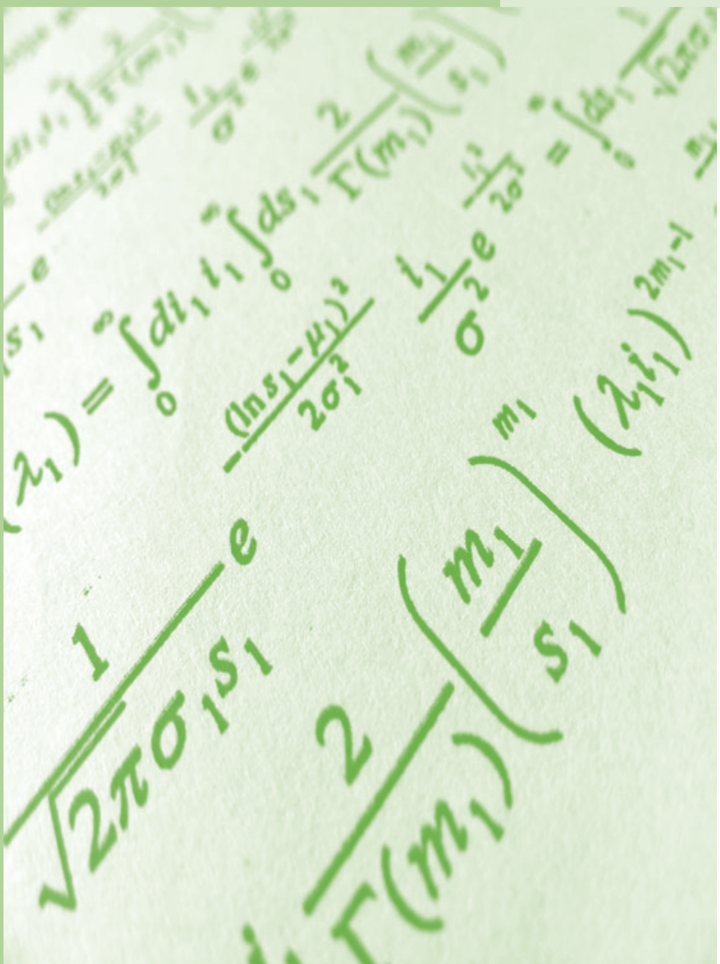
Chapter

11

Remainder and Factor Theorems



REMEMBER

Before beginning this chapter, you should be able to:

- Define polynomials and its functions

KEY IDEAS

After completing this chapter, you would be able to:

- Obtain zero of a polynomial
- Prove remainder theorem
- State and prove factor theorem
- Do factorization of polynomials using factor theorem
- Apply Horner's process for synthetic division of polynomials

INTRODUCTION

A real valued function $f(x)$ of the form $a_0x^n + a_1x^{n-1} + \cdots + a_n$, $(a_0 \neq 0)$ is called as a polynomial of degree n, where n is a non-negative integer. Here a_0, a_1, $\ldots, a_n$ are the coefficients of various powers of x.

Examples:

1. $4x^6 + 5x^5 + x^4 + x^2 - 1$ is a polynomial in x of degree 6.
2. $2x^3 + x^2 + 1$ is a polynomial in x of degree 3.

Note A constant is considered to be a polynomial of zero degree.

In earlier classes we have learnt the different operations on polynomials like addition, subtraction, multiplication and division. Here we shall learn two important theorems on polynomials.

REMAINDER THEOREM

If $p(x)$ is any polynomial and 'a' is any real number, then the remainder when $p(x)$ is divided by $(x - a)$ is given by $p(a)$.

Proof: Let $q(x)$ and $r(x)$ be the quotient and the remainder respectively when $p(x)$ is divided by $x - a$.

$\therefore$ By division algorithm

Dividend = quotient × divisor + remainder,

i.e., $p(x) = q(x)(x - a) + r(x)$

If $x = a$, then

$p(a) = q(a)(a - a) + r(a) \quad \Rightarrow \quad r(a) = p(a)$,

i.e., $p(x) = (x - a)\ q(x) + p(a)$

Thus the remainder is $p(a)$.

Notes

1. If $p(a) = 0$, we say that 'a' is a zero of the polynomial $p(x)$.
2. If $p(x)$ is a polynomial and 'a' is a zero of $p(x)$, then $p(x) = (x - a)\ q(x)$.
3. If $p(x)$ is divided by $ax + b$, then the remainder is given by $p\left(\dfrac{-b}{a}\right)$.
4. If $p(x)$ is divided by $ax - b$, then the remainder is given by $p\left(\dfrac{b}{a}\right)$.

EXAMPLE 11.1

Find the remainder when the polynomial $p(z) = z^3 - 3z + 2$ is divided by $z - 2$.

SOLUTION

Given $p(z) = z^3 - 3z + 2$

The remainder when $p(z)$ is divided by $z - 2$ is given by $p(2)$.

Now, $p(2) = (2)^3 - 3(2) + 2$

$= 8 - 6 + 2 = 4$

Hence, when $p(z)$ is divided by $z - 2$ the remainder is 4.

FACTOR THEOREM

If $p(x)$ is a polynomial of degree n (≥ 1) and a be any real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Proof: Let $q(x)$ be the quotient and $(x - a)(a \in R)$ be a divisor of $p(x)$

Given $p(a) = 0$

$\therefore$ By division algorithm

Dividend = quotient $\times$ divisor + remainder

$p(x) = q(x)(x - a) + p(a)$

$\Rightarrow \quad p(x) = q(x)(x - a)$ ($\because p(a) = 0$)

Therefore, $(x - a)$ is a factor of $f(x)$, which is possible only if $p(a) = 0$.

Hence, $(x - a)$ is a factor of $p(x)$ ($\because p(a) = 0$).

Notes

1. If $p(-a) = 0$, then $(x + a)$ is a factor of $p(x)$.
2. If $p\left(\frac{-b}{a}\right) = 0$, then $(ax + b)$ is a factor of $p(x)$.
3. If $p\left(\frac{b}{a}\right) = 0$, then $(ax - b)$ is a factor of $p(x)$.
4. If sum of all the coefficients of a polynomial is zero, then $(x - 1)$ is one of its factors.
5. If sum of the coefficients of odd powers of x is equal to the sum of the coefficients of even powers of x,then one of the factors of the polynomial is $(x + 1)$.

Examples:

1. Determine whether $x - 3$ is a factor of $f(x) = x^2 - 5x + 6$.

   Given $f(x) = x^2 - 5x + 6$

   Now $f(3) = (3)^2 - 5(3) + 6$

   $= 9 - 15 + 6$

   $= 0 \quad \Rightarrow \quad f(3) = 0.$

   Hence, by factor theorem we can say that $(x - 3)$ is a factor of $f(x)$.

2. Determine whether $(x - 1)$ is a factor of $x^3 - 6x^2 + 11x - 6$

   Let $f(x) = x^3 - 6x^2 + 11x - 6$

   Now $f(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

   $= 1 - 6 + 11 - 6 = 0 \quad \Rightarrow \quad f(1) = 0.$

   Hence, by factor theorem we can say that $(x - 1)$ is a factor of $f(x)$.

Factorization of Polynomials Using Factor Theorem

1. Factorize $x^2(y - z) + y^2(z - x) + z^2(x - y)$

   Let us assume the given expression as a polynomial in x, say $f(x)$

   $f(x) = x^2(y - z) + y^2(z - x) + z^2(x - y)$

Now put $x = y$ in the given expression

$\Rightarrow \quad f(y) = y^2(y - z) + y^2(z - y) + z^2(y - y)$

$= y^3 - zy^2 + y^2z - y^3 + 0 = 0 \quad \Rightarrow \quad f(y) = 0$

$\Rightarrow \quad x - y$ is a factor of the given expression.

