DPP - Daily Practice Problems

Date : ____ **Start Time :** ____ **End Time :** ____

MATHEMATICS CM04

SYLLABUS : Principle of Mathematical Induction

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. Let P(n) : "$2^n < (1 \times 2 \times 3 \times ... \times n)$". Then the smallest positive integer for which P(n) is true is
(a) 1 (b) 2
(c) 3 (d) 4

2. If $P(n)$: "$46^n + 16^n + k$ is divisible by 64 for $n \in$ N" is true, then the least negative integral value of k is.
(a) -1 (b) 1
(c) 2 (d) -2

3. Use principle of mathematical induction to find the value of k, where $(10^{2n-1} + 1)$ is divisible by k.
(a) 11 (b) 12
(c) 13 (d) 9

4. A student was asked to prove a statement $P(n)$ by induction. He proved that $P(k+1)$ is true whenever $P(k)$ is true for all $k > 5 \in$ N and also that $P(5)$ is true. On the basis of this he could conclude that P(n) is true
(a) for all $n \in$ N
(b) for all $n > 5$
(c) for all $n \geq 5$
(d) for all $n < 5$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. Let T(k) be the statement $1 + 3 + 5 + ... + (2k - 1) = k^2 + 10$
Which of the following is correct?
(a) T(1) is true
(b) T(k) is true $\Rightarrow$ T(k + 1) is true
(c) T(n) is true for all $n \in \mathbf{N}$
(d) All above are correct

6. Let $S(k) = 1 + 3 + 5 ... + (2k - 1) = 3 + k^2$. Then which of the following is true?
(a) Principle of mathematical induction can be used to prove the formula
(b) $S(k) \Rightarrow S(k+1)$
(c) $S(k) \not\Rightarrow S(k+1)$
(d) $S(1)$ is correct

7. For natural number n, $2^n(n-1)! < n^n$, if
(a) $n < 2$ (b) $n > 2$
(c) $n \geq 2$ (d) Never

8. For all positive integral values of n, $3^{2n} - 2n + 1$ is divisible by
(a) 2 (b) 4
(c) 8 (d) 12

9. For every natural number $n, n(n+1)$ is always
(a) Even (b) Odd
(c) Multiple of 3 (d) Multiple of 4

10. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + ...}}}$ having n radical signs then by methods of mathematical induction which is true?
(a) $a_n > 7 \; \forall \; n \geq 1$ (b) $a_n < 7 \; \forall \; n \geq 1$
(c) $a_n < 4 \; \forall \; n \geq 1$ (d) $a_n < 3 \; \forall \; n \geq 1$

11. For every positive integral value of n, $3^n > n^3$ when
(a) $n > 2$ (b) $n \geq 3$
(c) $n \geq 4$ (d) $n < 4$

12. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then P($n$) is true for
(a) $n \geq 1$ (b) $n > 0$
(c) $n < 0$ (d) $n \geq 2$

13. If $n \in N$, then $x^{2n-1} + y^{2n-1}$ is divisible by
(a) $x + y$ (b) $x - y$
(c) $x^2 + y^2$ (d) $x^2 + xy$

14. For a positive integer n,
Let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{(2^n) - 1}$. Then
(a) $a(100) \leq 100$ (b) $a(100) > 100$
(c) $a(200) \leq 100$ (d) $a(200) < 100$

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. $2^n > n^2$ when $n \in N$ such that

(a) $n > 2$ (b) $n > 3$

(c) $n < 5$ (d) $n \geq 5$

16. For every natural number n, $n(n^2 - 1)$ is divisible by

(a) 4 (b) 6

(c) 10 (d) None of these

17. If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in N$, then the least negative value of λ is

(a) –2 (b) –1

(c) –3 (d) – 4

18. If $n \in N$ and n is odd, then $n(n^2 - 1)$ is divisible by

(a) 24 (b) 16

(c) 32 (d) 19

19. For each $n \in N$, the correct statement is

(a) $2^n < n$ (b) $n^2 > 2n$

(c) $n^4 < 10^n$ (d) $2^{3n} > 7n + 1$

20. P(n) : $2.7^n + 3.5^n - 5$ is divisible by

(a) 24, $\forall$ n $\in$ N

(b) 21, $\forall$ n $\in$ N

(c) 35, $\forall$ n $\in$ N

(d) 50, $\forall$ n $\in$ N

21. By mathematical induction,

$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + ... + \frac{1}{n(n+1)(n+2)}$ is equal to

