

# 1. Real Numbers

## Exercise 1.1

### 1 A. Question

Using Euclid's division algorithm, find the HCF of 156 and 504

### Answer

Given numbers are 156 and 504

Here,  $504 > 156$

So, we divide 504 by 156

By using **Euclid's division lemma**, we get

$$504 = 156 \times 3 + 36$$

Here,  $r = 36 \neq 0$ .

On taking 156 as dividend and 36 as the divisor and we apply Euclid's division lemma, we get

$$156 = 36 \times 4 + 12$$

Here,  $r = 12 \neq 0$

So, on taking 36 as dividend and 12 as the divisor and again we apply Euclid's division lemma, we get

$$36 = 12 \times 3 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 12, the **HCF of 156 and 504 is 12.**

### 1 B. Question

Using Euclid's division algorithm, find the HCF of 135 and 225

### Answer

Given numbers are 135 and 225

Here,  $225 > 135$

So, we divide 225 by 135

By using **Euclid's division lemma**, we get

$$225 = 135 \times 1 + 90$$

Here,  $r = 90 \neq 0$ .

On taking 135 as dividend and 90 as the divisor and we apply Euclid's division lemma, we get

$$135 = 90 \times 1 + 45$$

Here,  $r = 45 \neq 0$

So, on taking 90 as dividend and 45 as the divisor and again we apply Euclid's division lemma, we get

$$90 = 45 \times 2 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 45, the **HCF of 135 and 225 is 45**.

### **1 C. Question**

Using Euclid's division algorithm, find the HCF of

455 and 42

#### **Answer**

Given numbers are 455 and 42

Here,  $455 > 42$

So, we divide 455 by 42

By using **Euclid's division lemma**, we get

$$455 = 42 \times 10 + 35$$

Here,  $r = 35 \neq 0$ .

On taking 42 as dividend and 35 as the divisor and we apply Euclid's division lemma, we get

$$42 = 35 \times 1 + 7$$

Here,  $r = 7 \neq 0$

So, on taking 35 as dividend and 7 as the divisor and again we apply Euclid's division lemma, we get

$$35 = 7 \times 5 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 7, the **HCF of 455 and 42 is 7.**

### **1 D. Question**

Using Euclid's division algorithm, find the HCF of  
8840 and 23120

#### **Answer**

Given numbers are 8840 and 23120

Here,  $23120 > 8840$

So, we divide 23120 by 8840

By using **Euclid's division lemma**, we get

$$23120 = 8840 \times 2 + 5440$$

Here,  $r = 5440 \neq 0$ .

On taking 8840 as dividend and 5440 as the divisor and we apply Euclid's division lemma, we get

$$8840 = 5440 \times 1 + 3400$$

Here,  $r = 3400 \neq 0$

On taking 5440 as dividend and 3400 as the divisor and again we apply Euclid's division lemma, we get

$$5440 = 3400 \times 1 + 2040$$

Here,  $r = 2040 \neq 0$ .

On taking 3400 as dividend and 2040 as the divisor and we apply Euclid's division lemma, we get

$$3400 = 2040 \times 1 + 1360$$

Here,  $r = 1360 \neq 0$

So, on taking 2040 as dividend and 1360 as the divisor and again we apply Euclid's division lemma, we get

$$2040 = 1360 \times 1 + 680$$

Here,  $r = 680 \neq 0$

So, on taking 1360 as dividend and 680 as the divisor and again we apply Euclid's division lemma, we get

$$1360 = 680 \times 2 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 680, the **HCF of 8840 and 23120 is 680.**

### **1 E. Question**

Using Euclid's division algorithm, find the HCF of  
4052 and 12576

#### **Answer**

Given numbers are 4052 and 12576

Here,  $12576 > 4052$

So, we divide 12576 by 4052

By using **Euclid's division lemma**, we get

$$12576 = 4052 \times 3 + 420$$

Here,  $r = 420 \neq 0$ .

On taking 4052 as dividend and 420 as the divisor and we apply Euclid's division lemma, we get

$$4052 = 420 \times 9 + 272$$

Here,  $r = 272 \neq 0$

On taking 420 as dividend and 272 as the divisor and again we apply Euclid's division lemma, we get

$$420 = 272 \times 1 + 148$$

Here,  $r = 148 \neq 0$

On taking 272 as dividend and 148 as the divisor and again we apply Euclid's division lemma, we get

$$272 = 148 \times 1 + 124$$

Here,  $r = 124 \neq 0$ .

On taking 148 as dividend and 124 as the divisor and we apply Euclid's division lemma, we get

$$148 = 124 \times 1 + 24$$

Here,  $r = 24 \neq 0$

So, on taking 124 as dividend and 24 as the divisor and again we apply Euclid's division lemma, we get

$$124 = 24 \times 5 + 4$$

Here,  $r = 4 \neq 0$

So, on taking 24 as dividend and 4 as the divisor and again we apply

Euclid's division lemma, we get

$$24 = 4 \times 6 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 4, the **HCF of 4052 and 12576 is 4.**

### **1 F. Question**

Using Euclid's division algorithm, find the HCF of

3318 and 4661

### **Answer**

Given numbers are 3318 and 4661

Here,  $4661 > 3318$

So, we divide 4661 by 3318

By using **Euclid's division lemma**, we get

$$4661 = 3318 \times 1 + 1343$$

Here,  $r = 1343 \neq 0$ .

On taking 3318 as dividend and 1343 as the divisor and we apply Euclid's division lemma, we get

$$3318 = 1343 \times 2 + 632$$

Here,  $r = 632 \neq 0$

So, on taking 1343 as dividend and 632 as the divisor and again we apply Euclid's division lemma, we get

$$1343 = 632 \times 2 + 79$$

Here,  $r = 79 \neq 0$

So, on taking 632 as dividend and 79 as the divisor and again we apply Euclid's division lemma, we get

$$632 = 79 \times 8 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 79, the **HCF of 3318 and 4661 is 79.**

### **1 G. Question**

Using Euclid's division algorithm, find the HCF of

250, 175 and 425

### **Answer**

Given numbers are 250, 175 and 425

$$\therefore 425 > 250 > 175$$

On applying **Euclid's division lemma** for 425 and 250, we get

$$425 = 250 \times 1 + 175$$

Here,  $r = 175 \neq 0$ .

So, again applying Euclid's division lemma with new dividend 250 and new divisor 175, we get

$$250 = 175 \times 1 + 75$$

Here,  $r = 75 \neq 0$

So, on taking 175 as dividend and 75 as the divisor and again we apply Euclid's division lemma, we get

$$175 = 75 \times 2 + 25$$

Here,  $r = 25 \neq 0$ .

So, again applying Euclid's division lemma with new dividend 75 and new divisor 25, we get

$$75 = 25 \times 3 + 0$$

Here,  $r = 0$  and divisor is 25.

So, HCF of 425 and 225 is 25.

Now, applying Euclid's division lemma for 175 and 25, we get

$$175 = 25 \times 7 + 0$$

Here, remainder = 0

So, **HCF of 250, 175 and 425 is 25.**

### **1 H. Question**

Using Euclid's division algorithm, find the HCF of

4407, 2938 and 1469

#### **Answer**

Given numbers are 4407, 2938 and 1469

$\therefore 4407 > 2938 > 1469$

On applying **Euclid's division lemma** for 4407 and 2938, we get

$$4407 = 2938 \times 1 + 1469$$

Here,  $r = 1469 \neq 0$ .

So, again applying Euclid's division lemma with new dividend 2938 and new divisor 1469, we get

$$2938 = 1469 \times 2 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this stage is 1469, the HCF of 4407 and 2938 is 1469.

Now, applying Euclid's division lemma for 1469 and 1469, we get

$$1469 = 1469 \times 1 + 0$$

Here, remainder = 0

So, **HCF of 4407, 2938 and 1469 is 1469.**

### **2. Question**

Show that every positive even integer is of the form  $2q$  and that every positive odd integer is of the form  $2q + 1$ , where  $q$  is some integer.

#### **Answer**

Let  $a$  and  $b$  be any two positive integers, such that  $a > b$ .

Then,  $a = bq + r$ ,  $0 \leq r < b$  ... (i) [by Euclid's division lemma]

On putting  $b = 2$  in Eq. (i), we get

$$a = 2q + r, 0 \leq r < 2 \text{ ... (ii)}$$

$\Rightarrow r = 0 \text{ or } 1$

When  $r = 0$ , then from Eq. (ii),  $a = 2q$ , which is divisible by 2

When  $r = 1$ , then from Eq. (ii),  $a = 2q + 1$ , which is not divisible by 2.

Thus, every positive integer is either of the form  $2q$  or  $2q + 1$ .

That means every positive integer is either even or odd. So, if  $a$  is a positive even integer, then  $a$  is of the form  $2q$  and if  $a$  is a positive odd integer, then  $a$  is of the form  $2q + 1$ .

### 3. Question

Show that any positive odd integer is of the form  $4q + 1$  or  $4q + 3$ , where  $q$  is some integer.

#### Answer

Let  $a$  be any positive odd integer. We apply the division algorithm with  $a$  and  $b = 4$ .

Since  $0 \leq r < 4$ , the possible remainders are 0, 1, 2 and 3.

i.e.  $a$  can be  $4q$ , or  $4q + 1$ , or  $4q + 2$ , or  $4q + 3$ , where  $q$  is the quotient.

As we know  $a$  is odd,  $a$  can't be  $4q$  or  $4q + 2$  because they both are divisible by 2.

Therefore, any positive odd integer is of the form  $4q + 1$  or  $4q + 3$ .

### 4. Question

There are 250 and 425 liters of milk in two containers. What is the maximum capacity of the container which can measure completely the quantity of milk in the two containers?

#### Answer

Given the capacities of the two containers are 250 L and 425 L.

Here,  $425 > 250$

Now, we divide 425 by 250.

We used **Euclid's division lemma**.

$$425 = 250 \times 1 + 175$$

Here, remainder  $r = 175 \neq 0$

So, the new dividend is 250 and the new divisor is 175, again we apply Euclid division algorithm.

$$250 = 175 \times 1 + 75$$

Here, remainder  $r = 75 \neq 0$

On taking the new dividend is 175 and the new divisor is 75, we apply Euclid division algorithm.

$$175 = 75 \times 2 + 25$$

Here, remainder  $r = 25 \neq 0$

On taking new dividend is 75 and the new divisor is 25, again we apply Euclid division algorithm.

$$75 = 25 \times 3 + 0$$

Here, remainder is zero and divisor is 25.

So, the HCF of 425 and 250 is 25.

Hence, **the maximum capacity of the required container is 25 L.**

### 5. Question

A rectangular surface has length 4661 meters and breadth 3318 meters. On this area, square tiles are to be put. Find the maximum length of such tiles.

### Answer

Given length and breadth are 4661 m and 3318 m respectively.

Here,  $4661 > 3318$

So, we divide 4661 by 3318

By using **Euclid's division lemma**, we get

$$4661 = 3318 \times 1 + 1343$$

Here,  $r = 1343 \neq 0$ .

On taking 3318 as dividend and 1343 as the divisor and we apply Euclid's division lemma, we get

$$3318 = 1343 \times 2 + 632$$

Here,  $r = 632 \neq 0$

So, on taking 1343 as dividend and 632 as the divisor and again we apply Euclid's division lemma, we get

$$1343 = 632 \times 2 + 79$$

Here,  $r = 79 \neq 0$

So, on taking 632 as dividend and 79 as the divisor and again we apply Euclid's division lemma, we get

$$632 = 79 \times 8 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 79, the HCF of 3318 and 4661 is 79.

**Hence, the maximum length of such tiles is 79 meters.**

## 6. Question

Find the least number of square tiles which can the floor of a rectangular shape having length and breadth 16 meters 58 centimeters and 8 meters 32.

### Answer

Firstly, we find the length of the largest tile so for that we have to find the HCF of 1658 and 832.

