

## PART # 03

### CALCULUS

#### EXERCISE # 01

#### SECTION-1 : (ONE OPTION CORRECT TYPE)

651.  $\int \frac{1 - 7 \cos^2 x}{\sin^7 x \cos^2 x} dx = \frac{f(x)}{(\sin x)^7} + C$ , then  $f(x)$  is equal to  
(A)  $\sin x$  (B)  $\cos x$  (C)  $\tan x$  (D)  $\cot x$
652. Let  $f(x) = (x + 1)(x + 2)(x + 3) \dots (x + 100)$  and  $g(x) = f(x) \cdot f''(x) - (f'(x))^2$ , then  $g(x) = 0$ , has  
(A) no solution (B) exactly one solution  
(C) exactly two solutions (D) minimum three solutions
653. Let  $f(x) = \min(\ln(\tan x), \ln(\cot x))$ , which of the following statement are incorrect  
(A)  $f(x)$  is continuous for  $x \in \left(0, \frac{\pi}{2}\right)$   
(B) Lagrange's mean value theorem is applicable on  $f(x)$  for  $x \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$   
(C) Rolle's theorem is not applicable on  $f(x)$  for  $x \in \left[\frac{\pi}{4}, \frac{3\pi}{8}\right]$   
(D) Rolle's theorem is applicable on  $f(x)$  for  $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$
654. Let 'n' be the number of elements in the Domain set of the function  $f(x) = \left| \ln \sqrt{x^2 + 4x} C_{2x^2+3} \right|$  and 'Y' be the global maximum value of  $f(x)$ , then  $[n + [Y]]$  is (where  $[.]$  = Greatest Integer function)  
(A) 4 (B) 5 (C) 6 (D) 7
655. The value of the integral  $\int_0^{\infty} e^{-2\theta} (\sin 2\theta + \cos 2\theta) d\theta$  is  
(A)  $\frac{1}{2}$  (B)  $\frac{2}{3}$  (C) does not exist (D) none of these
656. The value of  $\int_0^{\pi/3} [\sqrt{3} \tan x] dx$  (where  $[.]$  denotes the greatest integer function) is  
(A)  $\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)$  (B)  $\frac{5\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$  (C)  $\frac{\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$  (D) none of these
657. The value of  $\lim_{x \rightarrow \infty} \left( \frac{a_1^{1/x} + a_2^{1/x} + \dots + a_n^{1/x}}{n} \right)^{nx}$ ;  $a_i > 0, i = 1, 2, \dots, n$  is  
(A)  $a_1 + a_2 + \dots + a_n$  (B)  $e^{a_1 + a_2 + \dots + a_n}$  (C)  $\frac{a_1 + a_2 + \dots + a_n}{n}$  (D)  $a_1 a_2 a_3 \dots a_n$

658. The value of integral  $\int \cot^{-1} \left( \frac{\sqrt{4x-x^2}}{x-2} \right) dx$  is equal to
- (A)  $\frac{x-2}{2} \sin^{-1} \frac{x-2}{2} + \sqrt{4x-x^2} + c$  (B)  $(x-2) \sin^{-1} \left( \frac{x-2}{2} \right) + \sqrt{4x-x^2} + c$   
 (C)  $\frac{x-2}{2} \sin^{-1} \left( \frac{x-2}{2} \right) - \sqrt{\frac{4x-x^2}{2}} + c$  (D) none of these
659. Let  $f: [-2, 2] \rightarrow \mathbb{R}$ , where  $f(x) = x^3 + \sin x + \left[ \frac{x^2+1}{a} \right]$  be an odd function, then
- (A)  $a < 3$  (B)  $a > 5$  (C)  $a < 1$  (D)  $a < -2$
660.  $\lim_{n \rightarrow \infty} \left( \frac{3n+8}{3n+5} \right)^{5n+9}$  is equal to
- (A)  $3e^5$  (B)  $e^5$  (C)  $e^3$  (D) None of these
661. The differential equation of the orthogonal trajectories of the system of curves  $x + \tan^{-1}(y/x) = c$ , is
- (A)  $x dx + y dy = x^2 + y^2$  (B)  $x dy + y dx = x^2 + y^2$   
 (C)  $x dx + y dy = (x^2 + y^2) dy$  (D)  $x dx + y dy = (x^2 - y^2) dy$
662. The area bounded by the curves  $f(x) = x^2 - 2x + 2$  and its inverse i.e.  $f^{-1}$  is given by
- (A)  $\frac{1}{3}$  (B)  $\frac{2}{3}$  (C)  $\frac{5}{3}$  (D)  $\frac{4}{3}$
663. The value of the series  $\frac{C_0}{5} - \frac{C_1}{6} + \frac{C_2}{7} - \frac{C_3}{8} + \dots + (-1)^n \frac{C_n}{n+4}$
- (A)  $\int_0^1 x^2(1-x)^n dx$  (B)  $\int_0^1 x^3(1-x)^n dx$  (C)  $\int_0^1 x^4(1-x)^n dx$  (D) none of these
664. If  $f(n) = \frac{1}{n} [(n+1)(n+2) \dots (n+n)]^{1/n}$  then  $\lim_{n \rightarrow \infty} f(n)$  equals
- (A)  $e$  (B)  $1/e$  (C)  $2/e$  (D)  $4/e$
665. If  $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$  where  $y \neq 0$  for all  $x, y \in \mathbb{R}$  and  $f'(1) = 2$ . then the function  $f(x)$  is symmetric about
- (A)  $x$ -axis (B)  $y$ -axis (C) origin (D)  $y = x$
666. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be such that  $f(1) = 3$ ,  $f'(1) = 0$  and  $f''(1) = 6$ , then  $\lim_{x \rightarrow 0} \left[ \frac{f(1+x)}{f(1)} \right]^{1/x^2}$  equal to
- (A)  $e$  (B)  $e^{1/2}$  (C)  $e^2$  (D)  $e^3$
667. Let  $f(x)$  and  $g(x)$  be differentiable for  $0 \leq x \leq 1$  such that  $f(0) = 2$ ,  $g(0) = 0$ ,  $f(1) = 6$ . Let there exists a real numbers  $x$  in  $[0, 1]$  such that  $f'(c) = 2g'(c)$  then the value of  $g(1)$  must be
- (A) 1 (B) 2 (C) -1 (D) none of these
668. The area of the greatest circle inscribed in  $2|x| + 2|y| = 4$  is given by
- (A)  $\frac{\pi}{2}$  (B)  $\frac{\pi}{4}$  (C)  $\pi^2$  (D)  $\pi$

669.  $\int_0^{\pi/2} \sin x \log(\sin x) dx$  is equal to  
 (A)  $\log_e(e/2)$  (B)  $\log 2 - e$  (C)  $\log_e((2/e))$  (D)  $\log_e - 2$
670. If  $I_n = \int_0^{\pi/4} \tan^n x dx$ , then  $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}, \frac{1}{I_5 + I_7}$  are in  
 (A) A.P. (B) G.P. (C) H.P. (D) None of these
671.  $\int \frac{\ln(1+x^{2/3}+2x^{1/3})}{x+x^{2/3}} dx$  is equal to  
 (A)  $3 \ln(1+x^{1/3})^2 + c$  (B)  $\ln(1+x^{1/3}) + c$   
 (C)  $\ln(x^{1/3}-1) + c$  (D) none of these
672.  $\lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{x} \right]$  is equal to [.] represents G.I. function  
 (A) 1 (B) 0  
 (C) does not exist (D) none of these
673. The differential equation of all ellipses centred at origin is :  
 (A)  $y_2 + xy_1^2 - yy_1 = 0$  (B)  $xyy_2 + xy_1^2 - yy_1 = 0$   
 (C)  $yy_2 + xy_1^2 - xy_1 = 0$  (D) none of these
674. The solution of the differential equation  $2x \frac{dy}{dx} - y = 3$  represent  
 (A) straight lines (B) circles (C) parabolas (D) ellipses
675. If  $y = e^{4x} + 2e^{-x}$  satisfies the relation  $\frac{d^3y}{dx^3} + A \frac{dy}{dx} + By = 0$  then value of A and B respectively are:  
 (A) -13, 14 (B) -13, -12 (C) -13, 12 (D) 12, -13
676. A particle moves in a straight line with velocity given by  $\frac{dx}{dt} = x + 1$  (x being the distance described). The time taken by the particle to describe 99 metres is :  
 (A)  $\log_{10} e$  (B)  $2 \log_e 10$  (C)  $2 \log_{10} e$  (D)  $\frac{1}{2} \log_{10} e$
677. The acute angle between the curve  $y = |x^2 - 1|$  and  $y = |x^2 - 3|$  at their point of intersection is  
 (A)  $\frac{\pi}{4}$  (B)  $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$  (C)  $\tan^{-1}(4\sqrt{7})$  (D) None of these
678. The range of the function  $y = \sqrt{2\{x\} - \{x\}^2 - \frac{3}{4}}$  is (where  $\{.\}$  denotes fractional part)  
 (A)  $\left[-\frac{1}{4}, \frac{1}{4}\right]$  (B)  $\left[0, \frac{1}{2}\right]$  (C)  $\left[0, \frac{1}{4}\right]$  (D)  $\left[\frac{1}{4}, \frac{1}{2}\right]$
679. If  $f(x+y) = f(x) - f(y) + 2xy - 1 \forall x, y \in \mathbb{R}$ . Also if  $f(x)$  is differentiable and  $f'(0) = b$  also  $f(x) > 0 \forall x$ , then the set of values of b  
 (A)  $\phi$  (B)  $\{1\}$  (C)  $\{1, 2\}$  (D) none of these

680.  $\lim_{k \rightarrow \infty} \int_0^{k[x]} (kx - [kx])^k dx$ ;  $k \in \mathbb{N}$  is equal to (where  $[.]$  denotes the greatest integer function)
- (A)  $[kx]$  (B)  $[x]$  (C)  $\left[\frac{x}{k}\right]$  (D)  $[x^k]$
681. The equation of curve passing through (1, 1) in which the subtangent is always bisected at the origin is
- (A)  $y^2 = x$  (B)  $2x^2 - y^2 = 1$  (C)  $x^2 + y^2 = 2$  (D)  $x + y = 2$
682. If  $f'(3) = 5$  then  $\lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3-h^2)}{2h^2}$  is :
- (A) 5 (B)  $\frac{1}{5}$   
(C) 2 (D) None of these
683. If  $f$  is twice differentiable function then  $\lim_{h \rightarrow 0} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2}$  is :
- (A)  $2f'(a)$  (B)  $f''(a)$   
(C)  $f'(a)$  (D)  $f'(a) + f''(a)$
684. If  $f(x) = \sin x$ ,  $g(x) = x^2$ ,  $h(x) = \log x$  and  $F(x) = (\text{hogof})(x)$  then  $F''(x)$  is :
- (A)  $2\operatorname{cosec}^3 x$  (B)  $2\cot x^2 - 4x^2 \operatorname{cosec}^2 x^2$   
(C)  $2x \cot x^2$  (D)  $-2\operatorname{cosec}^2 x$
685. If  $x = \sec \theta - \cos \theta$ ,  $y = \sec^n \theta - \cos^n \theta$  then  $\left(\frac{dy}{dx}\right)^2$  is equal to :
- (A)  $\frac{n^2(y^2 + 4)}{x^2 + 4}$  (B)  $\frac{n^2(y^2 - 4)}{x^2}$  (C)  $n \frac{y^2 - 4}{x^2 - 4}$  (D)  $\left(\frac{ny}{x}\right)^2 - 4$
686. If  $y = f\left(\frac{2x-1}{x^2+1}\right)$  and  $f'(x) = \sin x^2$  then  $\frac{dy}{dx}$  is:
- (A)  $\cos x^2 \cdot f'(x)$  (B)  $-\cos x^2 \cdot f'(x)$   
(C)  $\frac{2(1+x-x^2)}{(x^2+1)^2} \sin\left(\frac{2x-1}{x^2+1}\right)^2$  (D) None of these
687. If  $y^2 = p(x)$ , a polynomial of degree 3 then  $2 \frac{d}{dx} \left( y^3 \frac{d^2 y}{dx^2} \right)$  is equal to:
- (A)  $p'''(x) + p'(x)$  (B)  $p'''(x) + p''(x)$   
(C)  $p(x) p'''(x)$  (D) a constant
688.  $I = \int \frac{(x + x^{\frac{2}{3}} + x^{\frac{1}{6}})}{x(1 + x^{\frac{1}{3}})} dx$  is equal to
- (A)  $\frac{3}{2} x^{\frac{2}{3}} + 6 \tan^{-1} \left( x^{\frac{1}{6}} \right) + c$  (B)  $\frac{3}{2} x^{\frac{2}{3}} - 6 \tan^{-1} \left( x^{\frac{1}{6}} \right) + c$   
(C)  $= \frac{3}{2} x^{\frac{2}{3}} + \tan^{-1} \left( x^{\frac{1}{6}} \right) + c$  (D) none of these

689.  $\int \frac{(\sqrt{x^2+1})\{\ln(x^2+1) - 2\ln x\}}{x^4} dx$  is equal to :
- (A)  $\frac{(x^2+1)\sqrt{x^2+1}}{x^3} \left[ 2 - 3\ln\left(\frac{x^2+1}{x^2}\right) \right] + c$  (B)  $\frac{1}{9} \frac{(x^2+1)\sqrt{x^2+1}}{x^3} \left[ 2 + 3\ln\left(\frac{x^2+1}{x^2}\right) \right] + c$
- (C)  $\frac{(x^2+1)\sqrt{x^2+1}}{x^3} \left[ 2 + 3\ln\left(\frac{x^2+1}{x^2}\right) \right] + c$  (D)  $\frac{1}{9} \frac{(x^2+1)\sqrt{x^2+1}}{x^3} \left[ 2 - 3\ln\left(\frac{x^2+1}{x^2}\right) \right] + c$
690. If the positive number  $x$  and  $y$  are connected by the relation  $x^2 - xy + y^2 = 12$ , then maximum value of  $2x + 3y$ , is
- (A)  $\frac{20}{\sqrt{19}}$  (B)  $\frac{74}{\sqrt{19}}$  (C)  $\frac{67}{\sqrt{19}}$  (D)  $\frac{76}{\sqrt{19}}$
691.  $f(x) = \text{Minimum}\{\tan x, \cot x\} \forall x \in \left(0, \frac{\pi}{2}\right)$ . Then  $\int_0^{\pi/3} f(x) dx$  is equal to :
- (A)  $\ln\left(\frac{\sqrt{3}}{2}\right)$  (B)  $\ln\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$  (C)  $\ln(\sqrt{2})$  (D)  $\ln(\sqrt{3})$
692. If  $f(x)$  is a function satisfying  $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$  for all non-zero  $x$ , then  $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x) dx$  equals
- (A)  $\sin\theta + \operatorname{cosec}\theta$  (B)  $\sin^2\theta$  (C)  $\operatorname{cosec}^2\theta$  (D) none of these
693. If  $\int_0^{100} f(x) dx = a$ , then  $\sum_{r=1}^{100} \left( \int_0^1 f(r-1+x) dx \right) =$
- (A)  $100a$  (B)  $a$  (C)  $0$  (D)  $10a$
694. The area bounded by the curves  $y = x(1 - \ln x)$ ;  $x = e^{-1}$  and a positive  $X$ -axis between  $x = e^{-1}$  and  $x = e$  is :
- (A)  $\left(\frac{e^2 - 4e^{-2}}{5}\right)$  (B)  $\left(\frac{e^2 - 5e^{-2}}{4}\right)$  (C)  $\left(\frac{4e^2 - e^{-2}}{5}\right)$  (D)  $\left(\frac{5e^2 - e^{-2}}{4}\right)$
695. The area bounded by  $y = x^2$ ,  $y = [x + 1]$ ,  $x \leq 1$  and the  $y$ -axis is :
- (A)  $1/3$  (B)  $2/3$  (C)  $1$  (D)  $7/3$
696. The area bounded by the curve  $y = f(x)$ ,  $x$ -axis and the ordinates  $x = 1$  and  $x = b$  is  $(b-1)\sin(3b+4)$ ,  $\forall b \in \mathbb{R}$ , then  $f(x) =$
- (A)  $(x-1)\cos(3x+4)$  (B)  $\sin(3x+4)$
- (C)  $\sin(3x+4) + 3(x-1)\cos(3x+4)$  (D) None of these
697. The areas of the figure into which curve  $y^2 = 6x$  divides the circle  $x^2 + y^2 = 16$  are in the ratio
- (A)  $\frac{2}{3}$  (B)  $\frac{4\pi - \sqrt{3}}{8\pi + \sqrt{3}}$  (C)  $\frac{4\pi + \sqrt{3}}{8\pi - \sqrt{3}}$  (D) none of these
698. If  $\sqrt{(x^2 + y^2)} = ae^{\tan^{-1}(y/x)}$ ,  $a > 0$ . Then  $y''(0)$ , equals
- (A)  $\frac{a}{2}e^{\pi/2}$  (B)  $ae^{\pi/2}$
- (C)  $-\frac{2}{a}e^{-\pi/2}$  (D)  $\frac{a}{2}e^{-\pi/2}$

699. The function  $f(\theta) = \frac{d}{d\theta} \int_0^\theta \frac{dx}{1 - \cos \theta \cos x}$  satisfies the differential equation

- (A)  $\frac{df}{d\theta} + 2f(\theta) \cot \theta = 0$  (B)  $\frac{df}{d\theta} - 2f(\theta) \cot \theta = 0$   
 (C)  $\frac{df}{d\theta} + 2f(\theta) = 0$  (D)  $\frac{df}{d\theta} - 2f(\theta) = 0$

700. The solution of the differential equation

$$(x^2 \sin^3 y - y^2 \cos x) dx + (x^3 \cos y \sin^2 y - 2y \sin x) dy = 0 \text{ is :}$$

- (A)  $x^3 \sin^3 y = 3y^2 \sin x + C$  (B)  $x^3 \sin^3 y + 3y^2 \sin x = C$   
 (C)  $x^2 \sin^3 y + y^3 \sin x = C$  (D)  $2x^2 \sin y + y^2 \sin x = C$

## SECTION-2 (MORE THAN ONE OPTION CORRECT TYPE)

701. A curve that passes through (2, 4) and having subnormal of constant length of 8 units can be;

- (A)  $y^2 = 16x - 8$  (B)  $y^2 = -16x + 24$   
 (C)  $x^2 = 16y - 60$  (D)  $x^2 = -16y + 68$

702. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ , such that  $f''(x) - 2f'(x) + f(x) = 2e^x$  and  $f'(x) > 0 \quad \forall x \in \mathbb{R}$ , then which of the following can be correct

- (A)  $|f(x)| = -f(x), \forall x \in \mathbb{R}$  (B)  $|f(x)| = f(x), \forall x \in \mathbb{R}$   
 (C)  $f(3) = -5$  (D)  $f(3) = 7$

703. Let  $|f(x)| \leq \sin^2 x, \forall x \in \mathbb{R}$ , then

- (A)  $f(x)$  is continuous at  $x = 0$   
 (B)  $f(x)$  is differentiable at  $x = 0$   
 (C)  $f(x)$  is continuous but not differentiable at  $x = 0$   
 (D)  $f(0) = 0$

704.  $f(x) = x^3 + x^2 f'(1) + x f''(2) = f'''(3)$

$\forall x \in \mathbb{R}$ , then

- (A)  $f(0) + f(2) = f(1)$  (B)  $f(0) + f(3) = 0$   
 (C)  $f(1) + f(3) = f(2)$  (D) none of these

705. The function  $f(x) = 9 + |\sin x|$  is

- (A) continuous every where (B) continuous nowhere  
 (C) differentiable at infinite number of points (D) not differentiable at infinite number of points

706. If  $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$ , then

- (A)  $a = -\frac{5}{2}$  (B)  $a = -\frac{3}{2}$   
 (C)  $a = -\frac{7}{2}$  (D)  $b = -\frac{3}{2}$

707. Let  $h(x) = \min \{x^2, x^4\}$  for every real number of  $a$ , then

- (A)  $h$  is not differentiable at two points (B)  $h$  is differentiable  $\forall x$   
 (C)  $h$  is continuous  $\forall x$  (D) none of these