Similarly if we consider the given expression as a polynomial in y we get $y - z$ is a factor of the given expression and we also get $z - x$ is a factor of the expression when we consider it as an expression in z.

Let $x^2(y - z) + y\,(z - x) + z^2(x - y) = k(x - y)(y - z)(z - x)$

For $x = 0$, $y = 1$ and $z = 2$, we get

$0^2(1 - 2) + 1^2(2 - 0) + 2^2(0 - 1) = k(0 - 1)(1 - 2)(2 - 0)$

$\Rightarrow \quad -2 = 2k \quad \Rightarrow \quad k = -1$

$\therefore$ The factors of the given expression are $x - y$, $y - z$ and $z - x$

2. Use factor theorem to factorize $x^3 + y^3 + z^3 - 3xyz$

Given expression is $x^3 + y^3 + z^3 - 3xyz$

Consider the expression as a polynomial in variable x say $f(x)$.

That is, $f(x) = x^3 + y^3 + z^3 - 3xyz$

Now $f\,[-(y + z)] = [-(y + z)]^3 + y^3 + z^3 - 3[-(y + z)]yz$

$= -(y + z)^3 + y^3 + z^3 + 3yz(y + z)$

$= -(y + z)^3 + (y + z)^3 = 0 \quad \Rightarrow \quad f[-(y + z)] = 0$

$\Rightarrow \quad$ According to factor theorem $x - [-(y + z)]$, i.e., $x + y + z$ is a factor of $x^3 + y^3 + z^3 - 3xyz$.

Now using the long division method we get the other factor as

$x^2 + y^2 + z^2 - xy - yz - zx$

$\therefore x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

Horner's Process for Synthetic Division of Polynomials

When a polynomial $f(x) = p_0x^n + p_1x^{n-1} + \cdots + p_{n-1}x + p_n$ is divided by a binomial $x - \alpha$, let the quotient be $Q(x)$ and remainder be r.

We can find quotient $Q(x)$ and remainder r by using Horner's synthetic division process as explained below.

α	p_0	p_1	p_2	$\ldots p_{n-1}$	p_n	…1st row
left (corner)		$q_0\alpha$	$q_1\alpha$	$\ldots q_{n-2}\,\alpha$	$q_{n-1}\,\alpha$	…2nd row
	q_0	q_1	q_2	$\ldots q_{n-1}$	r	…3rd row

Step 1: Write all the coefficients p_0, p_1, p_2, …, p_n of the given polynomial $f(x)$ in the order of descending powers of x as in the first row. When any term in $f(x)$ (as seen with descending powers of x) is missing we write zero for its coefficient.

Step 2: Divide the polynomial $f(x)$ by $(x - \alpha)$ by writing α in the left corner as shown above $(x - \alpha = 0 \quad \Rightarrow \quad x = \alpha)$.

Step 3: Write the first term of the third row as $q_0 = p_0$ then multiply q_0 by α to get $q_0\alpha$ and write it under p_1, as the first element of the second row.

Step 4: Add $q_0\alpha$ to p_1 to get q_1, the second element of the third row.

Step 5: Again multiply q_1 with α to get $q_1\alpha$ and write $q_1\alpha$ under p_2 and add $q_1\alpha$ to p_2 to get q_2 which is the third element of the third row.

Step 6: Continue this process till we obtain q_{n-1} in the third row. Multiply q_{n-1} with α and write $q_{n-1}\alpha$ under p_n and add $q_{n-1}\alpha$ to p_n to get r in third row as shown above.

In the above process the elements of the third row, i.e., $q_0, q_1, q_2, \ldots, q_{n-1}$ are the coefficients of the quotient $Q(x)$ in the same order of descending powers starting with x^{n-1}.

$\therefore$ $Q(x) = q_0x^{n-1} + q_1x^{n-2} + \cdots + q_{n-2}x + q_{n-1}$ and the remainder is r, i.e., the last element of the third row.

Note If the remainder $r = 0$ then α is one of the roots of $f(x) = 0$ or $x - \alpha$ is a factor of $f(x)$.

Example: Factorize $x^4 - 10x^2 + 9$.

Let $p(x) = x^4 - 10x^2 + 9$

Here sum of coefficients = 0, and also sum of coefficients of even powers of x = sum of coefficients of odd powers of x.

$\therefore$ $(x - 1)$ and $(x + 1)$ are the factors of $p(x)$.

Multiplier of $x - 1$ is 1 and $x + 1$ is -1

$\therefore$ The quotient is $x^2 - 9$

Hence $p(x) = (x - 1)(x + 1)(x^2 - 9)$

$\Rightarrow \quad p(x) = (x - 1)(x + 1)(x - 3)(x + 3)$.

1	1	0	−10	0	9
	0	1	1	−9	−9
−1	1	1	−9	−9	0
	0	−1	0	9	
	1	0	−9	0	

EXAMPLE 11.2

Find the value of a if $ax^3 - (a + 1)x^2 + 3x - 5a$ is divisible by $(x - 2)$.

SOLUTION

Let $p(x) = ax^3 - (a + 1)\,x^2 + 3x - 5a$

If $p(x)$ is divisible by $(x - 2)$, then its remainder is zero, i.e., $p(2) = 0$

$$\Rightarrow \quad a(2)^3 - (a + 1)(2)^2 + 3(2) - 5a = 0$$

$$\Rightarrow \quad 8a - 4a - 4 + 6 - 5a = 0$$

$$\Rightarrow \quad -a + 2 = 0$$

$$\Rightarrow \quad a = 2.$$

$\therefore$ The required value of a is 2.

EXAMPLE 11.3

If the polynomial $x^3 + ax^2 - bx - 30$ is exactly divisible by $x^2 - 2x - 15$. Find a and b and also the third factor.

SOLUTION

Let $p(x) = x^3 + ax^2 - bx - 30$

Given $p(x)$ is exactly divisible by $x^2 - 2x - 15$, i.e., $(x - 5)(x + 3)$

$\Rightarrow$ $p(x)$ is divisible by $(x + 3)$ and $(x - 5)$

$\therefore$ $p(-3) = 0$ and $p(5) = 0$

Consider $p(-3) = 0$

$$\Rightarrow (-3)^3 + a(-3)^2 - b(-3) - 30 = 0$$

$$\Rightarrow -27 + 9a + 3b - 30 = 0$$

$$\Rightarrow 9a + 3b - 57 = 0$$

$$\Rightarrow 3a + b - 19 = 0 \qquad (1)$$

Now consider $p(5) = 0$

That is, $53 + a(5)^2 - b(5) - 30 = 0$

$$\Rightarrow 125 + 25a - 5b - 30 = 0$$

$$\Rightarrow 25a - 5b + 95 = 0$$

$$\Rightarrow 5a - b + 19 = 0 \qquad (2)$$

Adding Eqs. (1) and (2), we get

$$8a = 0$$

$$\Rightarrow a = 0.$$

Substituting a in Eq. (1), we get $b = 19$.

$\therefore$ The required values of a and b are 0 and 19 respectively

$$\Rightarrow p(x) = x^3 + 0(x^2) - 19x - 30.$$

That is, $p(x) = x^3 - 19x - 30$.

