ii) then $k = -ct_1t_2t_3$

$\therefore$ Orthocentre is $\left(-\frac{c}{t_1t_2t_3}, -ct_1t_2t_3\right)$

which lies on $xy = c^2$

**412. B**

$\because x^2 + y^2 = 9 = 25 - 16$

which is director circle of the hyperbola

$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

Hence, angle between tangents must be $\pi/2$

**413. B**

Let $P \equiv (8, y_1)$

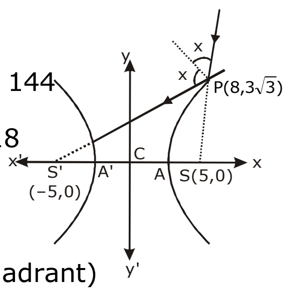
$\therefore\ 9(8)^2 - 16y_1^2 = 144$

$\Rightarrow 9 \times 8 - 2y_1^2 = 18$

$y_1 = \pm 3\sqrt{3}$

$\therefore P \equiv (8, 3\sqrt{3})$

($\because$ P lies in first quadrant)




Also, $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$\Rightarrow \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

$\therefore 3^2 = 4^2(e^2 - 1) \Rightarrow e = \frac{5}{4}$

then foci $\equiv (\pm 5, 0)$

$\therefore$ Equation of the reflected ray after first reflection passes through P, S' is

$y - 0 = \frac{3\sqrt{3} - 0}{8 + 5}(x + 5)$

$\Rightarrow 3\sqrt{3}x - 13y + 15\sqrt{3} = 0$

**414. C**

$y = z = 0$, then $x = 1$

suppose direction cosines of line of intersection are l,m,n. Then, $l + m - n = 0$

$2l - 3m + n = 0$ then $\frac{l}{2} = \frac{m}{3} = \frac{n}{5}$

$\therefore$ Equation of line in symmetric form is

$$\frac{x - 1}{2} = \frac{y}{3} = \frac{z}{5}$$

**415. C**

Line is parallel to the normal of the plane

$x - 2y - 3z = 7$

$\therefore$ Equation of line through (1, 1, – 1) is

$$\frac{x - 1}{1} = \frac{y - 1}{-2} = \frac{z + 1}{-3}$$

**416. B**

$r\cos\alpha = 9$, $r\cos\beta = 12$ and $r\cos\gamma = 8$

$\therefore r^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 81 + 144 + 64$

$r^2 . 1 = 289 \Rightarrow r = 17$

**417. B**

If DC's of line of intersection of planes

$4y + 6z = 5$ and $2x + 3y + 5z = 5$

are l, m, n

$\therefore 0 + 4m + 6n = 0$

$2l + 3m + 5n = 0$

$\Rightarrow \frac{l}{1} = \frac{m}{6} = \frac{n}{-4} = \frac{1}{\sqrt{53}}$

$\Rightarrow l = \frac{1}{\sqrt{53}}, m = \frac{6}{\sqrt{53}}, n = -\frac{4}{\sqrt{53}}$

Also, $6\left(\frac{1}{\sqrt{53}}\right) + 5\left(\frac{6}{\sqrt{53}}\right) + 9\left(-\frac{4}{\sqrt{53}}\right) = 0$

ie, line of intersection is perpendicular to normal of third plane. Hence, three planes have a line in common.

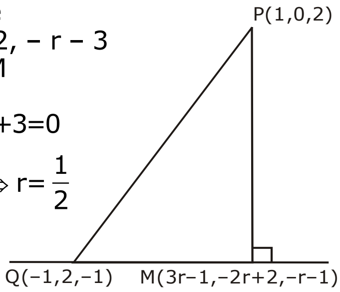
**418. B**

$$\frac{x + 1}{3} = \frac{y - 2}{-2} = \frac{z + 1}{-1} = r$$

$\therefore$ M(3r − 1, −2r + 2, − r − 1)
DR's of PM are
3r − 2, −2r + 2, − r − 3
DR's of line QM
are 3, −2, −1
$\therefore 9r-6+4r-4+r+3=0$

$\Rightarrow 14r-7 = 0 \Rightarrow r = \frac{1}{2}$

$\therefore M\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$

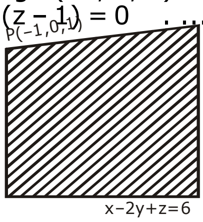



**419. C**

Point (3, 2, 1) and (2, −3, −1)
lies on 11x + my + nz = 28
ie, $33 + 2m + n = 28 \Rightarrow 2m + n = -5$ ...(i)
and $22 - 3m - n = 28 \Rightarrow -3m - n = 6$. ...(ii)
From Eqs. (i) and (ii), m = −1 and n = −3

**420. A**

Equation of plane through (−1, 0, 1) is
a(x + 1) + b(y − 0) + c (z − 1) = 0 ....(i)
which is parallel to given line and perpendicular to given plane




−a + 2b + 3c = 0...(ii)
& a − 2b + c = 0...(iii)
From Eqs. (ii) and (iii), c = 0 , a = 2b
From Eqs. (i), 2b (x + 1) + by = 0
$\Rightarrow 2x + y + 2 = 0$

**421. A**

$\vec{a}.\vec{b} = 0$

$\therefore$ $\vec{a}$ and $\vec{b}$ are mutually perpendicular

Also $|\vec{a}| = 2$ and $|\vec{b}| = 3$

Let $\vec{a} = 2\hat{i}$ then $\vec{b} = 2\hat{i}$

such that $\vec{a} = \hat{i}$ , $\vec{b} = \hat{j}$

$\therefore (\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})))=(2\hat{i}\times(2\hat{i}\times(2\hat{i}\times(2\hat{i}\times 3\hat{j})))$

$= 48\hat{j} = 48\vec{b}$

**422. D**

$3\vec{a} + 4\vec{b} + 5\vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar
No other conclusion can be drived from it.

**423. B**

$\vec{\alpha} = \vec{a} + \vec{b} + \vec{c} \qquad = 6\hat{i} + 12\hat{j}$

Let $\vec{\alpha} = x\vec{a} + y\vec{b} \qquad \Rightarrow 6x + 2y = 6$
$-3x - 6y = 12 \qquad \therefore x = 2, y = -3$
$\therefore \vec{\alpha} = 2\vec{a} - 3\vec{b}$

**424. B**

Given expression = $2(1 + 1 + 1) - 2\Sigma(\vec{a}\cdot\vec{b})$

$= 6 - 2\Sigma(\vec{a}\cdot\vec{b})$

But $(\vec{a} + \vec{b} + \vec{c})^2 \ge 0$

$\therefore (1 + 1 + 1) + 2\Sigma\, \vec{a}\cdot\vec{b} \ge 0$

$\therefore 3 \ge -2\Sigma\, \vec{a}\cdot\vec{b}$

From Eqs. (i) and (ii),
Given expression $\le 6 + 3 = 9$

**425. B**

$(\hat{i} \times \hat{j})\cdot\vec{c} \le |\hat{i} \times \hat{j}|\,|\vec{c}| \cos\frac{\pi}{6}$

$\Rightarrow -\frac{\sqrt{3}}{2} \le (\hat{i} \times \hat{j})\cdot\vec{c} \le \frac{\sqrt{3}}{2}$

**426. C**

$\because \begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

Then, $\begin{vmatrix} 1 & a-1 & a-1 \\ b & 1-b & 0 \\ c & 0 & 1-c \end{vmatrix} = 0$

Let a − 1 = A, b − 1 = B and c − 1 = C, then

$\begin{vmatrix} 1 & A & A \\ 1+B & -B & 0 \\ 1+C & 0 & -C \end{vmatrix} = 0$

$\Rightarrow 1(BC - 0) - A(-C - BC) + A(B + BC) = 0$

or $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = -2$ ....(i)

