DPP - Daily Practice Problems

Date : ____ **Start Time :** ____ **End Time :** ____

PHYSICS

CP08

SYLLABUS : Mechanical Properties of Fluids

Max. Marks : 74 **Time : 60 min.**

GENERAL INSTRUCTIONS

- The Daily Practice Problem Sheet contains 20 Questions divided into 5 sections.
  Section I has **6** MCQs with ONLY 1 Correct Option, **3** marks for each correct answer and **–1** for each incorrect answer.
  Section II has **4** MCQs with ONE or MORE THAN ONE Correct options.
  For each question, marks will be awarded in one of the following categories:
  Full marks: **+4** If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
  Partial marks: **+1** For darkening a bubble corresponding to each correct option provided NO INCORRECT option is darkened.
  Zero marks: If none of the bubbles is darkened.
  Negative marks: **–2** In all other cases.
  Section III has **4** Single Digit Integer Answer Type Questions, **3** marks for each Correct Answer and 0 marks in all other cases.
  Section IV has Comprehension Type Questions having **4** MCQs with ONLY ONE corect option, 3 marks for each Correct Answer and 0 marks in all other cases.
  Section V has **2** Matching Type Questions, **2** mark for the correct matching of each row and 0 marks in all other cases.
- You have to evaluate your Response Grids yourself with the help of Solutions.

Section I - Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

1. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option is

(a) $P(r=0)=0$

(b) $\frac{P(r=3R/4)}{P(r=2R/3)}=\frac{63}{80}$

(c) $\frac{P(r=3R/5)}{P(r=2R/5)}=\frac{5}{9}$

(d) $\frac{P(r=R/2)}{P(r=R/3)}=\frac{20}{27}$

RESPONSE GRID	1. ⓐⓑⓒⓓ

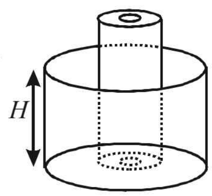
Space for Rough Work

2. A capillary tube with inner cross-section in the form of a square of side 'a' is dipped vertically in a liquid of density ρ and surface tension σ which wet the surface of capillary tube with angle of contact θ. The approximate height to which liquid will be raised in the tube is (Neglect the effect of surface tension at the corners capillary tube)

(a) $\dfrac{2\sigma\cos\theta}{a\rho g}$ (b) $\dfrac{4\sigma\cos\theta}{a\rho g}$

(c) $\dfrac{8\sigma\cos\theta}{a\rho g}$ (d) $\dfrac{4\sigma\sec\theta}{a\rho g}$

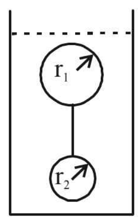
3. A hollow wooden cylinder of height h, inner radius R and outer radius $2R$ is placed in a cylindrical container of radius $3R$. When water is poured into the container, the minimum height H of the container for which cylinder can float inside freely is



(a) $\dfrac{h\rho_{water}}{\rho_{water}+\rho_{wood}}$ (b) $\dfrac{h\rho_{wood}}{\rho_{water}}$

(c) h (d) $\dfrac{h^2}{R}$

4. Two solid spherical balls of radii r_1 and r_2 ($< r_1$) and of density σ are tied up with a long string and released in a viscous liquid column of lesser density ρ with the string just taut as shown. The tension in the string when terminal velocity is attained, is



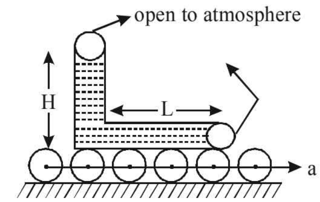
(a) $\dfrac{4}{3}\pi\left(\dfrac{r_2^4-r_1^4}{r_2-r_1}\right)(\sigma-\rho)g$

(b) $\dfrac{2}{3}\pi\left(r_2^3-r_1^3\right)(\rho-\sigma)g$

(c) $\dfrac{4}{3}\pi\left(r_2^3-r_1^3\right)(\sigma-\rho)g$

(d) $\dfrac{4}{3}\pi\left(\dfrac{r_2^4-r_1^4}{r_2+r_1}\right)(\sigma-\rho)g$

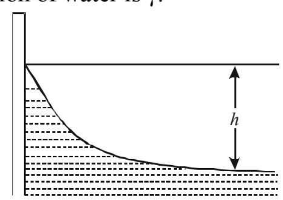
5. A narrow tube completely filled with a liquid is lying on a series of cylinders as shown in figure.



