Chapter

30 SETS

KEY FACTS

1. A **set** is a well defined collection of objects.
2. Sets are usually denoted by capital letters $A, B, C,$ etc. and their elements by small letters $a, b, c,$ etc.
3. If an element 'a' is a member of set A, then we write $\boldsymbol{a \in A}$.

   If an element 'a' is not a member of set A, then we write $\boldsymbol{a \notin A}$.
4. There are two main ways of expressing a set:

   (i) **Tabular form or Roster form:** In this form we list all the members of the set separating them by commas and enclosing them in only brackets.

   Ex. The set of the first ten perfect square numbers is written as

   $S = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

Note :	
	• The elements of a set can be written in any order.
	• An element of a set is not written more than once.

   (ii) **Set-builder form or Rule method:** In this method, instead of listing all elements of a set, we write the set by some special property or properties satisfied by all the elements and write it as:

   $$A = \{ x : P(x)\} \text{ or } A = \{ x \mid x \text{ has the property } P(x)\}$$

   Ex. The set of the first ten perfect square numbers is written as:

   $$S = \{x \mid x = n^2, 1 \le n \le 10, n \in N \}$$

5. **Types of Sets:**

   (i) **Finite set:** A set having no elements or a definite number of elements is called a finite set.

   Ex. V = The set of vowels of english alphabet = $\{ a, e, i, o, u\}$

   (ii) **Infinite set:** A set having unlimited number of elements is called an infinite set.

   Ex. N = The set of natural numbers = $\{ 1, 2, 3, 4 , ...\}$

   (iii) **Empty set:** A set containing no element is called the **empty** or **null** or **void** set. The symbol for empty set is ϕ.

   $$\phi = \{ \ \}$$

   (iv) **Singleton set:** A set containing only one element is called a singleton set.

   Ex. $\{2\}$, $\{a\}$, $\{0\}$

Note : $\{ \ \}$ is the empty set whereas $\{0\}$ is a singleton set.

6. **Cardinal number of a finite set:** The number of distinct elements in a finite set A is called the cardinal number of A and is denoted by $n(A)$

Ex. If $A = \{ a, e, i, o, u \}$, then $n(A) = \mathbf{5}$.

7. **Equal sets:** Two sets P and Q containing the same elements are called equal sets.

Here, $P = Q$ iff $a \in P \Rightarrow a \in Q$ and $a \in Q \Rightarrow a \in P$.

Ex. P = {letters of the word 'ramp'}

Q = {letters of the word 'pram '}

$\Rightarrow \quad P = Q$

8. **Equivalent sets:** Two sets A and B containing equal number of elements, which are not necessarily the same are called equivalent sets.

Ex. A = { letters of the word 'flower'}

$= \{ f, l, o, w, e, r \}$

$B = \{2, 4, 6, 8, 10, 12\}$

Here, $n(A) = 6$ and $n(B) = 6$, then $A \sim B$, where ~ is the symbol for equivalence.

> **Note :** All equal sets are equivalent but all equivalent sets are not necessarily equal.

9. **Subset and superset of a set:**

If every element of set A is also an element of set B, then A is a subset of B.

We write it symbolically as $A \subseteq B$ where '$\subseteq$' denotes **'is a subset of'.**

Ex. $A = \{4, 8, 12\}$, $B = \{ 2, 4, 6, 8, 10, 12\}$, then $A \subseteq B$.

$\Rightarrow$ A is contained in B

$\Rightarrow$ B contains A

$\Rightarrow$ B is a **superset** of A

$\Rightarrow$ $B \supseteq A$

> **Note :** (i) Every set is a subset of itself.
> (ii) Null set is a subset of every set.

10. **Proper subset of a set:**

All the subsets of a set, other than the set itself are known as proper subsets.

Ex. The subsets of set $A = \{ 1, 2, 3\}$ are { }, {1}, {2}, {3} , {1, 2}, {2, 3}, {1, 3} and {1, 2, 3}

Here all the subsets except the set {1, 2, 3} are proper subsets of A. The symbol '$\subseteq$' denotes ' **is a proper subset of '.** Thus, $\{ \} \subset A$, $\{1\} \subset A$, etc.

> **Note :** (i) $\not\subseteq$ denotes 'is not a subset of '.
> (ii) $\not\subset$ denotes 'is not a proper subset of '.

11. If there are n elements in a set P,

(i) the total number of subsets of P is 2^n.

(ii) the total number of proper subsets of P is $2^n–1$.

