Experiment - 5 : To determine young's modulus of elasticity of the material of a metallic wire.

Theory

Hooke's law states that if the stress developed in a body lies within a certain limit called the elastic limit, then the strain produced is directly proportional to the stress.

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

The constant is known as the modulus of elasticity. If the stress is in a wire, we call it the longitudinal stress and the corresponding strain is called longitudinal strain, then the constant of proportionality is called Young's modulus (Y).

$$Y = \frac{F}{A} \div \frac{l}{L}$$

where a wire of length L is stretched by a force F, its cross-section is A and it extends by l

$$Y = \frac{F \cdot L}{A \cdot l}$$

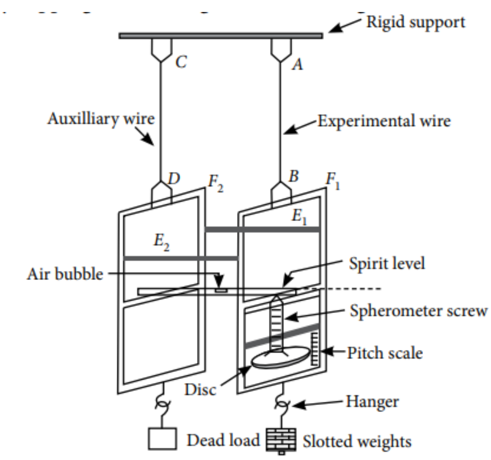
where $F = mg$ and $A = \pi r^2 \quad \Rightarrow \quad Y = \frac{mg \cdot L}{\pi r^2 l}$

Apparatus

Searle's apparatus, two long steel wires of same length and diameter, a meter scale, a screw gauge, eight 1/2 kg slotted weights and a 1 kg hanger.

Description of Searle's Apparatus

Construction : Searle's apparatus consists of two metal frames F_1 and F_2. Each frame has torsion head at the upper side and hook at the lower side. These frames are suspended from two wires AB and CD of same material, length and cross-section. The upper ends of the wires are screwed tightly in two torsion heads fixed in same rigid support. A constant weight of 1 kilogram is suspended from the hook of the frame F_2 attached to the auxiliary wire CD, which keeps the wire taut. A hanger H of 1 kilogram weight is suspended from the hook of the other frame F_1. The experimental wire AB can be loaded by slipping slotted weights and the hanger.

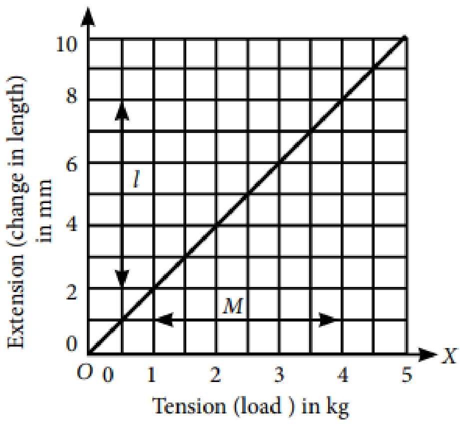


A spirit level rests horizontally with its one end hinged in the frame F_2. The other end of the spirit level rests on the tip of a spherometer screw fitted in the frame F_1. The spherometer screw can be rotated up and down along a vertical pitch scale marked in millimeter. The two frames are kept together by cross bars E_1 and E_2.

The Searle's apparatus is shown above :

Working : To perform the experiment, kinks are removed from the wire AB by loading and unloading it two or three times. All the weights are then removed from the hanger. The wire AB is kept taut under the weight of the hanger alone.

The spherometer screw is then rotated till the bubble comes in the middle of the spirit level. The spherometer disc reading is recorded for zero load. A half kilogram weight is now slipped in the hanger. The wire *AB*



extends and the frame F_1 moves down. The levelling is disturbed. The bubble is again brought in the middle by rotating the screw upwards. The distance by which screw is turned upwards gives the elongation of the wire due to half kilogram weight. A number of observations are taken by increasing the load on the hanger in steps of half kilogram each. The observations are then repeated by decreasing the load in the same order till all the weights are removed from the hanger. The mean of these observations is taken. A graph is plotted between load M and mean extension l. It is a straight line. From the graph, mean increase in length l for a load M kg is found. A sample graph is shown here.

