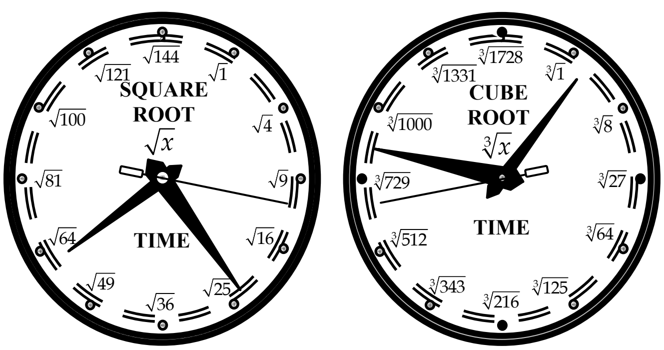
3 Square and Cube Roots

It is needful for carpenters, engineers, architects, construction workers, those who measure and mark land, artists, and designers. One time I observed construction people who were measuring and marking on the ground where a building would go. They had the sides marked, and they had a tape measure to measure the diagonals, and they asked me what the measure should be, because they couldn't quite remember how to do it. This diagonal check is to ensure that the building is really going to be a rectangle and not a trapezoid or some other shape.



The concept of a square root is a prerequisite to, and ties in with, many other concepts in mathematics:

Square root → 2nd degree equations → functions & graphing

Square root → Pythagorean theorem → trigonometry

Square root → fractional exponents → functions & graphing

Square root → irrational numbers → real numbers

Square Root: The **square root** of a number is a value that, when multiplied by itself, gives the number.

e.g.: Find the square root of 25, you want to find the number that when multiplied by itself gives you 25. The square root of 25, then, is 5. Following is a list of the first eleven perfect (whole number) square roots:

$\sqrt{0}=0 \quad \sqrt{16}=4 \quad \sqrt{64}=8$

$\sqrt{1}=1 \quad \sqrt{25}=5 \quad \sqrt{81}=9$

$\sqrt{4}=2 \quad \sqrt{36}=6 \quad \sqrt{100}=10$

$\sqrt{9}=3 \quad \sqrt{46}=7$

Example 1. What is the square root of 16?

Solution: $\sqrt{(16)}=\sqrt{(4\times 4)}=4$

Squares of Fractions: Remember that if a positive fraction with a value less than 1 is squared, the result is always smaller than the original fraction:

If $\quad 0<n<1$

Then $\quad n^2<n$

Try it.

Example 2. What are the values of the following fractions?

$$\left(\frac{2}{3}\right)^2 \text{ and } \left(\frac{1}{8}\right)^2$$

Solution: The answers are $\frac{4}{9}$ and $\frac{1}{64}$. respectively. Each of these is less than the original fraction. For example, $\frac{4}{9}<\frac{2}{3}$.

Special note

If no sign (or a positive sign) is placed in front of the square root, then the positive answer is required. Only if a negative sign is in front of the square root is the negative answer required. This notation is used in many texts and is adhered to in this book. Therefore,

$\sqrt{9}=3$ and $-\sqrt{9}=-3$

Short Method of Squaring Numbers

To calculate the square of a given integer we have to multiply the given integer with itself. For large numbers, multiplication may prove to be time consuming. In this section, we shall see how to find the square of two or three-digit numbers quickly without actual multiplication. For this, we follow the vedic Method.

Vedic Method

The method for squaring a two digit number uses the identity;

$(a+b)^2 = a^2+2ab+b^2$

Note

- If the number of digits in a perfect square 'n' is even, then the number of digits in its square root are $\frac{n}{2}$;
- If the number of digits in a perfect square 'n' is odd, then the number of digits in its square root are $\frac{n+1}{2}$.

To square a two digit number *ab* (where a is the tens digit and b is the units digit), we make three columns and we write $a^2, 2ab$ and b^2 respectively in these columns as follows:

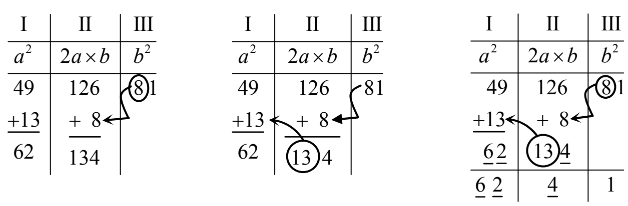
Example 3. As an example we take $ab = 79$.