Thus, the third factor is $x + 2$.

-3	1	0	-19	-30
	0	-3	9	30
5	1	-3	-10	0
	0	5	10	
	1	2	0	

EXAMPLE 11.4

Find the linear polynomial in x which when divided by $(x - 3)$ leaves 6 as remainder and is exactly divisible by $(x + 3)$.

SOLUTION

Let the linear polynomial be $p(x) = ax + b$

Given $p(3) = 6$ and $p(-3) = 0$.

$$\Rightarrow \quad a(3) + b = 6 \text{ and } a(-3) + b = 0$$

$$\Rightarrow \quad 3a + b = 6 \qquad (1)$$

$$\text{and } -3a + b = 0 \qquad (2)$$

Adding Eqs. (1) and (2),

$$2b = 6 \quad \Rightarrow \quad b = 3$$

Substituting the value of b in Eq. (1), we get $a = 1$

$\therefore$ The required linear polynomial is $x + 3$.

EXAMPLE 11.5

A quadratic polynomial in x leaves remainders as 4 and 7 respectively when divided by $(x + 1)$ and $(x - 2)$. Also it is exactly divisible by $(x - 1)$. Find the quadratic polynomial.

SOLUTION

Let the quadratic polynomial be $p(x) = ax^2 + bx + c$

Given $p(-1) = 4$, $p(2) = 7$ and $p(1) = 0$

$$p(-1) = a(-1)^2 + b(-1) + c = 4$$

$$\Rightarrow \quad a - b + c = 4 \qquad (1)$$

Now $p(1) = 0$ and $p(2) = 7$

$\therefore$

$$a(1)^2 + b(1) + c = 0 \text{ and}$$

$$a(2)^2 + b(2) + c = 7$$

$$\Rightarrow \quad a + b + c = 0 \qquad (2)$$

$$4a + 2b + c = 7 \qquad (3)$$

Subtracting Eq. (1) from Eq. (2), we have

$2b = -4 \quad \Rightarrow \quad b = -2$.

Subtracting Eq. (2) from Eq. (3), we have

$$3a + b = 7$$

$$\Rightarrow \quad 3a - 2 = 7 \ (\because \quad b = -2)$$

$$\Rightarrow \quad 3a = 9 \quad \Rightarrow \quad a = 3.$$

Substituting the values of a and b in (1), we get $c = -1$

Hence, the required quadratic polynomial is $3x^2 - 2x - 1$.

EXAMPLE 11.6

Find a common factor of the quadratic polynomials $3x^2 - x - 10$ and $2x^2 - x - 6$.

SOLUTION

Consider $p(x) = 3x^2 - x - 10$ and $q(x) = 2x^2 - x - 6$

Let $(x - k)$ be a common factor of $p(x)$ and $q(x)$

$$\therefore\ p(k) = q(k) = 0$$
$$\Rightarrow\quad 3k^2 - k - 10 = 2k^2 - k - 6$$
$$\Rightarrow\quad k^2 - 4 = 0$$
$$\Rightarrow\quad k^2 = 4$$
$$\Rightarrow\quad k = \pm 2.$$

$\therefore$ The required common factor is $(x - 2)$ or $(x + 2)$.

EXAMPLE 11.7

Find the remainder when x^{999} is divided by $x^2 - 4x + 3$.

SOLUTION

Let $q(x)$ and $mx + n$ be the quotient and the remainder respectively when x^{999} is divided by $x^2 - 4x + 3$.

$\therefore$
$$x^{999} = (x^2 - 4x + 3)\ q(x) + mx + n.$$

If $x = 1$,

$$1^{999} = (1 - 4 + 3)\ q(x) + m(1) + n$$
$$\Rightarrow\quad 1 = 0 \times q(x) + m + n$$
$$\Rightarrow\quad m + n = 1 \tag{1}$$

If $x = 3$,

$$3^{999} = (3^2 - 4(3) + 3)\ q(x) + 3m + n$$
$$\Rightarrow\quad 3^{999} = 0 \times q(x) + 3m + n$$
$$\Rightarrow\quad 3m + n = 3^{999} \tag{2}$$

Subtracting Eq. (1) from Eq. (2) we get

$$2m = 3^{999} - 1$$
$$m = \frac{1}{2}(3^{999} - 1).$$

Substituting m in Eq. (1), we have

$$n = 1 - \frac{1}{2}(3^{999} - 1) = 1 - \frac{1}{2}3^{999} + \frac{1}{2} = \frac{3}{2} - \frac{1}{2}3^{999}$$
$$n = \frac{3}{2}(1 - 3^{998}).$$

$\therefore$ The required remainder is $\frac{1}{2}(3^{999} - 1)x + \frac{3}{2}(1 - 3^{998})$.

EXAMPLE 11.8

Find the remainder when x^5 is divided by $x^3 - 4x$.

SOLUTION

Let $q(x)$ be the quotient and $lx^2 + mx + n$ be the remainder when x^5 is divided by $x^3 - 4x$

That is, $x(x - 2)(x + 2)$

$\therefore\ x^5 = (x^3 - 4x)\ q(x) + lx^2 + mx + n$

Put $x = 0$

$$\Rightarrow \quad 0 = 0 \times q(x) + l(0) + m(0) + n$$
$$\Rightarrow \quad n = 0.$$

Put $x = 2$

$$\Rightarrow \quad 2^5 = (8 - 8)\ q(x) + l(2)^2 + m(2) + n$$
$$\Rightarrow \quad 32 = 4l + 2m + n$$
$$\Rightarrow \quad 4l + 2m = 32 \ (\because \quad n = 0)$$
$$\Rightarrow \quad 2l + m = 16 \qquad (1)$$

Put $x = -2$

$$(-2)^5 = (-8 + 8)\ q(x) + l(-2)^2 + m\ (-2) + n$$
$$\Rightarrow \quad -32 = 4l - 2m + n$$
$$\Rightarrow \quad 4l - 2m = -32 \ (\because \quad n = 0)$$
$$\Rightarrow \quad 2l - m = -16 \qquad (2)$$

Adding Eqs. (1) and (2),

$$4l = 0$$
$$\Rightarrow \quad l = 0.$$

Substituting l in Eq. (1), we get

$$2(0) + m = 16$$
$$\Rightarrow \quad m = 16.$$

$\therefore$ The required remainder is $0(x^2) + 16x + 0$, i.e., $16x$.

EXAMPLE 11.9

If $f(x + 2) = x^2 + 7x - 13$, then find the remainder when $f(x)$ is divided by $(x + 2)$.

(a) -25 **(b)** -12 **(c)** -23 **(d)** -11

SOLUTION

Given, $f(x + 2) = x^2 + 7x - 13 \qquad (1)$

The remainder, when $f(x)$ is divided by$(x + 2)$ is $f(-2)$.

$\therefore$ Put $x = -4$ in Eq. (1)

$f(-4 + 2) = (-4)^2 + 7(-4) -13$

$\Rightarrow \quad f(-2) =16 - 28 - 13 = -25.$

EXAMPLE 11.10

If $(x - 2)$ and $(x - 3)$ are two factors of $f(x) = x^3 + ax + b$, then find the remainder when $f(x)$ is divided by $x - 5$.