Assuming no sliding between any surfaces, the value of acceleration of the cylinders for which liquid will not come out of the tube from anywhere is given by

(a) $\dfrac{gH}{2L}$ (b) $\dfrac{gH}{L}$ (c) $\dfrac{2gH}{L}$ (d) $\dfrac{gH}{\sqrt{2}L}$

6. Water of density ρ in a clean aquarium forms a meniscus, as illustrated in the figure. Calculate the difference in height h between the centre and the edge of the meniscus. The surface tension of water is γ.



(a) $\sqrt{\dfrac{2\gamma}{\rho g}}$ (b) $\sqrt{\dfrac{\gamma}{\rho g}}$ (c) $\dfrac{1}{2}\sqrt{\dfrac{\gamma}{\rho g}}$ (d) $2\sqrt{\dfrac{\gamma}{\rho g}}$

Section II - Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONE OR MORE** is/are correct.

7. An oil drop falls through air with a terminal velocity of 5×10^{-4} m/s. Viscosity of oil is 1.8×10^{-5} N s/m^2 and density of oil is 900 kg/m^3. Neglecting density of air as compared to that of the oil

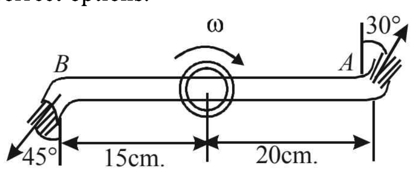
(a) radius of the drop is 6.20×10^{-2} m
(b) radius of the drop is 2.14×10^{-6} m
(c) terminal velocity of the drop at half of this radius is 1.25×10^{-4} m/s
(d) terminal velocity of the drop at half of this radius is 2.5×10^{-4} m/s

RESPONSE GRID	
	2. ⓐⓑⓒⓓ 3. ⓐⓑⓒⓓ 4. ⓐⓑⓒⓓ 5. ⓐⓑⓒⓓ 6. ⓐⓑⓒⓓ
	7. ⓐⓑⓒⓓ

Space for Rough Work

8. Water coming out of a horizontal tube at a speed v strikes normally a vetical wall close to the mouth of the tube and falls down vertically after impact. When the speed of water is increased to 2v
   (a) the thrust exerted by the water on the wall will be doubled
   (b) the thrust exerted by the water on the wall will be four times
   (c) the energy lost per second by water strike up the wall will also be four times
   (d) the energy lost per second by water striking the wall be increased eight times

9. A lawn sprinkler with two nozzles 0.5 cm. diameter each at 20cm. and 15cm. radii is connected across a tap capable of 6 litres/minute discharged. The nozzles discharge water upwards and outwards from the plane of rotation. Choose the correct options.



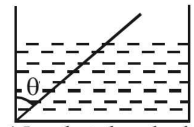
   (a) Total torque due to nozzles A and B is 0.355 Nm
   (b) If held stationary then angular velocity with which it will rotate freely is 9.72 rad/sec.
   (c) If held stationary then angular velocity with which it will rotate freely is 6.14 rad/sec.
   (d) Total torque due to nozzles A and B is 0.0355 Nm

10. An air bubble in a water tank rises from the bottom to the top. Which of the following statements are true :
   (a) Bubble rises upwards because pressure at the bottom is less than that at the top
   (b) Bubble rises upwards because pressure at the bottom is greater than that at the top
   (c) As the bubble rises, its size increases
   (d) As the bubble rises, its size decreases

Section III - Integer Type

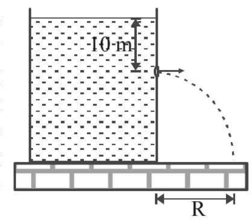
This section contains 4 questions. The answer to each of the questions is a single digit integer ranging from 0 to 9.

