Sets

Question1

Let S = {1, 2, 3,...,10}. Suppose M is the set of all the subsets of S, then the relation R = {(A, B) : A ∩ B ≠ φ ; A, B ∈ M} is :

[27-Jan-2024 Shift 1]

Options:

A.

symmetric and reflexive only

B.

reflexive only

C.

symmetric and transitive only

D.

symmetric only

Answer: D

Solution:

Let $S = \{1, 2, 3, \ldots, 10\}$

$R = \{(A, B) : A \cap B \neq \phi; A, B \in M\}$

For Reflexive,

M is subset of ' S '

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $A \cap B \neq \phi$,

$\Rightarrow$BRA $\Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If $A = \{(1, 2), (2, 3)\}$

$B = \{(2, 3), (3, 4)\}$

$C = \{(3, 4), (5, 6)\}$

ARB & BRC but A does not relate to C

So it not transitive

Question2

Let R be a relation on Z × Z defined by (a, b) R (c, d) if and only if ad - bc is divisible by 5 . Then R is

[29-Jan-2024 Shift 1]

Options:

A.

Reflexive and symmetric but not transitive

B.

Reflexive but neither symmetric not transitive

C.

Reflexive, symmetric and transitive

D.

Reflexive and transitive but not symmetric

Answer: A

Solution:

$(a, b)R(a, b)$ as $ab - ab = 0$

Therefore reflexive

Let $(a, b)R(c, d) \Rightarrow ad - bc$ is divisible by 5

$\Rightarrow \mathrm{bc} - \mathrm{ad}$ is divisible by $5 \Rightarrow (\mathrm{c}, \mathrm{d})\mathrm{R}(\mathrm{a}, \mathrm{b})$

Therefore symmetric

Relation not transitive as $(3, 1)\mathrm{R}(10, 5)$ and $(10, 5)\mathrm{R}(1, 1)$ but $(3, 1)$ is not related to $(1, 1)$

Question3

If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1, 2), (1, 3)} ⊂ R, then the number of elements in R is

[29-Jan-2024 Shift 2]

Options:

A.

10

B.

12

C.

8

D.

15

Answer: A

Solution:

Given set {1, 2, 3, 4}

Minimum order pairs are

(1, 1), (2, 2), (3, 3), (4, 4), (3, 1), (2, 1), (2, 3), (3, 2), (1, 3), (1, 2)

Thus no, of elements = 10

Question4

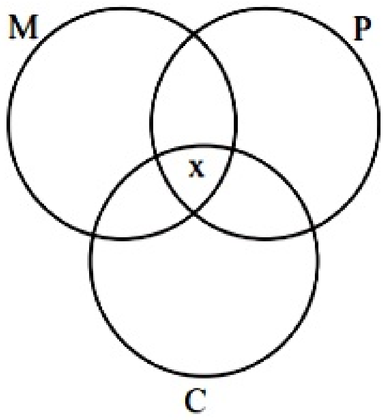
A group of 40 students appeared in an examination of 3 subjects -

Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is____

[30-Jan-2024 Shift 1]

Answer: 10

Solution:

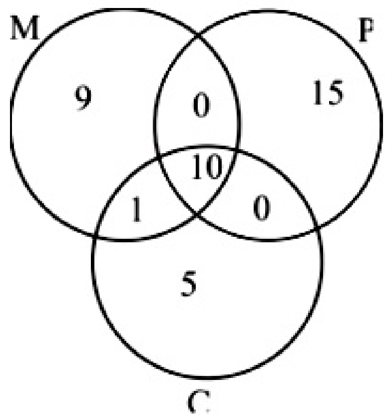


$11 - x \geq 0$ (Maths and Physics)

$x \leq 11$

x = 11 does not satisfy the data.

For x = 10



Hence maximum number of students passed in all the three subjects is 10 .

Question5

The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is___

[30-Jan-2024 Shift 2]

Answer: 960

Solution:

Total number of relation both symmetric and reflexive $= 2^{\frac{n^2-n}{2}}$

Total number of symmetric relation $= 2^{\left(\frac{n^2+n}{2}\right)}$

⇒ Then number of symmetric relation which are not reflexive

$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$

$\Rightarrow 2^{10} - 2^{6}$

$\Rightarrow 1024 - 64$

$= 960$

Question6

Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that R⊂S and the number of elements in S is n. Then, the minimum value of n is____

[31-Jan-2024 Shift 1]

Answer: 16

Solution:

All elements are included

Answer is 16

Question7

Let A = {1, 2, 3,........100}. Let R be a relation on A defined by (x, y) ∈ R if and only if 2x = 3y. Let R1 be a symmetric relation on A such that R⊂R1 and the number of elements in R1 is n. Then, the minimum value of n is

[31-Jan-2024 Shift 2]

Answer: 66

Solution:

R = {(3, 2), (6, 4), (9, 6), (12, 8),.........(99, 66)}

n(R) = 33

∴ 66

Question8

Let A = {1, 2, 3,...20}. Let R_1 and R_2 two relation on A such that
R1 = {(a, b) : b is divisible by a}
R2 = {(a, b) : a is an integral multiple of b}.
Then, number of elements in $R_1 - R_2$ is equal to_____

[1-Feb-2024 Shift 1]