Ex. If $A = \{ a, b, c, d\}$, then

number of subsets of $A = 2^4 = \mathbf{16}$

number of proper subsets of $A = 2^4 – 1 = 16 – 1 = \mathbf{15}$

12. **Power set:**

The set of all the subsets of a given set Q is called the **power set** of Q and is denoted by $P(Q)$

Ex. If $S = \{1\}$, then $P(S) = \{ \phi , s\}$

If $T = \{a, b\}$, then $P(T) = \{ \phi , \{a\}, \{b\}, T\}$

13. Comparable sets:

Two sets A and B are said to be comparable, if one of them is a subset of the other, *i.e.,* A and B are comparable if either $A \subseteq B$ or $B \subseteq A$.

Ex. A = {set of vowels}, B = {letters of english alphabet} are comparable as $A \subseteq B$.

14. Universal set:

The superset of all the sets for a particular discussion is called the **universal set.** It is denoted by $\cup$ or ξ.

Ex. The set of integers is the universal set for the set of positive numbers and negative numbers, also for prime numbers and composite numbers.

15. Complement of a set:

The set of elements of universal set, which are not in a given set (say P) is the complement of P, denoted by P'.

$P' = \{x \in \xi\ ; x \notin P)$

Ex. ξ = {set of natural numbers}, P = { set of even numbers}.

Then P' = {set of odd numbers}

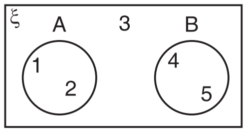
16. Venn diagrams:

A British mathematician venn pictorially represented universal sets, subsets, properties of sets and operations on sets by diagrams called venn diagrams.

The universal set is represented by a rectangle, the subsets by circles, ovals etc within the rectangle.

The elements of the sets are written inside the curve.

Ex. If $\xi = \{1, 2, 3, 4, 5\}$, $A = \{1,2\}$ and $B = \{4, 5\}$, then the venn diagram representing this information is :



17.

(i) **Overlapping sets:** Two sets are called overlapping if they have at least one element in common.

Ex. The sets {2, 3, 4} and {3, 6, 9} are overlapping as the element 3 is common to both of them.

(ii) **Disjoint sets:** If two sets A and B have no elements in common, they are called disjoint sets.

Ex. A ={ set of vowels} B = { set of consonants} are disjoint as they have no letter in common.

18. Operations on Sets.

(i) **Union of sets :** The **union of two sets** P and Q is the set of all the elements which belong to either P or Q or both.

The symbol for '**union of** ' is '**$\cup$**'.

Thus $\boldsymbol{P \cup Q = \{x : \text{either } x \in P \text{ or } x \in Q\}}$

Ex. $P = \{4, 6, 8, 16\}$, $Q = \{1, 4, 9, 16\}$

Then $P \cup Q = \{1, 4, 6, 8, 9, 16\}$

Properties of Union of Sets

(i) If A is any set, then (a) $A \cup \phi = A$ (b) $A \cup \xi = \xi$ (c) $A \cup A = A$ (d) $A \cup A' = \xi$

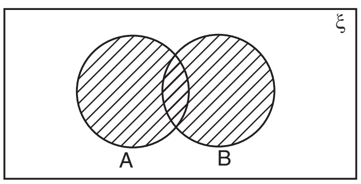
(ii) If A, B and C are any sets, then

(a) $A \cup B = B \cup A$ (Commutative law) (b) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

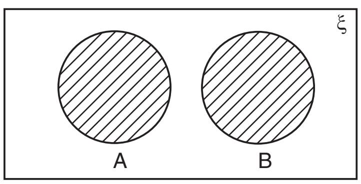
(c) $A \subseteq A \cup B$ and $B \subseteq A \cup B$ (d) If $A \subseteq B$, then $A \cup B = B$

(i) **Venn diagrams illustrating union of sets**

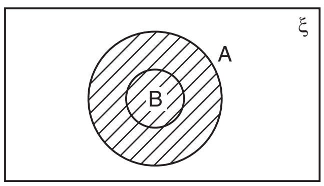
The shaded portions in the following diagrams illustrate the union of given sets.



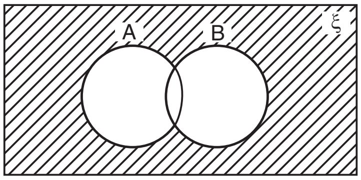
$A \cup B$
(Overlapping sets)



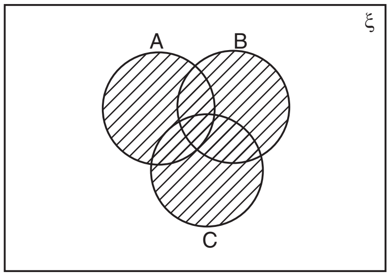
$A \cup B$
(Disjoint sets)



$A \cup B$
(B is a subset of A)



$(A \cup B)'$



$A \cup B \cup C$

(ii) Intersection of Sets

The intersection of two sets A and B is a set that contains elements that are in both A and B.

