10 Clocks

In clock short hand is called the hour needle and longer hand is called minute needle, some clocks having one more hand for seconds. All hands can rotate up to 360° angle: hour hand in 12 hours, minute hand in 60 minutes and seconds hand in 60 seconds subtend the 360° angle.

- The minute hand completes one revolution in one hour and in one day, the minute hand makes 24 revolutions.
- The hour hand completes one revolution in 12 hours and in one day, the hour hand makes two revolutions.
- The angle between the minute hand and the hour hand changes at the rate of 5.5° per minute.
- The angle between the minute hand and the hour hand at *t* o'clock is $(t \times 30)°$ when *t* is less than or equal to 6, and it is $[360° - (t \times 30)°]$ when t is greater than 6.
- The minute hand and the hour hand overlap 22 times in a day (or 11 times in a period of 12 hr). The time gap between two consecutive overlapping is 1 hr $5\frac{5}{11}$ min (or $\frac{12}{11}$ hr).
- Between *t* and (*t* + 1) o'clock, the two hands of a clock overlap at $t \times \frac{12}{11}$ hr or at $5 \times t\left(\frac{12}{11}\right)$ min past *t*.
- The angle between the minute hand and the hour hand is 180° on 11 occasions in a period of 12 hr. The angle between two hands is 90° on 22 occasions in a period of 12 hr.

Note

A clock is said to be running fast when the time shown by that clock is more than the time shown by the correct clock. Similarly, the clock is said to be running slow, when the time shown by that clock is less than the time shown by the correct clock.

If a clock indicates the time 8:20 when the correct time is 8:10, we say that the clock is running 10 min fast. Likewise, if a clock indicates 8:00, when the correct time is 8:10, we say that the clock is running 10 min show.

If a clock is running 5 min fast for each hour, we say that the clock is gaining time at the rate of 5 min per hour. Likewise, if a clock is running 2 min slow for each hour, we say that the clock is losing time at the rate of 2 min per hour.

Examples 1. At what time between 7 and 8 o'clock will the hands of a clock be in the same straight line, but not together?

Solution:

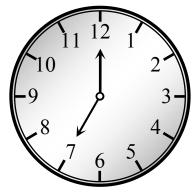


Fig (a)

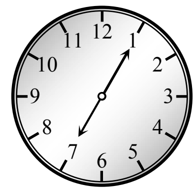


Fig (b)

Fig (a) shows the positions of the hands of the clock at 7 O'clock, and Fig (b) shows the position of hands of the clock when both the hands are in the opposite direction in a straight line (i.e., the angle between the two hand is 180°) .

The angle between the two hands at 7 o'clock is 150°. The two hands will be in a straight line, when the angle between them is 180º, that is when the angle increases by another 30°.

The angle changes at the rate of 5.5° per minute.

So, the time taken to change the angle by 30° is $\frac{1}{5.5} \times 30 = 5\frac{5}{11}$ min.

Therefore, the hands are in the same straight line (but not together), at $5\frac{5}{11}$ min past 7

Examples 2. At what time, between 4 and 5 o'clock, will the two hands of the clock overlap?

Solution: At 4 o'clock, the hour hand will be at 4 and the minute hand will be at 12.

So, the angle between the two hands at 4 o'clock is 120°.

The hands overlap when the angle between them reduces to zero.

The angle change by 5.5° in 1 min.

The angle changes by 120° in $\frac{1}{5.5} \times 20 = 21\frac{9}{11}$ min.

Therefore, the hands overlap at $21\frac{9}{11}$ min past 4.

Formula: Between *t* and (*t* + 1) o'clock, the two hands will overlap at $5 \times t\left(\frac{12}{11}\right)$ min past *t*. In this case, $5 \times 4\left(\frac{12}{11}\right) = 21\frac{9}{11}$ min past.

Note

Remember the direct formula and apply to get the answer much faster.

Examples 3. At what time, between 4 and 5 o'clock, are the hand 2 min space apart?