708. The value of the integral  $\int_0^{\pi} x f(\sin x) dx$  is
- (A)  $\pi \int_0^{\pi/2} f(\sin x) dx$  (B)  $\frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$  (C) 0 (D) none of these
709. If  $f(x) = |\log_{10} x|$ , then at  $x = 1$
- (A)  $f(x)$  is continuous and  $f'(1^-) = -\log_e 10$  (B)  $f(x)$  is continuous and  $f'(1^-) = -1$   
 (C)  $f(x)$  is differentiable on  $\mathbb{R} - \{1\}$  (D)  $f(x)$  is differentiable on  $\mathbb{R}$ .
710.  $\int_0^{\pi/2} \left( \frac{\pi}{2} - x \right) \sec x dx$  is equal to
- (A)  $-2 \int_0^1 \frac{\cot^{-1} x}{x} dx$  (B)  $\int_0^1 \tan^{-1} x dx$  (C)  $2 \int_0^1 \frac{\tan^{-1} x}{x} dx$  (D)  $\int_0^1 \cot^{-1} x dx$
711. For  $f(x) = \int_0^x 2|t| dt$ , then tangent parallel to bisector of positive co-ordinate axes are
- (A)  $y = x - \frac{1}{4}$  (B)  $y = x + \frac{1}{4}$  (C)  $y = x - \frac{3}{2}$  (D)  $y = x + \frac{3}{2}$
712. Let  $f(x) = \begin{cases} -2, & -3 \leq x \leq 0 \\ |x-2|, & 0 \leq x \leq 3 \end{cases}$  and  $g(x) = \int_{-3}^x f(t) dt$ , then
- (A)  $g(1) = -3$  (B)  $g(2) = -4$  (C)  $g'(1) = 1$  (D)  $g'(2)$  does not exist
713. The domain of the definition of the function  $f(x) = ([x] - |x-1|)^{-1/2} + \sec^{-1}[\cos x]$ , in the region  $[-\pi, 2\pi]$  where  $[.]$  denotes greater integer function lies in the interval
- (A)  $\left( \frac{\pi}{2}, \pi \right)$  (B)  $\left[ -\pi, -\frac{\pi}{2} \right) \cup \{2\pi\}$  (C)  $\left[ \pi, \frac{3\pi}{2} \right) \cup \{2\pi\}$  (D)  $[1, 2\pi]$
714. If  $f(x) = \sin \pi(x - [x])$  (where  $[.]$  denotes the greatest integer function), then
- (A)  $f(x)$  has period 1 (B)  $f(x)$  is non-differentiable at  $x = 1, 2, 3$   
 (C)  $\int_0^{100} f(x) dx = \frac{200}{\pi}$  (D)  $\int_0^{100} f(x) dx = 200\pi$
715. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3^{[x]} + 3^{-x}$ , (where  $[.]$  denotes the greatest integer function) then which of the following statements are current?
- (A)  $f(x)$  is many-one (B)  $f(x)$  is into  
 (C)  $f(x)$  is bijective (D) neither even nor odd
716. Let  $f(x) = (x + |x|)|x|$  then for all  $x$
- (A)  $f$  is continuous (B)  $f'$  is differentiable for all  $x$   
 (C)  $f'$  is continuous (D)  $f''$  is continuous
717. If  $\int \log(\sqrt{1-x} + \sqrt{1+x}) dx = x f(x) + Ax + B \sin^{-1} x + c$
- (A)  $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$  (B)  $A = -\frac{1}{2}$   
 (C)  $B = \frac{2}{3}$  (D)  $B = -\frac{1}{2}$

718. If  $f(x) = [x(x-1)] + |2x-1|$ , then  $f(x)$  is (where  $[.]$  denotes the greatest integer function)
- (A) continuous at  $x = 10$  (B) differentiable at  $x = 10$   
 (C) discontinuous at  $x = 10$  (D) nondifferentiable at  $x = 10$
719. If  $\lim_{x \rightarrow \infty} \left(1 + \frac{\lambda}{x} + \frac{\mu}{x^2}\right)^{2x} = e^2$ , then
- (A)  $\lambda = 1, \mu = 2$  (B)  $\lambda = 2, \mu = 1$  (C)  $\lambda = 1, \mu = \text{any } \mathbb{R}$  (D)  $\lambda = \mu = 1$
720. In which of the following intervals  $2x^3 - 24x + 5$  increases
- (A)  $(-2, 2)$  (B)  $(2, \infty)$  (C)  $(-\infty, -2)$  (D) None of these
721. If  $f(x) = \int_1^x \frac{\ln t}{1+t} dt$ , then
- (A)  $f\left(\frac{1}{x}\right) = -\int_1^x \frac{\ln t}{t(1+t)} dt$  (B)  $f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln t}{t(1+t)} dt$   
 (C)  $f(x) + f\left(\frac{1}{x}\right) = 0$  (D)  $f(x) + f\left(\frac{1}{x}\right) = \frac{1}{2}(\ln x)^2$
722. Let  $f$  and  $g$  be functions from the interval  $[0, \infty)$  to the interval  $[0, \infty)$   $f$  being an increasing function and  $g$  being a decreasing function. If  $f\{g(0)\} = 0$ , then
- (A)  $f\{g(x)\} \geq f\{g(0)\}$  (B)  $g\{f(x)\} \leq g\{f(0)\}$  (C)  $f\{g(1)\} = 0$  (D) none of these
723. If the line  $ax + by + c = 0$  is a normal to the curve  $xy = 1$ , then
- (A)  $a > 0, b > 0$  (B)  $a > 0, b < 0$  (C)  $a < 0, b > 0$  (D)  $a < 0, b < 0$
724. If  $y = x \log\left(\frac{x}{a+bx}\right)$ ,  $x^3 \frac{d^2 y}{dx^2} =$
- (A)  $\left(x \frac{dy}{dx} - y\right)^2$  (B)  $\frac{a^2 x^2}{(a+bx)^2}$  (C)  $\left(\frac{dy}{dx} - y\right)^2$  (D)  $\left(x \frac{dy}{dx} + y\right)^2$
725. On the interval  $I = [-2, 2]$  the function  $f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
- (A) is continuous for all  $x \in I - \{0\}$  (B) is continuous for all  $x \in I$   
 (C) assumes all intermediate values for  $f(-2)$  to  $f(2)$   
 (D) has a maximum values equal to  $3/e$
726. If  $I = \int_{-\pi/3}^{\pi/3} \frac{e^{\sec x} |\sin x| \sec^2 x}{(1 + e^{\cos x})} dx$ , then
- (A)  $I$  can be evaluated using the substitution  $\sec x = t$   
 (B)  $I$  is irrational number  
 (C)  $I = e^2 - e$  (D)  $I = e - 1$
727. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x+1) = \frac{f(x)-5}{f(x)-3} \quad \forall x \in \mathbb{R}$ . Then which of the following statement(s) is/are true
- (A)  $f(2008) = f(2004)$  (B)  $f(2006) = f(2010)$   
 (C)  $f(2006) = f(2002)$  (D)  $f(2006) = f(2018)$



728. If  $\lim_{x \rightarrow 0} \frac{ae^{2x} - b \cos 2x + ce^{-2x} - x \sin x}{x \sin x} = 1$  and  $f(t) = (a+b)t^2 + (a-b)t + c$ , then
- (A)  $a + b + c = 1$  (B)  $a + b + c = 2$  (C)  $f(1) = \frac{3}{4}$  (D)  $f(1) = 1$
729. If  $f(x) = \sec^{-1}\left(\frac{x+2}{2x-3}\right) + \sin^{-1}\left(\frac{2x-3}{x+2}\right)$  then in their domain of definition
- (A)  $f$  is non decreasing (B)  $f$  is non increasing  
(C)  $f'(1) = 10$  (D)  $f'(0) = 0$
730. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is decreasing and  $g: \mathbb{R} \rightarrow \mathbb{R}$  is increasing then which of the following function is increasing
- (A)  $f \circ f$  (B)  $g \circ g$  (C)  $f \circ g$  (D)  $g \circ f$
731. If  $f: \mathbb{R} \rightarrow \mathbb{R}^-$  (set of all negative reals) is decreasing and  $g: \mathbb{R} \rightarrow \mathbb{R}^-$  is increasing then which of the following is decreasing
- (A)  $f \circ f$  (B)  $g \circ f$  (C)  $f^2$  (D)  $g^2$
732. If  $f'(\sin x) = \cos^2 x$  for all  $x$  and  $f(1) = 1$  then
- (A)  $f$  is increasing (B)  $f$  is injective (C)  $f(0) = 1/3$  (D)  $f(-1) = -1/3$
733. If  $f(x + 1/x) = x^3 + 1/x^3$  ( $x \neq 0$ ) then
- (A)  $f(x)$  is increasing function (B)  $f(x)$  has a local maximum at  $x = -1$   
(C)  $f(x)$  is injective in its domain of definition  
(D) The equation  $f(x) = 3$  has a unique real root
734. The function  $f(x) = 4x^2 - 1/x$  increases over the interval
- (A)  $(0, \infty)$  (B)  $(-\infty, -1/2)$   
(C)  $(-1/2, 0)$  (D)  $(1, \infty)$
735. The function  $f(x) = 2 \ln |x| - x|x|$  decreases over the interval
- (A)  $(1, \infty)$  (B)  $(-\infty, -1)$  (C)  $(0, 1)$  (D)  $(-1, 0)$
736. The function  $f(x) = 2|x| + 1/x^2$  is increasing in the interval
- (A)  $(-\infty, -1)$  (B)  $(-1, 0)$  (C)  $(0, 1)$  (D)  $(1, \infty)$
737. Which of the following is/are true
- (A)  $e^\pi/\pi^e$  (B)  $(1 + \sin \pi/3)^{1 + \cos \pi/3} > (1 + \cos \pi/3)^{1 + \sin \pi/3}$   
(C)  $101^{202} > 202^{101}$  (D)  $(4/3)^{9/4} > (9/4)^{4/3}$
738. If  $f$  is differentiable at  $x = a$ ; then which of the following is FALSE
- (A) If  $f(a)$  is an extreme value of  $f(x)$ , then  $f'(a) = 0$   
(B) If  $f'(a) = 0$ , then  $f(a)$  is an extreme value of  $f(x)$   
(C) If  $f(a)$  is not an extreme value of  $f(x)$  then  $f'(a) \neq 0$   
(D) only one of these statement is false
739. If  $f(x) = \frac{x}{\sin x}$  and  $g(x) = \frac{x}{\tan x}$  where  $0 \leq x \leq 1$ , then in this interval
- (A) both  $f(x)$  and  $g(x)$  are increasing function (B) both  $f(x)$  and  $g(x)$  are decreasing function  
(C)  $f(x)$  is an increasing function (D)  $g(x)$  is a decreasing function
740. Which of the following DOESNOT hold Rolle's theorem in  $[-1, 1]$
- (A)  $f(x) = |x|$  (B)  $f(x) = x^2 - 1$  (C)  $f(x) = x^2 + x + 1$  (D)  $f(x) = \ln|x|$

741. Let  $f(x) = f(x) + f(1 - x)$  and  $f''(x) < 0$ ,  $0 \leq x \leq 1$ . Then
- (A)  $g(x)$  increases on  $\left[\frac{1}{2}, 1\right]$  (B)  $g(x)$  decreases on  $\left[\frac{1}{2}, 1\right]$
- (C)  $g(x)$  decreases on  $\left[0, \frac{1}{2}\right]$  (D)  $g(x)$  increases on  $\left[0, \frac{1}{2}\right]$
742. If  $f'(x) = g(x)(x - a)^2$ , where  $g(a) \neq 0$  and  $g$  is continuous at  $x = a$ , then  $f$  is
- (A) increasing in the nbd. of  $a$  if  $g(a) > 0$  (B) increasing in the nbd. of  $a$  if  $g(a) < 0$
- (C) decreasing in the nbd. of  $a$  if  $g(a) > 0$  (D) decreasing in the nbd. of  $a$  if  $g(a) < 0$
743. Let  $f(x) = 1 + 2^2x^2 + 3^2x^4 + 4^2x^6 + \dots + n^2x^{2n-2}$  then  $f(x)$  has
- (A) exactly one critical point (B) at least one maximum
- (C) exactly one minimum (D) None of these
744. Let  $f(x) = |x^2 - 3x - 4|$ ,  $-1 \leq x \leq 4$ . Then
- (A)  $f(x)$  is m.i. in  $\left[-1, \frac{3}{2}\right]$  (B)  $f(x)$  is m.d. in  $\left(\frac{3}{2}, 4\right)$
- (C) the maximum value of  $f(x)$  is  $25/4$  (D) the minimum value of  $f(x)$  is 0
745. A particle is moving in a straight line such that its distance at any time  $t$  is  $S = \frac{t^4}{4} - 2t^3 + 4t^2 + 7$ , then
- (A) velocity is max at  $t = \frac{(6 - 2\sqrt{3})}{3}$  (B) acceleration is min at  $t = 2$
- (C) the distance is min at  $t = 0, 4$  (D) None of these
746. Let  $f : \mathbb{R} \rightarrow (-1, 1)$  defined by  $f(x) = \frac{e^{x^3} + e^{-x^3}}{e^{x^3} - e^{-x^3}}$ , then  $f$  is
- (A) a one - one function (B) an increasing function
- (C) a decreasing function (D) onto function
747. Let  $f(x) = \frac{x^2 + 2}{[x]}$ ,  $1 \leq x \leq 4.9$ , where  $[x]$  denotes the integral part of  $x$ . Then
- (A)  $f(x)$  is m.i. in  $[1, 4.9]$  (B) least value of  $f(x) = 3$
- (C) greatest value of  $f(x) = 6.0075$  (D)  $f(x)$  is m.d. in  $[1, 4.9]$
748. Let  $f(x) = ax^3 + bx^2 + cx + 1$  have extrema at  $x = \alpha, \beta$  such that  $\alpha\beta < 0$  and  $f(\alpha), f(\beta) < 0$  then the equation  $f(x) = 0$  has
- (A) three equal real roots (B) three distinct real roots
- (C) one positive root if  $f(\alpha) < 0$  &  $f(\beta) > 0$  (D) one negative root if  $f(\alpha) > 0$  &  $f(\beta) < 0$
749. Let  $h(x) = \{f(x)\}^3 + \{f(x)\}^2 + 10f(x)$ . Then
- (A)  $h$  increases as  $f$  increases (B)  $h$  decreases as  $f$  decreases
- (C)  $h$  increases as  $f$  decreases (D) None of these
750. Let  $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$  for all real values of  $x$ . Then
- (A)  $h$  is increasing if  $f(x)$  is increasing (B)  $h$  is increasing if  $f'(x) < 0$
- (C)  $h$  is decreasing if  $f$  is decreasing
- (D) nothing can be said in general

751. Let  $f(x) = \cos x \sin 2x$  then

- (A)  $\min_{x \in (-\pi, \pi)} f(x) > -\frac{7}{9}$  (B)  $\min_{x \in (-\pi, \pi)} f(x) > -\frac{9}{7}$   
 (C)  $\min_{x \in [-\pi, \pi]} f(x) > -\frac{1}{9}$  (D)  $\min_{x \in [-\pi, \pi]} f(x) > -\frac{2}{9}$

752. If OT and ON are perpendiculars dropped from the origin to the tangent and normal to the curve  $x = a \sin^3 t$ ,  $y = a \cos^3 t$  at an arbitrary point, then

- (A)  $4OT^2 + ON^2 = a^2$   
 (B) the length of the tangent =  $\left| \frac{y}{\cos t} \right|$   
 (C) the length of the normal =  $\left| \frac{y}{\sin t} \right|$   
 (D) None of these

753. If  $F(x) = f(x)g(x)$  and  $f'(x)g'(x) = c$ , then

- (A)  $F' = c \left[ \frac{f}{f'} + \frac{g}{g'} \right]$  (B)  $\frac{F''}{F} = \frac{f''}{f} + \frac{g''}{g} + \frac{2c}{fg}$   
 (C)  $\frac{F'''}{F} = \frac{f'''}{f} + \frac{g'''}{g}$  (D)  $\frac{F'''}{F''} = \frac{f'''}{f''} + \frac{g'''}{g''}$

754. Let  $x^{\cos y} + y^{\cos x} = 5$ . Then

- (A) at  $x = 0, y = 0, y' = 0$  (B) at  $x = 0, y = 1, y' = 0$   
 (C) at  $x = y, y = 1, y' = 1$  (D) at  $x = 1, y = 0, y' = 1$

755. Let  $f(x) = (ax + b) \cos x + (cx + d) \sin x$  and  $f'(x) = x \cos x$  be an identity in  $x$ , then

- (A)  $a = 0$  (B)  $b = 1$   
 (C)  $c = 1$  (D)  $d = 0$

756. The function  $f(x) = \max\{(1-x), (1+x), 2\}$ ,  $x \in (-\infty, \infty)$  is

- (A) continuous for all  $x$   
 (B) differentiable for all  $x$   
 (C) except  $x = 1$  and  $x = -1$  differentiable for all  $x$   
 (D) None of these

757. If  $f_n(x) = e^{f_{n-1}(x)}$  for all  $n \in \mathbb{N}$  and  $f_0(x) = x$  then  $\frac{d}{dx} \{f_n(x)\}$  is equal to

- (A)  $f_n(x) \cdot \frac{d}{dx} \{f_{n-1}(x)\}$  (B)  $f_n(x) \cdot f_{n-1}(x)$   
 (C)  $f_n(x) \cdot f_{n-1}(x) \cdot \dots \cdot f_2(x) \cdot f_1(x)$   
 (D) None of these

758. Let  $f(x) = x^2 + xg'(1) + g''(2)$  and  $g(x) = f(1) \cdot x^2 + xf'(x) + f''(x)$  then

- (A)  $f'(1) + f'(2) = 0$  (B)  $g'(2) = g'(1)$   
 (C)  $g''(2) + f''(3) = 6$  (D) None of these

759. Let  $f(t) = \ln t$ . Then  $\frac{d}{dx} \left\{ \int_{x^2}^{x^3} f(t) dt \right\}$
- (A) has a value 0 when  $x = 0$   
 (B) has a value 0 when  $x = 1, x = \frac{4}{9}$   
 (C) has a value  $9e^2 - 4e$  when  $x = e$   
 (D) has a d.c.  $27e - 8$  when  $x = e$
760. If  $-1, f(x) = \begin{vmatrix} \sec \theta & \tan^2 \theta & 1 \\ \theta \sec x & \tan x & x \\ 1 & \tan x - \tan \theta & 0 \end{vmatrix}$  and  $6^n \sin\left(2x + \frac{n\pi}{2}\right) \cos\left(\frac{3x + n\pi}{2}\right)$  are in AP for all  $x, y$ , and  $y_n$ , then
- (A) 0  
 (B)  $y = \int_0^x f(t) \sin\{k(x-t)\} dt$   
 (C)  $y = \sin^2 \alpha + \cos^2(\alpha + \beta) + 2 \sin \alpha \sin \beta \cos(\alpha + \beta)$   
 (D)  $\frac{1}{\sqrt{1+9y^2}}$
761. If  $f(x) = \cos^{-1} \cos(x - \pi/4)$  then
- (A)  $f'\left(\frac{\pi}{2}\right) = 1$  (B)  $f'(0) = -1$   
 (C)  $f'(\pi) = 0$  (D)  $f'\left(\frac{\pi}{4}\right) = 0$
762. If  $f(x) = |\sin x - \cos x|$  then
- (A)  $f'\left(\frac{\pi}{2}\right) = 1$  (B)  $f(\pi) = -1$   
 (C)  $f'\left(\frac{3\pi}{4}\right) = -1$  (D)  $f'(0) = -1$
763. If  $f(x) = (ax + b) \sin x - (cx + d) \cos x$  and  $f'(x) = x \sin x$  then
- (A)  $a = d = 0$  (B)  $b = 1, c = -1$   
 (C)  $b = c = 1$  (D)  $a = 0, d = -1$
764. If  $f(x + y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$  and  $f'(0)$  exists then
- (A)  $f'(x) = f'(0)$   
 (B)  $f(x) = kx$   
 (C)  $f(x) = xf(1)$   
 (D)  $f(x)$  is an even as well as periodic function
765. If  $f(x - y), f(x) f(y)$  and  $f(x + y)$  are in A.P. for all  $x, y \in \mathbb{R}$  and  $f(0) \neq 0$ , then
- (A)  $f(x)$  is an even function (B)  $f'(1) + f'(-1) = 0$   
 (C)  $f'(2) - f'(-2) = 0$  (D)  $f(3) + f(-3) = 0$

766. For the function  $f(x) = \ell n (\sin^{-1} \log_2 x)$ ,

- (A) Domain is  $\left[\frac{1}{2}, 2\right]$  (B) Range is  $\left(-\infty, \ell n \frac{\pi}{2}\right]$   
 (C) Domain is  $(1, 2]$  (D) Range is  $\mathbb{R}$

767. A function 'f' from the set of natural numbers to integers defined by,

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases} \text{ is:}$$

- (A) one-one (B) many-one (C) onto (D) into

768. If  $F(x) = \frac{\sin \pi [x]}{\{x\}}$ , then  $F(x)$  is:

- (A) periodic with fundamental period 1  
 (B) even  
 (C) range is singleton  
 (D) identical to  $\operatorname{sgn} \left( \operatorname{sgn} \frac{\{x\}}{\sqrt{\{x\}}} \right) - 1$ , where  $\{x\}$  denotes fractional part function and  $[.]$  denotes greatest integer function and  $\operatorname{sgn}(x)$  is a signum function.