Column I	Column II	Column III
a^2 $(7^2 = 49)$	$2ab$ $(2\times7\times9)=126$	b^2 $(9\times9)=81$

Then we go through the following steps:

- **Step I:** Underline the units of b^2 (in column III) and add the tens digit of b^2, if any, to $2a\times b$ in column II.
- **Step II:** Underline the units digit in column II and add the remaining digits of 2ab, if any to a^2 in column I.
- **Step III:** Underline all the digits in column I.

Thus, the underlined digits give the required square, i.e., $79^2 = 6241$



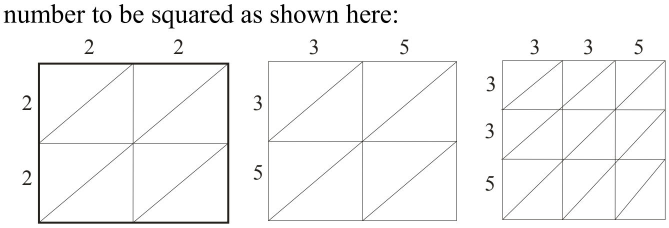
Step I Step II Step III

Diagonal Method

As the number of digits increases the column method becomes difficult, so we use the diagonal method which is fully explained in the following example.

For example, for squaring 22, 35 and 335, we go through the following steps:

Step I: First we form a square. Then we divide it into sub-squares and we draw some diagonals and write the digits of the number to be squared as shown here:

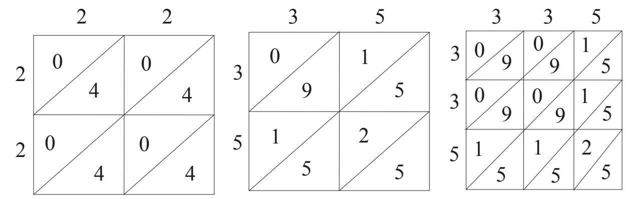


$(22)^2$ (i) $(35)^2$ (ii) $(335)^2$ (iii)

Step II: Now multiply each digit on the left of the square with each digit on top of the column one by one. Write the product in the corresponding sub-square. If the number so obtained is a single digit number, write it below the diagonal. If it is two digit number, write the tens digit above the diagonal and the units digit below the diagonal.

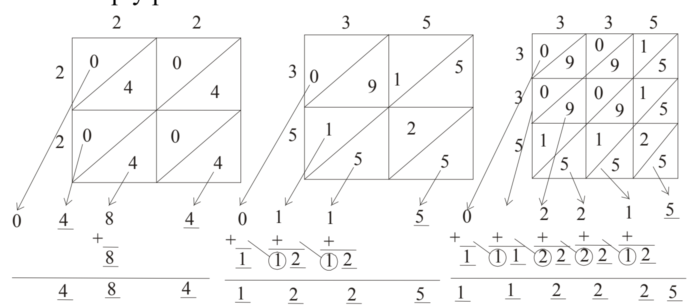
Here we multiply 2 by 2, 3 by 3, 3 by 5 and 5 by 5 and we get the products 4, 9, 15 and 25 respectively.

Clearly, we can observe that 15 and 25 are two-digit numbers, but 4 and 9 are single digit numbers, therefore to make them in two-digit numbers we write them as 04 and 09 respectively.



(i) (ii) (iii)

Step III: Starting below the lowest diagonal, sum the digits along the diagonals so obtained, underline the units digit of the sum, and take carry the remaining digits if any to the diagonal above. Units digits so underlined together with all the digits in the sum above the top-must diagonal give the square. Numbers in the empty places are taken as zero.



$\therefore\ 22^2 = 484$ $\therefore\ 35^2 = 1225$ $\therefore\ 335^2 = 112225$

(i) (ii) (iii)

Note: The diagonal method can be applied to find the square of any number irrespective of the number of digits.

Example 4. Find the squares of the following numbers using the identity.

$$(a-b)^2 = a^2 - 2ab + b^2$$

(i) 491 (ii) 189 (iii) 575

Solution:

(i) $491^2 = (500-9)^2$

$$=(500)^2 - 2\times500\times9 + 9^2 \left[\because (a-b)^2 = a^2 - 2ab + b^2\right]$$

$$= 250000 - 9000 + 81 = 241081$$

(ii) $189^2 = (200-11)^2 = 200^2 - 2\times200\times11 + 11^2$

$$= 40000 - 4400 + 121 = 35721$$

(iii) $575^2 = (600-25)^2 = 600^2 - 2\times600\times25+25^2$

$= 360000 - 30000 + 625 = 330625$

Example 5. Find the square root of 363609.