(a) $\frac{n(n+1)}{4(n+2)(n+3)}$ (b) $\frac{n(n+3)}{4(n+1)(n+2)}$

(c) $\frac{n(n+2)}{4(n+1)(n+3)}$ (d) None of these

22. For every positive integer n, $7^n - 3^n$ is divisible by

(a) 7 (b) 3

(c) 4 (d) 5

23. For all n $\in$ N, the sum of $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is

(a) a negative integer (b) a whole number

(c) a real number (d) a natural number

24. For $n \in N$, $x^{n+1} + (x+1)^{2n-1}$ is divisible by

(a) x (b) $x + 1$

(c) $x^2 + x + 1$ (d) $x^2 - x + 1$

25. If n is a positive integer, then $5^{2n+2} - 24n - 25$ is divisible by

(a) 574 (b) 575

(c) 674 (d) 576

26. For all $n \geq 1$,

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} =$

(a) $\frac{n}{n+1}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{n(n+1)}$ (d) None of these

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ			

27. By the principle of induction $\forall\ n \in N$, 3^{2n} when divided by 8, leaves remainder

(a) 2 (b) 3

(c) 7 (d) 1

28. **Statement-1 :** $1 + 2 + 3 + ... + n < \frac{1}{8}(2n + 1)^2$, $n \in N$.

Statement-2 : $n(n + 1)(n + 5)$ is a multiple of 3, $n \in N$.

(a) Only Statement-1 is true

(b) Only Statement-2 is true

(c) Both Statements are true

(d) Both Statements are false

29. **Statement-1 :** For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement-2 : For every natural number $n \geq 2$,

$\sqrt{n(n+1)} < n + 1$.

(a) Statement-1 is correct, Statement-2 is correct; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is correct, Statement-2 is correct; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is correct, Statement-2 is incorrect

(d) Statement-1 is incorrect, Statement-2 is correct.

30. For all $n \in N$, $41^n - 14^n$ is a multiple of

(a) 26 (b) 27

(c) 25 (d) None of these

RESPONSE GRID	
	27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 4 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	40	Qualifying Score	55
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(d)** Since $P(1): 2 < 1$ is false
$P(2): 2^2 < 1 \times 2$ is false
$P(3): 2^3 < 1 \times 2 \times 3$ is false
$P(4): 2^4 < 1 \times 2 \times 3 \times 4$ is true

2. **(a)** For $n = 1, P(1):\ 65 + k$ is divisible by 64.
Thus k, should be –1
Since $65 - 1 = 64$ is divisible by 64.

3. **(a)** Let P(n) be the statement given by
P(n) : $10^{2n-1} + 1$ is divisible by 11
For n = 1, P(1) : $10^{(2 \times 1)-1} + 1 = 11$,
which is divisible by 11.
So, P(1) is true.
Let P(k) be true, i.e. $10^{2k-1} + 1$ is divisible by 11
$\Rightarrow\ 10^{2k-1} + 1 = 11\lambda$, for some $\lambda \in N$ … (i)
We shall now show that P(k + 1) is true. For this, we have to show that $10^{2(k+1)-1} + 1$ is divisible by 11.
Now, $10^{2(k+1)-1} + 1 = 10^{2k-1}\, 10^2 + 1$
$= (11\lambda - 1)100 + 1$ [Using (i)]
$= 1100\lambda - 99 = 11(100\lambda - 9) = 11\mu$,
where $\mu = 100\lambda - 9 \in N$
$\Rightarrow\ 10^{2(k+1)-1} + 1$ is divisible by 11
$\Rightarrow$ P(k + 1) is true.
Thus, P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(k) is true for all $n \in N$, i.e. $10^{2n-1} + 1$ is divisible by 11 for all $n \in N$.

4. **(c)** Since $P(5)$ is true and $P(k+1)$ is true, whenever $P(k)$ is true.

5. **(b)** When k = 1, LHS = 1 but RHS = 1 + 10 = 11
$\therefore$ T(1) is not true
Let T(k) is true.
That is $1 + 3 + 5 + ... + (2k - 1) = k^2 + 10$

Now, $1+3+5+.....+(2k-1)+(2k+1)$

$$=k^2+10+2k+1=(k+1)^2+10$$

$\therefore T(k+1)$ is true.

That is T(k) is true $\Rightarrow T(k+1)$ is true.

But T(n) is not true for all $n \in \mathbf{N}$, as T(1) is not true.