Here,  $1658 > 832$

So, we divide 1658 by 832

By using **Euclid's division lemma**, we get

$$1658 = 832 \times 1 + 826$$

Here,  $r = 826 \neq 0$ .

On taking 832 as dividend and 826 as the divisor and we apply Euclid's division lemma, we get

$$832 = 826 \times 1 + 6$$

Here,  $r = 6 \neq 0$

So, on taking 826 as dividend and 6 as the divisor and again we apply Euclid's division lemma, we get

$$826 = 6 \times 137 + 4$$

Here,  $r = 4 \neq 0$

So, on taking 6 as dividend and 4 as the divisor and again we apply Euclid's division lemma, we get

$$6 = 4 \times 1 + 2$$

Here,  $r = 2 \neq 0$

So, on taking 4 as dividend and 2 as the divisor and again we apply Euclid's division lemma, we get

$$4 = 2 \times 2 + 0$$

The remainder has now become 0, so our procedure stops. Since the divisor at this last stage is 79, the HCF of 1658 and 832 is 2.

So, the length of the largest tile is 2 cm

$$\text{Area of each tile} = 2 \times 2 = 4\text{cm}^2$$

$$\text{The required number of tiles} = \frac{\text{Area of floor}}{\text{Area of tiles}}$$

$$= \frac{1658 \times 832}{2 \times 2}$$

$$= 344864$$

**Least number of square tiles are required are 344864**

## Exercise 1.2

### 1 A. Question

Express each of the following numbers as a product of its prime factors:

4320

**Answer**

Given number is 4320

Factorization of 4320 is

|   |      |
|---|------|
| 2 | 4320 |
| 2 | 2160 |
| 2 | 1080 |
| 2 | 540  |
| 2 | 270  |
| 3 | 135  |
| 3 | 45   |
| 3 | 15   |
| 5 | 5    |
|   | 1    |

Hence,  $4320 = 2^5 \times 3^3 \times 5$  (Product of its prime factors)

### 1 B. Question

Express each of the following numbers as a product of its prime factors:

7560

**Answer**

Given number is 7560

Factorization of 7560 is

|   |      |
|---|------|
| 2 | 7560 |
| 2 | 3780 |
| 2 | 1890 |
| 3 | 945  |
| 3 | 315  |
| 3 | 105  |
| 5 | 35   |
| 7 | 7    |
|   | 1    |

Hence,  $7560 = 2^3 \times 3^3 \times 5 \times 7$  (Product of its prime factors)

**1 C. Question**

Express each of the following numbers as a product of its prime factors:

140

**Answer**

Given number is 140

Factorization of 140 is

|   |     |
|---|-----|
| 2 | 140 |
| 2 | 70  |
| 5 | 35  |
| 7 | 7   |
|   | 1   |

Hence,  $140 = 2^2 \times 5 \times 7$  (Product of its prime factors)

**1 D. Question**

Express each of the following numbers as a product of its prime factors:

5005

**Answer**

Given number is 5005

Factorization of 5005 is

|    |      |
|----|------|
| 5  | 5005 |
| 7  | 1001 |
| 11 | 143  |
| 13 | 13   |
|    | 1    |

Hence,  $5005 = 5 \times 7 \times 11 \times 13$  (Product of its prime factors)

### 1 E. Question

Express each of the following numbers as a product of its prime factors:

32760

### Answer

Given number is 32760

Factorization of 32760 is

|    |       |
|----|-------|
| 2  | 32760 |
| 2  | 16380 |
| 2  | 8190  |
| 3  | 4095  |
| 3  | 1365  |
| 5  | 455   |
| 7  | 91    |
| 13 | 13    |
|    | 1     |

Hence,  $32760 = 2^3 \times 3^2 \times 5 \times 7 \times 13$  (Product of its prime factors)

### 1 F. Question

Express each of the following numbers as a product of its prime factors:

156

### Answer

Given number is 156

Factorization of 156 is

|    |     |
|----|-----|
| 2  | 156 |
| 2  | 78  |
| 3  | 39  |
| 13 | 13  |
|    | 1   |

Hence,  $156 = 2^2 \times 3 \times 13$  (Product of its prime factors)

### 1 G. Question

Express each of the following numbers as a product of its prime factors:

729

#### Answer

Given number is 729

Factorization of 729 is

|   |     |
|---|-----|
| 3 | 729 |
| 3 | 243 |
| 3 | 81  |
| 3 | 27  |
| 3 | 9   |
| 3 | 3   |
|   | 1   |

Hence,  $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$  (Product of its prime factors)

### 2. Question

Find the highest power of 5 in 23750.

#### Answer

To find the highest power of 5 in 23750, we have to factorize 23750

|    |       |
|----|-------|
| 2  | 23750 |
| 5  | 11875 |
| 5  | 2375  |
| 5  | 475   |
| 5  | 95    |
| 19 | 19    |
|    | 1     |

Hence,  $23750 = 2 \times 5^4 \times 19$

So, the highest power of 5 in 23750 is **4**.

### 3. Question

Find the highest power of 2 in 1440.

#### Answer

Given number is 1440

Factorization of 1440 is

|   |      |
|---|------|
| 2 | 1440 |
| 2 | 720  |
| 2 | 360  |
| 2 | 180  |
| 2 | 90   |
| 3 | 45   |
| 3 | 15   |
| 5 | 5    |
|   | 1    |

$$\text{Factors of } 1440 = 2^5 \times 3^2 \times 5$$

So, the highest power of 2 is **5**.

#### **4. Question**

If  $6370 = 2^m \cdot 5^n \cdot 7^k \cdot 13^p$ , then find  $m + n + k + p$ .

**Answer**

We have to factorize the 6370 to find the value of m, n, k and p

Factorization of 6370 is

|    |      |
|----|------|
| 2  | 6370 |
| 5  | 3185 |
| 7  | 637  |
| 7  | 91   |
| 13 | 13   |
|    | 1    |

$$6370 = 2 \times 5 \times 7^2 \times 13$$

On Comparing, we get

$$6370 = 2^m \cdot 5^n \cdot 7^k \cdot 13^p = 2^1 \times 5^1 \times 7^2 \times 13^1$$

$$\mathbf{m = 1}$$

$$\mathbf{n = 1}$$

$$\mathbf{k = 2}$$

$$\mathbf{p = 1}$$

$$\mathbf{\text{So, } m + n + k + p = 5}$$

#### **5 A. Question**

Which of the following is a pair of co-primes?

(32,62)

**Answer**

Given numbers are 32 and 62

For pairs to be co-primes there should be no common factor except 1

Factorization of 32 and 62 are

|   |    |
|---|----|
| 2 | 32 |
| 2 | 16 |
| 2 | 8  |
| 2 | 4  |
| 2 | 2  |
|   | 1  |

Factors of 32 =  $\textcircled{2} \times 2 \times 2 \times 2 \times 2 = 2^5$

|    |    |
|----|----|
| 2  | 62 |
| 31 | 31 |
|    | 1  |

Factors of 62 =  $\textcircled{2} \times 31$

Here, we can see that 2 is the common factor. So, (32,62) is not co-prime.

### 5 B. Question

Which of the following is a pair of co-primes?

(18,25)

**Answer**

Given numbers are 18 and 25

For pairs to be co-primes there should be no common factor except 1

Factorization of 18 and 25 are

|   |    |
|---|----|
| 2 | 18 |
| 3 | 9  |
| 3 | 3  |
|   | 1  |

Factors of 32 =  $2 \times 3 \times 3$

$$\begin{array}{r|l} 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Factors of 25 =  $5 \times 5$

Therefore, there is no common factor. So, (18,25) is co-prime.

### 5 C. Question

Which of the following is a pair of co-primes?

(31, 93)

### Answer

Given numbers are 31 and 93

For pairs to be co-primes there should be no common factor except 1

Factorization of 31 and 93 are

$$\begin{array}{r|l} 3 & 93 \\ \hline 31 & 31 \\ \hline 2 & 1 \end{array}$$

Factors of 93 =  $\textcircled{31} \times 3$

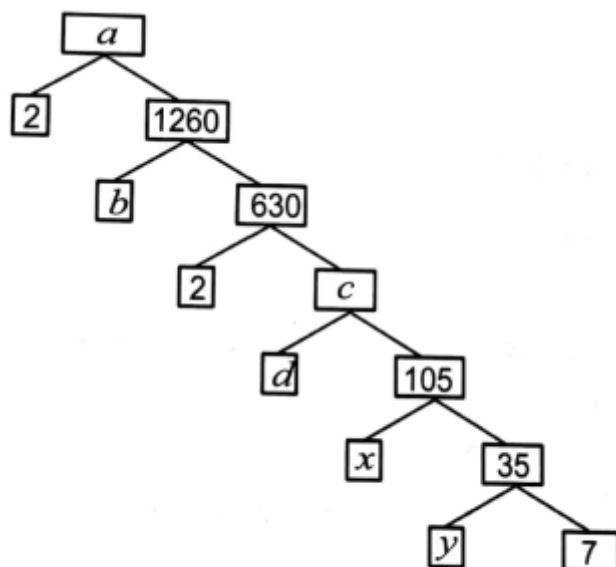
$$\begin{array}{r|l} 31 & 31 \\ \hline & 1 \end{array}$$

Factors of 31 =  $\textcircled{31} \times 1$

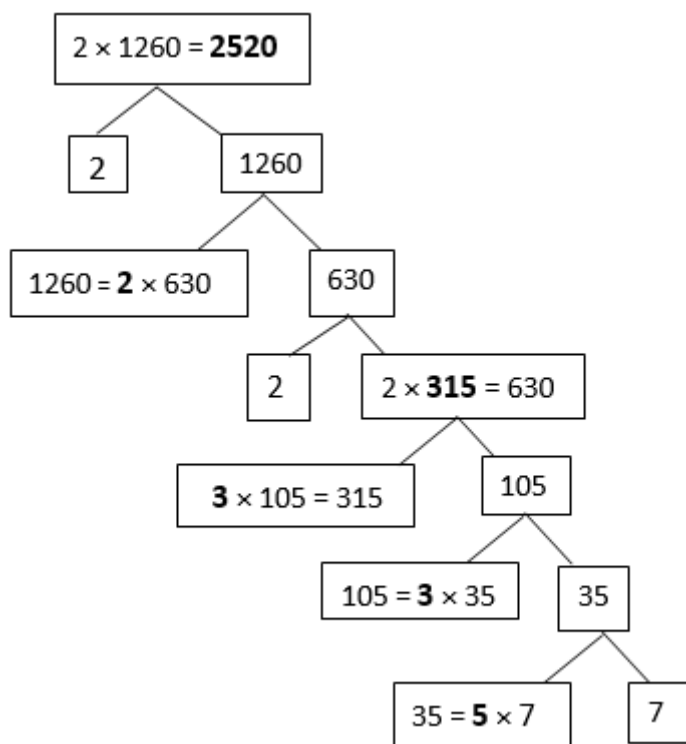
Here, we can see that 31 is the common factor. So, (31, 93) is not co-prime.