Solution: Given number is 363609.

Applying the long division method, we have

	6 0 3
6	$\overline{3}$ $\overline{3}$ $\overline{0}$
	3
12	0 3
	0 0
120	3 0
	3 0
	0

$\therefore \quad \sqrt{363609} = 603.$

Example 6. Find the greatest number of seven digits which is a perfect square. What is the square root of this number?

Solution: The greatest number of seven digits = 9999999

Now, we must find the least number which when subtracted from 9999999 gives a perfect square.

	3 1 6 2
3	9 9 9 9
	− 9
6	9
	− 6
62	3 9
	− 3 5
632	1439
	− 1264
	175

Thus, $(3162)^2 <$ 9999999 by 1755.

So, 1755 must be subtracted from 9999999 to get a perfect square.

$\therefore$ Required perfect square number

= (9999999 – 1755) = 9998244, and $\sqrt{9998244} = 3162$.

Example 7. Find the square root of 176.252176?

Solution: Here, the number of decimal places is already even. So, mark the periods and proceed as follows:

	13.276
1	1 $\overline{76}$ $.\overline{25}$ $\overline{21}$ $\overline{76}$
	1
23	76
	69
262	725
	524
2647	20121
	18529
26546	159276
	159276
	0

$\therefore \quad \sqrt{176.252176} = 13.276$

Example 8. Find the square root of 0.00059049.

Solution: Here, the number of decimal places is even. So, we mark the periods and find the square root as shown below:

	0.0243
2	0 $.\overline{00}$ $\overline{05}$ $\overline{90}$ $\overline{49}$
	4
44	190
	176
483	1449
	1449
	0

$\therefore \quad \sqrt{0.00059049} = 0.0243$

Table: 3.1

x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	50	7.071
2	1.414	51	7.141
3	1.732	52	7.211
4	2.000	53	7.280
5	2.236	54	7.348
6	2.449	55	7.416
7	2.646	56	7.483
8	2.828	57	7.550
9	3.000	58	7.616
10	3.162	59	7.681
11	3.317	60	7.746
12	3.464	61	7.810
13	3.606	62	7.874
14	3.742	63	7.937
15	3.873	64	8.000
16	4.000	65	8.062
17	4.123	66	8.124
18	4.243	67	8.185
19	4.359	68	8.246
20	4.472	69	8.307
21	4.583	70	8.367
22	4.690	71	8.426
23	4.796	72	8.485
24	4.899	73	8.544
25	5.000	74	8.602
26	5.099	75	8.660
27	5.196	76	8.718
28	5.292	77	8.775
29	5.385	78	8.832
30	5.477	79	8.888
31	5.568	80	8.994
32	5.657	81	9.000

Example 9. Find the square root of $52\frac{857}{2116}$

Solution: We have, $\sqrt{52\frac{857}{2116}} = \sqrt{\frac{110889}{2116}} = \frac{\sqrt{110889}}{2116}$

$$\begin{array}{r|l}
 & 333 \\ \hline
3 & \overline{11}\ \overline{08}\ \overline{89} \\
 & 9 \\ \hline
63 & 208 \\
 & 189 \\ \hline
663 & 1989 \\
 & 1989 \\ \hline
 & 0
\end{array}
\qquad
\begin{array}{r|l}
 & 46 \\ \hline
4 & \overline{21}\ \overline{16} \\
 & 16 \\ \hline
86 & 516 \\
 & 516 \\ \hline
 & 0
\end{array}$$

Thus, $\sqrt{110889} = 333$ and $\sqrt{2116} = 46$

$\therefore \quad \frac{\sqrt{110889}}{\sqrt{2116}} = \frac{333}{46}$ or $\sqrt{52\frac{857}{2116}} - \frac{333}{46} = 7.24.$

Cube Roots: The **cube root** of a number, some number that when multiplied by itself twice gives you the original number. In other words, to find the cube root of 8, you want to find the number that when multiplied by itself twice gives you 8. The cube root of 8, then, is 2, because $2\times2\times2=8.$ Notice that the symbol for cube root is the radical sign with a small three (called the *index*) above and to the left $\sqrt[3]{\ }$. Some perfect (whole number) cube roots are:

$\sqrt[3]{0}=0$ $\sqrt[3]{64}=4$ $\sqrt[3]{512}=8$

$\sqrt[3]{1}=1$ $\sqrt[3]{125}=5$ $\sqrt[3]{729}=9$

$\sqrt[3]{8}=2$ $\sqrt[3]{216}=6$ $\sqrt[3]{1000}=10$

$\sqrt[3]{27}=3$ $\sqrt[3]{343}=7$

Example 10. What is the cube root of 125?