$\therefore \frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1} = \frac{1+A}{A} + \frac{1+B}{B} + \frac{1+C}{C}$

$= \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + 3 = -2 + 3 = 1$ [from Eq. (i)]

**427. B**

Equation of line through (1, 2) is,
$y - 2 = m(x - 1)$...(i) $\Rightarrow mx - y - m + 2 = 0$

distance from (3, 1) = $\frac{3m - 1 - m + 2}{\sqrt{(m^2+1)}}$

Let D = $\frac{(2m+1)}{\sqrt{(m^2+1)}}$

For maximum or minimum $\frac{dD}{dm} = \frac{(2-m)}{(m^2+1)^{3/2}} = 0$

$\therefore$ m = 2, $\frac{d^2D}{dm^2}$ = −ve From Eq. (i), y = 2x

**428. A**

If remaining vertex is $(\alpha, \beta)$, then

$\frac{\alpha+3}{2} = -4, \frac{\beta+5}{2} = -3/2$
$\therefore \alpha = -11, \beta = -8$

and $\frac{\alpha-6}{2} = \frac{3-2}{2}, \frac{\beta-4}{2} = \frac{5+1}{2}$
$\therefore \alpha = 7, \beta = 10$

and also $\frac{\alpha-2}{2} = \frac{3-6}{2}, \frac{\beta+1}{2} = \frac{5-4}{2}$
$\therefore \alpha = -1, \beta = 0$
$\therefore$ Possibilities of remaining vertex are
(−11, −8) or (7, 10) or (−1, 0)

**429. B**

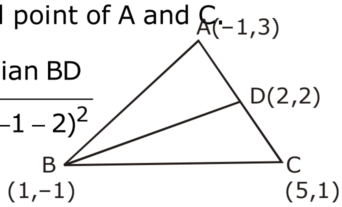
If D is the mid point of A and C

$\therefore$ D is (2, 2)

Length of median BD

$= \sqrt{(1-2)^2 + (-1-2)^2}$

$= \sqrt{10}$



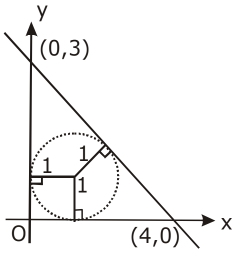

**430. B**

$\because$ 3x + 4y = 12

$\Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

Let coordinate of incentre is $(\alpha, \alpha)$

$\therefore$ Radius is also $\alpha$




Length of perpendicular from $(\alpha, \alpha)$ on

$(3x + 4y - 12 = 0) = \alpha \Rightarrow \frac{|3\alpha + 4\alpha - 12|}{5} = \alpha$

$\Rightarrow 7\alpha - 12 = \pm 5\alpha \;\; \therefore \alpha = 1$ and $\alpha = 6$

$\alpha \neq 6 \;\; \therefore \alpha = 1$ Then, incentre (1, 1).

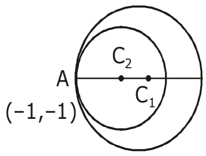
**431. B**

$x^2 + y^2 - 4x - 6y - 12 = 0$ .....(1)

Centre $C_1$ (2, 3)

Radius $r_1 = 5 = C_1 A$

If $C_2$ (h, k) is the centre of the circle of radius 3




which touches the circle (1) internally at the point A(–1, –1), then $r_2 = C_2A = 3$ and $C_1C_2 = C_1A - C_2A = 5 - 3 = 2$ Thus, $C_2$ (h, k) divide $C_1A$ in the ratio 2 : 3

internally $\therefore$ h = $\frac{2(-1)+3(2)}{2+3} = \frac{4}{5}$

& k = $\frac{2(-1)+3(3)}{2+3} = \frac{7}{5} \;\; \therefore C_2\left(\frac{4}{5}, \frac{7}{5}\right)$

**432. D**

$x^2 = y - 6$ ....(1)

$x^2 + y^2 + 16x + 12y + c = 0$ .....(2)

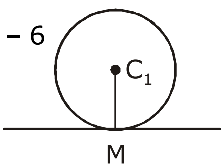
The tangent at P(1, 7) to the parabola (1)

is $\Rightarrow x(1) = \frac{1}{2}(y + 7) - 6$

$\Rightarrow y = 2x + 5$ ....(3)

which is also touches the circle (2)

By (2) & (3)




$\Rightarrow x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$ must have equal roots. Let, $\alpha$ & $\beta$ are the roots of the equal then, $\alpha + \beta = -12 \Rightarrow \alpha = -6$ ($\because \alpha = \beta$)

$\therefore$ x = –6 & y = 2x + 5 = –7

$\therefore$ Point of contat is (–6, –7)

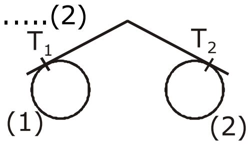
**433. C**

$x^2 + y^2 + 4x + 3 = 0$ .....(1)

p (h, k) $\Rightarrow$ locus = ?

$x^2 + y^2 - 6x + 5 = 0$ .....(2)

$\Rightarrow$ Given $\frac{PT_1}{PT_2} = 3$




**434. D**

y = |x| + c...(1), $x^2 + y^2 - 8|x| - 9 = 0$...(2)

both are symmetrical about y-axis for x > 0,

y = x + c ....(3)

equation of tangent to circle $x^2 + y^2 - 8x - 9 = 0$

which is parallel to line (3) is $y = x + (5\sqrt{2} - 4)$

for no solution $c > (5\sqrt{2} - 4) \;\therefore c \in (5\sqrt{2} - 4, \infty)$

**435. C**

$y^2 = 32x$

Length of focal chord

$a\left(t + \frac{1}{t}\right)^2$

$8\left(t + \frac{1}{t}\right)^2$

A.M. $\geq$ G.M

$\frac{t + \frac{1}{t}}{2} \geq \sqrt{t.\frac{1}{t}}$

$t + \frac{1}{t} \geq 2$

$\left(t + \frac{1}{t}\right)^2 \geq 4$

8 × 4 = 32

**436. A**

Equation of chord joining (2, 2) and (8, -4) is

$y - 2 = \frac{-4-2}{8-2}(x - 2) \Rightarrow y - 2 = -x + 2$

$\therefore$ x + y = 4 ...(i)

Let $R(\alpha^2, \alpha - 2)$

$\because$ R is interior to the parabola, O and R are on the same side of PQ

$\therefore (\alpha - 2)^2 - 2\alpha^2 < 0$

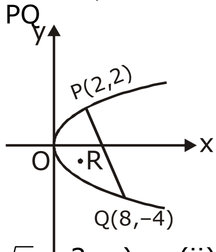
$\Rightarrow -\alpha^2 - 4\alpha + 4 < 0$

$\Rightarrow \alpha^2 + 4\alpha - 4 > 0$

$\Rightarrow (\alpha + 2)^2 > 8$

or $\alpha + 2 < -2\sqrt{2}$

and $\alpha + 2 > 2\sqrt{2}$




$\therefore \alpha \in (-\infty, -2 - 2\sqrt{2}) \cup (2\sqrt{2} + 2, \infty)$ ...(ii)

Also from equation (i) $\frac{\alpha - 2 + \alpha^2 - 4}{0 + 0 - 4} > 0$

$\Rightarrow \alpha^2 + \alpha - 6 < 0 \Rightarrow (\alpha + 3)(\alpha - 2) < 0$

$\therefore$ $-3 < \alpha < 2$ ...(iii)