Answer: 46

Solution:

$n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2 \quad + 2 + [\underbrace{1 + \ldots + 1}_{10 \text{ times}}]$

$n(R_1) = 66$

$R_1 \cap R_2 = \{(1, 1), (2, 2), \ldots (20, 20)\}$

$n(R_1 \cap R_2) = 20$

$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$

$= n(R_1) - 20$

$= 66 - 20$

$R_1 - R_2 = 46$ Pair

Question9

The number of elements in the set

S = {(x, y, z) : x, y, z ∈ Z, x + 2y + 3z = 42, x, y, z ≥ 0} equals ____

[1-Feb-2024 Shift 1]

Answer: 169

Solution:

$$x + 2y + 3z = 42, \qquad x, y, z \geq 0$$

$$
\begin{array}{ll}
z = 0 & x + 2y = 42 \Rightarrow 22 \\
z = 1 & x + 2y = 39 \Rightarrow 20 \\
z = 2 & x + 2y = 36 \Rightarrow 19 \\
z = 3 & x + 2y = 30 \Rightarrow 17 \\
z = 4 & x + 2y = 30 \Rightarrow 16 \\
z = 5 & x + 2y = 27 \Rightarrow 14 \\
z = 6 & x + 2y = 24 \Rightarrow 13 \\
z = 7 & x + 2y = 21 \Rightarrow 11 \\
z = 8 & x + 2y = 18 \Rightarrow 10 \\
z = 9 & x + 2y = 15 \Rightarrow 8 \\
z = 10 & x + 2y = 12 \Rightarrow 7 \\
z = 11 & x + 2y = 9 \Rightarrow 5 \\
z = 12 & x + 2y = 6 \Rightarrow 4 \\
z = 13 & x + 2y = 3 \Rightarrow 2 \\
z = 14 & x + 2y = 0 \Rightarrow 1
\end{array}
$$

Total : 169

Question10

Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all a,b,$\in$ R and (a, b) R_2 (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in$ N × N. Then

[1-Feb-2024 Shift 2]

Options:

A.

Only R_1 is an equivalence relation

B.

Only R_2 is an equivalence relation

C.

R_1 and R_2 both are equivalence relations

D.

Neither R_1 nor R_2 is an equivalence relation

Answer: B

Solution:

$aR_1b \Leftrightarrow a^2 + b^2 = 1; a, b \in R$

$(a, b)R_2(c, d) \Leftrightarrow a + d = b + c; (a, b), (c, d) \in N$

for R_1 : Not reflexive symmetric not transitive

for R_2 : R_2 is reflexive, symmetric and transitive

Hence only R_2 is equivalence relation.

Question11

The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d } so that it is an equivalence relation, is__
[24-Jan-2023 Shift 2]

Answer: 13

Solution:

Given R = {(a, b), (b, c), (b, d)}
In order to make it equivalence relation as per given set, R must be
{ (a, a), (b, b), (c, c), (d , d), (a, b), (b, a), (b, c), (c, b), (b, d), (d , b), (a, c), (a, d), (c, d), (d , c), (c, a), (d , a) }
There already given so 13 more to be added.

Question12

In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$ is :

[6-Apr-2023 shift 2]

Options:

A.

$\frac{\sqrt{129}}{12}$

B.

$\frac{\sqrt{117}}{12}$

C.

$\frac{\sqrt{119}}{12}$

D.

$\frac{3\sqrt{15}}{12}$

Answer: C

Solution:

Solution:

$n(A \cap B) = n(A) + n(B) - n(A \cap B)$

$n(A B B) = 75 + 40 - 100$

$n(A \cap B) = 15$

Only $E \rightarrow 60\ \alpha = 60$

Only $H \rightarrow 25\ \beta = 25$

Both $= 15$

$\frac{25x^2}{\alpha^2} + \frac{25y^2}{\beta^2} = 1$

$\frac{25x^2}{(60)^2} + \frac{(25y^2)}{(25)^2} = 1$

$e^2 = 1 - \left[\frac{25 \times 25}{(60)^2} \right]$

$e^2 = \frac{(60)^2 - (25)^2}{(60)^2}$

$e^2 = \frac{(60 - 25)(60 + 25)}{60 \times 60}$

$e^2 = \frac{(35)(85)}{60 \times 60} = \frac{119}{144}$

$e = \frac{\sqrt{119}}{12}$

Question13

Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:

[8-Apr-2023 shift 1]

Options:

A.

752

B.

772

C.

782

D.

792

Answer: D

Solution:

Solution:

$n(A \times B) = 10$

${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$

Question14

The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is

[10-Apr-2023 shift 1]

Answer: 6

Solution:

$-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 < 0$

$(n-5)^2 > 0$ $5 - 3\sqrt{2} < n < 5 + 3\sqrt{2}$

$N \in Z - \{5\}$ $n = \{2, 3, 4, 5, 6, 7, 8\}$

$\ldots$ (i) $\ldots$ (ii)

From (i) $\cap$ (ii)

$N = \{2, 3, 4, 5, 6, 8,\}$

Number of values of $n = 6$

Question15

The number of elements in the set S = {θ ∈[0, 2π] : $3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$} is :

[11-Apr-2023 shift 1]

Options:

A.

10

B.