Solution: Well, we just happen to know that $125=5\times5\times5$ (if you use 5 three times in a multiplication you will get 125)**. So, the answer is 5.**

Note

Perfect Cube: A natural number is said to be a perfect cube, if it is the cube of some natural number. In other words, a number '*n*' is a perfect cube if there exists a natural number p such that: $n = p^3$ and $-n = (-p)^3$

Table: 3.2

a	1	2	3	4	5	6	7	8
a^3	1	8	27	64	125	216	343	512
a	9	10	11	12	13	14	15	
a^3	729	1000	1331	1728	2197	2744	3375	

Finding the Cube of a Two-Digit Number (Vedic Method)

The cube of a number can be obtained by multiplying the number with itself two times. Thus, to find x^3, we may first find x^2 and then x^2x. Here we shall discuss a vedic method of finding x^3, where x is a two digit number.

Let $x = ab$, where a is the tens digit and b is the units digit. We use the Identity

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Example 11. A general wishing to daw up his 64025 men in the form of a solid square, found that he had 16 men over, find the number of men in the front row.

Solution: Number of men arranged in a solid square

$= (64025-16) = 64009.$

$\therefore$ Number of men in the front row $= \sqrt{64009} = 253$

Cube root of given number x is the number whose cube product of prime numbers choosing one each from three of the same type.

Example 12. Find the cube root of 9261.

Solution: Resolving 9261 into prime factors. We get

$9261 = 3\times3\times3\times7\times7\times7$

$\sqrt[3]{9261} = \sqrt[3]{(3\times3\times3\times7\times7\times7)}$

$=3\times7=21$

3	9261
3	3087
3	1027
7	343
7	49
	7

Way to Cube: The cube of a number is that number multiplied by itself twice. e.g.: 5 cubed–denoted as 5^3; is equal to $5\times5\times5=125.$ The technique for cubing any 2-digit number discussed below is based on the algebraic observation.

Step I: Write numbers in a row of 4 terms in such a way that the first one is the cube of the 1st digit, second one is the square of 1st number multiplied by 2nd digit, third one is the 1st digit multiplied by square of 2nd digit and the fourth one is the cube of the 2nd digit.

Step II: Write twice the values of 2nd and 3rd terms under the terms respectively in second row.

Step III: Add all the values column-wise and follow the carry over process.

Example 13. Find cube of 14.

Solution:

Step I: 1^3 $1^2\times 4$ 1×4^2 4^3

Worked-out row: 1 4 16 64

Step II: 8 32

Step III: Add the values column wise.

First Column from Right: Bring down 4 and carry over 6.

1	4 8	16 32	64
		6	
			4

Second Column from Right: 16 + 32 + 6 = 54. Bring down 4 and carry over 5.

1	4 8	16 32	64
	5	6	
		4	4

Third Column from Right: 4 + 8 + 5 = 17. Bring down 7 and carry over 1.

1	4 8	16 32	64
1	5	6	
	7	4	4

Fourth Column from Right: 1 + 1 = 2. Write down 2.

1	4 8	16 32	64
1	5	6	
2	7	4	4

This 2,744 is the cube of the number 14.

Example 14. Cube of 25 = ?

Solution:

Step-1		2^3	$2^2\times 5$	2×5^2	5^3
Worked-out		8	20	50	125
Step-2			40	100	
Step-3	Carries				
	Sum				

First Column from Right: Bring down 5 and carry over 12.

8	20 40	50 100	125
		12	
			5

Second Column from Right: 50 + 100 + 12 = 162. Bring down 2 and carry over 16.

8	20 40	50 100	125
	16	12	
		2	5

Third Column from Right: 20 + 40+ 16 = 76. Bring down 6 and carry over 7.

8	20 40	50 100	125
7	16	12	
	6	2	5

Fourth Column from Right: 8 + 7 = 15.
Write down 15.

8	20 40	50 100	125
7	16	12	
15	6	2	5

This 15,625 is the cube of the number 25.