From equations (ii) and (iii), we get

$\alpha \in (2\sqrt{2} - 2, 2)$ or $\alpha \in (-2 + 2\sqrt{2}, 2)$

**437. D**

$\because$ Normal at $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then,

$T = -t - \frac{2}{t}$ or $tT = -t^2 - 2$

$\Rightarrow t^2 + tT + 2 = 0 \quad \because$ t is real

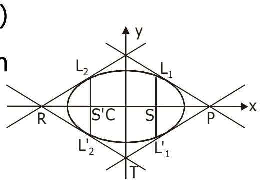
$\therefore T^2 - 4.1.2 \geq 0$ or $T^2 \geq 8$

**438. C**

Do yourself

**439. D**

$\frac{x^2}{9} + \frac{y^2}{5} = 1 \;....(1)$




Area of parallelogram PQRT

= 4 (Area of $\Delta$ PQC)

$= 4\left\{\frac{1}{2}(CP)(CQ)\right\}$

**440. C**

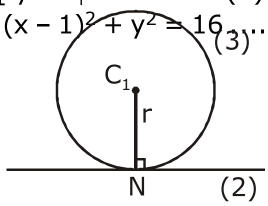
The equation of the tangent at the point $P(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi)$ to the ellipse

$16x^2 + 11y^2 = 256 \quad .....(1)$

$\Rightarrow 4x\cos\phi + \sqrt{11}\, y\sin\phi = 16 \quad .....(2)$

This touches the circle $(x-1)^2 + y^2 = 16 \;.....(3)$

$\Rightarrow C_1 N = r$




$\Rightarrow \phi = \pm\frac{\pi}{3}$

**441. A**

$ax^2 + 2bx + c = 0 \quad ....(1)$

Roots of equation (1) are not real, then

$\Rightarrow D = (2b)^2 - 4ac < 0 \Rightarrow b^2 < ac$

then the equation

$ax^2 + 2bxy + cy^2 + dx + ey + f = 0$

can represents an ellipse

**442. D**

$x^2 - 3y^2 - 4x - 6y - 11 = 0$

$\Rightarrow (x^2 - 4x + 4 - 4) - 3(y^2 + 2y + 1 - 1) = 11$

$\Rightarrow (x-2)^2 - 4 - 3(y+1)^2 + 3 = 11$

$\Rightarrow (x-2)^2 - 3(y+1)^2 = 12$

$\Rightarrow \frac{(x-2)^2}{12} - \frac{(y+1)^2}{4} = 1 \Rightarrow b^2 = a^2(e^2 - 1)$

$\Rightarrow 4 = 12(e^2 - 1) \Rightarrow e^2 = 1 + \frac{1}{3} \Rightarrow e = 2/\sqrt{3}$

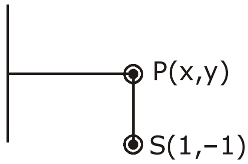
Distance between focii $2ae = 2 \times 2\sqrt{3} \times \frac{2}{\sqrt{3}} = 8$

**443. C**

Rectangular Hyperbola

$e = \sqrt{2}$

PS = ePM




$x - y + 1 = 0$

$(PS)^2 = e^2 (PM)^2$

$(x-1)^2 + (y+2)^2 = 2\left(\frac{x-y+1}{\sqrt{2}}\right)^2$

$x^2 - 2x + 1 + y^2 + 2y + 1$

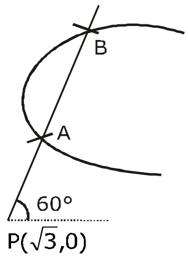
$= x^2 + y^2 + 1 - 2xy + 2x - 2y$

$2xy - 4x + 4y + 1 = 0$

**444. A**

$\therefore P \equiv (\sqrt{3}, 0)$

$\frac{x-\sqrt{3}}{\cos 60^\circ} = \frac{y-0}{\sin 60^\circ} = r$




or $x = \sqrt{3} + \frac{r}{2}$, $y = \frac{r\sqrt{3}}{2}$

or $\left(\sqrt{3} + \frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ lie on $y^2 = x + 2$, then

$\frac{3r^2}{4} = \sqrt{3} + \frac{r}{2} + 2 \Rightarrow \frac{3r^2}{4} - \frac{r}{2} - (2 + \sqrt{3}) = 0$

$\therefore$ PA. PB $= r_1 r_2 = \left|-\frac{(2+\sqrt{3})}{\frac{3}{4}}\right| = \frac{4}{3}(2+\sqrt{3})$

**445. D**

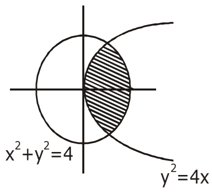
It is clear that point $(-2m, m + 1)$ lie inside the circle and parabola, then

$(-2m)^2 + (m+1)^2 - 4 < 0$

and $(m+1)^2 - 4(-2m) < 0$

$\therefore 5m^2 + 2m - 3 < 0$

and $m^2 + 10m + 1 < 0$




$\Rightarrow (m+1)(5m-3) < 0$

and $(m+5)^2 - 24 < 0$

$\Rightarrow -1 < m < \frac{3}{5}$ and $-5-2\sqrt{6} < m < -5 + 2\sqrt{6}$

Hence $-1 < m < -5 + 2\sqrt{6}$

**446.** **D**

Since, $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\cos 120°$

$= 1\,.\,2\left(-\frac{1}{2}\right) = -1$

$\because \{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$

$= \{3\vec{a} \times \vec{a} - \vec{a} \times \vec{b} + 9\vec{b} \times \vec{a} - 3\vec{b} \times \vec{b}\}^2$

$= [0 - \vec{a} \times \vec{b} - 9\vec{a} \times \vec{b} - 0]^2$

$= [-10(\vec{a} \times \vec{b})]^2 = 100(\vec{a} \times \vec{b})^2$

$= 100\{a^2b^2 - (\vec{a}.\vec{b})^2\}$

$= 100\{4 - 1\} = 300$

**447.** **C**

$|\vec{a} - \vec{b}| = \sqrt{|\vec{a}-\vec{b}|^2}$

$= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}} = \sqrt{9 + 16 - 2\vec{a}.\vec{b}}$

$= \sqrt{(25 - 2\vec{a}.\vec{b})}$ ...(i)

But $|\vec{a} + \vec{b}| + 5$ $\therefore |\vec{a} + \vec{b}|^2 = 25$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 25$

$9 + 16 + 2\vec{a}.\vec{b} = 25 \therefore \vec{a}.\vec{b} = 0$

From Eq. (i), $|\vec{a} - \vec{b}| = \sqrt{25 - 0} = 5$

**448.** **D**

$|(\vec{a} \times \vec{b}).\vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$

$|(\vec{a} \times \vec{b})|\,|\vec{c}|\cos\alpha = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$

$\{\alpha$ is the angle between $\vec{a} \times \vec{b}$ and $\vec{c}\}$

$\Rightarrow |\vec{a} \times \vec{b}|\cos\alpha = |\vec{a}|\,|\vec{b}|$

$\Rightarrow |\vec{a}|\,|\vec{b}|\sin\beta\cos\alpha = |\vec{a}|\,|\vec{b}|$

($\beta$ is the angle between $\vec{a}$ and $\vec{b}$)

$\therefore \cos\alpha\cos\beta = 1$

It is possible when $\cos\alpha = 1$, $\sin\beta = 1$

$\therefore \alpha = 0$ and $\beta = \pi/2$

$\therefore \vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0,\ \vec{c}.\vec{a} = 0$

**449.** **D**

Linear combination

$1.(\hat{i} + \hat{j} + \hat{k}) + \lambda(4\hat{i} + 3\hat{j} + 4\hat{k})$

$+ \mu(\hat{i} + \alpha\hat{j} + \beta\hat{k}) = 0.\hat{i} + 0.\hat{j} + 0.\hat{k}$

$1 + 4\lambda + \mu = 0$

and $|\vec{c}| = \sqrt{1 + \alpha^2 + \beta} = \sqrt{3}$

$1 + 3\lambda + a\mu = 0$ $\therefore \alpha^2 + \beta^2 = 2$

and $1 + 4\lambda + \beta\mu = 0$

Solve any two then putting the value in remaining third equation.