The symbol for '*intersection of*' is '***n***'.

$$\mathbf{A \cap B = \{\, x \mid x \in A \text{ and } x \in B\}}$$

Ex. A = { letters of the word 'fun' }

B = { letters of the word 'son' }

Then, $A \cap B = \{\, n\}$

Properties of Intersection of Sets

(i) If A is any set, then (*a*) $A \cap \phi = \phi$ (*b*) $A \cap \xi = A$ (*c*) $A \cap A = A$ (*d*) $A \cap A' = \phi$

(ii) If A, B and C be any sets, then

(a) $A \cap B = B \cap A$ (Commutative law)

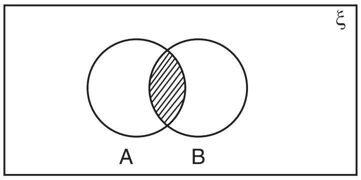
(b) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(c) If $A \subseteq B$, then $A \cap B = A$

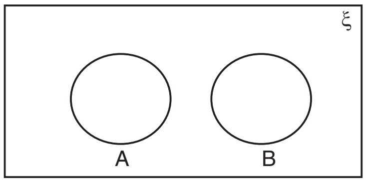
(d) For any sets A and B, we have $A \cap B \subseteq A$ and $A \cap B \subseteq B$

Venn diagrams illustrating the intersection of sets

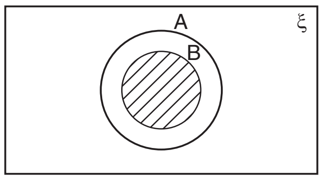
The shaded portions in the following diagrams illustrate the intersection of the given sets.



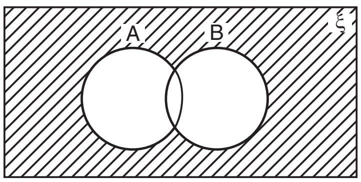
$A \cap B$ (Overlapping sets)



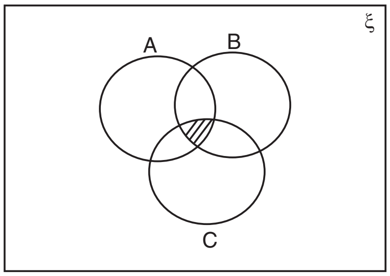
$A \cap B = \phi$ (Disjoint sets)



$A \cap B = B$ (B is a proper subset of A)



$(A \cap B)'$



$A \cap B \cap C$

Note : $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

(iii) Difference of two sets:

Let A and B be two sets, then $A - B$ is the sets of elements which belong to A but do not belong to B.

Thus, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

$B - A \; = \{x \mid x \in B \text{ and } x \notin B\}$

Ex. $A = \{2, 3, 5, 6\}, \; B = \{5, 6, 7, 8\}$

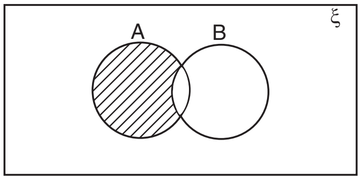
Then, $A - B \; = \{2, 3\}$

$B - A = \{7, 8\}$

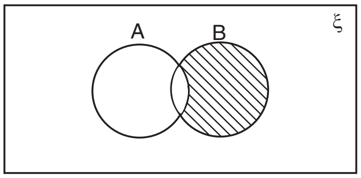
Note : In general, $A - B \neq B - A$

Venn diagrams illustrating the difference of sets.

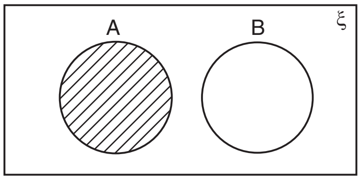
The shaded portions in the following diagrams show the difference of the given sets.