9

C.

8

D.

12

Answer: B

Solution:

Solution:

$$3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$$

$$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$$

$$\Rightarrow 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$$

$$\Rightarrow 3\cos^2\theta(\cos^2\theta - 1) + 2\sin^2\theta(\sin^4\theta - 1) = 0$$

$$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta(1 + \sin^2\theta)\cos^2\theta - 1$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2 + 2\sin^2\theta - 3) = 0$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta - 1) = 0$$

(C1) $\sin^2\theta = 0 \rightarrow 3$ solution; $\theta = \{0, \pi, 2\pi\}$

(C2) $\cos^2\theta = 0 \rightarrow 2$ solution; $\theta = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

(C3) $\sin^2\theta = \frac{1}{2} \rightarrow 4$ solution; $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

No. of solution $= 9$

Question16

An organization awarded 48 medals in event ' A ', 25 in event ' B ' and 18 in event ' C '. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events ?

[11-Apr-2023 shift 1]

Options:

A.

15

B.

9

C.

21

D.

10

Answer: C

Solution:

Solution:

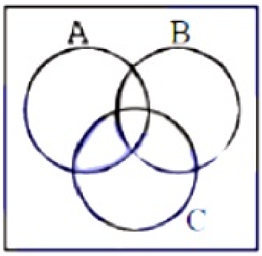
$|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$



$|A \cap B \cap C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C|$

$\Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60$

$= 36$

No. of men who received exactly 2 medals

$\Rightarrow \sum |A \cap B| - 3|A \cap B \cap C|$

$= 36 - 15$

$= 21$

Question17

The number of the relations, on the set {1,2,3} containing (1,2) and (2,3), which are reflexive and transitive

but not symmetric, is ________.

[12-Apr-2023 shift 1]

Answer: 3

Solution:

$A = \{1, 2, 3\}$

For Reflexive $(1, 1)(2, 2), (3, 3) \in R$

For transitive : $(1, 2)$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$

Not symmetric : $(2, 1)$ and $(3, 2) \notin R$

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$

Question18

The number of elements in the set $\{n \in N : 10 \le n \le 100.$ and $3^n - 3$ is a multiple of $7\}$ is ________

[15-Apr-2023 shift 1]

Answer: 15

Solution:

$n \in [10, 100]$

$3^n - 3$ is multiple of 7

$3^n = 7\lambda + 3$

$n = 1, 7, 13, 20, \ldots\ldots .97$

Number of possible values of $n = 15$

Question19

**Let $A = \{z \in C : 1 \le |z - (1 + i)| \le 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$.
Then, B :**
[24-Jun-2022-Shift-1]

Options:

A. is an empty set

B. contains exactly two elements

C. contains exactly three elements

D. is an infinite set

Answer: D

Solution:

Solution:

Let, $z = x + iy$

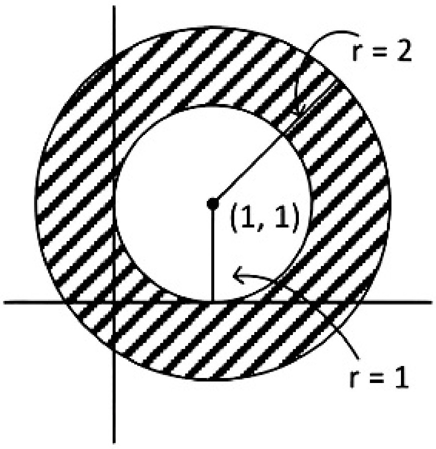
Given, $1 \leq |z - (1 + i)| \leq 2$

$\Rightarrow 1 \leq |x + iy - 1 - i| \leq 2$

$\Rightarrow 1 \leq |(x - 1) + i(y - 1)| \leq 2$

$\Rightarrow 1 \leq \sqrt{(x - 1)^2 + (y - 1)^2} \leq 2$

It represent two concentric circle both have center at $(1, 1)$ and radius 1 and 2 .



Also given,

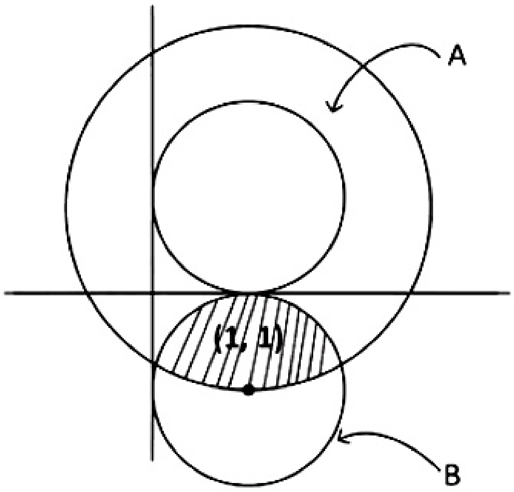
$|z - (1 - i)| = 1$

$\Rightarrow |x + iy - 1 + i| = 1$

$\Rightarrow |(x - 1) + i(y + 1)| = 1$

$\Rightarrow \sqrt{(x - 1)^2 + (y + 1)^2} = 1$

This represent a circle with center at $(1, -1)$ and radius $= 1$.