(a) 0 **(b)** 15 **(c)** 30 **(d)** 60

SOLUTION

Given $f(x) = x^3 + ax + b$

$f(2) = 0$ and $f(3) = 0$

$(2)^3 + 2a + b = 0 \quad \Rightarrow \quad 2a + b = -8$ (1)

$(3)^3 + 3a + b = 0 \quad \Rightarrow \quad 3a + b = -27$ (2)

On solving Eqs. (1) and (2), we get

$a = -19$ and $b = 30$

$f(x) = x^3 - 19x + 30.$

Now $f(5) = 5^3 - 19(5) + 30 = 125 - 95 + 30 = 60.$

EXAMPLE 11.11

If the polynomials $f(x) = x^2 + 5x - p$ and $g(x) = x^2 - 2x + 6p$ have a common factor, then find the common factor.

(a) $x + 2$ **(b)** x **(c)** $x + 4$ **(d)** Either (b) or (c)

SOLUTION

Given, $f(x) = x^2 + 5x - p$ and $g(x) = x^2 - 2x + 6p$

Let $x - k$ be the common factor of $f(x)$ and $g(x)$.

$\therefore f(k) = 0$ and $g(k) = 0$

$\Rightarrow \quad k^2 + 5k - p = 0$ (1)

$k^2 - 2k + 6p = 0$ (2)

From Eqs. (1) and (2), we get

$k = p$

Substitute $k = p$ in Eq. (1)

$p^2 + 5p - p = 0$

$p^2 + 4p = 0 \quad \Rightarrow \quad p = 0$

or $p = -4$.

$\therefore$ x or $x + 4$ is a common factor of $f(x)$ and $g(x)$.

EXAMPLE 11.12

When a fourth degree polynomial, $f(x)$ is divided by $(x + 6)$, the quotient is $Q(x)$ and the remainder is -6. And when $f(x)$ is divided by $[Q(x) + 1]$, the quotient is $(x + 6)$ and the remainder is $R(x)$. Find $R(x)$.

(a) $12 + x$ **(b)** $-(x + 12)$ **(c)** 0 **(d)** 3

SOLUTION

Given,

$f(x) = Q(x)(x + 6) - 6$

$\Rightarrow \quad Q(x)(x + 6) = f(x) + 6$ (1)

And also given,

$f(x) = (x + 6)[Q(x) + 1] + R(x)$

$\Rightarrow \quad f(x) = (x + 6)\ Q(x) + x + 6 + R(x)$

$\Rightarrow \quad f(x) = f(x) + 6 + x + 6 + R(x)$ (from Eq. (1))

$\Rightarrow \quad R(x) = -(x + 12).$

EXAMPLE 11.13

Given $f(x)$ is a cubic polynomial in x. If $f(x)$ is divided by $(x + 3)$, $(x + 4)$, $(x + 5)$ and $(x + 6)$, then it leaves the remainders 0, 0, 4 and 6 respectively. Find the remainder when $f(x)$ is divided by $x + 7$.

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

SOLUTION

From the given data $x + 3$ and $x + 4$ are two factors of $f(x)$.

Let other factor be $ax + p$

$\therefore f(x) = (x + 3)(x + 4)(ax + p)$

And also given,

$f(-5) = 4$ and $f(-6) = 6$

$\Rightarrow \quad (-2)(-1)(-5a + p) = 4$

$\Rightarrow \quad -5a + p = 2 \qquad (1)$

and $(-3)(-2)\ (-6a + p) = 6$

$\Rightarrow \quad -6a + p = 1 \qquad (2)$

On solving Eqs. (1) and (2), we get

$a = 1$ and $p = 7$.

$\therefore f(x) = (x + 3)(x + 4)(x + 7)$

$\therefore f(-7) = 0$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Let $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$ $(a_0 \neq 0)$ be a polynomial of degree n. If $x + 1$ is one of its factors, then ______.

2. If a polynomial $f(x)$ is divided by $(x + a)$, then the remainder obtained is ______.

3. If $a - b$ is a factor of $a^n - b^n$, then n is ______.

4. If $f(x) = x^3 + 2$ is divided by $x + 2$, then the remainder obtained is ______.

5. The condition for which $ax^2 + bx + a$ is exactly divisible by $x - a$ is ______.

6. If $x + 1$ is a factor of $x^m + 1$, then m is ______.

7. The remainder when $f(x) = x^3 + 5x^2 + 2x + 3$ is divided by x is ______.

8. The remainder when $(x - a)^2 + (x - b)^2$ is divided by x is ______.

9. The remainder when $x^6 - 4x^5 + 8x^4 - 7x^3 + 3x^2 + 2x - 7$ is divided by $x - 1$ is ______.

10. For two odd numbers x and y, if $x^3 + y^3$ is divisible by 2^k, $k \in N$, then $x + y$ is divisible by 2^k. [True/False]

11. One of the factors of $2x^{17} + 3x^{15} + 7x^{23}$ is ______. $(x^{17}/x^{15}/x^{23})$

12. If $(x - 2)^2$ is the factor of an expression of the form $x^3 + bx + c$, then the other factor is ______.

13. What should be added to $3x^3 + 5x^2 - 6x + 3$ to make it exactly divisible by $x - 1$?

14. The remainder when $2x^6 - 5x^3 - 3$ is divided by $x^3 + 1$ is ______.

15. The remainder when $f(x)$ is divided by $g(x)$ is $f\left(-\frac{3}{2}\right)$, then $g(x)$ is necessarily $2x + 3$. [True/False]

16. Find the remainder when the polynomial $x^2 + 13x + 11$ is divided by $x - 1$.

17. Find the value of the polynomial $a^2 - \frac{1}{6}a + \frac{3}{2}$ when $a = \frac{1}{2}$.

18. The polynomial $7x^2 - 11x + a$ when divided by $x + 1$ leaves a remainder of 8. Then find the value of 'a'.

19. If $x + 2$ is a factor of $f(x)$ and $f(x) = x^3 + 4x^2 + kx - 6$, then find the value of k.

20. Find the values of a if $x^3 - 5x(a - 1) - 3(x + 1) + 5a$ is divisible by $x - a$.

21. Find the value of a if $x - a$ is a factor of the polynomial $x^5 - ax^4 + x^3 - ax^2 + 2x + 3a - 2$.

22. Find the remainder when $x^3 + 3px + q$ is divided by $(x^2 - a^2)$ without actual division.

23. The remainder obtained when $x^2 + 3x + 1$ is divided by $(x - 5)$ is ______.

24. If the polynomial $3x^4 - 11x^2 + 6x + k$ is divided by $x - 3$, it leaves a remainder 7. Then the value of k is ______.

25. $(7x - 1)$ is a factor of $7x^3 + 6x^2 - 15x + 2$. (True/False)

26. If $ax^2 + bx + c$ is exactly divisible by $2x - 3$, then the relation between a, b and c is ______.

27. If $x^2 + 5x + 6$ is a factor of $x^3 + 9x^2 + 26x + 24$, then find the remaining factor.

28. If $(2x - 1)$ is a factor of $2x^2 + px - 2$, then the other factor is ______.

29. The expression $x^{m^n} - 1$ is divisible by $x + 1$, only if M is ______. (even/odd)

30. If $x + m$ is one of the factors of the polynomial $x^2 + mx - m + 4$, then the value of m is ______.

Short Answer Type Questions

31. For what values of m and n is $2x^4 - 11x^3 + mx + n$ is divisible by $x^2 - 1$?

32. Find a linear polynomial which when divided by $(2x + 1)$ and $(3x + 2)$ leaves remainders 3 and 4 respectively.

33. Prove that $x^m + 1$ is a factor of $x^{mn} - 1$ if n is even.

34. The remainders of a polynomial $f(x)$ in x are 10 and 15 respectively when $f(x)$ is divided by$(x - 3)$ and $(x - 4)$. Find the remainder when $f(x)$ is divided by $(x - 3)(x - 4)$.