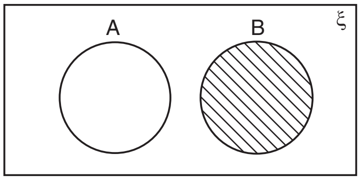
$A - B$
(Overlapping sets)



$B - A$
(Overlapping sets)

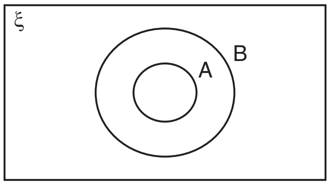


$A - B$

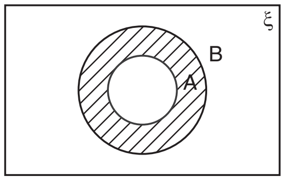


$B - A$

(Disjoint sets)



$A - B = \phi$



$B - A$

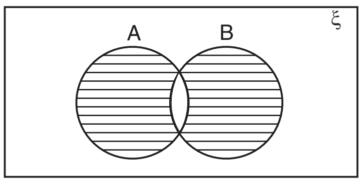
(When A is a proper subset of B)

(iv) **Symmetric difference of two sets:**

The symmetric difference of two sets A and B denoted by $A \,\Delta\, B$ is the set $(A - B) \cup (B - A)$

$$A \,\Delta\, B = (A - B) \cup (B - A)$$
$$= \{x \mid x \in A \cap B\}$$

Represented diagramatically, it is shown by the shaded part as



Ex. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$

Then $A - B = \{1, 2\}$ and $B - A = \{5, 6\}$

$\therefore$ $A \,\Delta\, B = (A - B) \cup (B - A)$

$= \{1, 2\} \cup \{5, 6\} = \{1, 2, 5, 6\}$

Cartesian Product of Sets

Let A and B be non-empty sets. The cartesian product of A and B is denoted by $A \times B$ (read 'A cross B') and is defined as the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. symbolically.

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Ex. $A = \{3, 5, 7\}$ and $B = \{a, b\}$

Then, $A \times B = \{(3, a), (5, a), (7, a), (3, b), (5, b), (7, b)\}$

$B \times A = \{(a, 3), (a, 5), (a, 7), (b, 3), (b, 5), (b, 7)\}$.

> **Note :** $A \times B \times C = \{(a, b, c) : a \in A, \ b \in B, \ c \in C\}$

SOME IMPORTANT RESULTS

1. $A \cup (B \cap B') = (A \cup B) \cap (A \cup B')$
2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
3. $A - (B \cup C) = (A - B) \cap (A - C)$
4. $A - (B \cap C) = (A - B) \cup (A - C)$
5. $A - B = (A \cup B) - B$
6. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
7. $n(A \cup B \cup C) = n(A) + n(B) + n(c) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
8. $n(A - B) = n(A) - n(A \cap B)$
9. $n(B - A) = n(B) - n(A \cap B)$
10. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$
11. $n(A' \cup B') = n(\xi) - n(A \cap B)$
12. $n(A' \cap B') = n(\xi) - n(A \cup B)$

Solved Examples

Ex. 1. ***Which of the following sets are comparable?***

(a) $A = \{2, 4, 6\}$ *and* $B = \{1, 3, 5, 6\}$

(b) $A = \{x : x = 4n, n \in N, \ n \le 3\}$ *and* $B = \{x : x = 2n, n \in N$ *and* $n \le 6\}$

(c) $A = \{a, e, i, o, u\}$ *and* $B = \{a, e, i, \{o, u\}\}$

(d) *None of these*

Sol. $A = \{x : x = 4n, n \in N$ and $n \le 3\}$

$= \{4, 8, 12\}$

$B = \{x : x = 2n, n \in N$ and $n \le 6\}$

$= \{2, 4, 6, 8, 10, 12\}$

Here $A \subset B$

$\Rightarrow$ A and B are comparable.

Ex. 2. ***Given*** $\xi = \{x : x$ *is a natural number*$\}$

$A = \{x : x$ *is an even number*, $x \in N\}$

$B = \{x : x$ *is an odd number*, $x \in N\}$

Then $(B \cap A) - (x - A\} = ...$

Sol. $A = \{2, 4, 6, 8, ...\}$

$B = \{1, 3, 5, 7, ...\}$

$B \cap A = \{2, 4, 6, 8, ...\} \cap \{1, 3, 5, 7, ...\} = \phi$

$\xi - A = \{1, 2, 3, 4, 5, 6, ...\} - \{2, 4, 6, 8, ...\}$

$= \{1, 3, 5, 7, ...\}$

$\therefore$ $B \cap A - (\xi - A) = \phi - \{1, 3, 5, 7, ...\}$

$= \phi$

Ex. 3. ***If*** $\xi = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$A = \{3, 5, 7, 9, 11\}$

$B = \{7, 8, 9, 10, 11\}$ *then find* $(A - B)'$.