769.  $D \equiv [-1, 1]$  is the domain of the following functions, state which of them are injective.

- (A)  $f(x) = x^2$  (B)  $g(x) = x^3$  (C)  $h(x) = \sin 2x$  (D)  $k(x) = \sin (\pi x/2)$

770. If  $f(x) = \sqrt{x}$  and  $g(x) = x - 1$ , then

- (A)  $\operatorname{fog}$  is continuous on  $[0, \infty)$  (B)  $\operatorname{gof}$  is continuous on  $[0, \infty)$   
 (C)  $\operatorname{fog}$  is continuous on  $[1, \infty)$  (D) none of these

771. The function  $f(x) = \begin{cases} x^m \sin \frac{1}{x}, & x > 0 \\ 0, & x = 0 \end{cases}$  is continuous at  $x = 0$  if

- (A)  $m \geq 0$  (B)  $m > 0$  (C)  $m < 1$  (D)  $m \geq 1$

772. Let  $f(x) = \frac{1}{[\sin x]}$  ( $[.]$  denotes the greatest integer function) then

- (A) domain of  $f(x)$  is  $(2n\pi + \pi, 2n\pi + 2\pi) \cup \{2n\pi + \pi/2\}$   
 (B)  $f(x)$  is continuous when  $x \in (2n\pi + \pi, 2n\pi + 2\pi)$   
 (C)  $f(x)$  is continuous at  $x = 2n\pi + \pi/2$  (D)  $f(x)$  has the period  $2\pi$

773. Let  $f(x) = [x] + \sqrt{x - [x]}$ , where  $[x]$  denotes the greatest integer function. Then

- (A)  $f(x)$  is continuous on  $\mathbb{R}^+$  (B)  $f(x)$  is continuous on  $\mathbb{R}$   
 (C)  $f(x)$  is continuous on  $\mathbb{R} - \mathbb{I}$  (D) discontinuous at  $x = 1$

774. Let  $f(x)$  and  $g(x)$  be defined by  $f(x) = [x]$  and  $g(x) = \begin{cases} 0, & x \in \mathbb{I} \\ x^2, & x \in \mathbb{R} - \mathbb{I} \end{cases}$  (where  $[.]$  denotes the greatest integer function) then

- (A)  $\lim_{x \rightarrow 1} g(x)$  exists, but  $g$  is not continuous at  $x = 1$   
 (B)  $\lim_{x \rightarrow 1} f(x)$  does not exist and  $f$  is not continuous at  $x = 1$   
 (C)  $\operatorname{gof}$  is continuous for all  $x$   
 (D)  $\operatorname{fog}$  is continuous for all  $x$

775. Which of the following function(s) defined below has/have single point continuity.

$$(A) \quad f(x) = \begin{cases} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases} \quad (B) \quad g(x) = \begin{cases} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{cases}$$

$$(C) \quad h(x) = \begin{cases} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases} \quad (D) \quad k(x) = \begin{cases} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{cases}$$

776. Two functions  $f$  &  $g$  have first & second derivatives at  $x = 0$  & satisfy the relations,

$$f(0) = \frac{2}{g(0)}, \quad f'(0) = 2g'(0) = 4g(0), \quad g''(0) = 5f''(0) = 6f(0) = 3 \text{ then:}$$

$$(A) \quad \text{if } h(x) = \frac{f(x)}{g(x)} \text{ then } h'(0) = \frac{15}{4} \quad (B) \quad \text{if } k(x) = f(x) \cdot g(x) \sin x \text{ then } k'(0) = 2$$

$$(C) \quad \lim_{x \rightarrow 0} \frac{g'(x)}{f'(x)} = \frac{1}{2} \quad (D) \quad \text{none}$$

777. If  $f_n(x) = e^{f_{n-1}(x)}$  for all  $n \in \mathbb{N}$  and  $f_0(x) = x$ , then  $\frac{d}{dx} \{f_n(x)\}$  is equal to:

$$(A) \quad f_n(x) \cdot \frac{d}{dx} \{f_{n-1}(x)\} \quad (B) \quad f_n(x) \cdot f_{n-1}(x) \\ (C) \quad f_n(x) \cdot f_{n-1}(x) \dots \dots \dots f_2(x) \cdot f_1(x) \\ (D) \quad \text{none of these}$$

778. If  $f$  is twice differentiable such that  $f''(x) = -f(x)$  and  $f'(x) = g(x)$ . If  $h(x)$  is a twice differentiable function such that  $h'(x) = [f(x)]^2 + [g(x)]^2$ . If  $h(0) = 2$ ,  $h(1) = 4$ , then the equation  $y = h(x)$  represents:

- (A) a curve of degree 2
- (B) a curve passing through the origin
- (C) a straight line with slope 2
- (D) a straight line with y intercept equal to 2.

779. Given  $f(x) = -\frac{x^3}{3} + x^2 \sin 1.5a - x \sin a \cdot \sin 2a - 5 \sin^{-1}(a^2 - 8a + 17)$  then:

$$(A) \quad f'(x) = -x^2 + 2x \sin 6 - \sin 4 \sin 8 \quad (B) \quad f'(\sin 8) > 0 \\ (C) \quad f'(x) \text{ is not defined at } x = \sin 8 \quad (D) \quad f'(\sin 8) < 0$$

780.  $P(x)$  is a fourth degree polynomial such that

$$(a) \quad P(-x) = P(x) \quad (b) \quad P(-x) > 0 \quad \forall x \in \mathbb{R} \quad (c) \quad P(0) = 1 \\ (d) \quad P(x) \text{ has exactly two local minima at } x_1 \text{ and } x_2 \text{ such that } |x_1 - x_2| = 2$$

The line  $y = 1$  touches the curve at a certain point  $Q$  and the enclosed area between the line and the

curve is  $\frac{8\sqrt{2}}{15}$ . Let  $g(x) = Ax^2 + Bx + C$  ( $A \neq 0$ ) such that  $\lim_{x \rightarrow 0} \frac{P(x) - g(x) - g(-x)}{x^2}$  is finite and is

equal to the slope of the tangent of  $g(x)$  at  $x = -1$ . Also  $P(x)$  and  $g(x)$  have common tangent at  $Q$  parallel to  $x$ -axis, Then

- (A) the value of  $A$  is  $\frac{-1}{2}$  (B) the value of  $B + C$  is  $\frac{-1}{2}$
- (C) the value of  $A + C$  is 1 (D) the value of  $A + B + C$  is
- 781.** If  $f(x) = (ax + b) \sin x + (cx + d) \cos x$ , then the values of  $a, b, c$  and  $d$  such that  $f'(x) = x \cos x$  for all  $x$  are  
 (A)  $a = d = 1$  (B)  $b = 0$  (C)  $c = 0$  (D)  $b = c$
- 782.** If  $f(x) = \sum_{k=0}^n a_k |x|^k$ , where  $a_i$ 's are real constants, then  $f(x)$  is  
 (A) continuous at  $x = 0$  for all  $a_i$  (B) differentiable at  $x = 0$  for all  $a_i \in \mathbb{R}$   
 (C) differentiable at  $x = 0$  for all  $a_{2k+1} = 0$  (D) None of these
- 783.** Consider the curve  $f(x) = x^{1/3}$ , then  
 (A) the equation of tangent at  $(0, 0)$  is  $x = 0$   
 (B) the equation of normal at  $(0, 0)$  is  $y = 0$   
 (C) normal to the curve does not exist at  $(0, 0)$   
 (D)  $f(x)$  and its inverse meet at exactly 3 points.
- 784.** The equation of normal to the curve  $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$  ( $n \in \mathbb{N}$ ) at the point with abscissa equal to ' $a$ ' can be:  
 (A)  $ax + by = a^2 - b^2$  (B)  $ax + by = a^2 + b^2$   
 (C)  $ax - by = a^2 - b^2$  (D)  $bx - ay = a^2 - b^2$
- 785.** If the line,  $ax + by + c = 0$  is a normal to the curve  $xy = 2$ , then:  
 (A)  $a < 0, b > 0$  (B)  $a > 0, b < 0$   
 (C)  $a > 0, b > 0$  (D)  $a < 0, b < 0$
- 786.** In the curve  $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ , at point  $(2, -1)$   
 (A) length of subtangent is  $7/6$ . (B) slope of tangent =  $6/7$   
 (C) length of tangent =  $\sqrt{85}/6$  (D) None of these
- 787.** If  $y = f(x)$  be the equation of a parabola which is touched by the line  $y = x$  at the point where  $x = 1$ . Then  
 (A)  $f'(1) = 1$  (B)  $f'(0) = f'(1)$   
 (C)  $2f(0) = 1 - f'(0)$  (D)  $f(0) + f'(0) + f''(0) = 1$
- 788.** If the tangent to the curve  $2y^3 = ax^2 + x^3$  at the point  $(a, a)$  cuts off intercepts  $\alpha, \beta$  on co-ordinate axes, where  $\alpha^2 + \beta^2 = 61$ , then the value of ' $a$ ' is equal to:  
 (A) 20 (B) 25  
 (C) 30 (D) -30
- 789.** The curves  $ax^2 + by^2 = 1$  and  $Ax^2 + By^2 = 1$  intersect orthogonally, then  
 (A)  $\frac{1}{a} + \frac{1}{A} = \frac{1}{b} + \frac{1}{B}$  (B)  $\frac{1}{a} - \frac{1}{A} = \frac{1}{b} - \frac{1}{B}$   
 (C)  $\frac{1}{a} + \frac{1}{b} = \frac{1}{B} - \frac{1}{A}$  (D)  $\frac{1}{a} - \frac{1}{b} = \frac{1}{A} - \frac{1}{B}$

790. The set of values of  $a$  for which the function  $f(x) = x^2 + ax + 1$  is an increasing function on  $[1, 2]$  is  $I_1$  and decreasing in  $[1, 2]$  is  $I_2$ , then :
- (A)  $I_1 : a \in (2, \infty)$  (B)  $I_2 : a \in (-\infty, -4)$   
 (C)  $I_2 : a \in (-\infty, -4]$  (D)  $I_1 : a \in [-2, \infty)$
791. If  $f$  is an even function then
- (A)  $f^2$  increases on  $(a, b)$  (B)  $f$  cannot be monotonic  
 (C)  $f^2$  need not increase on  $(a, b)$  (D)  $f$  has inverse
792. Let  $g(x) = 2f(x/2) + f(1 - x)$  and  $f''(x) < 0$  in  $0 \leq x \leq 1$  then  $g(x)$  :
- (A) decreases in  $\left[0, \frac{2}{3}\right]$  (B) decreases  $\left[\frac{2}{3}, 1\right]$   
 (C) increases in  $\left[0, \frac{2}{3}\right]$  (D) increases in  $\left[\frac{2}{3}, 1\right]$
793. On which of the following intervals, the function  $x^{100} + \sin x - 1$  is strictly increasing
- (A)  $(-1, 1)$  (B)  $[0, 1]$  (C)  $[\pi/2, \pi]$  (D)  $[0, \pi/2]$
794. The function  $y = \frac{2x-1}{x-2}$  ( $x \neq 2$ ) :
- (A) is its own inverse (B) decreases for all values of  $x$   
 (C) has a graph entirely above  $x$ -axis (D) is bound for all  $x$ .
795. Let  $f$  and  $g$  be two functions defined on an interval  $I$  such that  $f(x) \geq 0$  and  $g(x) \leq 0$  for all  $x \in I$  and  $f$  is strictly decreasing on  $I$  while  $g$  is strictly increasing on  $I$  then
- (A) the product function  $fg$  is strictly increasing on  $I$   
 (B) the product function  $fg$  is strictly decreasing on  $I$   
 (C)  $fog(x)$  is monotonically increasing on  $I$   
 (D)  $fog(x)$  is monotonically decreasing on  $I$
796. Let  $f(x) = 40/(3x^4 + 8x^3 - 18x^2 + 60)$ , consider the following statement about  $f(x)$ .
- (A)  $f(x)$  has local minima at  $x = 0$   
 (B)  $f(x)$  has local maxima at  $x = 0$   
 (C) absolute maximum value of  $f(x)$  is not defined  
 (D)  $f(x)$  is local maxima at  $x = -3, x = 1$
797. Maximum and minimum values of the function,
- $$f(x) = \frac{2-x}{\pi} \cos \pi(x+3) + \frac{1}{\pi^2} \sin \pi(x+3) \quad 0 < x < 4 \text{ occur at :}$$
- (A)  $x = 1$  (B)  $x = 2$  (C)  $x = 3$  (D)  $x = \pi$
798. If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} [f(x)]$  ( $[ \cdot ]$  denotes the greater integer function) and  $f(x)$  is non-constant continuous function, then
- (A)  $\lim_{x \rightarrow a} f(x)$  is integer (B)  $\lim_{x \rightarrow a} f(x)$  is non-integer  
 (C)  $f(x)$  has local maximum at  $x = a$  (D)  $f(x)$  has local minima at  $x = a$
799. If the derivative of an odd cubic polynomial vanishes at two different values of ' $x$ ' then
- (A) coefficient of  $x^3$  &  $x$  in the polynomial must be same in sign  
 (B) coefficient of  $x^3$  &  $x$  in the polynomial must be different in sign  
 (C) the values of ' $x$ ' where derivative vanishes are closer to origin as compared to the respective roots on either side of origin.  
 (D) the values of ' $x$ ' where derivative vanishes are far from origin as compared to the respective roots on either side of origin.



800. Let  $f(x) = \ln(2x - x^2) + \sin \frac{\pi x}{2}$ . Then
- (A) graph of  $f$  is symmetrical about the line  $x = 1$   
 (B) graph of  $f$  is symmetrical about the line  $x = 2$   
 (C) maximum value of  $f$  is 1 (D) minimum value of  $f$  does not exist
801. The curve  $y = \frac{x+1}{x^2+1}$  has:
- (A)  $x = 1$ , the point of inflection (B)  $x = -2 + \sqrt{3}$ , the point of inflection  
 (C)  $x = -1$ , the point of minimum (D)  $x = -2 - \sqrt{3}$ , the point of inflection
802. If the function  $y = f(x)$  is represented as,  $x = \phi(t) = t^3 - 5t^2 - 20t + 7$   
 $y = \psi(t) = 4t^3 - 3t^2 - 18t + 3$  ( $-2 < t < 2$ ), then:
- (A)  $y_{\max} = 12$  (B)  $y_{\max} = 14$  (C)  $y_{\min} = -67/4$  (D)  $y_{\min} = -69/4$
803. The maximum and minimum values of  $y = \frac{ax^2 + 2bx + c}{Ax^2 + 2Bx + C}$  are those for which
- (A)  $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$  is equal to zero  
 (B)  $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$  is a perfect square  
 (C)  $\frac{dy}{dx} = 0$  and  $\frac{d^2y}{dx^2} \neq 0$   
 (D)  $ax^2 + 2bx + c - y(Ax^2 + 2Bx + C)$  is not a perfect square
804. If  $\int \frac{(x-1)dx}{x^2 \sqrt{2x^2 - 2x + 1}}$  is equal to  $\frac{\sqrt{f(x)}}{g(x)} + c$  then
- (A)  $f(x) = 2x^2 - 2x + 1$  (B)  $g(x) = x + 1$   
 (C)  $g(x) = x$  (D)  $f(x) = \sqrt{2x^2 - 2x}$
805.  $\int \frac{dx}{5 + 4 \cos x} = I \tan^{-1}\left(m \tan \frac{x}{2}\right) + C$  then:
- (A)  $I = 2/3$  (B)  $m = 1/3$  (C)  $I = 1/3$  (D)  $m = 2/3$
806. If  $\int \frac{3 \cot 3x - \cot x}{\tan x - 3 \tan 3x} dx = p f(x) + q g(x) + c$  where 'c' is a constant of integration, then
- (A)  $p = 1$ ;  $q = \frac{1}{\sqrt{3}}$ ;  $f(x) = x$ ;  $g(x) = \ln \left| \frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x} \right|$   
 (B)  $p = 1$ ;  $q = -\frac{1}{\sqrt{3}}$ ;  $f(x) = x$ ;  $g(x) = \ln \left| \frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x} \right|$   
 (C)  $p = 1$ ;  $q = -\frac{2}{\sqrt{3}}$ ;  $f(x) = x$ ;  $g(x) = \ln \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right|$   
 (D)  $p = 1$ ;  $q = -\frac{1}{\sqrt{3}}$ ;  $f(x) = x$ ;  $g(x) = \ln \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right|$

807.  $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$  is equal to:  
 (A)  $\cot^{-1}(\cot^2 x) + c$  (B)  $-\cot^{-1}(\tan^2 x) + c$  (C)  $\tan^{-1}(\tan^2 x) + c$  (D)  $-\tan^{-1}(\cos 2x) + c$
808. If  $f(x)$  is integrable over  $[1, 2]$ , then  $\int_1^2 f(x) dx$  is equal to  
 (A)  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right)$  (B)  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$   
 (C)  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r+n}{n}\right)$  (D)  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$
809. If  $f(x) = \int_0^x (\cos^4 t + \sin^4 t) dt$ ,  $f(x + \pi)$  will be equal to  
 (A)  $f(x) + f(\pi)$  (B)  $f(x) + 2f(\pi)$  (C)  $f(x) + f\left(\frac{\pi}{2}\right)$  (D)  $f(x) + 2f\left(\frac{\pi}{2}\right)$
810. The value of  $\int_0^1 \frac{2x^2 + 3x + 3}{(x+1)(x^2 + 2x + 2)} dx$  is:  
 (A)  $\frac{\pi}{4} + 2 \ln 2 - \tan^{-1} 2$  (B)  $\frac{\pi}{4} + 2 \ln 2 - \tan^{-1} \frac{1}{3}$   
 (C)  $2 \ln 2 - \cot^{-1} 3$  (D)  $-\frac{\pi}{4} + \ln 4 + \cot^{-1} 2$
811. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal  
 (A) is linear (B) is homogeneous  
 (C) has separable variables (D) is none of these
812. The solution of  $x^2 y_1'^2 + xy_1' - 6y^2 = 0$  are  
 (A)  $y = Cx^2$  (B)  $x^2 y = C$  (C)  $\frac{1}{2} \log y = C + \log x$  (D)  $x^3 y = C$
813. The orthogonal trajectories of the system of curves  $\left(\frac{dy}{dx}\right)^2 = a/x$  are  
 (A)  $9a(y + c) = 4x^3$  (B)  $y + C = \frac{-2}{3\sqrt{a}} x^{3/2}$  (C)  $y + C = \frac{2}{3\sqrt{a}} x^{3/2}$  (D) None
814. The solution of  $\left(\frac{dy}{dx}\right)(x^2 y^3 + xy) = 1$  is  
 (A)  $1/x = 2 - y^2 + Ce^{-y^2}/2$   
 (B) the solution of an equation which is reducible to linear equation.  
 (C)  $2/x = 1 - y^2 + e^{-y}/2$  (D)  $\frac{1-2x}{x} = -y^2 + Ce^{-y^2}/2$

## SECTION-3 (COMPREHENSION TYPE)

### COMPREHENSION-1

#### Paragraph for Questions Nos. 815 to 817

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function such that  $f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2$

815.  $f(3)$  is equal to

(A)  $f(0)$

(B)  $4 + f(0)$

(C)  $9 + f(0)$

(D)  $16 + f(0)$

816. The equation  $f(x) - x - f(0) = 0$  have exactly

(A) No solution

(B) One solution

(C) Two solution

(D) infinite solution

817.  $f'(0)$  is equal to

(A) 0

(B) 1

(C)  $f(0)$

(D)  $-f(0)$

### COMPREHENSION-2

#### Paragraph for Questions Nos. 818 to 820

If  $f(x) = \max. (|x^2 - 1|, |x - 1|)$  and  $g(x) = \int_a^x f(t) dt$ ,  $x \in \mathbb{R}$ .