6. **(b)** $S(k) = 1+3+5+...+(2k-1) = 3+k^2$

$S(1): 1 = 3+1$, which is not true

$\because$ $S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(k)$ is true, i.e.

$$1+3+5....+(2k-1)=3+k^2$$

$$\Rightarrow 1+3+5....+(2k-1)+2k+1$$

$$=3+k^2+2k+1=3+(k+1)^2$$

$\therefore S(k) \Rightarrow S(k+1)$

7. **(b)** Check through option, the condition $2^n(n-1)! < n^n$ is satisfied for $n > 2$

8. **(a)** Putting $n = 2$ in $3^{2n} - 2n + 1$ then,

$3^{2\times2} - 2\times2+1 = 81-4+1 = 78$, which is divisible by 2.

9. **(a)** The product of two consecutive numbers is always even.

10. **(b)** $a_1 = \sqrt{7} < 7$. Let $a_m < 7$

Then $a_{m+1} = \sqrt{7+a_m} \Rightarrow a^2_{m+1}$

$= 7 + a_m < 7 + 7 < 14$.

$\Rightarrow a_{m+1} < \sqrt{14} < 7$; So by the principle of mathematical induction $a_n < 7 \ \forall\ n$.

11. **(c)** Check through option, the condition $3^n > n^3$ is true when $n \geq 4$.

12. **(d)** Let $P(n): \dfrac{4^n}{n+1} < \dfrac{(2n)!}{(n!)^2}$

For $n = 2$,

$$P(2): \frac{4^2}{2+1} < \frac{4!}{(2)^2} \Rightarrow \frac{16}{3} < \frac{24}{4}$$

which is true.

Let for $n = m \geq 2$, $P(m)$ is true.

i.e. $\dfrac{4^m}{m+1} < \dfrac{(2m)!}{(m!)^2}$

Now, $\dfrac{4^{m+1}}{m+2} = \dfrac{4^m}{m+1} \cdot \dfrac{4(m+1)}{m+2}$

$$< \frac{(2m)!}{(m!)^2} \cdot \frac{4(m+1)}{(m+2)}$$

$$= \frac{(2m)!(2m+1)(2m+2)4(m+1)(m+1)^2}{(2m+1)(2m+2)(m!)^2(m+1)^2(m+2)}$$

$$= \frac{[2(m+1)]!}{[(m+1)!]^2} \cdot \frac{2(m+1)^2}{(2m+1)(m+2)}$$

$$< \frac{[2(m+1)]!}{[(m+1)!]^2}$$

Hence, for $n \geq 2$, P(n) is true.

13. **(a)** $x^{2n-1} + y^{2n-1}$ is always contain equal odd power. So it is always divisible by $x + y$.

14. **(a)** It can be proved with the help of mathematical induction that $\dfrac{n}{2} < a(n) \leq n$.

$\therefore$ $\dfrac{200}{2} < a(200) \Rightarrow a(200) > 100$ and

$a(100) \leq 100$.

15. **(d)** Let the given statement be P (n), then

P (1) $\Rightarrow 2^1 > 1^2$ which is true

P (2) $\Rightarrow 2^2 > 2^2$ which is false

P (3) $\Rightarrow 2^3 > 3^2$ which is false

P (4) $\Rightarrow 2^4 > 4^2$ which is false

P (5) $\Rightarrow 2^5 > 5^2$ which is true

P (6) $\Rightarrow 2^6 > 6^2$ which is true

$\therefore$ P (n) is true when $n \geq 5$

16. **(b)** $n(n^2-1) = (n-1)(n)(n+1)$

It is product of three consecutive natural numbers, so according to Langrange's theorem it is divisible by 3! i.e., 6

17. **(b)** For $n = 1$, we have

$49^n + 16n + \lambda = 49 + 16 + \lambda = 65 + \lambda$

$= 64 + (\lambda + 1)$, which is divisible by 64 if $\lambda = -1$

For $n = 2$, we have

$49^n + 16n + \lambda = 49^2 + 16 \times 2 + \lambda = 2433 + \lambda$

$= 64 \times 38 + (\lambda + 1)$, which is divisible by 64 if $\lambda = -1$

Hence, $\lambda = -1$

18. **(a)** Let P (n) = n ($n^2 - 1$) then

P(1) = 1 (0) = 0 which is divisible by every $n \in$ N

P (3) = 3 (8) = 24 which is divisible by 24 and 8

P (5) = 5 (24) = 120 which is divisible by 24 and 8

Hence P (n) is divisible by 24.

19. **(c)** Let n = 1, then option (a), (b) and (d) eliminated. Only option (c) satisfied.