Sol. $A - B = \{3, 5\}$

$(A - B)' = \{2, 4, 6, 7, 8, 9, 10, 11\}$

Ex. 4. ***If P and Q any two sets, then Q – P =***

(a) $Q \cup P'$ ***(b)*** $Q \cap P'$ ***(c)*** $Q' \cap P'$ ***(d)*** $Q' \cap P'$

Sol. For all $x \in Q - P \Rightarrow x \in Q$ and $x \notin P$

$\Rightarrow x \in Q$ and $x \in P'$

$\Rightarrow x \in Q \cap P'$

$\therefore \quad Q - P = Q \cap P'$

Ex. 5. ***Let P and Q be two sets, then what is*** $(P \cap Q') \cup (P \cup Q)'$ ***equal to ?***

Sol.
$$\begin{aligned}(P \cap Q') \cup (P \cup Q)' &= (P \cap Q') \cap (P' \cap Q') \\ &= (P \cup P') \cap (P \cup Q') \cap (Q' \cup P') \cap (Q' \cup Q') \\ &= \xi \cap \{Q' \cup (P \cap P')\} \cap Q' \\ &= \xi \cap \{Q' \cup \xi) \cap Q' \\ &= \xi \cap Q' \cap Q' = \xi \cap Q' = \boldsymbol{\xi}\end{aligned}$$

Ex. 6. ***If n (A) = 120, n (B) = 250 and n (A – B) = 52 , then find n*** $(A \cup B)$.

Sol. $n(A - B) = n(A) - n(A \cap B)$

$\Rightarrow 52 = 120 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 120 - 52 = 68$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 250 - 68$

$= \mathbf{302}$

Ex. 7. ***If A = {x, y} and B = { 3, 4, 5, 7, 9} and C = { 4, 5, 6, 7 } , find*** $A \times (B \cap C)$.

Sol. $B \cap C = \{3, 4, 5, 7, 9\} \cap \{4, 5, 6, 7\}$

$= \{4, 5, 7\}$

$\therefore A \times (B \cap C) = \{x, y\} \times \{4, 5, 7\}$

$= \mathbf{\{(x, 4), (x, 5), (x, 7), (y, 4), (y, 5), (y, 7)\}}$

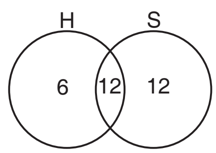
Ex. 8. ***In a certain group of 36 people, only 18 are wearing hats and only 24 are wearing sweaters. If six people are wearing neither a hat nor a sweater, then how many people are wearing both a hat and a sweater?***

Sol. Number of people wearing a hat or a sweater or both = 36 – 6 = 30

$n(H \cup S) = n(H) + n(S) - n(H \cap S)$

$30 = 18 + 24 - n(H \cap S)$

$\Rightarrow n(H \cap S) + 42 - 30 = \mathbf{12.}$



Question Bank–30

1. If A, B and C are three finite sets, then what is $[(A \cup B) \cap C]'$ equal to ?

(a) $(A' \cup B') \cap C'$ (b) $A' \cap (B' \cap C')$

(c) $(A' \cap B') \cup C'$ (d) $(A \cap B) \cap C$

2. If X and Y are any two non-empty sets, then what is $(X - Y)'$ equal to?

(a) $X' - Y'$ (b) $X' \cap Y$

(c) $X' \cup Y$ (d) $X - Y'$

3. Out of 32 persons, 30 invest in National Savings Certificates and 17 invest in Shares. What is the number of persons who invest in both?

(a) 13 (b) 15

(c) 17 (d) 19

4. If $A = P(\{1, 2\})$, where P denotes the power set, then which one of the following statements is correct?

(a) $\{1, 2\} \subset A$ (b) $1 \in A$

(c) $\phi \notin A$ (d) $\{1, 2\} \in A$

5. Which one of the following statements is correct?

(a) $A \cup P(A) = P(A)$ (b) $A \cap P(A) = A$

(c) $A - P(A) = A$ (d) $P(A) - \{A\} = P(A)$

where $P(A)$ denotes the power set of A.

6. If ξ is the universal set and P is a subset of ξ, then what is $P \cap \{(P - \xi) \cup (\xi - P)\}$ equal to

(a) ϕ (b) P'

(c) ξ (d) P

7. If $F(n)$ denotes the set of all divisions of n except 1, what is the least value of y satisfying $[F(20) \cap F(16) \subseteq F(y)$?

(a) 1 (b) 2

(c) 4 (d) 8

8. Consider the following for any three non empty sets A, B and C.

1. $A - (B \cup C) = (A - B) \cup (A - C)$

2. $A - B = A - (A \cap B)$

3. $A = (A \cap B) \cup (A - B)$

which of the above is /are correct?

(a) Only 1 (b) 2 and 3

(c) 1 and 2 (d) 1 and 3

9. Let A and B be two non-empty subsets of a set X. If $(A - B) \cup (B - A) = A \cup B$, then which one of the following is correct?