818. The value of  $f(x)$  is

$$(A) \quad f(x) = \begin{cases} x^2 - 1, & x \leq -2 \\ 1 - x, & -2 < x \leq 0 \\ 1 - x^2, & 0 < x < 1 \\ x^2 - 1, & x > 1 \end{cases}$$

$$(B) \quad f(x) = \begin{cases} x^2 - 1, & x \leq -2 \\ 1 - x^2, & -2 < x \leq 0 \\ 1 - x, & 0 < x \leq 1 \\ x - 1, & x > 1 \end{cases}$$

$$(C) \quad f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ x - 1, & x > 1 \end{cases}$$

$$(D) \quad f(x) = \begin{cases} x - 1, & x \leq 1 \\ x^2 - 1, & x > 1 \end{cases}$$

819. The function  $f(x)$  is continuous for  $x$  belongs to

(A)  $\mathbb{R} - \{0, 1\}$

(B)  $\mathbb{R} - \{-2, 0, 1\}$

(C)  $\mathbb{R}$

(D) none of these

820. The function  $g(x)$  is differentiable for

(A)  $\mathbb{R} - \{0, 1\}$

(B)  $\mathbb{R} - \{-2, 0, 1\}$

(C)  $\mathbb{R}$

(D) None of these

### COMPREHENSION-3

#### Paragraph for Questions Nos. 821 to 823

$$\text{Let } f(x) = \log_{\{x\}}[x]$$

$$g(x) = \log_{[x]}\{x\}$$

$$h(x) = \log_{\{x\}}\{x\}$$

where  $[.]$ ,  $\{.\}$  denotes the greatest integer function and fractional part.

**821.** For  $x \in (1, 5)$  the  $f(x)$  is not defined at how many points

- |       |       |
|-------|-------|
| (A) 5 | (B) 4 |
| (C) 3 | (D) 2 |

**822.** If  $A = \{x: x \in \text{domain of } f(x)\}$  and  $B = \{x: x \in \text{domain of } g(x)\}$  then  $A - B$  will be

- |              |                   |
|--------------|-------------------|
| (A) $(2, 3)$ | (B) $(1, 3)$      |
| (C) $(1, 2)$ | (D) none of these |

**823.** Domain of  $h(x)$  is

- |                               |                                 |
|-------------------------------|---------------------------------|
| (A) $\mathbb{R}$              | (B) $\mathbb{I}$                |
| (C) $\mathbb{R} - \mathbb{I}$ | (D) $\mathbb{R}^+ - \mathbb{I}$ |

### COMPREHENSION-4

#### Paragraph for Questions Nos. 824 to 826

Let a function  $f(x)$  satisfies the condition  $f(x+y) = \frac{f(x)+f(y)}{f(x)}$  such that  $f'(0) = 2$  and  $f(x) \geq 0$ . Using the above information answer the following:

**824.** The curve  $y = f(x)$  is

- |                         |                         |
|-------------------------|-------------------------|
| (A) $y = \sqrt{2(x+1)}$ | (B) $y = 2\sqrt{(x+1)}$ |
| (C) $y = \ln(x+1)$      | (D) $y = \ln(x-1)$      |

**825.** Area bounded between  $y = f(|x|)$  and  $y = 7 - |x|$  is

- |                             |                             |
|-----------------------------|-----------------------------|
| (A) $\frac{23}{6}$ sq. unit | (B) $\frac{11}{6}$ sq. unit |
| (C) $\frac{86}{6}$ sq. unit | (D) 7 sq. unit              |

**826.** The number of points where  $g(x) = \max. \{f(x), 6, 7 - |x|\}$  is non differentiable  $\forall x \in [-10, 10]$  are

- |       |       |
|-------|-------|
| (A) 5 | (B) 6 |
| (C) 7 | (D) 8 |

## COMPREHENSION-5

### Paragraph for Questions Nos. 827 to 829

Let  $f : [2, \infty) \rightarrow [1, \infty)$  defined by  $f(x) = 2^{x^4 - 4x^2}$  and  $g : \left[\frac{\pi}{2}, \pi\right] \rightarrow A$  defined by  $g(x) = \frac{\sin x + 4}{\sin x - 2}$  be two invertible functions, then

827.  $f^{-1}(x)$  is equal to

- (A)  $-\sqrt{2 + \sqrt{4 + \log_2 x}}$  (B)  $\sqrt{2 + \sqrt{4 + \log_2 x}}$   
 (C)  $\sqrt{2 - \sqrt{4 + \log_2 x}}$  (D) None of these

828. The set A is equal to

- (A)  $[-5, -2]$  (B)  $[2, 5]$  (C)  $[-5, 2]$  (D)  $[-3, -2]$

829. The domain of  $f^{-1}g^{-1}(x)$  is

- (A)  $[-5, \sin 1]$  (B)  $\left[-5, \frac{\sin 1}{2 - \sin 1}\right]$  (C)  $\left[-5, -\frac{(4 + \sin 1)}{2 - \sin 1}\right]$  (D)  $\left[-\frac{(4 + \sin 1)}{2 - \sin 1}, -2\right]$

## COMPREHENSION-6

### Paragraph for Questions Nos. 830 to 832

A function is defined as the approaching value of the expression  $\frac{1 + 2(x)^{2n}}{1 + x^{2n}}$  as  $x$  approaches to infinity.

830. The domain and range of the function is

- (A)  $(-\infty, 1) \cup (1, \infty), \left\{1, -1, \frac{3}{2}, -\frac{3}{2}, 2, -2\right\}$  (B)  $(-\infty, \infty), \left\{1, \frac{3}{2}, 2\right\}$   
 (C)  $(1, \infty), \{1, -1\}$  (D) None of these

831. The points of discontinuity of the function are

- (A)  $1, -1$  (B)  $1, 0, -1$  (C)  $1, 3$  (D) None of these

832. The composition of the function with  $y = |x|$  is

- (A)  $\begin{cases} 1, & |x| < 1 \\ \frac{3}{2}, & |x| = 1 \\ 2, & |x| > 1 \end{cases}$  (B)  $\begin{cases} -2, & x \leq -1 \\ -\frac{3}{2}, & -1 < x < 0 \\ 1, & 0 \leq x < 1 \\ 2, & x \geq 1 \end{cases}$   
 (C)  $\begin{cases} -2, & x \leq -1 \\ -\frac{3}{2}, & x = -1 \\ -1, & -1 < x < 0 \\ 1, & 0 \leq x < 1 \\ \frac{3}{2}, & x = 1 \\ 2, & x > 1 \end{cases}$  (D) None of these

## COMPREHENSION-7

### Paragraph for Questions Nos. 833 to 835

Let  $f(x)$  be a real valued function not identically zero, such that  
 $f(x + y^n) = f(x) + (f(y))^n \quad \forall x, y \in \mathbb{R}$  where  $n \in \mathbb{N}$  ( $n \neq 1$ ) and  $f'(0) \geq 0$ .

- 833.** The value of  $f'(0)$  is  
(A) 1 (B)  $1 + n$   
(C)  $n$  (D) 2
- 834.** The value of  $f(5)$  is  
(A) 2 (B) 3  
(C)  $5n$  (D) 5
- 835.**  $\int_0^1 f(x) dx$  is equal to  
(A)  $\frac{1}{2n}$  (B)  $2n$   
(C)  $\frac{1}{2}$  (D) 2

## COMPREHENSION-8

### Paragraph for Questions Nos. 836 to 838

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be function defined as

$$f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

and  $g(x) = f(x - 1) + f(x + 1) \quad \forall x \in \mathbb{R}$ .

- 836.** For  $x \in [-1, 1]$ ,  $g(|x|)$  is equal to  
(A)  $x$  (B)  $-|x|$   
(C)  $|x|$  (D)  $2 + |x|$
- 837.** Value of  $g\left(-\frac{3}{2}\right)$  is equal to  
(A) 0 (B)  $\frac{1}{2}$   
(C)  $\frac{7}{2}$  (D)  $\frac{3}{2}$
- 838.** The number of points at which  $y = |g(x)|$  is non differentiable is  
(A) 3 (B) 4  
(C) 5 (D) 6

### COMPREHENSION-9

#### Paragraph for Questions Nos. 839 to 841

Let  $y = f(x)$  be a function continuous and differentiable every where also,

$$g_1(x) = \min\{|f(x)|, |f(x-1)|\}$$

$$g_2(x) = f(|x|)$$

$$g_3(x) = -f(|x|)$$

If  $f(x) = x - 1$ , then

**839.** The area bounded by  $y = g_1(x)$ , x-axis and lines  $x = 0$  and  $x = 3$  is equal to

- (A) 1 (B) 2  
(C)  $\frac{5}{4}$  (D) None of these

**840.** The area bounded by  $y = g_2(x)$  and  $y = g_3(x)$  is equal to

- (A) 2 (B) 4  
(C) 1 (D) none of these

**841.** The area bounded by  $y = g_3(x)$  and  $y = \ln(|x|)$  is equal to

- (A) 2 (B) 3  
(C) 4 (D) None of these

### COMPREHENSION-10

#### Paragraph for Questions Nos. 842 to 844

Let  $f$  be a polynomial function such that  $f(x)f(y) + 2 = f(x) + f(y) + f(xy) \quad \forall x, y \in \mathbb{R}^+ \cup \{0\}$  and  $f(x)$  is one-one  $\forall x \in \mathbb{R}^+$  with  $f(0) = 1$  and  $f'(1) = 2$ .

**842.** The function  $y = f(x)$  is given by

- (A)  $x^{1/3} - 1$  (B)  $1 + \frac{2x^3}{3}$   
(C)  $1 + x^2$  (D)  $1 - x^2$

**843.** Area bounded between the curve  $y = x^2$  and  $y = g(x)$  where  $g(x) = \frac{2}{f(x)}$  and x-axis is

- (A)  $\frac{\pi}{2} - \frac{1}{3}$  (B)  $\pi - \frac{1}{3}$   
(C)  $\frac{\pi}{2} - \frac{1}{6}$  (D)  $\pi - \frac{2}{3}$

**844.** If  $h(x) = \min\left\{\frac{2}{f(x)}, x^2, |1 - |x||\right\}$ , then the number of points of non-differentiability of  $h(x)$  is/are

- (A) 3 (B) 4  
(C) 5 (D) 6

## COMPREHENSION-11

### Paragraph for Questions Nos. 845 to 847

Let  $f(x)$  be a function such that its derivative  $f'(x)$  is continuous in  $[a, b]$  and derivable in  $(a, b)$ . Consider a function  $\phi(x) = f(b) - f(x) - (b - x)f'(x) - (b - x)^2 A$ . If Rolle's theorem is applicable to  $\phi(x)$  on  $[a, b]$ , answer following questions

- 845.** If there exist some number  $c$  ( $a < c < b$ ) such that  $\phi'(c) = 0$  and  $f(b) = f(a) + (b - a)f'(a) + \lambda(b - a)^2 f''(c)$ , then  $\lambda$  is

(A) 1 (B) 0 (C)  $\frac{1}{2}$  (D)  $-\frac{1}{2}$

- 846.** Let  $f(x) = x^3 - 3x + 3$ ,  $a = 1$  and  $b = 1 + h$ . If there exists  $c \in (1, 1 + h)$  such that  $\phi'(c) = 0$  and  $\frac{f(1+h) - f(1)}{h^2} = \lambda c$ , then  $\lambda =$

(A)  $\frac{1}{2}$  (B) 2 (C) 3 (D) does not exist

- 847.** Let  $f(x) = \sin x$ ,  $a = \alpha$  and  $b = \alpha + h$ . If there exists a real number  $t$  such that  $0 < t < 1$ ,  $\phi'(\alpha + th) = 0$  and  $\frac{\sin(\alpha + h) - \sin \alpha - h \cos \alpha}{h^2} = \lambda \sin(\alpha + th)$ , then  $\lambda =$

(A)  $\frac{1}{2}$  (B)  $-\frac{1}{2}$  (C)  $\frac{1}{4}$  (D)  $\frac{1}{3}$

## COMPREHENSION-12

### Paragraph for Questions Nos. 848 to 850

Sometimes we can find the sum of series by use of differentiation. If we know the sum of a series

e.g. if  $f(x) = f_1(x) + f_2(x) + \dots$

$f'(x) = f_1'(x) + f_2'(x) + \dots$

e.g.  $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \dots$

$|x| < 1$

Hence the sum of the AGP

$1 + 2x + 3x^2 + \dots = (1 - x)^{-2}$  (By differentiation both the sides)

Now answer the question that follows

- 848.** The sum of the series  $\frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \dots$  upto  $\infty$  is

(A)  $4e - 1$  (B)  $5e$  (C)  $5e - 1$  (D)  $4e$

- 849.** Sum of the series  $1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \dots$  upto  $\infty$  is

(A)  $\frac{1}{2} - \ln 2$  (B)  $1 - \ln 2$  (C)  $\infty$  (D)  $\frac{3}{2} - \ln 2$

- 850.** Sum of the series  $1 + 1 + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \dots$  upto infinite terms, is

(A) 4 (B) 2 (C) 1 (D)  $\frac{1}{4}$



### COMPREHENSION-13

#### Paragraph for Questions Nos. 851 to 853

If  $y = \int_{u(x)}^{v(x)} f(t) dt$ , let us define  $\frac{dy}{dx}$  in a different manner as  $\frac{dy}{dx} = v'(x) f(v(x)) - u'(x) f(u(x))$  and the

equation of the tangent at  $(a, b)$  as  $y - b = \left( \frac{dy}{dx} \right)_{(a, b)} (x - a)$

851. If  $y = \int_x^{x^2} t^2 dt$ , then equation of tangent at  $x = 1$  is  
(A)  $y = x + 1$  (B)  $x + y = 1$  (C)  $y = x - 1$  (D)  $y = x$
852. If  $F(x) = \int_1^x e^{t^2/2} (1 - t^2) dt$ , then  $\frac{d}{dx} F(x)$  at  $x = 1$  is  
(A) 0 (B) 1 (C) 2 (D) -1
853. If  $y = \int_{x^3}^{x^4} \ln t dt$ , then  $\lim_{x \rightarrow 0^+} \frac{dy}{dx}$  is  
(A) 0 (B) 1 (C) 2 (D) -1

### COMPREHENSION-14

#### Paragraph for Questions Nos. 854 to 856

Let  $f$  be a function defined so that every element of the codomain has at most two pre-images and there is at least one element in the co-domain which has exactly two pre-images we shall call this function as "two-one" function. A two-one function is definitely a many one function but vice-versa is not true. For example,  $y = |e^x - 1|$  is a "two-one" function.  $y = x^3 - x$  is a many one function but not a "two-one" function. In the light of above definition answer the following questions:

854. In the following functions which one is a "two-one" function :-  
(A)  $y = |\ln|x||$  (B)  $y = x^2 \sin x$   
(C)  $y = x^3 + 3x + 1$  (D)  $y = x^4 - x + 1$
855. Let  $f(x) = \{x\}$  be the fractional part function. For what domain is the function "two-one"?  
(A)  $\left[ \frac{1}{2}, \frac{5}{2} \right]$  (B)  $\left[ -\frac{1}{2}, \frac{3}{2} \right]$   
(C)  $[1, 2)$  (D) None of these
856. A continuous "two-one" function defined for  $x \in (a, b)$  has  
(A) atmost one point of extremum  
(B) atleast two points of extrema  
(C) exactly one point of extremum  
(D) none of these

## COMPREHENSION-15

### Paragraph for Questions Nos. 857 to 859

Continuous Probability Distributions. A continuous distribution is one in which the variate may take any value between certain limits  $a$  and  $b$ ,  $a < b$ . Suppose that the probability of the variate  $X$  falling in the infinitesimal interval  $x - \frac{1}{2} dx$  to  $x + \frac{1}{2} dx$  is expressible as  $f(x) dx$ , where  $f(x)$  is a continuous function of  $x$ .

Symbolically,  $P(x - \frac{1}{2} dx \leq X \leq x + \frac{1}{2} dx) = f(x) dx$

where  $f(x)$  is called the probability density function (abbreviated as p.d.f.) or simply density function. The continuous curve  $y = f(x)$  is called probability curve ; and when this is symmetrical, the distribution is said to be symmetrical. Clearly, the probability density function possesses the following properties:

(i)  $f(x) \geq 0$  for every  $x$  in the interval  $[a, b]$ ,  $a < b$

(ii)  $\int_a^b f(x) dx = 1$ ,  $a, b > 0$

since the total area under the curve is unity.

(iii) Furthermore, we define for any  $[c, d]$ , where  $c, d \in [a, b]$ ,  $c < d$ ;

$$P(c \leq X \leq d) = \int_c^d f(x) dx \quad \dots\dots\dots(i)$$

We define  $F(x)$ , the cumulative distribution function (abbreviated as c.d.f.) of the random variate  $X$  where

$$F(x) = P(X \leq x)$$

or 
$$F(x) = \int_a^x f(x) dx . \quad \dots\dots\dots(ii)$$

**857.** If  $f(x) = \begin{cases} 2x & ; 0 \leq x \leq 1 \\ 0 & ; x > 1 \end{cases}$  then the probability that  $x \leq \frac{1}{2}$  is

- (A)  $\frac{1}{2}$                       (B)  $\frac{1}{4}$                       (C)  $\frac{3}{4}$                       (D)  $\frac{1}{8}$

**858.** In Q. No. 857, probability that  $x \geq \frac{3}{4}$  given  $x \geq \frac{1}{2}$  is

- (A)  $\frac{7}{16}$                       (B)  $\frac{3}{4}$                       (C)  $\frac{3}{7}$                       (D)  $\frac{7}{12}$

**859.** Suppose the life in hours ( $x$ ) of a certain kind of radio tube has the probability density function  $f(x) = \frac{100}{x^2}$  when  $x > 100$  and zero when  $x < 100$ . Then the probability that none of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation, is

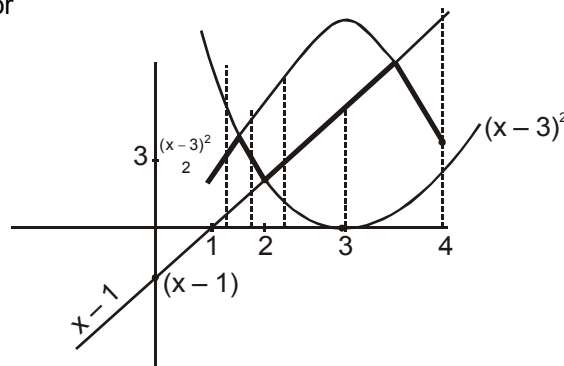
- (A)  $\frac{1}{27}$                       (B)  $\frac{8}{27}$                       (C)  $\frac{1}{225}$                       (D)  $\frac{26}{27}$

## COMPREHENSION-16

### Paragraph for Questions Nos. 860 to 862

If  $f(x) = \text{Mid} \{g(x), h(x), p(x)\}$  means the function which will be second in order when values of the three function at a particular value of  $x$  are arranged, then for

$$f(x) = \text{Mid} \left\{ x-1, (x-3)^2, 3 - \frac{(x-3)^2}{2} \right\}, x \in [1, 4]$$



- 860.** Numerical value of difference between the LHD and RHD at the point  $x = 2$  for  $f(x)$  in  $x \in [1, 4]$  will be  
 (A) 0 (B) 2 (C) 3 (D) 1
- 861.** The greatest value of  $f(x)$  in  $[1, 4]$  will be  
 (A)  $1 + \sqrt{3}$  (B)  $2 + \sqrt{3}$  (C)  $3 + \sqrt{3}$  (D) N.O.T.
- 862.** Rate of change of  $x$  w.r.t.  $f(x)$  at  $x = 3$  will be  
 (A) 1 (B)  $\frac{3}{2}$  (C) 2 (D)  $-\frac{3}{2}$

## COMPREHENSION-17

### Paragraph for Questions Nos. 863 to 865

A function  $f(x)$  having the following properties;