Question20

Let A = {x ∈ R: | x + 1 | <2} and B = {x ∈ R: | x − 1 | ≥2}. Then which one of the following statements is NOT true?
[25-Jun-2022-Shift-2]

Options:

A. A − B = (−1, 1)

B. B − A = R − (−3, 1)

C. A ∩ B = (−3, −1]

D. A ∪ B = R − [1, 3)

Answer: B

Solution:

Solution:

$A = (-3, 1)$ and $B = (-\infty, -1] \cup [3, \infty)$

So, $A - B = (-1, 1)$

$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

$A \cap B = (-3, -1]$

and $A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3)$

So Option B is not True

Question21

Let A = { n ∈ N : H.C.F. (n, 45) = 1 } and
Let B = {2k : k ∈ {1, 2, ..., 100}}. Then the sum of all the elements of A ∩ B is______
[26-Jun-2022-Shift-1]

Answer: 5264

Solution:

Solution:

Sum of all elements of A∩B=2 [Sum of natural numbers upto 100 which are neither divisible by 3 nor by 5]

$$= 2\left[\frac{100\times 101}{2} - 3\left(\frac{33\times 34}{2}\right) - 5\left(\frac{20\times 21}{2}\right) + 15\left(\frac{6\times 7}{2}\right) \right]$$

$$= 10100 - 3366 - 2100 + 630$$

$$= 5264$$

Question22

Let a set $A = A_1 \cup A_2 \cup \ldots\ldots \cup A_k$, where $A_i \cap A_j = \varphi$ for $i \neq j$, $1 \leq j, j \leq k$. Define the relation R from A to A by R = { (x, y) : $y \in A_i$. if and only if .$x \in A_i$, $1 \leq i \leq k$ }. Then, R is :
[29-Jun-2022-Shift-1]

Options:

A. reflexive, symmetric but not transitive.

B. reflexive, transitive but not symmetric.

C. reflexive but not symmetric and transitive.

D. an equivalence relation.

Answer: D

Solution:

R = {(x, y) : $y \in A_i$, iff $x \in A_i 1 \leq i \geq k$}
(1) Reflexive
(a, a) ⇒ $a \in A_i$ iff $a \in A_i$
(2) Symmetric
(a, b) ⇒ $a \in A_i$ iff $b \in A_i$
(b, a) ∈R as $b \in A_i$ iff $a \in A_i$
(3) Transitive
(a, b) ∈ R&(b, c) ∈ R
⇒$a \in A_i$ iff $b \in A_i$&$b \in A_i$ iff $c \in A_i$
⇒$a \in A_i$ iff $c \in A_i$
⇒(a, c) ∈ R.
⇒ RElation is equivalnece.

Question23

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A : \text{either } 1 \notin T \text{ or } 2 \in T\}$ and $C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number $\}$. Then the number of elements in the set $B \cup C$ is____
[25-Jul-2022-Shift-2]

Answer: 107

Solution:

$\because (B \cup C)' = B' \cap C'$
B' is a set containing sub sets of A containing element 1 and not containing 2.
And C' is a set containing subsets of A whose sum of elements is not prime.
So, we need to calculate number of subsets of $\{3, 4, 5, 6, 7\}$ whose sum of elements plus 1 is composite.
Number of such 5 elements subset = 1
Number of such 4 elements subset = 3 (except selecting 3 or 7)
Number of such 3 elements subset = 6 (except selecting $\{3, 4, 5\}$, $\{3, 6, 7\}$, $\{4, 5, 7\}$ or $\{5, 6, 7\}$)
Number of such 2 elements subset = 7 (except selecting $\{3, 7\}$, $\{4, 6\}$, $\{5, 7\}$)
Number of such 1 elements subset = 3 (except selecting $\{4\}$ or $\{6\}$)
Number of such 0 elements subset = 1
$n(B' \cap C') = 21 \Rightarrow n(B \cup C) = 2^7 - 21 = 107$

Question24

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \varphi\}$ is _______.
[26-Jul-2022-Shift-2]

Answer: 112

Solution:

$A = \{7, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$

Total subset of $A = 2^7 = 128$

$C \cap B = \varphi$ when set C contains the element 1, 2, 4, 5

$\therefore S = \{C \subseteq A; C \cap B \neq \varphi\}$

$= \text{Total} - (C \cap B = \varphi)$

$= 128 - 2^4 = 112$

Question25

Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $aR_1b \Leftrightarrow ab \geq 0$ and $aR_2b \Leftrightarrow a \geq b$ Then,
[27-Jul-2022-Shift-1]

Options:

A. R_1 is an equivalence relation but not R_2

B. R_2 is an equivalence relation but not R_1

C. both R_1 and R_2 are equivalence relations

D. neither R_1 nor R_2 is an equivalence relation

Answer: D

Solution:

$R_1 = \{xy \geq 0, x, y \in R\}$
For reflexive $x \times x \geq 0$ which is true.
For symmetric
If $xy \geq 0 \Rightarrow yx \geq 0$
If $x = 2, y = 0$ and $z = -2$
Then $x \cdot y \geq 0 \& y \cdot z \geq 0$ but $x \cdot z \geq 0$ is not true
⇒ not transitive relation.
$\Rightarrow R_1$ is not equivalence
R_2 if $a \geq b$ it does not implies $b \geq a$
$\Rightarrow R_2$ is not equivalence relation