20. **(a)** P(n) : $2.7^n + 3.5^n - 5$ is divisible by 24.

For n = 1,

P(1) : 2.7 + 3.5 – 5 = 24, which is divisible by 24.

Assume that P(k) is true,

i.e. $2.7^k + 3.5^k - 5 = 24q$, where $q \in$ N ... (i)

Now, we wish to prove that P(k + 1) is true whenever P(k) is true, i.e. $2.7^{k+1} + 3.5^{k+1} - 5$ is divisible by 24.

We have,

$2.7^{k+1} + 3.5^{k+1} - 5 = 2.7^k \cdot 7^1 + 3.5^k \cdot 5^1 - 5$

$= 7[2.7^k + 3.5^k - 5 - 3.5^k + 5] + 3.5^k \cdot 5 - 5$

$= 7[24q - 3.5^k + 5] + 15.5^k - 5$

$= (7 \times 24q) - 21.5^k + 35 + 15.5^k - 5$

$= (7 \times 24q) - 6.5^k + 30 = (7\times 24q) - 6(5^k - 5)$

$= (7 \times 24q) - 6(4p)$ [$\because$ $(5^k - 5)$ is a multiple of 4]

$= (7 \times 24q) - 24p = 24(7q - p)$

$= 24 \times r$; $r = 7q - p$, is some natural number ... (ii)

Thus, P(k + 1) is true whenever P(k) is true.

Hence, by the principle of mathematical induction, P(n) is true for all $n \in$ N.

21. (b) Let P(n) : $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+.....+\frac{1}{n(n+1)(n+2)}$

$=\frac{n(n+3)}{4(n+1)(n+2)}$

For n = 1,

L.H.S. $=\frac{1}{1\cdot2\cdot3}=\frac{1}{6}$

and R.H.S. $=\frac{1(1+3)}{4(1+1)(1+2)}=\frac{1}{6}$

∴ P(1) is true.

Let P(k) is true, then

P(k) : $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+.....+\frac{1}{k(k+1)(k+2)}$

$=\frac{k(k+3)}{4(k+1)(k+2)}$ ... (i)

For n = k + 1,

P(k + 1) : $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+.....$

$+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{(k+1)(k+4)}{4(k+2)(k+3)}$

L.H.S. $=\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+.....$

$+\frac{1}{k(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$

$=\frac{k(k+3)}{4(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$ [from (i)]

$=\frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)}=\frac{(k+1)(k+4)}{4(k+2)(k+3)}=$ R.H.S.

Hence, P(k + 1) is true.

Hence, by principle of mathematical induction for all n ∈ N, P(n) is true.

22. (c) Let P(n) : $7^n - 3^n$ is divisible by 4.

For n = 1,

P(1) : $7^1 - 3^1 = 4$, which is divisible by 4. Thus, P(n) is true for n = 1.

Let P(k) be true for some natural number k,

i.e. P(k) : $7^k - 3^k$ is divisible by 4.

We can write $7^k - 3^k = 4d$, where d ∈ N ... (i)

Now, we wish to prove that P(k + 1) is true whenever P(k) is true, i.e. $7^{k+1} - 3^{k+1}$ is divisible by 4.

Now, $7^{(k+1)} - 3^{(k+1)} = 7^{(k+1)} - 7.3^k + 7.3^k - 3^{(k+1)}$

$= 7(7^k - 3^k) + (7-3)3^k = 7(4d) + 4.3^k$ [using (i)]

$= 4(7d + 3^k)$, which is divisible by 4.

Thus, P(k + 1) is true whenever P(k) is true. Therefore, by the principle of mathematical induction the statement is true for every positive integer n.

23. (d) Let the statement P(n) be defined as

P(n) : $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number for all n ∈ N.

Step I : For n = 1,

P(1) : $\frac{1}{5}+\frac{1}{3}+\frac{7}{15}=1\in N$

Hence, it is true for n = 1.

Step II : Let it is true for n = k,

i.e. $\frac{k^5}{5}+\frac{k^3}{3}+\frac{7k}{15}=\lambda\in N$ ... (i)

Step III : For n = k + 1,

$\frac{(k+1)^5}{5}+\frac{(k+1)^3}{3}+\frac{7(k+1)}{15}$

$=\frac{1}{5}(k^5+5k^4+10k^3+10k^2+5k+1)$

$+\frac{1}{3}(k^3+3k^2+3k+1)+\frac{7}{15}k+\frac{7}{15}$

$=\left(\frac{k^5}{5}+\frac{k^3}{3}+\frac{7}{15}k\right)+(k^4+2k^3+3k^2+2k)$

$+\frac{1}{5}+\frac{1}{3}+\frac{7}{15}$

$=\lambda+k^4+2k^3+3k^2+2k+1$

[using equation (i)]

which is a natural number, since λ k ∈ N.