- (i)  $f(x)$  is continuous except at  $x = 3$
- (ii)  $f(x)$  is differentiable except at  $x = -2$  and  $x = 3$
- (iii)  $f(0) = 0$ ,  $\lim_{x \rightarrow 3} f(x) \rightarrow -\infty$ ,  $\lim_{x \rightarrow -\infty} f(x) = 3$ ,  $\lim_{x \rightarrow \infty} f(x) = 0$
- (iv)  $f'(x) > 0 \forall x \in (-\infty, -2) \cup (3, \infty)$  and  $f'(x) \leq 0 \forall x \in (-2, 3)$
- (v)  $f''(x) > 0 \forall x \in (-\infty, -2) \cup (-2, 0)$  and  $f''(x) < 0 \forall x \in (0, 3) \cup (3, \infty)$

then answer the following questions

- 863.** Range of  $f(x)$  is  
 (A)  $(-\infty, \infty)$  (B)  $(-\infty, 3]$   
 (C)  $(-\infty, 3)$  (D)  $(-\infty, f(-2)]$
- 864.** Graph of function  $y = f(-|x|)$  is  
 (A) differentiable for all  $x$ , if  $f'(0) = 0$   
 (B) continuous but not differentiable at two points, if  $f'(0) = 0$   
 (C) continuous but not differentiable at one points, if  $f'(0) = 0$   
 (D) discontinuous at two points, if  $f'(0) = 0$
- 865.**  $f(x) + 3x = 0$  has five solutions if  
 (A)  $f(-2) > 6$  (B)  $f'(0) < -3$  and  $f(-2) > 6$   
 (C)  $f'(0) > -3$  (D)  $f'(0) > -3$  and  $f(-2) > 6$

## COMPREHENSION-18

### Paragraph for Questions Nos. 866 to 868

$l, m, n$  are real numbers and  $x_0$  be an arbitrary real number in  $[p, q]$  and  $f$  is a real valued function such that

$$l^2 [f(a-x) - f(a+x)] + 4l [f(x) + f(-x)] + \left\{ \lim_{x \rightarrow x_0} f(x) - f(x_0) \right\} = 0,$$

$$m^2 [f(a-x) - f(a+x)] + 4m [f(x) + f(-x)] + \left\{ \lim_{x \rightarrow x_0} f(x) - f(x_0) \right\} = 0,$$

$$\& n^2 [f(a-x) - f(a+x)] + 4n [f(x) + f(-x)] + \left\{ \lim_{x \rightarrow x_0} f(x) - f(x_0) \right\} = 0,$$

866. The function  $f$  is

- (A) periodic with period 'a' (B) periodic with period '4a'  
(C) periodic with a period  $2a$  (D) non periodic

867.  $x_1, x_2, \dots, x_n \in (p, q)$  and for some  $\xi \in (p, q)$ ,  $(f(\xi) \neq 0) =$

- (A)  $n$  (B)  $n+1$  (C)  $n-1$  (D)  $2n$

868. If  $f(x) > 0 \forall x \in [0, 2a]$  then  $\frac{\int_0^{2a} f(x) dx}{\int_0^a f(x) dx} =$

- (A) 1 (B) 2 (C) 3 (D) 4

## COMPREHENSION-19

### Paragraph for Questions Nos. 869 to 872

Let  $f(x) = x^3 + ax^2 + bx + c$  be a cubic polynomial where  $a, b, c \in \mathbb{R}$ . Now  $f'(x) = 3x^2 + 2ax + b$  and let  $D = 4a^2 - 12b$  be the discriminant of the equation  $f'(x) = 0$ . If  $D > 0$ ,  $f'(x) = 0$  has two real roots.  $\alpha, \beta (\alpha < \beta)$ , then  $x = \alpha$  will be point of local maxima and  $x = \beta$  will be a point of local minima of  $f(x)$ , also

If  $f(\alpha)f(\beta) > 0$ , then  $f(x) = 0$  would have just one real root.

$f(\alpha)f(\beta) < 0$ , then  $f(x) = 0$  would have three real and distinct roots.

$f(\alpha)f(\beta) = 0$ , then  $f(x) = 0$  would have three real roots.

869. If the function  $f(x) = x^3 - 9x^2 + 24x + k$  has three real and distinct roots  $x_1, x_2, x_3$  where  $x_1 < x_2 < x_3$ . Then the possible value of  $k$  will be

- (A)  $k < -20$  (B)  $k > 20$  (C)  $16 < k < 20$  (D)  $-20 < k < -16$

870. In the question No. 869,  $[x_1] + [x_3]$  is equal to {where  $[x]$  is greatest integer function}

- (A) 2 (B) 3 (C) 4 (D) 5

871. In the question No. 869,  $x_2$  lies in the interval

- (A)  $(-2, 0)$  (B)  $(0, 2)$  (C)  $(2, 4)$  (D) none of these

872. If  $f(x) = ax^3 + bx^2 + cx + d$  has it non-zero local minimum and maximum values at  $x = 2$  and  $x = 1$  respectively. If  $a$  be the root of the equation  $x^2 - 2x - 15 = 0$ , then  $a$  is equal to

- (A) -3 (B) 5 (C) both (a) and (b) (D) none of these

## COMPREHENSION-20

### Paragraph for Questions Nos. 873 to 875

One of the most famous functions in calculus is the Dirichlet's function, viz.

$D(x) = \begin{cases} 1, & x \in Q \\ 0, & x \notin Q \end{cases}$ . This function is one of the rare functions whose graph cannot be drawn. A number of functions were later defined by imitating Dirichlet's function.

$$\text{Let } f(x) = \begin{cases} x^3 + 2x^2, & x \in Q \\ -x^3 + 2x^2 + ax, & x \notin Q \end{cases}$$

873. The value of  $a$  so that this function is differentiable at  $x = 0$  is  
(A) 1 (B) -1 (C) 0 (D) none of these
874. For the value of  $a$  obtained in above question,  $f(x)$  is  
(A) one-one and onto (B) many-one and onto  
(C) one-one and into (D) many one and into
875.  $\lim_{x \rightarrow 0} |f'(x)|$   
(A) equals 1 (B) equals 2 (C) equals 3 (D) does not exist

## COMPREHENSION-21

### Paragraph for Questions Nos. 876 to 878

$$\text{Let } f(x) = \begin{cases} e^{\{x^2\}} - 1, & x > 0 \\ \frac{\sin x - \tan x + \cos x - 1}{2x^2 + \ln(2+x) + \tan x}, & x < 0, \\ 0, & x = 0 \end{cases}$$

where  $\{ \}$  represents fractional part function. Lines  $L_1$  and  $L_2$  represent tangent and normal to curve  $y = f(x)$  at  $x = 0$ . Consider the family of circles touching both the lines  $L_1$  and  $L_2$

876. Ratio of radii of two circles belonging to this family cutting each other orthogonally is  
(A)  $2 + \sqrt{3}$  (B)  $\sqrt{3}$   
(C)  $2 + \sqrt{2}$  (D)  $2 - \sqrt{2}$
877. A circle having radius unity is inscribed in the triangle formed by  $L_1$  and  $L_2$  and a tangent to it. Then the minimum area of the triangle possible is  
(A)  $3 + \sqrt{2}$  (B)  $3 - \sqrt{2}$   
(C)  $3 + 2\sqrt{2}$  (D)  $3 - 2\sqrt{2}$
878. If centers of circles belonging to family having equal radii ' $r$ ' are joined, the area of figure formed is  
(A)  $2r^2$  (B)  $4r^2$   
(C)  $8r^2$  (D)  $r^2$

## COMPREHENSION-22

### Paragraph for Questions Nos. 879 to 881

While finding the Sine of a certain angle  $x$ , an absent minded professor failed to notice that his calculator was not in the correct angular mode. However he was lucky to get the right answer. The two least positive values of  $x$  for which the Sine of  $x$  degrees is the same as the Sine of  $x$  radians were found by him as  $\frac{m\pi}{n-\pi}$  and  $\frac{p\pi}{q+\pi}$  where  $m, n, p$  and  $q$  are positive integers. Suppose  $\frac{mn}{pq}$  be denoted by the quantity 'L'. Now answer the following questions.

879. The value of  $(m + n + p + q)$  is equal to  
(A) 720 (B) 900  
(C) 1080 (D) 1260
880. If  $x$  is measured in radians and  $\lim_{x \rightarrow \infty} \left( \sqrt{Ax^2 + Bx} - Cx \right) = L$ , the value of  $\frac{BC}{A}$  equals ( $A, B, C \in \mathbb{R}$ )  
(A) 4 (B) 2 (C)  $\frac{1}{2}$  (D) none
881. Assume that  $f$  is differentiable for all  $x$ . The sign of  $f'$  is as follows:  
 $f'(x) > 0$  on  $(-\infty, -4)$   
 $f'(x) < 0$  on  $(-4, 6)$   
 $f'(x) > 0$  on  $(6, \infty)$   
Let  $g(x) = f(10 - 2x)$ . The value of  $g'(L)$  is  
(A) Positive  
(B) negative  
(C) zero  
(D) the function  $g$  is not differentiable at  $x = 5$

## COMPREHENSION-23

### Paragraph for Questions Nos. 882 to 884

Consider a family of curves, where the ordinate is proportional to the cube of the abscissa and let  $A$  be a fixed point in the plane which has coordinates  $(a, b)$ .

882. If tangents be drawn through  $A$  to the members of family of curves then the locus of the point of contact is  
(A)  $xy + bx - 3ay = 0$  (B)  $xy - 4bx + 3ay = 0$   
(C)  $2xy + bx - 3ay = 0$  (D)  $2xy - 4bx + 3ay + 2 = 0$
883. If the tangent through  $A$  to a curve cuts the curve again at a point  $B$  then the locus of  $B$  is  
(A)  $xy + bx - 3ay = 0$  (B)  $xy - 4bx + 3ay = 0$   
(C)  $x^2 - 3y^2 = ax - 3by$  (D)  $x^2 + 3y^2 = ax + 3by$
884. If the tangent through  $A$  to a curve cuts the curve again at a point  $B$  then the locus of  $B$  is  
(A)  $xy - 4bx + 3ay = 0$  (B)  $2xy + bx - 3ay = 0$   
(C)  $x^2 - 3y^2 = ax - 3by$  (D)  $a^2x^2 + b^2y^2 = 1$

## COMPREHENSION-24

### Paragraph for Questions Nos. 885 to 887

A chemical manufacturing company has 1000 kl holding tank which it uses to control the release of pollutants into a sewage system. Initially the tank has 360 kl of water containing 2 kg of pollutant per kl. Water containing 3 kg of pollutant per kl enters the tank at the rate 80 kl per hour and is uniformly mixed with the water already in the tank. Simultaneously, water is released from the tank at the rate of 40 kl per hours.

885. If  $P(t)$  denotes the amount of pollutant at any given time 't' inside the tank, then the rate at which pollutant is leaving the tank is

(A)  $\frac{P(t)}{9-t}$  (B)  $\frac{P(t)}{9+t}$  (C)  $\frac{P(t)}{10+t}$  (D)  $\frac{P(t)}{10-t}$

886. The differential equation giving pollutant at any instant 't' is given by

(A)  $\frac{dP}{dt} + \frac{P}{9+t} = 240$  (B)  $\frac{dP}{dt} - \frac{P}{9+t} = 240$   
(C)  $\frac{dP}{dt} + \frac{P}{10+t} = 240$  (D)  $\frac{dP}{dt} - \frac{P}{10+t} = 240$

887. The amount of pollutant at any time 't' is given by

(A)  $P(t) = 120(9-t) - \frac{3240}{9+t}$  (B)  $P(t) = 120(9+t) + \frac{3240}{9+t}$   
(C)  $P(t) = 120(10-t) + \frac{3240}{10-t}$  (D)  $P(t) = 120(9+t) - \frac{3240}{9+t}$

## COMPREHENSION-25

### Paragraph for Questions Nos. 888 to 890

Let the derivative of  $f(x)$  be defined as  $D^* f(x) = \lim_{h \rightarrow 0} \frac{f^2(x+h) - f^2(x)}{h}$ , where  $f^2(x) = \{f(x)\}^2$ .

888. If  $u = f(x)$ ,  $v = g(x)$ , then the value of  $D^*(u \cdot v)$  is

(A)  $(D^* u) v + (D^* v) u$  (B)  $u^2 D^* v + v^2 D^* u$   
(C)  $D^* u + D^* v$  (D)  $uv D^*(u+v)$

889. If  $u = f(x)$ ,  $v = g(x)$  then the value of  $D^* \left\{ \frac{u}{v} \right\}$  is

(A)  $\frac{u^2 D^* v - v^2 D^* u}{v^4}$  (B)  $\frac{u D^* v - v D^* u}{v^2}$  (C)  $\frac{v^2 D^* u - u^2 D^* v}{v^4}$  (D)  $\frac{v D^* u - u D^* v}{v^2}$

890. The value of  $D^* c$ , where  $c$  is constant, is

(A) non-zero constant (B) 2  
(C) does not exist (D) zero

## COMPREHENSION-26

### Paragraph for Questions Nos. 891 to 893

Consider the implicit equation  $x^2 + 5xy + y^2 - 2x + y - 6 = 0$  .....(i)

891. The value of  $\frac{dy}{dx}$  at (1, 1) is

(A)  $\frac{5}{8}$

(B)  $-\frac{5}{8}$

(C)  $\frac{8}{5}$

(D)  $-\frac{8}{5}$

892. The value of  $\frac{d^2y}{dx^2}$  at (1, 1) is

(A)  $\frac{111}{256}$

(B)  $-\frac{111}{256}$

(C)  $\frac{256}{111}$

(D)  $-\frac{256}{111}$

893. The equation of normal to the conic (i) at (1, 1) is

(A)  $5x - 8y - 3 = 0$

(B)  $8y - 5x - 3 = 0$

(C)  $8x - 5y - 3 = 0$

(D)  $8x - 5y + 3 = 0$

## COMPREHENSION-27

### Paragraph for Questions Nos. 894 to 896

If  $f : [0, 2] \rightarrow [0, 2]$  is a bijective function defined by  $f(x) = ax^2 + bx + c$ , where a, b, c are non zero real numbers, then

894.  $f(2)$  is equal to

(A) 2

(B)  $\alpha$  where  $\alpha \in (0, 2)$

(C) 0

(D) cannot be determined

895. Which of the following is one of the roots  $f(x) = 0$  ?

(A)  $\frac{1}{a}$

(B)  $\frac{1}{b}$

(C)  $\frac{1}{c}$

(D)  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

896. Which of the following is not a value of 'a'

(A)  $a = -\frac{1}{4}$

(B)  $a = \frac{1}{2}$

(C)  $a = -\frac{1}{2}$

(D)  $a = 1$

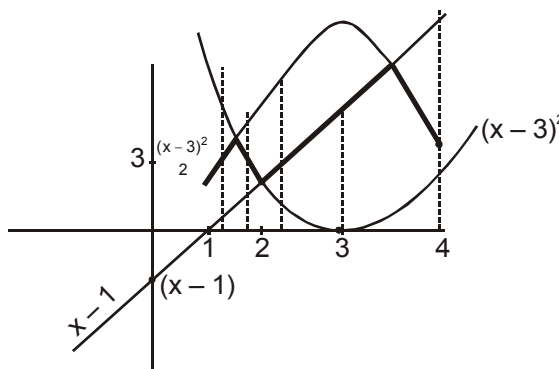


## COMPREHENSION-28

### Paragraph for Questions Nos. 897 to 899

If  $f(x) = \text{Mid} \{g(x), h(x), p(x)\}$  means the function which will be second in order when values of the three function at a particular  $x$  are arranged?

$$f(x) = \text{Mid} \left\{ x-1, (x-3)^2, 3 - \frac{(x-3)^2}{2} \right\}, x \in [1, 4]$$



- 897.** Numerical value of difference between the LHD and RHD at the point  $x = 2$  for  $f(x)$  in  $x \in [1, 4]$  will be  
 (A) 0 (B) 2 (C) 3 (D) 1
- 898.** The greatest value of  $f(x)$  in  $[1, 4]$  will be  
 (A)  $1 + \sqrt{3}$  (B)  $2 + \sqrt{3}$   
 (C)  $3 + \sqrt{3}$  (D) N.O.T.
- 899.** Rate of change of  $x$  w.r.t.  $f(x)$  at  $x = 3$  will be  
 (A) 1 (B)  $\frac{3}{2}$  (C) 2 (D)  $-\frac{3}{2}$

## COMPREHENSION-29

### Paragraph for Questions Nos. 900 to 902

$$\text{Let } f(x) = \begin{cases} 2x + a & : x \geq -1 \\ bx^2 + 3 & : x < -1 \end{cases}$$

$$\text{and } g(x) = \begin{cases} x + 4 & : 0 \leq x \leq 4 \\ -3x - 2 & : -2 < x < 0 \end{cases}$$

functions

- 900.**  $g(f(x))$  is not defined if  
 (A)  $a \in (10, \infty), b \in (5, \infty)$  (B)  $a \in (4, 10), b \in (5, \infty)$   
 (C)  $a \in (10, \infty), b \in (1, 5)$  (D)  $a \in (4, 10), b \in (1, 5)$
- 901.** If domain of  $g(f(x))$  is  $[-1, 4]$ , then  
 (A)  $a = 0, b > 5$  (B)  $a = 2, b > 7$   
 (C)  $a = 2, b > 10$  (D)  $a = 0, b \in \mathbb{R}$
- 902.** If  $a = 2$  and  $b = 3$  then range of  $g(f(x))$  is  
 (A)  $(-2, 8]$  (B)  $(0, 8]$   
 (C)  $[4, 8]$  (D)  $[-1, 8]$

### COMPREHENSION-30

#### Paragraph for Questions Nos. 903 to 905

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a function satisfying  $f(2-x) = f(2+x)$  and  $f(20-x) = f(x)$ ,  $\forall x \in \mathbb{R}$ . For this function  $f$  answer the following.

903. If  $f(0) = 5$ , then minimum possible number of values of  $x$  satisfying  $f(x) = 5$ , for  $x \in [0, 170]$ , is  
(A) 21 (B) 12 (C) 11 (D) 22
904. Graph of  $y = f(x)$  is  
(A) symmetrical about  $x = 18$  (B) symmetrical about  $x = 5$   
(C) symmetrical about  $x = 8$  (D) symmetrical about  $x = 20$
905. If  $f(2) \neq f(6)$ , then  
(A) fundamental period of  $f(x)$  is 1 (B) fundamental period of  $f(x)$  may be 1  
(C) period of  $f(x)$  can't be 1 (D) fundamental period of  $f(x)$  is 8

### COMPREHENSION-31

#### Paragraph for Questions Nos. 906 to 908

If  $f : (0, \infty) \rightarrow (0, \infty)$  satisfy  $f(xf(y)) = x^2y^a$  ( $a \in \mathbb{R}$ ), then

906. Value of  $a$  is  
(A) 4 (B) 2 (C)  $\sqrt{2}$  (D) 1
907.  $\sum_{r=1}^n f(r) {}^nC_r$  is  
(A)  $n \cdot 2^{n-1}$  (B)  $n(n-1) 2^{n-2}$   
(C)  $n \cdot 2^{n-1} + n(n-1) 2^{n-2}$  (D) 0
908. Number of solutions of  $2f(x) = e^x$  is  
(A) 1 (B) 2 (C) 3 (D) 4

### COMPREHENSION-32

#### Paragraph for Questions Nos. 909 to 911

Consider two functions  $f(x) = \lim_{n \rightarrow \infty} \left( \cos \frac{x}{\sqrt{n}} \right)^n$  and  $g(x) = -x^{4b}$  where  $b = \lim_{x \rightarrow \infty} \left( \sqrt{x^2 + x + 1} - \sqrt{x^2 + 1} \right)$ .

Then

909.  $f(x)h$  is  
(A)  $e^{-x^2}$  (B)  $e^{\frac{-x^2}{2}}$  (C)  $e^{x^2}$  (D)  $e^{\frac{x^2}{2}}$
910.  $g(x)$  is  
(A)  $-x^2$  (B)  $x^2$  (C)  $x^4$  (D)  $-x^4$
911. Number of solutions of  $f(x) + g(x) = 0$  is  
(A) 2 (B) 4 (C) 0 (D) 1

### COMPREHENSION-33

#### Paragraph for Questions Nos. 912 to 914

Let  $f(x) = \lim_{n \rightarrow \infty} \left( \cos \sqrt{\frac{x}{n}} \right)^n$ ,  $g(x) = \lim_{n \rightarrow \infty} \left( 1 - x + x \sqrt[n]{e} \right)^n$ . Now, consider the function  $y = h(x)$ , where  $h(x) = \tan^{-1} (g^{-1} f^{-1}(x))$ .