What would happen to the result of cubing a number if that number is a negative integer or a rational number? Rational numbers and negative integers are just a form of numbers and their repeated multiplication (thrice for a cube) can easily be calculated keeping in mind the different sign conventions.

Let's see some of the sign conventions followed in Maths.
Sign convention

- $(-)\times(+)=(-)$

  i.e., when we multiply a negative integer and a positive integer, the result would always have a negative sign.
- $(-)\times(-)=(+)$

  i.e., when we multiply two negative integers, the result would always have a positive sign.
- $(+)\times(-)=(-)$

  i.e., when we multiply a positive integer and a negative integer, the result would always have a negative sign.
- $(+)\times(+)=(+)$

  i.e., when we multiply two positive integers, the result would have a positive sign.

Multiple Choice Questions

1. $\sqrt{(3+\sqrt{5})}$ is equal to

a. $\sqrt{5}+1$ **b.** $\sqrt{3}+\sqrt{2}$

c. $(\sqrt{5}+1)/\sqrt{2}$ **d.** $\frac{1}{2}(\sqrt{5}+1)$

2. $\sqrt{[10-\sqrt{(24)}-\sqrt{(40)}+\sqrt{(60)}]}=$

a. $\sqrt{5}+\sqrt{3}+\sqrt{2}$ **b.** $\sqrt{5}+\sqrt{3}-\sqrt{2}$

c. $\sqrt{5}-\sqrt{3}-\sqrt{2}$ **d.** $\sqrt{2}+\sqrt{3}-\sqrt{5}$

3. $\sqrt[4]{(17+12\sqrt{2})}=$

a. $\sqrt{2}+1$ **b.** $2^{1/4}(\sqrt{2}+1)$

c. $2\sqrt{2}+1$ **d.** None of these

4. $\sqrt[3]{(61-46\sqrt{5})}=$

a. $41-2\sqrt{5}$ **b.** $-41-\sqrt{5}$

c. $2-\sqrt{5}$ **d.** None of these

5. Which of the following is not a perfect square?

a. 36 **b.** 196 **c.** 181 **d.** 169

6. In a perfect square number, the last digit is given. Which of the following cannot be the last digit?

a. 1 **b.** 0 **c.** 5 **d.** 7

7. Which of the following letters best represents the location of *x–y,* where $x=\sqrt{169}$ and $y=\sqrt{64}$?

a. B **b.** A **c.** D **d.** E

8. The value of *x,* if $5^{x-3}\cdot 3^{2x-8}=225$, is:

a. 1 **b.** 2 **c.** 3 **d.** 5

9. If *m* is the square of a natural number *n*, then *n* is

a. the square of *m* **b.** greater than *m*

c. equal to *m* **d.** equal to $\sqrt{m}$

10. The value of expression $\sqrt{248+\sqrt{52+\sqrt{144}}}$ is

a. 14 **b.** 12 **c.** 16 **d.** 13

11. It is given that $\sqrt{4761}=69$, then the value of $\sqrt{4761}+\sqrt{47\cdot 61}+\sqrt{0.4761}$ is

a. 77 **b.** 75.59 **c.** 76.59 **d.** 70.59

12. The two other numbers forming a Pythagorean triplet, whose third number is 5, are

a. (3, 4) **b.** (– 3, – 4) **c.** (6, 4) **d.** (3, 7)

13. If $\sqrt{2+\sqrt{x}}=3$, then the value of *x* is

a. 1 **b.** $\sqrt{7}$ **c.** $\sqrt{49}$ **d.** 49

14. The smallest square number which is exactly divisible by each of the numbers 6, 9 and 15, is

a. 100 **b.** 400 **c.** 900 **d.** 1024

15. If $\sqrt{1+\frac{27}{169}}=\left(1+\frac{x}{13}\right)$, then the value of *x* is

a. 1 **b.** 3 **c.** 5 **d.** 7

16. What least number should be added to the number 6800, so that the resultant number is a perfect square?

a. 24 **b.** 76 **c.** 89 **d.** 256

17. The greatest five-digit number, which is a perfect square, is

a. 99746 **b.** 99856 **c.** 90456 **d.** 99999

18. The square root of $\frac{1.69}{0.0036}\times\frac{1.44}{6.76}\times\frac{0.25}{1.21}$ is

a. 4.54 **b.** 4.75 **c.** 4.95 **d.** 4.99

19. Match the following:

Column-I	**Column-II**
(A) Smallest perfect square	**1.** 17
(B) Sum of first 8 odd numbers	**2.** 64
(C) Area of square is 144 cm^2, its perimeter is	**3.** 1
(D) The least number added to sum of squares of first 5 prime numbers to form perfect square	**4.** 48

Code:

a. A→1; B→2; C→3; D→4

b. A→3; B→2; C→4; D→1

c. A→3; B→4; C→2; D→1

d. A→2; B→4; C→3; D→1

20. **Assertion (A):** 1 + 3 + 5 + 7 + 9 + 11= 36.