Solution: The hands are two minutes space apart is the other way of saying that the angle between the hand is 12° (one minutes space apart is equal to 6°). At 4 o'clock the angel between the hands is 120°. When this angle changes by 180° (120° – 12°) and 132° (120° + 12°), the angle between the hands will be 12°.

The angle between the hands changes by 5.5° in 1 min.

The angle changes by 108° in $\frac{1}{5.5}\times 108 = 19\times\frac{7}{11}$ min.

The angle changes by 132° in $\frac{1}{5.5}\times 132 = 24$ min.

Thus, the hands are 2 min space apart, once at $19\times\frac{7}{11}$ min past 4 and again at 24 min past 4 (i.e., at 4 hr, 19 min, 38 s and at 4 hr, 24 min).

Formula: Between t and $(t + 1)$ o'clock, the two hands will be a minutes apart at $(5t \pm a)\times\frac{12}{11} = 18\times\frac{12}{11}$ or $22\times\frac{12}{11} = 19\times\frac{7}{11}$ or 24 min.

Therefore, they will be 2-min space apart at $19\times\frac{7}{11}$ min past 4 and 24 min past 4.

Examples 4. A minute hand of a clock overtakes the hour hand at intervals of 63 min of correct time. How much time does the clock lose or gain per day?

Solution: In any clock, the minute and hour hands overlap once in every $65\times\frac{5}{11}$ min, as per that clock. In the given clock the minute and hour hands are overlapping once in 63 min of correct time.

Thus the clock is gaining $65\times\frac{5}{11} - 63 = 2\frac{5}{11} = \frac{27}{11}$ min in 63 min.

$\therefore$ Time gained in 24 hr $= \frac{27\times 60\times 24}{11\times 63} = 56\frac{8}{77}$ min.

Formula: Gain or loss in 24 hr (1 day)

$$= \left[\frac{720}{11} - \text{given interval in minutes}\right]\times 60\times\frac{24}{\text{given interval in minutes}}$$

$$= \frac{65\frac{5}{11} - \text{given interval}}{\text{given minutes}}\times 60\times 24$$

If the sign is positive (+), the clock is gaining time, and if the sign is negative (–), the clock is losing time.

Here, $\left[\frac{720}{11} - 63\right]\times 60\times\frac{24}{63}$ min

$= \frac{27}{11}\times 60\times\frac{24}{63}$ min $= 56\frac{8}{77}$ min

Since the sing is +ve, there is again of $56\frac{8}{77}$ min per day.

Examples 5. At what time between 5 and 6 o'clock will the minute and the hour hand be perpendicular to each other?

Solution: When the angle between the two hands is 90°, the hands are perpendicular to each other.

The angle between the minute hand and the hour hand at 5 o'clock is 150°.

The angle becomes 90°, when it changes by 60° and 240°, that is (150° – 90°) and (150° + 90).

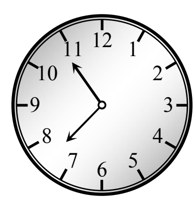
The angel changes by 5.5° in 1 min

The angle change by 60° in $\frac{1}{5.5}\times 60 = 10\frac{10}{11}$ min.

The angle changes by 240° in $\frac{1}{5.5}\times 240 = 43\frac{7}{11}$ min.

Therefore, the two hands are perpendicular at 5 hr $10\frac{10}{11}$ min, and angle at 5 hr $43\frac{7}{11}$ min.

Examples 6. Helen's watch needs repairing. She sets it correctly at 4:12 pm but three hours later it shows 8:00 pm. After a further two hours she notices that it shows 10:32 pm. She goes to bed early and gets up when her watch shows 6:46 am. What time is it really?



Solution: [3:42 am]: The watch gains 16 minutes per hour.

Examples 7. Deepa notices the reflection of a wall clock in a mirror. The time shown by this clock when see in the mirror is 9 hours 30 minutes. What is the actual time shown on the clock:

a. 5 hours 30 minutes
b. 2 hours 30 minutes
c. 4 hours 20 minutes
d. 3 hours 20 minutes

Solution: (b) According to the question. The time shown by the clock when see in the mirror = 9 hours 30 minutes.