11. A wooden plank of length 1m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. The angle θ (in degree) is 15 x that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0°). Find the value of x.



12. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. If the ratio P_B/P_A, where P_A and P_B pressure in bubbles A and B, respectively is $\frac{x}{3}$. Find the value of x. [Neglect the effect of gravity.]

13. A large tank is filled with water (density = 10^3 kg/m^3). A small hole is made at a depth 10 m below water surface. The range of water issuing out of the hole is R on ground. What extra pressure (in atm) must be applied on the water surface so that the range becomes $2R$ (take 1 atm = 10^5 Pa and g = 10 m/s^2)



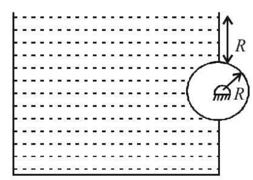
14. A cube of wood supporting 200 g mass just floats in water. When the mass is removed, the cube rises by 2cm. The side of the cube (in cm) is $2x$. Find the value of x.

Section IV - Comprehension Type

Directions (Qs. 15-18) : Based upon the given paragraphs, 4 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

PARAGRAPH-1

A cylinder of radius R is kept embedded along the wall of a dam as shown. Take density of water as ρ. Take length as L.



RESPONSE GRID	
	8. ⓐⓑⓒⓓ 9. ⓐⓑⓒⓓ 10. ⓐⓑⓒⓓ
	11. ⓪①②③④⑤⑥⑦⑧⑨ 12. ⓪①②③④⑤⑥⑦⑧⑨
	13. ⓪①②③④⑤⑥⑦⑧⑨ 14. ⓪①②③④⑤⑥⑦⑧⑨

Space for Rough Work

15. The vertical force exerted by water on the cylinder is

(a) $\rho\pi R^2 Lg$ (b) $\rho\pi R^2 Lg/2$
(c) zero (d) Zero

16. The net torque exerted by liquid on the cylinder is

(a) $\frac{2\rho R^3 Lg}{3}$ (b) $\frac{\rho R^3 Lg}{3}$
(c) $\frac{\rho R^3 Lg}{2}$ (d) Zero

PARAGRAPH-2

A tube of length ℓ and radius R carries a steady flow of fluid whose density is ρ and viscosity η. The velocity v of flow is given by $v = v_0\left(\frac{R^2 - r^2}{R^2}\right)$, where r is the distance of the flowing fluid from the axis.

17. Volume of fluid, flowing across the section of the tube, in unit time is

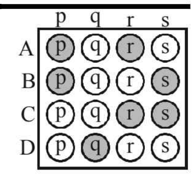
(a) $v_0 \pi R^2$ (b) $\frac{\pi v_0 R^2}{2}$
(c) $\frac{\pi v_0 R^2}{3}$ (d) $\frac{\pi v_0 R^2}{4}$

18. Kinetic energy of the fluid within the volume of the tube is

(a) $\frac{\pi \ell \rho v_0^2 R^2}{2}$ (b) $\frac{\pi \ell \rho v_0^2 R^2}{4}$
(c) $\frac{\pi \ell \rho v_0^2 R^2}{6}$ (d) $\frac{\pi \ell \rho v_0^2 R^2}{3}$

Section V - Matrix-Match Type

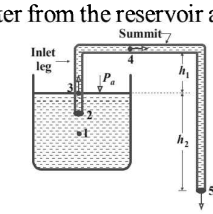
This section contains 2 questions. It contains statements given in two columns, which have to be matched. Statements in column I are labelled as A, B, C and D whereas statements in column II are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A-p, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following:



19. Bucket A contains only water, an identical bucket B contains water, but also contains a solid object in the water. Consider the following four situations. Which bucket weighs more