Reason (R): Sum of first *n* consecutive odd numbers is n^2. Which of the following is true?

a. Both (A) and (R) are true and (R) is correct explanation of (A)

b. Both (A) and (R) are true but (R) is not correct explanation of (A)

c. (A) is true and (R) is false

d. (A) is false and (R) is true

21. If $\sqrt{188+\sqrt{53+\sqrt{y}}}=14$, then the value of y is

a. 121 **b.** 11 **c.** 1331 **d.** 161

22. The value of $\sqrt{6\sqrt{6\sqrt{6\sqrt{6}}}}$ is

a. 6^4 **b.** $6^{1/4}$ **c.** 3^6 **d.** $6^{15/16}$ s

23. A group of students in a class collects R 9216. The amount contributed by each student is equivalent to the number of students in the class. Then, total number of students is:

a. 43 **b.** 53 **c.** 96 **d.** 66

24. If $a=\sqrt{2}+1$ and $b=\sqrt{2}-1$, then the value of expression $\dfrac{a^2-ab+b^2}{a^2+ab+b^2}$ is:

a. $32-4\sqrt{2}$ **b.** $32+4\sqrt{2}$

c. 0 **d.** $\dfrac{5}{7}$

25. If three numbers are in the ration 1:2:3 and the sum of their square is 224. Then, the difference between the squares of greatest and least numbers, is:

a. 80 **b.** 160 **c.** 128 **d.** 240

26. A person borrowed some money from a friend and promised him to pay daily for 1 month. He will pay like R 1 for first day, R 3 for second day, R 5 for third day and so on for 30 days. If he paid an interest of R 150 included in the above amount, then the money borrowed by him is:

a. 600 **b.** 750 **c.** 900 **d.** 1200

27. State 'T' for true of 'F' for false.

I. The sum of two perfect squares is a perfect square.

II. The sum of first n even numbers is n^2.

III. When a square number ends in 6, the number whose square it is, will have either 4 or 7 in unit's place.

IV. The general from of Pythagorean triplet is $m^2-1, m^2+1, 2m+1$.

V. The square of a prime number is a prime number.

	I	II	III	IV	V
a.	F	F	F	F	F
b.	T	F	T	F	T
c.	T	T	T	T	T
d.	F	T	F	T	F

28. Fill in the blanks with the help of options, given in the box.

(i) 169 **(ii)** 144 **(iii)** 200 **(iv)** 400
(v) $2n+1$ **(vi)** 8 **(vii)** 16 **(viii)** 9
(ix) 15, **(x)** 1 **(xi)** 4

I. $1+3+5+7+9+11+13+15+17+19+21+23$ is__(find without adding)

II. The value of 101^2-99^2 is ____

III. There are ___ natural numbers between n^2 and $(n+1)^2$.

IV. The sides of a right angled triangle, whose hypotenuse is 17 cm, are __and __.

	I	II	III	IV
a.	(ii)	(iv)	(v)	(iv) , (ix)
b.	(i)	(ii)	(iv)	(iv), (v)
c.	(vii)	(viii)	(ix)	(x), (xi)
d.	(ii)	(iii)	(iv)	(v), (vi)

29. $\sqrt[3]{(729)^{2.5}}=$

a. $\dfrac{1}{81}$ **b.** 81

c. 243 **d.** 729

30. $\sqrt[4]{\sqrt[3]{x^2}}=$

a. x **b.** $x^{\frac{1}{2}}$

c. $x^{\frac{1}{3}}$ **d.** $x^{\frac{1}{6}}$

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	b	a	a	c	d	d	d	d	c
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	a	d	c	a	c	b	a	b	a
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
a	d	c	d	c	b	a	a	c	d

SOLUTIONS

1. **(c)** Let $\sqrt{3+\sqrt{5}}=\sqrt{x}+\sqrt{y}$

$3+\sqrt{5}=x+y+2\sqrt{xy}$.