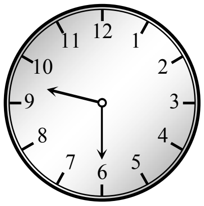


Fig. (a) Time in mirror

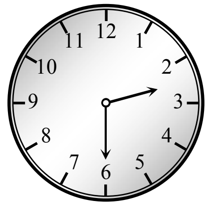


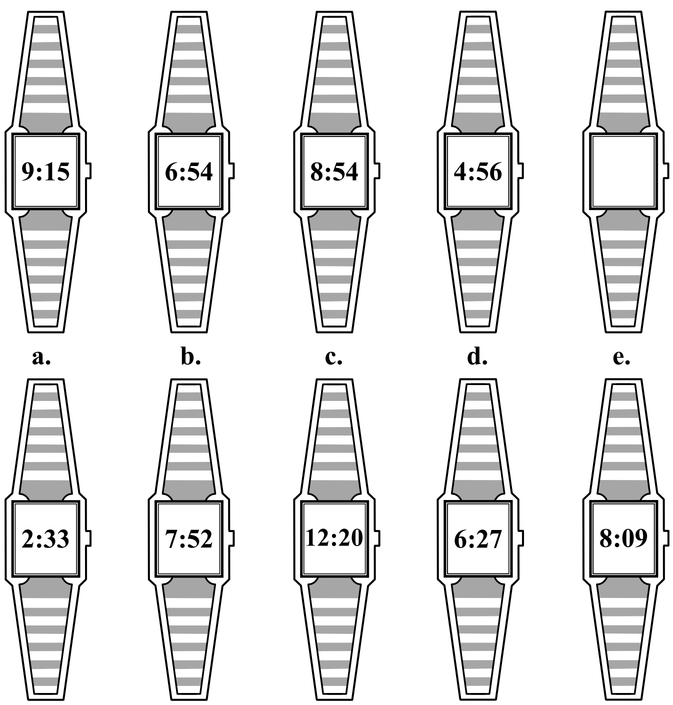
Fig. (b) Actual time

Therefore, the actual times shown on the clock

= (12 – 9 hours 30 minutes)

= 2 hours 30 minutes

Examples 8. Which watch fits on the end of this sequence?



Solution: (d) On each watch the digits add up to 15. (like $9+1+5=15,\ 6+5+4=15$ etc.)

Examples 9. How much does the clock gain or lose per day if its hands coincide every 69 minutes?

a. Gains in $80\frac{80}{341}$ minutes

b. Gains in $92\frac{244}{253}$ 30 minutes

c. Loses in $80\frac{80}{253}$ minutes

d. Loses $73\frac{251}{253}$ minutes

Solution: (d) As we know that in a correct clock, the hands of a clock coincide every $65\frac{5}{11}$ minutes. But in case of incorrect clocks, if the hands of a clock coincide is greater than $65\frac{5}{11}$ minutes, then the clock lose time.

So, Loss in 69 minutes $=\left(69-65\frac{5}{11}\right)=\left\{69-\left(65+\frac{5}{11}\right)\right\}$

$=\left(4-\frac{5}{11}\right)=\frac{39}{11}$ minutes

So, Loss in day

i.e., 24 hours

$=24\times 60\times\frac{39}{11}\times\frac{1}{69}$

$=\frac{1870}{253}=73\frac{251}{253}$

Therefore, the clock loses in a day i.e., in 24 hours.

Use the Formula: If $x>65\frac{5}{11}$, then total times lost in T hours

$=(T\times 60)\left\{x-65\frac{5}{11}\right\}/x$ minutes.

Where x is the time in which the hands of the incorrect clock coincide.