35. If x^{555} is divided by $x^2 - 4x + 3$, then find its remainder.

36. If $(x^2 - 1)$ is a factor of $ax^3 - bx^2 - cx + d$, then find the relation between a and c.

37. When $x^4 - 3x^3 + 4x^2 + p$ is divided by $(x - 2)$, the remainder is zero. Find the value of p.

38. Find the common factors of the expressions $a_1x^2 + b_1x + c_1$ and $a_2x^2 + b_2x + c_1$ where $c_1 \neq 0$.

39. If $(x - 3)$ is a factor of $x^2 + q$ (where $q \in Q$), then find the remainder when $(x^2 + q)$ is divided by$(x - 2)$.

40. If $p + q$ is a factor of the polynomial $p^n - q^n$, then n is

41. The expression $x^{4005} + y^{4005}$ is divisible by ______.

42. The value of a for which $x - 7$ is a factor of $x^2 + 11x - 2a$ is ______.

43. If a polynomial $f(x)$ is divided by $(x - 3)$ and $(x - 4)$ it leaves remainders as 7 and 12 respectively, then find the remainder when $f(x)$ is divided by $(x - 3)(x - 4)$.

44. Find the remainder when $5x^4 - 11x^2 + 6$ is divided by $5x^2 - 6$.

45. If $f(x - 2) = 2x^2 - 3x + 4$, then find the remainder when $f(x)$ is divided by $(x - 1)$.

Essay Type Questions

46. Factorize $x^4 - 2x^3 - 9x^2 + 2x + 8$ using remainder theorem.

47. Find the remainder when x^{29} is divided by $x^2 - 2x - 3$.

48. If $x^2 - 2x - 1$ is a factor of $px^3 + qx^2 + 1$, (where p, q are integers) then find the value of $p + q$.

49. If $x^2 - x + 1$ is a factor of $x^4 + ax^2 + b$, then the values of a and$b$ are respectively ______.

50. If $lx^2 + mx + n$ is exactly divisible by $(x - 1)$ and $(x + 1)$ and leaves a remainder 1 when divided by $x + 2$, then find m and n.

CONCEPT APPLICATION

Level 1

1. The value of a for which the polynomial $y^3 + ay^2 - 2y + a + 4$ in y has $(y + a)$ as one of its factors is ______.

(a) $\frac{-3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

2. If the expression $2x^3 - 7x^2 + 5x - 3$ leaves a remainder of $5k - 2$ when divided by $x + 1$, then find the value of k.

(a) 3 (b) −3

(c) 5 (d) −5

3. Find the remainder when $x^{2003} + y^{6009}$ is divided by $x + y^3$.

(a) y^{4006} (b) 1

(c) 0 (d) Cannot be determined

4. Find the remainder when $x^6 - 7x^3 + 8$ is divided by $x^3 - 2$.

(a) −2 (b) 2

(c) 7 (d) 1

5. If both the expressions $x^{1248} - 1$ and $x^{672} - 1$, are divisible by $x^n - 1$, then the greatest integer value of n is ______.

(a) 48 (b) 96

(c) 54 (d) 112

6. When $x^2 - 7x + 2$ is divided by $x - 8$, then the remainder is ______.

(a) 122 (b) 4

(c) 45 (d) 10

7. If $ax^2 + bx + c$ is exactly divisible by $4x + 5$, then

(a) $25a - 5b + 16c = 0$

(b) $25a + 20b + 16c = 0$

(c) $25a - 20b - 16c = 0$

(d) $25a - 20b + 16c = 0$

8. The expression $2x^3 + 3x^2 - 5x + p$ when divided by $x + 2$ leaves a remainder of $3p + 2$. Find p.

(a) −2 (b) 1

(c) 0 (d) 2

9. $3x - 4$ is a factor of ______.

(a) $18x^4 - 3x^3 - 28x^2 - 3x + 4$

(b) $3x^4 - 10x^3 - 7x^2 + 38x - 24$

(c) $9x^4 - 6x^3 + 5x^2 - 15$

(d) $9x^4 + 36x^3 + 17x^2 - 38x - 24$

10. Which of the following is a factor of $5x^{20} + 7x^{15} + x^9$?

(a) x^{20} (b) x^{15}

(c) x^9 (d) x^{24}

11. If $(x + 3)^2$ is a factor of $f(x) = ex^3 + kx + 6$, then find the remainder obtained when $f(x)$ is divided by $x - 6$.

(a) 1 (b) 0

(c) 5 (d) 4

12. The expression $x^{mn} + 1$ is divisible by $x + 1$, only if

(a) n is odd.

(b) m is odd.

(c) both m and n are even.

(d) Cannot say

13. If both the expressions $x^{1215} - 1$ and $x^{945} - 1$, are divisible by $x^n - 1$, then the greatest integer value of n is ______.

(a) 135 (b) 270

(c) 945 (d) None of these

14. If $(x - 2)$ is a factor of $x^2 + bx + 1$ (where $b \in Q$), then find the remainder when $(x^2 + bx + 1)$ is divided by $2x + 3$.

(a) 7 (b) 8

(c) 1 (d) 0

15. When $x^3 + 3x^2 + 4x + a$ is divided by $(x + 2)$, the remainder is zero. Find the value of a.

(a) 4 (b) 6

(c) −8 (d) −12

16. If $(x + 1)$ and $(x - 1)$ are the factors of $ax^3 + bx^2 + cx + d$, then which of the following is true?

(a) $a + b = 0$ (b) $b + c = 0$

(c) $b + d = 0$ (d) None of these

17. Find the remainder when x^5 is divided by $x^2 - 9$.

(a) $81x$ (b) $81x + 10$

(c) $3^5x + 34$ (d) None of these

18. The remainder when $x^{45} + x^{25} + x^{14} + x^9 + x$ divided by $x^2 - 1$ is ______.

(a) $4x - 1$ (b) $4x + 2$

(c) $4x + 1$ (d) $4x - 2$

19. For what values of a and b is the expression $x^4 + 4x^3 + ax^2 - bx + 3$ a multiple of $x^2 - 1$?

(a) $a = 1, b = 7$ (b) $a = 4, b = -4$

(c) $a = 3, b = -5$ (d) $a = -4, b = 4$

20. When the polynomial $p(x) = ax^2 + bx + c$ is divided by $(x - 1)$ and $(x + 1)$, the remainders obtained are 6 and 10 respectively. If the value of $p(x)$ is 5 at $x = 0$, then the value of $5a - 2b + 5c$ is ______.

(a) 40 (b) 44

(c) 21 (d) 42

21. If $p - q$ is a factor of the polynomial $p^n - q^n$, then n is ______.

(a) a prime number

(b) an odd number

(c) an even number

(d) All of these

22. When the polynomial $f(x) = ax^2 + bx + c$ is divided by x, $x - 2$ and $x + 3$, remainders obtained are 7, 9 and 49 respectively. Find the value of $3a + 5b + 2c$.