**450.** **A**

Given, $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ and $|\vec{a}| = 1, |\vec{b}| = 1$

and $|\vec{c}| = 1$ $\because [2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]$

$(2\vec{a} - \vec{b}).\{(2\vec{b}-\vec{c}) \times (2\vec{c}-\vec{a})\}$

$= (2\vec{a} - \vec{b}).\{4\vec{b} \times \vec{c}) - 2\vec{b} \times \vec{a} - 2\vec{c} \times \vec{c} + \vec{a})\}$

$= (2\vec{a} - \vec{b}).\{4\vec{b} \times \vec{c}) + 2(\vec{a} \times \vec{b}) - 0 + (\vec{c} \times \vec{a})\}$

$= 8\,\vec{a}.(\vec{a} \times \vec{c}) + 4\vec{a}.(\vec{a} \times \vec{b}) + 2\vec{a}.(\vec{c} \times \vec{a})$

$-4\vec{b}.(\vec{b} \times \vec{c}) - 2\vec{b}.(\vec{a} \times \vec{b}) - \vec{b}.(\vec{c} \times \vec{a})$

$= 8[\vec{a}\ \vec{b}\ \vec{c}] + 0 + 0 - 0 - 0 - [\vec{a}\ \vec{b}\ \vec{c}]$

$= 7[\vec{a}\ \vec{b}\ \vec{c}] = 0$ $(\because [\vec{a}\ \vec{b}\ \vec{c}] = 0)$

**451.** **C**

Let $\vec{r} = r_1\hat{i} + r_2\hat{j} + r_3\hat{k}$

$\therefore r_1.\hat{i} = r_1,\ \vec{r}.\hat{j} = r_2,\ \vec{r}.\hat{k} = r_3$

and $\vec{r} \times \hat{i} = 0 + r_2(\hat{j} \times \hat{i}) + r_3(\hat{k} \times \hat{i})$

$= -r_2\hat{k} + r_3\hat{j}$

$\therefore (\vec{r}.\hat{i})(\vec{r} \times \hat{i}) = -r_1r_2\hat{k} + r_3r_2\hat{k}$

similarly $(\vec{r}.\hat{j})(\vec{r} \times \hat{j}) = -r_2r_3\hat{i} + r_2r_1\hat{k}$

and $(\vec{r}.\hat{k})(\vec{r} \times \hat{i}) = -r_3r_1\hat{j} + r_2r_3\hat{i}$

$\therefore (\vec{r}.\hat{i})(\vec{r} \times \hat{i}) + (\vec{r}.\hat{j})(\vec{r} \times \hat{j})$

$+ (\vec{r}.\hat{k})(\vec{r} \times \hat{k}) = 0$

**452.** **B**

$\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\therefore |(\vec{a} \times \vec{b}) \times \vec{c}| = |(\vec{a} \times \vec{b})|\,|\vec{c}|\sin 30°$

$= \sqrt{2^2 + (-2)^2 + 1^2}\ |\vec{c}|\ \frac{1}{2} = \frac{3}{2}|\vec{c}|$

Given, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ $(\vec{c} - \vec{a})^2 = 8$

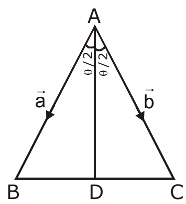
$\Rightarrow (\vec{c})^2 + (\vec{a})^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$\Rightarrow (|\vec{c}|)^2 - 1)^2 = 0$

$\therefore |\vec{c}| = 1$ $\therefore |\vec{a} \times \vec{b}) \times \vec{c}| = \frac{3}{2} \times 1 = \frac{3}{2}$

**453.** **C**




Now, in $\Delta$ ABC

$\frac{BD}{DC} = \frac{a}{b}$

$\therefore BD = ak,\ DC = bk$

$BC = (a + b)k$

$(BC)^2 = (AB)^2 + (AC)^2 - 2AB \cdot AC\cos\theta$

$\Rightarrow (a + b)^2k^2 = a^2 + b^2 - 2ab\cos\theta$

$\therefore k^2 = \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}$

In $\Delta$ ADC and $\Delta$ ABD

$\cos\left(\frac{\theta}{2}\right) = \frac{b^2 + (AD)^2 - b^2k^2}{2bAD}$

$= \frac{a^2 + (AD)^2 - a^2k^2}{2aAD}$

$\Rightarrow (AD)^2 = ab(1 - k^2)$

$\therefore = ab\left\{1 - \frac{a^2 + b^2 - 2ab\cos\theta}{(a+b)^2}\right\}$ [from Eq. (i)]

$= \frac{4a^2b^2\cos^2\theta/2}{(a+b)^2} \therefore AD = \frac{2ab\cos\theta/2}{(a+b)}$

$\therefore\ \overrightarrow{AD} = \pm\dfrac{(\vec{a}b+\vec{b}a)}{(a+b)} = \pm\dfrac{ab}{(a+b)}\left(\dfrac{\vec{a}}{a}+\dfrac{\vec{b}}{b}\right)$

$= \pm\dfrac{ab}{(a+b)}(\hat{a}+\hat{b})$

$\therefore\ \dfrac{\overrightarrow{AD}}{AD} = \pm\dfrac{(\hat{a}+\hat{b})}{2\cos\theta/2}$

**454. D**

If angle between $\vec{b}$ and $\vec{c}$ is $\alpha$

then $|\vec{b}\times\vec{c}| = \sqrt{15}$

$\Rightarrow |\vec{b}|\,|\vec{c}|\sin\alpha = \sqrt{15} \Rightarrow \sin\alpha = \dfrac{\sqrt{15}}{4}$

$\therefore \cos\alpha = 1/4 \qquad \vec{b} - 2\vec{c} = \lambda\vec{a}$

$\Rightarrow (\vec{b} - 2\vec{c})^2 = \lambda^2(\vec{a})^2$

$\Rightarrow (\vec{b})^2 + 4(\vec{c})^2 - 4\vec{b}.\vec{c} = \lambda^2(\vec{a})^2$

$\Rightarrow\ 16 + 4 - 4\{|\vec{b}|\,|\vec{c}|\cos\alpha\} = \lambda^2$

$\therefore\ \lambda^2 = 16 \Rightarrow \lambda = \pm 4$

**455. D**

Given, $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}$

$\vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$

and $\vec{r} = \vec{a} - 4\vec{b} + 2\vec{c}$

Given, $V_1 = [\vec{a}\ \vec{b}\ \vec{c}]$ . ...(i)

$\therefore\ V_2 = [\vec{p}\ \vec{q}\ \vec{r}] = \begin{vmatrix} 1 & 1 & -2 \\ 3 & -2 & 1 \\ 1 & -4 & 2 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore\ V_2 = 12\ [\vec{a}\ \vec{b}\ \vec{c}]$

from Eqs. (i) and (ii), $V_2 : V_1 = 15 : 1$

**456. B**

Equation of line joining

$6\vec{a} - 4\vec{b} - 5\vec{c}$, and $-4\vec{c}$ is

$\vec{r} = (6\vec{a} - 4\vec{b} - 5\vec{c}) + \lambda(-6\vec{a} + 4\vec{b} + \vec{c})$

$= \vec{a}(6 - 6\lambda) + \vec{b}(-4 + 4\lambda) + \vec{c}(-5 + \lambda)$ ...(i)

and equation of line joining

$-\vec{a} - 2\vec{b} - 3\vec{c}$ and $\vec{a} + 2\vec{b} - 5\vec{c}$ is

$\vec{r}\ (-\vec{a} - 2\vec{b} - 3\vec{c}) + \mu(2\vec{a} + 4\vec{b} - 2\vec{c})$