### 6 A. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



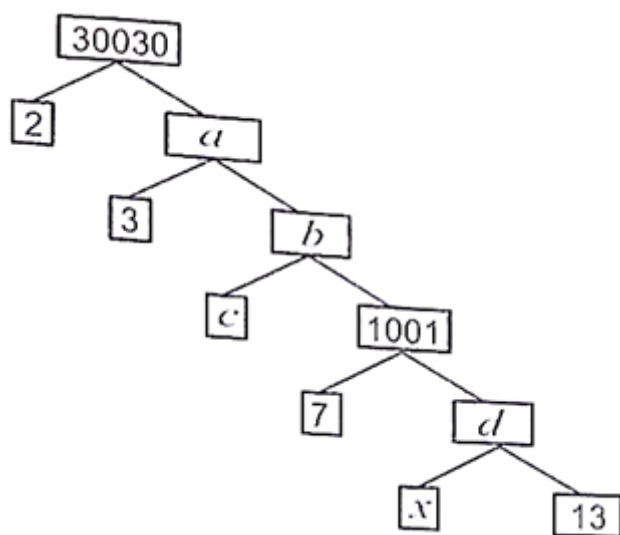
**Answer**



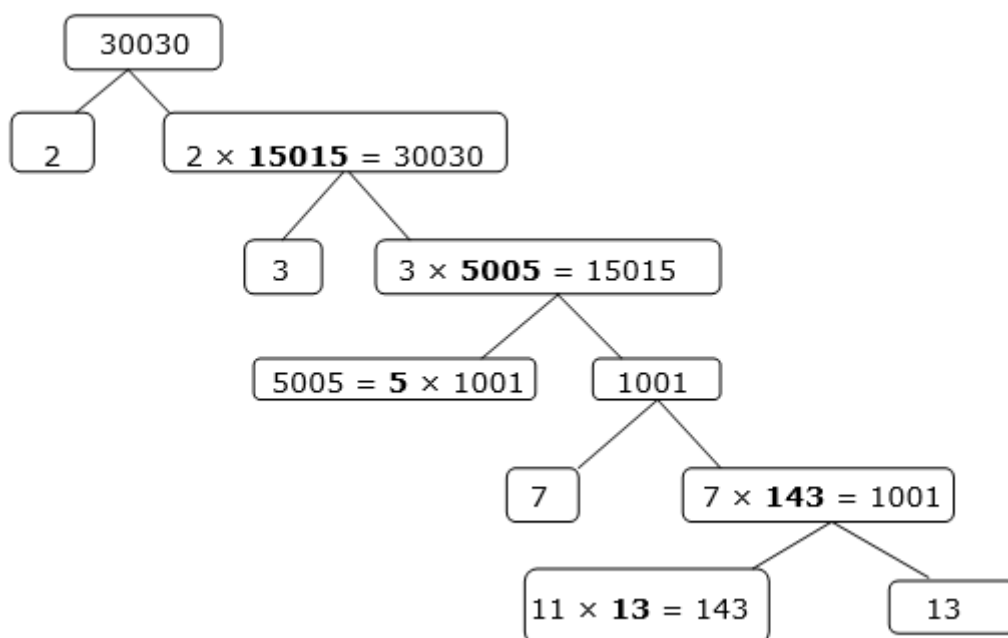
Here,  $a = 2520$ ;  $b = 2$ ;  $c = 315$ ;  $d = 3$ ;  $x = 3$ ;  $y = 5$

## 6 B. Question

Write down the missing numbers  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $x$ ,  $y$  in the following factor tree :



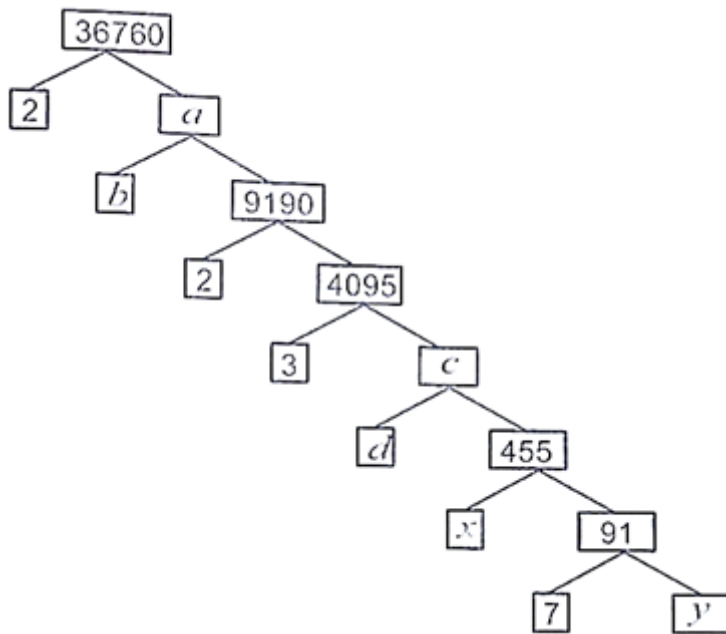
**Answer**



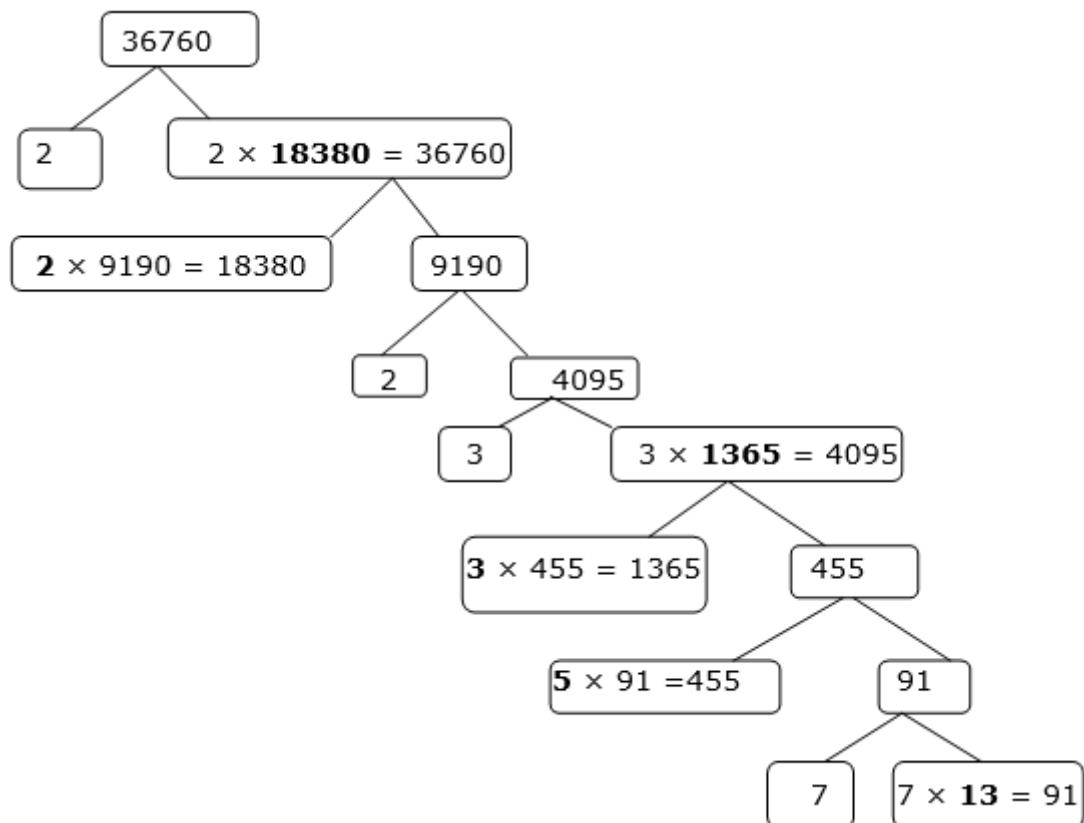
Here,  $a = 15015$ ;  $b = 5005$ ;  $c = 5$ ;  $d = 143$ ;  $x = 13$

### 6 C. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



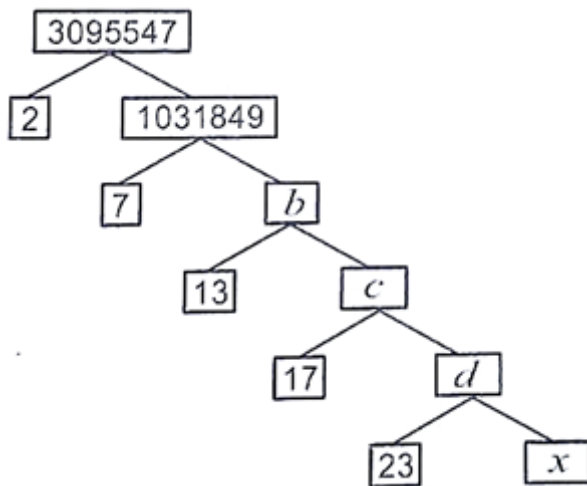
**Answer**



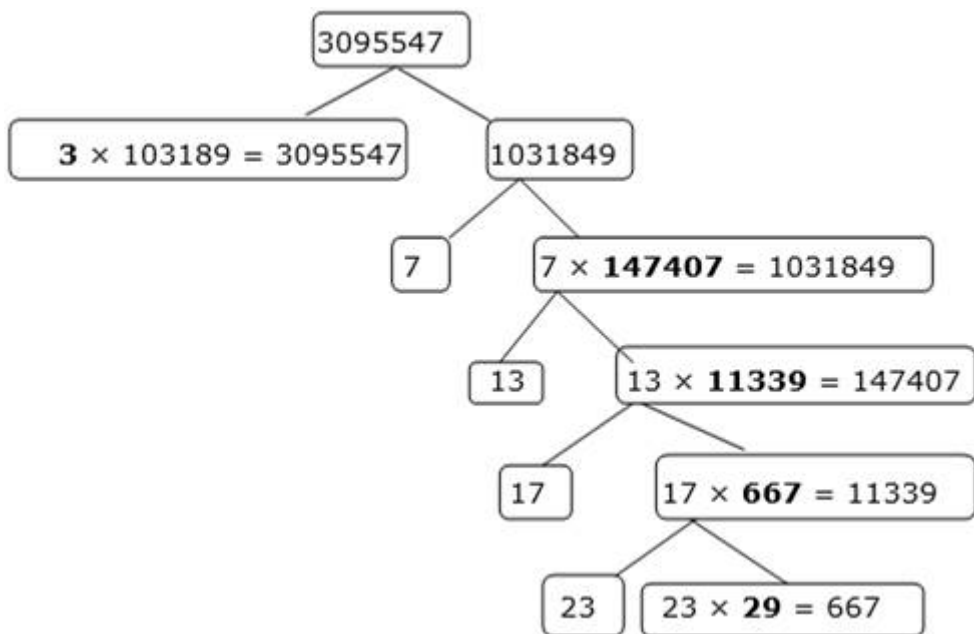
Here,  $a = 18380$ ;  $b = 2$ ;  $c = 1365$ ;  $d = 3$ ;  $x = 5$ ;  $y = 13$

#### 6 D. Question

Write down the missing numbers a, b, c, d, x, y in the following factor tree :



**Answer**



Here,  $a = 3$ ;  $b = 147407$ ;  $c = 11339$ ;  $d = 667$ ;  $x = 29$

### 7 A. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

96 and 404

**Answer**

Given numbers are 96 and 404.

The prime factorization of 96 and 404 gives:

|   |    |
|---|----|
| 2 | 96 |
| 2 | 48 |
| 2 | 24 |
| 2 | 12 |
| 2 | 6  |
| 3 | 3  |
|   | 1  |

$$96 = 2^5 \times 3 = (2 \times 2) \times 2 \times 2 \times 2 \times 3$$

|     |     |
|-----|-----|
| 2   | 404 |
| 2   | 202 |
| 101 | 101 |
|     | 1   |

$$404 = 2^2 \times 101 = (2 \times 2) \times 101$$

Here,  $2^2$  is the smallest power of the common factor 2.

Therefore, the **H.C.F** of these two integers is  $2 \times 2 = 4$

$2^5 \times 3^1 \times 101^1$  are the greatest powers of the prime factors 2, 3 and 101 respectively involved in the given numbers.

Now, **L.C.M** of 96 and 404 is  $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 101 = \mathbf{9696}$

## 7 B. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

6 and 20

### Answer

Given numbers are 6 and 20

The prime factorization of 6 and 20 gives:

|   |   |
|---|---|
| 2 | 6 |
| 3 | 3 |
|   | 1 |

$$6 = (2) \times 3$$

|   |    |
|---|----|
| 2 | 20 |
| 2 | 10 |
| 5 | 5  |
|   | 1  |

$$20 = \textcircled{2} \times 2 \times 5$$

Here,  $2^1$  is the smallest power of the common factor 2.