Question26

For $\alpha \in N$, consider a relation R on N given by R. = { (x, y) : $3x + \alpha y$ is a multiple of 7}. The relation R is an equivalence relation if and only if :
[28-Jul-2022-Shift-1]

Options:

A. $\alpha = 14$

B. α is a multiple of 4

C. 4 is the remainder when α is divided by 10

D. 4 is the remainder when α is divided by 7

Answer: D

Solution:

R = { (x, y) : $3x + \alpha y$ is multiple of 7}, now R to be an equivalence relation
(1) R should be reflexive : $(a, a) \in R\ \forall a \in N$
$\therefore 3a + a\alpha = 7k$
$\therefore (3 + \alpha)a = 7k$
$\therefore 3 + \alpha = 7k_1 \Rightarrow \alpha = 7k_1 - 3$
$= 7k_1 + 4$
(2) R should be symmetric : $aRb \Leftrightarrow bRa$
$aRb : 3a + (7k - 3)b = 7m$
$\Rightarrow 3(a - b) + 7kb = 7m$
$\Rightarrow 3(b - a) + 7ka = 7m$
So, $aRb \Rightarrow bRa$
$\therefore$ R will be symmetric for $a = 7k_1 - 3$
(3) Transitive : Let $(a, b) \in R$, $(b, c) \in R$
$\Rightarrow 3a + (7k - 3)b = 7k_1$ and
$3b + (7k_2 - 3)c = 7k_3$
Adding $3a + 7kb + (7k_2 - 3)c = 7(k_1 + k_3)$

$3a + (7k_2 - 3)c = 7m$
$\therefore (a, c) \in R$
$\therefore$R is transitive
$\therefore \alpha = 7k - 3 = 7k + 4$

Question27

Let R be a relation from the set {1, 2, 3, ..., 60} to itself such that R = { (a, b) : b = pq, where p, q ≥ 3 are prime numbers }. Then, the number of elements in R is:
[29-Jul-2022-Shift-1]

Options:

A. 600

B. 660

C. 540

D. 720

Answer: B

Solution:

b can take its values as 9, 15, 21, 33, 39, 51, 57, 25, 35, 55, 49
b can take these 11 values and a can take any of 60 values
So, number of elements in $R = 60 \times 11 = 660$

Question28

Let $A = \{ n \in N : n$ is a 3 -digit number $\}$ $B = \{9k + 2 : k \in N \}$ and $C = \{9k + I : k \in N \}$ for some $I (0 < 1 < 9)$
If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to
[2021, 24 Feb. Shift-1]

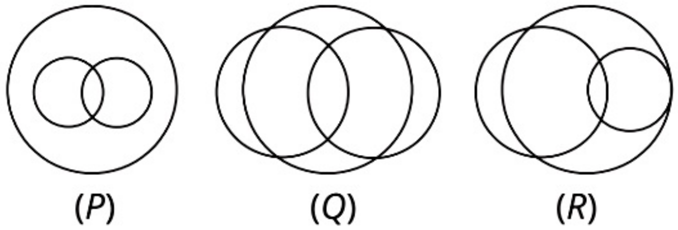
Answer: 5

Solution:

Given, $A = \{ n \in N : n$ is a 3 -digit number $\}$
$B = \{9k + 2 : k \in N \}$
$C = \{9k + 1 : k \in N \}$
∵3 digit number of the form $3k + 2$ are $\{101, 109, ... 992\}$
$\Rightarrow \text{ Sum } = \frac{100}{2}[101 + 992] = \frac{100 \times 1093}{2}$
Similarly, 3-digit number of the form $9k + 5$ is
$\frac{100}{2}[104 + 995] = \frac{100 \times 1099}{2}$
[∵ numbers are $104, 113, \ldots, 995$] Their sum $= \frac{100 \times 1093}{2} + \frac{100 \times 1099}{2}$
$= 100 \times 1096 = 400 \times 274$
Hence, we can say the value of $I = 5$ as the second series of numbers obtained by set C is of the form $9k + 5$.
∴ Required value of $I = 5$

Question29

In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagram can justify the above statement?



[2021, 17 March Shift-1]

Options:

A. P and Q

B. P and R

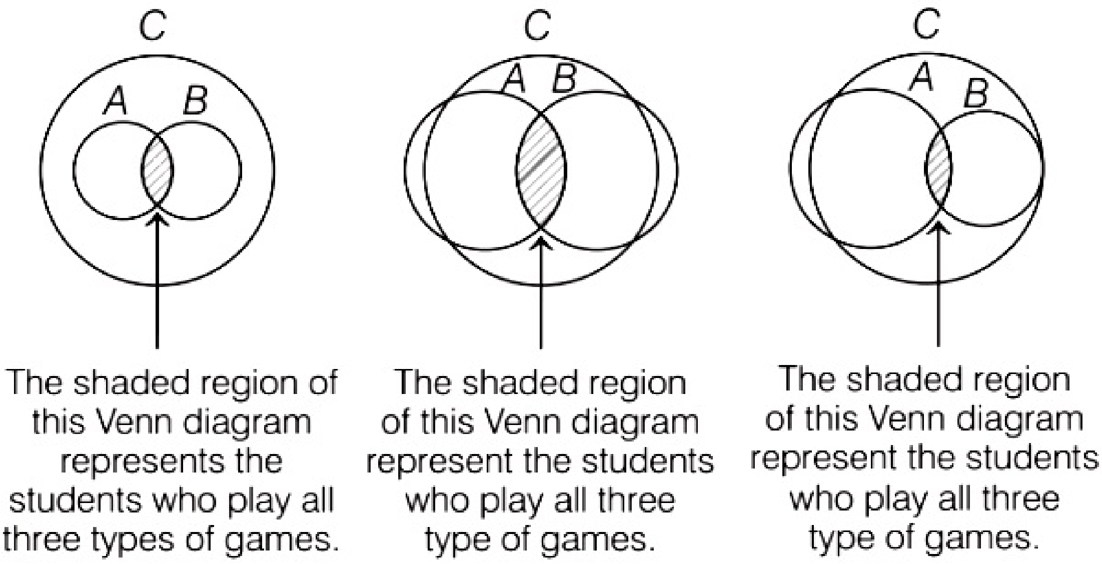
C. None of these

D. 0 and R

Answer: C

Solution:

Solution:



Question30

Let A = $\{n \in N \mid n^2 \leq n + 10,000\}$,
B = $\{3k + 1 \mid k \in N\}$ and C = $\{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to
[2021, 27 July Shift-II]

Answer: 832

Solution:

Solution:
Let $A = \{n \in N \mid n^2 \leq n + 10000\}$
$n^2 \leq n + 10000$
$n^2 - n \leq 10000$
$\Rightarrow n(n-1) \leq 100 \times 100$
$\Rightarrow A = \{1, 2, 3, \ldots\ldots, 100\}$
Now, $B = \{3k + 1 \mid k \in N\}$

$B = \{4, 7, 10, 13, \ldots\}$
and $C = \{2k \mid k \in N\}$
$C = \{2, 4, 6, 8, \ldots\}$
So, $B - C = \{7, 13, 19, \ldots\ldots, 97, \ldots\}$
So, $A \cap (B - C) = \{7, 13, 19, \ldots\ldots, 97\}$
This form an AP with common difference (d=6)
$\Rightarrow 97 = 7 + (n-1)6$
$n = \frac{97-7}{6} + 1 = 16 \quad [\because a_n = a + (n-1)d]$
Hence, sum $= \frac{16}{2}[7 + 97]$
$= 832 \quad \left\{ \because S_n = \frac{n}{2}(a + 1) \right\}$

Question31

If $A = \{x \in R: |x - 2| > 1\}$,
$B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$ and
$C = \{x \in R: |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is
[2021, 27 Aug. Shift-I]

Answer: 256

Solution:

$A = \{x \in R: |x - 2| > 1\}$
$\Rightarrow A = (-\infty, 1) \cup (3, \infty)$
$B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$
$\Rightarrow B = (-\infty, -2) \cup (2, \infty)$
$C = \{x \in R: |x - 4| \geq 2\}$
$\Rightarrow C = (-\infty, 2] \cup [6, \infty)$
$\Rightarrow A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$
$\Rightarrow (A \cap B \cap C)^C = [-2, 6)$
$\therefore (A \cap B \cap C)^C \cap Z = \{-2, -1, 0, 1, 2, 3, 4, 5\}$
Number of subsets of $(A \cap B \cap C)^C \cap Z$
$= 2^8 = 256$

Question32

Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set
[2021, 26 Aug. Shift-1]

Options:

A. $\{80, 83, 86, 89\}$

B. $\{84, 86, 88, 90\}$

C. $\{79, 81, 83, 85\}$

D. $\{84, 87, 90, 93\}$

Answer: C

Solution:

Solution:
Let $A =$ Patient suffering from heart ailment and $B =$ Set of patient suffering from lungs infection
Given, $n(A) = 89\%$ and $n(B) = 98\%$
$n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$
$\Rightarrow 100 \geq 89 + 98 - n(A \cap B)$
$\Rightarrow 87 \leq n(A \cap B)$
Also, $n(A \cap B) = \min\{n(A), n(B)\}$
$\Rightarrow n(A \cap B) \leq 89$
$\therefore 87 \leq n(A \cap B) \leq 89$
So, $n(A \cap B) \notin \{79, 81, 83, 85\}$.

Question33

Let $X = \{n \in N : 1 \leq n \leq 50\}$. If $A = \{n \in X : n$ is a multiple of $2\}$ and $B = \{n \in X : n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is
[Jan. 7, 2020 (II)]

Answer: 29

Solution:

From the given conditions, $n(A) = 25, n(B) = 7$ and $n(A \cap B) = 3$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 25 + 7 - 3 = 29$

Question34

Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is
[Sep. 06, 2020 (I)]

Answer: 28

Solution:

$2^m = 112 + 2^n \Rightarrow 2^m - 2^n = 112$
$\Rightarrow 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$
$\therefore m = 7, n = 4 \Rightarrow mn = 28$

Question35

A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:
[Sep. 05, 2020 (I)]

Options:

A. 63

B. 36

C. 54

D. 38

Answer: B

Solution:

Given, $n(C) = 73$, $n(T) = 65$, $n(C \cap T) = x$
$\therefore 65 \geq n(C \cap T) \geq 65 + 73 - 100$
$\Rightarrow 65 \geq x \geq 38 \Rightarrow x \neq 36$

Question36

A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:
[Sep. 04, 2020 (I)]

Options:

A. 29

B. 37

C. 65

D. 55

Answer: D

Solution:

Let $n(U) = 100$, then $n(A) = 63$, $n(B) = 76$ $n(A \cap B) = x$
Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) \leq 100$
$= 63 + 76 - x \leq 100$
$\Rightarrow x \geq 139 - 100 \Rightarrow x \geq 39$
$\because n(A \cap B) \leq n(A)$
$\Rightarrow x \leq 63$
$\therefore 39 \leq x \leq 63$

Question37

Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i' s and exactly 6 of sets Y_i's, then n is equal to
[Sep. 04, 2020 (II)]

Options:

A. 15

B. 50

C. 45

D. 30

Answer: D

Solution:

$\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$
$\because n(X_i) = 10, n(Y_i) = 5$
So, $\bigcup_{i=1}^{50} X_i = 500, \bigcup_{i=1}^{n} Y_i = 5n$
$\Rightarrow \frac{500}{20} = \frac{5n}{6} \Rightarrow n = 30$

Question38

Let $S = \{1, 2, 3, ..., 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is :
[Jan. 12, 2019 (I)]

Options:

A. $2^{100} - 1$

B. $2^{50}(2^{50} - 1)$

C. $2^{50} - 1$

D. $2^{50} + 1$

Answer: B

Solution:

∵ Product of two even number is always even and product of two odd numbers is always odd.
∴ Number of required subsets
= Total number of subsets − Total number of subsets having only odd numbers
$= 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

Question39

Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is :
[Jan. 12, 2019 (II)]

Options:

A. 2^{15}

B. 2^{18}

C. 2^{12}

D. 2^{10}

Answer: A

Solution:

(a) Let $x \in A$, then
∵ $2^{(x+2)(x^2-5x+6)} = 1 \Rightarrow (x+2)(x-2)(x-3) = 0$
$x = -2, 2, 3$
$A = \{-2, 2, 3\}$
Then, $n(A) = 3$
Let $x \in B$, then
$-3 < 2x - 1 < 9$

$-1 < x < 5$ and $x \in Z$
$\therefore$ $B = \{0, 1, 2, 3, 4\}$
$n(B) = 5$
$n(A \times B) = 3 \times 5 = 15$
Hence, Number of subsets of $A \times B = 2^{15}$

Question40

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is:
[Jan. 10, 2019 (II)]

Options:

A. 102

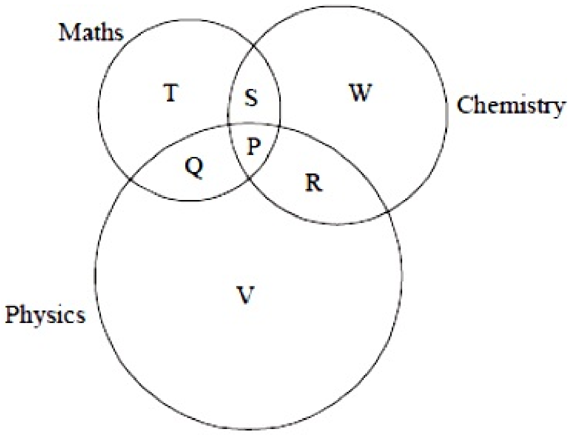
B. 42

C. 1

D. 38

Answer: D

Solution:



$P = \{30, 60, 90, 120\}$
$\Rightarrow n(P) = 4$
$Q = \{6n : n \in N, 1 \le n \le 23\} - P$
$\Rightarrow n(Q) = 19$
$R = \{15n : n \in N, 1 \le n \le 9\} - P$
$\Rightarrow n(R) = 5$
$S = \{10n : n \in N, 1 \le n \le 14\} - P$
$\Rightarrow n(S) = 10$
$n(T) = 70 - n(P) - n(Q) - n(S) = 70 - 33 = 37$
$n(V) = 46 - n(P) - n(Q) - n(R) = 46 - 28 = 18$
$n(W) = 28 - n(P) - n(R) - n(S) = 28 - 19 = 9$
$\Rightarrow$ Number of required students $= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$
$= 140 - 102 = 38$

Question41

Let A, B and C be sets such that $\varphi \neq A \cap B \subseteq C$. Then which of the following statements is not true? [April 12, 2019 (II)]

Options:

A. $B \cap C \neq \varphi$

B. If $(A - B) \subseteq C$, then $A \subseteq C$

C. $(C \cup A) \cap (C \cup B) = C$

D. If $(A - C) \subseteq B$, then $A \subseteq B$

Answer: D

Solution:

Solution:
(1),(2) and (4) are always correct
In (3) option,
If $A = C$ then $A - C = \varphi$
Clearly, $\varphi \subset eqB$ but $A \subset eqB$ is not always true.