Column I	Column II
(A) The object floats in bucket B, and the buckets have the same water level.	(p) Bucket A
(B) The object floats in bucket B, and the buckets have the same volume of water.	(q) Bucket B
(C) The object sinks completely in bucket B, and the buckets have the same water level.	(r) Both buckets have the same weight
(D) The object sinks completely in bucket B, and the buckets have the same volume of water.	(s) Bucket's weight can't be compaired

20. Figure shows a siphon. It is a long pipe which is used to drain water from the reservoir at higher level to a reservoir at lower level. Regarding the siphon match the following columns :



Column-I	Column-II
(A) Pressure is more than atmospheric pressure at	(p) 1
(B) Pressure is less than atmospheric pressure at	(q) 2
(C) Pressure is highest of all the five points at	(r) 3
(D) Pressure is least of all the five points	(s) 4

RESPONSE GRID	
	15. ⓐⓑⓒⓓ 16. ⓐⓑⓒⓓ 17. ⓐⓑⓒⓓ 18. ⓐⓑⓒⓓ
	19. A - ⓟⓠⓡⓢ; B - ⓟⓠⓡⓢ; C - ⓟⓠⓡⓢ; D - ⓟⓠⓡⓢ
	20. A - ⓟⓠⓡⓢ; B - ⓟⓠⓡⓢ; C - ⓟⓠⓡⓢ; D - ⓟⓠⓡⓢ

DAILY PRACTICE PROBLEM DPP CP08 - PHYSICS

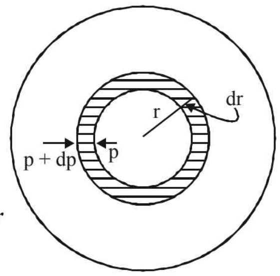
Total Questions	20	Total Marks	74
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	24	Qualifying Score	35

$$\text{Net Score} = \sum_{i=I}^{V}\left[\left(\text{correct}_i \times MM_i\right) - \left(In_i - NM_i\right)\right]$$

Space for Rough Work

1. (b) Let us consider an elemental mass dm shown in the shaded portion.

Here



$$P\,4\pi r^2-(P+dP)\,4\pi r^2=\frac{GMr}{R^3}\,\rho\,(4\pi r^2)\,dr$$

$$\therefore\quad -\int_0^P dp=\frac{GM\rho}{R^3}\int_R^r r\,dr$$

$$\therefore\quad P=\frac{GM\rho}{2R^3}\left[R^2-r^2\right]$$

$$\therefore\quad \frac{P\,(\mathrm{r}=3\mathrm{R}/4)}{P\,(\mathrm{r}=2\mathrm{R}/3)}=\frac{\left[R^2-\frac{9R^2}{16}\right]}{\left[R^2-\frac{4R^2}{9}\right]}=\frac{\frac{7R^2}{16}}{\frac{5R^2}{9}}=\frac{63}{80}$$

$$\text{and}\quad \frac{P\,(\mathrm{r}=3\mathrm{R}/5)}{P\,(\mathrm{r}=2\mathrm{R}/5)}=\frac{\left[R^2-\frac{9R^2}{25}\right]}{\left[R^2-\frac{4R^2}{25}\right]}=\frac{16}{21}$$

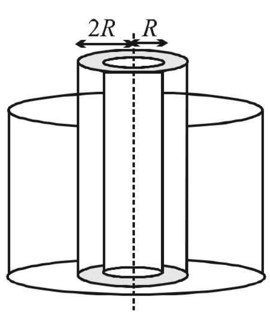
B and C are correct options.

2. (b) Upward force by capillary tube on top surface of liquid is $f_{up}=4\sigma a\cos\theta$.