(a) $A \subset B$ (b) $A \subset (X - B)$

(c) $A = B$ (d) $B \subset A$

10. Which one of the following is correct?

(a) $A \cup (B - C) = A \cup (B \cap C')$

(b) $A - (B \cup C) = (A \cup B') \cap C'$

(c) $A - (B \cap C) = (A \cap B') \cap C$

(d) $A \cap (B - C) = (A \cap B) \cap C$

11. If $A = \{1, 2, 3\}$, $B = \{1, 2\}$ and $C = \{2, 3\}$, which one of the following is correct?

(a) $(A \times B) \cap (B \times A) = (A \times C) \cap (B \times C)$

(b) $(A \times B) \cap (B \times A) = (C \times A) \cap (C \times B)$

(c) $(A \times B) \cup (B \times A) = (A \times B) \cup (B \times C)$

(d) $(A \times B) \cup (B \times A) = (A \times B) \cup (A \times C)$

12. If A and B are finite sets. which of the following is the correct statement.

(a) $n(A - B) = n(A) - n(B)$

(b) $n(A - B) = n(B - A)$

(c) $n(A - B) = n(A) - n(A \cap B)$

(d) $n(A - B) = n(B) - n(A \cap B)$

13. Out of 40 children, 30 can swim, 27 can play chess and 5 can do neither. How many children can swim only?

(a) 30 (b) 22

(c) 12 (d) 8

14. Which one of the following is correct?

(a) $A \times (B - C) = (A - B) \times (A - C)$

(b) $A \times (B - C) = (A \times B) - (A \times C)$

(c) $A \cap (B \cup C) = (A \cap B) \cup C$

(d) $A \cup (B \cap C) = (A \cup B) \cap C$

15. Consider the following statements for non-empty sets A, B and C.

1. $A - (B - C) = (A - B) \cup C$

2. $A - (B \cup C) = (A - B) - C$

which of the statements given above is /are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

16. Out of 600 students in a school, 125 played cricket, 220 played football and 300 played hockey of the total, 28 played both hockey and football, 70 played cricket and football and 32 played cricket and hockey, 26 played all the three games. What is the number of students who did not play any game?

(a) 240 (b) 169

(c) 259 (d) 171

17. If $n(A) = 65$, $n(B) = 32$ and $n(A \cap B) = 14$, then $n(A \Delta B)$ equals

(a) 65 (b) 47

(c) 97 (d) 69

18. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are

(a) 7, 6 (b) 6, 3

(c) 5, 1 (d) 8, 7

19. U is a universal set and $n(U) = 160$. A, B and C are subset of U. If $n(A) = 50$, $n(B) = 70$, $n(B \cup C) = \phi$, $n(B \cap C) = 15$ and $A \cup B \cup C = U$, then $n(C)$ equals.

(a) 40 (b) 50

(c) 55 (d) 60

20. Match List-I with List-II and select the correct answer using the codes given below for the lists:

List-I	List-II
(A) $(E - A) \cup (E - A')$	1. ϕ
(B) $(E - [(A \cup A') - (A \cap A')]$	2. A
(C) $(E \cap (A - A') \cup A$	3. A'
(D) $[(E - \phi) \cup (\phi - E)] - A$	4. E

Here A' is the complement set of A, E is the universal set and ϕ is an empty set.

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	3	2	1
(c)	2	3	4	1
(d)	2	1	4	3

Answers

1. (c)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (b)
11. (c)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (a)

Hints and Solutions

1. (c) Given, A, B and C are three finite sets, then

$$[(A \cup B) \cap C]' = (A \cup B)' \cup C'$$
$$= (A' \cup B') \cup C'$$

2. (c) $X - Y = \{x : x \in x, x \notin y\}$

$$= \{x : x \in x, x \notin y'\}$$
$$\Rightarrow \{x : x \in X \cap Y'\}$$
$$\Rightarrow \{X - Y)' = (X \cap Y'\}'$$
$$= X' \cup (Y') = X' \cup Y$$

3. (b) Use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

4. (a) Given, $A = P(\{1, 2\})$, then

$$A = [\phi, \{1\}, \{2\}, \{1, 2\}]$$
$$\Rightarrow \{1, 2\} \in A$$

5. (b) Here, $P(A)$ denotes the power set of A.

Hence, A is a subset of $P(A)$.

$\therefore$ $A \cap P(A) = A$

6. (a) Given set $= P \cap \{(P - \xi) \cup (\xi - P)\}$

$$= P \cap \{\phi \cup P'\}$$
$$= P \cap P' = \phi$$

7. (c) $F(20) \cap F(16) \subseteq F(y)$

$\Rightarrow [\{2, 4, 5, 10, 20\} \cap \{2, 4, 8, 16\}] \subseteq F(y)$

$\Rightarrow [\{2, 4\}] \subseteq F(y)$

$\therefore$ The least value of y is 4.