Therefore, P(k + 1) is true, when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

24. (c) For $n=1$, we have

$x^{n+1}+(x+1)^{2n-1}=x^2+(x+1)=x^2+x+1$,

which is divisible by x^2+x+1

For $n=2$, we have

$x^{n+1}+(x+1)^{2n-1}=x^3+(x+1)^3=(2x+1)(x^2+x+1)$,

which is divisible by x^2+x+1.

Hence, option (c) is true.

25. (d) Let P(n) be the statement given by

P(n) : $5^{2n+2} - 24n - 25$ is divisible by 576.

For n = 1,

P(1) : $5^{2+2} - 24 - 25 = 625 - 49 = 576$,

which is divisible by 576.

∴ P(1) is true.

Let P(k) be true,

i.e. P(k) : $5^{2k+2} - 24k - 25$ is divisible by 576.

⇒ $5^{2k+2} - 24k - 25 = 576\lambda$ ... (i)

We have to show that P(k + 1) is true,

i.e. $5^{2k+4} - 24k - 49$ is divisible by 576

Now, $5^{2k+4} - 24k - 49$

$= 5^{2k+2+2} - 24k - 49 = 5^{2k+2}\cdot5^2 - 24k - 49$

$= (576\lambda + 24k + 25)\cdot25 - 24k - 49$ [from (i)]

$= 576.25\lambda + 600k + 625 - 24k - 49$

$= 576.25\lambda + 576k + 576$

$= 576\{25\lambda + k + 1\}$, which is divisible by 576.

∴ P(k + 1) is true whenever P(k) is true.

So, P(n) is true for all n ∈ N.

26. **(a)** Let us write the statement

$$P(n): \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

we note that $P(1): \frac{1}{1.2}=\frac{1}{1+1} \Rightarrow \frac{1}{2}=\frac{1}{2}$ is true thus P(n) is true for n = 1

Suppose that P(k) is true for some natural number 'k'

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{k(k+1)}=\frac{k}{k+1} ......(1)$$

Now, $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$

$$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)} \quad \text{[From (1)]}$$

$$=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}$$

$$=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}=\frac{k+1}{(k+1)+1}$$

Thus P(k + 1) is true whenever P(k) is true. Hence. by the principle of mathematical induction P(n) is true for all natural numbers.

27. **(d)** Let P(n) be the statement given by

P(n) : 3^{2n} when divided by 8, the remainder is 1.

or P(n) : $3^{2n} = 8\lambda + 1$ for some $\lambda \in N$

For n = 1,

P(1) : $3^2 = (8 \times 1) + 1 = 8\lambda + 1$, where $\lambda = 1$

∴ P(1) is true.

Let P(k) be true.

Then, $3^{2k} = 8\lambda + 1$ for some $\lambda \in N$ ... (i)

We shall now show that P(k + 1) is true, for which we have to show that $3^{2(k+1)}$ when divided by 8, the remainder is 1.

Now, $3^{2(k+1)} = 3^{2k} \cdot 3^2 = (8\lambda + 1) \times 9$ [Using (i)]

$= 72\lambda + 9 = 72\lambda + 8 + 1 = 8(9\lambda + 1) + 1$

$= 8\mu + 1$, where $\mu = 9\lambda + 1 \in N$

⇒ P(k + 1) is true.

Thus, P(k + 1) is true, whenever P(k) is true.

Hence, by the principle of mathematical induction P(n) is true for all n ∈ N.

28. **(c)** I. Let the statement P(n) be defined as

$$P(n) : 1 + 2 + 3 + ... + n < \frac{1}{8}(2n + 1)^2$$

Step I : For n = 1,

$$P(1) : 1 < \frac{1}{8}(2.1 + 1)^2 \Rightarrow 1 < \frac{1}{8} \times 3^2$$

$\Rightarrow 1 < \frac{9}{8}$, which is true.