If liquid is raised to a height h then we use

$$4\sigma a\cos=ha^2\rho g\quad\text{or}\quad h=\frac{4\sigma\cos\theta}{a\rho g}$$

3. (b)



$$mg=v\rho_{wood}\,g=(3\pi R^2h)\,\rho_{wood}\,g$$

$$(3\pi R^2h)\,\rho_{wood}\,g=(3\pi R^2H)\,\rho_w g$$

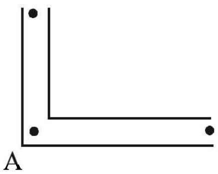
$$\frac{h\rho_{wood}}{\rho_{water}}=H$$

4. (d) $\frac{4}{3}\pi\left(r_1^3+r_2^3\right)(\sigma-\rho)\,g=6\pi\eta(r_1+r_2)\,v$

and $\frac{4}{3}\pi r_2^3(\sigma-\rho)g=T+6\pi\eta r_1 v$

$$\Rightarrow T=\frac{4}{3}\pi\frac{(r_2^4-r_1^4)(\sigma-\rho)g}{(r_1+r_2)}$$

5. (a) No sliding $\Rightarrow$ pure rolling



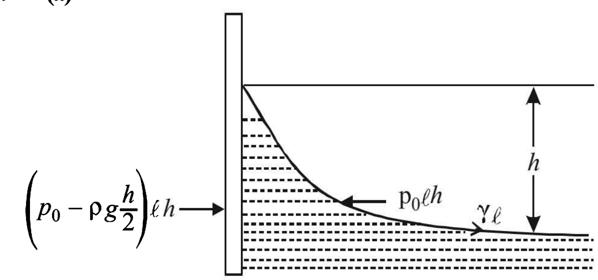
Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

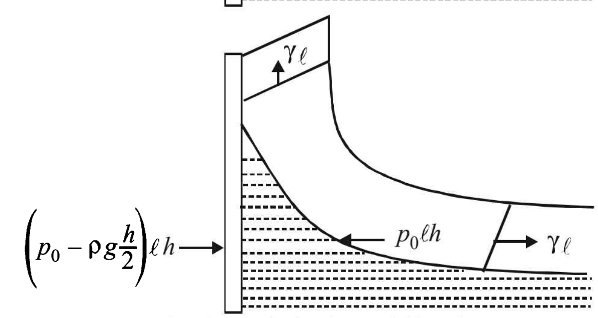
$P_A=P_{atm}+\rho(2a)L$ (From horizontal limb)

Also, $P_A=P_{atm}+\rho gh$ (From vertical lmb)

$$\Rightarrow\quad a=\frac{gH}{2L}$$

6. (a)





Balancing forces in horizontal direction

$$\left(p_0-\rho g\frac{h}{2}\right)\ell h+\gamma\ell=p_0\ell h\ \Rightarrow\ h=\sqrt{\frac{2\gamma}{\rho g}}$$

7. (b, c) As $v_t\propto r^2$

$$\therefore\quad \frac{v_1}{v_2}=\frac{(r)^2}{(r/2)^2}\Rightarrow v_2=\frac{v_1}{4}.$$

8. (b, d) $F=\rho vQ=\rho Av^2$,

Clearly when velocity becomes two times, the thrust becomes four times.

Energy lost per second, $P=Fv=\rho Av^3$, so it becomes eight times.

9. (b, d) Assume the discharge to be equally divided between the two nozzles.

$$Q_A=Q_B=\frac{3\times10^{-3}}{60}=50\times10^{-6}\ \mathrm{m^3/s}$$

$$v_A = v_B = \frac{50 \times 10^{-6}}{\pi / 4 \times 0.005^2} = 2.54 \text{ m/s}$$

$v_{\theta A} = 2.54 \cos 30° = 2.2$ m/s

$v_{\theta B} = 2.54 \cos 45° = 1.8$ m/s

Assume that the water entering the sprinkler through a tap does not involve any angular momentum.

When stationary, the torque due to nozzles action,

for nozzle A, $\tau_A = 1000 \times 50 \times 10^{-6} \times 2.2 \times 0.20$
$= 0.0220$ Nm

for nozzle B, $\tau_B = 1000 \times 50 \times 10^{-6} \times 1.8 \times 1.5$
$= 0.0135$ Nm

Total torque due to nozzles A and B,

$\tau = 0.0355$ Nm

When rotating free, let the angular velocity be ω.