Therefore, the **H.C.F** of these two integers = **2**

$2^2 \times 3^1 \times 5^1$  are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given numbers.

Now, **L.C.M** of 6 and 20 =  $2 \times 2 \times 3 \times 5 = \mathbf{60}$

### 7 C. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

26 and 91

#### Answer

Given numbers are 26 and 91.

The prime factorization of 26 and 91 gives:

$$\begin{array}{r|l} 2 & 26 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$26 = 2 \times \textcircled{13}$$

$$\begin{array}{r|l} 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$91 = 7 \times \textcircled{13}$$

Here,  $13^1$  is the smallest power of the common factor 13.

Therefore, the **H.C.F** of these two integers = **13**

$2^1 \times 7^1 \times 13^1$  are the greatest powers of the prime factors 2, 7 and 13 respectively involved in the given numbers.

Now, **L.C.M** of 6 and 21 is  $2 \times 7 \times 13 = \mathbf{182}$

### 7 D. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

87 and 145

**Answer**

Given numbers are 87 and 145.

The prime factorization of 87 and 145 gives:

$$\begin{array}{r|l} 3 & 87 \\ \hline 29 & 29 \\ \hline & 1 \end{array}$$

$$87 = 3 \times (29)$$

$$\begin{array}{r|l} 5 & 145 \\ \hline 29 & 29 \\ \hline & 1 \end{array}$$

$$145 = 5 \times (29)$$

Here,  $29^1$  is the smallest power of the common factor 29.

Therefore, the **H.C.F** of these two integers = **29**

$3^1 \times 5^1 \times 29^1$  are the greatest powers of the prime factors 3, 5 and 29 respectively involved in the given numbers.

Now, **L.C.M** of 87 and 145 =  $3 \times 5 \times 29 = \mathbf{435}$

**7 E. Question**

Find the LCM and HCF of the following integers by applying the prime factorization method :

1485 and 4356

**Answer**

Given numbers are 1485 and 4356.

The prime factorization of 1485 and 4356 gives:

$$\begin{array}{r|l} 3 & 1485 \\ \hline 3 & 495 \\ \hline 3 & 165 \\ \hline 5 & 55 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$$1485 = (3 \times 3) \times 3 \times 5 \times (11)$$

|    |      |
|----|------|
| 2  | 4356 |
| 2  | 2178 |
| 3  | 1089 |
| 3  | 363  |
| 11 | 121  |
| 11 | 11   |
|    | 1    |

$$4356 = 2 \times 2 \times (3 \times 3 \times 11) \times 11$$

Here,  $3^2 \times 11$  is the smallest power of the common factors 3 and 11.

Therefore, the **H.C.F** of these two integers =  $3 \times 3 \times 11 = 99$

$2^2 \times 3^3 \times 5^1 \times 11^2$  are the greatest powers of the prime factors 2, 3 and 7 respectively involved in the given numbers.

Now, **L.C.M** of 1485 and 4356 =  $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11 \times 11 = 65430$

### 7 F. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

1095 and 1168

#### Answer

Given numbers are 1095 and 1168.

The prime factorization of 1095 and 1168 gives:

|    |      |
|----|------|
| 3  | 1095 |
| 5  | 365  |
| 73 | 73   |
|    | 1    |

$$1485 = 3 \times 5 \times (73)$$

|    |      |
|----|------|
| 2  | 1168 |
| 2  | 584  |
| 2  | 292  |
| 2  | 146  |
| 73 | 73   |
|    | 1    |

$$4356 = 2 \times 2 \times 2 \times 2 \times (73)$$

Here,  $73^1$  is the smallest power of the common factor 73.

Therefore, the **H.C.F** of these two integers = **73**

$2^4 \times 3^1 \times 5^1 \times 73^1$  are the greatest powers of the prime factors 2, 3, 5 and 73 respectively involved in the given numbers.

Now, **L.C.M** of 1485 and 4356 =  $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 73 = \mathbf{17520}$

### 7 G. Question

Find the LCM and HCF of the following integers by applying the prime factorization method :

6 and 21

#### Answer

Given numbers are 6 and 21.

The prime factorization of 6 and 21 gives:

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$6 = 2 \times \textcircled{3}$$

$$\begin{array}{r|l} 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$21 = \textcircled{3} \times 7$$

Here,  $3^1$  is the smallest power of the common factor 3.

Therefore, the **H.C.F** of these two integers = **3**

$2^1 \times 3^1 \times 7^1$  are the greatest powers of the prime factors 2, 3 and 7 respectively involved in the given numbers.

Now, **L.C.M** of 6 and 21 is  $2 \times 3 \times 7 = \mathbf{42}$

### 8 A. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

96 and 404

#### Answer

Given numbers are 96 and 404

The prime factorization of 96 and 404 gives:

$$96 = 2^5 \times 3 \text{ and } 404 = 2^2 \times 101$$

Therefore, the H.C.F of these two integers =  $2^2 = 4$

Now, the L.C.M of 96 and 404 =  $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 101 = 9696$

Now, we have to verify

$$\text{L.C.M (a, b)} \times \text{H.C.F (a, b)} = \text{Product of two numbers (a} \times \text{b)}$$

$$\text{L.H.S} = \text{L.C.M} \times \text{H.C.F} = 9696 \times 4 = 38784$$

$$\text{R.H.S} = \text{Product of two numbers} = 96 \times 404 = 38784$$

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

### 8 B. Question

Find the LCM and HCF of the following pair of integers and verify that LCM X HCF = Product of two numbers :

852 and 1491

#### Answer

Given numbers are 852 and 1491

The prime factorization of 852 and 1491 gives:

$$852 = 2 \times 2 \times 3 \times 71 \text{ and } 1491 = 3 \times 7 \times 71$$

Therefore, the H.C.F of these two integers =  $3 \times 71 = 213$

Now, the L.C.M of 96 and 404 =  $2 \times 2 \times 3 \times 7 \times 71 = 5964$

Now, we have to verify

$$\text{L.C.M (a, b)} \times \text{H.C.F (a, b)} = \text{Product of two numbers (a} \times \text{b)}$$

$$\text{L.H.S} = \text{L.C.M} \times \text{H.C.F} = 5964 \times 213 = 1270332$$

$$\text{R.H.S} = \text{Product of two numbers} = 852 \times 1491 = 1270332$$

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

### 8 C. Question

Find the LCM and HCF of the following pair of integers and verify that  $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$  :

777 and 1147

**Answer**

Given numbers are 777 and 1147

The prime factorization of 777 and 1147 gives:

$$777 = 3 \times 7 \times 37 \text{ and } 1147 = 31 \times 37$$

Therefore, the H.C.F of these two integers = **37**

Now, the L.C.M of 96 and 404 =  $3 \times 7 \times 31 \times 37 = \mathbf{24087}$

Now, we have to verify

$$\text{L.C.M (a, b)} \times \text{H.C.F (a, b)} = \text{Product of two numbers (a} \times \text{b)}$$

$$\text{L.H.S} = \text{L.C.M} \times \text{H.C.F} = 24087 \times 37 = 891219$$

$$\text{R.H.S} = \text{Product of two numbers} = 777 \times 1147 = 891219$$

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

### **8 D. Question**

Find the LCM and HCF of the following pair of integers and verify that  $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$  :

36 and 64

**Answer**

Given numbers are 36 and 64

The prime factorization of 36 and 64 gives:

$$36 = 2 \times 2 \times 3 \times 3 \text{ and } 64 = 2^6$$

Therefore, the H.C.F of these two integers =  $2 \times 2 = \mathbf{4}$

Now, the L.C.M of 36 and 64 =  $3 \times 3 \times 2^6 = \mathbf{576}$

Now, we have to verify

$$\text{L.C.M (a, b)} \times \text{H.C.F (a, b)} = \text{Product of two numbers (a} \times \text{b)}$$

$$\text{L.H.S} = \text{L.C.M} \times \text{H.C.F} = 576 \times 4 = 2304$$

$$\text{R.H.S} = \text{Product of two numbers} = 36 \times 64 = 2304$$

Hence,  $\text{L.H.S} = \text{R.H.S}$

So, the product of two numbers is equal to the product of their HCF and LCM.

### 8 E. Question

Find the LCM and HCF of the following pair of integers and verify that  $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$  :

32 and 80

#### Answer

Given numbers are 32 and 80

The prime factorization of 32 and 80 gives:

$$32 = 2^5 \text{ and } 80 = 2^4 \times 5$$

Therefore, the H.C.F of these two integers  $= 2^4 = \mathbf{16}$

Now, the L.C.M of 32 and 80  $= 5 \times 2^5 = \mathbf{160}$

Now, we have to verify

$$\boxed{\text{L.C.M (a, b)} \times \text{H.C.F (a, b)} = \text{Product of two numbers (a} \times \text{b)}}$$

$$\text{L.H.S} = \text{L.C.M} \times \text{H.C.F} = 160 \times 16 = 2560$$

$$\text{R.H.S} = \text{Product of two numbers} = 32 \times 80 = 2560$$

Hence,  $\text{L.H.S} = \text{R.H.S}$

So, the product of two numbers is equal to the product of their HCF and LCM.

### 8 F. Question

Find the LCM and HCF of the following pair of integers and verify that  $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$  :

902 and 1517

#### Answer

Given numbers are 902 and 1517

The prime factorization of 902 and 1517 gives:

$$902 = 2 \times 11 \times 41 \text{ and } 1517 = 37 \times 41$$

Therefore, the H.C.F of these two integers = **41**

Now, the L.C.M of 902 and 1517 =  $2 \times$

$$11 \times 37 \times 41 = \mathbf{33374}$$

Now, we have to verify

$$\mathbf{L.C.M (a, b) \times H.C.F (a, b) = Product of two numbers (a \times b)}$$

$$L.H.S = L.C.M \times H.C.F = 33374 \times 41 = 1368334$$

$$R.H.S = Product of two numbers = 902 \times 1517 = 1368334$$

Hence, L.H.S = R.H.S

So, the product of two numbers is equal to the product of their HCF and LCM.

### 9 A. Question

Find LCM and HCF of the following integers by using prime factorization method:

6, 72 and 120

**Answer**

Given numbers are 6, 72 and 120

Factorization of 6, 72 and 120

|                                                                      |                                                                                                                         |                                                                                                                           |
|----------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{c c} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$ | $\begin{array}{c c} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$ | $\begin{array}{c c} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$ |
|----------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|

$$\begin{array}{lcl} 6 & = & 2 \times 3 = 2^1 \times 3^1 \\ 72 & = & 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 \\ 120 & = & 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1 \end{array}$$

Here,  $2^1 \times 3^1$  are the smallest powers of the common factors 2 and 3, respectively.

$$\text{So, } \mathbf{HCF (6, 72, 120) = 2 \times 3 = 6}$$

$2^3 \times 3^2 \times 5^1$  are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given three numbers .

**LCM** of these three integers =  $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

### 9 B. Question

Find LCM and HCF of the following integers by using prime factorization method:

8, 9, and 25

#### Answer

Given numbers are 8, 9 and 25

Factorization of 8, 9 and 25

|   |   |   |   |   |    |
|---|---|---|---|---|----|
| 2 | 8 | 3 | 9 | 5 | 25 |
| 2 | 4 | 3 | 3 | 5 | 5  |
| 2 | 2 |   | 1 |   | 1  |
|   | 1 |   |   |   |    |

$$8 = 2 \times 2 \times 2 \times 1 = 2^3 \times 1$$

$$9 = 3 \times 3 \times 1 = 3^2 \times 1$$

$$25 = 5 \times 5 \times 1 = 5^2 \times 1$$

Here,  $1^1$  is the smallest power of the common factor 1.