(a) −2 (b) 2

(c) 5 (d) −5

23. If $f(x + 1) = 2x^2 + 7x + 5$, then one of the factors of $f(x)$ is ______.

(a) $2x + 3$ (b) $2x^2 + 3$

(c) $3x + 2$ (d) None of these

24. If $(x - p)$ and $(x - q)$ are the factors of $x^2 + px + q$, then the values of p and q are respectively ______.

(a) 1, −2 (b) 2, −3

(c) $\frac{-1}{3}, \frac{-2}{3}$ (d) None of these

PRACTICE QUESTIONS

25. Let $f\left(x-\dfrac{1}{x}\right)=x^2+\dfrac{1}{x^2}$, find the remainder when $f(x)$ is divided by $x-3$.

(a) $\dfrac{82}{9}$ (b) $\dfrac{8}{3}$

(c) 10 (d) 11

26. If $(x-2)^2$ is a factor of $f(x)=x^3+px+q$, then find the remainder when $f(x)$ is divided by $x-1$.

(a) 4 (b) −4

(c) −5 (d) 5

27. A quadratic polynomial in x leaves remainders 4, 4 and 0 respectively when divided by $(x-1)$, $(x-2)$ and $(x-3)$. Find the quadratic polynomial.

(a) $-2x^2+6x+3$ (b) $-2x^2+6x$

(c) $-2x^2+6x+5$ (d) $-2x^2+6x-5$

28. If $f(x+3)=x^2+x-6$, then one of the factors of $f(x)$ is ______.

(a) $x-3$ (b) $x-4$

(c) $x-5$ (d) $x-6$

29. If $(x-1)^2$ is a factor of $f(x)=x^3+bx+c$, then find the remainder when $f(x)$ is divided by $(x-2)$.

(a) 2 (b) −3

(c) 4 (d) −4

30. For what values of m and n, the expression $2x^2-(m+n)x+2n$ is exactly divisible by $(x-1)$ and $(x-2)$?

(a) $m=5,\ n=2$

(b) $m=3,\ n=4$

(c) $m=4,\ n=2$

(d) $m=2,\ n=4$

Level 2

31. The ratio of the remainders when the expression x^2+bx+c is divided by $(x-3)$ and $(x-2)$ respectively is 4 : 5. Find b and c, if $(x-1)$ is a factor of the given expression.

(a) $b=\dfrac{-11}{3},\ c=\dfrac{14}{3}$

(b) $b=\dfrac{-14}{3},\ c=\dfrac{11}{3}$

(c) $b=\dfrac{14}{3},\ c=\dfrac{-11}{3}$

(d) None of these

32. If the polynomials $f(x)=x^2+9x+k$ and $g(x)=x^2+10x+l$ have a common factor, then $(k-l)^2$ is equal to ______.

(a) $9l-10k$ (b) $10l-9k$

(c) Both (a) and (b) (d) None of these

33. When $f(x)$ is divided by $(x-2)$, the quotient is $Q(x)$ and the remainder is zero. And when $f(x)$ is divided by $[Q(x)-1]$, the quotient is $(x-2)$ and the remainder is $R(x)$. Find the remainder $R(x)$.

(a) $x+2$ (b) $-x+2$

(c) $x-2$ (d) Cannot be determined

34. Find the values of m and n, if $(x-m)$ and $(x-n)$ are the factors of the expression x^2+mx-n.

(a) $m=-1,\ n=-2$

(b) $m=0,\ n=1$

(c) $m=\dfrac{-1}{2},\ n=\dfrac{1}{2}$

(d) $m=-1,\ n=2$

35. Let $f\left(x+\dfrac{1}{x}\right)=x^2+\dfrac{1}{x^2}$, find the remainder when $f(x)$ is divided by $2x+1$.

(a) $\dfrac{-7}{4}$ (b) $\dfrac{9}{4}$

(c) $\dfrac{-9}{4}$ (d) $\dfrac{11}{4}$

36. A polynomial $f(x)$ leaves remainders 10 and 14 respectively when divided by $(x-3)$ and $(x-5)$. Find the remainder when $f(x)$ is divided by $(x-3)(x-5)$.

(a) $2x+6$ (b) $2x-4$

(c) $2x+4$ (d) $2x-6$

37. If $f(x+3)=x^2-7x+2$, then find the remainder when $f(x)$ is divided by $(x+1)$.

(a) 8 (b) −4

(c) 20 (d) 46

38. A polynomial $f(x)$ when divided by $(x - 5)$ and $(x - 7)$ leaves remainders 6 and 16 respectively. Find the remainder when $f(x)$ is divided by $(x - 5)(x - 7)$.

(a) $5x + 7$ (b) $5x - 7$

(c) $5x + 19$ (d) $5x - 19$

39. A polynomial $p(x)$ leaves remainders 75 and 15 respectively, when divided by $(x - 1)$ and $(x + 2)$.

Then the remainder when $f(x)$ is divided by $(x - 1)(x + 2)$ is ______.

(a) $5(4x + 11)$ (b) $5(4x - 11)$

(c) $5(3x + 11)$ (d) $5(3x - 11)$

40. The leading coefficient of a polynomial $f(x)$ of degree 3 is 2006. Suppose that $f(1) = 5$, $f(2) = 7$ and $f(c) = 9$. Then find $f(x)$.

(a) $2006\ (x - 1)(x - 2)(x - 3) + 2x + 3$

(b) $2006\ (x - 1)(x - 2)(x - 3) + 2x + 1$

(c) $2006\ (x - 1)(x - 2)(x - 3) + 2x - 1$

(d) $2006\ (x - 2)(x - 3)(x - 1) - (2x - 3)$

41. The ratio of the remainders when the expression $x^2 + ax + b$ is divided by $(x - 2)$ and $(x - 1)$ respectively is 4 : 3. Find a and b if $(x + 1)$ is a factor of the expression.

(a) 9, −10 (b) −9, 10

(c) 9, 10 (d) −9, −10

42. If $x^3 - ax^2 + bx - 6$ is exactly divisible by $x^2 - 5x + 6$, then $\frac{a}{b}$ is ______.

(a) $\frac{6}{11}$ (b) $\frac{-6}{11}$

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

43. If $f(x) = x^2 + 5x + a$ and $g(x) = x^2 + 6x + b$ have a common factor, then which of the following is true?

(a) $(a - b)^2 + 5(a - b) + b = 0$

(b) $(a + b)^2 + 5(a + b) + a = 0$

(c) $(a + b)^2 + 6(a + b) + b = 0$

(d) $(a - b)^2 + 6(a - b) + b = 0$

44. If $ax^4 + bx^3 + cx^2 + dx$ is exactly divisible by $x^2 - 4$, then $\frac{a}{c}$ is ______.

(a) $\frac{1}{4}$ (b) $\frac{-1}{4}$

(c) $\frac{-1}{8}$ (d) $\frac{1}{8}$

45. If $x^2 + x + 1$ is a factor of $x^4 + ax^2 + b$, then the values of a and b respectively are

(a) 2, 4 (b) 2, 1

(c) 1, 1 (d) None of these

46. If $(x + 1)$ and $(x - 1)$ are the factors of $x^3 + ax^2 - bx - 2$, then find the other factor of the given polynomial.

The following are the steps involved in solving the problem given above. Arrange them in the sequential order.

(A) Put $x = 1$ and $x = -1$ in the given polynomial and obtain the equations in a and b.

(B) Substitute a and b in the given polynomial.

(C) Factorize the polynomial.

(D) Solve the equations in a and b.

(a) ADCB (b) ADBC

(c) ABCD (d) ABDC

47. The following are the steps involved in finding the value of a when $x - 2$ is a factor of $3x^2 - 7x + a$. Arrange them in sequential order.