Examples 10. The reflex angle between the hands of a clock at 10.25 is:

a. $192\frac{1}{2}^{\circ}$

b. 195°

c. 180°

d. $197\frac{1}{2}^{\circ}$

Solution: (d) Angle traced by hour hand in 12 hrs = 360°

So, Angle traced by hour hand in $\frac{125}{12}$ hrs.

$=\left(\frac{360}{12}\times\frac{125}{12}\right)^{\circ}=312\frac{1}{2}^{\circ}$

Angle traced by minute hand in 60 minutes = 360°

So, Angle traced by minute hand in 25 minutes

$=\left(\frac{360}{12}\times 25\right)=150^{\circ}$

Therefore, reflex angle

$=360^{\circ}-\left(312\frac{1}{2}^{\circ}-150^{\circ}\right)$

$=360^{\circ}-162\frac{1}{2}^{\circ}=197\frac{1}{2}^{\circ}$

Multiple Choice Questions

1. At what angle the hands of a clock are inclined at 15 minutes past 5?

a. $72\frac{1°}{2}$ **b.** $67\frac{1°}{2}$ **c.** $58\frac{1°}{2}$ **d.** 64°

2. How many times are the hands of a clock at right angles in a day?

a. 22 **b.** 44 **c.** 11 **d.** 24

3. How many times do the hands of a clock point towards each other in a day?

a. 11 **b.** 48 **c.** 24 **d.** 22

4. At what angle the hands of a clock are inclined at 30 minutes past 6?

a. 45° **b.** 15° **c.** 30° **d.** 20°

5. At what time between 5 and 6 are the hands of a clock coincident?

a. 22 minutes past 5 **b.** 30 minutes past 5

c. $22\frac{8}{11}$ minutes past 5 **d.** $27\frac{3}{11}$ minutes past 5

6. What is the angle between the two hands of a clock, when the clock shows 5 hours 30 minutes?

a. 45° **b.** 30° **c.** 60° **d.** 15°

7. At what time between 5.30 and 6 will the hands of a clock be at right angles?

a. $43\frac{5}{11}$ minutes past 5 **b.** $43\frac{7}{11}$ minutes past 5

c. 40 minutes past 5 **d.** 45 minutes past 5

8. At what time between 9 and 10 o'clock will the hands of a watch be together?

a. 35 minutes past 9 **b.** $49\frac{1}{11}$ minutes past 9

c. $53\frac{2}{11}$ minutes past 9 **d.** $45\frac{1}{11}$ minutes past 9

9. If the actual times shown on the clock is 6 hours 45 minutes, then what time does it shown on the mirror?

a. 5 hours 25 minutes **b.** 9 hours 15 minutes

c. 9 hours 25 minutes **d.** 5 hours 15 minutes

10. How many degrees does an hour hand covers in 20 minutes?

a. 5° **b.** 10° **c.** 15° **d.** 20°

11. How many degrees does the minute hand covers, in the same time in which the second hand covers 180°?

a. 18° **b.** 6° **c.** 10° **d.** 3°

12. The angle between the two hands of a clock is 30°, when the hour hand is between 5 and 6. What time does the watch show?

a. 5 hours $20\frac{20}{11}$ minutes **b.** 5 hours $27\frac{3}{11}$ minutes

c. 5 hours $30\frac{30}{11}$ minutes **d.** **B**oth (a) and (c)

13. A clock is set right at 8 a.m. the clock uniformly loses 24 minutes in a day. What will be the right time when the clock indicates 4 pm on the next day?

a. 4.50 pm **b.** 4.30 pm

c. 4.50 am **d.** 4.32 am

14. A watch, which gains uniformly, is 2 min. show at noon on Monday, and is 4 min. 48 seconds fast at 2 p.m. on the following Monday. What was it correct?

a. 2 p.m. on Tuesday **b.** 2p.m. on Wednesday

c. 3 p.m. on Thursday **d.** 1 p.m. on Friday

15. How much does the clock gain or lose per day if its hands coincide every 63 minutes?

a. Loses $56\frac{8}{77}$ minutes **b.** Gains $56\frac{8}{77}$ minutes

c. Loses $56\frac{56}{77}$ minutes **d.** Gains $75\frac{75}{341}$ minutes

16. A clock, which loses uniformly is 15 minutes fast at 9 am on 3rd of the December and is 25 minutes less than the correct time at 3 pm on 6^{th} of the same month. At what time it was correct?