So, **HCF (8, 9, 25) = 1**

$2^3 \times 3^2 \times 5^2$  are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given three numbers .

**LCM** of these three integers =  $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

### 9 C. Question

Find LCM and HCF of the following integers by using prime factorization method:

12, 15, and 21

#### Answer

Given numbers are 12, 15 and 21

Factorization of 12, 15 and 21

|   |    |   |    |   |    |
|---|----|---|----|---|----|
| 2 | 12 | 3 | 15 | 3 | 21 |
| 2 | 6  | 5 | 5  | 7 | 7  |
| 3 | 3  |   | 1  |   | 1  |
|   | 1  |   |    |   |    |

$$\begin{array}{lcl}
 12 & = 2 \times 2 \times \textcircled{3} & = 2^2 \times 3^1 \\
 15 & = \textcircled{3} \times 5 & = 3^1 \times 5^1 \\
 21 & = \textcircled{3} \times 7 & = 3^1 \times 7^1
 \end{array}$$

Here,  $3^1$  is the smallest power of the common factor 3.

So, **HCF (12, 15, 21) = 3**

$2^2 \times 3^1 \times 5^1 \times 7^1$  are the greatest powers of the prime factors 2, 3, 5 and 7 respectively involved in the given three numbers .

**LCM** of these three integers =  $2 \times 2 \times 3 \times 5 \times 7 = \mathbf{420}$

### 9 D. Question

Find LCM and HCF of the following integers by using prime factorization method:

36, 45, and 72

**Answer**

Given numbers are 36, 45 and 72

Factorization of 36, 45 and 72

|   |    |   |    |   |    |
|---|----|---|----|---|----|
| 2 | 36 | 3 | 45 | 2 | 72 |
| 2 | 18 | 3 | 15 | 2 | 36 |
| 3 | 9  | 5 | 5  | 2 | 18 |
| 3 | 3  |   | 1  | 3 | 9  |
|   | 1  |   |    | 3 | 3  |
|   |    |   |    |   | 1  |

$$36 = \boxed{2 \times 2} \times \textcircled{3 \times 3} = 2^2 \times 3^2$$

$$45 = \textcircled{3 \times 3} \times 5 = 3^2 \times 5^1$$

$$72 = \boxed{2 \times 2} \times 2 \times \textcircled{3 \times 3} = 2^3 \times 3^2$$

Here,  $3^2$  is the smallest power of the common factor 3.

So, **HCF (36, 45, 72) =  $3 \times 3 = 9$**

$2^3 \times 3^2 \times 5^1$  are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the given three numbers .

**LCM** of these three integers =  $2 \times 2 \times 2 \times 3 \times 3 \times 5 = \mathbf{360}$

### 9 E. Question

Find LCM and HCF of the following integers by using prime factorization method:

42, 63 and 140

**Answer**

Given numbers are 42, 63 and 140

Factorization of 42, 63 and 140

|   |    |   |    |   |     |
|---|----|---|----|---|-----|
| 2 | 42 | 3 | 63 | 2 | 140 |
| 3 | 21 | 3 | 21 | 2 | 70  |
| 7 | 7  | 7 | 7  | 5 | 35  |
|   | 1  |   | 1  | 7 | 7   |
|   |    |   |    |   | 1   |

$$42 = \boxed{2} \times \boxed{3} \times \textcircled{7} = 2^1 \times 3^1 \times 7^1$$

$$63 = 3 \times \boxed{3} \times \textcircled{7} = 3^2 \times 7^1$$

$$140 = \boxed{2} \times 2 \times 5 \times \textcircled{7} = 2^2 \times 5^1 \times 7^1$$

Here,  $7^1$  is the smallest power of the common factor 7.

So, **HCF (42, 63, 140) = 7**

$2^2 \times 3^2 \times 5^1 \times 7^1$  are the greatest powers of the prime factors 2, 3, 5 and 7 respectively involved in the given three numbers .

**LCM** of these three integers =  $2 \times 2 \times 3 \times 3 \times 5 \times 7 = \mathbf{1260}$

## 9 F. Question

Find LCM and HCF of the following integers by using prime factorization method:

48, 72 and 108

**Answer**

Given numbers are 48, 72 and 108

Factorization of 48, 72 and 108

|   |    |   |    |   |     |
|---|----|---|----|---|-----|
| 2 | 48 | 2 | 72 | 2 | 108 |
| 2 | 24 | 2 | 36 | 2 | 54  |
| 2 | 12 | 2 | 18 | 3 | 27  |
| 2 | 6  | 3 | 9  | 3 | 9   |
| 3 | 3  | 3 | 3  | 3 | 3   |
|   | 1  |   | 1  |   | 1   |

$$\begin{aligned}
 48 &= 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3 \\
 72 &= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 \\
 108 &= 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3
 \end{aligned}$$

Here,  $2^2 \times 3^1$  are the smallest powers of the common factors 2 and 3 respectively.

So, **HCF (48, 72, 108) =  $2 \times 2 \times 3 = 12$**

$2^4 \times 3^3$  are the greatest powers of the prime factors 2 and 3 respectively involved in the given three numbers .

**LCM** of these three integers =  $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 432$

### 10 A. Question

If HCF (96, 404) and 4, then, find LCM (96, 404)

**Answer**

Given: HCF (96, 404) = 4

To Find: LCM (96, 404)

We use the formula

**L.C.M (a,b)  $\times$  H.C.F (a,b) = Product of two numbers (a **$\times$ b)****

LCM (96, 404)  $\times$  HCF (96, 404) =  $96 \times 404$

$\Rightarrow$  LCM (96, 404)  $\times 4 = 96 \times 404$  [ $\because$  HCF (96, 404) = 4]

$$\Rightarrow \text{LCM (96, 404)} = \frac{96 \times 404}{4}$$

$\Rightarrow$  **LCM (96, 404) = 9696**

### 10 B. Question

If LCM (72, 126) = 504, find HCF (72, 126)

**Answer**

Given: LCM (72, 126) = 504

To Find: HCF (72, 126)

We use the formula

**L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)**

$$\text{LCM (72, 126)} \times \text{HCF (72, 126)} = 72 \times 126$$

$$\Rightarrow 504 \times \text{HCF (72, 126)} = 72 \times 126 [\because \text{LCM(72,126)}=504]$$

$$\Rightarrow \text{HCF (72, 126)} = \frac{72 \times 126}{504}$$

$$\Rightarrow \text{HCF (72 , 126)} = 18$$

#### **10 C. Question**

If HCF (18, 504) = 18, find LCM (18, 504)

**Answer**

Given: HCF (18, 504) = 18

To Find: LCM (18, 504)

We use the formula

**L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)**

$$\text{LCM (18, 504)} \times \text{HCF (18, 504)} = 18 \times 504$$

$$\Rightarrow \text{LCM (18, 504)} \times 18 = 18 \times 504 [\because \text{HCF(18, 504)} = 18]$$

$$\Rightarrow \text{LCM (18, 504)} = \frac{18 \times 504}{18}$$

$$\Rightarrow \text{LCM (18, 504)} = 504$$

#### **10 D. Question**

If LCM (96, 168) = 672, find HCF (96, 168)

**Answer**

Given: LCM (96, 168) = 672

To Find: HCF (96, 168)

We use the formula

**L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)**

$$\text{LCM (96, 168)} \times \text{HCF (96, 168)} = 96 \times 168$$

$$\Rightarrow 672 \times \text{HCF (96, 168)} = 96 \times 168 [\because \text{LCM(96, 168)}=672]$$

$$\Rightarrow \text{HCF (96, 168)} = \frac{96 \times 168}{672}$$

$$\Rightarrow \text{HCF (96, 168)} = 24$$

### 10 E. Question

If HCF (306, 657) = 9, find LCM (306, 657)

**Answer**

Given: HCF (306, 657) = 9

To Find: LCM (306, 657)

We use the formula

**L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)**

$$\text{LCM (306, 657)} \times \text{HCF (306, 657)} = 306 \times 657$$

$$\Rightarrow \text{LCM (306, 657)} \times 9 = 306 \times 657 [\because \text{HCF(306,657)= 9}]$$

$$\Rightarrow \text{LCM (306, 657)} = \frac{306 \times 657}{9}$$

$$\Rightarrow \text{LCM (306, 657)} = 22338$$

### 10 F. Question

If HCF (36, 64) = 4, find LCM (36, 64)

**Answer**

Given: HCF (36, 64) = 4

To Find: LCM (36, 64)

We use the formula

**L.C.M (a,b) × H.C.F (a,b) = Product of two numbers (a×b)**

$$\text{LCM (36, 64)} \times \text{HCF (36, 64)} = 36 \times 64$$

$$\Rightarrow \text{LCM (36, 64)} \times 4 = 36 \times 64 [\because \text{HCF (36, 64)= 4}]$$

$$\Rightarrow \text{LCM (36, 64)} = \frac{36 \times 64}{4}$$

$$\Rightarrow \text{LCM (36, 64)} = 576$$

### 11 A. Question

Examine whether  $(15)^n$  can end with the digit 0 for any  $n \in \mathbb{N}$ .

**Answer**

If  $(15)^n$  end with the digit 0, then the number should be divisible by 2 and 5.

As  $2 \times 5 = 10$

$\Rightarrow$  This means the prime factorization of  $15^n$  should contain prime factors 2 and 5.

But  $(15)^n = (3 \times 5)^n$  and it does not have the prime factor 2 but have 3 and 5.

$\therefore$ , 2 is not present in the prime factorization, there is no natural number nor which  $15^n$  ends with digit zero.

So,  $15^n$  cannot end with digit zero.

**11 B. Question**

Examine whether  $(24)^n$  can end with the digit 5 for any  $n \in \mathbb{N}$ .

**Answer**

If  $(24)^n$  end with the digit 5, then the number should be divisible by 5.

$\Rightarrow$  This means the prime factorization of  $24^n$  should contain prime factors 5.

But  $(24)^n = (2^3 \times 3)^n$  and it does not have the prime factor 5 but have 3 and 2.

$\therefore$ , 5 is not present in the prime factorization, there is no natural number nor which  $24^n$  ends with digit 5.

So,  $24^n$  cannot end with digit 5.

**11 C. Question**

Examine whether  $(21)^n$  can end with the digit 0 for any  $n \in \mathbb{N}$ .

**Answer**

If  $(21)^n$  end with the digit 0, then the number should be divisible by 2 and 5.

As  $2 \times 5 = 10$

$\Rightarrow$  This means the prime factorization of  $21^n$  should contain prime factors 2 and 5.

But  $(21)^n = (3 \times 7)^n$  and it does not have the prime factor 2 and 5 but have 3 and 7.

$\therefore$ , 2 and 5 is not present in the prime factorization, there is no natural number nor which  $21^n$  ends with digit zero.

So,  $21^n$  cannot end with digit zero.

### 11 D. Question

Examine whether  $(8)^n$  can end with the digit 5 for any  $n \in \mathbb{N}$ .

#### Answer

If  $(8)^n$  end with the digit 5, then the number should be divisible by 5.

$\Rightarrow$  This means the prime factorization of  $8^n$  should contain prime factor 5.

But  $(8)^n = (2^3)^n$  and it does not have the prime factor 5 but have 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of  $8^n$ .

$\therefore$ , 5 is not present in the prime factorization, there is no natural number nor which  $8^n$  ends with digit 5.

So,  $8^n$  cannot end with digit 5.

### 11 E. Question

Examine whether  $(4)^n$  can end with the digit 0 for any  $n \in \mathbb{N}$ .

#### Answer

If  $(4)^n$  end with the digit 0, then the number should be divisible by 5.

As  $2 \times 5 = 10$

$\Rightarrow$  This means the prime factorization of  $4^n$  should contain prime factor 5.