$= \vec{a}(-1+2\mu) + \vec{b}(-2+4\mu) + \vec{c}(-3-2\mu)$ ...(ii)

Comparing Eqs. (i) and (ii), then

$6 - 6\lambda = -1 + 2\mu$

$-4 + 4\lambda = -2 + 4\mu$

and $-5 + \lambda = -3 - 2\mu$

After solving, we get

$\lambda = 1$ and $\mu = \dfrac{1}{2}$

Substituting the value of $\lambda$ in Eq. (i)

then point of intersection is $\vec{r} = -4\vec{a}$

**457. B**

$\therefore\ \vec{a} = (x, y, z)$

$\vec{a}$ makes an obtuse angle with y-axis

$\therefore\ y < 0$ and given $\dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} = \dfrac{\vec{a}.\vec{c}}{|\vec{a}|.|\vec{c}|}$

$\Rightarrow \dfrac{\vec{a}.\vec{b}}{|\vec{b}|} = \dfrac{\vec{a}.\vec{c}}{|\vec{c}|}$

$\Rightarrow \dfrac{xy - 2yz + 3zx}{\sqrt{(y^2 + 4z^2 + 9x^2)}} = \dfrac{2zx + 3xy - yz}{\sqrt{(4z^2 + 9x^2 + y^2)}}$

$\Rightarrow$ xy − 2yz + 3zx = 2zx + 3xy − yz

$\Rightarrow$ 2xy + yz − zx = 0 . ...(i)

and given $\vec{a}\cdot\vec{d} = 0$

$\therefore x - y + 2z = 0 \Rightarrow z = \dfrac{y-x}{2}$ .....(ii)

from Eqs. (i) and (ii), $2xy + \dfrac{(y-x)^2}{2} = 0$

$\therefore\ (y + x)^2 = 0$

$\therefore\ y = -x$ ....(iii)

$\therefore\ z = -x$ [from Eq. (ii)]

$\therefore\ \vec{a} = (x, -x, -x)$

$\therefore\ |\vec{a}| = \sqrt{x^2 + x^2 + x^2} = x\sqrt{3} = 2\sqrt{3}$

$\therefore$ x = 2, y = −2, z = −2

$\therefore\ \vec{a} = (2, -2, -2)$

**458. B**

$\therefore$ AD $\perp$ BC $\quad\therefore (\vec{a} - \vec{d}).(\vec{b} - \vec{c}) = 0$

and BD $\perp$ AC $\quad\therefore (\vec{b} - \vec{d}).(\vec{c} - \vec{a}) = 0$

$\therefore$ D is orthocentre.

**459. B**

Let image of P w.r.t the given line be $Q(\alpha, \beta, \gamma)$. Then mid point of PQie,

$\left(\dfrac{\alpha+7}{2}, \dfrac{\beta-1}{2}, \dfrac{\gamma+2}{2}\right)$

lies on the line

$\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k}) = 6$

$\therefore\ \dfrac{\alpha+7}{2}\hat{i} + \dfrac{\beta-1}{2}\hat{j} + \dfrac{\gamma+2}{2}\hat{k}$

$= 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$

On comparing

$$\left.\begin{aligned} &\frac{\alpha+7}{2} = 9 + \lambda \Rightarrow \alpha = 11 + 2\lambda \\ &\frac{\beta-1}{2} = 5 + 3\lambda \Rightarrow \beta = 11 + 6\lambda \\ \text{and}\quad &\frac{\gamma+2}{2} = 5 + 5\lambda \Rightarrow \gamma = 8 + 10\lambda \end{aligned}\right\} \ldots\text{(i)}$$

Also, PQ and given line are perpendicular

ie, $(\alpha - 7).1 + (\beta + 1).3 + (\gamma - 2).5 = 0$

$\Rightarrow\ \alpha - 7 + 3\beta + 3 + 5\gamma - 10 = 0$

$\therefore\ \alpha + 3\beta + 5\gamma = 14$ ....(ii)

From Eqs. (i) and (ii),

$11 + 2\lambda + 3(11 + 6\lambda) + 5(8 + 10\lambda) = 14$

$\therefore\ \lambda = -1$

From Eq. (i), $(\alpha, \beta, \gamma) = (9, 5, -2)$.

**460.** $(3\vec{a} - 5\vec{b}).(2\vec{a} + \vec{b}) = 0$

$(\vec{a} + 4\vec{b}).(\vec{b} - \vec{a}) = 0$

**461. A**

$\because$ 3a + 2b + 6c = 0 $\quad\therefore\ c = -\dfrac{(3a+2b)}{6}$

$\therefore\ ax + by + c = 0 \Rightarrow ax + by - \dfrac{(3a+2b)}{6} = 0$

$\Rightarrow 6ax + 6by - 3a - 2b = 0$
$\Rightarrow 3a(2x - 1) + 2b(3y - 1) = 0$

$\Rightarrow (2x - 1) + \frac{2b}{3a}(3y - 1) = 0$

$P + \lambda Q = 0$, $\therefore P = 0, Q = 0$

Then, $2x - 1 = 0$, $3y - 1 = 0$; $x = 1/2$, $y = 1/3$

Hence, fixed points is $\left(\frac{1}{2}, \frac{1}{3}\right)$

**462. B**

Equation of any line through the point of intersection of the given lines is $(3x + y - 5) + \lambda(x - y + 1) = 0$ since this line is perpendicular to one of the given lines $\frac{3+\lambda}{\lambda-1} = -1$ or $\frac{1}{3}$

$\Rightarrow \lambda = -1$ or $-5$, therefore the required straight line is $x + y - 3 = 0$ or $x - 3y + 5 = 0$

**463. C**

Let $A \equiv (p + 1, 1)$, $B \equiv (2p + 1, 3)$, and $C \equiv (2p + 2, 2p)$

$\therefore$ Slope of AB = Slope of AC

$$\Rightarrow \frac{3-1}{2p+1-p-1} = \frac{2p-1}{2p+2-p-1} \quad \therefore p = 2, -1/2$$

**464. C**

Since $m_1$ and $m_2$ are the roots of the equation $x^2 + (\sqrt{3} + 2)x + (\sqrt{3} - 1) = 0$

then, $m_1 + m_2 = -(\sqrt{3} + 2)$, $m_1 m_2 = (\sqrt{3} - 1)$

$$\therefore m_1 - m_2 = \sqrt{(m_1 + m_2)^2 - 4m_1 m_2}$$

$$= \sqrt{(3 + 4 + 4\sqrt{3} - 4\sqrt{3} + 4)} = \sqrt{11}$$

and coordinates of the vertices of the given triangle are $(0, 0)$, $(c/m_1, c)$ and $(c/m_2, c)$. Hence, the required area of triangle

$$= \frac{1}{2}\left|\frac{c}{m_1} \times c - \frac{c}{m_2} \times c\right| = \frac{1}{2} c^2 \left|\left(\frac{1}{m_1} - \frac{1}{m_2}\right)\right|$$

$$= \frac{1}{2} c^2 \left|\frac{m_2 - m_1}{m_1 m_2}\right| = \frac{1}{2} c^2 \frac{\sqrt{11}}{(\sqrt{3} - 1)}$$

$$= \frac{1}{2} c^2 \frac{\sqrt{11}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \left(\frac{\sqrt{33} + \sqrt{11}}{4}\right) c^2$$

On comparing, $a = \frac{\sqrt{33}}{4}$, $b = \frac{\sqrt{11}}{4}$

or $a = \frac{\sqrt{11}}{4}$, $b = \frac{\sqrt{33}}{4}$

$$\therefore a^2 + b^2 = \frac{33}{16} + \frac{11}{16} = \frac{44}{16} = \frac{11}{4}$$