Now the absolute velocities of the nozzle-discharge in the circumferential direction are

for nozzle A, $v_{\theta A} = (2.2 - 0.2\,\omega)$ m/s

for nozzle B, $v_{\theta B} = (1.8 - 0.15\,\omega)$ m/s

There being no external moment, the angular momentum should be conserved,

$$\rho\, Q_A (2.2 - 0.2\omega) \times -0.2 + \rho\, Q_B (1.8 - 0.15\omega) \times 0.15 = 0$$

Cancelling ρ and using $Q_A = Q_B$, $\omega = 9.72$ rad/s.

10. **(b, c)** $P_{top} V \quad = \quad P_{bottom} V'$

$P_{bottom} \quad = \quad (P_{top} + \rho g h)$

As $P_{bottom} > P_{top}$; $\therefore$ $V > V'$.

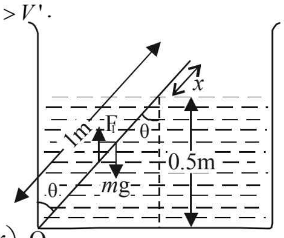
11. **3**

For equilibrium

$F_{net} = 0$

$\tau_{net} = 0$

Taking moment about O



$$mg \times \frac{\ell}{2} \sin\theta = F_T \left(\frac{\ell - x}{2}\right) \sin\theta \quad \text{... (i)}$$

Also F_T = weight of fluid displaced.

$F_T = [(\ell - x)A] \times \rho g$ ... (ii)

and $m = (\ell A) 0.5 \rho_w$ ... (iii)

where A is the area of cross section of the rod.

From (i), (ii) and (iii)

$$(\ell A) 0.5 \rho_w g \times \frac{\ell}{2} \sin\theta = [(u - x)A] \rho_w g \times \left(\frac{\ell - x}{2}\right) \sin\theta$$

Here, $\ell = 1$

$\therefore$ $(1 - x)^2 = 0.5$

$\therefore$ $1 - x = 0.707$

$\Rightarrow$ $x = 0.293$ m

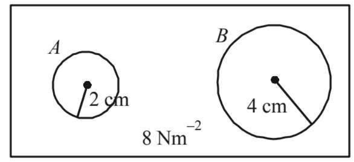
From the diagram

$$\cos\theta = \frac{0.5}{1 - x} = \frac{0.5}{0.707}$$

$\Rightarrow$ $\theta = \mathbf{45°}$

12. **4** **For bubble A:**

If P_A is the pressure inside the bubble then



$$P_A - 8 = \frac{4T}{R_A} = \frac{4 \times 0.04}{0.02} = 8 \Rightarrow P_A = 16 N/m^2$$

For bubble B:

If P_B is the pressure inside the bubble then

$$P_B - 8 = \frac{4T}{R_B} = \frac{4 \times 0.04}{0.04} = 4 \Rightarrow P_B = 12 N/m^2$$

$$\therefore \quad \frac{P_B}{P_A} = \frac{4}{3}$$

13. **3** Range, $R = vt = \sqrt{2gh} \times t$ ...(i)

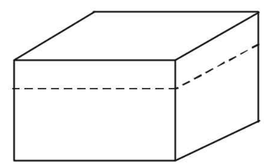
Also2R $= \sqrt{2gh'} \times t$ ...(ii)

From above equations, we get $h' = 4h = 40m$

$\therefore$ Extra height of water = 30 m = 3 atm.