912.  $\lim_{x \rightarrow 0} \frac{\ln(f(x))}{\ln(g(x))}$  is equal to

- (A)  $\frac{1}{2}$  (B)  $-\frac{1}{2}$  (C) 0 (D) 1

913. Domain of the function  $y = h(x)$  is

- (A)  $(0, \infty)$  (B)  $\mathbb{R}$   
(C)  $(0, 1)$  (D)  $[0, 1]$

914. Range of the function  $y = h(x)$  is

- (A)  $\left(0, \frac{\pi}{2}\right)$  (B)  $\left(-\frac{\pi}{2}, 0\right)$  (C)  $\mathbb{R}$  (D)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

### COMPREHENSION-34

#### Paragraph for Questions Nos. 915 to 917

If  $f(x)$  approaches to zero as  $x$  approaches to 'a' then

$$\lim_{x \rightarrow a} \frac{\sin(f(x))}{f(x)} = 1, \lim_{x \rightarrow a} \frac{\tan(f(x))}{f(x)} = 1, \lim_{x \rightarrow a} \frac{\ln(1+f(x))}{f(x)} = 1$$

$$\lim_{x \rightarrow a} \frac{K^{f(x)} - 1}{f(x)} = \ln(K), K > 0 \quad (K \text{ is independent of } x)$$

915.  $\lim_{x \rightarrow 0} \frac{\tan(\sin x)}{\sin x}$  is

- (A) 0 (B) 1 (C)  $\frac{1}{2}$  (D) -1

916.  $\lim_{x \rightarrow \infty} x \sin \left( \frac{2}{x} \right)$  is

- (A) 2 (B) 1 (C)  $\frac{1}{2}$  (D) 0

917.  $\lim_{x \rightarrow 0} \frac{\sin([x^2])}{x^2}$ , where  $[.]$  denote the greatest integer function

- (A) is 1 (B) is 0  
(C) does not exist (D) none of these

## SECTION - 4 (MATRIX MATCH Type)

918. Match the following:

List – I	List – II
(A) $\int_0^{\pi/2} \frac{dx}{1 + \tan x}$	(i) $\frac{1}{117}$
(B) If $\int_0^{x^2(1+x^5+7x^{12})} f(t)dt = x$ , then $f(3)$ is equal to	(ii) $\frac{\pi}{2} - \log 2$
(C) $\int_0^{\infty} e^{-2x} (\sin 2x + \cos 2x) dx$ is equal to	(iii) $\frac{1}{2\sqrt{2}}$
(D) $\int_0^1 \cot^{-1}(1+x^2-x) dx$ is equal to	(iv) $\frac{\pi}{4}$

919. Match the following:

List – I	List – II
(A) $\lim_{x \rightarrow -\infty} \frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{1 +  x ^3}$ is equal to	(i) $-24$
(B) $\lim_{x \rightarrow \infty} \left(\frac{x+8}{x+3}\right)^{x+6}$ is equal to	(ii) $\frac{1}{e}$
(C) $\lim_{x \rightarrow \pi/3} \frac{\tan^3 x - 3 \tan x}{\cos\left(x + \frac{\pi}{6}\right)}$ is equal to	(iii) $-1$
(D) $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$ is equal to	(iv) $e^5$

920. Match the following:

List – I	List – II
(A) $\int_{\pi/6}^{\pi/3} \frac{1}{1 + \sqrt{\tan x}} dx =$	(i) $\frac{3\pi}{2}$
(B) The value of $\alpha$ which satisfy $\int_{\pi/2}^{\alpha} \sin x dx = \sin 2\alpha$ is	(ii) $\frac{\pi}{2}$
(C) $I = \int_0^{\pi/2} \frac{f(x)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx$ , then value of $2I$ is	(iii) $-\frac{\pi}{2}$
(D) $\int_{-1}^1 \left\{ \frac{d}{dx} \left( \tan^{-1} \frac{1}{x} \right) \right\} dx =$	(iv) $\frac{\pi}{12}$

921. Match the following:

List – I	List – II
(A) If $x \cdot e^{xy} = y + \sin^2 x$ , then $\left[ \frac{dy}{dx} \right]_{x=0}$ is equal to	(i) 4
(B) Derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is	(ii) 0
(C) Let $f(x) = \begin{cases} -x-2, & \text{for } -3 \leq x \leq 0 \\ x-2, & \text{for } 0 < x \leq 3 \end{cases}$ , $g(x) =  f(x)  + f( x )$ , then the number of points of non-differentiability of $g(x)$ is	(iii) 1
(D) Let $F(x) = f(x)g(x)h(x) \forall$ real $x$ , where $f, g$ and $h$ are differentiable functions. At some point $x_0$ , $F'(0) = 21 F(x_0)$ , $f'(x_0) = 4f(x_0)g'(x_0) = -7g(x_0)$ and $h'(x_0) = kh(x_0)$ , then $\frac{k}{12}$ is equal to	(iv) 2

922. Match the following

List – I	List – II
(A) Domain of ${}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$	(i) $\left\{-1, 2, \frac{1}{2}\right\}$
(B) Range of $f(x) = \lim_{n \rightarrow \infty} \frac{1+x^{2n}}{2x^{2n}-1}$	(ii) $\{-1, 1\}$
(C) Domain of $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$	(iii) $\phi$
(D) Domain of $f(x) = \frac{1}{\sqrt{x- x }}$	(iv) $\{2, 3\}$

923. If  $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ , then match the following :

List – I	List – II
(A) $\int_0^{\infty} \frac{\sin 5x}{x} dx$	(i) 0
(B) $\int_0^{\infty} \frac{\sin ax \cos bx}{x} dx$ ( $a > b > 0$ )	(ii) $\frac{\pi}{2}$
(C) $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$	(iii) $\frac{\pi}{4}$
(D) $\int_0^{\infty} \frac{\sin^3 x}{x} dx$	(iv) $\pi$

924. Match the following :

List – I	List – II
(A) $x^{100} + \sin x - 1$ is decreasing in	(i) $(-\infty, \infty)$
(B) Domain of $\log_4 \log_5 \log_3 (18x - x^2 - 80)$	(ii) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$
(C) Range of $x^3 + 3x^2 + 10x + 2\sin x$	(iii) $(8, 10)$
(D) $\left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$ decreases for all $x$ the set of values of $a$	(iv) none of these
	(v) $\left(\frac{\pi}{2}, \pi\right)$

925. Match the following:

List – I	List – II
(A) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a periodic function such that $f(T + x) = 1 + [1 - 3f(x) + 3(f(x))^2 - (f(x))^3]^{1/3}$ , where $T$ is fixed positive number, then period of $f(x)$ is $AT$ , where $A =$	(i) 3
(B) The area between the curve $y = 2x^4 - x^2$ , the $x$ -axis and the ordinates of two minima of the curve is $\frac{B}{120}$ where $B$ is	(ii) 2
(C) $\int_0^4 \frac{f(x)}{f(x) + f(4-x)} dx =$	(iii) 7
(D) $f(x) = \begin{cases} \frac{1 - \sin x}{(\pi - 2x)^2} \log \sin x \\ k, \end{cases}$ is continuous at $x = \frac{\pi}{2}$  $= \frac{\pi}{2}$ then $-\frac{1}{k}$ equals to	(iv) 64

926. Match the list:

List – I	List – II
(A) $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \sqrt{\frac{n+r}{n-r}}$	(i) $\frac{\pi}{2}$
(B) $\lim_{x \rightarrow \infty} \left[ \frac{1}{\sqrt{n^2-1}} + \frac{1}{n^2-2^2} + \cdots + \frac{1}{\sqrt{n^2-(n-1)^2}} \right]$	(ii) $\frac{\pi}{2} + 1$
(C) $\lim_{x \rightarrow \infty} \left( \frac{n!}{n^n} \right)^{1/n}$	(iii) $2\pi$
(D) $\int_0^{2\pi} e^{\cos x} \cos(\sin x) dx$	(iv) $\frac{1}{e}$
	(v) $e$

927. Match the following:

List I (Expression)		List II (Value)	
I.	If $f(x + y) = f(x) f(y)$ ( $x, y$ are independent $\forall x, y \in \mathbb{R}$ and $f(2) = f'(2) = 3$ then $f'(4) =$	(A)	0
II.	If $f(xy) = f(x) f(y)$ and $f'(4) = 2f'(8)$ , then $f(2) =$	(B)	10
III.	If $f(x)$ is a diff. function such that $f(xy) = f(x) + f(y) \forall x, y \in \mathbb{R}$ then $f(e) + f\left(\frac{1}{e}\right) =$	(C)	9
IV.	If $f$ is a twice diff. function Such the $f'(x) = -f(x)$ If $h(x) = (f(x))^2 + (g(x))^2$ And $h(5) = 10$ . Then $h(10) =$	(D)	1

928. Match the following:

- (A)  $\frac{\sin 1}{\sin 2} - \frac{\sin 5}{\sin 6}$  (1) positive
- (B)  $\tan \frac{3}{2} - \frac{9}{4}$  (2) negative
- (C)  $\lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{x} \right]$  (where  $[.]$  denotes the greatest integer function) (3) 1
- (D) If  $f'(\alpha) = 0$  and  $f'(x) > 0 \forall x \in \mathbb{R} - \{\alpha\}$ , then  $f''(\alpha)$  is (4) does not exist  
(5) 0

929. If  $y = \cos^{-1} \left( \frac{a \cos x + b}{a + b \cos x} \right) - 2 \tan^{-1} \left( \sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right)$ , then match the following :

- | Column-I  | Column-II         |
|---|-------------------|
| (A) If $a = 4, b = 3$ , then $\frac{dy}{dx}$ at $x = 0$ | (P) 0             |
| (B) Number of points of local minima                    | (Q) $\frac{4}{5}$ |
| (C) For $a = 4, b = 3$ , value of $y$ at $x = 0$        | (R) $\frac{3}{5}$ |
| (D) Number of tangents parallel to the $y$ axis         | (S) 2             |

930. Let  $f(x) = ax^2 + bx + c$ , Given that  $f'(1) = 8, f(2) + f''(2) = 33$  and  $2a + 3b + 6c = 14$ , then match the following

- | Column-I   | Column-II       |
|--|-----------------|
| (A) Global maximum value of $f(x)$                               | (P) Not defined |
| (B) If global minimum value of $f(x) = k$ then $28k$ is equal to | (Q) 48          |
| (C) Number of real roots of $f(x) = 0$                           | (R) 0           |

- (D) Number of real roots of  $f(x) = 3$  (S) 2

931.

	Column I		Column II
(A)	The number of non-differentiability points on the curve $y =  e^{ x } - 3 $ is/are	(p)	1
(B)	Length of the latus-rectum of the parabola defined by $x = \cos t - \sin t$ and $y = \sin 2t$	(q)	0
(C)	The number of real solution of the equation $x^{2\log_x(x+3)} = 16$ is	(r)	3
(D)	If in a triangle $2R + r = r_1$ , then $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right)$ is equal to	(s)	2

932.

(A)	Area of the rectangle formed by asymptotes of the hyperbola $xy - 3y - 2x = 0$ and co-ordinate axes is	(p)	32
(B)	Area bounded by $\min( x ,  y ) = 1$ and $\max( x ,  y ) = 3$ is,	(q)	2
(C)	The number of common tangents of the two circles $x^2 + y^2 - 10x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 2y - 11 = 0$ are	(r)	5
(D)	The greatest value of $f(x) = 2\cos(2xe^x + 7x^4 - \log(1 + x^2))$	(s)	6

933. List I with List II and select the correct answers using the codes given below the lists :

List I	List II
LIMIT	VALUE
1. $\lim_{x \rightarrow 0} \frac{(x)}{\tan(x)}$	(A) $-\log_{16}e$
2. $\lim_{x \rightarrow 0} \frac{\sqrt{1-x+x^2} - \sqrt{1+x^2}}{4^x - 1}$	(B) $e^{-1}$
3. $\lim_{x \rightarrow 0} \frac{2e^{\sin x} - (1 + \sin x)^2}{2(\tan^{-1}(\sin x))^2}$	(C) 1
4. $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}$	(D) 0

934.

List I	List II
Limits	Value
I. $\lim_{x \rightarrow 2} \frac{\sqrt{1 + \sqrt{2+x}} - \sqrt{3}}{x - 2}$	(A) $-\frac{4}{3}$
II. $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{3-x} - 3^{x/2}}$	(B) $\frac{1}{2}$
III. $\lim_{x \rightarrow 2} \left( \frac{x^3 - 4x}{x^3 - 8} \right)^{-1} - \left( \frac{x + \sqrt{2}x}{x - 2} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1}$	(C) 8
IV. $\lim_{x \rightarrow 2} \frac{2^x + 2^{3/2} - 6}{\sqrt{2-x} + 2^{1-x}}$	(D) $\frac{1}{4(3 + \sqrt{3})}$



935.

List I

Function  $f(x)$ 

- I.  $f(x) = \frac{1 + \cos 5x}{1 - \cos 4x}$  in  $0 \leq x \leq \pi$
- II.  $f(x) = [\cos x + \sin x]$   $0 < x < 2\pi$
- III.  $f(x) = 4x + 7[x] + 2\log(1 + x)$
- IV.  $f(x) = \int_0^x t \sin \frac{1}{t} dt$  where  $0 < x < \pi$

936.

List I

(Function)

- I.  $f(x) = \left[ \frac{x}{\sin(x)} \right]$
- II.  $f(x) = \frac{a^{[x]+x} - 1}{[x] + x}$
- III.  $f(x) = \frac{\sin[\cos x]}{1 + [\cos x]}$
- IV.  $f(x) = \frac{1}{x} \left( \int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt \right)$

Where  $[.]$  denotes G.I.F..

937.

List I

(Function)

- I.  $f(x) = \begin{cases} 1, & x \leq 0 \\ 1 + \sin x, & 0 \leq x < \frac{\pi}{2} \end{cases}$
- II.  $= x(\sqrt{x} - \sqrt{x+1})$
- III.  $= x^3 \operatorname{sgn} x$
- IV.  $g(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases}$
- $f(x) = g(|x|) + |g(x)|$

List II

The No. of points of discontinuity

- (A) 0
- (B) Infinite
- (C) 5
- (D) 3

List II

(Limit at  $x = 0$ )

- (A)  $\lim_{x \rightarrow 0} f(x) = 0$
- (B)  $\lim_{x \rightarrow 0} f(x) = e^{\sin^2 y}$
- (C)  $\lim_{x \rightarrow 0^-} f(x) = 1 - \frac{1}{a}$
- (D)  $\lim_{x \rightarrow 0} f(x) = 1$

List II

(Derivative)

- (A)  $L\text{-}\lim_{x \rightarrow 0} f(x) = -1, R\text{-}\lim_{x \rightarrow 0} f(x) = 0$
- (B)  $f'(0) = 0$
- (C)  $f'(0) = 1$
- (D)  $f'(0) = -1$  does not exist

938. Match the columns -

Column - I	Column - II
(a) The number of values of $c$ for which $\int_0^1  (c-x)  dx = \frac{1}{2}$ is	(P) 2
(b) If $\int \frac{\operatorname{cosec}\left(2x - \frac{5\pi}{6}\right)}{\sin\left(2x - \frac{\pi}{6}\right)} dx = \frac{k}{\sqrt{3}} \ln \left( \frac{\sin\left(2x - \frac{5\pi}{6}\right)}{\sin\left(2x - \frac{\pi}{6}\right)} \right) + c$ , then $k =$	(Q) 1
(c) Area of the region bounded by the $x^2 + y^2 - 2x \leq 0$ , $x + y \leq 1$ , $y \geq 0$ is	(R) $\frac{\pi}{8}$
(d) $\frac{1}{\pi} \int_{-\pi}^{\pi} \frac{x \sin x}{e^x + 1} dx$ is equal to	(S) 8

939. Match the following

Column - I	Column - II
(a) The number of solutions of the equation $x \cdot 2^x = x + 1$ is	(P) 4
(b) $\lim_{n \rightarrow \infty} \left( \frac{1 + \sqrt[n]{4}}{2} \right)^n$ is equal to	(Q) 8
(c) The number of points at which $g(x) = \frac{1}{1 + \frac{2}{f(x)}}$ is not differentiable where $f(x) = \frac{1}{1 + \frac{1}{x}}$ , is :	(R) 2
(d) $f\left(x + \frac{1}{2}\right) + f\left(x - \frac{1}{2}\right) = f(x)$ for all $x \in \mathbb{R}$ , then period of $f(x)$ is	(S) 3

940. Match the following

Column - I	Column - II
(a) The least positive integral solution of $x^2 - 4x > \cot^{-1} x$	(P) 1
(b) The least positive integral value of $x$ for which $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ is increasing	(Q) 2
(c) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that $f(a) = 1, f'(a) = 2$ , then $\lim_{x \rightarrow 0} \left( \frac{f^2(a+x)}{f(a)} \right)^{\frac{1}{x}} = e^k$ , then $k =$	(R) 5
(d) Number of integral values of $x$ which satisfy equation $\sin^{-1}((3x-x)(x-1)) + \sin^{-1}(2- x ) = \frac{\pi}{2}$ is/are	(S) 4

941. Consider  $f(x) = t^{|x^2 - 4x + 3|}$ , where  $t$  is a real number greater than 1. Then

**Column I**

- (A)  $f(x)$  increases in the interval (P)  
 (B)  $f(x)$  decreases in the interval (Q)  
 (C) Local maxima of  $f(x)$  occurs in the interval (R)  
 (D)  $f(x)$  has a local minima in the interval (S)

**Column II**

- $(-\infty, 1)$   
 $(0, 2)$   
 $(1, 2) \cup (3, \infty)$   
 $(1, 3)$

942. Match the column

**Column I**

- (A) Let  $a, b, c$  be positive and  $a + b + c = abc$  the maximum value of square of least among  $a, b, c$  is (P)  
 (B) The fundamental period of the function  $y = \sin^2\left(\frac{\sqrt{2}t + 3}{6\pi}\right)$  is  $\lambda\pi^2$  then the value of  $\frac{\lambda}{\sqrt{2}}$  is (Q)  
 (C) If  $(x + y)^m$  has three consecutive coefficients in A.P. ( $m \in \mathbb{N}$ ) for which the sum of first 'n' values of  $m$  is  $an^3 + bn^2 + cn + d$ . The value of greatest integer of  $\left(\frac{a + b + c + d}{2}\right)$  is (R)

- (D) If equation of tangent to the curve  $y = \int_{x^2}^{\frac{x^3}{\sqrt{1+t^2}}} dt$  at  $x = 1$  is  $\sqrt{2}x = by + \sqrt{2}$  then value of  $\frac{b}{2}$  is (S)

**Column II**

- 1  
 2  
 3  
 Number of solution of  $\cos x + \cos \sqrt{2}x = 2$   
 Number of values of  $x$  for which  $f(x) = \frac{1}{\ln|x|}$  is not defined

943. Match the column

**Column I**

- (A) The area of the quadrilateral formed by the tangents from the point  $(4, 0)$  to the circle  $x^2 + y^2 - 2x + 4y - 4 = 0$  and the pair of radii through the points of contact of the tangent is  
 (B) The number of points at which the function  $f(x) = \frac{1}{\ln|x|}$  is discontinuous is  
 (C) Let  $f(x + y) = f(x) \cdot f(y)$  for all  $x, y \in \mathbb{R}$  if  $f(5) = 2$  and  $f'(0) = 3$ , then  $f'(5) =$   
 (D) If the normal to the curve  $y = f(x)$  at the point  $(3, 4)$  makes an angle  $\frac{3\pi}{4}$  with the positive  $x$ -axis, then  $f'(3) =$

**Column II**

- (P) 1  
 (Q) 3  
 (R) 6  
 (S) 4

944. Match the following

Column-I

- (a) If the point (6, k) is closest to the curve  $x^2 = 2y$  at (2, 2), then k =  
 (b) If the curve  $y = px^2 + qx + r$  passes through the point (1, 2) and touches the line  $y = x$  at the origin, then the value of  $p - q + r =$   
 (c) Let  $f(x) = kx^3 + 9kx^2 + 9x + 3$  be a strictly increasing function and has non stationary point. The greatest value of k is

(d) Let  $0 < a < b < \frac{\pi}{2}$ . If  $f(x) = \begin{bmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{bmatrix}$ , then minimum

Column-II

- (P) 0  
 (Q) -2  
 (R) 1  
 (S) does not exist

possible number of roots of  $f'(x) = 0$  lying in (a, b) is

945. Column I

- (A) if  $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$  where  $g(x) = \int_0^{\cos x} (1 + \sin t^2) dt$   
 then the value of  $f'(\pi/2)$   
 (B) If  $f(x)$  is a non zero differentiable function such that  
 $\int_0^x f(t)dt = (f(x))^2$  for all x, then  $f(2)$  equals  
 (C) If  $\int_a^b (2 + x - x^2) dx$  is maximum then (a + b) is equal to  
 (D) If  $\lim_{x \rightarrow 0} \left( \frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = 0$  then (3a + b) has the value equals to

Column II

- (P) 3  
 (Q) 2  
 (R) 1  
 (S) -1

946.