a. 2.15 am on 3rd **b.** 2.15 pm on 4th

c. 2.15 pm on 3rd **d.** 2.15 am on 4th

17. Two clocks are set correctly at 10 am on Sunday. One clock loses 3 minutes in an hour while the other gain 2 minutes in an hour. By how many minutes do the two clocks differ at 4 pm on the same day?

a. 25 minutes **b.** 20 minutes

c. 35 minutes **d.** 30 minutes

18. How many times do the hands ofa clock point towards each other in a day?

a. 24 **b.** 20

c. 21 **d.** 22

19. A watch, which gains uniformly, is 3 minutes slow at 12 noon on Sunday and is 5 minutes 36 seconds fast at 4 pm on the next Sunday. At what time it was correct?

a. 12 am on same day **b.** 12 pm on Monday

c. 12 am on Tuesday **d.** 12 am on Wednesday

20. An accurate clock shows 8 O' clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 O'clock in the afternoon?

a. 150° **b.** 168° **c.** 144° **d.** 180°

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	b	d	b	d	d	b	b	d	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
d	d	b	b	b	b	d	d	d	d

SOLUTIONS

1. **(b)** At 15 minutes past 5, the minute hand is at 3 and hours hand slightly advanced from 5. Angle between their 3^{rd} and 5^{th} position is $60°$ + the hour hand moves in 15 minutes from 5 towards 6.

2. **(b)** As we know that the hands of a clock are at right angle twice in every hour. So, in a period of 12 hours, the hands of a clock are at right angle = 22 times.

[∵ Two positions of the hands of a clock i.e., at 3 o'clock and 9 o'clock are same.]

Therefore, the hands of a clock are at right angles with each other in a day i.e., in 24 hours = (2 × 2) times = 44 times.

3. **(d)** As we know that the hands of a clock points towards each other in every 12 hours = 11 times.

[∵ The hands of a clock point towards each other once at 6 o'clock between 5 and 7]

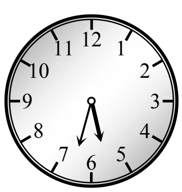
Therefore, the hands of a clock point towards each other in a day i.e., in 24 hours $=11\times2$ times = 22 times.

4 **(b)** From the formula

$\theta = 30h - \frac{11}{2}m$ $\quad\left[\because 30h > \frac{11}{2}m\right]$

According to the question, Here, h = 6, m = 30.

So, $\theta = 30\times6 - \frac{11}{2}\times30 = 180 - 165 = 15$



Therefore, the hands of a clock are inclined at 30 minutes past 6 at 15°.

5. **(d)** At 5 O'clock, the minute hand is 25 minute spaces apart. To be coincident, it must gain 25 minute spaces. Now, 55 minutes are gained in 60 minutes.

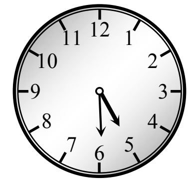
25 minutes will be gained in $\left(\frac{60}{55}\times25\right)$ min or $27\frac{3}{11}$ min,

So, the hands are coincident at $27\frac{3}{11}$ min past 5.

6. **(d)** From the formula

$\theta = \frac{11}{2}m - 30h$

$\left[\because \frac{11}{2}m > 30h\right]$



where θ = angle,

m = minutes and

h = hours

According to the question

Here, m = 30 minutes, h = 5 hours

So, $\theta = \frac{11}{2}\times30 - 30\times5 = 11\times15 - 150 = 165 - 150 = 15°$

Therefore, the angle between the two hands of a clock at 5 hours 30 minutes = 15°

7. **(b)** At 5 O'clock, the hands are 25 min. spaces apart. To be at right angles and that too between 5.30 and 6, the min. hand has to gain (25 + 15) or 40 min, spaces. Now 55 min spaces are gained in 60 min.