Obviously $x+y=3$ and $4xy=5$

So $(x-y)^2=9-5=4$ or $(x-y)=2$

After solving $x=\dfrac{5}{2}, y=\dfrac{1}{2}$

Hence $\sqrt{3+\sqrt{5}}=\sqrt{\dfrac{5}{2}}+\sqrt{\dfrac{1}{2}}=\dfrac{\sqrt{5}+1}{\sqrt{2}}$

2. **(b)** Let $10-\sqrt{24}-\sqrt{40}+\sqrt{60}$

$=(\sqrt{a}-\sqrt{b}+\sqrt{c})^2+23$

$10-\sqrt{24}-\sqrt{40}+\sqrt{60}\ =a+b+c-2\sqrt{ab}$

$-2\sqrt{bc}+2\sqrt{ca}, a, b, c, > 0.$

Then $a+b+c=10,\ ab=6,\ bc=10,\ ca=15$

$a^2b^2c^2=900 \Rightarrow abc=30(\neq \pm 30)$

So, $a=3, b=2, c=5$

Therefore, $\sqrt{(10-\sqrt{24}-\sqrt{40}+\sqrt{60})}=\pm(\sqrt{3}+\sqrt{5}-\sqrt{2})$

3. **(a)** $\sqrt{(17+12\sqrt{2})}$

$=\sqrt{[3^2+(2\sqrt{2})^2+2.3.2\sqrt{2}]}=3+2\sqrt{2}$

$\therefore\ \sqrt[4]{(17+12\sqrt{2})}=\sqrt{(3+2\sqrt{2})}=\sqrt{2}+1$

4. **(a)** $\sqrt[3]{61-46\sqrt{5}}\ =a-\sqrt{b}$

$\Rightarrow\ 61-46\sqrt{5}=(a-\sqrt{b})^3=a^3+3ab-(3a^2+b)\sqrt{b}$

$\Rightarrow\ 61=a^3+3ab, 46\sqrt{5}=(3a^2+b)\sqrt{b}$

$\Rightarrow\ 61(a^2+3b)a, 23\sqrt{20}=(3a^2+b)\sqrt{b}$

So $a=1, b=20$

Therefore $\sqrt[3]{61-46\sqrt{5}}\ =1-\sqrt{20}\ =1-2\sqrt{5}$

5. **(c)** Here, $\sqrt{36}=6$

$\sqrt{196}=14$

$\sqrt{169}=13\ \ \sqrt{181}=13.45$

Hence,181is not a perfect square.

6. **(d)** 7 cannot be the last digit (unit place) for perfect number.

7. **(d)**

$\because\ x=\sqrt{169}=13$ and $y=\sqrt{64}=8$

$\therefore\ x-y=13-8=5=E$

9. **(d)**

$\because\ m=n^2$

$\therefore\ n=\sqrt{m}$

10. **(c)** $\sqrt{248+\sqrt{52+\sqrt{144}}}$

$=\sqrt{248+\sqrt{52+12}}$

$=\sqrt{248+\sqrt{64}}$

$=\sqrt{248+8}\ =\sqrt{256}=16$

11. **(c)** Here, $\sqrt{4761}=69$

$\sqrt{47.61}=\sqrt{\dfrac{4761}{100}}=\dfrac{69}{10}=6.9$

$\sqrt{0.4761}=\sqrt{\dfrac{4761}{10000}}=\dfrac{69}{100}=0.69$

$\therefore\ \sqrt{4761}+\sqrt{47.61}+\sqrt{0.4761}\ =69+6.9+0.69\ \ \ =76.59$

12. **(a)** In general, Pythagorean triplet is, $(2n, n^2-1, n^2+1)$.

$\therefore\ 5=n^2+1\ \ \Rightarrow n=2$

$\therefore\ 2n=4\ \ \Rightarrow\ n^2-1=4-1=3$

So, the other numbers are 3 and 4.

13. **(d)**

$\therefore\ \sqrt{2+\sqrt{x}}=3$

Squaring on both sides,

$2+\sqrt{x}=9$

$\Rightarrow\ \sqrt{x}=7$

$\therefore\ x=(7)^2=49$

14. **(c)** LCM of $6, 9, 15=3\times 2\times 3\times 5=90$

But we see that, 90 is not a perfect square.

$\sqrt{90}=3\sqrt{10}$

For a perfect square,

$3\sqrt{10}\times\sqrt{10}=30$

$\therefore$ Square of 30 = 900

So, smallest number which is a perfect square, is 900.