This is not possible because  $(4)^n = (2^{2n})$ , so the only prime in the factorization of  $4^n$  is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of  $4^n$ .

$\therefore$ , 5 is not present in the prime factorization, there is no natural number nor which  $4^n$  ends with digit zero.

So,  $4^n$  cannot end with digit zero.

### 11 F. Question

Examine whether  $(7)^n$  can end with the digit 5 for any  $n \in \mathbb{N}$ .

### Answer

If  $(7)^n$  end with the digit 5, then the number should be divisible by 5.

$\Rightarrow$  This means the prime factorization of  $7^n$  should contain prime factor 5.

But  $(7)^n$  does not have the prime factor 5. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of  $7^n$ .

$\therefore$ , 5 is not present in the prime factorization, there is no natural number  $n$  for which  $7^n$  ends with digit 5.

So,  $7^n$  cannot end with digit 5.

### 12 A. Question

Explain why  $7 \times 11 \times 13 \times 17 + 17$  is a composite number.

### Answer

Composite number: A whole number that can be divided exactly by numbers other than 1 or itself.

$$\text{Given } 7 \times 11 \times 13 \times 17 + 17 \Rightarrow 17 (7 \times 11 \times 13 \times 17 + 1)$$

$$\Rightarrow 17 (7 \times 11 \times 13 \times 17 + 1)$$

$$\Rightarrow 17 (17017 + 1)$$

$$\Rightarrow 17 (17018)$$

$$\Rightarrow 17 (2 \times 8509)$$

$$\Rightarrow 17 \times 2 \times 8509$$

So, given number is the composite number because it is the product of more than one prime numbers.

### 12 B. Question

Explain why  $5 \times 7 \times 13 + 5$  is a composite number.

### Answer

Composite number: A whole number that can be divided exactly by numbers other than 1 or itself.

$$5 \times 7 \times 13 + 5$$

$$\Rightarrow 5 (1 \times 7 \times 13 + 1)$$

$$\Rightarrow 5 (91 + 1)$$

$$\Rightarrow 5 (92)$$

$$\Rightarrow 5 (2^2 \times 23)$$

$$\Rightarrow 5 \times 2 \times 2 \times 23$$

So, given number is the composite number because it is the product of more than one prime numbers.

### 12 C. Question

Show that  $5 \times 7 \times 11 \times 13 + 55$  is a composite number.

#### Answer

Composite number: A whole number that can be divided exactly by numbers other than 1 or itself.

$$5 \times 7 \times 11 \times 13 + 55$$

$$\Rightarrow 5 (1 \times 7 \times 11 \times 13 + 11)$$

$$\Rightarrow 5 \times 11 (91 + 1)$$

$$\Rightarrow 5 \times 11 (92)$$

$$\Rightarrow 5 \times 11 (2^2 \times 23)$$

$$\Rightarrow 5 \times 11 \times 2 \times 2 \times 23$$

So, given number is the composite number because it is the product of more than one prime numbers.

### 13. Question

Three measuring rods 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured exact number of times using anyone of the above rods.

#### Answer

Lengths of three measuring rods = 64cm, 80cm and 96cm

Least Length of cloth that can be measured = LCM (64, 80, 96)

|   |    |
|---|----|
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8  |
| 2 | 4  |
| 2 | 2  |
|   | 1  |

|   |    |
|---|----|
| 2 | 80 |
| 2 | 40 |
| 2 | 20 |
| 3 | 10 |
| 5 | 5  |
|   | 1  |

|   |    |
|---|----|
| 2 | 96 |
| 2 | 48 |
| 2 | 24 |
| 2 | 12 |
| 2 | 6  |
| 3 | 3  |
|   | 1  |

$$64 = 2^6$$

$$80 = 2^3 \times 3 \times 5$$

$$96 = 2^5 \times 3$$

So,  $2^6 \times 3 \times 5$  are the greatest powers of the prime factors 2, 3 and 5

$$\text{LCM} (64, 80, 96) = 2^6 \times 3 \times 5 = 960$$

Least Length of cloth that can be measured is 960 cm

#### 14. Question

Three containers contain 27 litres, 36 litres and 72 litres of milk. What biggest measure can measure exactly the milk in the three containers?

#### Answer

Milk in three containers = 27L, 36L, 72L

Biggest measure which can exactly measure the milk = HCF (27, 36, 72)

|   |    |
|---|----|
| 2 | 36 |
| 2 | 18 |
| 3 | 9  |
| 3 | 3  |
|   | 1  |

|   |    |
|---|----|
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9  |
| 3 | 3  |
|   | 1  |

|   |    |
|---|----|
| 3 | 27 |
| 3 | 9  |
| 3 | 3  |
|   | 1  |

$$27 = 3^3$$

$$36 = 2^2 \times 3^2$$

$$72 = 2^3 \times 3^2$$

Here,  $3^2$  is the smallest power of the common factor of the prime 3

$$\text{HCF} (27, 36, 72) = 9$$

So, biggest measure which can exactly measure the milk = 9L

#### 15. Question

Three different containers contain different quantities of mixtures of milk and water, whose measurements are 403 kg, 434 kg and 465 kg, what biggest measure can measure all the different quantities exactly.

**Answer**

Mixtures of milk and water in three containers = 403kg, 434kg, 465kg

Biggest measure which can exactly measure different quantities = HCF (403, 434, 465)

|    |     |    |     |    |     |
|----|-----|----|-----|----|-----|
| 13 | 403 | 2  | 434 | 3  | 465 |
| 31 | 31  | 7  | 217 | 5  | 155 |
|    | 1   | 31 | 31  | 31 | 31  |
|    |     | 1  | 1   | 1  | 1   |

$$403 = 13 \times \textcircled{31}$$

$$434 = 2 \times 7 \times \textcircled{31}$$

$$465 = 3 \times 5 \times \textcircled{31}$$

Here,  $31^1$  is the smallest power of the common factor.

$$\text{HCF (403, 434, 465)} = 31$$

So, biggest measure which can exactly measure the milk = 31L

**Exercise 1.3**

**1. Question**

Prove that  $\sqrt{2}$  is irrational.

**Answer**

Let us assume that  $\sqrt{2}$  is rational. So, we can find integers p and q ( $\neq 0$ ) such that  $\sqrt{2} = \frac{p}{q}$ .

Suppose p and q have a common factor other than 1.

Then, we divide by the common factor to get  $\sqrt{2} = \frac{a}{b}$ , where a and b are coprime.

$$\text{So, } b\sqrt{2} = a.$$

Squaring on both sides, we get

$$2b^2 = a^2$$

Therefore, 2 divides  $a^2$ .

Now, by Theorem which states that **Let  $p$  be a prime number. If  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer,**

$\Rightarrow$  2 divides  $a^2$ .

So, we can write  $a = 2c$  for some integer  $c$

Substituting for  $a$ , we get  $2b^2 = 4c^2$ , i.e.  $b^2 = 2c^2$ .

This means that 2 divides  $b^2$ , and so 2 divides  $b$  (again using the above Theorem with  $p = 2$ ). Therefore,  $a$  and  $b$  have at least 2 as a common factor.

But this contradicts the fact that  $a$  and  $b$  have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that 2 is rational.

**So, we conclude that  $\sqrt{2}$  is irrational.**

## 2. Question

Prove that  $\sqrt{3}$  is irrational.

### Answer

Let us assume that  $\sqrt{3}$  is rational.

Hence,  $\sqrt{3}$  can be written in the form  $\frac{a}{b}$

where  $a$  and  $b$  ( $\neq 0$ ) are co-prime (no common factor other than 1).

Hence,  $\sqrt{3} = \frac{a}{b}$

So,  $b\sqrt{3} = a$ .

Squaring on both sides, we get

$$3b^2 = a^2$$

$$\frac{a^2}{3} = b^2$$

Hence, 3 divides  $a^2$ .

By theorem: **Let  $p$  is a prime number and  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer,**

$\Rightarrow$  3 divides  $a$  also ...(1)

Hence, we can say  $a = 3c$  for some integer  $c$

Now, we know that  $3b^2 = a^2$

Putting  $a = 3c$

$$3b^2 = (3c)^2$$

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Hence, 3 divides  $b^2$

By theorem: **Let  $p$  is a prime number and  $p$  divides  $a^2$ , then  $p$  divides  $a$ , where  $a$  is a positive integer,**

So, 3 divides  $b$  also ...(2)

By (1) and (2)

3 divides both  $a$  and  $b$

Hence, 3 is a factor of  $a$  and  $b$

So,  $a$  and  $b$  have a factor 3

Therefore,  $a$  and  $b$  are not co-prime.

Hence, our assumption is wrong

Therefore, by contradiction  **$\sqrt{3}$  is irrational.**

### 3. Question

Prove that  $\frac{1}{\sqrt{5}}$  is irrational.

#### Answer

Let us assume that  $\frac{1}{\sqrt{5}}$  be a rational number.

Then, it will be of the form  $\frac{a}{b}$  where  $a$  and  $b$  are co-prime and  $b \neq 0$ .

$$\text{Now, } \frac{a}{b} = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{a}{b} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$\Rightarrow \frac{5a}{b} = \sqrt{5}$$

Since,  $5a$  is an integer and  $b$  is also an integer

So,  $\frac{5a}{b}$  is a rational number

$\Rightarrow \sqrt{5}$  is a rational number

But this contradicts to the fact that  $\sqrt{5}$  is an irrational number.

Therefore, our assumption is wrong.

Hence,  $\frac{1}{\sqrt{5}}$  is an irrational number.

#### 4 A. Question

Prove that following numbers are not rational :

$$(6)^{1/3}$$

#### Answer

Suppose  $6^{1/3}$  is rational.

Then,  $6^{1/3} = \frac{n}{m}$  for some integers  $n$  and  $m$  which are co-prime.

$$\text{So, } 6 = \frac{n^3}{m^3}$$

$$\Rightarrow 6m^3 = n^3$$

So,  $n^3$  must be divisible by 6

$\Rightarrow n$  must be divisible by 6.

Let  $n = 6p$  for some integer  $p$

This gives

$$6 = \frac{(6p)^3}{m^3}$$

$$1 = \frac{6^2 p^3}{m^3}$$

$\Rightarrow m^3$  is divisible by 6

Hence,  $m$  must be divisible by 6.

But  $n$  and  $m$  were co-prime.

So, we have a contradiction.

Hence,  $(6)^{1/3}$  is irrational

#### 4 B. Question

Prove that following numbers are not rational :

$$3\sqrt{3}$$

#### Answer

Let us assume that  $3\sqrt{3}$  be a rational number.

Then, it will be of the form  $\frac{a}{b}$  where a and b are co-prime and  $b \neq 0$ .

$$\text{Now, } \frac{a}{b} = 3\sqrt{3}$$

$$\Rightarrow \frac{a}{3b} = \sqrt{3}$$

Since, a is an integer and 3b is also an integer ( $3b \neq 0$ )

So,  $\frac{a}{3b}$  is a rational number

$\Rightarrow \sqrt{3}$  is a rational number

But this contradicts to the fact that  $\sqrt{3}$  is an irrational number.

Therefore, our assumption is wrong.

Hence,  $3\sqrt{3}$  is an irrational number.

#### 4 C. Question

Prove that following numbers are not rational :

$$5\sqrt{3}$$

#### Answer

Let us assume that  $5\sqrt{3}$  be a rational number.

Then, it will be of the form  $\frac{a}{b}$  where a and b are co-prime and  $b \neq 0$ .

$$\text{Now, } \frac{a}{b} = 5\sqrt{3}$$

$$\Rightarrow \frac{a}{5b} = \sqrt{3}$$

Since, a is an integer and 5b is also an integer ( $5b \neq 0$ )

So,  $\frac{a}{5b}$  is a rational number

$\Rightarrow \sqrt{3}$  is a rational number

But this contradicts to the fact that  $\sqrt{3}$  is an irrational number.