$\Rightarrow 2008(a^2 + b^2) = 2008 \times \frac{11}{4}$

$= 502 \times 11 = 5522$

**465. C**

$x = a + m$...(1), $y = -2$...(2), $y = mx$ ...(3)

Point of intersection of (1) & (2)

$(a + m, -2)$ is lies on (3)

$\Rightarrow -2 = m(a + m) \Rightarrow -2 = am + m^2$

$\Rightarrow m^2 + am + 2 = 0 \quad \because m$ is real

$\Rightarrow D \geq 0 \Rightarrow a^2 - 8 \geq 0 \Rightarrow a^2 \geq 8 \Rightarrow |a| \geq 2\sqrt{2}$

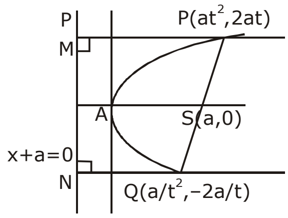
**466. A**

$\therefore SP = PM = a + at^2$, $SQ = QN = a + \frac{a}{t^2}$

$\frac{1}{SP} = \frac{1}{a(1+t^2)}$

$\therefore \frac{1}{SQ} = \frac{t}{a(1+t^2)}$

$\therefore \frac{1}{SP} + \frac{1}{SQ} = a$




**467. D**

Since y-axis is major axis $\Rightarrow f(4a) < f(a^2 - 5)$

$\Rightarrow 4a > a^2 - 5$ ($\because$ f is decreasing)

$\Rightarrow a^2 - 4a - 5 < 0 \Rightarrow a \in (-1, 5)$

**468. C**

$\because$ Equation of tangent of $y^2 = 4ax$ in terms of slope (m) is $y = mx + \frac{a}{m}$

Which is also tangent of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

then $\left(\frac{a}{m}\right)^2 = a^2m^2 + b^2$

$$\Rightarrow a^2\left(\frac{1}{m^2} - m^2\right) = b^2 \Rightarrow \left(\frac{1}{m^2} - m^2\right) = \frac{b^2}{a^2}$$

$$\Rightarrow \frac{(1+m^2)(1-m^2)}{m^2} = \frac{b^2}{a^2}$$

$$\Rightarrow \left(\frac{1-m^2}{m^2}\right) = \frac{b^2}{a^2(1+m^2)} > 0 \Rightarrow \left(\frac{1-m^2}{m^2}\right) > 0$$

$$\Rightarrow \frac{m^2-1}{m^2} < 0 \Rightarrow 0 < m^2 < 1$$

$\therefore m \in (-1, 0) \cup (0, 1)$

for positive values of m set is $m \in (0, 1)$

**469. A**

Let the point be $(\alpha, \beta) \Rightarrow \beta = \alpha + c$

Chord of contact of hyperbola $T = 0$

$$\therefore \frac{x\alpha}{2} - \frac{y\beta}{1} = 1 \Rightarrow \frac{x\alpha}{2} - y(\alpha + c) = 1$$

$$\Rightarrow \left(\frac{x}{2} - y\right)\alpha - (yc + 1) = 0$$

Since, this passes through point $(x_1, y_1)$

$\therefore x_1 = 2y_1$ and $y_1 c + 1 = 0$

$\therefore y_1 = \frac{x_1}{2}$ hence, $\frac{x_1}{y_1} = 2$

**470. B**

If eccentricities of ellipse and hyperbola are $e$ and $e_1$ $\therefore$ Foci $(\pm ae, 0)$ and $(\pm a_1e_1, 0)$

Here, $ae = a_1e_1 \Rightarrow a^2e^2 = a_1^2 e_1^2$

$\Rightarrow a^2\left(1-\frac{b^2}{a^2}\right) = a_1^2\left(1+\frac{b_1^2}{a_1^2}\right)$

$\Rightarrow a^2 - b^2 = a_1^2 + b_1^2$

$\Rightarrow 25 - b^2 = \frac{144}{25} + \frac{81}{25} = 9 \quad \therefore b^2 = 16$

$\therefore \cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{\lambda^2+\lambda^2-\frac{\lambda^2}{4}}{2\lambda^2} = 1 - \frac{1}{8} = \frac{7}{8}$

**471. A**

$a \to s,\ b \to p,\ c \to p,\ d \to r$

(a) $\frac{y}{y'} = c \Rightarrow \frac{y'}{y} = \frac{1}{c} \Rightarrow \log y = \frac{x}{c} + d$

$y = Ae^{x/c}$ Exponential curve

(b) $yy' = c \Rightarrow y^2 = 2cx + d$ [Parabola]

(c) $\frac{y}{y'} = 2x \Rightarrow \frac{2y'}{y} = \frac{1}{x}$

$\Rightarrow \ell ny^2 = \ell nx + \ell nc \Rightarrow y^2 = cx$ [Parabola]

(d) $yy' = 2x \Rightarrow \frac{y^2}{2} = x^2 + c$ [Hyperbola]

**472. D**

Let direction ratios of PQ are a,b,c

$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-3}{-3}$

Then, $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z+2}{c} = r$

(Let PQ = r)

$Q \equiv (ar + 2, br + 1, cr - 2)$

Which lie on $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-3}{-3}$, then

$\frac{ar+2-1}{2} = \frac{br+1+1}{2} = \frac{cr-2-3}{-3}$

$\Rightarrow \frac{ar+1}{2} = \frac{br+2}{2} = \frac{cr-5}{-3} = \lambda$ (say)

$\therefore a = \frac{2\lambda-1}{r},\ b = \frac{2\lambda-2}{r},\ c = \frac{-3\lambda+5}{r}$

Given, PQ is parallel to $x + 2y + z = 4$, then

$a + 2b + c = 0$

or $\frac{2\lambda-1}{r} + \frac{4\lambda-4}{r} + \frac{-3\lambda+5}{r} = 0$

$\Rightarrow 3\lambda = 0 \Rightarrow \lambda = 0$

then, $a = -\frac{1}{r},\ b = -\frac{2}{r},\ c = \frac{5}{r}$

$\therefore Q = (1, -1, 3)$

$\therefore PQ \equiv \sqrt{(2-1)^2+(1+1)^2+(3+2)^2} = \sqrt{30}$

**473. B**

Let DC's of shortest distance line are l, m, n which is perpendicular to both the given lines

$\therefore 2l - 7m + 5n = 0$ . ............(i)

and $2l + m - 3n = 0$ ...(ii)

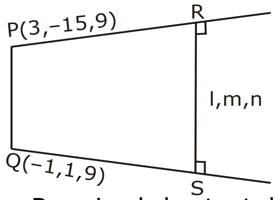
From Eqs. (i) and (ii).

| l | m | n | l | m |
|---|---|---|---|---|
| 2 | −7 | 5 | 2 | −7 |
| 2 | 1 | −3 | 2 | 1 |

$\therefore \frac{l}{16} = \frac{m}{16} = \frac{n}{16}$

$\Rightarrow \frac{l}{1} = \frac{m}{1} = \frac{n}{1} = \frac{\sqrt{(l^2+m^2+n^2)}}{\sqrt{(1^2+1^2+1^2)}} = \frac{1}{\sqrt{3}}$

$\Rightarrow l = \frac{1}{\sqrt{3}},\ m = \frac{1}{\sqrt{3}},\ n = \frac{1}{\sqrt{3}}$