14. **5**

Let the edge of cube be ℓ. When mass is on the cube of wood



$$200g + \ell^3 d_{wood} g = \ell^3 d_{H_2O} g$$

$$\Rightarrow \quad 200 + \ell^3 d_{wood} = \ell^3 d_{H_2O}$$

$$\Rightarrow \quad \ell^3 d_{wood} = \ell^3 d_{H_2O} - 200 \quad \text{... (i)}$$

when the mass is removed

$$\ell^3 d_{wood} = (\ell - 2)\ell^2 d_{H_2O} \quad \text{... (ii)}$$

From (i) and (ii)

$$\ell^3 d_{H_2O} - 200 = (\ell - 2)\ell^2 d_{H_2O}$$

But $d_{H_2O} = 1$

$\therefore$ $\ell^3 - 200 = \ell^2(\ell - 2)$

$\therefore$ $\ell^3 - 200 = \ell^3 - 2\ell^2 \Rightarrow \ell = \mathbf{10\ cm}$

15. **(b)**

16. **(d)**

17. **(b)** Volume of fluid crossing the section of the tube of length ℓ and radius R in unit time

$$V = \int 2\pi r v dr = \int_0^R 2\pi v_0 r \left(\frac{R^2 - r^2}{R^2}\right) dr$$

$$= 2\pi v_0 \left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{v_0 R^2}{2}$$

18. (c) Kinetic energy of the fluid within the volume of the tube

$$= \int \frac{1}{2}(dm)\, v^2 = \int \frac{1}{2}(2\pi r dr \ell \rho)\, v^2$$

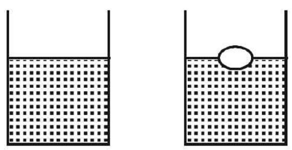
$$= \int_0^R \pi r \ell \rho v_0^2 \left(\frac{R^2 - r^2}{R^2}\right)^2 dr$$

$$= \pi \rho v_0^2 \ell \int_0^R \left(r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right) dr$$

$$= \pi \rho v_0^2 \ell \left(\frac{R^2}{2} + \frac{R^2}{6} - \frac{R^2}{2}\right) = \frac{\pi \ell \rho v_0^2 R^2}{6}$$

19. (A) → r; (B) → q; (C) → q; (D) → q

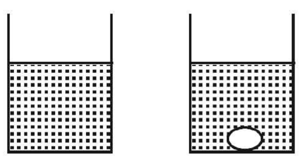
In A : Same water level implies



Wt. of fluid displaced is the same as that of object hence both buckets have equal weight.

In B, C : Mass of water in both buckets is equal and B has additional mass of solid object hence B is heavier.

In D : Same water level and object sinks $\rho_0 > \rho_\omega$ i.e. some volume of ρ_ω is replaced by same volume of ρ_0 mass increases.

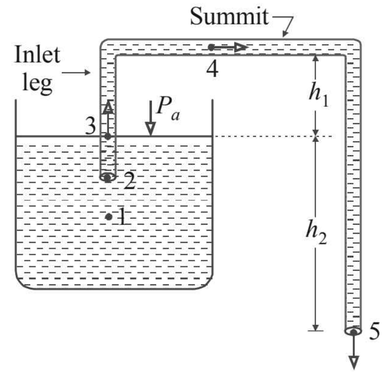


∴ B is heavier.

20. (A) → p; (B) → q,r,s; (C) → p; (D) → s

For pipe of uniform cross-section $v_3 = v_4 = v_5 = v$

Applying Bernoulli's equation between (1) and (5) , we have



$$P_a = P_a + \frac{1}{2}\rho v_2^2 - \rho g h_2$$

$$\Rightarrow \quad v_2 = \sqrt{2gh_2}$$

Thus for $v_2 > 0$, $h_2 > 0$

Also $\quad P_a = P_3 + \frac{1}{2}\rho v^2 = P_4 + \frac{1}{2}\rho v^2 + \rho g h_1$

$$= P_a + \frac{1}{2}\rho v^2 - \rho g h_2$$

From the above equation following conclusion can be made

(i) $P_4 < P_3 < P_a$

(ii) $\rho g(h_1 + h_2) = P_a - P_4$

$$\Rightarrow (h_1 + h_2) = \frac{P_a - P_4}{\rho g}$$

or $\quad h_1 + h_2 < \frac{P_a}{\rho g}$