Column I

- (A) Number of integers which do not lie in the range of the function  $f(x) = \sec \left( 2 \sin^{-1} \frac{1}{x} \right)$   
 (B) Let  $f : (0, \infty)$  onto  $(0, \infty)$  be a derivable function for which there exists its primitive F such that  $2(F(x) - f(x)) = f^2(x)$  for any real positive x. Then  $\lim_{x \rightarrow \infty} \frac{f(x)}{x}$  equals  
 (C) How many of the following derivatives are correct (on their domains)?  
 I.  $\frac{d}{dx} \ln |\sec x| = \tan x$  II.  $\frac{d}{dx} \ln(x + e^x) = 1 + \frac{1}{x}$   
 III.  $\frac{d}{dx} x^{\ln x} = (\ln x) x^{\ln(x)-1}$   
 (D) A differentiable function satisfies  $f'(x) = f(x) 2e^x$  with initial conditions  $f(0) = 0$ . The area enclosed  $f(x)$  and the x-axis is

Column II

- (P) 0  
 (Q) 1  
 (R) 2  
 (S) 3

947. Column I

Column II

(A) Suppose,  $f(n) = \log_2(3) \cdot \log_3(4) \cdot \log_4(5) \dots \log_{n-1}(n)$  then the sum  $\sum_{k=2}^{100} f(2^k)$  equals (P) 5010

(B) Let  $f(x) = \sqrt{1+x} \sqrt{1+(x+1)} \sqrt{1+(x+2)} \dots \sqrt{1+(x+99)}$  then  $\int_0^{100} f(x) dx$  is (Q) 5050

(C) In an A.P. the series containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is (R) 5100

(D)  $\lim_{x \rightarrow 0} \frac{\prod_{r=1}^{100} (1+rx) - 1}{x}$  equals (S) 5049

948. Column - I

Column - II

(A) Let  $f$  be continuous and the function  $F$  is defined as (P) 0

$F(x) = \int_0^x \left( t^2 \cdot \int_1^t f(u) du \right) dt$  where  $f(1) = 3$ , then  $F'(1) + F''(1)$  has the value equal to (Q) 1

(B) For each value of  $x$  a function  $f(x)$  is defined as (R) 2

$\min \{2x + 3, \frac{(x+4)}{3}, 3(6-x)\}$  Maximum value of  $f(x)$  is .

(C)  $\lim_{x \rightarrow 1^+} (\ln x)^{\frac{1}{(x-1)\tan x}}$  (S) 3

(D) Exponent of 2 in the binomial coefficient  ${}^{500}C_{212}$  is

949.

Column I

Column II

(A) If three normals can be drawn to the curve  $y^2 = x$  from the point  $(c, 0)$ , then  $c$  can be equal to (P) 1  
(Q) 0

(B) Subnormal length to  $xy = c^2$  at any point varies directly as (R)  $\frac{5}{4}$

(C) If the sides and angles of a plane triangle vary in such a way that its circum radius remains constant, then (S) Cube of ordinate  
(T) 2

$$\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} =$$

where  $da, db, dc$  are small increments in the sides  $a, b, c$ , respectively

950.	Column-I	Column-II
(A)	$\int_{-\pi/2}^{\pi/2} \frac{\ln(\cos x)}{1+e^x \cdot e^{\sin x}} dx =$	(P) $-2\pi \ln 2$
(B)	$\int_0^{2\pi} \ln(1+\sin x) dx =$	(Q) $-\frac{\pi}{4} \ln 2$
(C)	$\int_{-\pi/4}^{\pi/4} \ln \sqrt{1+\sin 2x} dx =$	(R) $-\pi \ln 2$
(D)	$\int_{-\infty}^0 \frac{xe^{-x}}{\sqrt{1-e^{-2x}}} dx.$	(S) $-\frac{\pi}{2} \ln 2$

951.	Column I	Column II
(A)	The function $f(x) - (x - [x])^2$ , (where $[x]$ is greatest integer function $\leq x$ ) is	(P) periodic (Q) non - periodic
(B)	The function $f(x) = \log_a \left( x + \sqrt{x^2 + 1} \right)$ ; $a > 0, a \neq 1$ is (assume it to be an onto)	(R) one - one (S) many one
(C)	The function $f(x) = \cos(5x + 2)$ is	(T) invertible

952.	Column I	Column II
(A)	The area bounded by the curve $\max\{ x ,  y \} = 1$ is	(P) 0
(B)	If the point $(a, a)$ lies between the lines $ x + y  = 6$ , then [ a ] is (where [.] denotes the greatest integer function)	(Q) 1 (R) 2
(C)	Number of integral values of $b$ for which the origin and the point $(1, 1)$ lie on the same side of the st. line $a^2x + aby + 1 = 0$ for all $a \in \mathbb{R} \sim \{0\}$	(S) 3 (T) 4

953.	Column I	Column II
(A)	The slope of the curve $2y^2 = ax^2 + b$ at $(1, -1)$ is -1, then	(P) $a - b = 2$
(B)	If $(a, b)$ be the point on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes, then	(Q) $a - b = \frac{7}{2}$ (R) $a - b = \frac{4}{3}$
(C)	If the tangent at any point $(1, 2)$ on the curve $y = ax^2 + bx + \frac{7}{2}$ be parallel to the normal at $(-2, 2)$ on the curve $y = x^2 + 6x + 10$ , then	(S) $\alpha +  b  = \frac{20}{3}$ (T) $5a + 2b = 0$

954. Match the column

**Column – I**

**Column – II**

- |   |       |
|---|-------|
| (A) The number of possible values of k if<br>fundamental period of $\sin^{-1}(\sin kx)$ is $\frac{\pi}{2}$                          | (p) 1 |
| (B) Numbers of points in the domain of<br>$f(x) = \tan^{-1}x + \sin^{-1}x + \sec^{-1}x$   | (q) 2 |
| (C) $f(x) = \sin\left(\frac{\pi x}{2}\right) \cdot \operatorname{cosec}\left(\frac{\pi x}{2}\right)$ is periodic with period        | (r) 3 |
| (D) If range of the function $f(x) = \cos^{-1}[5x]$ where $[.]$ denotes<br>greatest integer, is $\{a, b, c\}$ , then $a + b + c$ is | (s) 4 |

955. Match the column

**Column – I**

**Column – II**

$[.]$  and  $\{.\}$  represent the greatest integer and fractional part functions respectively.

- |  |       |
|--|-------|
| (A) Number of solutions of $[x] = \cos^{-1}x$                          | (p) 3 |
| (B) Number of solutions of $\sin^{-1}x = \operatorname{sgn}(x)$        | (q) 2 |
| (C) Number of solutions of $\{x\} = e^{x^2}$                           | (r) 1 |
| (D) Number of solutions of $\frac{\sin^{-1}x + \cos^{-1}x}{2} = \{x\}$ | (s) 0 |

956. Match the column

**Column – I**

**Column – II**

- |   |                      |
|---|----------------------|
| (A) Smallest positive integral value of x for<br>which $x^2 - x + \sin^{-1}(\sin 2) < 0$ is   | (p) $\frac{3\pi}{2}$ |
| (B) Number of solution of $2[x] = x + 2\{x\}$ is<br>where $[x]$ , $\{x\}$ are greatest integer and least integer<br>functions respectively. | (q) 3                |
| (C) If $x^2 + y^2 = 1$ , then maximum value of $x + y$ is   | (r) 1                |
| (D) $f\left(x + \frac{1}{2}\right) + f\left(x - \frac{1}{2}\right) = f(x)$ for all $x \in \mathbb{R}$ ,<br>then period of $f(x)$ is         | (s) 2                |

957. Match the column

**Column – I**

- (A) If function  $f(x)$  is defined in  $[-2, 2]$ , then domain of  $f(|x| + 1)$  is
- (B) If range of the function  $f(x) = \frac{\sin^{-1} x + \cos^{-1} x + \tan^{-1} x}{\pi}$  is
- (C) Range of the function  $f(x) = 3 |\sin x| - 4 |\cos x|$  is
- (D) Range of  $f(x) = (\sin^{-1} x) \sin x$  is

**Column – II**

- (p)  $\left[ \frac{1}{4}, \frac{3}{4} \right]$
- (q)  $[-1, 1]$
- (r)  $[-4, 3]$
- (s)  $\left[ 0, \frac{\pi}{2} \sin 1 \right]$

958. **Column - I**

- (A) Range of  $\text{sgn} \{x\}$  is (where  $\{.\}$  represents fractional part function)
- (B) Domain of  $\sin^{-1} x + \sin^{-1} (1 - x)$  is
- (C) Range of  $\sqrt{\frac{2 \tan^{-1} x}{\pi}}$  is
- (D) Range of  $\frac{2}{\pi} \sin^{-1} [x^2 + x + 1]$  is (where  $[.]$  represent greatest integer function)

**Column - II**

- (p)  $\{1\}$
- (q)  $[0, 1)$
- (r)  $\{0, 1\}$
- (s)  $[0, 1]$

959. **Column – I**

- (A) Domain of  $f(x) = \sin^{-1} \left( \frac{2-x}{2x} \right)$  is
- (B) Range of  $f(x) = \frac{2x^2 - 2}{3x^2 + 1}$  is
- (C) Set of all values of  $p$  for which the function  $f(x) = px + \sin x$  is bijective is
- (D) If  $f : (-\infty, 1] \rightarrow A$  is defined by  $f(x) = x^2 - 3x$ , then set  $A$  for which  $f(x)$  becomes invertible, is

**Column – II**

- (p)  $[-2, \infty)$
- (q)  $(-\infty, -1] \cup [1, \infty)$
- (r)  $(-\infty, -2] \cup [2/3, \infty)$
- (s)  $[-2, 2/3]$



960. Match the column

Column – I

Column – II

(A)  $\lim_{x \rightarrow \frac{\pi}{2}^+} \frac{\cos(\tan^{-1}(\tan x))}{x - \frac{\pi}{2}}$

(p)  $\frac{3\pi}{2}$

(B) The number of solutions of the equation

(q)  $-\frac{1}{\sqrt{2}}$

$2 \cos x = |\sin x|$   $0 \leq x \leq 4\pi$  is

(C) If  $y = \tan^{-1} \sqrt{\frac{1 - \sin x}{1 + \sin x}}$ , then the value

(r) 1

of  $\frac{dy}{dx}$  at  $x = \frac{\pi}{6}$  is

(D) If  $f(x) = e^{(2x)} + \sin 2\pi x$ , the period of  $f(x)$  is  
 $\{ \}$  represents fractional part function

(s) Does not exist

961. Match the column

Column – I

Column – II

(A) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function and

(p) 0

$f(1) = 1, f'(1) = 3$ . Then the value of  $\lim_{x \rightarrow 1} \int_1^{x^2} \frac{(f(t) - t)}{(x - 1)^2} dt$  is

(B)  $\lim_{n \rightarrow \infty} \left( \frac{1 + \sqrt[n]{4}}{2} \right)^n$  is equal to

(q) -1

(C) If  $f(x) = \lim_{n \rightarrow \infty} \frac{2x}{\pi} \cdot \tan^{-1}(nx)$ ,  $x > 0$

(r) 2

then  $\lim_{x \rightarrow 0^+} [f(x) - 1]$  is,  
 $\{ \}$  where  $[ ]$  represents greatest integer function

(D)  $\lim_{n \rightarrow \infty} \left[ \sum_{r=1}^n \frac{1}{2^r} \right]$ ,

(s) 4

where  $[ ]$  denotes the greatest integer function

(t) 1

962.	Column - I	Column - II
(A)	Number of points of discontinuity of $f(x) = \tan^2 x - \sec^2 x$ in $(0, 2\pi)$ is	(p) 1
(B)	Number of points at which $f(x) = \sin^{-1}x + \tan^{-1}x + \cot^{-1}x$ is non-differentiable in $(-1, 1)$ is	(q) 2
(C)	Number of points of discontinuity of $y = [\sin x]$ , $x \in [0, 2\pi)$ where $[.]$ represents greatest integer function	(r) 0
(D)	Number of points where $y =  (x-1)^3  +  (x-2)^5  +  x-3 $ is non-differentiable	(s) 3

963.	Column - I	Column - II
	For $x \in \mathbb{R}$ ,	
(A)	$f(x) = \{\sin(\pi x)\}$ is discontinuous for $x \in$	(p) $[0, 1)$
(B)	$g(x) = \left\{ \frac{\sin x}{x} \right\}$ is discontinuous for $x \in$	(q) $\{1, 2\}$
(C)	$h(x) = \frac{\{\sin x\}}{\{x\}}$ is non-differentiable for $x \in$	(r) $\{0\}$
(D)	$u(x) = \frac{(\sin x)}{[x]}$ is discontinuous function for $x \in$	(s) $\left\{ \frac{1}{2} \right\}$

964.	Column - I	Column - II
(A)	Point of discontinuity of $y = \frac{1}{t^2 - t - 2}$ where $t = \frac{1}{x+1}$	(p) $-\frac{1}{2}$
(B)	Points of continuity of $y = [x] + [-x]$	(q) $-2$
(C)	$y = [\sin(\pi x)]$ is non differentiable at	(r) $-1$
(D)	$f(x) =  2x + 1  +  x + 2  -  x + 1  -  x - 4 $ is non differentiable at	(s) 4

965. Match the column

**Column – I**

**Column – II**

(A)  $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$  is equal to

(p) does not exist

(B) If  $f(x) = \log_{x^2}(\log x)$ , then  $f'\left(\frac{1}{2}\right)$  is equal to

(q) 0

(C) For the function  $f(x) = \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$   
if  $\frac{dy}{dx} = \sec x + p$ , then  $p$  is equal to

(r) 28

(D)  $\lim_{x \rightarrow 0} \frac{1}{x} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$  is equal to

(s) 4

966. Match the column

**Column – I**

**Column – II**

(A) If  $y = \cos^{-1}(\cos x)$ , then  $y'$  at  $x = 5$  is equal to

(p) -1

(B) The value of  $\frac{1}{2^{11}} \sum_{0 \leq i \leq j \leq 8} i \cdot {}^8C_j$  is

(q)  $-\frac{1}{2}$

(C) The derivative of  $\tan^{-1}\left(\frac{1+x}{1-x}\right)$  at  $x = 1$  is

(r)  $\frac{1}{2}$

(D) The derivative of  $\frac{\log|x|}{x}$  at  $x = -1$  is

(s) 1

## SECTION-5 (INTEGER TYPE)

967. Let  $f(x) = \max\{x^2, (1-x)^2, 2x(1-x)\}$ , where  $0 \leq x \leq 1$ . Then the integral part of area of the region bounded by the curves  $y = f(x)$ ,  $x$ -axis  $x = 0$  and  $x = 1$  is \_\_\_\_\_
968. If  $g(x) = 2 + \cos x \cos\left(x + \frac{\pi}{3}\right) - \left(\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right)\right)$  and  $f\left(\frac{5}{4}\right) = 9$ , then the value of  $f(g(x))$  is \_\_\_\_\_
969. Let  $f(x)$  be a polynomial of degree 3 if the curve  $y = f(x)$  has relative extremities at  $x = \pm \frac{2}{\sqrt{3}}$  and passes through  $(0, 0)$  and  $(1, -2)$  dividing the circle  $x^2 + y^2 = 4$  in two parts. Then the integral part of areas of these two parts is \_\_\_\_\_
970.  $I = \int_0^{1.5} [x^2] dx$  where  $[.]$  is greatest integer function then the value of  $[I]$  is \_\_\_\_\_
971. From a point A on the curve  $x = 3y^2 - 2y + 7$ , subnormal and subtangent are drawn. If they measure 1 unit each, distance of A from  $(4, 1)$  is \_\_\_\_\_
972. The value of  $\frac{8\sqrt{2}}{\pi} \int_0^1 \left( \frac{1-x^2}{1+x^2} \right) \frac{dx}{\sqrt{1+x^4}}$  is \_\_\_\_\_
973. Let A be the area of the region bounded by the curve  $a^4 y^2 = (2a - x)x^5$  and B be the area of the circle whose radius is  $\frac{a}{2}$ , then  $\frac{A}{B}$  is \_\_\_\_\_
974. The area bounded by curves  $y = \left[ 6 + x \left[ \frac{1}{x} \right] \right]$ ,  $y^2 - 18x + 18 = 0$  and  $6x - 5y - 6 = 0$ , (where  $[.]$  denotes the greatest integer function) is \_\_\_\_\_.
975.  $\left[ \int_{-1}^{\sqrt{3}} \frac{\sin^{-1} \frac{2x}{1+x^2}}{1+x^2} dx \right] =$  \_\_\_\_\_; where  $[.]$  denotes G.I.F.
976. The value of  $\lim_{x \rightarrow 0} \frac{16 - 16 \cos(1 - \cos x)}{x^4}$  is \_\_\_\_\_
977. The altitude of a right circular cone of minimum volume circumscribed about a sphere of radius 2 is \_\_\_\_\_.
978.  $\int 7 \left\{ \frac{x^2(x^4 - 4x - 3)}{(x^3 - 1)^2} \cos x - \frac{(x^4 + 1)}{(x^3 - 1)} \sin x \right\} dx =$  \_\_\_\_\_.
979. The value of  $\lim_{x \rightarrow \pi/2} 12 \tan^2 x \left[ \sqrt{6 + 3 \sin x - 2 \cos^2 x} - \sqrt{3 + 6 \sin x - \cos^2 x} \right]$  is \_\_\_\_\_
980. If  $\int_0^{\pi/2} \frac{dx}{(\sqrt{\cos x} + \sqrt{\sin x})^4} = A$ . Then the values of  $6A$  is \_\_\_\_\_

981. A closed right circular cylinder has volume 2156 cubic units. The radius of its base so that its total surface area may be minimum is \_\_\_\_\_.
982. If  $g(x)$  is a polynomial satisfying  $g(x)g(y) = g(x) + g(y) + g(xy) - 2 \forall x, y \in \mathbb{R}$  and  $g(2) = 5$ , then the value of  $g(3)$  is \_\_\_\_\_.
983. Let  $A$  be the area of the region in the first quadrant bounded by the  $x$ -axis the line  $2y = x$  and the ellipse  $\frac{x^2}{9} + \frac{y^2}{1} = 1$ . Let  $A'$  be the area of the region in the first quadrant bounded by the  $y$ -axis, the line  $y = kx$  and the ellipse. The value of  $9k$  such that  $A$  and  $A'$  are equal is \_\_\_\_\_.
984. If  $A$  be the area bounded by  $y = f(x)$ ,  $y = f^{-1}(x)$  and line  $4x + 4y - 5 = 0$  where  $f(x)$  is a polynomial of 2<sup>nd</sup> degree passing through the origin and having maximum value of  $1/4$  at  $x = 1$ , then  $96A$  is equal to \_\_\_\_\_.
985. Let  $y = g(x)$  be the image of  $f(x) = x + \sin x$  about the line  $x + y = 0$ . If the area bounded by  $y = g(x)$ ,  $x$ -axis,  $x = 0$  and  $x = 2\pi$  is  $A$ , then  $\frac{A}{\pi^2}$  is \_\_\_\_\_.
986. If  $I_n = \int_0^{\infty} e^{-x} (\sin x)^n dx$  ( $n > 1$ ), then the value of  $\frac{101I_{10}}{I_8}$  is equal to \_\_\_\_\_.
987. The number of solutions of the equation  $[\sin^{-1} x] = x - [x]$ , where  $[.]$  denotes the greatest integer function is \_\_\_\_\_.
988. Find the value of  $a + c$  so that:  

$$\lim_{x \rightarrow \infty} \left( \sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d} \right) = 4$$
989. Find the value of limits using expansion :  $2 \lim_{x \rightarrow 0} \left[ \frac{\ell n(1+x)^{(1+x)}}{x^2} - \frac{1}{x} \right]$
990. Evaluate  $3 \lim_{x \rightarrow 0^+} \left\{ n \lim_{n \rightarrow \infty} \left( \frac{[1^2 (\sin x)^x] + [2^2 (\sin x)^x] + \dots + [n^2 (\sin x)^x]}{n^3} \right) \right\}$ ,  
 where  $[.]$  denotes the greatest integer function.
991. Evaluate the following limit  $\lim_{x \rightarrow \infty} \log_{x-1}(x) \cdot \log_x(x+1) \cdot \log_{x+1}(x+2) \cdot \log_{x+2}(x+3) \dots \log_k(x^5)$ ;  
 where  $k = x^5 - 1$ .
992. Let  $P_n = \frac{2^3 - 1}{2^3 + 1} \cdot \frac{3^3 - 1}{3^3 + 1} \cdot \frac{4^3 - 1}{4^3 + 1} \dots \frac{n^3 - 1}{n^3 + 1}$ . find the value of  $\lim_{n \rightarrow \infty} 6P_n$ .
993.  $\lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+3)!}$ ,  $n \in \mathbb{N} =$
994. If  $f(x) = \begin{cases} x-1, & x \geq 1 \\ 2x^2 - 2, & x < 1 \end{cases}$ ,  $g(x) = \begin{cases} x+1, & x > 0 \\ -x^2 + 1, & x \leq 0 \end{cases}$ , and  $h(x) = |x|$  then find  $\lim_{x \rightarrow 0} f(g(h(x)))$

- 995.** Let  $[x]$  denote the greatest integer function &  $f(x)$  be defined in a neighbourhood of 2 by

$$f(x) = \begin{cases} \frac{\exp \left\{ (x+2) \frac{1}{4} [x+1] \ln 4 \right\} - 16}{4^x - 16} & , x < 2 \\ A \frac{1 - \cos(x-2)}{(x-2) \tan(x-2)} & , x > 2 \end{cases}$$

Find the value of  $A + 2f(2)$  in order that  $f(x)$  may be continuous at  $x = 2$ .