8. (b) **1.** $A - (B \cup C) = (A - B) \cup (A - C)$

$(A - B) \cup (A - C)$

$\Rightarrow$ For all ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \notin C$)

$\Rightarrow$ For all ($x \in A$ and $x \notin B$ and $x \notin C$)

$\Rightarrow x \in A - (B \cap C)$

Hence not correct.

2. $x \in (A - (A \cap B)) \Rightarrow x \in A$ and $x \notin A \cap B$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

3. $x \in [(A \cap B) \cup (A - B)]$

$\Rightarrow$ ($x \in A$ and $x \notin B$) or ($x \in A$ and $x \in B'$)

$\Rightarrow x \in A$ and $x \in B$ and $x \in B'$

$\Rightarrow x \in A$ and $x \in B \cap B'$

$\Rightarrow x \in A$ and $x \in \phi$

$\Rightarrow x \in A \cup \phi \Rightarrow x \in A$

Hence (1) is incorrect.

10. (b) **A.** $A \cup (B - C) = A \cup (B \cap C')$

B. $A - (B \cup C) = A \cap (B \cup C)'$

$$= A \cap (B' \cup C')$$

C. $A - (B \cap C) = A \cap (B \cap C)'$

$$= A \cap (B' \cup C')$$

D. $A \cap (B - C) = A \cap (B \cap C')$

Hence B is the correct option.

11. (c) $(A \times B) = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

$(B \times A) = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

$(B \times C) = \{(1,2), (1,3), (2,2), (2,3)\}$

$(C \times B) = \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times C) = \{(1,2), (1,3), (2,2), (2,3), (3,2)\ (3,3)\}$

$(C \times A) = \{(2,1), (2,2), (2,3), (3,1), (3,2)\ (3,3)\}$

$\therefore$ $(A \times B) \cup (B \times A) = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\ (1,3), (2,3)\}$

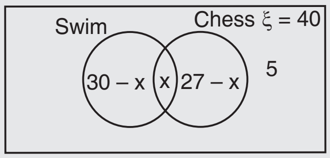
and $(A \times B) \cup (B \times C) = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (1,3), (2,3)\}$

$\therefore$ $(A \times B) \cup (B \times A) = (A \times B) \cup (B \times C)$

12. (c) $n(A - B)$ = No. of these elements of A which are not common in A and B

$$= n(A) - n(A \cap B)$$

13. (d) Let the number of children who can swim and play chess both are x



Then $30 - x + x + 27 - x = 40 - 5 = 35$

$\Rightarrow$ $57 - x = 35 \Rightarrow x = 22$

$\therefore$ The number of children who can swim only are $30 - 22 = \mathbf{8}$.

14. (b) We known that the cartesian product of two sets is defined as

$$X \times Y = \{(x, y) : x \in x \text{ and } x \in y\}$$

$\therefore$ $A \times (B - C) = (A \times B) - (A \times C)$

15. (b) **1.** $A - (B - C) = A - (B \cap C')$

$$= A \cap (B \cap C')'$$
$$= A \cap (B' \cup (C')')$$
$$= A \cap (B' \cup C)$$

$(A - B) \cup C = (A \cap B') \cup C$

Thus, $A - (B - C) \neq (A - B) \cup C$

2. $A - (B \cup C) = A \cap (B \cap C)'$

$$= A \cap (B' \cap C')$$

$(A - B) - C = (A \cap B') - C$

$$= A \cap B' \cap C'$$

$\Rightarrow$ $A - (B \cup C) = (A - B) - C$

Associative property.

16. (c) Here, $n(c) = 125$, $n(F) = 220$, $n(H) = 300$

$n(H \cap F) = 28$, $n(C \cap F) = 70$, $n(C \cap H) = 32$ and $n(C \cap F \cap H) = 26$

$\therefore$ Number of students who did not play any game

$= n(C' \cap F' \cap H')$

$= n((C \cup F \cup H)')$

$= n(\xi) - n(C \cup F \cup H)$

$= n(\xi) - [n(C) + n(F) + n(H) - n(C \cap F) - n(H \cap F) - n(C \cap H) + n(C \cap F \cap H)]$

$= 800 - [125 + 220 + 300 - 70 - 28 - 32 - 26]$

$= 800 - 541 = 259$

17. (d) $n(A \Delta B) = n(A - B) + n(B - A)$

$\therefore\ A \Delta B = (A - B) \cup (B - A)$

$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$

$= n(A) + n(B) - 2n(A \cap B)$

$= 65 + 32 - 2 \times 14 = \mathbf{69}.$

18. (b) $\because$ The number of subsets of a set containing p elements $= 2^p$,

Here, $2^m - 2^n = 56$

The values of m and n satisfying the given equations from the given options are 6, 3.