Therefore, our assumption is wrong.

Hence,  $5\sqrt{3}$  is an irrational number.

### 5 A. Question

Prove that following numbers are irrational :

$$6 + \sqrt{2}$$

#### Answer

Let us assume  $6 + \sqrt{2}$  is rational

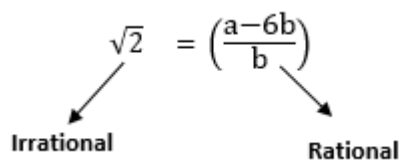
$\Rightarrow 6 + \sqrt{2}$  can be written in the form  $\frac{a}{b}$  where a and b are co-prime.

$$\text{Hence, } 6 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

$$\sqrt{2} = \frac{a-6b}{b}$$

$$\sqrt{2} = \left( \frac{a-6b}{b} \right)$$



Since, rational  $\neq$  irrational

This is a contradiction.

$\therefore$ , Our assumption is incorrect.

Hence,  $6 + \sqrt{2}$  is irrational.

### 5 B. Question

Prove that following numbers are irrational :

$$5 - \sqrt{3}$$

### Answer

Let us assume  $5 - \sqrt{3}$  is rational

$\Rightarrow 5 - \sqrt{3}$  can be written in the form  $\frac{a}{b}$  where a and b are co-prime.

$$\text{Hence, } 5 - \sqrt{3} = \frac{a}{b}$$

$$-\sqrt{3} = \frac{a}{b} - 5$$

$$-\sqrt{3} = \frac{a-5b}{b}$$

$$\sqrt{3} = -\left(\frac{a-5b}{b}\right)$$

$$\begin{array}{ccc} & \sqrt{3} = \left(\frac{5b-a}{b}\right) & \\ \swarrow & & \searrow \\ \text{Irrational} & & \text{Rational} \end{array}$$

Since, rational  $\neq$  irrational

This is a contradiction.

$\therefore$ , Our assumption is incorrect.

Hence,  $5 - \sqrt{3}$  is irrational.

### 5 C. Question

Prove that following numbers are irrational :

$$2 + \sqrt{2}$$

### Answer

Let us assume  $2 + \sqrt{2}$  is rational

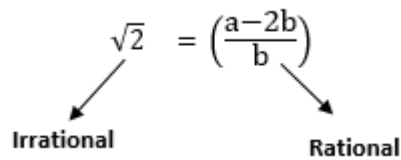
$\Rightarrow 2 + \sqrt{2}$  can be written in the form  $\frac{a}{b}$  where a and b are co-prime.

$$\text{Hence, } 2 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 2$$

$$\sqrt{2} = \frac{a-2b}{b}$$

$$\sqrt{2} = \left(\frac{a-2b}{b}\right)$$



Since, rational  $\neq$  irrational

This is a contradiction.

$\therefore$ , Our assumption is incorrect.

Hence,  $2 + \sqrt{2}$  is irrational.

### 5 D. Question

Prove that following numbers are irrational :

$$3 + \sqrt{5}$$

### Answer

Let us assume  $3 + \sqrt{5}$  is rational

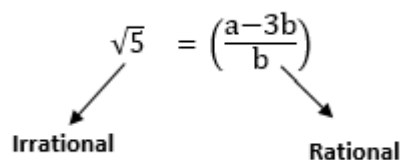
$\Rightarrow 3 + \sqrt{5}$  can be written in the form  $\frac{a}{b}$  where a and b are co-prime.

$$\text{Hence, } 3 + \sqrt{5} = \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{a-3b}{b}$$

$$\sqrt{5} = \left(\frac{a-3b}{b}\right)$$



Since, rational  $\neq$  irrational

This is a contradiction.

$\therefore$ , Our assumption is incorrect.

Hence,  $3 + \sqrt{5}$  is irrational.

### 5 E. Question

Prove that following numbers are irrational :

$$\sqrt{3} - \sqrt{2}$$

**Answer**

Let us assume  $\sqrt{3} - \sqrt{2}$  is rational

$$\text{Let, } \sqrt{3} - \sqrt{2} = \frac{a}{b}$$

Squaring both sides, we get

$$(\sqrt{3} - \sqrt{2})^2 = \frac{a^2}{b^2}$$

$$5 - 2\sqrt{6} = \frac{a^2}{b^2}$$

$$2\sqrt{6} = \frac{a^2}{b^2} - 5$$

$$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$$

$$\sqrt{6} = \left( \frac{a^2 - 5b^2}{2b^2} \right)$$

$$\begin{array}{ccc} \swarrow \sqrt{6} & = & \left( \frac{a^2 - 5b^2}{2b^2} \right) \\ \text{Irrational} & & \searrow \text{Rational} \end{array}$$

Since, rational  $\neq$  irrational

This is a contradiction.

$\therefore$ , Our assumption is incorrect.

Hence,  $\sqrt{3} - \sqrt{2}$  is irrational.

### 5 F. Question

Prove that following numbers are irrational :

$$\sqrt{7} - \sqrt{5}$$

**Answer**

Let us assume  $\sqrt{7} - \sqrt{5}$  is rational

$$\text{Let, } \sqrt{7} - \sqrt{5} = \frac{a}{b}$$

Squaring both sides, we get

$$(\sqrt{7} - \sqrt{5})^2 = \frac{a^2}{b^2}$$

$$12 - 2\sqrt{35} = \frac{a^2}{b^2}$$

$$2\sqrt{35} = \frac{a^2}{b^2} - 12$$

$$2\sqrt{35} = \left( \frac{a^2 - 12b^2}{b^2} \right)$$

$$\sqrt{35} = \left( \frac{a^2 - 12b^2}{2b^2} \right)$$

$\sqrt{35} = \left( \frac{a^2 - 12b^2}{2b^2} \right)$

Irrational                      Rational

Since, rational  $\neq$  irrational

This is a contradiction.

$\therefore$ , Our assumption is incorrect.

Hence,  $\sqrt{7} - \sqrt{5}$  is irrational.

## Exercise 1.4

### 1 A. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{17}{8}$$

**Answer**

Given rational number is  $\frac{17}{8}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$17 = 17 \times 1$$

$$8 = 2 \times 2 \times 2$$

$\Rightarrow$  17 and 8 have no common factors

Therefore, 17 and 8 are co-prime.

Now, we have to check that  $q$  is in the form of  $2^n 5^m$

$$8 = 2^3$$

$$= 1 \times 2^3$$

$$= 5^0 \times 2^3$$

So, denominator is of the form  $2^n 5^m$  where  $n = 3$  and  $m = 0$

Thus,  $\frac{17}{8}$  is a **terminating** decimal.

### 1 B. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{3}{8}$$

**Answer**

Given rational number is  $\frac{3}{8}$

$\frac{p}{q}$  is terminating if

a)  $p$  and  $q$  are co-prime &

b)  $q$  is of the form of  $2^n 5^m$  where  $n$  and  $m$  are non-negative integers.

Firstly, we check co-prime

$$3 = 3 \times 1$$

$$8 = 2 \times 2 \times 2$$

$\Rightarrow$  3 and 8 have no common factors

Therefore, 3 and 8 are co-prime.

Now, we have to check that  $q$  is in the form of  $2^n 5^m$

$$8 = 2^3$$

$$= 1 \times 2^3$$

$$= 5^0 \times 2^3$$

So, denominator is of the form  $2^n 5^m$  where  $n = 3$  and  $m = 0$

Thus,  $\frac{3}{8}$  is a **terminating** decimal.

### 1 C. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{29}{343}$$

#### Answer

Given rational number is  $\frac{29}{343}$

$\frac{p}{q}$  is terminating if

a)  $p$  and  $q$  are co-prime &

b)  $q$  is of the form of  $2^n 5^m$  where  $n$  and  $m$  are non-negative integers.

Firstly, we check co-prime

$$29 = 29 \times 1$$

$$343 = 7 \times 7 \times 7$$

$\Rightarrow$  29 and 343 have no common factors

Therefore, 29 and 343 are co-prime.

Now, we have to check that  $q$  is in the form of  $2^n 5^m$

$$343 = 7^3$$

So, denominator is not of the form  $2^n 5^m$

Thus,  $\frac{29}{343}$  is a **non-terminating repeating** decimal.

### 1 D. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{13}{125}$$

#### Answer

Given rational number is  $\frac{13}{125}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$13 = 13 \times 1$$

$$125 = 5 \times 5 \times 5$$

$\Rightarrow$  13 and 125 have no common factors

Therefore, 13 and 125 are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$125 = 5^3$$

$$= 1 \times 2^3$$

$$= 2^0 \times 5^3$$

So, denominator is of the form  $2^n 5^m$  where  $n = 0$  and  $m = 3$

Thus,  $\frac{13}{125}$  is a **terminating** decimal.

### 1 E. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{27}{8}$$

### Answer

Given rational number is  $\frac{27}{8}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$27 = 3 \times 3 \times 3$$

$$8 = 2 \times 2 \times 2$$

$\Rightarrow$  27 and 8 have no common factors

Therefore, 27 and 8 are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$8 = 2^3$$

$$= 1 \times 2^3$$

$$= 5^0 \times 2^3$$

So, denominator is of the form  $2^n 5^m$  where  $n = 3$  and  $m = 0$

Thus,  $\frac{27}{8}$  is a **terminating** decimal.

### 1 F. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{7}{80}$$

### Answer

Given rational number is  $\frac{7}{80}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b)  $q$  is of the form of  $2^n 5^m$  where  $n$  and  $m$  are non-negative integers.

Firstly, we check co-prime

$$7 = 7 \times 1$$

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

$\Rightarrow$  7 and 80 have no common factors

Therefore, 7 and 80 are co-prime.

Now, we have to check that  $q$  is in the form of  $2^n 5^m$

$$80 = 2^4 \times 5$$

So, denominator is of the form  $2^n 5^m$  where  $n = 4$  and  $m = 1$

Thus,  $\frac{7}{80}$  is a **terminating** decimal.

### 1 G. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{64}{455}$$

**Answer**

Given rational number is  $\frac{64}{455}$

$\frac{p}{q}$  is terminating if

a)  $p$  and  $q$  are co-prime &

b)  $q$  is of the form of  $2^n 5^m$  where  $n$  and  $m$  are non-negative integers.

Firstly, we check co-prime

$$64 = 2^6$$

$$455 = 5 \times 7 \times 13$$

$\Rightarrow$  64 and 455 have no common factors

Therefore, 64 and 455 are co-prime.

Now, we have to check that  $q$  is in the form of  $2^n 5^m$

$$455 = 5 \times 7 \times 13$$

So, denominator is not of the form  $2^n 5^m$

Thus,  $\frac{64}{455}$  is a **non-terminating repeating** decimal.

### 1 H. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{6}{15}$$

**Answer**

Given rational number is  $\frac{6}{15}$

$$\frac{6}{15} = \frac{2}{5}$$

$\frac{p}{q}$  is terminating if

a)  $p$  and  $q$  are co-prime &

b)  $q$  is of the form of  $2^n 5^m$  where  $n$  and  $m$  are non-negative integers.

Firstly, we check co-prime

$\Rightarrow$  2 and 5 have no common factor

Therefore, 2 and 5 are co-prime.

Now, we have to check that  $q$  is in the form of  $2^n 5^m$

$$5 = 5^1 \times 1$$

$$= 5^1 \times 2^0$$

So, denominator is of the form  $2^n 5^m$  where  $n = 0$  and  $m = 1$

Thus,  $\frac{6}{15}$  is a **terminating** decimal.

### 1 I. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{35}{50}$$

**Answer**

Given rational number is  $\frac{35}{50}$

$$\frac{35}{50} = \frac{7}{10}$$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$7 = 1 \times 7$$

$$10 = 2 \times 5$$

$\Rightarrow$  7 and 10 have no common factor

Therefore, 7 and 10 are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$10 = 5^1 \times 2^1$$

So, denominator is of the form  $2^n 5^m$  where n = 1 and m = 1

Thus,  $\frac{35}{50}$  is a **terminating** decimal.