- 996.** Let the greatest and the least values of the function  $f(x)$  be respectively  $a$  and  $b$

$$f(x) = \text{minimum of } \{3t^4 - 8t^3 - 6t^2 + 24t; 1 \leq t \leq x\}, 1 \leq x < 2.$$

$$\text{maximum of } \left\{ 3t + \frac{1}{4} \sin^2 \pi t + 2; 2 \leq t \leq x \right\}, 2 \leq x \leq 4. \text{ Then find the value of } a + b.$$

- 997.** Find the area of the largest rectangle with lower base on the  $x$ -axis & upper vertices on the curve  $y = 12 - x^2$ .
- 998.** The cosine of the angle at the vertex of an isosceles triangle having the greatest area for the given constant length  $\ell$  of the median drawn to its lateral side is  $p$ . Find  $100p$ .
- 999.** The fuel charges for running a train are proportional to the square of the speed generated in m.p.h. & costs Rs. 48/- per hour at 16 mph. What is the most economical speed if the fixed charges i.e. salaries etc. amount to Rs. 300/- per hour.
- 1000.** A figure is bounded by the curves,  $y = x^2 + 1, y = 0, x = 0$  &  $x = 1$ . At a point  $(a, b)$ , a tangent should be drawn to the curve,  $y = x^2 + 1$  for it to cut off a trapezium of the greatest area from the figure. Find  $2a + 12b$

# JEE ADVANCED REVISION PACKAGE - Answer Key (MATHEMATICS)

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
1	A	51	AB	101	AD	151	A-(iii), B-(ii), C-(i), D-(iv)
2	C	52	AC	102	CD	152	A-(3), B-(2), C-(1), D-(4)
3	C	53	BD	103	ACD	153	A-(iii), B-(i), C-(ii), D-(iv)
4		54	ABCD	104	AB	154	A-(iii), B-(ii), C-(iv), D-(i)
5	C	55	BD	105	C	155	A-(R), B-(S), C-(Q), D-(P)
6	C	56	AC	106	C	156	A-(Q), B-(Q), C-(Q), D-(P)
7	C	57	ABC	107	B	157	A-(r), B-(s), C-(q), D-(p)
8	C	58	AB	108	C	158	A-(q), B-(p), C-(s), D-(r)
9	D	59	ABCD	109	B	159	A-(q), B-(p), C-(s), D-(r)
10	A	60	AB	110	A	160	A-(q), B-(p), C-(s), D-(r)
11	C	61	BD	111	C	161	A-(r), B-(p), C-(s), D-(q)
12	A	62	ABCD	112	D	162	A-(q), B-(r), C-(p)
13	C	63	CD	113	A	163	A-(p), B-(q), C-(r), D-(s)
14	A	64	ABCD	114	D	164	A-(q), B-(r), C-(s), D-(s)
15	D	65	AC	115	C	165	A-(q,s), B-(p,q,r), C-(p), D-(p,s)
16	B	66	ABC	116	D	166	A-(P), B-(S), C-(Q), D-(Q)
17	A	67	ABCD	117	B	167	A-(R), B-(Q), C-(S), D-(P)
18	B	68	ABC	118	C	168	A-(q), B-(p), C-(r), D-(r)
19	B	69	BC	119	C	169	A-(p), B-(p,q,r), C-(p,q,r,s), D-(p)
20	A	70	AC	120	D	170	A-(q), B-(s), C-(r), D-(p)
21	D	71	ABCD	121	A	171	15
22	C	72	AC	122	A	172	52
23	B	73	BC	123	A	173	50
24	D	74	BCD	124	C	174	3
25	C	75	ABD	125	D	175	1
26	B	76	ABC	126	D	176	729
27	A	77	ABC	127	A	177	4
28	B	78	ABC	128	C	178	7
29	C	79	BD	129	B	179	9
30	B	80		130	B	180	1
31	B	81	CD	131	C	181	210
32	A	82	ABCD	132	A	182	192
33	A	83	AB	133	A	183	142857
34	B	84	ACD	134	A	184	44
35	C	85	AC	135	C	185	0
36	B	86	AD	136	C	186	70
37	A	87	ABCD	137	A	187	(2, 1) or (2/5, -1/5)
38	A	88	ABCD	138	A	188	4
39	D	89	BC	139	B	189	16
40	B	90	AB	140	C	190	730
41	B	91	ABCD	141	A-(q), B-(q), C-(q), D-(p)	191	8315
42	C	92	ABCD	142	A-(ii), B-(iii), C-(iv), D-(i)	192	24
43	B	93	BC	143	A-(iii), B-(i), C-(iv), D-(ii)	193	40
44	D	94	AD	144	A-(ii), B-(i), C-(iv), D-(iii)	194	0
45	C	95	AD	145	A-(iii), B-(i), C-(ii), D-(iv)	195	1
46	D	96	AC	146	A-(ii), B-(iv), C-(ii), D-(iv)	196	n=9
47	C	97	BD	147	A-(v), B-(i), C-(ii), D-(i)	197	280
48		98	BCD	148	A-(iii), B-(ii), C-(iv), D-(i)	198	1260
49	C	99	CD	149	A-(iii), B-(iv), C-(i), D-(ii)	199	11
50	D	100	ABC	150	A-(I),(ii), B-(i), C-(iii), D-(iii)	200	744

# Answer Key

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
201	A	251	AC	301	B	351	D
202	C	252	AB	302	C	352	C
203	B	253		303	D	353	D
204	B	254	AB	304	B	354	C
205	C	255	AB	305	B	355	A
206	C	256	BD	306	B	356	C
207	C	257	ABD	307	C	357	B
208	D	258	ABD	308	B	358	D
209	D	259	BD	309	A	359	B
210	C	260	ABD	310	A	360	A-(Q), B-(P), C-(S), D-(R)
211	D	261	ABCD	311	A	361	
212	B	262	AB	312	B	362	
213	D	263	CD	313	D	363	
214	A	264	BD	314	C	364	
215	A	265	AC	315	B	365	A-(S), B-(S), C-(Q), D-(P)
216	C	266	BC	316	B	366	A-(S), B-(R), C-(Q), D-(S)
217	C	267	ABC	317	B	367	A-(R), B-(Q), C-(S), D-(P)
218	B	268	BCD	318	D	368	A-(Q), B-(S), C-(R), D-(P)
219	A	269	AD	319	D	369	A-(P), B-(Q), C-(R), D-(P)
220	C	270	ABC	320	C	370	A-(q), B-(p), C-(s), D-(r)
221	C	271	AB	321	C	371	A-(s), B-(p), C-(q), D-(r)
222	A	272	BD	322	B	372	A-(r), B-(p), C-(q), D-(s)
223	A	273	BD	323	A	373	A-(s), B-(p), C-(q), D-(r)
224	B	274	ACD	324	C	374	A-(r), B-(p), C-(s), D-(q)
225	A	275	AD	325	C	375	A-(q), B-(r), C-(q), D-(q)
226	A	276	BC	326	B	376	2
227	A	277	ACD	327	C	377	1
228	B	278	BC	328	A	378	8
229	C	279	AD	329	D	379	13
230	B	280	AB	330	A	380	82
231	C	281	ABC	331	A	381	6
232	D	282	AB	332	B	382	98
233	A	283	AD	333	C	383	3
234	D	284	BC	334	C	384	156
235	C	285	ABCD	335	A	385	40
236	B	286	BC	336	A	386	18
237	A	287	AC	337	B	387	15
238	C	288	CD	338	A	388	55
239	A	289	AC	339	D	389	55
240	B	290	CD	340	B	390	8
241	A	291	ABCD	341	C	391	16
242	C	292	B	342	A	392	134
243	A	293	C	343	D	393	1
244	C	294	B	344	BC	394	2
245	B	295	B	345	A	395	3
246	ABC	296	B	346	A	396	27
247	ABCD	297	C	347	B	397	0
248	AB	298	D	348	D	398	8
249	BCD	299	B	349	B	399	1
250	AB	300	C	350	B	400	1



# Answer Key

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
401	C	451	BC	501	A-(iii), B-(iii), C-(ii), D-(iv)	551	C
402		452	AC	502	A-(ii), B-(i), C-(iii), D-(iv)	552	A
403	B	453	BC	503	A-(ii), B-(iii), C-(ii)	553	D
404	C	454	BC	504	A-(iv), B-(v), C-(i), D-(ii)	554	D
405	C	455	BD	505	A-(ii), B-(iii), C-(i), D-(ii)	555	B
406	C	456	ABC	506	A-(ii), B-(i), C-(i), D-(iii)	556	AD
407	D	457	ABCD	507	A-(iv), B-(iii), C-(iv), D-(ii)	557	AB
408	B	458	ACD	508	A-(ii), B-(iii), C-(iii), D-(iii)	558	AC
409	C	459	ABCD	509	A-(2), B-(3), C-(3), D-(3)	559	D
410	A	460	ACD	510	A-(PR), B-(PR), C-(Q), D-(S)	560	AC
411	B	461	ACD	511	2	561	AB
412	C	462	BC	512	4	562	AD
413	C	463	AC	513	28	563	AC
414	B	464	BC	514	6	564	BC
415	C	465	AD	515	76	565	AD
416		466	A	516	5	566	AC
417	B	467	B	517	1	567	BC
418	A	468	C	518	52	568	AC
419	B	469	D	519	216	569	BC
420	A	470	D	520	15	570	AC
421	A	471	C	521	11	571	ABCD
422	D	472	C	522	3	572	AB
423	A	473	C	523	1	573	BD
424		474	D	524	1	574	BC
425		475	B	525	2	575	ABC
426	ABCD	476	D	526	2	576	B
427	ACD	477	D	527	7	577	A
428	AC	478	C	528	0	578	D
429	ABC	479	B	529	27	579	B
430	AD	480	B	530	200	580	A
431	AB	481	A	531	B	581	B
432	ABC	482	A	532	B	582	B
433	ABCD	483	B	533	C	583	C
434	ABC	484	A	534	D	584	C
435	AB	485	C	535	C	585	B
436	AB	486	B	536	A	586	D
437	ABC	487	A	537	D	587	D
438	ABD	488	A	538	B	588	
439	ABCD	489	B	539	D	589	
440	BC	490	A	540	A	590	
441	ABCD	491	B	541	C	591	C
442	AB	492	B	542	A	592	A
443	AC	493	B	543	B	593	B
444	BC	494	A	544	D	594	B
445	AB	495	C	545	C	595	A
446	AB	496	A-(iii), B-(iv), C-(i), D-(ii)	546	A	596	B
447	BCD	497	A-(iv), B-(iii), C-(i), D-(ii)	547	B	597	A
448	BD	498	A-(iv), B-(iii), C-(i), D-(ii)	548	A	598	B
449	AD	499	A-(iv), B-(iii), C-(ii), D-(i)	549	C	599	A
450	ABC	500	A-(iii), B-(iii), C-(ii), D-(iv)	550	B	600	A

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
601		651	C	701	AB	751	AB
602	A	652	A	702	BD	752	ABC
603	C	653	D	703	ABD	753	ABC
604	A	654	B	704	ABC	754	BC
605	C	655	A	705	AD	755	ABC
606	D	656	A	706	AD	756	AC
607	B	657	D	707	AC	757	AC
608	C	658	B	708	AB	758	ABC
609	B	659	B	709	AC	759	BC
610	C	660	B	710	AC	760	AC
611	D	661	C	711	AB	761	AB
612	C	662	A	712	BC	762	ABD
613	A	663	C	713	AC	763	AC
614	A	664	D	714	ABC	764	ABC
615	C	665	B	715	ABC	765	AB
616	B	666	A	716	AC	766	BC
617	D	667	B	717	AB	767	AC
618	C	668	A	718	CD	768	ABCD
619	B	669	C	719	ACD	769	BD
620	C	670	A	720	BC	770	BC
621	A-(ii), B-(iii), C-(iv), D-(i)	671	A	721	BD	771	BD
622	A-(i), B-(i), C-(iii), D-(ii)	672	C	722	AB	772	ABD
623	A-(ii), B-(ii), C-(iv), D-(iii)	673	B	723	AD	773	ABC
624	A-(ii), B-(i), C-(iv), D-(iii)	674	C	724	AB	774	ABC
625	A-(ii), B-(i), C-(iv), D-(iv)	675	B	725	ACD	775	BCD
626	A-(), B-(), C-(), D-()	676	B	726	ABC	776	ABC
627		677	B	727		777	AC
628	A-(1), B-(2), C-(1), D-(3)	678	B	728	AC	778	CD
629	A-(i), B-(ii), C-(ii), D-(ii)	679		729	AB	779	AD
630	A-(iv), B-(i), C-(ii), D-(iii)	680	B	730	AB	780	ABD
631		681	A	731	ABD	781	ABC
632	A-(iv), B-(iii), C-(ii), D-(i)	682	A	732	CD	782	AC
633		683	B	733	ACD	783	ABD
634	5	684	D	734	ACD	784	AC
635	3	685	A	735	ABD	785	AB
636	8	686	C	736	AD	786	ABC
637	16	687	C	737	ABC	787	AC
638	2	688	A	738	BC	788	CD
639	4	689	D	739	CD	789	BD
640	17	690	D	740	ACD	790	CD
641	1	691	D	741	BD	791	BC
642	90	692	D	742	AD	792	BC
643	3	693	B	743	AC	793	ABD
644	40	694	B	744	ABCD	794	AB
645	2	695	B	745	ABC	795	AD
646	6	696	C	746	ACD	796	ACD
647	4	697	C	747	BC	797	AC
648	4	698	C	748	BCD	798	AD
649	3	699	A	749	AB	799	BC
650	15	700	A	750	AC	800	ACD

# Answer Key

Qs.	Ans.	Qs.	Ans.	Qs.	Ans.	Qs.	Ans.
801	ABD	851	C	901	A	951	A-(PS), B-(QRT), C-(PS)
802	BD	852	A	902	C	952	A-(T), B-(PQR), C-(S)
803	BD	853	A	903	A	953	A-(P), B-(R), C-(QT)
804	AC	854	D	904	A	954	A-(s), B-(q), C-(q), D-(s)
805	ABD	855	B	905	C	955	A-(s), B-(p), C-(s), D-(q)
806	AD	856	D	906	A	956	A-(q), B-(q), C-(q), D-(q)
807	ABCD	857	B	907	C	957	A-(q), B-(p), C-(r), D-(s)
808	BC	858	D	908	C	958	A-(r), B-(s), C-(q), D-(r)
809	AD	859	B	909	B	959	A-(r), B-(s), C-(q), D-(p)
810	AD	860	C	910	A	960	A-(r), B-(s), C-(p), D-(q)
811	AB	861	A	911	D	961	A-(s), B-(r), C-(q), D-(p)
812	ACD	862	A	912	B	962	A-(q), B-(r), C-(q), D-(s)
813	ABC	863	D	913	C	963	A-(q), B-(p), C-(s), D-(p)
814	ABD	864	B	914	D	964	A-(p,q,r), B-(p), C-(q,r,s), D-(p,q,r,s)
815	D	865	D	915	B	965	A-(q), B-(q), C-(r), D-(p)
816	C	866	B	916	A	966	A-(q), B-(p), C-(r), D-(s)
817	A	867	A	917	B	967	0
818	A	868	B	918	A-(iv), B-(i), C-(iii), D-(ii)	968	9
819	C	869	D	919	A-(iii), B-(iv), C-(i), D-(ii)	969	6
820	B	870	D	920	A-(iv), B-(iii), C-(ii), D-(iii)	970	0
821	C	871	C	921	A-(iii), B-(i), C-(iv), D-(iv)	971	4
822	C	872	A	922	A-(iv), B-(i), C-(ii), D-(iii)	972	2
823	C	873	C	923	A-(ii), B-(ii), C-(ii), D-(iii, i)	973	5
824	B	874		924	A-(v), B-(iv), C-(i), D-(ii)	974	11
825	C	875		925	A-(ii), B-(iii), C-(ii), D-(iv)	975	0
826	A	876		926	A-(ii), B-(i), C-(iv), D-(iii)	976	2
827	B	877	C	927	A-(C), B-(D), C-(A), D-(B)	977	8
828	A	878	B	928	A-(2), B-(1), C-(4), D-(5)	978	24
829	C	879	B	929	A-(P), B-(P), C-(P), D-(P)	979	1
830	B	880	A	930	A-(P), B-(Q), C-(R), D-(S)	980	2
831	A	881		931	A-(r), B-(p), C-(q), D-(s)	981	7
832	A	882	C	932	A-(s), B-(p), C-(q), D-(q)	982	10
833	A	883	D	933	A-(C), B-(A), C-(D), D-(B)	983	2
834	D	884	A	934	A-(D), B-(A), C-(B), D-(C)	984	34
835	C	885	B	935	A-(D), B-(C), C-(B), D-(A)	985	2
836	C	886	A	936	A-(D), B-(C), C-(A), D-(B)	986	90
837	B	887	D	937	A-(D), B-(C), C-(B), D-(A)	987	1
838	B	888	B	938	A-(P), B-(Q), C-(R), D-(Q)	988	7
839	C	889	C	939	A-(R), B-(R), C-(S), D-(S)	989	1
840	A	890	D	940	A-(R), B-(P), C-(S), D-(Q)	990	1
841	B	891	B	941	A-(R), B-(P), C-(S), D-(Q)	991	5
842	C	892	A	942	A-(RT), B-(RT), C-(RT), D-(PS)	992	4
843	D	893	C	943	A-(R), B-(Q), C-(R), D-(P)	993	0
844	D	894	C	944	A-(P), B-(P), C-(S), D-(R)	994	0
845	C	895	A	945	A-(S), B-(R), C-(R), D-(Q)	995	2
846	C	896	D	946	A-(R), B-(Q), C-(Q), D-(R)	996	22
847	B	897	C	947	A-(S), B-(R), C-(S), D-(Q)	997	32 sq.
848	C	898	A	948	A-(S), B-(S), C-(P), D-(P)	998	80
849		899	A	949	A-(P,R,T), B-(S), C-(Q)	999	40 mph
850	A	900		950	A-(S), B-(P), C-(Q), D-(S)	1000	16