MCQs Corner

Experiment – 5

22. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is

(a) 4.0 mm (b) zero (c) 5.0 mm (d) 3.0 mm

23. A thin 1 m long rod has a radius of 5 mm. A force of 50p kN is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false ?

(a) The maximum value of Y that can be determined is 10^{14} N/m^2.
(b) $\frac{\Delta Y}{Y}$ gets minimum contribution from the uncertainty in the length.
(c) $\frac{\Delta Y}{Y}$ gets its maximum contribution from the uncertainty in strain.
(d) The figure of merit is the largest for the length of the rod.

24. A solid cylindrical steel column is 4 m long and 9 cm in diameter. ($Y_{steel} = 1.9 \times 10^{11}$ N m^{-2}). The decrease in length of the column, while carrying a load of 80000 kg is

(a) 1.8 mm (b) 2.6 mm (c) 3.2 mm (d) 4.4 mm

25. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional

change in the radius of the sphere, $\frac{\delta R}{R}$ is

(a) $\frac{Mg}{KA}$ (b) $\frac{Mg}{2KA}$ (c) $\frac{Mg}{3KA}$ (d) $\frac{Mg}{4KA}$

Answer Key

22. (d) 23. (a) 24. (b) 25. (c)

Hints & Explanation

22. (d) : Let ρ and σ be the density of the liquid and material of the load respectively.
In first case, the extension in the wire is
$x = Mg/k = V\rho g/k$ ...(i)
When the load is immersed in the liquid,
upthrust + internal force due to extension in wire
= weight of the load
$\Rightarrow \quad V\sigma g + kx_1 = V\rho g \Rightarrow x_1 = Vg(\rho - \sigma)/k$ ... (ii)
Using (i) and (ii),

$$x_1 = \frac{Vgx}{Vg\rho}(\rho - \sigma) = x\left(1 - \frac{\sigma}{\rho}\right) = 4 \times \left(1 - \frac{2}{8}\right) = 3\text{ mm}$$

23. (a) : Here, $L = 1$ m, $r = 5$ mm $= 5 \times 10^{-3}$ m
$F = 50\pi$ kN, L.C. of all lengths = 0.01 mm
$Y = ?$

$$Y = \frac{\text{Stress}}{\text{Stain}} = \frac{FL}{Al} = \frac{FL}{\pi r^2 l}$$

$$Y = \frac{50\pi \times 10^3}{\pi(5 \times 10^{-3})^2} \times \frac{L}{l} = 2 \times 10^9 \times \frac{L}{l} = \frac{2 \times 10^9}{0.01 \times 10^{-3}}$$

$= 2 \times 10^{14}$ N/m^2

24. (b) : $A = \pi r^2 = (3.14)(0.045\text{ m})^2 = 6.36 \times 10^{-3}\text{ m}^2$

From $Y = \dfrac{FL}{Al}$

$$l = \frac{(8 \times 10^4 \times 9.8\text{ N}) \cdot (4\text{m})}{(6.36 \times 10^{-3}\text{ m}^2) \cdot (1.9 \times 10^{11}\text{ Nm}^{-2})}$$

$= 2.6 \times 10^{-3}$ m $= 2.6$ mm

25. (c) : $K = \dfrac{P}{\left(\dfrac{\delta V}{V}\right)} = \dfrac{Mg}{A\left(\dfrac{\delta V}{V}\right)}$ or $\dfrac{\delta V}{V} = \dfrac{Mg}{KA}$

Now, $V = \dfrac{4}{3}\pi R^3$

$\ln V = \ln\left(\dfrac{4}{3}\pi\right) + 3\ln R$

Differentiating, $\dfrac{\delta V}{V} = \dfrac{3\delta R}{R} = \dfrac{Mg}{KA} \Rightarrow \dfrac{\delta R}{R} = \dfrac{Mg}{3KA}$