15. **(a)**

$\therefore\ \sqrt{1+\dfrac{27}{169}}=\left(1+\dfrac{x}{13}\right)$

$\Rightarrow\ \sqrt{\dfrac{169+27}{169}}=1+\dfrac{x}{13}$

$\Rightarrow\ \dfrac{14}{13}=1+\dfrac{x}{13}$

$\Rightarrow\ 1+\dfrac{1}{13}=1+\dfrac{x}{13}$

$\therefore\ x=1$

16. **(c)**

$\therefore\ 80^2=6400$ and $85^2=7225$

So, 6800 lies between 80 and 85.

Also, $81^2=6561$

$82^2=6724$

$83^2=6889...$ near to 6800.

Hence, 89 must be added.

17. **(b)** We know that, Greatest five-digit number $=99999$

where, $\sqrt{99999}=316.2261$

But we have to find perfect square,

So $316\times 316=99856$

18. **(a)** We have, $\sqrt{\frac{1.69}{0.0036}\times\frac{1.44}{6.76}\times\frac{0.25}{1.21}}$

$=\sqrt{\frac{1.3\times1.3}{0.06\times0.06}\times\frac{1.2\times1.2}{2.6\times2.6}\times\frac{0.5\times0.5}{1.1\times1.1}}$

$=\frac{1.3}{0.06}\times\frac{1.2}{2.6}\times\frac{0.5}{1.1}=\frac{5}{1.1}=4.54$

21. **(a)** $\sqrt{188+\sqrt{53+\sqrt{y}}}=14$

On squaring both sides,

$188+\sqrt{53+\sqrt{y}}=196$

$\Rightarrow\quad \sqrt{53+\sqrt{y}}=196-188$

$\Rightarrow\quad \sqrt{53\sqrt{y}}=8$ Again, squaring on both sides,

$53+\sqrt{y}=64$

$\Rightarrow\quad \sqrt{y}=64-53=11$

Again, squaring on both sides, $y=121$

22. **(d)** Consider,

$\sqrt{6\sqrt{6\sqrt{6\sqrt{6}}}}$

$=\sqrt{6\sqrt{6\sqrt{6\times6^{1/2}}}}=\sqrt{6\sqrt{6\sqrt{6^{3/2}}}}$

$=\sqrt{6\sqrt{6\times6^{3/4}}}=\sqrt{6\sqrt{6^{7/4}}}$

$=\sqrt{6\times6^{7/8}}=6^{15/16}$

23. **(c)** Total amount collected

= Rs. 9216

Let the number of students be x.

Then, amount contributed by each student

= Rs. x

According to the question,

$\Rightarrow\quad x\times x=9216$

$\Rightarrow\quad x^2=9216$

$\Rightarrow\quad x=\sqrt{9216}=96$

$\therefore$ Total number of students =96

24. **(d)** Given, $a=\sqrt{2}+1$ and $b=\sqrt{2}-1$

$\therefore\quad a+b=2\sqrt{2}$

and $\quad a-b=2$

Now, $ab=2-1=1$

$[\because (a+b)(a-b)=a^2-b^2]$

$\therefore\quad \frac{a^2-ab+b^2}{a^2+ab+b^2}=\frac{(a-b)^2+ab}{(a+b)^2-ab}$

$=\frac{(2)^2+1}{(2\sqrt{2})^2-1}$

$=\frac{4+1}{8-1}=\frac{5}{7}$

25. **(c)** Since, numbers are in the ratio 1:2:3.

Let the number be x, 2 x and 3x.

According to the question.

$x^2+(2x)^2+(3x)^2=224$

$\Rightarrow\quad x^2+4x^2+9x^2=224$

$\Rightarrow\quad 14x^2=224$

$\Rightarrow\quad x^2=16$

$\Rightarrow\quad x=4$

$\therefore$ Difference between squares of greatest and smallest numbers

$=(3x)^2-(x)^2=8x^2=8\times16=128$

26. **(b)** Total money paid

= 1 + 3 + 5 + 7 + 9 + … + 30 terms

$=30^2=900$

[since, sum of n consecutive numbers is n^2]

Interest paid = Rs. 150

$\therefore$ Amount borrowed = Rs. (900 – 150)

= Rs. 750

27. **(a)**

I. False **II.** False **III.** False **IV.** False

28. **(a)**

I. 144 **II.** 400 **III.** $2n+1$ **IV.** 8, 15

□□□