∴ 40 min space are gained in $\left(\frac{60}{55}\times40\right)$ min

or $43\frac{7}{11}$ min.

So, the hands are at right angles at $43\frac{7}{11}$ min past 5.

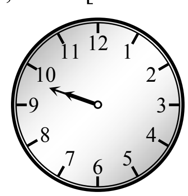
8. **(b)** From the formula

$\theta = \frac{11}{2}m - 30h$

$\left[\because \frac{11}{2}m > 30h\right]$

According to the question

Here, θ = 0°, h = 9 [∵ the hands of a clock coincide at 0°]



So, $0 = \frac{11}{2}m - 30h$

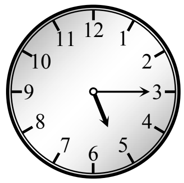
Or $\frac{11}{2}m = 30h$

Or, $m = \frac{60\text{ h}}{11} = \frac{60\times9}{11} = \frac{540}{11} = 49\frac{1}{11}$ minutes past 9

Therefore, the two hands of the clock between 9 and 10 o'cock are together at

$= 49\frac{1}{11}$ minutes past 9.

9. **(d)** According to the question, the actual time shown on the clock = 6 hours 45 minutes.

Therefore, the time shown on the mirror by this clock = (15 – 6 hours 45 minutes) = 5 hours 15 minutes.

10. **(b)** As we know that the hour hand moves through $=\frac{1^\circ}{2}$ in each minute i.e., 1 minute $=\frac{1^\circ}{2}$.

Therefore, the hour hand moves in 20 minutes $=\frac{1^\circ}{2}\times 20 = 10^\circ$.

11. **(d)** As we know that the second hand covers 360° in 1 minute i.e., 60 seconds.

So, the second hand covers 180° in $=\frac{60}{360^\circ}\times 180^\circ = 30$ seconds.

Also, we know that the minute hand covers 6° in 1 minute i.e., in 60 seconds.

Therefore, the minute hand covers in 30 seconds $=\frac{6^\circ}{2}\times 3^\circ$

12. **(d)** From the formula,

$\theta = \frac{11}{2}\text{m} - 30\text{h}$ $\left[\text{when } \frac{11}{2}\text{m} > 30\text{h}\right]$

According to the question,

Here $\theta = 30^\circ, \text{h} = 5$.

So $30 = \frac{11}{2}\text{m} - 30\times 5$

or $\text{m} = \frac{30\times 2 + 30\times 5\times 2}{11}$

or $\text{m} = \frac{60+300}{11} = \frac{360}{11}$

or, $\text{m} = 30\frac{30}{11}$ minutes past 5 o'clock.

Also, $\theta = 30\text{h} - \frac{11}{2}\text{m}$ $\left[\text{when } 30\text{h} > \frac{11}{2}\text{m}\right]$

So, $30 = 30\times 5 - \frac{11}{2}\text{m}$

or $\text{m} = \frac{30\times 5\times 2 - 30\times 2}{11}$

or, $\text{m} = \frac{300-60}{11} = \frac{240}{11}$

or, $\text{m} = 20\frac{20}{11}$ minutes past 5 o'clock.

Therefore, the angle between the two hands is 30°, when the clock shows = 5 hours $30\frac{30}{11}$ minutes and 5 hours $20\frac{20}{11}$ minutes.

13. **(b)** According to the question, Total time from 8 am of a particular day to 4 pm on the next day = 32 hours.

∴ The clock loses 24 minutes in 24 hours

So, 23 hours 36 minutes of this clock = 24 hours of the correct clock.

or, $23\frac{36}{60}$ i.e., $23\frac{3}{5}$

or, $\left(23+\frac{3}{5}\right)$ of this clock = 24 hours of the correct clock.

or $\frac{118}{5}$ of the clock = 24 hours of the correct clock.

So, 32 hours of this clock $=\left(\frac{32\times 5\times 24}{118}\right)$ hours of the correct clock.