(A) $12 - 14 + a = 0 \Rightarrow a = 2$

(B) By factor theorem, $f(2) = 0 \Rightarrow 3(2)^2 - 7(2) + a = 0$

(C) Let $f(x) = 3x^2 - 7x + a$

(a) CBA (b) BCA

(c) CAB (d) BAC

48. If $px^3 + qx^2 + rx + s$ is exactly divisible by $x^2 - 1$, then which of the following is/are necessarily true?

(A) $p = r$ (B) $q = s$

(C) $p = -r$ (D) $q = -s$

(a) Both (A) and (B) (b) Both (C) and (D)

(c) Both (A) and (D) (d) Both (B) and (C)

49. Which of the following is a factor of $x^3 + 3px^2 - 3pqx - q^3$? (where p and q are constants.)

(a) $x + p$ (b) $x + q$

(c) $x - p$ (d) $x - q$

50. If $(x - k)$ is a common factor of $x^2 + 3x + a$ and $x^2 + 4x + b$, then find the value of k in terms of a and b.

(a) $a + b$ (b) $a - b$

(c) $2a + 3b$ (d) $2a - 3b$

Level 3

51. Find the remainder when x^{33} is divided by $x^2 - 3x - 4$.

(a) $\left(\frac{4^{33}-1}{5}\right)x+\left(\frac{4^{33}-4}{5}\right)$

(b) $\left(\frac{4^{33}+1}{5}\right)x+\left(\frac{4^{33}-4}{5}\right)$

(c) $\left(\frac{4^{33}-4}{5}\right)x+\left(\frac{4^{33}+1}{5}\right)$

(c) $\left(\frac{4^{33}+4}{5}\right)x+\left(\frac{4^{33}-1}{5}\right)$

52. If $6x^2 - 3x - 1$ is a factor of $ax^3 + bx - 1$ (where a, b are integers), then find the value of b.

(a) 1 (b) 3

(c) −5 (d) −7

53. If the polynomials $f(x) = x^2 + 6x + p$ and $g(x) = x^2 + 7x + q$ have a common factor, then which of the following is true?

(a) $p^2 + q^2 + 2pq + 6p - 7q = 0$

(b) $p^2 + q^2 - 2pq + 7p - 6q = 0$

(c) $p^2 + q^2 - 2pq + 6p - 7q = 0$

(d) $p^2 + q^2 + 2pq + 7p - 6q = 0$

54. A polynomial of degree 2 in x, when divided by $(x + 1)$, $(x + 2)$ and $(x + 3)$, leaves remainders 1, 4 and 3 respectively. Find the polynomial.

(a) $\frac{1}{2}(x^2+9x+6)$

(b) $\frac{1}{2}(x^2-9x+6)$

(c) $\frac{-1}{2}(x^2-9x+6)$

(d) $\frac{-1}{2}(x^2+9x+6)$

55. When a third degree polynomial $f(x)$ is divided by $(x - 3)$, the quotient is $Q(x)$ and the remainder is zero. Also when $f(x)$ is divided by $[Q(x) + x + 1]$, the quotient is $(x - 4)$ and remainder is $R(x)$. Find the remainder $R(x)$.

(a) $Q(x) + 3x + 4 + x^2$

(b) $Q(x) + 4x + 4 - x^2$

(c) $Q(x) + 3x + 4 - x^2$

(d) Cannot be determined

56. If the expression $x^2 + 3x - 3$, is divided by $(x - p)$, then it leaves remainder 1. Find the value of p.

(a) 1 (b) −3

(c) −4 (d) Either (a) or (c)

57. If $ax^3 - 5x^2 + x + p$ is divisible by $x^2 - 3x + 2$, then find the values of a and p.

(a) $a = 2, p = 2$ (b) $a = 2, p = 3$

(c) $a = 1, p = 3$ (d) $a = 1, p = 2$

58. Which of the following should be added to $9x^3 + 6x^2 + x + 2$ so that the sum is divisible by $(3x + 1)$?

(a) −4 (b) −3

(c) −2 (d) −1

59. If the expression $6x^2 + 13x + k$ is divisible by $2x + 3$, then which of the following is the factor of the expression?

(a) $3x + 1$ (b) $3x + 4$

(c) $3x + 2$ (d) $3x + 5$

60. Given $ax^2 + bx + c$ is a quadratic polynomial in x and leaves remainders 6, 11 and 18 respectively when divided by $(x + 1)$, $(x + 2)$ and $(x + 3)$. Find the value of $a + b + c$.

(a) 1 (b) 2

(c) 3 (d) 4

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $a_1 + a_3 + a_5 + \ldots = a_0 + a_2 + a_4 + \ldots$
2. $f(-a)$
3. $n \in N$
4. -6
5. $a = 0$ or $a^2 + b + 1 = 0$
6. odd
7. 3
8. $a^2 + b^2$
9. -4
10. True
11. x^{15}
12. $x + 4$
13. -5
14. 4
15. False
16. 25
17. $\frac{5}{3}$
18. -10
19. 1
20. 1 and 3
21. $\frac{2}{5}$
22. $(a^2 + 3p)x + q$.
23. 41
24. -155
25. True
26. $9a + 6b + 4c = 0$
27. $(x + 4)$.
28. $x + 2$
29. even number
30. 4

Short Answer Type Questions

31. $m = 11$ and $n = -2$
32. $-6x$
34. $5(x - 1)$
35. $\frac{1}{2}(3^{555} - 1)x + \frac{3}{2}(1 - 3^{554})$
36. $a = c$
37. -8
38. $\left(x + \frac{b_1 - b_2}{a_1 - a_2}\right)$
39. -5
40. 42
41. $x + y$
42. 63
43. $5x - 8$
44. 0
45. 13

Essay Type Questions

46. $(x - 1)\ (x + 1)\ (x + 2)\ (x - 4)$.
47. $\left(\frac{3^{29} + 1}{4}\right)x + \left(\frac{3^{29} - 3}{4}\right)$
48. -3
49. 1, 1
50. $m = 0, n = \frac{-1}{3}$

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (b)
21. (c)	**22.** (a)	**23.** (a)	**24.** (a)	**25.** (d)	**26.** (d)	**27.** (b)	**28.** (c)	**29.** (c)	**30.** (c)

Level 2

31. (b)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (a)
41. (d)	**42.** (a)	**43.** (d)	**44.** (b)	**45.** (c)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (d)	**50.** (b)

Level 3

51. (b)	**52.** (c)	**53.** (b)	**54.** (d)	**55.** (c)	**56.** (d)	**57.** (a)	**58.** (c)	**59.** (c)	**60.** (b)

CONCEPT APPLICATION

Level 1

1. Use factor theorem.
2. Use remainder theorem.
3. Use remainder theorem.
4. Use remainder theorem.
5. The greatest possible value of n is the HCF of 1278 and 672.
6. Use remainder theorem.
7. Use factor theorem.
8. Use remainder theorem.
9. Use factor theorem.
10. $5x^{20} + 7x^{15} + x^9 = x^9(5x^{11} + 7x^6 + 1)$
11. Since the coefficient of x^2 is zero, the sum of the roots is zero.
12. Use factor theorem.
13. Largest possible value of n is the HCF of 1215 and 945.

17. Use division algorithm.
18. Use division algorithm.
19. $(x + 1)$ and $(x - 1)$ are the factors of the given expression.
20. $P(1) = 6$, $P(-1) = 10$ and $P(0) = 5$.
21. Use division algorithm.
22. $f(0) = 7$, $f(2) = 9$ and $f(-3) = 49$.
23. Put $x = x - 1$ in $f(x + 1)$ to get $f(x)$.