$\therefore$ Required shortest distance = Projection of PQ on RS

$= |(3 - (-1))\, l + (-15 - 1)m + (9 - 9)n|$

$= |4l - 16m| = \left|\frac{-12}{\sqrt{3}}\right| = 4\sqrt{3}$

**474. A**

$x - y + 2z = 5,\ 3x + y + z = 6$

Let the direction ratio of lies are a,b,c

$a - b + 2c = 0,\ 3a + b + c = 0$

$\frac{a}{\begin{vmatrix}-1 & 2\\ 1 & 1\end{vmatrix}} = \frac{b}{\begin{vmatrix}2 & 1\\ 1 & 3\end{vmatrix}} = \frac{c}{\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}} \Rightarrow \frac{a}{-3} = \frac{b}{5} = \frac{c}{4}$

Equation of line $\frac{x-1}{-3} = \frac{y-2}{-2} = \frac{z-3}{4}$

**475. A**

$\frac{x+3}{3} = \frac{y-2}{-2} = \frac{z+1}{1} = r$

$M(3r - 3, -2r + 2, r - 1)$

$\Rightarrow 4x + 5y + 3z - 5 = 0$

$\Rightarrow 4'(3r - 3) + 5(-2r + 2) + 3(r - 1) - 5 = 0$

$\Rightarrow 12r - 12 - 10r + 10 + 3r - 3 - 5 = 0$

$\Rightarrow 5r = 10 \Rightarrow r = 2 \Rightarrow M(3, -1, 1)$

**476. D**

$\alpha = 45^o,\ \beta = 60^o \Rightarrow \ell = \cos\alpha = \cos 45^o = \frac{1}{\sqrt{2}}$

$\&\ m = \cos\beta = \cos 60^o = \frac{1}{2}$

$\Rightarrow \ell^2 + m^2 + n^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{2} \Rightarrow \cos^2\gamma = \frac{1}{4}$

$\Rightarrow \cos\gamma = \pm\frac{1}{2} \Rightarrow \cos\gamma = \frac{1}{2}\ \&\ \cos\gamma = -\frac{1}{2}$

$\therefore \gamma = 60^o, 120^o$

**477. D**

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$

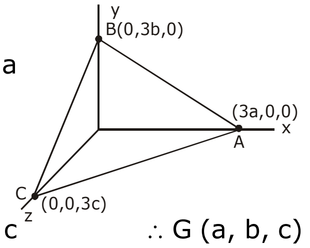
Let centroids of $\Delta ABC$ is

$G(x, y, z)$

$$x = \frac{3a+0+0}{3} = a$$

$$y = \frac{0+3b+0}{3} = b$$

$$z = \frac{0+0+3c}{3} = c$$




$\therefore$ G (a, b, c)

**478. C**

$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ & $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$

are coplanar

$$\Rightarrow \begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 + 2k) + 1\,(1 + k^2) - 1(2 - k) = 0$

$\Rightarrow 1 + 2k + 1 + k^2 - 2 + k = 0$

$\Rightarrow k^2 + 3k = 0 \Rightarrow k = 0\,,\, -3$

**479. B**

$\overrightarrow{OP} = \vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}\,,\ \overrightarrow{OQ} = \vec{b} = 3\hat{i} - \hat{j} - 2\hat{k}$

$\frac{PM}{MQ} = \frac{OP}{OQ} = \frac{\sqrt{14}}{\sqrt{14}} = \frac{1}{1}$

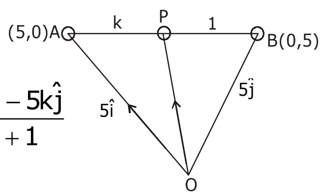
$M\left(\frac{3+1}{2}, \frac{3-1}{2}, \frac{-2-2}{2}\right) \Rightarrow M(2, 1, -2)$

$\overrightarrow{OM} = 2i + j - 2\hat{k}$

**480. B**

$\overrightarrow{OP} = \frac{5k\,\hat{j} + 5\hat{i}}{k+1}$

$\overrightarrow{PO} = \vec{b} = \frac{-5\hat{i} - 5k\hat{j}}{k+1}$




$\Rightarrow |b| \le \sqrt{37}$

$\Rightarrow \frac{25 + 25k^2}{(k+1)^2} \le 37 \Rightarrow 25 + 25k^2 \le 37k^2 + 74k + 37$

$\Rightarrow 12k^2 + 74k + 12 \ge 0 \Rightarrow 6k^2 + 37\,k + k + 6 \ge 0$

$\Rightarrow 6k\,(k + 6) + 1\,(k + 6) \ge 0 \Rightarrow (k + 6)(6k+1) \ge 0$

$\therefore k \in (-\infty, -6] \cup [-\frac{1}{6}, \infty)$

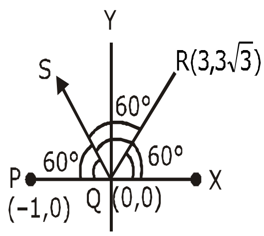
**481. C**

Slope of bisector

= tan 120°

$= -\sqrt{3}$

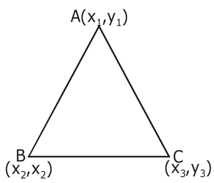
Equation $y = -\sqrt{3}\,x$




**482. A**

$$\text{Area } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

= A rational number




$\because$ ($x_1$, $x_2$, $x_3$, $y_1$, $y_2$, $y_3$, are integers)

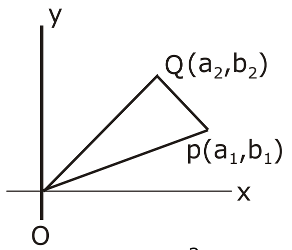
But Area of equilateral triangle

$= \frac{\sqrt{3}}{4}(BC)^2$ = irrational

**483. A**

cos $\angle$POQ

$$= \frac{OP^2 \quad OQ^2 - PQ^2}{2OP.OQ}$$




**484. B**

$x_2 = x_1 r\ ,\ y_2 = y_1 r \Rightarrow x_3 = x_1\,r^2,\ y_3 = y_1 r^2$

$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3}$ i.e. point lies on $\frac{y}{x} = k \Rightarrow y = kx$

**485. A**

$$x + 2y + \left(\frac{1(9) + 6(-5)}{1+6}\right) = 0 \Rightarrow x + 2y - 3 = 0$$

**486. D**

$|\vec{a}| = 1,\ |\vec{b}| = 5,\ |\vec{c}| = 3$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{c} \Rightarrow |\vec{a} \times (\vec{a} \times \vec{b})| = |\vec{c}|$

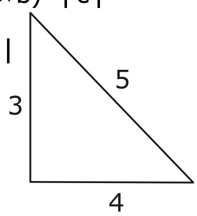
$\Rightarrow |\vec{a}| \times |\vec{a} \times \vec{b}|\sin 90^\circ = |\vec{c}|$

$\Rightarrow |\vec{a}||\vec{a}||\vec{b}|\sin\theta = |\vec{c}|$

$\Rightarrow 5 \sin\theta = 3$

$\Rightarrow \sin\theta = 3/5$

$\therefore \tan\theta = 3/4$




**487. D**

$|\vec{a}| = |\vec{c}| = 1,\quad |\vec{b}| = 4$

$\Rightarrow \vec{a} \times \vec{b} = 2\vec{a} \times \vec{c} \Rightarrow \vec{b} = \lambda\vec{a} + 2\vec{c}$

$\Rightarrow \frac{\vec{b} - 2\vec{c}}{\lambda} = \vec{a} \Rightarrow \vec{a} \times \vec{b} - 2\vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times 2\vec{c} = 0 \Rightarrow \vec{a} \times (\vec{b} - 2\vec{c}) = 0$

$\therefore \vec{a}$ is collinear to $\vec{b} - 2\vec{c}$

$\Rightarrow \vec{b} - 2\vec{c} = \lambda\vec{a} \Rightarrow (\vec{b} - 2\vec{c}) = \lambda^2\,|\vec{a}|^2$