$2^6 - 2^3 = 64 - 8 = \mathbf{56}.$

19. (c) $A \cup B \cup C = U$

and $A \cap (B \cup C) = \phi$

$\Rightarrow (B \cup C) = A' = U - A$

$\Rightarrow n(B \cup C) = n(U) - n(A)$

$= 160 - 50 = 110$

Now, $n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$\Rightarrow 110 = 70 + n(C) - 15$

$\Rightarrow n(C) = 110 + 15 - 70 = \mathbf{55}.$

20. (a) (A) $(E - A) \cup (E - A') = A' \cup A = E$

(B) $E - \{(A \cup A') - (A \cap A')\}$

$= E - \{E - \phi\}$

$= E - E + \phi = \phi$

(C) $\{E \cap (A - A')\} \cup A$

$= \{E \cap A\} \cup A = A \cup A = A$

(D) $\{(E - \phi) \cup (\phi - E)\} - A$

$= \{E \cup \phi\} - A = E - A = A'$

Self Assessment Sheet–29

1. If A and B are subsets of a set X, then what is $(A \cap (X - B)) \cup B$ equal to ?

(a) $A \cup B$ (b) $A \cap B$

(c) A (b) B

2. A set contains n elements. The power set of this set contains:

(a) n^2 elements (b) $2^{\lambda/2}$ elements

(c) 2^n elements (b) n elements

3. For non empty subsets A, B and C of a set X such that $A \cup B = B \cap C$, which one of the following is the strongest inference that can be derived?

(a) $A = B = C$ (b) $A \subseteq B = C$

(c) $A = B \subseteq C$ (b) $A \subseteq B \subseteq C$

4. Let ξ = the set of all triangles , P = the set of all isosceles triangles, Q = the set of all equilateral triangles, R = the set of all right angled triangles. What do the sets $P \cap Q$ and $R - P$ represent respectively?

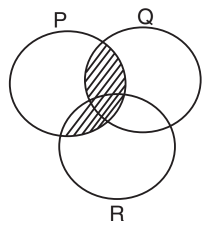
(a) The set of isosceles triangles; the set of non-isosceles right angled triangles.

(b) The set of isosceles triangles; the set of right-angled triangles.

(c) The set of equilateral triangles; the set of right-angled triangles.

(b) The set of isosceles triangles; the set of equilateral triangles.

5. What does the shaded region represent in the figure given below?



(a) $(P \cup Q) - (P \cap Q)$

(b) $P \cap (Q \cup R)$

(c) $(P \cap Q) \cap (P \cap R)$

(d) $(P \cap Q) \cup (P \cap R)$

6. Let $A = \{(n, 2n) : n \in z\}$ and

$B = \{(2n, 3n) : n \in z)\}$, what is $A \cap B$ equal to

(a) $\{(n, 6n) : n \in z\}$

(b) $\{(2n, 6n) : n \in z\}$

(c) $\{(n, 3n) : n \in z\}$

(d) ϕ

7. While preparing the progress reports of the students, the class teacher found that 70% of the students passed in Hindi, 80% passed in English and only 65% passed in both the subjects. Find out the percentage of students who failed in both the subjects.

(a) 15% (b) 20%
(c) 30% (d) 35%

8. One hundred twenty five (125) aliens descended on a set of film as Extra Terrestrial beings. 40 had two noses, 30 had three legs, 20 had four ears, 10 had two noses, and three legs, 12 had 3 legs and four ears, 5 had two noses and four ears and 3 had all the unusual features. How many were there without any of these unusual features?

(a) 5 (b) 35
(c) 80 (d) None of these

9. If A and B are non empty sets and A' and B' represents their compliments respectively, then

(a) $A - B = A' - B'$ (b) $A - A' = B - B'$
(c) $A - B = B' - A'$ (d) $A - B' = A' - B$

10. Let Z_N be the set of non-negative integers, Z_p be the set of non-positive intergers, Z the set of integers, E the set of even integers and P the set of prime numbers. Then,

(a) $E \cap P = \phi$ (b) $Z_N \cap Z_P = \phi$
(c) $Z - Z_N = Z_P$ (d) $Z_N \Delta Z_P = Z - \{0\}$

Answers

1. (a)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (c)	**10.** (d)