    (i) Write $2x^2 + 7x + 5$ in terms of $x + 1$.

    (ii) Replace $x + 1$ by x.

    (iii) Apply remainder theorem.

24. (i) $x^2 + px + q = (x - p)(x - q)$.

    (ii) Compare the terms in LHS and RHS.

25. (i) $f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^2 + 2.$

    (ii) Replace $\left(x - \frac{1}{x}\right)$ with x.

    (iii) Use remainder theorem to obtain remainder.

26. (i) Since the coefficient of x^2 is 0, the sum of the roots is '0'.

    $\Rightarrow$ Third root is -4.

    (ii) Apply remainder theorem for $f(x) = (x - 2)^2 (x + 4)$.

27. (i) Let $f(x) = ax^2 + bx + c$. $f(1) = 4$; $f(2) = 4$; $f(3) = 0$

    (ii) Solve for a, b, and c.

28. (i) Put $x = x - 3$ in $f(x + 3)$ to get $f(x)$.

    (ii) Apply factor theorem.

29. (i) Coefficient of x^2 is 0, therefore sum of roots is 0.

    $\therefore$ Third root $= -2$.

    (ii) Apply factor theorem.

    (iii) To obtain the remainder, use the remainder theorem.

30. (i) Take the given polynomial as $f(x)$.

    (ii) $f(1) = 0$, $f(2) = 0$.

Level 2

31. $\frac{f(3)}{f(2)} = \frac{4}{5}$ and $f(1) = 0$.
32. (i) Let the common factor be $x - a$ and find $f(a)$, and $g(a)$.

    (ii) Obtain the value of a in terms of k and l.

33. Dividend = Divisor × Quotient + Remainder.
34. (i) $x^2 + mx - n = (x - m)\ (x - n)$.

    (ii) Equate the corresponding terms.

35. (i) $f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2.$

    (ii) Replace $x + \frac{1}{x}$ by x.

    (iii) Put $x = \frac{1}{2}$.

36. (i) $f(3) = 10$, $f(5) = 14$

    (ii) Dividend = Divisor × Quotient + Remainder.

39. (i) $f(1) = 75$, $f(-2) = 15$.

(ii) Dividend = Divisor × Quotient + Remainder.

40. Verify from the options whether $f(1) = 5$, $f(2) = 7$ and $f(3) = 9$ by using remainder theorem.

41. $\frac{f(2)}{f(1)} = \frac{4}{0}$ and $f(-1) = 0$.

42. (i) $x^2 - 5x + 6 = (x - 2)(x - 3)$

(ii) $f(2) = 0$, $f(3) = 0$.

43. (i) Let the common factor be $(x - a)$, then $f(a) = g(a)$, obtain value of 'a'.

(ii) Substitute value of 'a' in $f(x)$.

44. $f(2) = 0$ and $f(-2)= 0$.

45. $x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$.

46. ADBC is the required sequential order.

47. CBA is the required sequential order.

48. Let $f(x) = px^3 + qx^2 + rx + s$

Given $x^2 - 1$ is a factor of $f(x)$.

$\therefore$ $(x + 1)$ and $(x - 1)$ are factors of $f(x)$

$\therefore$ $f(-1) = 0$ and $f(1) = 0$

$-p + q - r + s = 0$

$\Rightarrow \quad p + r = q + s$ (1)

$p + q + r + s = 0$ (2)

From Eqs. (1) and (2), we have

$p + r = 0$ and $q + s = 0$

$\Rightarrow \quad p = -r$ and $q = -s$.

49. Let $f(x) = x^3 + 3px^2 - 3pqx - q^3$

From the options

$f(q) = q^3 + 3pq^2 - 3pq^2 - q^3 = 0$

$x - q$ is a factor of $f(x)$.

50. Given $x - k$ is a common factor of $x^2 + 3x + a$ and $x^2 + 4x + b$

$\therefore$ $(k)^2 + 3k + a = 0$ and $(k)^2 + 4k + b = 0$

$\Rightarrow \quad k^2 + 4k + b = k^2 + 3k + a$

$\Rightarrow \quad k = a - b$.

Level 3

53. (i) Let the common factor be $(x - a)$, then make $f(a) = g(a)$, and get the value of 'a'.

(ii) Substitute value of 'a' in $f(x)$.

54. Let $f(x) = ax^2 + bx + c$, given $f(-1)$

$= 1$, $f(-2) = 4$ and $f(-3) = 3$.

55. Dividend = Divisor × Quotient + Remainder.

56. Let $f(x) = x^2 + 3x - 3$

Given, $f(p) = 1$

$\Rightarrow \quad p^2 + 3p - 3 = 1$

$\Rightarrow \quad p^2 + 3p - 4 = 0$

$\Rightarrow \quad (p + 4)(p - 1) = 0$

$\Rightarrow \quad p = -4$ or 1.

57. Let $f(x) = ax^3 - 5x^2 + x + p$

Given, $f(x)$ is divisible $x^2 - 3x + 2$, i.e.,

$\Rightarrow \quad f(x)$ is divisible by $(x - 1)$ and $(x - 2)$

$f(1) = 0$ and $f(2) = 0 \quad \Rightarrow \quad a + p - 4 = 0$

$\Rightarrow \quad a + p = 4$ (1)

$8a + p = 18 = 0 \quad \Rightarrow \quad 8a + p = 18$ (2)

On solving Eqs. (1) and (2), we get $a = p = 2$.

58. Let k should be added to the given expression so that the sum is divisible by $(3x + 1)$.

Let $f(x) = 9x^3 + 6x^2 + x + 2 + k$

Given $f\left(-\frac{1}{3}\right) = 0$

$$\Rightarrow \quad 9\left(-\frac{1}{3}\right)^3 + 6\left(-\frac{1}{3}\right)^2 - \frac{1}{3} + 2 + k = 0$$

$$\Rightarrow \quad -\frac{1}{3} + \frac{2}{3} - \frac{1}{3} + 2 + k = 0 \quad \Rightarrow \quad 2 + k = 0$$

$\Rightarrow \quad k = -2$.

59. Let $f(x) = 6x^2 + 13x + k$

Given $2x + 3$ is a factor of $f(x)$

By factor theorem,

$$f\left(-\frac{3}{2}\right) = 0$$

$$\Rightarrow \quad 6\left(-\frac{3}{2}\right)^2 + 13\left(-\frac{3}{2}\right) + k = 0$$

$$\Rightarrow \quad \frac{27}{2} - \frac{39}{2} + k = 0 \quad \Rightarrow \quad k = 6$$

$\therefore f(x) = 6x^2 + 13x + 6$

$= (2x + 3)(3x + 2)$

$\therefore$ The other factor is $3x + 2$.

60. Let $f(x) = ax^2 + bx + c$.

Given, $f(-1) = 6$, $f(-2) = 11$ and $f(-3) = 18$

$\Rightarrow \quad a - b + c = 6 \quad (1)$

$\Rightarrow \quad 4a - 2b + c = 11 \quad (2)$

$\Rightarrow \quad 9a - 3b + c = 18 \quad (3)$

Eq. (2) − Eq. (1) $\Rightarrow \quad 3a - b = 5 \quad (4)$

Eq. (3) − Eq. (2) $\Rightarrow \quad 5a - b = 7 \quad (5)$

on solving Eqs. (4) and (5), we get

$a = 1$, $b = -2$

substitute $a = 1$ and $b = -2$ in Eq. (1).

$\Rightarrow \quad c = 3$

Now $a + b + c = 2$.