$\Rightarrow |\vec{b}|^2 + 4|\vec{c}|^2 - 4\,|\vec{b}|^2\,|\vec{c}|^2\,\cos\theta = \lambda^2\,|\vec{a}|^2$

$\Rightarrow 16 + 4\,(1)^2 - 4 \times 4 \times 1 \times \frac{1}{4} = \lambda^2$

$\Rightarrow 16 + 4 - 4 = \lambda^2 \Rightarrow \lambda = \pm 4$

**488. B**

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\Rightarrow [\vec{a} + \vec{b} + \vec{c} \quad \vec{a} + \vec{b} \quad \vec{b} + \vec{c}]$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c})\,.\ \{(\vec{a} + \vec{b}) \times (\vec{b} \times \vec{c})\}$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c})\,.\ \{(\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c})\}$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{a}\ \vec{c}] + [\vec{c}\ \vec{a}\ \vec{b}]$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] - [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$

$$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2 = 1 \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = \pm 1$

**489. C**

Given, $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=\alpha\vec{\delta}$ ...(i), $b$ ...(ii)

From Eqs.(i)$\Rightarrow \vec{\alpha}+\beta+\vec{\gamma}+\delta = (a+1)\vec{\delta}$ ...(iii)

From Eqs.(ii)$\Rightarrow \vec{\alpha}+\beta+\vec{\gamma}+\delta = (b+1)\vec{\alpha}$ ...(iv)

From Eqs.(iii)&(iv)$\Rightarrow (a+1)\vec{\delta} = (b+1)\vec{\alpha}$ ...(v)

Since $\vec{\alpha}$ is not parallel to $\vec{\delta}$

$\therefore$ From Eq. (v) $\Rightarrow$ a + 1 = 0 and b + 1 = 0

From Eq. (iii) $\Rightarrow \vec{\alpha}+\beta+\vec{\gamma}+\delta = 0$

**490. C**

We have, z = 0 for the point where the line intersects the curve $\therefore \frac{x-2}{3} = \frac{+}{2} = \frac{}{-1}$

$\Rightarrow \frac{x-2}{3} = 1$ and $\frac{+}{2} = 1 \Rightarrow x = 5$ & $y = 1$

Put these values in $xy = c^2$

we get, $5 = c^2 \Rightarrow c = \pm\sqrt{5}$

**491. A**

The given lines are

$(a + b)x + (a - b)y - 2ab = 0$ ...(i)

$(a - b)x + (a + b)y - 2ab = 0$ ...(ii)

$x + y = 0$ ...(iii)

The triangle formed by the lines (i), (ii) and (iii) is an isosceles triangle if the internal bisector of the vertical angle is perpendicular to the third side. Now equations of bisectors of the angle between lines (i) and (ii) are

$$\frac{+ \quad + \quad - \quad -}{\sqrt{[(a+b)^2+(a-b)^2]}} = \pm\frac{- \quad + \quad + \quad -}{\sqrt{[(a-b)^2+(a+b)^2]}}$$

or $x - y = 0$ (iv)

and $x + y = 2b$ (v)

Obviously the bisector (iv) is perpendicular to the third side of the triangle. Hence, the given lines form an isosceles triangle.

**492. C**

Equation of chord of contact from $A(x_1, y_1)$ is

$xx_1 + yy_1 - a^2 = 0$

$xx_2 + yy_2 - a^2 = 0$

$xx_3 + yy_3 - a^2 = 0$

i.e., $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \Rightarrow$ A, B, C are collinear.

**493. A**

Here $(O_1O_2)^2 = t^2 + (t^2+1)^2 = t^4 + 3t^2 + 1 \geq 0$

$\Rightarrow O_1O_2 \geq 1$ and $|r_1 - r_2| = 1$

$\Rightarrow O_1O_2 \geq |r_1 - r_2|$ hence the two circles have at least one common tangent.

**494. A**

Let normals at points $A(at^2_1, 2at_1)$ and $C(at^2_3, 2at_3)$ meets the parabola again at points $B(at^2_2, 2at_2)$ and $D(at_4^2, 2at_4)$,`

then $t_2 = -t_1 - \frac{2}{t_1}$ and $t_4 = -t_3 - \frac{2}{t_3}$

Adding $t_2 + t_4 = -t_1 - t_3 - \frac{2}{t_1} - \frac{2}{t_3}$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = -\frac{2}{t_1} - \frac{2}{t_3}$

$\Rightarrow \frac{1}{t_1} + \frac{1}{t_3} = 0 \qquad \Rightarrow t_1 + t_3 = 0$

Now, point of intersection of tangent at A and C will be $(at_1t_3, a(t_1 + t_3))$

Since $t_1 + t_3 = 0$, so this point will lie on x-axis, which is axis of parabola.

**495. D**

Let $C_1$, $C_2$ the centres and $r_1$, $r_2$ be the radii of the two circles. Let $S_1=0$ lies completely inside the circle. $S_2 = 0$. Let C and r be centre and radius of the variable circle, respectively.

then $CC_2 = r_2 - r$ and

$C_1C = r_1 + r$

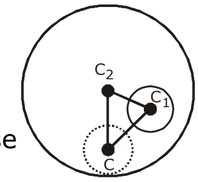
$\Rightarrow C_1C + C_2C = r_1 + r_2$ (constant)

$\Rightarrow$ Locus of C is an ellipse

$\Rightarrow S_2$ is true

Statement 1 is false

(two circles are intersecting).




**496. D**

We have $\sqrt{(\lambda-3)^2+16} - 4 = 1$

$\Rightarrow \lambda = 0$ or 6

**497. B**

$\vec{r}\cdot\vec{a} = \vec{r}\cdot\vec{b} = \vec{r}\cdot\vec{c} = 0$ only if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar. $\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = 0$

Hence, Statement 2 is true.

Also, $[\vec{a}-\vec{b} \;\; \vec{b}-\vec{c} \;\; \vec{c}-\vec{a}] = 0$

even if $[\vec{a}\,\vec{b}\,\vec{c}] \neq 0$.

Hence, statement 2 is not the correct expanation for statement 1

**498. B**

Let $\vec{d} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\vec{c}$

$\Rightarrow [\vec{d}\,\vec{a}\,\vec{b}] = \lambda_3[\vec{c}\,\vec{a}\,\vec{b}] \Rightarrow \lambda_3 = 1$

$[\vec{c}\,\vec{a}\,\vec{b}] = 1$ (because $\vec{a}, \vec{b}$, and $\vec{c}$ are three mutually perpendicular unit vectors)

Similarly, $\lambda_1 = \lambda_2 = 1 \Rightarrow \vec{d} = \vec{a} + \vec{b} + \vec{c}$

Hence Statement 1 and Statement 2 are correct, but Statement 2 does not explain Statement 1 as it does not give the value of dot products.

**499. A**

Any point on the first line is $(2x_1 + 1, x_1 - 3, -3x_1 + 2)$

Any point on the second line is $(y_1 + 2, -3y_1 + 1, 2y_1 - 3)$.

If two lines are coplanar, then $2x_1 - y_1 = 1$, $x_1 + 3y_1 = 4$ and $3x_1 + 2y_1 = 5$ are consistent.

**500. C**

Any point on the line $\frac{x-1}{1} = \frac{}{-1} = \frac{+}{2}$ is $B(t + 1, -t, 2t - 2)$, $t \in R$.

Also, AB is perpendicular to the line, where A is (1, 2, −4).

$\Rightarrow 1(t) - (-t - 2) + 2(2t + 2) = 0$

$\Rightarrow 6t + 6 = 0 \qquad \Rightarrow t = -1$

Point B is (0, 1, −4)

Hence, $AB = \sqrt{1+1+0} = \sqrt{2}$