### 1 J. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{129}{2^2 5^7 7^5}$$

**Answer**

Given rational number is  $\frac{129}{2^2 \times 5^7 \times 7^5}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$129 = 3 \times 43$$

$$\text{Denominator} = 2^2 \times 5^7 \times 7^5$$

$\Rightarrow$  129 and  $2^2 \times 5^7 \times 7^5$  have no common factors

Therefore, 129 and  $2^2 \times 5^7 \times 7^5$  are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$\text{Denominator} = 2^2 \times 5^7 \times 7^5$$

So, denominator is not of the form  $2^n 5^m$

Thus,  $\frac{129}{2^2 \times 5^7 \times 7^5}$  is a **non-terminating repeating** decimal.

### 1 K. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{2^2 \times 7}{5^4}$$

**Answer**

Given rational number is  $\frac{2^2 \times 7}{5^4}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$28 = 7 \times 2^2$$

$$625 = 5^4$$

$\Rightarrow$  28 and 625 have no common factors

Therefore, 28 and 625 are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$625 = 5^4 \times 1$$

$$= 5^4 \times 2^0$$

So, denominator is of the form  $2^n 5^m$  where  $n = 0$  and  $m = 4$

Thus,  $\frac{2^2 \times 7}{5^4}$  is a **terminating** decimal.

### 1 L. Question

Without actually performing the long division, state whether the following rational numbers have terminating or non-terminating repeating (recurring) decimal expansion.

$$\frac{29}{243}$$

### Answer

Given rational number is  $\frac{29}{243}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly, we check co-prime

$$29 = 29 \times 1$$

$$243 = 3^5$$

$\Rightarrow$  29 and 243 have no common factors

Therefore, 29 and 243 are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$243 = 3^5$$

So, the denominator is not of the form  $2^n 5^m$

Thus,  $\frac{29}{243}$  is a **non-terminating repeating** decimal.

## 2 A. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{17}{8}$$

**Answer**

$$\text{We know, } \frac{17}{8} = \frac{17}{2^3 \times 5^0}$$

Multiplying and dividing by  $5^3$

$$= \frac{17 \times 5^3}{2^3 \times 5^0 \times 5^3}$$

$$= \frac{17 \times 125}{2^3 \times 1 \times 5^3}$$

$$= \frac{2125}{(2 \times 5)^3}$$

$$= \frac{2125}{(10)^3}$$

$$= \frac{2125}{1000}$$

$$= 2.125$$

## 2 B. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{3}{8}$$

**Answer**

$$\text{We know, } \frac{3}{8} = \frac{3}{2^3 \times 5^0}$$

Multiplying and dividing by  $5^3$

$$= \frac{3 \times 5^3}{2^3 \times 5^0 \times 5^3}$$

$$= \frac{3 \times 125}{2^3 \times 1 \times 5^3}$$

$$= \frac{375}{(2 \times 5)^3}$$

$$= \frac{375}{(10)^3}$$

$$= \frac{375}{1000}$$

$$= 0.375$$

## 2 C. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{29}{343}$$

### Answer

We know,  $\frac{29}{343} = \frac{29}{7^3}$

Given rational number is  $\frac{29}{343}$

$\frac{p}{q}$  is terminating if

a) p and q are co-prime &

b) q is of the form of  $2^n 5^m$  where n and m are non-negative integers.

Firstly we check co-prime

$$29 = 29 \times 1$$

$$343 = 7 \times 7 \times 7$$

$\Rightarrow$  29 and 343 have no common factors

Therefore, 29 and 343 are co-prime.

Now, we have to check that q is in the form of  $2^n 5^m$

$$343 = 7^3$$

So, the denominator is not of the form  $2^n 5^m$

Thus,  $\frac{29}{343}$  is a **non-terminating repeating** decimal.

## 2 D. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{13}{125}$$

**Answer**

We know,  $\frac{13}{125} = \frac{13}{2^0 \times 5^3}$

Multiplying and dividing by  $2^3$

$$= \frac{13 \times 2^3}{2^0 \times 5^3 \times 2^3}$$

$$= \frac{13 \times 8}{1 \times 2^3 \times 5^3}$$

$$= \frac{104}{(2 \times 5)^3}$$

$$= \frac{104}{(10)^3}$$

$$= \frac{104}{1000}$$

$$= \mathbf{0.104}$$

## 2 E. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{27}{8}$$

**Answer**

We know,  $\frac{27}{8} = \frac{3^3}{2^3 \times 5^0}$

Multiplying and dividing by  $5^3$

$$= \frac{27 \times 5^3}{2^0 \times 2^3 \times 5^3}$$

$$= \frac{27 \times 125}{1 \times 2^3 \times 5^3}$$

$$= \frac{3375}{(2 \times 5)^3}$$

$$= \frac{3375}{(10)^3}$$

$$= \frac{3375}{1000}$$

$$= 3.375$$

## 2 F. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{7}{80}$$

**Answer**

$$\text{We know, } \frac{7}{80} = \frac{7}{2^4 \times 5^1}$$

Multiplying and dividing by  $5^3$

$$= \frac{7 \times 5^3}{2^4 \times 5^1 \times 5^3}$$

$$= \frac{7 \times 125}{2^4 \times 5^4}$$

$$= \frac{875}{(2 \times 5)^4}$$

$$= \frac{875}{(10)^4}$$

$$= \frac{875}{10000}$$

$$= 0.0875$$

## 2 G. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{64}{455}$$

**Answer**

We know,  $\frac{64}{455} = \frac{2^6}{5 \times 7 \times 13}$

Since the denominator is not of the form  $2^n 5^m$

$\frac{64}{455}$  has a non-terminating repeating decimal expansion.

## 2 H. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{6}{15}$$

**Answer**

We know,  $\frac{6}{15} = \frac{2}{5} = \frac{2}{2^0 \times 5}$

Multiplying and dividing by  $2^1$

$$= \frac{2 \times 2}{2^0 \times 5^1 \times 2}$$

$$= \frac{4}{1 \times 10}$$

$$= \frac{4}{10}$$

$$= 0.4$$

## 2 I. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{35}{50}$$

**Answer**

We know,  $\frac{35}{50}$

$$= \frac{7}{10}$$

$$= \frac{7}{2 \times 5}$$

$$= \frac{7}{10}$$

$$= 0.7$$

## 2 J. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{129}{2^2 5^7 7^5}$$

**Answer**

Given rational number is  $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form  $2^n 5^m$

$\frac{129}{2^2 \times 5^7 \times 7^5}$  has a non-terminating repeating decimal expansion.

## 2 K. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{2^2 \times 7}{5^4}$$

**Answer**

We know,  $\frac{2^2 \times 7}{5^4} = \frac{2^2 \times 7}{2^0 \times 5^4}$

Multiplying and dividing by  $2^6$

$$= \frac{2^2 \times 7 \times 2^6}{2^0 \times 5^4 \times 2^6}$$

$$= \frac{2^2 \times 7 \times 2^6}{1 \times 5^4 \times 2^6}$$

$$= \frac{7 \times 2^6}{(2 \times 5)^4}$$

$$= \frac{448}{(10)^4}$$

$$= \frac{448}{10000}$$

$$= 0.0448$$

## 2 L. Question

Write down the decimal expansions of the following numbers which have terminating decimal expansions.

$$\frac{29}{243}$$

**Answer**

Given rational number is  $\frac{29}{243}$

$$\frac{29}{243} = \frac{29}{3^5}$$

Since the denominator is not of the form  $2^n 5^m$ .

$\frac{29}{243}$  has a non-terminating repeating decimal expansion.

### 3. Question

The following real numbers have decimal expansions as given below. In each case examine whether they are rational or not. If they are a rational number of the form  $p/q$ , what can be said about  $q$ ?

(i) 7.2345

(ii)  $5.\overline{234}$

(iii) 23.245789

(iv)  $7.\overline{3427}$

(v) 0.120120012000120000...

(vi) 23.142857

(vii) 2.313313313331...

(viii) 0.02002000220002...

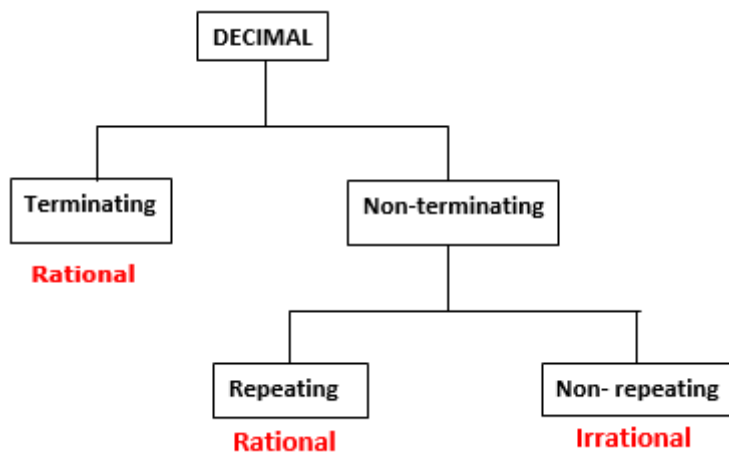
(ix) 3.300030000300003...

(x) 1.7320508...

(xi) 2.645713

(xii) 2.8284271...

**Answer**



(i) 7.2345

Here, 7.2345 has terminating decimal expansion.

So, it represents a rational number.

$$\text{i.e. } 7.2345 = \frac{7.2345}{10000} = \frac{p}{q}$$

Thus,  $q = 10^4$ , those factors are  $2^3 \times 5^3$

(ii)  $5.\overline{234}$

$5.\overline{234}$  is non-terminating but repeating.

So, it would be a rational number.

In a non-terminating repeating expansion of  $\frac{p}{q}$ ,

$q$  will have factors other than 2 or 5.

(iii) 23.245789

23.245789 is terminating decimal expansion

So, it would be a rational number.

$$\text{i.e. } 23.245789 = \frac{23.245789}{1000000} = \frac{p}{q}$$

Thus,  $q = 10^6$ , those factors are  $2^5 \times 5^5$

In a terminating expansion of  $\frac{p}{q}$ ,  $q$  is of the form  $2^n 5^m$

So, prime factors of  $q$  will be either 2 or 5 or both.

(iv)  $7.\overline{3427}$

$7.\overline{3427}$  is non-terminating but repeating.

So, it would be a rational number.

In a non-terminating repeating expansion of  $\frac{p}{q}$ ,

q will have factors other than 2 or 5.

(v) 0.120120012000120000...

0.120120012000120000... is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(vi) 23.142857

23.142857 is terminating expansion.

So, it would be a rational number.

$$\text{i.e. } 23.142857 = \frac{23.142857}{1000000} = \frac{p}{q}$$

Thus,  $q = 10^6$ , whose factors are  $2^5 \times 5^5$

In a terminating expansion of  $\frac{p}{q}$ , q is of the form  $2^n 5^m$

So, prime factors of q will be either 2 or 5 or both.

(vii) 2.313313313331...

2.313313313331... is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(viii) 0.02002000220002...

0.02002000220002... is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(ix) 3.300030000300003...

3.300030000300003... is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(x) 1.7320508...

1.7320508... is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.

(xi) 2.645713

2.645713 is terminating expansion

So, it would be a rational number.

$$\text{i.e. } 2.645713 = \frac{2.645713}{1000000} = \frac{p}{q}$$

Thus,  $q = 10^6$ , those factors are  $2^5 \times 5^5$

In a terminating expansion of  $\frac{p}{q}$ ,  $q$  is of the form  $2^n 5^m$

So, prime factors of  $q$  will be either 2 or 5 or both.

(xii) 2.8284271...

2.8284271... is non-terminating and non-repeating.

So, it is not a rational number as we see in the chart.