Question42

Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: [April. 09, 2019 (II)]

Options:

A. 13.9

B. 12.8

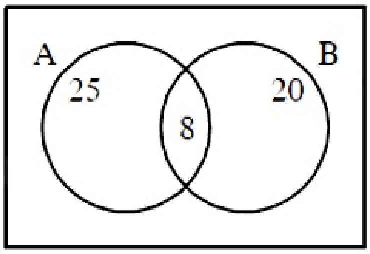
C. 13

D. 13.5

Answer: A

Solution:

Solution:



% of people who reads A only $= 25 - 8 = 17\%$
% of people who read B only $= 20 - 8 = 12\%$
% of people from A only who read advertisement $= 17 \times 0.3 = 5.1\%$
% of people from B only who read advertisement $= 12 \times 0.4 = 4.8\%$
% of people from A&B both who read advertisement $= 8 \times 0.5 = 4\%$
∴ total % of people who read advertisement $= 5.1 + 4.8 + 4 = 13.9\%$

Question43

Let $S = \{ x \in R : x \geq 0$ and $2 \mid \sqrt{x} - 3 \mid + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$. Then S [2018]

Options:

A. contains exactly one element.

B. contains exactly two elements.

C. contains exactly four elements.

D. is an empty set

Answer: B

Solution:

Solution:
Case-I: $x \in [0, 9]$
$2(3 - \sqrt{x}) + x - 6\sqrt{x} + 6 = 0$
$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$
$\Rightarrow x = 16, 4$
Since $x \in [0, 9]$
$\therefore\ x = 4$
Case-II: $x \in [9, \infty]$
$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$
$\Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$
Since $x \in [9, \infty]$
$\therefore x = 16$
Hence, $x = 4 \& 16$

Question44

If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S [2016]

Options:

A. contains exactly two elements.

B. contains more than two elements.

C. is an empty set.

D. contains exactly one element.

Answer: A

Solution:

Solution:

$f(x) + 2f\left(\frac{1}{x}\right) = 3x \quad \ldots..(1)$

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$

Adding (1) and (2)

$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = x + \frac{1}{x}$

Substracting (1) from (2)

$\Rightarrow f(x) - f\left(\frac{1}{x}\right) = \frac{3}{x} - 3x\ldots$

On adding (3) and (4)

$\Rightarrow f(x) = \frac{2}{x} - x$

$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x \Rightarrow x = \frac{2}{x}$

$x^2 = 2$ or $x = \sqrt{2}, -\sqrt{2}$

Question45

Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then: [Online April 10, 2016]

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Options:

A. $P \subset Q$ and $Q - P \neq \varphi$

B. Q not $\subset P$

C. $P = Q$

D. P not $\subset Q$

Answer: C

Solution:

Solution:

$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \cos\theta + \sqrt{2}\cos\theta$

$= (\sqrt{2}+1)\cos\theta = \left(\frac{2-1}{\sqrt{2}-1}\right)\cos\theta$
$\Rightarrow (\sqrt{2}-1)\sin\theta = \cos\theta$
$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$
$\therefore$ P = Q

Question46

In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements:
(A) 5% families own both a car and a phone
(B) 35% families own either a car or a phone
(C) 40,000 families live in the town
Then,
[Online April 10, 2015]

Options:

A. Only (A) and (C) are correct.

B. Only (B) and (C) are correct.

C. All (A), (B) and (C) are correct.

D. Only (A) and (B) are correct.

Answer: C

Solution:

Solution:
$n(P) = 25\%$
$n(C) = 15\%$
$n(P' \cup C') = 65\%$
$\Rightarrow n(P \cup C)' = 65\%$
$n(P \cup C) = 35\%$
$n(P \cap C) = n(P) + n(C) - n(P \cup C)$
$25 + 15 - 35 = 5\%$
$x \times 5\% = 2000$
$x = 40,000$

Question47

A relation on the set $A = \{x : |x| < 3, x \in Z\}$ where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is:
[Online April 12, 2014]

Options:

A. 32

B. 16

C. 8

D. 64

Answer: B

Solution:

Solution:
(b) $A = \{x : |x| < 3, x \in Z\}$
$A = \{-2, -1, 0, 1, 2\}$
$R = \{(x, y) : y = |x|, x \neq -1\}$
$R = \{(-2, 2), (0, 0), (1, 1), (2, 2)\}$
R has four elements Number of elements in the power set of R
$= 2^4 = 16$

Question48

Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is : [2012]

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Options:

A. 5^2

B. 3^5

C. 2^5

D. 5^3

Answer: B

Solution:

Solution:
Let $X = \{1, 2, 3, 4, 5\}$ $n(x) = 5$
Each element of x has 3 options. Either in set Y or set Z or none.
$(\because Y \cap Z = \varphi)$
So, number of ordered pairs $= 3^5$

Question49

If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then [2009]

Options:

A. $A = C$

B. $B = C$

C. $A \cap B = \varphi$

D. $A = B$

Answer: B

Solution:

Finding the value:
$A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$
$\Rightarrow (A \cap C) \cup (B \cap C) = C$
$\Rightarrow (A \cap B) \cup (B \cap C) = C \;\ldots .\,(i) \;\; (\because A \cap C = A \cap B)$
$\Rightarrow A \cup B = A \cup C$
$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B$
$\Rightarrow B = (A \cap B) \cup (C \cap B)$
$= (A \cap B) \cup (B \cap C) \;\; \cdots\cdots\cdot (ii)$
From (i) and (ii)
$B = C$