Step II : Let it is true for n = k.

$$1 + 2 + 3 + ... + k < \frac{1}{8}(2k + 1)^2 \quad ... (i)$$

Step III : For n = k + 1,

$$(1+2+3+.....+k)+(k+1)<\frac{1}{8}(2k+1)^2+(k+1)$$

[using equation (i)]

$$=\frac{(2k+1)^2}{8}+\frac{k+1}{1}=\frac{(2k+1)^2+8k+8}{8}$$

$$=\frac{4k^2+1+4k+8k+8}{8}$$

$$=\frac{4k^2+12k+9}{8}=\frac{(2k+3)^2}{8}$$

$$=\frac{(2k+2+1)^2}{8}=\frac{[2(k+1)+1]^2}{8}$$

$$\Rightarrow 1+2+3+.....+k+(k+1)<\frac{[2(k+1)+1]^2}{8}$$

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

II. Let the statement P(n) be defined as

P(n) : n(n + 1) (n + 5) is a multiple of 3.

Step I : For n = 1,

P(1) : 1(1 + 1) (1 + 5) = 1 × 2 × 6 = 12 = 3 × 4, which is a multiple of 3, that is true.

Step II : Let it is true for n = k,

i.e. $k(k + 1)(k + 5) = 3\lambda$

⇒ $k(k^2 + 5k + k + 5) = 3\lambda$

⇒ $k^3 + 6k^2 + 5k = 3\lambda$ ... (i)

Step III : For n = k + 1, (k + 1) (k + 1 + 1) (k + 1 + 5)

$= (k + 1)(k + 2)(k + 6) = (k^2 + 2k + k + 2)(k + 6)$

$= (k^2 + 3k + 2)(k + 6)$

$= k^3 + 6k^2 + 3k^2 + 18k + 2k + 12$

$= k^3 + 9k^2 + 20k + 12$

$= (3\lambda - 6k^2 - 5k) + 9k^2 + 20k + 12$

[using equation (i)]

$= 3\lambda + 3k^2 + 15k + 12$

$= 3(\lambda + k^2 + 5k + 4)$, which is a multiple of 3.

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

Hence, both the statements are true.

29. **(a)** **Statement-1 :** Let $P(n) : \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+.....+\frac{1}{\sqrt{n}}>\sqrt{n}$

For n = 2,

$P(2) : \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}>\sqrt{2}$, which is true.

Assume P(k) is true,

i.e. $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+.....+\frac{1}{\sqrt{k}}>\sqrt{k}$ ... (i)

For n = k + 1, we have to show that

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+.....+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1} \quad ... (ii)$$

$$\text{L.H.S.} = \left(\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}}\right) + \frac{1}{\sqrt{k+1}} \quad ...(iii)$$

Statement-2 : For n = k,

$\sqrt{k(k+1)} < k + 1$

$\Rightarrow \sqrt{k}\sqrt{k+1} < \sqrt{k+1}\sqrt{k+1}$

$\Rightarrow \sqrt{k} < \sqrt{k+1}$

$\because \sqrt{k+1} > \sqrt{k}$ for $k \geq 2$

$\Rightarrow 1 > \frac{\sqrt{k}}{\sqrt{k+1}}$

$\Rightarrow \sqrt{k} > \frac{k}{\sqrt{k+1}}$, (Multiplying by $\sqrt{k}$)

$\Rightarrow \sqrt{k} > \frac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \sqrt{k+1} - \frac{1}{\sqrt{k+1}}$

$$\Rightarrow \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad ...(iv)$$

From (iii) and (iv),

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k}$$

$$+ \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad \text{[Using (i)]}$$

Hence, (ii) is true for n = k + 1

Hence, P(n) is true for $n \geq 2$

So, Statement-1 and Statement-2 are correct and Statement-2 is the correct explanation of Statement-1.

30. (b) Let P(n) be the statement given by

P(n) : $41^n - 14^n$ is a multiple of 27

For n = 1,

i.e. $P(1) = 41^1 - 14^1 = 27 = 1 \times 27$,

which is a multiple of 27.

$\therefore$ P(1) is true.

Let P(k) be true, i.e. $41^k - 14^k = 27\lambda$ ... (i)

For n = k + 1,

$41^{k+1} - 14^{k+1} = 41^k\, 41 - 14^k\, 14$

$= (27\lambda + 14^k) 41 - 14^k\, 14$ [using (i)]

$= (27\lambda \times 41) + (14^k \times 41) - (14^k \times 14)$

$= (27\lambda \times 41) + 14^k (41 - 14)$

$= (27\lambda \times 41) + (14^k \times 27)$

$= 27(41\lambda + 14^k)$,

which is a multiple of 27.

Therefore, P(k + 1) is true when P(k) is true. Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.