= 32 hours 30 minutes approximately.

Therefore, the right time is 32 hours 30 minutes (approx.)

After 8 am = 4 hours 30 minutes = 4.30 pm

14. **(b)** Time from Monday noon to 2 p.m. on following Monday = 7 days 2 hour

= 170 hours.

The watch gain $\left(2+4\frac{4}{5}\right)$ or $\frac{34}{5}$ min. in 170 hours.

∴ It will gain 2 min in $\left(\frac{170\times 5}{34}\times 2\right)$ hrs.

= 50 hrs = 2 days 2hrs

So, the watch is correct 2 days 2 hours after Monday noon, i.e., at 2 pm on Wednesday.

15. **(b)** As we know that in correct clock, the hands of a clock coincide every $65\frac{5}{11}$ minutes but in case of incorrect clocks, if the hands of a clock coincide is less than $65\frac{5}{11}$ minutes, then the clock gains time.

So, Gain in 63 minutes

$$= \left(65\frac{5}{11} - 63\right) = \left(65 + \frac{5}{11} - 63\right) = \left(2 + \frac{5}{11}\right)$$

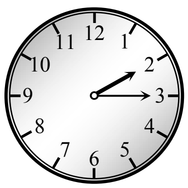
$$= \frac{27}{11} \text{ minutes } \left[\therefore 65\frac{5}{11} \text{ can be written as } 65 + \frac{5}{11}\right]$$

So, Gain in a day i.e., 24 hours

$$= 24 \times 60 \times \frac{27}{11} \times \frac{1}{63} = \frac{4320}{77} = 56\frac{8}{77} \text{ minutes.}$$

Therefore, the clock gains in a day i.e., 24 hours $= 56\frac{8}{77}$

16. **(b)** According to the question, total time from 9 am on 3rd of the December to 3 pm on 6th of the December = 3 days 6 hours = 78 hours.



Also, the clock loses in 78 hours = (15 + 25) = 40 minutes.

So, the clock loses 15 minutes in $= \frac{15}{40} \times 78 = 29$ hours 15 minutes.

Therefore, the clock is correct after 29 hours 15 minutes form 9 am on 3rd December = 2.15 pm on 4th December.

17. **(d)** According to the question,

One clock loses = 3 minutes per hour

Other clock gains = 2 minutes per hour

So, the difference in minutes between these two clocks in one hour = 2 – (–3) (2 + 3) minutes = 5 minutes. [∵ Loss in minutes = negative sign]

Total time from 10 am to 4 pm on Sunday = 6 hours.

Therefore, the difference in minutes between these two clocks from 10 am to 4 pm on Sunday = (5 × 6) minutes = 30 minutes.

18. **(d)** The hands of a clock point towards each other 11 times in every 12 hours (because between 5 and 7, at 6 O' clock only they point towards each other).

19. **(d)** According to the question, Total time from Sunday noon (12 pm) to 4 pm the next Sunday = 7 days 4 hours = 172 hours. Also, the watch gains in 172 hours

= 3 minutes + 5 minutes 36 seconds

$$= 3 + 5\frac{36}{60} = \left(3 + 5\frac{3}{5}\right) \text{ minutes } = \frac{43}{5} \text{ minutes.}$$

So, the watch gains 3 minutes in $= \left(\frac{3}{43/5}\right) \times 172$ hours

$$= \left(\frac{3 \times 5 \times 172}{43}\right) \text{ hours.} = 60 \text{ hours} = 2 \text{ days } 12 \text{ hours.}$$

Therefore, the watch is correct after 2 days 12 hours from Sunday 12 noon = 12 am on Wednesday.

[∵ 2 days 12 hours + 12 hours

= 2 days 24 hours = 3 days]

20. **(d)** Time difference between 8 am and 2 pm= 6 hrs. Required rotation by the hour hand in 6 hours

$$= \left(\frac{360}{12} \times 6\right)^{\circ} = 180^{\circ}$$

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